Dynamic Modeling And Inverse Dynamic Control Of Mobile Robot

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***Abstract*—This paper presents a dynamic modeling and control studies of mobile robot. The robot is mainly built all around three wheels and platform. A mathematical model of unicycle mobile robot is determined by kinematic and dynamic model. The dynamic modelling equations are based on lagrangian formulation. The motion control strategy is based on the inverse of dynamic control. This leads to accurate tracking of trajectory. The goal to reach in this paper is to improve the performances of inverse dynamic control of mobile robot. The validity of the proposed controller is demonstrated by the simulation of two wheels mobile robots case.**

Keywords: modeling dynamic, inverse dynamic control, mobile robot.

I. Introduction

Mobile robots can be utilized in many applications such as industry, medicine and agriculture, which is capable of autonomous motion. Then, it can be found in many configurations. In mobile robots, there are unicycle robots, tricycle, car-like, omnidirectional robots and spherical mobile robot. Mobile robots are very complicated in mathematical statics and dynamics model, electronics, control theory and manufacturing. Many advanced researches are interested in the area of modeling and control of mobile robotics and their localization [2]. In mathematical model, there are many methods. This enables us to calculate dynamics models such as formula lagrangian, Newton-Euler methodologies, Kanes method and other formulations. In the Newton-Euler method, the dynamic model can be based on two kinds of forces. The first, is given forces and the second, constraint forces. The given forces contain the external forces which are applied to the actuators. But the constraint forces are the forces of interaction between the robot platform and ground through the wheels [1]. The methodology lagrange is the derivation of the kinetic energy and the potential. The kinetic energy is expressed in function of the speed of rotation, speed of translation and the forces exercised on the system [1].

In fact, the dynamic model of mobile robot is generally nonlinear, a complex form and multivariable system.To overcome these problems, there are several control techniques that can solve this problem, such as: the indirect adaptive control, robust control method, inverse dynamics control and other methods.Then, the adaptive control may have some advantages. For instance the hugeness of calculation capability and its complication. But the robust control can make the system unstable [14] and [15]. This paper discussed the dynamic model in the form of a second-order vector differential equation. The dynamics model is based on the Lagrange formula because it can easily be applied. But the control strategies using the inverse dynamics control can avoid the nonlinear system, decoupling and the realization of tracking trajectory of the system[3].

II. Kinematics and Dynamics Model of mobile robot

A. Kinematics Model

In this paper we focused on a mobile robot that has two independent wheels and single freewheel to ensure the stability of the system. The coordinate of mobile robot is given by (Fig. 1):

The position of the mobile robot wheels, in this environment

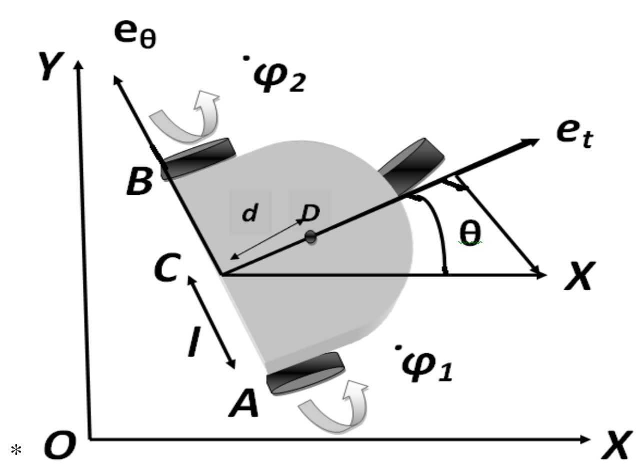

Fig. 1. Coordinate of mobile robot.

has two different coordinate systems (inertial coordinate

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system, robot coordinate system).
$R_1 = (O, \vec{X}, \vec{Y}, \vec{Z})$: inertial coordinate system is a global reference frame.
$R_2 = (D, \vec{e}t, \vec{e}\theta, \vec{Z})$: robot coordinate system is moving frame.
Where:
q_1 and q_2 two coordinates that define the position of the origin D of the mobile robot.

