

Chapter 1

Formation channels of single and binary stellar-mass black holes

Michela Mapelli

Abstract These are exciting times for binary black hole (BBH) research. LIGO and Virgo detections are progressively drawing a spectacular fresco of BBH masses, spins and merger rates. In this review, we discuss the main formation channels of BBHs from stellar evolution and dynamics. Uncertainties on massive star evolution (e.g., stellar winds, rotation, overshooting and nuclear reaction rates), core-collapse supernovae and pair instability still hamper our comprehension of the mass spectrum and spin distribution of black holes (BHs), but substantial progress has been done in the field over the last few years. On top of this, the efficiency of mass transfer in a binary system and the physics of common envelope substantially affect the final BBH demography. Dynamical processes in dense stellar systems can trigger the formation of BHs in the mass gap and intermediate-mass BHs via hierarchical BH mergers and via multiple stellar collisions. Finally, we discuss the importance of reconstructing the cosmic evolution of BBHs.

Key words: black holes – gravitational waves – pair instability – massive stars – binary systems – binary black holes – intermediate-mass black holes – stellar dynamics – star clusters – merger rates

1.1 Introduction: Observational facts about gravitational waves

On 2015 September 14, the LIGO interferometers [1] captured the gravitational wave (GW) signal from a binary black hole (BBH) merger [5]. This event, named GW150914, is the first direct detection of GWs, about hundred years after Ein-

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stein's prediction. Over the last five years, LIGO and Virgo [21] witnessed a rapidly growing number of GW events: the second gravitational wave transient catalogue (GWTC-2, [14, 15, 16]) consists of 50 binary compact object mergers from the first (O1), the second (O2) and the first part of the third observing run (O3a) of the LIGO–Virgo collaboration (LVC). Based on the results of independent pipelines, [383], [350], [356] and [261] claimed several additional GW candidates from O1 and O2. Furthermore, several dozens of public triggers from the second part of the third observing run (O3b) can be retrieved at <https://gracedb.ligo.org/>.

Among the aforementioned detections, GW170817, the first binary neutron star (BNS) merger detected during O2, is the first and the only GW event unquestionably associated with an electromagnetic counterpart to date [11, 12, 150, 311, 241, 87, 318, 78, 88, 259, 276, 24].

Several other exceptional events were observed during O3a: the first unequal-mass BBH, GW190412 [17]; ii) the second BNS, GW190425 [6]; iii) the first black hole (BH) – neutron star (NS) candidate, GW190814 [19], and iv) GW190521, which is the most massive system ever observed with GWs [18, 20]. Also, GW190521 is the first BBH event with a possible electromagnetic counterpart [157]. This growing sample represents a “Rosetta stone” to unravel the formation of binary compact objects.

Astrophysicists have learned several revolutionary concepts about compact objects from GW detections. Firstly, GW150914 has confirmed the existence of BBHs, i.e. binary systems composed of two BHs. BBHs have been predicted a long time ago (e.g. [349, 344, 315, 192, 316, 56, 280, 85, 53]), but GW150914 is their first observational confirmation [2]. Secondly, GW detections show that a number of BBHs are able to merge within a Hubble time. Thirdly, GW170817 has confirmed the connection between short gamma-ray bursts, kilonovae and BNS mergers [11].

Finally, most of the LIGO–Virgo BHs observed so far host BHs with mass in excess of $20 M_{\odot}$. The very first detection, GW150914, has component masses equal to $m_1 = 35.6^{+4.7}_{-3.1} M_{\odot}$ and $m_2 = 30.6^{+3.0}_{-4.4} M_{\odot}$ [3]. This was a genuine surprise for the astrophysicists [2], because the only stellar BHs for which we have a dynamical mass measurement, i.e. about a dozen of BHs in X-ray binaries, have mass $\leq 20 M_{\odot}$ [264, 266, 249]. Moreover, most theoretical models available five years ago did not predict the existence of BHs with mass $m_{\text{BH}} > 30 M_{\odot}$ (but see [378, 164, 225, 232, 48, 133, 233, 386, 322] for a few exceptions). Thus, the first GW detections have urged the astrophysical community to deeply revise the models of BH formation and evolution.

The recently published O3a events add complexity to this puzzle. In particular, the secondary component of GW190814, with a mass $m_2 = 2.59^{+0.08}_{-0.09} M_{\odot}$ [19], is either the lightest BH or the most massive NS ever observed, questioning the proposed existence of a mass gap between 2 and $5 M_{\odot}$ [266, 116].

The merger product of GW190521 ($m_f = 142^{+28}_{-16} M_{\odot}$, [18, 20]) is the first intermediate-mass BH (IMBH) ever detected by LIGO and Virgo and is the first BH with mass in the range $\sim 100 - 1000 M_{\odot}$ ever observed not only with GWs but also in the electromagnetic spectrum. Moreover, the mass of the primary compo-

nent, $m_1 = 85^{+21}_{-14} M_\odot$, falls inside the predicted pair instability mass gap, as we will discuss later in this review. According to the re-analysis of [260] (see also [120]), GW190521 could be an intermediate-mass ratio inspiral with primary (secondary) mass $168^{+15}_{-61} M_\odot$ ($16^{+33}_{-3} M_\odot$), when assuming a uniform in mass-ratio prior. This interpretation avoids a violation of the mass gap, but requires the formation of an IMBH to explain the primary component.

LIGO and Virgo do not observe only masses, they also allow us to extract information on spins and merger rates. The local BBH merger rate density inferred from GWTC-2 is $23.9^{+14.9}_{-8.6}$ (58^{+54}_{-29}) $\text{Gpc}^{-3} \text{ yr}^{-1}$ within the 90% credible interval, when GW190814 is included in (excluded from) the sample of BBHs [15]. The LVC also estimated an upper limit for the merger rate density of BH–NS binaries ($\mathcal{R}_{\text{BHNS}} < 610 \text{ Gpc}^{-3} \text{ yr}^{-1}$, [8]) and inferred a BNS merger rate density $\mathcal{R}_{\text{BNS}} = 320^{+490}_{-240} \text{ Gpc}^{-3} \text{ yr}^{-1}$ within the 90% credible interval [15].

The uncertainties on the spins of each binary component are still too large to draw strong conclusions, apart from very few cases (for example, the spin of the primary component of GW190814 is close to zero, see Figure 6 of [19]). However, LIGO and Virgo allow to give an estimate of two spin combinations, which are called effective spin (χ_{eff}) and precessing spin (χ_p). The effective spin is defined as

$$\chi_{\text{eff}} = \frac{(m_1 \chi_1 + m_2 \chi_2)}{m_1 + m_2} \cdot \frac{\mathbf{L}}{L}, \quad (1.1)$$

where m_1 and m_2 (χ_1 and χ_2) are the masses (dimensionless spin parameters) of the primary and secondary component of the binary, respectively and \mathbf{L} is the Newtonian orbital angular momentum vector of the binary. The dimensionless spin parameters are defined as $\chi_i \equiv S_i c / (G m_i^2)$, where S_i is the magnitude of the spin vector, c the speed of light and G the gravity constant. Hence, the effective spin can take any values between -1 and 1 , where $\chi_{\text{eff}} = 1$ (-1) means that the two BHs are both maximally rotating and perfectly aligned (anti-aligned) with respect to the orbital angular momentum of the BBH, while $\chi_{\text{eff}} = 0$ indicates either that the two BHs are perfectly non-rotating or that both spins lie in the plane of the BBH orbit.

The precessing spin is defined as

$$\chi_p = \frac{c}{B_1 G m_1^2} \max(B_1 S_{1\perp}, B_2 S_{2\perp}), \quad (1.2)$$

where $B_1 = 2 + 3q/2$ and $B_2 = 2 + 3/(2q)$, $q = m_2/m_1 \leq 1$, $S_{1\perp}$ and $S_{2\perp}$ are the components of the spin vectors perpendicular to the orbital angular momentum. Hence, χ_p can take values between 0 (no spin components in the orbital plane) and 1 (at least one spin being maximal and lying in the orbital plane). χ_p is called precessing spin because spin components misaligned with respect to the orbital angular momentum of the binary drive precession [313].

In current GW observations, small values of χ_{eff} are preferred and χ_p is unconstrained [8, 7], with a few exceptions. GW151226, GW170729, GW190412 and GW190425, together with a few candidate events (e.g., GW190517_055101) show

support for positive values of χ_{eff} [8, 17, 14]. If we look at the overall population of GWTC-2 BBHs [15], $\sim 12\%$ to 44% of BBH systems have spins tilted by more than 90° with respect to their orbital angular momentum, supporting a negative effective spin parameter.

The precessing spin of GW190814 has a strong upper bound $\chi_p < 0.07$ [19]. Finally, GW190521 shows mild evidence for a non-zero precessing spin ($\chi_p = 0.68^{+0.25}_{-0.37}$ within the 90% credible interval, [20]). In contrast, spin measurements in X -ray binaries point to a range of spin magnitudes, including high spins [248, 249].

This review discusses the formation channels of BHs and BBHs in light of the challenges posed by recent GW detections. This field has witnessed an exponential growth of publications and models in the last few years. While I will try to give an overview as complete as possible of the main models and connected issues, it would be impractical to mention every interesting study in the span of this review.

1.2 The formation of compact remnants from single stellar evolution and supernova explosions

Black holes (BHs) and neutron stars (NSs) are expected to form as remnants of massive ($\gtrsim 8 M_\odot$) stars. An alternative theory predicts that BHs can also form from gravitational collapse in the early Universe (the so called primordial BHs, e.g. [72, 58, 71, 173]). In this review, we will focus on BHs of stellar origin.

The mass function of BHs is highly uncertain, because it may be affected by a number of barely understood processes. In particular, stellar winds and supernova (SN) explosions both play a major role on the formation of compact remnants. Processes occurring in close binary systems (e.g. mass transfer and common envelope) are a further complication and will be discussed in the next Section.

1.2.1 Stellar winds and stellar evolution

Stellar winds are outflows of gas from the atmosphere of a star. In cold stars (e.g. red giants and asymptotic giant branch stars) they are mainly induced by radiation pressure on dust, which forms in the cold outer layers (e.g. [354]). In massive hot stars (O and B main sequence stars, luminous blue variables and Wolf-Rayet stars), stellar winds are powered by the coupling between the momentum of photons and that of metal ions present in the stellar photosphere. A large number of strong and weak resonant metal lines are responsible for this coupling (see e.g. [191] for a review).

Understanding stellar winds is tremendously important for the study of compact objects, because mass loss determines the pre-SN mass of a star (both its total mass and its core mass), which in turn affects the outcome of an SN explosion [132, 134, 225, 232, 48].

Early work on stellar winds (e.g. [13, 190, 200]) highlighted that the mass loss of O and B stars depends on metallicity as $\dot{m} \propto Z^\alpha$ (with $\alpha \sim 0.5 - 1.0$, depending on the model). However, such early work did not account for multiple scattering, i.e. for the possibility that a photon interacts several times before being absorbed or leaving the photosphere. Vink et al. (2001, [360]) accounted for multiple scatterings and found a universal metallicity dependence $\dot{m} \propto Z^{0.85} v_\infty^p$, where v_∞ is the terminal velocity and $p = -1.23$ ($p = -1.60$) for stars with effective temperature $T_{\text{eff}} \gtrsim 25000$ K (12000 K $\lesssim T_{\text{eff}} \lesssim 25000$ K).

