COLLEGE OF ENGINEERING SPCC

ECE EE CPE

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HINGA MUNA!!!

IEE1-(Vector Analysis)
Review Materials for MATH subject

VECTOR ANALYSIS

Scalar quantities – quantities which are completely specified when their magnitude are given.
for ex. speed, distance, work, volume, mass, specific heat, gravitational potential, time, etc.

Vector quantities – quantities which require both magnitude and direction in order to be completely specified.
for ex. velocity, displacement, momentum, weight, torque, centrifugal force, electric field intensity, etc.

Vector Representation:

a. Graphically
For example,

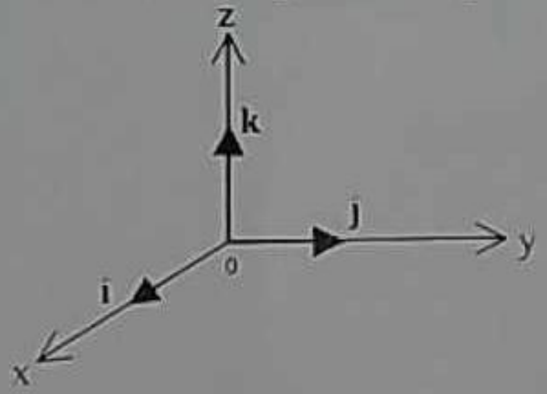
length - represents the magnitude
arrowhead – represents the direction

b. Analytically
for ex. vector A can be represented as $\vec{A}$ or **A**

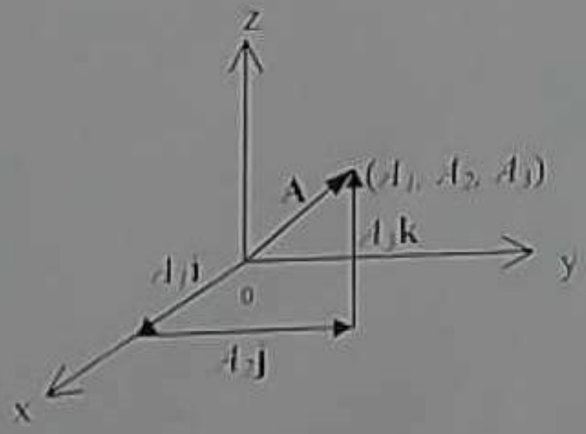
Note:
$-\vec{A}$ is a vector with the same magnitude as $\vec{A}$ but opposite in direction.

Unit Vector – is a vector having unit magnitude

Cartesian Unit vectors i, j, k in 3D-space:



Vector in 3D-space:



$\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k} = \langle A_1, A_2, A_3 \rangle$

where A_1, A_2, and A_3 are scalar components of **A**

i, j and **k** are unit vectors in the direction of increasing value of x, y and z respectively
A_1**i**, A_2**j**, and A_3**k** are vector components of **A**

Note

1. Zero or Null Vector – it has zero magnitude and direction is undefined or no specific direction.
2. Equality of Vectors – two vectors are equal if and only if their corresponding components are also equal.

Magnitude or length of A in denoted by |A| or A.

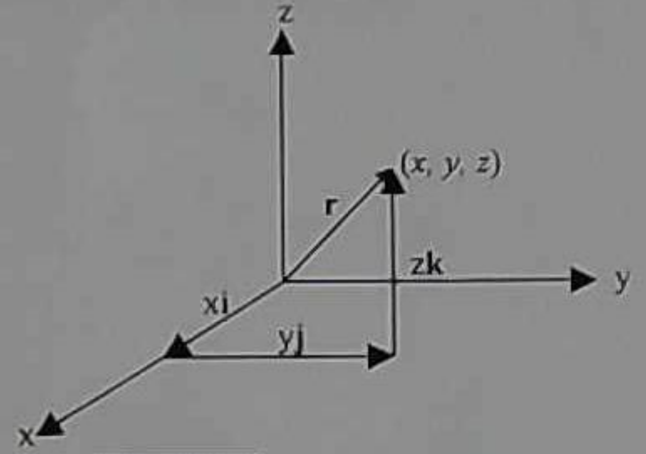
$$|\mathbf{A}| = \sqrt{A_1^2 + A_2^2 + A_3^2}$$

Unit Vector in the direction of A is denoted by ∂_A:

$$\partial_A = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}}{\sqrt{A_1^2 + A_2^2 + A_3^2}}$$

Position/Radius Vector – any vector starts from the origin which is usually represented by **r**.