$$q_1 = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \quad and \quad q_2 = \begin{bmatrix} et \\ e\theta \\ \theta \end{bmatrix} \tag{1}$$

The two coordinates q_1 and q_2 are related by the transformation given with (2), (3) and (4) :

$$\begin{cases} et = x\cos(\theta) + y\sin(\theta) \\ e\theta = -x\sin(\theta) + y\cos(\theta) \end{cases} \tag{2}$$

$$\begin{cases} x = \cos(\theta) \times et - \sin(\theta) \times e\theta \\ y = \sin(\theta) \times et + \cos(\theta) \times e\theta \end{cases} \tag{3}$$

$$q_1 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \times q_2 \tag{4}$$

With:

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$R(\theta)$: Orthogonal rotation matrix according to the axis Z [1]. The movement of mobile robot is described by two constraint equations, which are obtained by two the following assumptions:

1) *The first constraint pure rolling:* The contact between the wheels and the ground during locomotion is punctual. So the speed of the wheel is relative to the ground at the point of contact, it is equal to zero. The wheels are non-deformable with a constant radius r. The contact between the wheels and the ground is given by (Fig. 2):
The speed of the wheel is relative to the ground at the point P is given by the equation (6):

$$\vec{V}_P = \vec{V}_A + \vec{\omega}\hat{}\vec{A}P = \vec{0} \tag{6}$$

$$(\dot{x}\vec{X} + \dot{y}\vec{Y}) + \dot{\varphi}_1(\sin(\theta)\vec{X} - \cos(\theta)\vec{Y})\hat{}(-r\vec{Z}) = \vec{0} \tag{7}$$

$$(\dot{x} + r\dot{\varphi}_1\cos(\theta))\vec{X} + (\dot{y} + r\dot{\varphi}_1\sin(\theta))\vec{Y} = \vec{0} \tag{8}$$

$$\begin{cases} \dot{x} + r\dot{\varphi}_1\cos(\theta) = 0 \\ \dot{y} + r\dot{\varphi}_1\sin(\theta) = 0 \end{cases} \tag{9}$$

$$\begin{cases} \sin(\theta)\dot{x} + r\dot{\varphi}_1\sin(\theta)\cos(\theta) = 0 \\ \cos(\theta)\dot{y} + r\dot{\varphi}_1\cos(\theta)\sin(\theta) = 0 \end{cases} \tag{10}$$

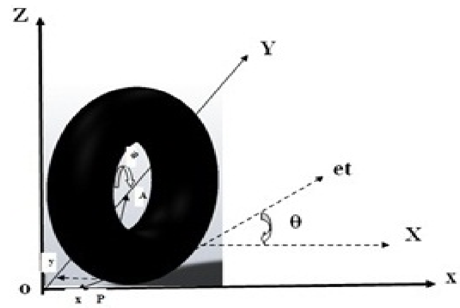

Fig. 2. Rolling contact Characterization.

Then:

$$-\sin(\theta)\dot{x} + \cos(\theta)\dot{y} = 0 \tag{11}$$

2) *The second constraint has no lateral slip motion:* The wheel must not drag orthogonally to the axis of the wheel. So the movement is along the tangential axis ($\dot{e}\theta$= 0) [1], [4], [9], [10] and [13].

$$\begin{cases} \cos(\theta)\dot{x} + r\dot{\varphi}_1\cos(\theta)\cos(\theta) = 0 \\ \sin(\theta)\dot{y} + r\dot{\varphi}_1\sin(\theta)\sin(\theta) = 0 \end{cases} \tag{12}$$

Then:

$$\cos(\theta)\dot{x} + \sin(\theta)\dot{y} = -r\dot{\varphi}_1 \tag{13}$$

$$V_A = r\dot{\varphi}_1 = \overline{CA} \times \dot{\theta} + V_C = \overline{BA} \times \dot{\theta} + V_B \tag{14}$$

$$V_A = \overline{BA} \times \dot{\theta} + V_B = r\dot{\varphi}_1 = 2l \times \dot{\theta} + r\dot{\varphi}_2 \tag{15}$$