The situation is more uncertain for post-main sequence stars. For Wolf-Rayet (WR) stars, i.e. naked helium cores, [362] predict a similar trend with metallicity $\dot{m} \propto Z^{0.86}$. With a different numerical approach (which accounts also for wind clumping), [154] find a strong dependence of WR mass loss on metallicity but also on the electron-scattering Eddington factor $\Gamma_e = \kappa_e L / (4\pi c G m)$, where κ_e is the cross section for electron scattering, L is the stellar luminosity, c is the speed of light, G is the gravity constant, and m is the stellar mass. The importance of Γ_e has become increasingly clear in the last few years [155, 361, 359], but only few stellar evolution models include this effect.

For example, [336, 75] adopt a mass loss prescriptions for massive hot stars (O and B stars, luminous blue variables and WR stars) that scales as $\dot{m} \propto Z^\alpha$, where $\alpha = 0.85$ if $\Gamma_e < 2/3$, $\alpha = 2.45 - 2.4\Gamma_e$ if $2/3 \leq \Gamma_e \leq 1$ and $\alpha = 0.05$ if $\Gamma_e > 1$. This simple formula accounts for the fact that metallicity dependence tends to vanish when the star is close to be radiation pressure dominated, as clearly shown by figure 10 of [154]. Figure 1.1 shows the mass evolution of a star with zero-age main sequence (ZAMS) mass $m_{\text{ZAMS}} = 90 M_\odot$ for seven different metallicities, as obtained with the SEVN code [322]. At the end of its life, a solar-metallicity star (here we assume $Z_\odot = 0.02$) has lost more than $2/3$ of its initial mass, while the most metal-poor star in the Figure ($Z = 0.005 Z_\odot$) has retained almost all its initial mass.

Other aspects of massive star evolution also affect the pre-SN mass of a star. For example, surface magnetic fields appear to strongly quench stellar winds by magnetic confinement [141, 275, 180]. In particular, [275] show that a non-magnetic star model with metallicity $\sim 0.1 Z_\odot$ and a magnetic star model with solar metallicity and Alfvén radius $R_A \sim 4R_\odot$ undergo approximately the same mass loss according to this model. This effect cannot be neglected because surface magnetic fields are detected in ~ 10 per cent of the hot stars [366], but is currently not included in models of compact-object formation.

Finally, rotation affects the evolution of a massive star in several ways (e.g. [217, 76, 202, 203, 77]). As a general rule of thumb, rotation increases the stellar luminosity. This implies that mass loss is generally enhanced if rotation is accounted for. On the other hand, rotation also induces chemical mixing, which leads to the formation of larger helium and carbon-oxygen cores. While enhanced mass loss implies smaller pre-SN masses, the formation of bigger cores has strong implications for the final fate of a massive star, as we discuss in the following Section.

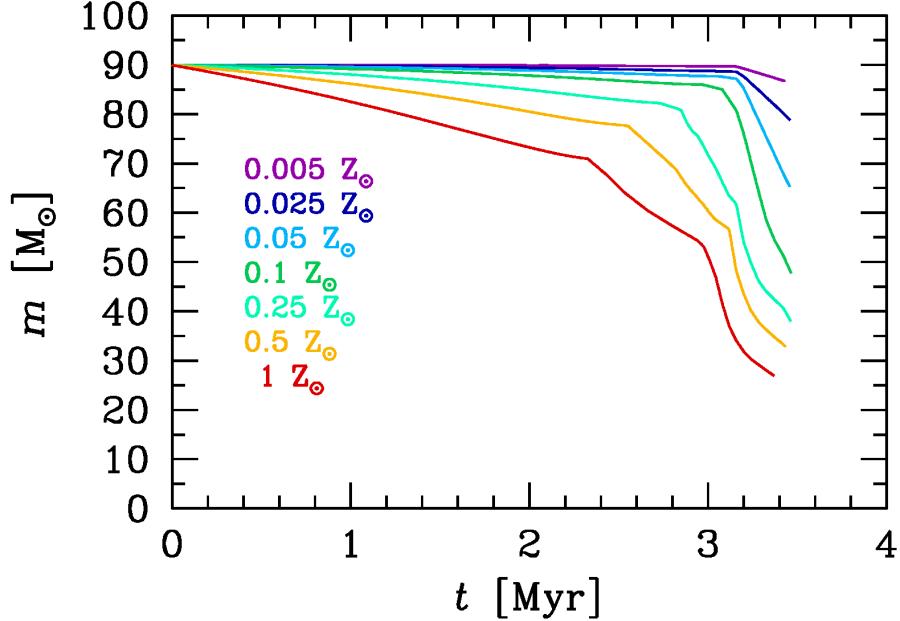


Fig. 1.1 Evolution of stellar mass as a function of time for a star with ZAMS mass $m_{\text{ZAMS}} = 90 \text{ M}_\odot$ and seven different metallicities, ranging from 0.005 Z_\odot up to Z_\odot (we assumed $\text{Z}_\odot = 0.02$). These curves were obtained with the SEVN population-synthesis code [322], adopting PARSEC stellar evolution tracks [75].

1.2.2 Core-collapse supernova (SN) or direct collapse

Whether a star undergoes a successful core-collapse SN or a failed SN is the first key question to address, in order to assess the properties of the final compact object. A star undergoing a successful core-collapse SN explosion will leave a NS or a light BH, while stars that end their life with a failed SN will become rather massive BHs ($> 20 \text{ M}_\odot$), because most (if not all) of the final mass of the star collapses into a BH directly. Addressing this question is a challenge for several reasons.

The mechanisms triggering iron core-collapse SNe are still highly uncertain. The basic framework and open issues are the following. As the mass of the central degenerate core reaches the Chandrasekhar mass [73], the degeneracy pressure of relativistic electrons becomes insufficient to support it against collapse. Moreover, electrons are increasingly removed, because protons capture them producing neutrons and neutrinos. This takes the core into a new state, where matter is essentially composed of neutrons, which support the core against collapse by their degeneracy pressure. To reach this new equilibrium, the core collapses from a radius of few thousand km down to a radius of few ten km in less than a second. The gravitational

energy gained from the collapse is $W \sim 5 \times 10^{53} \text{ erg} (m_{\text{PNS}}/1.4 M_{\odot})^2 (10 \text{ km}/R_{\text{PNS}})$, where m_{PNS} and R_{PNS} are the mass and radius of the proto-neutron star (PNS).

The main problem is to explain how this gravitational energy can be –at least partially– transferred to the stellar envelope triggering the SN explosion [84, 57]. Several mechanisms have been proposed, including rotationally-driven SNe and/or magnetically-driven SNe (see e.g. [178, 121] and references therein). The most commonly investigated mechanism is the convective SN engine (see e.g. [133]). According to this model, the collapsing core drives a bounce shock. For the SN explosion to occur, this shock must reverse the supersonic infall of matter from the outer layers of the star. Most of the energy in the shock consists in a flux of neutrinos. As soon as neutrinos are free to leak out (because the shock has become diffuse enough), their energy is lost and the shock stalls. The SN occurs only if the shock is revived by some mechanism. In the convective SN scenario, the region between the PNS surface and the shock stalling radius can become convectively unstable (e.g. because of a Rayleigh–Taylor instability). Such convective instability can convert the energy leaking out of the PNS in the form of neutrinos to kinetic energy pushing the convective region outward. If the convective region overcomes the ram pressure of the infalling material, the shock is revived and an explosion is launched. If not, the SN fails.

While this is the general idea of the convective engine, fully self-consistent simulations of core collapse with a state-of-the-art treatment of neutrino transport do not lead to explosions in spherical symmetry except for the lighter SN progenitors ($\lesssim 10 M_{\odot}$, [121, 110]). Simulations which do not require the assumption of spherical symmetry (i.e. run at least in 2D) appear to produce successful explosions from first principles for a larger range of progenitor masses (see e.g. [255, 254]). However, 2D and 3D simulations are still computationally challenging and cannot be used to make a study of the mass distribution of compact remnants.

Thus, in order to study compact-object masses, SN explosions are artificially induced by injecting in the pre-SN model some amount of kinetic energy (kinetic bomb) or thermal energy (thermal bomb) at an arbitrary mass location. The evolution of the shock is then followed by means of 1D hydrodynamical simulations with some relatively simplified treatment for neutrinos. This allows to simulate hundreds of stellar models.

Following this approach, O’Connor & Ott (2011, [262]) propose a criterion to decide whether a SN is successful or not, based on the compactness parameter:

$$\xi_m = \frac{m/M_{\odot}}{R(m)/1000 \text{ km}}, \quad (1.3)$$

where $R(m)$ is the radius which encloses a given mass m . Usually, the compactness is defined for $m = 2.5 M_{\odot}$ ($\xi_{2.5}$). [262] measure the compactness at core bounce¹ in their simulations and find that the larger $\xi_{2.5}$ is, the shorter is the time to form a BH (as shown in their Figure 6). This means that stars with a larger value of $\xi_{2.5}$ are more likely to collapse to a BH without SN explosion. The work by Ugliano et

¹ [351] show that $\xi_{2.5}$ is not significantly different at core bounce or at the onset of collapse.

al. (2012, [351]) and Horiuchi et al. (2014, [171]) indicate that the best threshold between exploding and non-exploding models is $\xi_{2.5} \sim 0.2$.

Ertl et al. (2016, [110]) indicate that a single criterion (e.g. the compactness) cannot capture the complex physics of core-collapse SN explosions. They introduce a two-parameter criterion based on

$$M_4 = \frac{m(s=4)}{M_\odot} \quad \text{and} \quad \mu_4 = \left[\frac{dm/M_\odot}{dR/1000\text{ km}} \right]_{s=4}, \quad (1.4)$$

where M_4 is the mass (at the onset of collapse) where the dimensionless entropy per baryon is $s = 4$, and μ_4 is the spatial derivative at the location of M_4 . This choice is motivated by the fact that, in their 1D simulations, the explosion sets shortly after M_4 has fallen through the shock and well before the shell enclosing $M_4 + 0.3 M_\odot$ has collapsed. They show that exploding models can be distinguished from non-exploding models in the μ_4 versus $M_4 \mu_4$ plane (see their Figure 6) by a linear fit

$$y(x) = k_1 x + k_2 \quad (1.5)$$

where $y(x) = \mu_4$, $x = M_4 \mu_4$, and k_1 and k_2 are numerical coefficients which depend on the model (see Table 2 of [110]). The reason of this behaviour is that μ_4 scales with the rate of mass infall from the outer layers (thus the larger μ_4 is, the lower the chance of the SN to occur), while $M_4 \mu_4$ scales with the neutrino luminosity (thus the larger $M_4 \mu_4$ is, the higher the chance of a SN explosion). Finally, [110] find that fallback is quite inefficient ($< 0.05 M_\odot$) when the SN occur.

The models proposed by O'Connor & Ott (2011, [262]) and Ertl et al. (2016, [110], see also [334, 333, 273, 111]) are sometimes referred to as the “islands of explodability” scenario, because they predict a non-monotonic behaviour of SN explosions with the stellar mass. This means, for example, that while a star with a mass $m = 25 M_\odot$ and a star with a mass $m = 29 M_\odot$ might end their life with a powerful SN explosion, another star with intermediate mass between these two (e.g. with a mass $m = 27 M_\odot$) is expected to directly collapse to a BH without SN explosion. Thus, these models predict the existence of islands of explodability, i.e. ranges of mass where a star is expected to explode, surrounded by mass intervals in which the star will end its life with a direct collapse.

The models discussed so far depend on quantities $(\xi_{2.5}, M_4, \mu_4)$ which can be evaluated no earlier than the onset of core collapse. Thus, stellar evolution models are required which integrate a massive star till the iron core has formed. This is prohibitive for most stellar evolution models (with few remarkable exceptions, e.g. FRANEC [76] and MESA [272]).