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$



$$|\mathbf{r}| = r = \sqrt{x^2 + y^2 + z^2}$$

ENGINEERING

Laws of Vector Algebra:

If **A**, **B**, and **C** are vectors and m and n are scalars then,

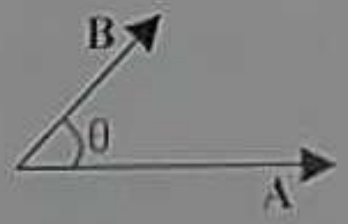
1. $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
2. $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$
3. $m\mathbf{A} = \mathbf{A}m$
4. $m(n\mathbf{A}) = (mn)\mathbf{A}$
5. $(m + n)\mathbf{A} = m\mathbf{A} + n\mathbf{A}$
6. $m(\mathbf{A} + \mathbf{B}) = m\mathbf{A} + m\mathbf{B}$

Multiplication of Vectors:

I. DOT OR SCALAR PRODUCT

By definition,

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos\theta$$



where θ - smaller angle between **A** and **B**

$$0 \leq \theta \leq \pi$$

Important Laws (Dot Product):

1. $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
2. $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$

3. $m(\mathbf{A} \cdot \mathbf{B}) = (m\mathbf{A}) \cdot \mathbf{B} = \mathbf{A} \cdot (m\mathbf{B}) = (\mathbf{A} \cdot \mathbf{B})m$
   where m is a scalar
4. since **i**, **j**, and **k** are orthogonal
   $\mathbf{i} \cdot \mathbf{j} = \mathbf{i} \cdot \mathbf{k} = \mathbf{j} \cdot \mathbf{k} = (1)(1) \cos 90° = 0$
   $\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = (1)(1) \cos 0 = 1$

5. If $\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$ and $\mathbf{B} = B_1\mathbf{i} + B_2\mathbf{j} + B_3\mathbf{k}$,
   Then $\mathbf{A} \cdot \mathbf{B} = A_1B_1 + A_2B_2 + A_3B_3$
6. $\mathbf{A} \cdot \mathbf{A} = A_1(A_1) + A_2(A_2) + A_3(A_3) = A_1^2 + A_2^2 + A_3^2 = |\mathbf{A}|^2$

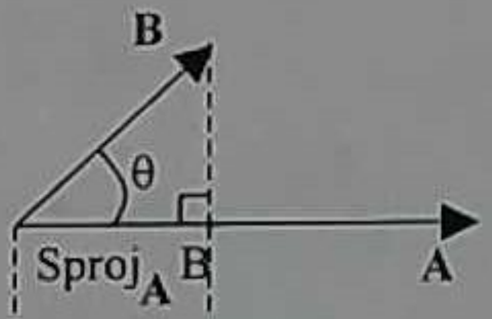
also,

$\mathbf{B} \cdot \mathbf{B} = B_1(B_1) + B_2(B_2) + B_3(B_3) = B_1^2 + B_2^2 + B_3^2 = |\mathbf{B}|^2$

7. If $\mathbf{A} \cdot \mathbf{B} = 0$, where **A** and **B** are not null vectors, then **A** and **B** are perpendicular

Sample Application of Dot Product:

Scalar Projection of B unto A (S_{proj_A} B):



$$\cos\theta = \frac{S_{proj_A}\mathbf{B}}{|\mathbf{B}|}$$

$$S_{proj_A}\mathbf{B} = |\mathbf{B}|\cos\theta$$

$$\text{but } \cos\theta = \frac{\mathbf{A}\bullet\mathbf{B}}{|\mathbf{A}||\mathbf{B}|}$$

therefore,

$$S_{proj_A}\mathbf{B} = \frac{\mathbf{A}\bullet\mathbf{B}}{|\mathbf{A}|} = \frac{\mathbf{B}\bullet\mathbf{A}}{|\mathbf{A}|} = \mathbf{B}\bullet\partial_A$$