Then:

$$\dot{\theta} = \frac{r}{2l}(\dot{\varphi}_1 - \dot{\varphi}_2) \tag{16}$$

$$V_B = r\dot{\varphi}_2 = \overline{CB} \times \dot{\theta} + V_C \tag{17}$$

$$V_C = \frac{1}{2}(V_A + V_B) = \frac{r}{2}(\dot{\varphi}_2 + \dot{\varphi}_1) = \dot{e}t \tag{18}$$

$$\dot{q}_2 = \begin{bmatrix} \dot{e}t \\ \dot{e}\theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ 0 & 0 \\ \frac{r}{2l} & -\frac{r}{2l} \end{bmatrix} \times \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} \tag{19}$$

Where:

$$X_D = X_c + d\cos(\theta), Y_D = Y_c + d\sin(\theta) \tag{20}$$

$$X_A = X_c + l\sin(\theta), Y_A = Y_c - l\cos(\theta) \tag{21}$$

$$X_B = X_c - l\sin(\theta), Y_B = Y_c + l\cos(\theta) \tag{22}$$

$$\dot{X}_D = \dot{X}_c - d\dot{\theta}\sin(\theta), \dot{Y}_D = \dot{Y}_c + d\dot{\theta}\cos(\theta) \tag{23}$$

$$\dot{X}_A = \dot{X}_c + l\dot{\theta}\cos(\theta), \dot{Y}_A = \dot{Y}_c + l\dot{\theta}\sin(\theta) \tag{24}$$

$$\dot{X}_B = \dot{X}_c - l\dot{\theta}\cos(\theta), \dot{Y}_B = \dot{Y}_c - l\dot{\theta}\sin(\theta) \tag{25}$$

B. Dynamics Model

To determine the dynamic model of the system, we must apply the formula of Lagrange. It can be written by the equation (26):

$$\frac{d}{dt}\left(\frac{dl}{d\dot{q}_i}\right)-\frac{dl}{dq_i}=Q_i-\tau_i \tag{26}$$

With:
l: Lagrangian of the system.
Lagrangian equation is given by the equation (27):

$$l=E-U \tag{27}$$

E: Total kinetic energy of the system.
U : Total energy potential of the system.
Let's consider a moving rigid body ($U=0$)
Then, l is given by (28)

$$l=E \tag{28}$$

The mobile robot is composed of four bodies: two motorized wheels (left wheel, right wheel), a freewheeling and a platform. The kinetic energy for each corresponding body is presented by (29)-(32)[1], [12]and [15]:

$$E_{c_{rouelibre}}=0 \tag{29}$$

$$E_{crA}=\frac{1}{2}m_r(\dot{X}_A^2+\dot{Y}_A^2)+\frac{1}{2}I_W\dot{\varphi}_1^2+\frac{1}{2}I_m\dot{\theta}^2 \tag{30}$$

$$E_{crB}=\frac{1}{2}m_r(\dot{X}_B^2+\dot{Y}_B^2)+\frac{1}{2}I_W\dot{\varphi}_2^2+\frac{1}{2}I_m\dot{\theta}^2 \tag{31}$$

$$E_{c_{platforme}}=\frac{1}{2}m_d(\dot{X}_d^2+\dot{Y}_d^2)+\frac{1}{2}I_d\dot{\theta}^2 \tag{32}$$

We add equations (29), (30), (31) and (32) the following expression is obtained:

$$\begin{aligned}E_{c_{totale}}&=\tfrac{1}{2}m_d(\dot{X}_d^2+\dot{Y}_d^2)+\tfrac{1}{2}I_d\dot{\theta}^2+\tfrac{1}{2}m_r(\dot{X}_A^2+\dot{Y}_A^2)\\&+\tfrac{1}{2}m_r(\dot{X}_B^2+\dot{Y}_B^2)+\tfrac{1}{2}I_W\dot{\varphi}_1^2+\tfrac{1}{2}I_W\dot{\varphi}_2^2+I_m\dot{\theta}^2\end{aligned} \tag{33}$$