Fryer et al. (2012, [133]) propose a simplified approach (see also [132, 134, 51]). They suggest that the mass of the compact remnant depends mostly on two quantities: the carbon-oxygen core mass m_{CO} and the total final mass of the star m_{fin} . In particular, m_{CO} determines whether the star will undergo a core-collapse SN or will collapse to a BH directly (namely, stars with $m_{\text{CO}} > 11 M_\odot$ collapse to a BH directly), whereas m_{fin} determines the amount of fallback on the PNS. In this formalism, the only free parameter is the time to launch the shock. The explosion energy

is significantly reduced if the shock is launched $\gg 250$ ms after the onset of the collapse (*delayed* SN explosion) with respect to an explosion launched in the first ~ 250 ms (*rapid* SN explosion, [133]).

While this approach is quite simplified with respect to other prescriptions, [203] and [77] show that there is a strong correlation between the final carbon-oxygen mass and the compactness parameter $\xi_{2.5}$ at the onset of collapse, regardless of the rotation velocity of the progenitor star (see figure 1 of [236]). Thus, we can conclude that the simplified models by [133] can effectively describe the overall trend of a collapsing star, although they do not take into account several details of the stellar structure at the onset of collapse.

Recently, [269] propose an interesting alternative approach. They integrate a large grid of naked carbon-oxygen (CO) cores to the onset of core collapse and estimate the explodability of each model with the compactness $\xi_{2.5}$ [262] and with the parameter M_4 [111]. Naked CO cores are faster and simpler to evolve than full stellar models (with hydrogen and helium) and are less sensitive to metallicity. They made available their grid of simulations for implementation into population-synthesis codes. Other works (e.g. [82, 221, 222]) highlight the stochasticity of the final direct collapse or core-collapse SN. Finally, even the most advanced formalisms to derive the explodability of massive stars should be taken *with a grain of salt*, because of the complexity of the processes involved in core collapse SNe and because of the simplifications still included in the models [68].

Even if we were in the conditions to tell if a given star undergoes a failed SN instead of a successful SN, this would not mean we can automatically infer the final mass of the compact object. In the case of a failed SN, the main uncertainty on the final compact object mass is represented by the fate of the envelope [236]. In fact, the envelope of a massive giant star is rather loosely bound and even a small energy injection can unbind a fraction of it.

Fernandez et al. (2018, [118]) show that a $0.1 - 0.5 M_\odot$ neutrino emission during the PNS phase causes a decrease in the gravitational mass of the core, resulting in an outward going pressure wave (sound pulse) that steepens into a shock as it travels out through the star. This might cause the ejection of a fraction of the loosely bound stellar envelope. According to [118], the ejected mass is a monotonically decreasing function of the envelope compactness (Figure 6 of [118]), defined as

$$\xi_{\text{env}} \equiv \frac{M_{\text{cc}}/M_\odot}{R_{\text{cc}}/R_\odot}, \quad (1.6)$$

where M_{cc} and R_{cc} are the total mass and radius of the star at the onset of core-collapse. With this formalism, the ejected mass is up to a few M_\odot for red super-giant stars, and $1 M_\odot$ for more compact stars like blue super-giant stars and WR stars.

The possibility of a direct collapse is supported by observations. A survey conducted with the Large Binocular Telescope to find quietly disappearing stars [183, 142] reported evidence for the disappearance of a $\sim 25 M_\odot$ red super-giant star [23]. In addition, surveys of SNe indicate a dearth of red super-giant progenitors with mass $> 20 M_\odot$ associated with Type IIP SNe [182].

1.2.3 Pair instability and the mass gap

If the helium core of a star grows above $\sim 30 M_{\odot}$ and the core temperature is $\gtrsim 7 \times 10^8$ K at the end of carbon burning, the process of electron-positron pair production becomes effective. It removes photon pressure from the core producing a sudden contraction of the carbon-oxygen core, before the formation of an iron core [123, 42, 288, 375]. For $m_{\text{He}} > 135 M_{\odot}$, the contraction cannot be reversed and the star collapses directly into a BH [375]. If $135 \gtrsim m_{\text{He}} \gtrsim 64 M_{\odot}$, the collapse triggers an explosive burning of heavier elements, especially oxygen and silicon. This leads to a pair instability SN (PISN): the star is completely disrupted, leaving no compact remnant [164]. For $64 \gtrsim m_{\text{He}} \gtrsim 32 M_{\odot}$, pair production induces a series of pulsations of the core (pulsational pair instability), which trigger an enhanced mass loss [375]. At the end of this instability phase, the star finds a new equilibrium and evolves towards core-collapse: a compact object with non-zero mass is produced, less massive than we would expect without pulsational mass loss.

The main effect of (pulsational) pair instability is to open a gap in the mass spectrum of BHs between approximately $\sim 50^{+20}_{-10} M_{\odot}$ and $\sim 120 M_{\odot}$ [49, 375, 376, 321, 147, 145, 240, 239, 330, 355]. The large uncertainty on the edges of the mass gap is due to our poor understanding of the physics of massive stars. In particular, [114] and [113] integrate pure-He stars with MESA [270, 271, 272] and show that the uncertainties on the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate [92] are responsible for a change in the lower-edge of the mass gap of more than $\sim 15 M_{\odot}$.

Mapelli et al. (2020, [236]) show that assuming that the hydrogen envelope collapses to the final BH can lead to an increase of the lower-edge of the mass gap from $\sim 40 M_{\odot}$ to $\sim 65 M_{\odot}$. Finally, [86] combine the uncertainties on the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate with the uncertainties on the collapse of the hydrogen envelope and find that the mass gap progressively reduces from $\sim 80 - 150 M_{\odot}$ (for a rate computed with the standard $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate -1σ) to $\sim 92 - 110 M_{\odot}$ (for a standard rate -2σ) and even disappears (for a standard rate -3σ) as an effect of convection and envelope undershooting. These uncertainties leave open the possibility that the primary mass of GW190521 is the result of stellar evolution [86, 52, 340, 339].

1.2.4 The mass of compact remnants

The previous Sections suggest that our knowledge of the compact remnant mass is hampered by severe uncertainties, connected with both stellar winds and SNe. Thus, models of the mass spectrum of compact remnants must be taken *with a grain of salt*. However, a few robust features can be drawn.

Figure 1.2 is a simplified version of Figures 2 and 3 of Heger et al. (2003 [163]). The final mass of a star and the mass of the compact remnant are shown as a function of the ZAMS mass. The left and the right-hand panels show the case of solar metallicity and metal-free stars, respectively. In the case of solar metallicity stars, the final mass of the star is much lower than the initial one, because stellar winds

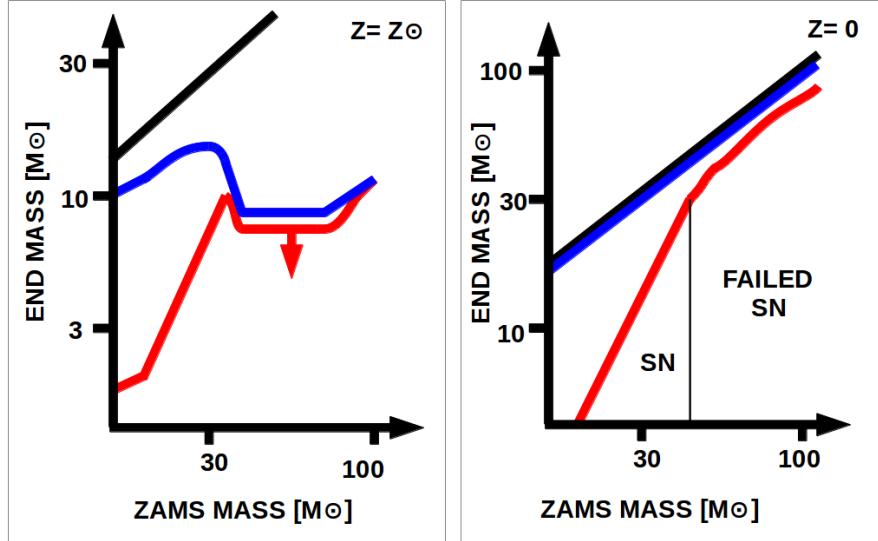


Fig. 1.2 Final mass of a star (m_{fin} , blue lines) and mass of the compact remnant (m_{rem} , red lines) as a function of the ZAMS mass of the star. The thick black line marks the region where $m_{\text{fin}} = m_{\text{ZAMS}}$. Left-hand panel: solar metallicity stars. Right-hand panel: metal-free stars. The red arrow on the left-hand panel is an upper limit for the remnant mass. Vertical thin black line in the right-hand panel: approximate separation between successful and failed SNe at $Z = 0$. This cartoon was inspired by Figures 2 and 3 of Heger et al. (2003 [163]).

are extremely efficient. The mass of the compact remnant is also much lower than the final mass of the star because a core-collapse SN always takes place.

In contrast, a metal-free star (i.e., a population III star) loses a negligible fraction of its mass by stellar winds (the blue and the black line in Figure 1.2 overlap). As for the mass of the compact remnant, Figure 1.2 shows that there are two regimes: below a given threshold ($\approx 30 - 40 M_{\odot}$) the SN explosion succeeds even at zero metallicity and the mass of the compact remnant is relatively small. Above this threshold, the mass of the star (in terms of both core mass and envelope mass) is sufficiently large that the SN fails. Most of the final stellar mass collapses to a BH, whose mass is significantly larger than in the case of a SN explosion. The only exception is represented by the pair instability window: single metal-free stars with ZAMS mass $m_{\text{ZAMS}} \sim 140 - 260 M_{\odot}$ undergo a PISN and are completely destroyed, while single metal-free stars with $m_{\text{ZAMS}} \sim 110 - 140 M_{\odot}$ undergo pulsational pair instability and leave smaller compact objects. In this simplified cartoon, we neglect the existence of islands of explodability.

What happens at intermediate metallicity between solar and zero? Predicting what happens to a metal-free star is relatively simple, because its evolution does not depend on the interplay between metals and stellar winds. The fate of a solar metallicity star is more problematic, because we must account for line-driven stellar

winds, but most observational data about stellar winds are for nearly solar metallicity stars and allow us to calibrate our models for such high metallicity. In contrast, modelling intermediate metallicities is significantly more complicated, because the details depend on the interplay between metals and stellar winds and only limited data are available for calibration (mostly data for the Large and Small Magellanic Clouds, e.g. [307]).

As a rule of thumb (see e.g. [133, 322]), we can draw the following considerations. If the zero-age main sequence (ZAMS) mass of a star is large ($m_{\text{ZAMS}} \gtrsim 30 M_{\odot}$), then the amount of mass lost by stellar winds is the main effect which determines the mass of the compact remnant. At low metallicity ($\lesssim 0.1 Z_{\odot}$) and for a low Eddington factor ($\Gamma_e < 0.6$), mass loss by stellar winds is not particularly large. Thus, the final mass m_{fin} and the carbon-oxygen mass m_{CO} of the star may be sufficiently large to avoid a core-collapse SN explosion: the star may form a massive BH ($\gtrsim 20 M_{\odot}$) by direct collapse, unless a pair-instability or a pulsational-pair instability SN occurs. At high metallicity ($\approx Z_{\odot}$) or large Eddington factor ($\Gamma_e > 0.6$), mass loss by stellar winds is particularly efficient and may lead to a small m_{fin} and m_{CO} : the star is expected to undergo a core-collapse SN and to leave a relatively small remnant.

If the ZAMS mass of a star is relatively low ($8 < m_{\text{ZAMS}} < 30 M_{\odot}$), then stellar winds are not important (with the exception of asymptotic giant branch stars), regardless of the metallicity. In this case, the details of the SN explosion (e.g. energy of the explosion and amount of fallback) are crucial to determine the final mass of the remnant. This general sketch may be affected by several factors, such as pair-instability SNe, pulsational pair-instability SNe (e.g. [375]) and an *island scenario* for core-collapse SNe (e.g. [110]).

