Automatic Control course project

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1 Project rules

In this document you can find the task for the final project of the Automatic Control course. The project is an **individual work**, and it is worth **4 points** out of the 30 total points of the exam.

You are asked to answer the questions presented in this document, properly motivating them, in a **written report** in PDF format. **Please add your personal ID number on top of the first page of the report**. There is no layout requirement and it can be written in any way you prefer. You will be given 4 questions, worth a total of 4 points. Additionally, writing the report in LaTeX will give you an extra 0.5 points. The extra points can be used to reach the 4/4 grade in case some answers are wrong, but **it will not be possible to obtain more than 4 points in total for the final grade**. Note, additionally, that there is a **strict upper limit** to the number of pages of the report of **6 pages maximum**. You are not required to fill all the 6 pages; answering correctly to the questions is enough to get the full points. Together with the written report, you must submit your **Matlab code** (with comments on the key steps).

When you want to deliver your project, send an e-mail to riccardo.ballaben@unitn.it. Please put as object of the e-mail *'Project Delivery - [name] [surname]'* and attach a **ZIP file** containing the Matlab code and the report, named *'[year].[month].[day] - [surname] [name]'*, where year, month, and day correspond to the delivery date in number.

The project has to be delivered **by the end of the first day of the month when you want to verbalize the exam** (note that it is possible to send the e-mail at any day of the previous months). For example, if you want to verbalize your grade in July, you have until 23:59 of the 1st of July to send the e-mail. The grade you will get for the project is valid until February 2025, so you do not need to send your file multiple times. If you are unhappy with the grade of the project you can submit it again. Note, however, that the project deliveries are accepeted only **once in the summer/autumn session** and **once in the winter session**. This means that you can redo the project at most once.

2 System description: 2D model of an overhead crane

In this project you will design a controller for an overhead crane, illustrated in Figure 1, to stabilize a target configuration of the system. The crane is made of a trolley (in blue in the figure), running on rails mounted on the bridge (in yellow in the figure). The trolley houses the motor that lifts the load.

To simplify the problem, we will consider only the motion in the plane of the crane. The relevant physical quantities we can use to model the system are then

- s, the trolley position along the bridge in $[m]$;
- θ, the swing angle in $[rad]$;
- ℓ, the rope length in $[m]$;

The meaning of these variables is illustrated in Figure 2. The control inputs are the force driving the trolley along the bridge F_s and the force along the rope pulling the load F_ℓ.

The nonlinear equations of motion of the system shown in Figure 2 are

$$\begin{aligned}
&(M+m)\ddot{s} - m\ddot{\ell}\sin(\theta) - m\ell\ddot{\theta} - 2m\dot{\ell}\cos(\theta) - m\ell\dot{\theta}^2\sin(\theta) + c\dot{s} = F_s, \\
&-m\ddot{s}\sin(\theta) + m\ddot{\ell} - m\ell\dot{\theta}^2 - mg\cos(\theta) = F_\ell, \\
&m\ell\ddot{\theta} + 2m\dot{\ell}\dot{\theta} - m\ddot{s}\cos(\theta) + mg\sin(\theta) = 0,
\end{aligned} \tag{1}$$

Figure 1: A double girder overhead crane, used to lift and move heavy loads inside factories or warehouses.

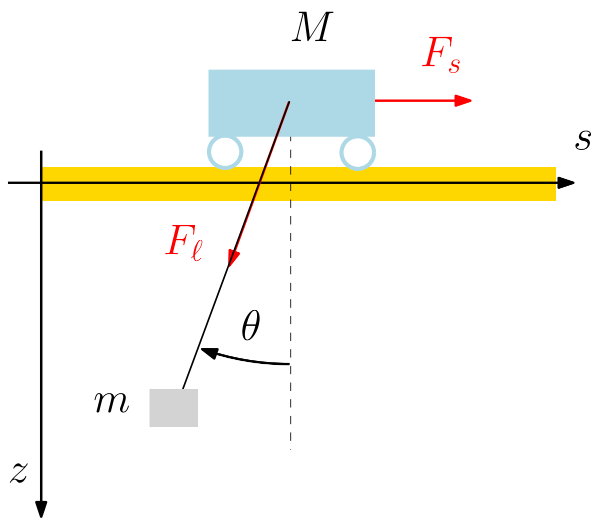

Figure 2: 2D model of an overhead crane. M and m denote, respectively, the mass of the trolley (in blue) and of the load (in gray).

where c is the damping coefficient of the viscous friction corresponding to the translational motion of the trolley along the bridge and g is the gravitational constant.