Similarly for scalar projection of **A** unto **B** is,

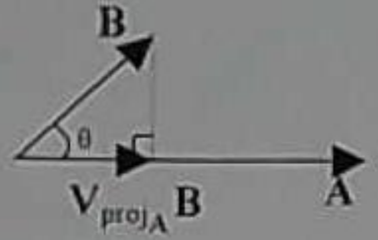
$$\boxed{S_{proj_B}\mathbf{A} = \mathbf{A}\bullet\partial_B}$$

$Sproj_A B = B \cdot \partial_A$

Note: if $90° < \theta \leq 180°$ (–Scalar projection)

if $0 \leq \theta < 90°$ (+Scalar projection)

Vector Projection of B unto A (V_{proj_A} B):



$$V_{proj_A} \mathbf{B} = (S_{proj_A}\mathbf{B})(\partial_A) = (\mathbf{B} \bullet \partial_A)(\partial_A)$$

Similarly for Vector Projection of A unto **B** is,

$$\boxed{V_{proj_B}\mathbf{A} = (\mathbf{A} \bullet \partial_B)\partial_B}$$

II. CROSS OR VECTOR PRODUCT:

By definition,

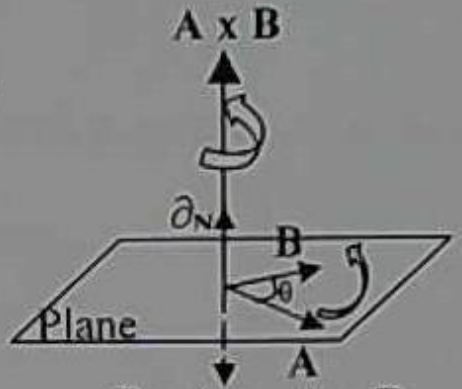
$\mathbf{A} \times \mathbf{B} = |\mathbf{A}||\mathbf{B}|\sin\theta \, \partial_n$

where $\partial_n = \dfrac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$

then,

$$\mathbf{A} \times \mathbf{B} = |\mathbf{A}||\mathbf{B}|(\sin\theta)\left(\frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}\right)$$

$$|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin\theta$$



$\mathbf{B} \times \mathbf{A} = -\mathbf{A} \times \mathbf{B}$

Important Laws(Cross Product)

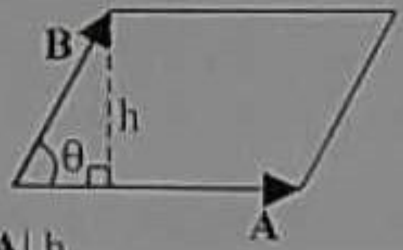
1. $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$
2. $\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$
3. $m(\mathbf{A} \times \mathbf{B}) = (m\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (m\mathbf{B}) = (\mathbf{A} \times \mathbf{B})m$,
   where m is a scalar
4. $\mathbf{i} \times \mathbf{i} = 0 \qquad \mathbf{i} \times \mathbf{j} = \mathbf{k} \qquad \mathbf{i} \times \mathbf{k} = -\mathbf{j}$
   $\mathbf{j} \times \mathbf{j} = 0 \qquad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \qquad \mathbf{j} \times \mathbf{k} = \mathbf{i}$
   $\mathbf{k} \times \mathbf{k} = 0 \qquad \mathbf{k} \times \mathbf{i} = \mathbf{j} \qquad \mathbf{k} \times \mathbf{j} = -\mathbf{i}$
5. If $\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$ and $\mathbf{B} = B_1\mathbf{i} + B_2\mathbf{j} + B_3\mathbf{k}$
   $\mathbf{A} \times \mathbf{B} = (A_2B_3 - A_3B_2)\mathbf{i} + (A_3B_1 - A_1B_3)\mathbf{j} + (A_1B_2 - A_2B_1)\mathbf{k}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