The effect of pair-instability and pulsational pair-instability SNe is clearly shown in Figure 1.3. The top panel was obtained accounting only for stellar evolution and core-collapse SNe. In contrast, the bottom panel also includes pair-instability and pulsational pair-instability SNe. This figure shows that the mass of the compact remnant strongly depends on the metallicity of the progenitor star if $m_{\text{ZAMS}} \gtrsim 30 M_{\odot}$. In most cases, the lower the metallicity of the progenitor is, the larger the maximum mass of the compact remnant [163, 225, 48, 232, 233, 322, 321]. However, for metal-poor stars ($Z < 10^{-3}$) with ZAMS mass $230 > m_{\text{ZAMS}}/M_{\odot} > 110$ pair instability SNe lead to the complete disruption of the star and no compact remnant is left. Only very massive ($m_{\text{ZAMS}} > 230 M_{\odot}$) metal-poor ($Z < 10^{-3}$) stars can collapse to a BH directly, producing intermediate-mass BHs (i.e., BHs with mass $\gtrsim 100 M_{\odot}$).

If $Z < 10^{-3}$ and $110 > m_{\text{ZAMS}} \gtrsim 60 M_{\odot}$, the star enters the pulsational pair-instability SN regime: mass loss is enhanced and the final BH mass is smaller ($m_{\text{BH}} \sim 30 - 55 M_{\odot}$, bottom panel of Fig. 1.3) than we would have expected from direct collapse ($m_{\text{BH}} \sim 50 - 100 M_{\odot}$, top panel of Fig. 1.3).

Finally, the mass spectrum of relatively low-mass stars ($8 < m_{\text{ZAMS}} < 30 M_{\odot}$) is not significantly affected by metallicity. The assumed core-collapse SN model is the most important factor in this mass range [133].

The models presented here do not take into account stellar rotation. Studied in [203, 236, 239], stellar rotation produces larger cores because of chemical mixing.

This shifts the minimum ZAMS mass for a star to undergo pulsational pair instability and PISN to lower values, because of the larger He core masses. The net result is a decrease of the maximum BH mass for fast-rotating stars ($\geq 150 \text{ km s}^{-1}$) with respect to low-rotating stars [236].

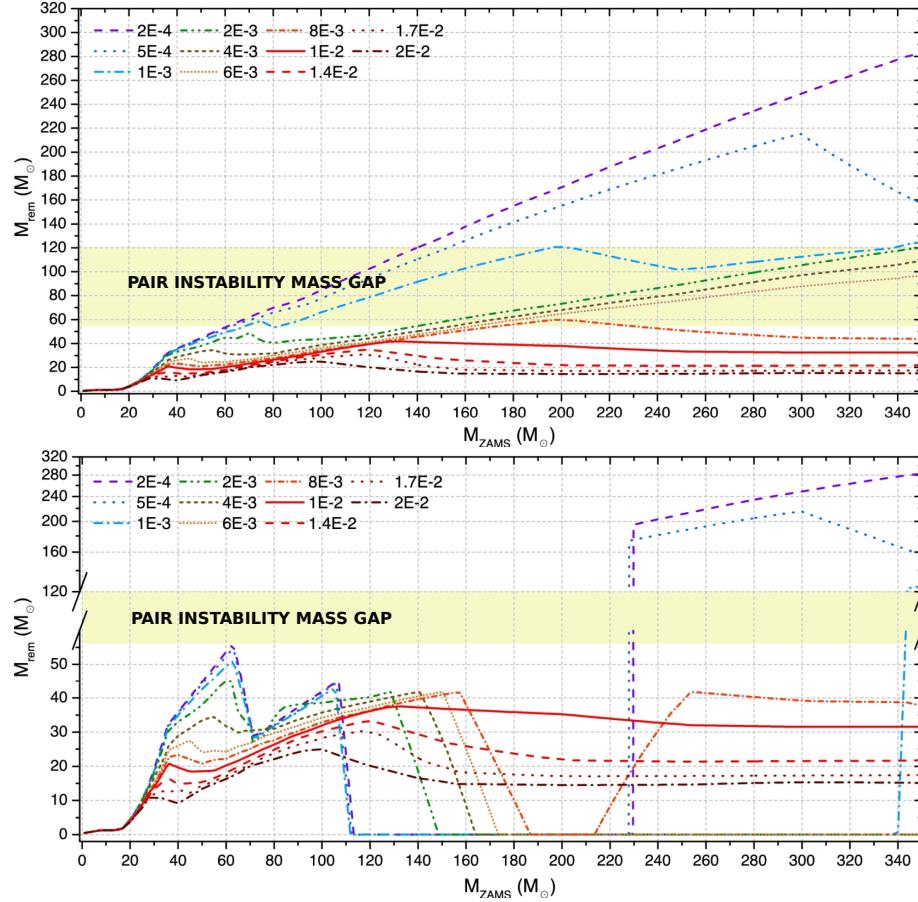


Fig. 1.3 Mass of the compact remnant (m_{rem}) as a function of the ZAMS mass of the star (m_{ZAMS}). Lower (upper) panel: pulsational pair-instability and pair-instability SNe are (are not) included. In both panels: dash-dotted brown line: $Z = 2.0 \times 10^{-2}$; dotted dark orange line: $Z = 1.7 \times 10^{-2}$; dashed red line: $Z = 1.4 \times 10^{-2}$; solid red line: $Z = 1.0 \times 10^{-2}$; short dash-dotted orange line: $Z = 8.0 \times 10^{-3}$; short dotted light orange line: $Z = 6.0 \times 10^{-3}$; short dashed green line: $Z = 4.0 \times 10^{-3}$; dash-double dotted green line: $Z = 2.0 \times 10^{-3}$; dash-dotted light blue line: $Z = 1.0 \times 10^{-3}$; dotted blue line: $Z = 5.0 \times 10^{-4}$; dashed violet line: $Z = 2.0 \times 10^{-4}$. A delayed core-collapse SN mechanism has been assumed, following the prescriptions of [133]. This Figure was adapted from Figures 1 and 2 of Spera & Mapelli (2017, [321]).

1.2.5 Compact object spins

In the previous Sections, we have seen that the connection between the mass of a BH and the properties of its progenitor star is still highly uncertain. Our knowledge on the origin of BH spins is even more uncertain. It is reasonable to assume that a compact object inherits the spin of its progenitor (or at least of the core of its progenitor) if the progenitor collapses to a BH directly, without any SN explosion. In contrast, mass ejection during a SN explosion can significantly dissipate part of the final spin of the progenitor star. Hence, if the final spin of the progenitor star is not negligible, we would expect large birth spins for the most massive BHs, which form from direct collapse, and low birth spins for NSs and light BHs, which form from successful SN explosions.

Observations of Galactic pulsars seem to confirm the idea that NSs are born with relatively low χ . Young pulsars (i.e., pulsars which did not have much time to slow down or to be recycled) have a value of the spin parameter $\chi \lesssim 0.01$ [185, 219]. This indicates that most of the spin of the progenitor was lost either before or during the SN. In contrast, millisecond pulsars are significantly spun up by mass accretion from a companion star: the fastest millisecond pulsar (B1937+21, [166]) has $\chi \lesssim 0.4$, but this value has almost nothing to do with the initial spin of the NS.

On the other hand, the observed spins of BHs make us “scratch our head”. Relatively low-mass BHs in high-mass X-ray binaries seem to show very large spins [152, 153, 204, 293, 249]. Considering that these BHs did not have enough time to spin up by mass accretion (for the short lifetime of their companion stars), this suggests large spins at birth. In contrast, LIGO–Virgo results support low values of χ_{eff} for most of the BBHs [7], which can be interpreted either as strongly misaligned spins with respect to the binary angular momentum or as very low values of χ .

From a theoretical perspective, the key question is then: what is the final spin of the progenitors of BHs and NSs? The final spin of a massive single star depends on mass loss and angular momentum transport. A mildly efficient transport by meridional currents (as adopted in, e.g., [106, 203]), leads to a non-negligible final spin of the star and to a high spin of the BH, if born from direct collapse [52, 236]. In contrast, [136] investigate efficient angular momentum transport via the magnetic Tayler-Spruit instability [327, 137] and find extremely slow spins ($\chi \sim 10^{-2}$) for BHs born from single stars.

If the magnetic Tayler-Spruit instability is the dominant process for angular momentum transfer in massive stars, BH spins larger than $\chi \sim 10^{-2}$ can be produced only by tidal torques in binary stars [195, 285, 45, 47] or by chemically homogeneous evolution [216, 377, 382]. Overall, the initial rotation speed of very massive stars and the process of angular momentum transfer remain uncertain, hampering the predictive power of theoretical models on the spin distribution of BHs.

1.2.6 Natal kicks

Compact objects are expected to receive a natal kick from the parent SN explosion, because of asymmetries in the neutrino flux and/or in the ejecta (see [178] for a review). The natal kick has a crucial effect on the evolution of a binary compact object, because it can either unbind the binary or change its orbital properties. For example, a SN kick can increase the orbital eccentricity or misalign the spins of the two members of the binary.

Unfortunately, it is extremely difficult to quantify natal kicks from state-of-the-art SN simulations and measurements of natal kicks are scanty, especially for BHs. As to NSs, indirect observational estimates of SN kicks give contrasting results. Hobbs et al. (2005, [169]) found that a single Maxwellian with root mean square $\sigma_{\text{CCSN}} = 265 \text{ km s}^{-1}$ can match the proper motions of 73 young single pulsars in the Milky Way. Other works suggest a bimodal velocity distribution, with a first peak at low velocities (e.g. $\sim 0 \text{ km s}^{-1}$ according to [131] or $\sim 90 \text{ km s}^{-1}$ according to [34]) and a second peak at high velocities ($> 600 \text{ km s}^{-1}$ according to [131] or $\sim 500 \text{ km s}^{-1}$ for [34]). Similarly, Verbunt et al. (2017, [357]) indicate that a double Maxwellian distribution provides a significantly better fit to the observed velocity distribution than a single Maxwellian. Finally, the analysis of Beniamini & Piran (2016, [55]) shows that low kick velocities ($\lesssim 30 \text{ km s}^{-1}$) are required to match the majority of Galactic BNSs, especially those with low eccentricity.

A possible interpretation of these observational results is that natal kicks depend on the SN mechanism (e.g. electron-capture versus core-collapse SN, e.g. [146]) or on the binarity of the NS progenitor. For example, if the NS progenitor evolves in a close binary system (i.e., in a binary system where the two stars have exchanged mass with each other, see Section 1.3.1) it might undergo an ultra-stripped SN (see [341] and references therein for more details). A star can undergo an ultra-stripped SN explosion only if it was heavily stripped by mass transfer to a companion [342, 343]. The natal kick of an ultra-stripped SN should be low [343], because of the small mass of the ejecta ($\lesssim 0.1 M_\odot$). Low kicks ($\lesssim 50 \text{ km s}^{-1}$) for ultra-stripped core-collapse SNe are also confirmed by recent hydrodynamical simulations [335, 177].

As to BHs, the only indirect measurements of natal kicks arise from spatial distributions, proper motions and orbital properties of BHs in X-ray binaries (e.g. [250]). Evidence for a relatively small natal kick has been found for both GRO J1655–40 [372] and Cygnus X-1 [374], whereas H 1705–250 [291, 292] and XTE J1118 + 480 [251, 128] require high kicks ($> 100 \text{ km s}^{-1}$). By analysing the position of BHs in X-ray binaries with respect to the Galactic plane, Repetto et al. (2012, [291]) suggest that BH natal kicks should be as high as NS kicks. Repetto et al. (2017, [292]) perform a similar analysis but accounting also for binary evolution, and find that at least two BHs in X-ray binaries (H 1705–250 and XTE J1118 + 480) require high kicks.