The total kinetic energy can be written as:

$$E_c=\frac{1}{2}m_t(\dot{X}_c^2+\dot{Y}_c^2)+\frac{1}{2}I_t\dot{\theta}^2+\frac{1}{2}I_W\dot{\varphi}_1^2+\frac{1}{2}I_W\dot{\varphi}_2^2 \tag{34}$$

With:

$$\mathrm{m}_t=\mathrm{m}_d+2\mathrm{m}_r \tag{35}$$

And

$$I_t=m_d d^2+2m_r l^2+I_d+2I_m \tag{36}$$

Where m_t is the total vehicle mass, m_d is the mass of the platform without actuators, m_r is the mass of the wheel with actuators, I_d is the moment of inertia of the platform about the vertical axis through the center of mass, I_w is the moment of inertia of wheel with a motor about the wheel axis and I_m is the moment of inertia of wheel with a motor about the wheel diameter [1].
The matrix form of the system can be expressed as:

$$H(q)\ddot{q}+K(\dot{q},q)\dot{q}=\tau(q) \tag{37}$$

τ: is the load torque.
$H(q)$: is the mobile robot inertia matrix.
$K(q)$: is the matrix of centrifugal and coriolis torques.
With :

$$H\,(\mathrm{q})=\begin{bmatrix} I_W+\frac{r^2}{4l^2}(m_t l^2+I_t) & \frac{r^2}{4l^2}(m_t l^2-I_t)\\ \frac{r^2}{4l^2}(m_t l^2-I_t) & I_W+\frac{r^2}{4l^2}(m_t l^2+I_t)\end{bmatrix} \tag{38}$$

$$K(\dot{q},q)=\begin{bmatrix} 0 & \frac{r^2}{2l}m_d d\dot{\theta}\\ \frac{r^2}{2l}m_d d\dot{\theta} & 0\end{bmatrix} \tag{39}$$

$$\dot{q}=\begin{bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\end{bmatrix} \tag{40}$$

III. INVERSE DYNAMICS CONTROL OF MOBILE ROBOT

The aim of inverse dynamics control is to make the system linear and decoupled and also ensure the desired trajectory tracking.The dynamic of mobile robot is given by (37).We choose the acceleration as a control input given by (41):

$$H(q)\times u_1+K(\dot{q},q)\dot{q}=\tau(q) \tag{41}$$

u_1 Can now be written by the following expression:

$$u_1(t)=-K_0q(t)-K_1\dot{q}(t)+R(t) \tag{42}$$

K_1 and K_0 Are constant, diagonal and positive matrix

$$K_1=diag\left\{2w_i\xi_i\right\} \tag{43}$$

And

$$K_0=diag\left\{w_i^2\right\} \tag{44}$$

In the following expression $R(t)$ is chosen as the reference input:

$$R(t)=\ddot{q}_d(t)+K_1\dot{q}_d(t)+K_0q_d(t) \tag{45}$$

By combining the equation (41) with (37) the following expression of the tracking error e = q-q_d is described by :

$$\ddot{e}+K_1\dot{e}+K_0e=0 \tag{46}$$

The closed loop system is identical to a system of a second order. The closed-loop system is globally decoupled and asymptotically stable provided that K_1 and K_0 are symmetric matrix [7], [8] and [14].

IV. SIMULATION RESULT

In order to demonstrate the performance of the inverse dynamics controller from the simulation results in matlab simulink we suppose the desired trajectories of the wheels positions are opted to be q_1= $sin(\theta)\times t^2$ (rad) and q_2= $cos(\theta)\times exp(-$ t) (rad).
A numerical simulation was made out on a model of unicycle mobile robot.

$$K_0=\begin{bmatrix}18 & 0\\ 0 & 18\end{bmatrix} \tag{47}$$

And

$$K_1 = \begin{bmatrix} 23 & 0 \\ 0 & 23 \end{bmatrix} \quad (48)$$

The figures Fig. 3 - 6 show that the real trajectory of position and speed of the robot is following the desired trajectory perfectly.The figures Fig. 8-9 shows the actual trajectory of position of the left and the right wheel of the unicycle in the presence of external disturbances of the model dynamic $F_1 = sin(-100 \times t) \times t^5(rad)$ and $F_2 = 10^5 \times cos(t) \times t^2(rad)$ that not could track the reference trajectory very accurately and perfectly.This explains that The controller based on inverse dynamic has not good performances and not robustness when the modeling dynamic of mobile robot including a disturbance vector.