Taking as the state of the nonlinear system $x = (x1, x2, x3, x4, x5, x6) = (s, \ell, \theta, \dot{s}, \dot{\ell}, \dot{\theta})$, one can write the nonlinear state space model corresponding to equations (1) as

$$
\begin{aligned}
\dot{x}_1 &= x_4, \\
\dot{x}_2 &= x_5, \\
\dot{x}_3 &= x_6, \\
\dot{x}_4 &= \frac{F_x - cx_4 + F_t \sin(x_3) + mx_4 \sin^2(x_3) + 2mx_2x_6^2 \sin(x_3)}{-m\cos^2(x_3) + M + m} \\
\dot{x}_5 &= \frac{mx_2x_6^2 + F_t + gm\cos(x_3) + mx_4 \sin(x_3)}{m} \\
\dot{x}_6 &= \frac{F_x \cos(x_3) - cx_4 \cos(x_3) - gm\sin(x_3) + F_t \cos(x_3)\sin(x_3)}{x_2\left(-m\cos^2(x_3) + M + m\right)} \\
&\quad + \frac{-2Mx_5x_6 - 2mx_5x_6 - Mg\sin(x_3)}{x_2\left(-m\cos^2(x_3) + M + m\right)} \\
&\quad + \frac{gm\cos^2(x_3)\sin(x_3) + mx_4\cos(x_3)\sin^2(x_3) + 2mx_5x_6\cos^2(x_3) + 2mx_2x_6^2\cos(x_3)\sin(x_3)}{x_2\left(-m\cos^2(x_3) + M + m\right)}
\end{aligned} \tag{2}
$$

Linearizing the nonlinear equations (2) around the operating point identified by the configuration $(s_0, \ell_0, 0, 0, 0, 0) = (2, 1, 0, 0, 0, 0)$ and input $(F_x, F_\ell) = (0, -mg)$ (i.e., the the crane is holding the load still at a constant height) yields the dynamical system

$$
\begin{aligned}
\dot{x} &= Ax + Bu, \\
y &= Cx + Du,
\end{aligned} \tag{3}
$$

where the state of the linear system is $x = (s - s_0, \ell - \ell_0, \theta, \dot{s}, \dot{\ell}, \dot{\theta}) \in \mathbb{R}^6$, the input u is $u = (F_x, F_\ell - mg)$ the measured output is $y = (s, \ell)$, i.e., the position of the trolley along the bridge and the length of the rope, and matrices the in (3) are

$$
\begin{aligned}
A &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{gm}{M} & -\frac{c}{M} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{g(M+m)}{M\ell_0} & -\frac{c}{M\ell_0} & 0 & 0 \end{bmatrix}, \\
B &= \begin{bmatrix} 0 & 0 & 0 & \frac{1}{M} & 0 & \frac{1}{M\ell_0} \\ 0 & 0 & 0 & 0 & \frac{1}{m} & 0 \end{bmatrix}^\top, \\
C &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
D &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.
\end{aligned} \tag{4}
$$

The values of the parameters in (4) are

- $M = 2 \cdot 10^3$ $[kg]$;
- $m = 10^4$ $[kg]$;
- $c = 300$ $[Ns/m]$;
- $g = 9.81$ $[m/s^2]$

3 Questions

1. **Question**: Is the system (3), (4) controllable? Is it observable? Based on your answers, is it possible to design a static feedback control of the form $u = Kx$ to stabilize the linearized system (3)? (1pt)

2. **Question:** Design an observer so that the estimate of the state converges to the real value of the state x with a convergence rate of at least $\alpha = 6$. (1pt)

3. **Question:** Find a feedback selection $u = K_i x$, $i \in \{1, 2, 3\}$, rendering the closed-loop system $\dot{x} = (A + BK_i)x$ asymptotically stable with convergence speed $\alpha_i = i, i \in \{1, 2, 3\}$, while minimizing the norm of the matrix K. Simulate the linearized system starting from the initial condition $x_0 = (0.2, 0.2, -\frac{\pi}{20}, 0, 0, 0)$, showing the time history of the state, of its estimate, and of the control input. (1pt)

   Hint: When computing the control input, remember that the only measurements of the state x available are those coming from the output y.

4. **Question:** Simulate the nonlinear system (2) using the three stabilizers found in the above question. Based on the results of the simulations, which controller would you use for the real system? (1pt)

   Hint: Remember the relation between the state and control input of the linearized system (3), (4) and the state and control input of the nonlinear system (2).