Most models of BBH evolution assume that natal kicks of BHs are drawn from the same distribution as NS kicks, but reduced by some factor. For example, linear momentum conservation suggests that

$$v_{\text{BH}} = \frac{\langle m_{\text{NS}} \rangle}{m_{\text{BH}}} v_{\text{NS}}, \quad (1.7)$$

where v_{BH} is the natal kick of a BH with mass m_{BH} , $\langle m_{\text{NS}} \rangle \approx 1.33 M_{\odot}$ is the average mass of NSs [267] and v_{NS} is randomly drawn from the NS kick distribution (e.g., from a single or double Maxwellian).

Alternatively, the natal kick can be reduced by the amount of fallback, under the reasonable assumption that fallback quenches the initial asymmetries. Following [133],

$$v_{\text{BH}} = (1 - f_{\text{fb}}) v_{\text{NS}}, \quad (1.8)$$

where f_{fb} quantifies the fallback ($f_{\text{fb}} = 0$ for no fallback and $f_{\text{fb}} = 1$ for direct collapse). Most studies assume that BHs born from direct collapse receive no kick [133].

The model by [148] can unify BH kicks and NS kicks, naturally accounting for ultra-stripped SNe and electron-capture SNe, which mostly lead to reduced kicks (e.g., [335, 341]). According to this toy model, the kick of a compact object can be described as

$$v_k = f_{\text{H05}} \frac{\langle m_{\text{NS}} \rangle}{m_{\text{rem}}} \frac{m_{\text{ej}}}{\langle m_{\text{ej}} \rangle}, \quad (1.9)$$

where m_{rem} is the mass of the considered compact remnant, m_{ej} is the mass of the ejecta, $\langle m_{\text{ej}} \rangle$ is the ejecta mass of a SN that leaves a single NS with mass $\langle m_{\text{NS}} \rangle$, and f_{H05} is a number randomly drawn from a Maxwellian probability density curve with one-dimensional root-mean square velocity dispersion $\sigma_{\text{1D}} = 265 \text{ km s}^{-1}$ [169]. To derive this model, [148] assume that the Maxwellian distribution fitted by [169] is a good proxy to the distribution of the kicks of NSs born from single stars and that kicks are the effect of asymmetries in mass ejecta ($\propto m_{\text{ej}}$, see also [65, 66, 337]), modulated by linear momentum conservation ($\propto m_{\text{rem}}^{-1}$). This simple formalism seems to solve the tension between the local merger rate of BNSs derived from gravitational-wave interferometers [7, 6] and the proper motions of Galactic young pulsars [169].

In summary, natal kicks are one of the most debated issues about compact objects. Their actual amount has dramatic implications on the merger rate and on the properties (spin and mass distribution) of merging compact objects.

1.3 Binaries of stellar black holes

Naively, one could think that if two massive stars are members of a binary system, they will eventually become a BBH and the mass of each BH will be the same as if its progenitor star was a single star. This is true only if the binary system is sufficiently wide (detached binary) for its entire evolution. If the binary is tight enough, it will evolve through several processes which might significantly change its final fate.

The so-called binary population-synthesis codes have been used to investigate the effect of binary evolution processes on the formation of BBHs in isolated binaries (e.g. [283, 172, 277, 51, 233, 245, 107, 109, 228, 329, 147, 43, 145, 146, 188, 108, 323, 148, 340]). These are semi-analytic codes which combine a description of stellar evolution with prescriptions for SN explosions and with a formalism for binary evolution processes. In the following, we mention some of the most important binary-evolution processes and we briefly discuss their treatment in the most used population-synthesis codes.

1.3.1 Mass transfer

If two stars exchange matter to each other, it means they undergo a mass transfer episode. This might be driven either by stellar winds or by an episode of Roche-lobe filling. When a massive star loses mass by stellar winds, its companion might be able to capture some of this mass. This will depend on the amount of mass which is lost and on the relative velocity of the wind with respect to the companion star. Based on the Bondi & Hoyle (1944, [61]) formalism, Hurley et al. (2002, [172]) describe the mean mass accretion rate by stellar winds as

$$\dot{m}_2 = \frac{1}{\sqrt{1-e^2}} \left(\frac{Gm_2}{v_w^2} \right)^2 \frac{\alpha_w}{2a^2} \frac{1}{[1+(v_{\text{orb}}/v_w)^2]^{3/2}} |\dot{m}_1|, \quad (1.10)$$

where e is the binary eccentricity, G is the gravitational constant, m_2 is the mass of the accreting star, v_w is the velocity of the wind, $\alpha_w \sim 3/2$ is an efficiency constant, a is the semi-major axis of the binary, $v_{\text{orb}} = \sqrt{G(m_1+m_2)/a}$ is the orbital velocity of the binary (m_1 being the mass of the donor), and $|\dot{m}_1|$ is the mass loss rate by the donor. Since $|\dot{m}_1|$ is usually quite low ($|\dot{m}_1| < 10^{-3} M_\odot \text{ yr}^{-1}$) and v_w is usually quite high ($> 1000 \text{ km s}^{-1}$ for a line-driven wind) with respect to the orbital velocity, this kind of mass transfer is usually rather inefficient. However, most of the observed high-mass X-ray binaries [352, 353] and in particular all the systems with a WR star companion [112] are wind-fed systems.

Mass transfer by Roche lobe overflow is usually more efficient than wind accretion. The Roche lobe of a star in a binary system is a teardrop-shaped equipotential surface surrounding the star. The Roche lobes of the two members of the binary are connected in just one point, which is the Lagrangian L1 point. A widely used approximate formula for the Roche lobe is [104]

$$R_{L,1} = a \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1+q^{1/3})}, \quad (1.11)$$

where a is the semi-major axis of the binary and $q = m_1/m_2$ (m_1 and m_2 are the masses of the two stars in the binary). This formula describes the Roche lobe of a star with mass m_1 , while the corresponding Roche lobe of a star with mass m_2 ($R_{L,2}$)

is obtained by swapping the subscripts. A star overfills (underfills) its Roche lobe when its radius is larger (smaller) than the Roche lobe. If a star overfills its Roche lobe, a part of its mass flows toward the companion star which can accrete (a part of) it.

Mass transfer obviously changes the mass of the two stars in a binary, and thus the final mass of the compact remnants of such stars, but also the orbital properties of the binary. If mass transfer is non conservative, which is the most realistic case, it leads to an angular momentum loss, which in turn affects the semi-major axis. Recently, [63] show that the hypothesis of a highly non-conservative mass transfer (mass accretion efficiency $f_{\text{MT}} \leq 0.5$) is in tension with LVC data, if we assume that all BBHs observed by the LVC form via isolated binary evolution.

A crucial information about Roche lobe overflow is whether it is stable or unstable and on which timescale. The most commonly used approach can be described as follows [369, 283, 348, 172, 105]. Let us assume that the stellar radius and mass are connected by a simple relation $R \propto m^{\zeta}$. Thus, the variation of the donor's radius during Roche lobe is

$$\frac{dR_1}{dt} = \frac{\partial R_1}{\partial t} + \zeta \frac{R_1}{m_1} \frac{dm_1}{dt}. \quad (1.12)$$

In the above equation, the term $\frac{\partial R_1}{\partial t}$ is due to nuclear burning, while the term with ζ measures the adiabatic or thermal response of the donor star to mass loss. Note that $\frac{dm_1}{dt}$ is the mass loss from the donor; hence it is always negative.

Similarly, the change of the size of the Roche lobe of the donor $R_{L,1}$ can be estimated as

$$\frac{dR_{L,1}}{dt} = \frac{\partial R_{L,1}}{\partial t} + \zeta_L \frac{R_{L,1}}{m_1} \frac{dm_1}{dt}, \quad (1.13)$$

where $\frac{\partial R_{L,1}}{\partial t}$ depends on tides and GW radiation, while ζ_L describes the response of the Roche lobe to mass loss: the Roche lobe might shrink or expand. If $\zeta_L > \zeta$, the Roche lobe shrinks faster than the radius of the star does and the mass transfer is unstable, otherwise it remains stable until the radius changes significantly by nuclear burning.

Mass transfer can be unstable either on a dynamical timescale (if ζ describes the adiabatic response of the donor and $\zeta < \zeta_L$) or on a thermal timescale (if ζ describes the thermal response of the donor and $\zeta < \zeta_L$). If mass transfer is dynamically unstable or both stars overfill their Roche lobe, then the binary is expected to merge – if the donor lacks a steep density gradient between the core and the envelope –, or to enter common envelope (CE) – if the donor has a clear distinction between core and envelope.

1.3.2 Common envelope (CE)

If two stars enter CE, their envelope(s) stop co-rotating with their cores. The two stellar cores (or the compact object and the core of the companion star, if the binary

is already single degenerate) are embedded in the same non-corotating envelope and start spiralling in as an effect of gas drag exerted by the envelope. Part of the orbital energy lost by the cores as an effect of this drag is likely converted into heating of the envelope, making it more loosely bound. If this process leads to the ejection of the envelope, then the binary survives, but the post-CE binary is composed of two naked stellar cores (or of a compact object and a naked stellar core). Moreover, the orbital separation of the two cores (or the orbital separation of the compact object and the stellar core) is considerably smaller than the initial orbital separation before the CE, as an effect of the spiral in². This circumstance is crucial for the fate of a BBH. In fact, if the binary which survives a CE phase evolves into a BBH, this BBH will have a short semi-major axis ($a \lesssim 100 R_\odot$), much shorter than the sum of the maximum radii of the progenitor stars, and may be able to merge by GW emission within a Hubble time.

In contrast, if the envelope is not ejected, the two cores (or the compact object and the core) spiral in till they eventually merge. This premature merger of a binary during a CE phase prevents the binary from evolving into a BBH. The cartoon in Figure 1.4 summarizes these possible outcomes.

The α formalism [368] is the most common formalism adopted to describe a common envelope. The basic idea of this formalism is that the energy needed to unbind the envelope comes uniquely from the loss of orbital energy of the two cores during the spiral in. The fraction of the orbital energy of the two cores which goes into unbinding the envelope can be expressed as

$$\Delta E = \alpha (E_{b,f} - E_{b,i}) = \alpha \frac{G m_{c1} m_{c2}}{2} \left(\frac{1}{a_f} - \frac{1}{a_i} \right), \quad (1.14)$$

where $E_{b,i}$ ($E_{b,f}$) is the orbital binding energy of the two cores before (after) the CE phase, a_i (a_f) is the semi-major axis before (after) the CE phase, m_{c1} and m_{c2} are the masses of the two cores, and α is a dimensionless parameter that measures which fraction of the removed orbital energy is transferred to the envelope. If the primary is already a compact object (as in Figure 1.4), m_{c2} is the mass of the compact object.

The binding energy of the envelope is

$$E_{\text{env}} = \frac{G}{\lambda} \left[\frac{m_{\text{env},1} m_1}{R_1} + \frac{m_{\text{env},2} m_2}{R_2} \right], \quad (1.15)$$

where m_1 and m_2 are the masses of the primary and the secondary member of the binary, $m_{\text{env},1}$ and $m_{\text{env},2}$ are the masses of the envelope of the primary and the secondary member of the binary, R_1 and R_2 are the radii of the primary and the secondary member of the binary, and λ is the parameter (or the function) which

² A short-period (from a few hours to a few days) binary system composed of a naked helium core and BH might be observed as an X-ray binary, typically a WR X-ray binary. In the local Universe, we know a few (~ 7) WR X-ray binaries, in which a compact object (BH or NS) accretes mass through the wind of the naked stellar companion (see e.g. [112] for more details). These rare X-ray binaries are thought to be good progenitors of merging compact-object binaries.