6. $\mathbf{A} \times \mathbf{A} = 0$ and $\mathbf{B} \times \mathbf{B} = 0$
   Also $\mathbf{A} \times 0 = 0$
   $0 \times \mathbf{A} = 0$
7. If $\mathbf{A} = m\mathbf{B}$, it means $\mathbf{A}$ is parallel to $\mathbf{B}$,
   then $\theta = 0$ or π and we define $\mathbf{A} \times \mathbf{B} = 0$

Sample Application of Cross Product:

Area of a Parallelogram with vector sides A and B;



$A_{\square} = |\mathbf{A}|\,h$

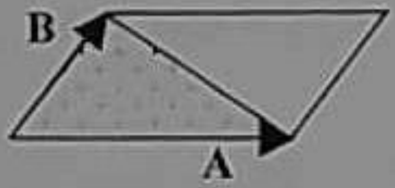
$A_{\square} = |\mathbf{A}||\mathbf{B}|\sin\theta$

but $|\mathbf{A}||\mathbf{B}|\sin\theta = |\mathbf{A} \times \mathbf{B}|$

therefore,

$$\boxed{A_{\square} = |\mathbf{A} \times \mathbf{B}|}$$

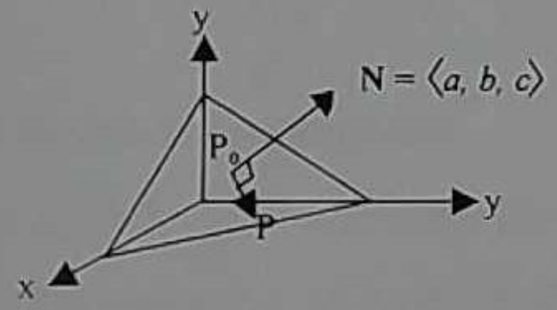
Note: the area of triangle with vector sides **A** and **B** is ½ of the area of parallelogram



thus,

$$A_{\triangle} = \tfrac{1}{2}\,|\mathbf{A} \times \mathbf{B}|$$

PLANE IN 3D-SPACE:



$P_o \longrightarrow (x_o, y_o, z_o)$
$P \longrightarrow (x, y, z)$
$\overrightarrow{P_oP} = \langle x - x_o, y - y_o, z - z_o \rangle$
$N \perp \overrightarrow{P_oP}$, then $N \bullet \overrightarrow{P_oP} = 0$
$\langle a, b, c \rangle \bullet \langle x - x_o, y - y_o, z - z_o \rangle = 0$
$a(x - x_o) + b(y - y_o) + c(z - z_o) = 0$ (Standard Equation of a Plane)
$ax + by + cz + (-ax_o - b_{y_o} - cz_o) = 0$
$ax + by + cz + d = 0$ (General Equation of a Plane)

SCALAR TRIPLE PRODUCT:

for ex

1. $A \bullet (B \times C) = A \bullet B \times C$
2. $B \bullet (C \times A) = B \bullet C \times A$
3. $C \bullet (A \times B) = C \bullet A \times B$

Note

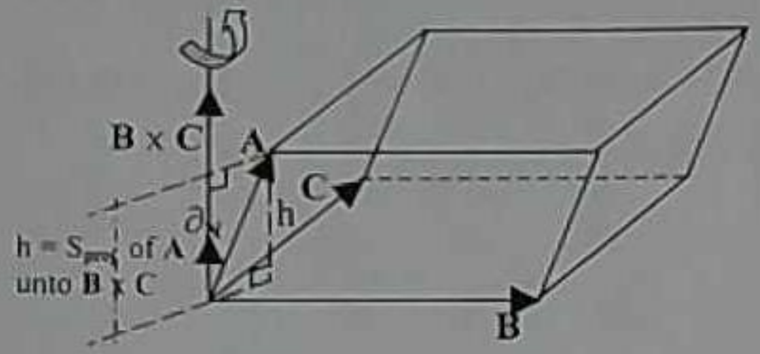
1. The cross product must be evaluated first
2. The parenthesis used in scalar triple product is not necessary
3. The dot product and the cross symbol can be interchanged

