HINGA MUNA!!!

IEE1-(Differential Equation)
Review Materials for MATH subject

SECOND ORDER DIFFERENTIAL EQUATIONS:

$$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$$

where a, b, and c are constant and $a \neq 0$.

$$\text{Let } D=\frac{d}{dx}$$

$$aD^2y+bDy+cy=0$$

$$(aD^2+bD+c)y=0$$

Auxiliary equation:

$$am^2+bm+c=0$$

→ by factoring

→ by Q. E. F., $m=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

CASE 1: When the roots m_1 and m_2 are real and unequal.

The general solution is,

$$y=C_1e^{m_1x}+C_2e^{m_2x}$$

CASE 2: When the roots m_1 and m_2 are real and equal.

The general solution is,

$$y = C_1 e^{mx} + C_2 x e^{mx}$$

or

$$y = (C_1 + C_2 x) e^{mx}$$

CASE 3: When the roots m_1 and m_2 are imaginary and unequal.

$$m = \frac{-b \pm j\sqrt{4ac - b^2}}{2a} = A \pm jB$$

then the general solution is,

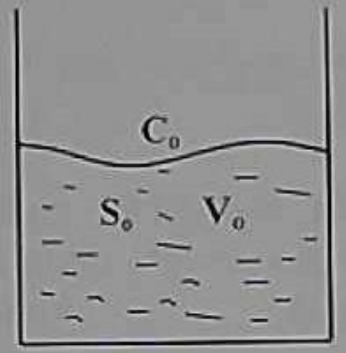
$$y = e^{Ax}\left(C_3 \cos Bx + C_4 \sin Bx\right)$$

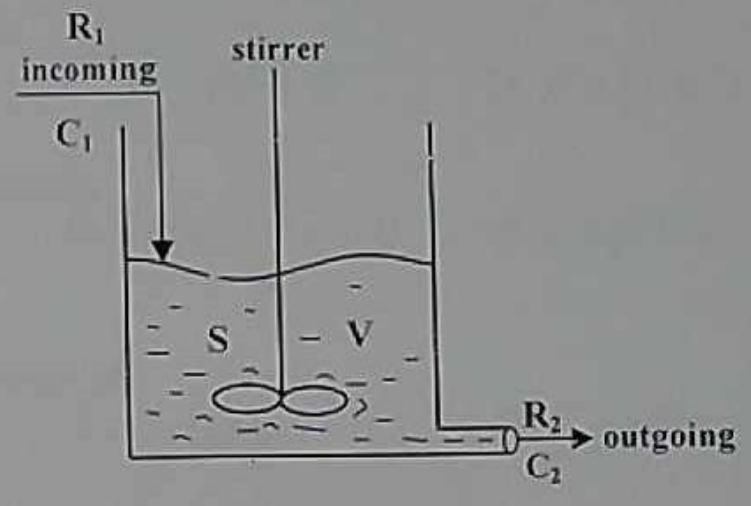
CHEMICAL SOLUTIONS:

"The rate of change of the pure substance in a tank is equal to the difference of the Rate at which the pure substance flows to the tank and the rate at which the pure substance flows Out of the tank."

ILLUSTRATION:

Initially, t = 0





Let S = amount of substance in the tank at anytime, t

S_o = amount of substance in the tank at time t = 0

$\frac{dS}{dt}$ = rate of change of S

V = volume of mixture or solution at anytime, t

V_o = original volume of mixture or solution

C_1 = concentration of incoming solution
C_2 = concentration of outgoing solution
} $\frac{\text{weight}}{\text{unit volume}}$

R_1 = rate of inflow
R_2 = rate of outflow
} $\frac{\text{volume}}{\text{unit time}}$

Note:

If $R_1 > R_2$, $V > V_o$ ⟶ it will overflow

If $R_1 = R_2$, $V = V_o$ ⟶ constant

If $R_1 < R_2$, $V < V_o$ ⟶ it will be emptied later

Formulas:

$$\frac{dS}{dt} = \text{rate of } S_{gained} - \text{rate of } S_{lost} = C_1R_1 - C_2R_2$$

In general at anytime "t", $C = \frac{S}{V}$, $V = V_o + (R_1 - R_2)t$

since $C_2 = C = \frac{S}{V} = \frac{S}{V_0 + (R_1 - R_2)t}$

Therefore, $\frac{dS}{dt} = C_1R_1 - \frac{R_2S}{V_o + (R_1 - R_2)t}$

Note: If $R_1 = R_2$ (V. S.)