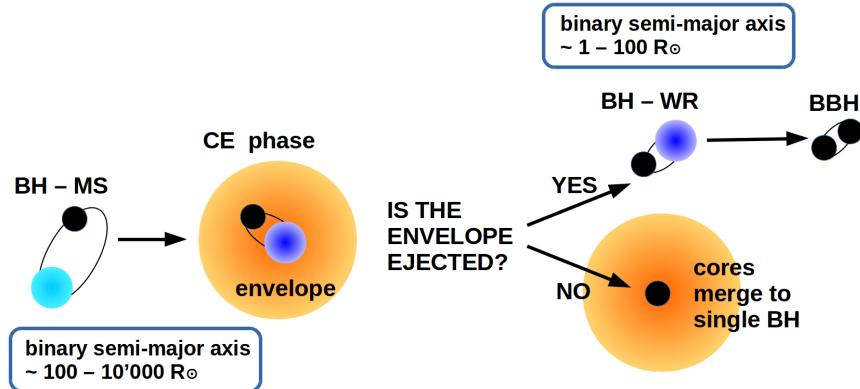


Fig. 1.4 Schematic representation of the evolution of a BBH through CE. The companion of the BH is initially in the main sequence (MS). In the cartoon, the BH is indicated by a black circle, while the MS companion is indicated by the light blue circle. When the companion evolves off the MS, becoming a giant star, it overfills its Roche lobe. The BH and the giant star enter a CE (the CE is indicated in orange, while the core of the giant is represented by the dark blue circle). The core of the giant and the BH spiral in because of the gas drag exerted by the envelope. If the envelope is ejected, we are left with a new binary, composed of the BH and the naked helium core of the giant. The new binary has a much smaller orbital separation than the initial binary. If the naked helium core becomes a BH and its natal kick does not disrupt the binary, then a BBH is born, possibly with a small semi-major axis. In contrast, if the envelope is not ejected, the BH and the helium core spiral in, till they merge together. A single BH is left in this case.

measures the concentration of the envelope (the smaller λ is, the more concentrated is the envelope).

By imposing $\Delta E = E_{\text{env}}$ we can derive the value of the final semi-major axis a_f for which the envelope is ejected. This means that the larger (smaller) α is, the larger (smaller) the final orbital separation. If the resulting a_f is lower than the sum of the radii of the two cores (or than the sum of the Roche lobe radii of the cores), then the binary will merge during CE, otherwise the binary survives.

Actually, we have known for a long time (see [175] for a review) that this simple formalism is a poor description of the physics of CE, which is considerably more complicated. A healthy treatment of CE should take into account not only the orbital energy of the cores and the binding energy of the envelope, but also i) the thermal energy of the envelope, which is the sum of radiation energy and kinetic energy of gas particles [161], ii) the recombination energy (as the envelope expands it cools down, the plasma recombines and some atoms even form molecules, releasing binding energy, [189]), iii) tidal heating/cooling from stellar spin down/up [175], iv) nuclear fusion energy [176], v) the enthalpy of the envelope [174], and vi) the accretion energy, which might drive outflows and jets [319, 213, 212, 320, 207, 89].

Moreover, the envelope concentration parameter λ cannot be the same for all stars. It is expected to vary wildly not only from star to star but also during different evolutionary stages of the same star. Several authors [379, 206] have estimated E_{env} directly from their stellar models, significantly improving this formalism. However, even in this case, we cannot get rid of the α parameter.

Thus, it is essential to model the physics of CE with analytic models and numerical simulations. A lot of effort has been put on this in the last few years, but the problem remains largely unconquered. Several recent studies investigate the onset of CE, when an unstable mass transfer prevents the envelope from co-rotating with the core and leads to the plunge-in of the companion inside the envelope [210, 211, 208, 209, 358].

Several hydrodynamical simulations model the fast spiral in phase after plunge-in [294, 295, 268, 263], when the two cores spiral in on a dynamical time scale (≈ 100 days). At the end of this dynamical spiral in only a small fraction of the envelope ($\sim 25\%$, [263]) appears to be ejected in most simulations. When the two cores are sufficiently close that they are separated only by a small gas mass, the spiral in slows down and the system evolves on the Kelvin-Helmholtz timescale of the envelope ($\approx 10^{3-5}$ years).

Simulating the system for a Kelvin-Helmholtz timescale is prohibitive for current three-dimensional simulations (e.g., [198]). Fragos et al. (2019, [129]) reduce the complexity of the problem by simulating the entire CE evolution in just one dimension, with the hydrodynamic stellar evolution code MESA [270, 271, 272]. They evolve a binary system composed of a NS and a $12 M_{\odot}$ red super-giant star for a thermal timescale. In their model, envelope ejection is mostly driven by the thermal energy of the envelope and by the orbital energy, while recombination contributes to $\lesssim 10\%$ of the total energy required to eject the envelope. The final system is a NS – naked helium star system with an orbital separation of a few R_{\odot} , i.e. a good progenitor for a merging BNS. Their simulated system can be reproduced by the α formalism if $\alpha \approx 5$. The apparently unphysical value of $\alpha > 1$ is motivated by the fact that orbital energy is only one of the energy sources that participate in envelope ejection.

1.3.3 Alternative evolution to CE

Massive fast rotating stars can have a chemically homogeneous evolution (CHE): they do not develop a chemical composition gradient because of the mixing induced by rotation. This is particularly true if the star is metal poor, because stellar winds are not efficient in removing angular momentum. If a binary is very tight, the spins of its members are even increased during stellar life, because of tidal synchronisation. The radii of stars following CHE are usually much smaller than the radii of stars developing a chemical composition gradient [91, 220]. This implies that even very tight binaries (few tens of solar radii) can avoid CE.

Marchant et al. (2016, [238]) simulate tight binaries whose components are fast rotating massive stars. A number of their simulated binaries evolve into contact binaries where both binary components fill and even overfill their Roche volumes. If metallicity is sufficiently low and rotation sufficiently fast, these binaries may evolve as “over-contact” binaries: the over-contact phase differs from a classical CE phase because co-rotation can, in principle, be maintained as long as material does not overflow the L2 point. This means that spiral-in can be avoided, resulting in a stable system evolving on a nuclear timescale.

Such over-contact binaries maintain relatively small stellar radii during their evolution (few ten solar radii) and may evolve into a BBH with a very short orbital period. This scenario predicts the formation of merging BHs with relatively large masses ($> 20 M_{\odot}$), nearly equal mass ($q = 1$), and with large aligned spins (i.e., large χ_{eff}). While a large positive χ_{eff} is not consistent with most of the LIGO–Virgo BBHs, the CHE model is a viable alternative to explain systems with large and aligned spins. Moreover, mass loss during the naked helium core phase might reduce the progenitor’s spin and lead to lower BH spins than predicted by early models [296].

1.3.4 BBH spins in the isolated binary evolution model

Most evolutionary processes in an isolated binary star (tides, mass transfer, CE) lead to the alignment of the spins of the components to the orbital angular momentum of the binary [172]. The only³ evolutionary process that can significantly misalign BH spins with respect to the orbital angular momentum is the SN explosion [172, 144, 299, 143, 52].

Under the assumption that the system has no time to re-align spins between the first and the second SN and that SNe do not change the direction of compact-object spins but only the direction of the orbital angular momentum of the binary, we can derive the angle between the direction of the spins of the two compact objects and that of the orbital angular momentum of the binary system as [144, 299]

$$\cos \theta = \cos(\nu_1) \cos(\nu_2) + \sin(\nu_1) \sin(\nu_2) \cos(\phi), \quad (1.16)$$

where ν_i (with $i = 1, 2$) is the angle between the new (\mathbf{L}_{new}) and the old (\mathbf{L}) orbital angular momentum after a SN ($i = 1$ corresponding to the first SN, $i = 2$ corresponding to the second SN), so that $\cos(\nu) = \hat{\mathbf{L}}_{\text{new}} \cdot \hat{\mathbf{L}}_{\text{old}}$, while ϕ is the phase of the projection of $\hat{\mathbf{L}}$ into the orbital plane. As shown by [299], the most commonly adopted SN kick models fail producing a significant misalignment.

Hence, we expect that the isolated binary evolution model has a preference for BH spins aligned with the orbital angular momentum of the binary, especially in the case of massive BHs which undergo just a failed SN.

³ Stegmann & Antonini (2021, [328]) recently proposed a possible spin flip mechanism during mass transfer.

1.3.5 Summary of the isolated binary formation channel

In this Section, we have highlighted the most important aspects and the open issues of the *isolated binary formation scenario*, i.e. the model which predicts the formation of merging BHs through the evolution of isolated binaries. For isolated binaries we mean stellar binary systems which are not perturbed by other stars or compact objects.

To summarize, let us illustrate schematically the evolution of an isolated stellar binary (see e.g. [50, 228, 329, 147]) which can give birth to merging BHs like GW150914 and the other massive BHs observed by the LVC [5, 4, 3, 10, 8, 7]. In the following discussion, several details of stellar evolution have been simplified or skipped to facilitate the reading for non specialists.

Figure 1.5 shows the evolution of an isolated binary system composed of two massive stars. These stars are gravitationally bound since their birth. Initially, the two stars are both on the main sequence (MS). When the most massive one leaves the MS, its radius starts inflating and can grow by a factor of a hundreds. The most massive star becomes a giant star with a helium core and a large hydrogen envelope. If its radius equals the Roche lobe (equation 1.11), the system starts a stable mass-transfer episode. Some mass is lost by the system, some is transferred to the companion. After several additional evolutionary stages, the primary collapses to a BH. A direct collapse is preferred with respect to a SN explosion if we want the BH mass to be $> 20 M_{\odot}$ (as most BBHs observed by the LVC) and if we want the binary to remain bound (direct collapse implies almost no kick, apart from neutrino mass loss). At this stage, the semi-major axis of the system is still quite large: hundreds to thousands of solar radii.

When the secondary also leaves the MS, growing in radius, the system enters a CE phase: the BH and the helium core spiral in. If the orbital energy and the thermal energy of the envelope are not sufficient to unbind the envelope, then the BH merges with the helium core leaving a single BH. In contrast, if the envelope is ejected, we are left with a new binary system, composed of the BH and of a stripped naked helium star. The new binary system has a much smaller orbital separation (tens of solar radii) than the pre-CE binary, because of the spiral in. If this new binary system remains bound after the naked helium star undergoes a SN explosion or if the naked helium star is sufficiently massive to directly collapse to a BH, the system evolves into a tight BBH, which might merge within a Hubble time.