Some Important Laws in Scalar Triple Product:

1. $\mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = \mathbf{A} \cdot \mathbf{B} \times \mathbf{C}$
2. $\mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = \mathbf{C} \cdot \mathbf{A} \times \mathbf{B}$
3. $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{B} \cdot \mathbf{C} \times \mathbf{A} = \mathbf{C} \cdot \mathbf{A} \times \mathbf{B}$
4. $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = -\mathbf{B} \cdot \mathbf{A} \times \mathbf{C} = -\mathbf{C} \cdot \mathbf{B} \times \mathbf{A} = -\mathbf{A} \cdot \mathbf{C} \times \mathbf{B}$
5. $\mathbf{A} \cdot \mathbf{A} \times \mathbf{C} = \mathbf{A} \times \mathbf{A} \cdot \mathbf{C} = 0$, since $\mathbf{A} \times \mathbf{A} = 0$
6. $\mathbf{A} \cdot \mathbf{A} \times \mathbf{C} = \mathbf{A} \cdot \mathbf{C} \times \mathbf{A} = 0$, since $\mathbf{A} \times \mathbf{A} = 0$

VOLUME OF A PARALLELEPIPED:

ABC ABC



$$V = A_{\square}(h)$$
$$= |\mathbf{B} \times \mathbf{C}|(\text{Scalar proj of } \mathbf{A} \text{ unto } \mathbf{B} \times \mathbf{C})$$
$$= |\mathbf{B} \times \mathbf{C}|\,(\mathbf{A} \cdot \partial_n)$$
$$= |\mathbf{B} \times \mathbf{C}|\left(\mathbf{A} \cdot \frac{\mathbf{B} \times \mathbf{C}}{|\mathbf{B} \times \mathbf{C}|}\right) = \mathbf{A} \cdot \mathbf{B} \times \mathbf{C}$$

$$V = |\mathbf{A} \cdot \mathbf{B} \times \mathbf{C}| = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

VOLUME OF TETRAHEDRON:

$$\text{Vol. of Pyramid} = \frac{1}{3}\ (\text{Area of its base})(\text{Height})$$

$$\text{Vol. of Tetrahedron} = \frac{1}{3}\ \left(\frac{1}{2}\, A_{\square}\right)(\text{Sproj of } \mathbf{A} \text{ unto } \mathbf{B} \times \mathbf{C})$$

$$= \frac{1}{6}\ \text{vol. of Parallelepiped}$$

$$\boxed{\text{Vol. of Tetrahedron} = \frac{1}{6}\ |\mathbf{A} \bullet \mathbf{B} \times \mathbf{C}|}$$

VECTOR TRIPLE PRODUCT:

for ex.

$\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$, $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$, etc

Important Laws of Vector Triple Product:

1. $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \neq (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$

   Showing the need for parentheses in $\mathbf{A} \times \mathbf{B} \times \mathbf{C}$ to avoid ambiguity

2. $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \bullet \mathbf{C})\mathbf{B} - (\mathbf{A} \bullet \mathbf{B})\mathbf{C}$

   also,

   $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = (\mathbf{A} \bullet \mathbf{C})\mathbf{B} - (\mathbf{B} \bullet \mathbf{C})\mathbf{A}$

3. $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = -(\mathbf{B} \times \mathbf{C}) \times \mathbf{A}$

DIFFERENTIAL OPERATOR DEL OR NEBLA (∇)

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

If $\phi(x, y, z)$ is a differential scalar field. Then the gradient of ϕ is

$$\nabla\phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$$

THE DIVERGENCE:

If **V** (x, y, z) is a vector function with components whose first derivatives are continuous in the domain of **V**, the divergence of **V** is given by

$$\nabla \cdot \mathbf{V} = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \cdot (\mathbf{i}V_1 + \mathbf{j}V_2 + \mathbf{k}V_3)$$

$$\nabla \cdot \mathbf{V} = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}$$

GRADIENT OR NEBLA (∇)