If $R_1 \neq R_2$ (L. D. E.)

1. State the order and degree of $y''' + 2(y'')^2 + y' = \cos x$

A. 3, 2 B. 2, 1 C. 3, 1 D. 2, 2

2. State the order and degree of $Ry'' = [1 + (y')^2]^{3/2}$

A. 2, 2 B. 3, 2 C. 2, 3 D. 2, 1

REE – Sept. 2015

3. Find the general solution of $y' + \frac{x}{y} = 0$

A. $x^2 + 2y^2 = C$ B. $x^2 + y^2 = C$ C. $x^2 - 2y^2 = C$ D. $x^2 - y^2 = C$

4. For $xdy + ydx = 2x^2ydx$, the general solution is

A. $\ln xy^2 = x + C$ B. $\ln xy = x^2 + C$ C. $\ln (xy)^2 = x + C$ D. $\ln xy^2 = x^2 + C$

REE – Apr. 2012/Apr. 2016

5. The equation y^2 = cx is the general solution of

A. $y' = 2y/x$ B. $y' = 2x/y$ C. $y' = y/2x$ D. $y' = x/2y$

REE – Sept. 2004

6. Solve the particular solution of $\frac{dx}{dt} = \frac{x}{2}$ if $x(0) = 1$

A. $x^3 = e^t$ B. $x^2 = e^t$ C. $x = e^t$ D. $x^4 = te^t$

7. Solve $(x^2 + y^2)dx + 2xydy = 0$

A. $x^3 + 3xy^2 = C$ B. $x^3 + 2xy^2 = C$ C. $x^3 + 3x^2y^2 = C$ D. $x^3 + 5x^2y^2 = C$

REE – Sept. 2003/Apr. 2004/Sept. 2006

8. Find the particular solution of the differential equation $(dy/dx) - 3y/x = (x \text{ cubed})$; if $y(1) = 4$.

A. $y = (x \text{ to the } 4^{th} \text{ power}) + 3(x \text{ cubed})$
B. $y = (x \text{ to the } 4^{th} \text{ power})$
C. $y = (x \text{ to the } 4^{th} \text{ power}) + 3xy$
D. $y = (x \text{ cubed}) + (x \text{ squared}) + 4x$

9. Solve $y' - \frac{2y}{x} = 4xy^2$.

A. $x^2 = y(C - x^4)$ B. $x^2 = y^2(C + x^4)$ C. $x^2 = y(C + x^4)$ D. $x^2 = y^2(C - x^4)$

10. Given the differential equation below, solve for P as a function of x.

$$\frac{dP}{dx} - \frac{P}{x} = 2p^2$$

A. $P = x^2 - x + C$ B. $P = C - x^2$ C. $P = \frac{x^2}{C - x}$ D. $P = \frac{x}{C - x^2}$

REE – Apr. 2013

11. A certain population of bacteria grows such that its rate of change is always proportional to the amount present. It doubles in 2 years. If in 3 years there are 20,000 of bacteria present, how much is present initially?

A. 9,071 B. 10,071 C. 7,071 D. 8,071

REE – Apr. 2007

12. A population P(t) of small rodents has birth rate $\beta = (0.001)P$ (births per month per rodent) and a constant death rate δ. If $P(0) = 100$ and $P'(0) = 8$, how long (in months) will it take his population to double to 200 rodents?

A. $50 \ln(9/7)$ B. $50 \ln(9/8)$ C. $48 \ln(7/8)$ D. $56 \ln(9/7)$

REE – Apr. 2007

13. Given that the half-life of radium is 1690 years, how much in milligrams will remain of one gram of radium after 1000 years?

A. 627.2 B. 589.3 C. 663.6 D. 547.8

REE – Sept. 2014

14. A steel ball at 120 deg C cools in 20 minutes to 80 deg C in a room at 25 deg C. Find the temperature of the ball after half an hour.