The most critical quantities in this scenario are the masses of the two stars and also their initial orbital separation (with respect to the stellar radii): a BBH can merge within a Hubble time only if its initial orbital separation is tremendously small (few tens of solar radii, unless the eccentricity is rather extreme, [274]), but a massive star ($> 20 M_{\odot}$) can reach a radius of several thousand solar radii during its evolution. Thus, if the initial orbital separation of the binary star is tens of solar radii, the binary system merges before it can become a BBH. On the other hand, if the initial orbital separation is too large, the two BHs will never merge. In this scenario, the two BHs can merge only if the initial orbital separation of the progenitor binary star is in the range which allows the binary to enter CE and then to leave

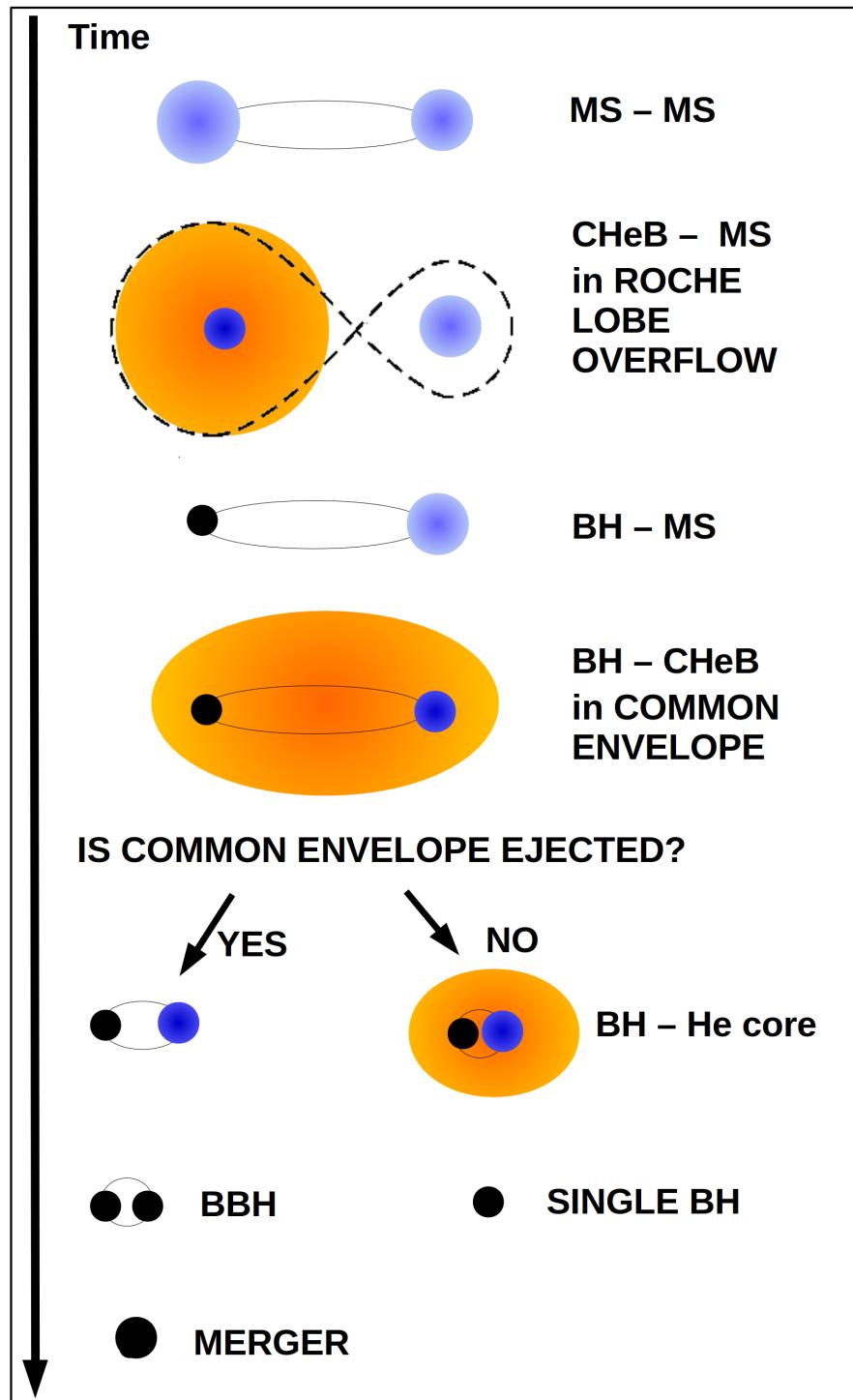


Fig. 1.5 Schematic evolution of an isolated binary star which can give birth to a BBH merger (see e.g. [50, 228, 329, 147]).

a short period BBH. This range of initial orbital separations dramatically depends on CE efficiency and on the details of stellar mass and radius evolution. Two possible alternatives to CE are very efficient stable mass transfer [145, 46] and CHE [238, 220].

There is one more interesting comment to add. Some stellar wind models predict a maximum BH mass of $\sim 65 - 70 M_{\odot}$ from single stellar evolution and from the collapse of the residual hydrogen envelope at solar metallicity (e.g., [147, 145]). This means that the maximum total mass of a BBH is $m_{\text{TOT}} = m_1 + m_2 \sim 130 - 140 M_{\odot}$. However, the maximum total mass of a BBH which merges within a Hubble time is only $m_{\text{TOT}} \sim 80 - 90 M_{\odot}$ [145], as shown in Figure 1.6. The reason for this difference is that if the initial orbital separation is sufficiently small ($a \sim 10^2 - 10^4 R_{\odot}$, [323]) to allow mass transfer and CE, then the two stars lose almost all their hydrogen envelope: they might become a BBH that merges within a Hubble time, but their total mass is at most equal to the total mass of the two naked helium cores. In contrast, if the two stars are metal poor ($Z \lesssim 0.0002$) and if the initial orbital separation is too large to induce mass transfer and CE, the binary star becomes a BBH almost without mass loss, with a total mass up to $140 M_{\odot}$, but the final semi-major axis is too large to allow coalescence by GW emission.

1.4 The dynamics of binary black holes (BBHs)

In the previous Sections of this review, we discussed the formation of BBHs as isolated binaries. There is an alternative channel for BBH formation: the dynamical evolution scenario.

1.4.1 Dynamically active environments

Collisional dynamics is important for the evolution of binaries only if they are in a dense environment ($\gtrsim 10^3$ stars pc^{-3}), such as a star cluster. On the other hand, astrophysicists believe that the vast majority of massive stars (which are BH progenitors) form in star clusters ([196, 370, 371, 282], but see also [367]).

There are several different flavours of star clusters. *Globular clusters* [158] are old stellar systems (~ 12 Gyr), mostly very massive (total mass $M_{\text{SC}} \geq 10^4 M_{\odot}$) and dense (central density $\rho_c \geq 10^4 M_{\odot} \text{ pc}^{-3}$). They are sites of intense dynamical processes, such as the gravothermal catastrophe. Globular clusters represent a small fraction of the baryonic mass in the local Universe ($\lesssim 1$ per cent, [162]). Most studies of dynamical formation of BBHs focus on globular clusters (e.g. [316, 280, 224, 98, 99, 54, 338, 67, 305, 300, 299, 36, 304, 303, 35, 30, 124, 298, 301, 186]).

Young star clusters are young ($\lesssim 100$ Myr), relatively dense ($\rho_c > 10^3 M_{\odot} \text{ pc}^{-3}$) stellar systems, and are thought to be the most common birthplace of massive stars [196, 282]. When they disperse by gas evaporation or are disrupted by the tidal

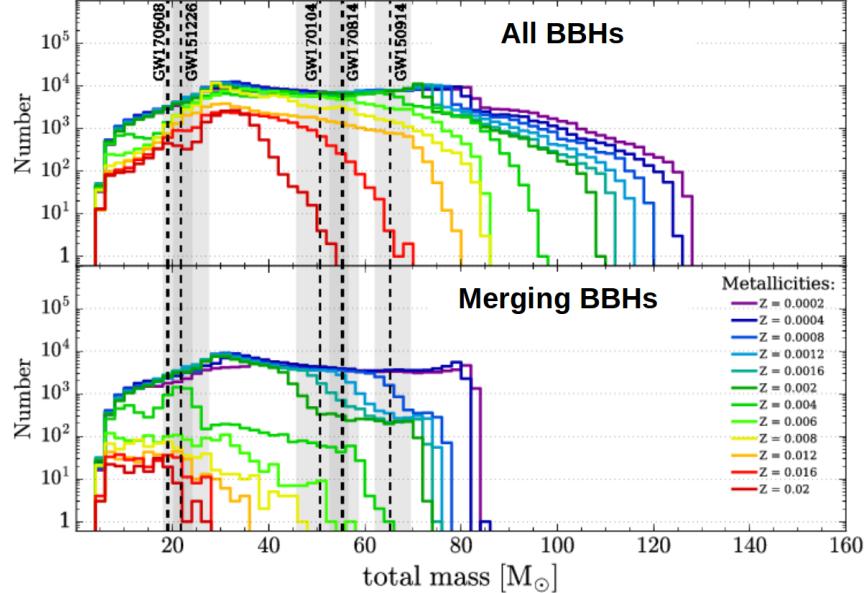


Fig. 1.6 Upper panel: total mass ($m_{\text{TOT}} = m_1 + m_2$) of all BBHs formed from a stellar population of 2×10^7 massive binary stars simulated with MOBSE and with different metallicities (from 0.0002 to 0.02) [145]. Lower panel: total mass of the BBHs merging within a Hubble time from the same simulations. See <https://mobse-webpage.netlify.app/> for more details on MOBSE and on these runs. Courtesy of Nicola Giacobbo.

field of their host galaxy, their stellar content is released into the field. Thus, it is reasonable to expect that a large fraction of BBHs which are now in the field may have formed in young star clusters, where they participated in the dynamics of the cluster. A fraction of young star clusters might survive gas evaporation and tidal disruption and evolve into older *open clusters*, like M67. Figure 1.7 shows a snapshot of an N-body simulation of a young star cluster. Population-synthesis prescriptions are included in this simulation to follow the evolution of binary stars and the formation of BBHs. Studies of BBHs in young and open clusters include [280, 40, 386, 223, 181, 38, 39, 135, 193, 194, 41, 93, 94, 95].

Finally, *nuclear star clusters* lie in the nuclei of galaxies. Nuclear star clusters are rather common in galaxies (e.g. [60, 119, 156, 258]), are usually more massive and denser than globular clusters, and may co-exist with super-massive BHs (SMBHs). Stellar-mass BHs formed in the innermost regions of a galaxy could even be “trapped” in the accretion disc of the central SMBH, triggering their merger (see e.g. [331, 44, 244]). These features make nuclear star clusters unique among star clusters, for the effects that we will describe in the next Sections.

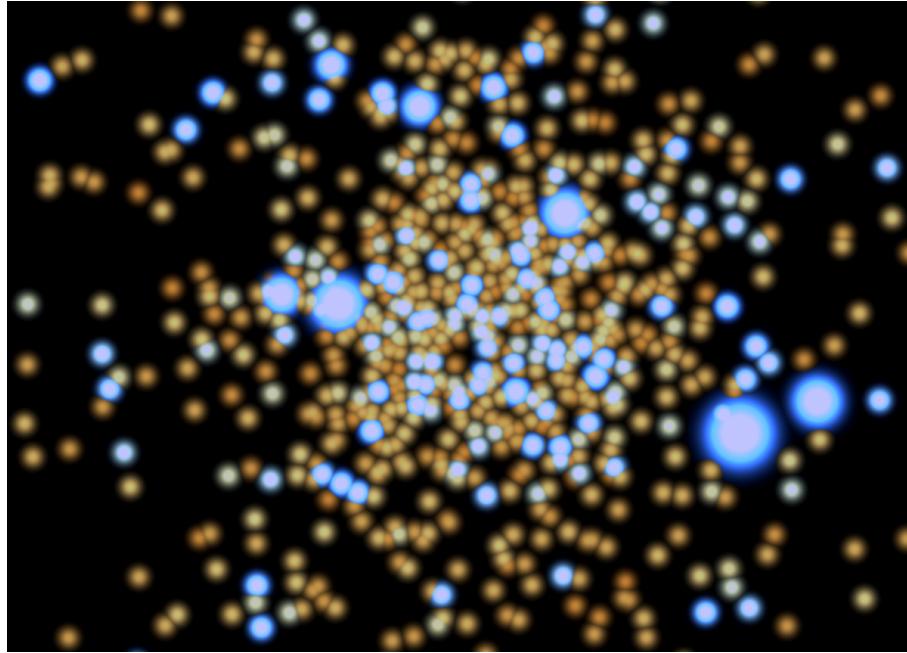


Fig. 1.7 Snapshot of an N-body simulation of a young star cluster. Different particle sizes and colors refer to different star luminosities and temperatures, as estimated by population-synthesis calculations. This simulation has been included in [233].