$f(x, y, z) = x^2 + y^2 + z^2$, $P(1, 2, 3)$

$\nabla(f) = i(2x) + j(2y) + k(2z)$

$= i(2(1)) + j(2 \times 2) + k(2(3))$

$\nabla = 2i + 4j + 6k$

Evaluating $\nabla|r|^3$

$\rightarrow \vec{\nabla} r^n = nr^{n-2} r$

Maximum rate of change of ∇ at point P in the direction — point substitute to $\bar{\nabla}$

Maximum rate of change at P $\rightarrow |\bar{\nabla} v| = 2^2 + 4^2 + 6^2$

Rate of change of ∇ at point P towards origin $\rightarrow \bar{U} = \frac{\vec{PO}}{|\vec{PO}|}$ then $(\nabla V \cdot \bar{U})$

$\vec{PO}$ — neg given na $P(1, 2, 3) = i + 2j + 3k$

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REE – Apr. 2007

1. Find the length of the vector (2, 4, 4)

A. 5 B. 6 C. 4 D. 8

2. Express in forms of the unit vectors **i**, **j**, **k** the force of 200 N that starts at the point (2, 5, - 3) and passes through the point (–3, 2, 1)

A. –141.42**i** + 84.85**j** + 113.14**k**
B. 141.42**i** – 84.85**j** + 113.14**k**
C. 141.42**i** + 84.85**j** + 113.14**k**
D. –141.42**i** – 84.85**j** + 113.14**k**

3. If **a** and **b** are non-collinear vector and **A** = (x + 4y)**a** + (2x + y + 1)**b** and **B** = (y – 2x + 2)**a** + (2x – 3y – 1)**b**. Find x and y such that 3**A** = 2**B**.

A. 2, –1 B. 2, –3 C. 5, 1 D. 3, 1

REE – Oct. 1994

4. Displacement **A** is 2 meters north, displacement **B** is 3 meters south. Find the magnitude and direction of **B** – **A**.

A. 1 S B. 1 N C. 5 S D. 5 N

REE – Apr. 2015

5. Find **a** • **b** if $|\mathbf{a}| = 26$ and $|\mathbf{b}| = 17$ and the angle between them is pi/3

A. 221 B. 212 C. 383 D. 338

6. Given $\mathbf{A} = (y - 1)\mathbf{a}_x + 2x\mathbf{a}_y$, find the vector at (2, 2, 1) and its projection on **B** where $\mathbf{B} = 5\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z$

A. $\mathbf{a}_x + 4\mathbf{a}_y, \frac{1}{6}\mathbf{a}_x + \frac{1}{30}\mathbf{a}_y + \frac{1}{5}\mathbf{a}_z$

B. $\mathbf{a}_x + 4\mathbf{a}_y, \frac{1}{6}\mathbf{a}_x + \frac{1}{30}\mathbf{a}_y + \frac{1}{15}\mathbf{a}_z$

C. $\mathbf{a}_x + 4\mathbf{a}_y, \frac{1}{6}\mathbf{a}_x - \frac{1}{30}\mathbf{a}_y + \frac{1}{15}\mathbf{a}_z$

D. $\mathbf{a}_x + 4\mathbf{a}_y, \frac{1}{6}\mathbf{a}_x - \frac{1}{30}\mathbf{a}_y + \mathbf{a}_z$

7. Find the area of the triangle whose vertices are (0, 1, 2), (–1, 2, 1) and (5, 1, 2).

A. $5\sqrt{2}$ B. $2\sqrt{2}$ C. $\frac{4}{3}\sqrt{2}$ D. $\frac{5}{2}\sqrt{2}$

REE – Sept. 2001

8. The 3 vectors described by 10cm/at 120k degrees, k = 0, 1, 2 encompass the sides of an equilateral triangle. Determine the magnitude of the vector cross product

0.5[(10/ at 0 deg) x (10/ at 120 deg)]

A. 86.6 B. 25.0 C. 50.0 D. 43.3

REE – Sept. 2011 / Sept. 2016

9. There is a vector **v** = 7**j**, another vector **u** starts from the origin with a magnitude of 5 rotates in the xy plane. Find the maximum magnitude of **u x v**