The mobile robot parameters are given by table I

TABLE I
TABLE OF MOBILE ROBOT PARAMETRS

parameter	lift-wheel	right-wheel	platform
m_r (kg)	0.3	0.3	-
m_d (kg)	-	-	0.45
I_w (Kgm^2)	0.12	0.12	-
I_d (Kgm^2)	-	-	0.2
I_m (Kgm^2)	0.143	0.143	-
r (m)	0.125	0.125	-
l (m)	-	-	0.2
d (m)	-	-	0.15

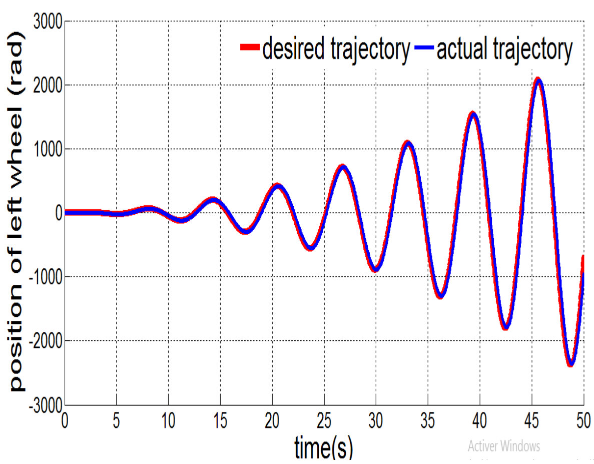

Fig. 3. Trajectory of position of left wheel.

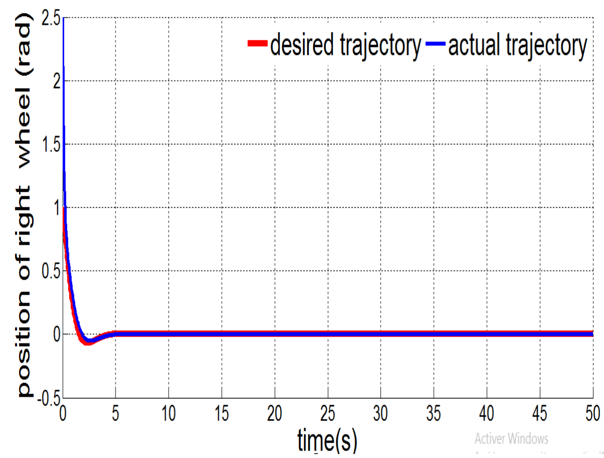

Fig. 4. Trajectory of position of right wheel.

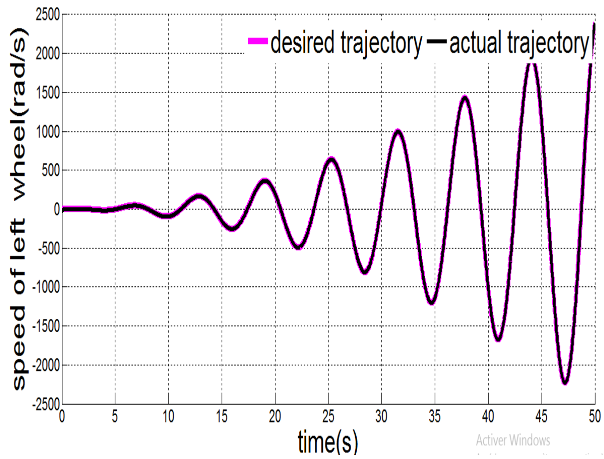

Fig. 5. Trajectory of speed of left wheel.