A. 40.96 deg C B. 45.96 deg C C. 66.85 deg C D. 55.96 deg C

$-\frac{d\theta}{dt} =$

REE – May 2010

15. An object falls from rest in e medium offering a resistance The velocity of the object before the object reaches the ground is given by the differential equation dv/dt plus v per 10 equals 32 ft per sec squared What is the velocity of the object one second after it falls?

A. 34.12 B. 40.54 C. 30.45 D. 38.65

REE – Sept. 2007

16. An arrow is shot straight upward from the ground with an initial velocity of 160 ft/sec. It experience both the deceleration of gravity and deceleration (v squared)/800 due to air resistance. How high in the air does it go?
A. 314.11 ft B. 289.31 ft C. 277.26 ft D. 254.84 ft

REE – Sept. 2008

17. A 400-gal tank initially contains 100 gal of brine combining 50 lb of salt. Brine containing 1 lb of salt per gallon enters the tank at the rate of 5 gal/s, and the well-mixed brine in the tank flows out at the rate of 3 gal/s. How much salt will the tank contain when it is full of brine?
A. 393.75 lb B. 389.65 lb C. 426.35 lb D. 435.85 lb

REE – Sept. 2006

18. A water tank has the shape obtained by revolving the curve y = x raised to (4/3) around the y-axis. A plug at the bottom is removed at 12 noon when the depth of water in the tank is 12 ft. At 1 PM, the depth of the water is 6 ft. When will the tank be empty?
A. 1:26 PM B. 1:15 PM C. 1:38 PM D. 1:20 PM

19. Find the particular solution of $y'' + 3y' + 2y = 0$ when $x = 0$, $y = 0$, $y' = 1$
A. $y = e^{x} - e^{-x}$ B. $y = e^{x} - 1$ C. $y = 2e^{x} - e^{-x}$ D. $y = e^{-x} - e^{-2x}$

REE – Apr. 2013 / Sept. 2015 / Apr. 2017

20. What is the general solution of $(D^4 - 1)y(t) = 0$?
A. $y = C_1e^{t} + C_2e^{-t} + C_3\cos t + C_4\sin t$
B. $y = C_1e^{t} + C_2e^{-t} + C_3te^{t} + C_4te^{-t}$
C. $y = C_1e^{t} + C_2e^{-t}$
D. $y = C_1e^{t} + C_2te^{-t}$

REE – Sept. 2012/Sept. 2015

21. Find the general solution of $y'' + 10y = 0$

A. $y = C_1 \cos(\text{sqrt. of } 10x) + C_2 \sin(\text{sqrt. of } 10x)$

B. $y = C_1 \cos(\text{sqrt. of } 5x) + C_2 \sin(\text{sqrt. of } 5x)$

C. $y = C \cos(\text{sqrt. of } 10x)$

D. $y = C \sin(\text{sqrt. of } 10x)$

REE – April 2005

22. Given $y = e^{mx}$, what value of m (– infinity to + infinity) will satisfy the relationship $6y'' - y' - y = 0$

A. –1/3, 1/2 B. 1/3, –1/2 C. –1/3, –1/2 D. 1/3, 1/2

REE – Apr. 2003

23. Which of the following is the solution of $y''' - 3y'' + 3y' - y = 0$

I. y = (e to the x)

II. y = x (e to the x)

III. y = (e to the – x)

A. I and II B. III only C. I only D. II only

REE – Sept. 2012

24. Find the general solution of $y'' + 6y' + 9y = x + 1$

A. $y = (C_1x + C_2x^2)\, e^{-3x} + 1/27 + x/9$

B. $y = (C_1 + C_2x)\, e^{-3x} + 1/27 + x/9$

C. $y = (C_1x + C_2x^2)\, e^{3x} + 1/27 + x/9$

D. $y = (C_1 + C_2x)\, e^{3x} + 1/27 + x/9$

REE – Sept. 2008

25. Suppose that a crossbow bolt is shot straight upward with initial velocity 288 ft/s. If its deceleration due to air resistance is (0.04v), then its height x(t) satisfies the initial value problem $x'' = -32 - (0.04)x'$, $x(0) = 0$, $x'(0) = 288$. Find the time required for it to reach the maximum height.

A. 7.9 s B. 7.2 s C. 7.0 s D. 7.7 s