A. 24 B. 70 C. 12 D. 35

REE – Apr. 2007

10. The magnitude of vector product of two vectors is sqrt of 3 times their scalar product. The angle between the vector is

A. pi/4 B. pi/2 C. pi/6 D. pi/3

REE – Apr. 2013

perpendicular |A×B|

11. What is the vector which is orthogonal both to 9i + 9j and 9i + 9k?

A. 81i + 81j – 81k B. 81i – 81j – 81k C. 81i – 81j + 81k D. 81i + 81j + 81k

REE – Sept. 2014 / Sept. 2015

12 What is the unit vector orthogonal both to 9**i** + 9**j** and 9**i** + 9**k**?

A. $\frac{i}{\sqrt{3}}+\frac{j}{\sqrt{3}}+\frac{k}{\sqrt{3}}$ B. $\frac{i}{3}+\frac{j}{3}+\frac{k}{3}$ C. $\frac{i}{\sqrt{3}}-\frac{j}{\sqrt{3}}-\frac{k}{\sqrt{3}}$ D. $\frac{i}{3}-\frac{j}{3}-\frac{k}{3}$

REE – Oct. 1996

13 Find the equation of the plane passing thru the points P(2, –3, 1), P'(5, –3, –5) and perpendicular to the plane x – 2y + 5z + 20 = 0

A. x – 2y + 5z – 15 = 0 C. x – 2y + 5z + 15 = 0
B. 4x + 7y + 2z + 11 = 0 D. 4x + 7y + 2z – 11 = 0

14. Find the volume of the parallelepiped having **i** + 3**j** + 2**k**, 2**i** + **j** – **k** and **i** – 2**j** + **k** be the edges

A. 18 c.u B. 19 c.u C. 20 c.u D. 21 c.u

15 Find the value of m that makes vectors **A** = **i** + **j** – **k**, **B** = 2**i** – **j** + **k** and **C** = m**i** – **j** + m**k** coplanar

A. 0 B. 1 C. 2 D. 3

– 1(2m–m) + (–1)(–2–(–m))

derivative

REE – Apr. 1999

16 Determine the gradient of the function $f(x, y, z) = x^2 + y^2 + z^2$ at the point (1, 2, 3). Give the magnitude of the gradient of f

A. 7.21 units B. 8.25 units C. 6.00 units D. 7.48 units

17 Evaluate $\nabla|\mathbf{r}|^3$

A. $r\mathbf{r}$ B. $2r\mathbf{r}$ C. $4r\mathbf{r}$ D. $3r\mathbf{r}$

REE – May 2008

18 The electric potential V at (x, y, z) is V = x(squared) + 4(y squared) + 9(z squared). Find the direction that produce the maximum rate of change of V at point P(2, –1, 3) in the direction of

A. 8i – 4j +54k B. 4i – 8j +54k C. 8i + 4j +54k D. 8i + 4j – 54k

REE – Sept. 2009

19 The electric potential V at (x, y, z) is V = x(squared) + 4(y squared) + 9(z squared). What is the maximum rate of change at P? (2, -1, 3)

A. 54.8 B. 85.4 C. 45.8 D. 84.5

REE – Apr. 2004

20 The electric potential V at (x, y, z) is V = x(squared) + 4(y squared) + 9(z squared). Find the rate of change of V at point P(2, –1, 3) towards the origin

A. 164/(sq. rt. of 15) B. 175/(sq. rt. of 14) C. –178/(sq. rt. of 14) D. –164/(sq. rt. of 15)

REE – Apr. 2001

21 Determine the divergence of the vector:

V = **i**(x^2) + **j**(–xy) + **k**(xyz) at the point (3, 2, 1)

A. 13.00 B. 9.00 C. 11.00 D. 7.00

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22 A point travels as described by the following parametric equations x = 10t + 10cos(πt), y = 10t + 10sin(πt), z = 10t, where x, y, z are in meters, t in seconds, all angles are in radians. The vector locating the body at anytime is **r** = ix + jy + **k**z. Determine the magnitude of the velocity of the body in meters per second at time t = 0.25

A. 33.07 B. 34.57 C. 35.87 D. 33.85