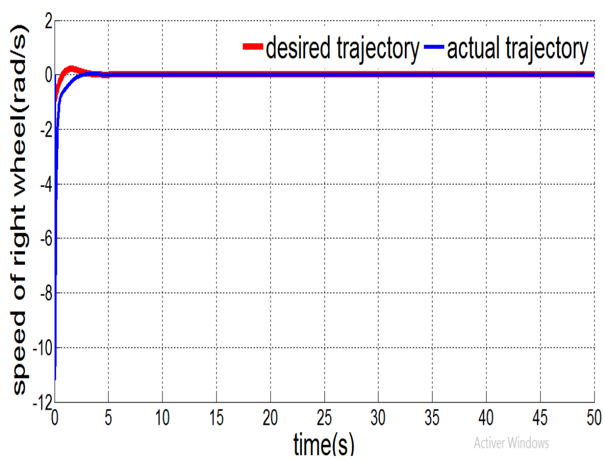

Fig. 6. Trajectory of speed of right wheel.

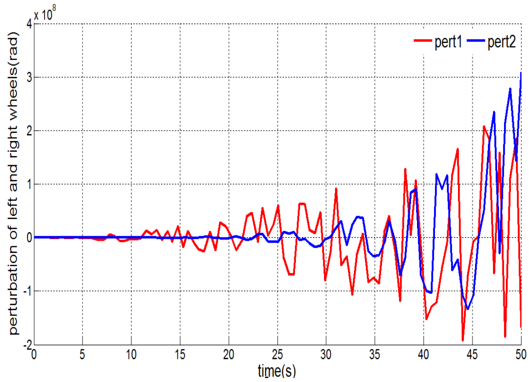

Fig. 7. Trajectory of perturbation of left and right wheel.

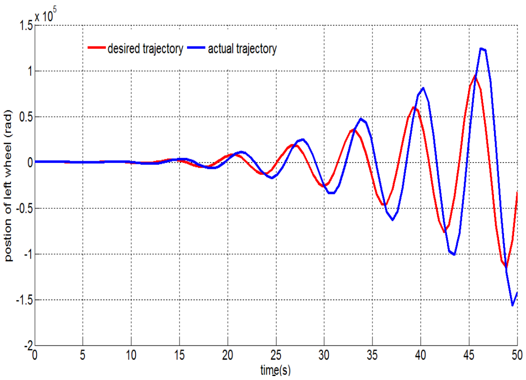

Fig. 8. Trajectory of position of left wheel.

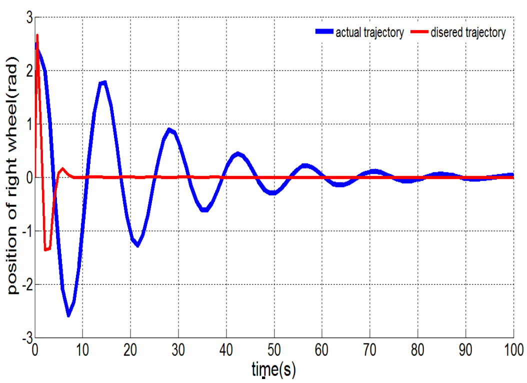

Fig. 9. Trajectory of position of right wheel.

V. Conclusion

In this paper, the modeling kinematic and dynamic of unicycle mobile robot is discussed. So the kinematic model of mobile robot motion was established using constraint pure rolling and constraint with no lateral slip motion and the dynamics model was developed using the formula lagrangian. The problem of decoupling and nonlinear model dynamics for unicycle-type vehicles is solved by using the inverse dynamic control. The simulation result of control we have already proposed in this paper has good responses which allow to achieve the trajectory tracking when there is no disturbances . But when the modeling dynamic of unicycle robot has the external perturbation, the performances of proposed control are not excellent.So we need in our search to find other more robust methods to avoid the disturbance effect on the system. Then we are going to deal with the experimental tests in the purpose of evaluating the performances of the conceived system and its control law by applying new methods.

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