1.4.2 Two-body encounters, dynamical friction and core-collapse

The main driver of the dynamics of star clusters is gravity force. Gravitational two-body encounters between stars lead to local fluctuations in the potential of the star cluster and drive major changes in the internal structure of the star cluster over a two-body relaxation timescale [325, 326]:

$$t_{\text{rlx}} = 0.34 \frac{\sigma^3}{G^2 \langle m \rangle \rho \ln \Lambda}, \quad (1.17)$$

where σ is the local velocity dispersion of the star cluster, $\langle m \rangle$ is the average stellar mass in the star cluster, ρ is the local mass density, G is the gravity force and $\ln \Lambda \sim 10$ is the Coulomb logarithm. The two-body relaxation timescale is the time needed for a typical star in the stellar system to completely lose memory of its initial velocity due to two-body encounters. In star clusters, t_{rlx} is much shorter than the Hubble time ($t_{\text{rlx}} \sim 10 - 100$ Myr in young star clusters, [282]), while in galaxies and large-scale structures it is much longer than the lifetime of the Universe. Hence, close encounters are common in dense star clusters.

Dynamical friction is another consequence of gravity force: a massive body of mass M orbiting in a sea of lighter particles feels a drag force that slows down its motion over a timescale [74]:

$$t_{\text{DF}}(M) = \frac{3}{4(2\pi)^{1/2} G^2 \ln \Lambda} \frac{\sigma^3}{M \rho(r)}. \quad (1.18)$$

It is apparent that two-body relaxation and dynamical friction are driven by the same force and are related by

$$t_{\text{DF}}(M) \sim \frac{\langle m \rangle}{M} t_{\text{rlx}}, \quad (1.19)$$

i.e., dynamical friction happens over a much shorter timescale than two-body relaxation and leads to mass segregation (or mass stratification) in a star cluster. This process speeds up the collapse of the core of a star cluster and can trigger the so-called Spitzer's instability [324]. Two-body relaxation, dynamical friction and their effects play a crucial role in shaping the demography of BBHs in star clusters, as we discuss below.

1.4.3 Binary – single encounters

We now review what are the main dynamical effects which can affect a BBH, starting from binary – single star encounters. Binaries have a energy reservoir, their internal energy:

$$E_{\text{int}} = \frac{1}{2} \mu v^2 - \frac{G m_1 m_2}{r}, \quad (1.20)$$

where $\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass of the binary (whose components have mass m_1 and m_2), v is the relative velocity between the two members of the binary, and r is the distance between the two members of the binary. As shown by Kepler's laws, $E_{\text{int}} = -E_b = -G m_1 m_2 / (2a)$, where E_b is the binding energy of the binary system, a being its semi-major axis.

The internal energy of a binary can be exchanged with other stars only if the binary undergoes a close encounter with a star, so that its orbital parameters are perturbed by the intruder. This happens only if a single star approaches the binary by few times its orbital separation. We define this close encounter between a binary and a single star as a *three-body encounter*. For this to happen with a non-negligible frequency, the binary must be in a dense environment, because the rate of three-body encounters scales with the local density of stars. Three-body encounters have crucial effects on BH binaries, such as *hardening*, *exchanges* and *ejections*.

1.4.4 Hardening

If a BBH undergoes a number of three-body encounters during its life, we expect that its semi-major axis will shrink as an effect of the encounters. This process is called dynamical *hardening*.

Following [165], we call hard binaries (soft binaries) those binaries with binding energy larger (smaller) than the average kinetic energy of a star in the star cluster. According to Heggie's law [165], hard binaries tend to harden (i.e., to become more and more bound) via binary–single encounters. In other words, a fraction of the internal energy of a hard binary can be transferred into kinetic energy of the intruder and of the centre-of-mass of the binary during three body encounters. This means that the binary loses internal energy and its semi-major axis shrinks.

Most BBHs are expected to be hard binaries, because BHs are among the most massive bodies in star clusters. Thus, BBHs are expected to harden as a consequence of three-body encounters. The hardening process may be sufficiently effective to shrink a BBH till it enters the regime where GW emission is efficient: a BBH which is initially too loose to merge may then become a GW source thanks to dynamical hardening. The hardening rate for hard binaries with semi-major axis a can be estimated as [165]

$$\frac{d}{dt} \left(\frac{1}{a} \right) = 2\pi G \xi \frac{\rho}{\sigma}, \quad (1.21)$$

where $\xi \sim 0.1 - 10$ is a dimensionless hardening parameter (which has been estimated through numerical experiments, [167, 286]), ρ is the local mass density of stars, σ is the local velocity dispersion and G is the gravity constant.

Dynamical hardening is the main responsible for the shrinking of a binary, till it reaches a semi-major axis short enough for GW emission to become effective, which can be derived with the following equation [274]:

$$\frac{da}{dt} = -\frac{64}{5} \frac{G^3 m_1 m_2 (m_1 + m_2)}{c^5 (1 - e^2)^{7/2}} a^{-3}. \quad (1.22)$$

By combining equations 1.21 and 1.22, it is possible to make a simple analytic estimate of the evolution of the semi-major axis of a BBH which is affected by three-body encounters and by GW emission:

$$\frac{da}{dt} = -2\pi \xi \frac{G\rho}{\sigma} a^2 - \frac{64}{5} \frac{G^3 m_1 m_2 (m_1 + m_2)}{c^5 (1 - e^2)^{7/2}} a^{-3}, \quad (1.23)$$

This equation holds under the assumption that the binary star is hard, the total mass of the binary star is much larger than the average mass of a star in the star cluster (exchanges are unlikely) and that most three-body encounters have a small impact parameter. The first part of the right-hand term of equation 1.23 accounts for the effect of three-body hardening on the semi-major axis. It scales as $da/dt \propto -a^2$, indicating that the larger the binary is, the more effective the hardening. This can be easily understood considering that the geometric cross section for three body

interactions with a binary scales as a^2 . The second part of the right-hand term of equation 1.23 accounts for energy loss by GW emission. It is the first-order approximation of the calculation by Peters (1964, [274]). It scales as $da/dt \propto -a^{-3}$ indicating that GW emission becomes efficient only when the two BHs are very close to each other.

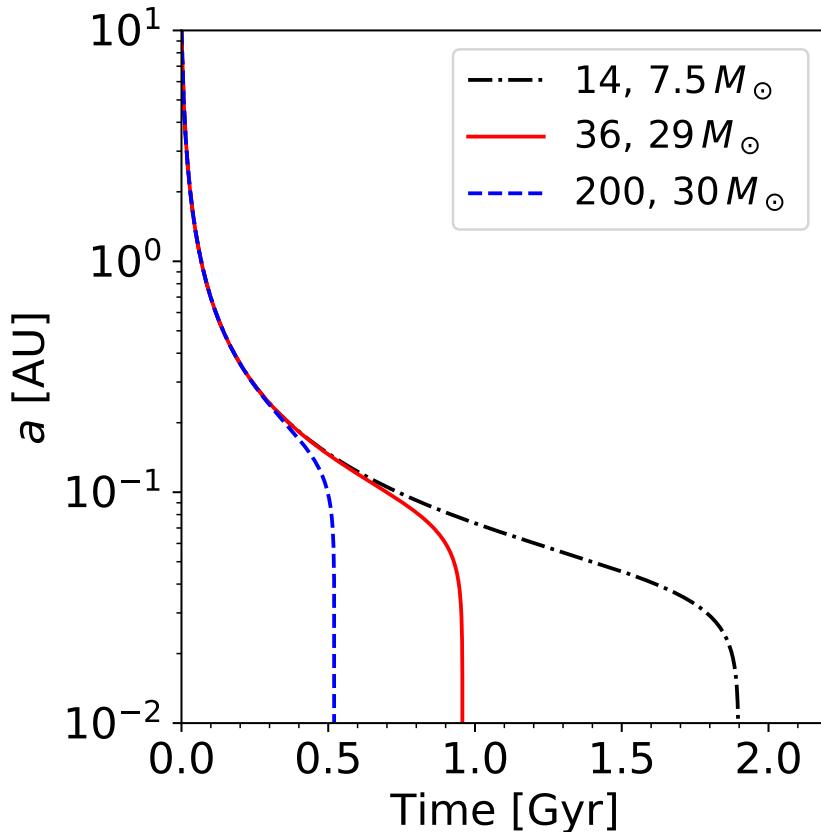


Fig. 1.8 Time evolution of the semi-major axis of three BH binaries estimated from equation 1.23. Blue dashed line: BH binary with masses $m_1 = 200 M_\odot$, $m_2 = 30 M_\odot$; red solid line: $m_1 = 36 M_\odot$, $m_2 = 29 M_\odot$; black dot-dashed line: $m_1 = 14 M_\odot$, $m_2 = 7.5 M_\odot$. For all BH binaries: $\xi = 1$, $\rho = 10^5 M_\odot \text{ pc}^{-3}$, $\sigma = 10 \text{ km s}^{-1}$, $e = 0$ (here we assume that ρ , σ and e do not change during the evolution), initial semi-major axis of the BH binary $a_i = 10 \text{ AU}$.

In Figure 1.8 we solve equation 1.23 numerically for three BBHs with different mass. All binaries evolve through i) a first phase in which hardening by three body encounters dominates the evolution of the binary, ii) a second phase in which the semi-major axis stalls because three-body encounters become less efficient as the semi-major axis shrink, but the binary is still too large for GW emission to become

efficient, and iii) a third phase in which the semi-major axis drops because the binary enters the regime where GW emission is efficient.

1.4.5 Exchanges

Dynamical exchanges are three-body encounters during which one of the members of the binary is replaced by the intruder. Exchanges may lead to the formation of new BBHs: if a binary composed of a BH and a low-mass star undergoes an exchange with a single BH, this leads to the formation of a new BBH. This is a fundamental difference between BHs in the field and in star clusters: a BH which forms as a single object in the field has negligible chances to become member of a binary system, while a single BH in the core of a star cluster has good chances of becoming member of a binary by exchanges.

Exchanges are expected to lead to the formation of many more BBHs than they can destroy, because the probability for an intruder to replace one of the members of a binary is ≈ 0 if the intruder is less massive than both binary members, while it suddenly jumps to ~ 1 if the intruder is more massive than one of the members of the binary [168]. Since BHs are among the most massive bodies in a star cluster after their formation, they are very efficient in acquiring companions through dynamical exchanges. Thus, exchanges are a crucial mechanism to form BH binaries dynamically. By means of direct N-body simulations, Ziosi et al. (2014, [386]) show that > 90 per cent of BBHs in young star clusters form by dynamical exchange. Moreover, BBHs formed via dynamical exchange will have some distinctive features with respect to field BBHs (see e.g. [386]):

- BBHs formed by exchanges will be (on average) more massive than isolated BBHs, because more massive intruders have higher chances to acquire companions;
- exchanges trigger the formation of highly eccentric BBHs; eccentricity is then significantly reduced by circularisation induced by GW emission, if the binary enters the regime where GW emission is effective;
- BBHs born by exchange will likely have misaligned spins: exchanges and other dynamical interactions tend to lead to isotropically distributed spin directions with respect to the binary orbital plane, because dynamical interactions remove any memory of previous alignments.

Zevin et al. (2017, [384]) compare a set of simulations of field binaries with a set of simulations of globular cluster binaries, run with the same population-synthesis code. The most striking difference between merging BH binaries in their globular cluster simulations and in their population-synthesis simulations is the dearth of merging BHs with mass $< 10 M_\odot$ in the globular cluster simulations. This is likely due to the fact that exchanges tend to destroy binaries composed of light BHs.

Figure 1.9 compares the masses of simulated BBH mergers in isolated binaries and in young star clusters. While the maximum total mass of BBH mergers in iso-

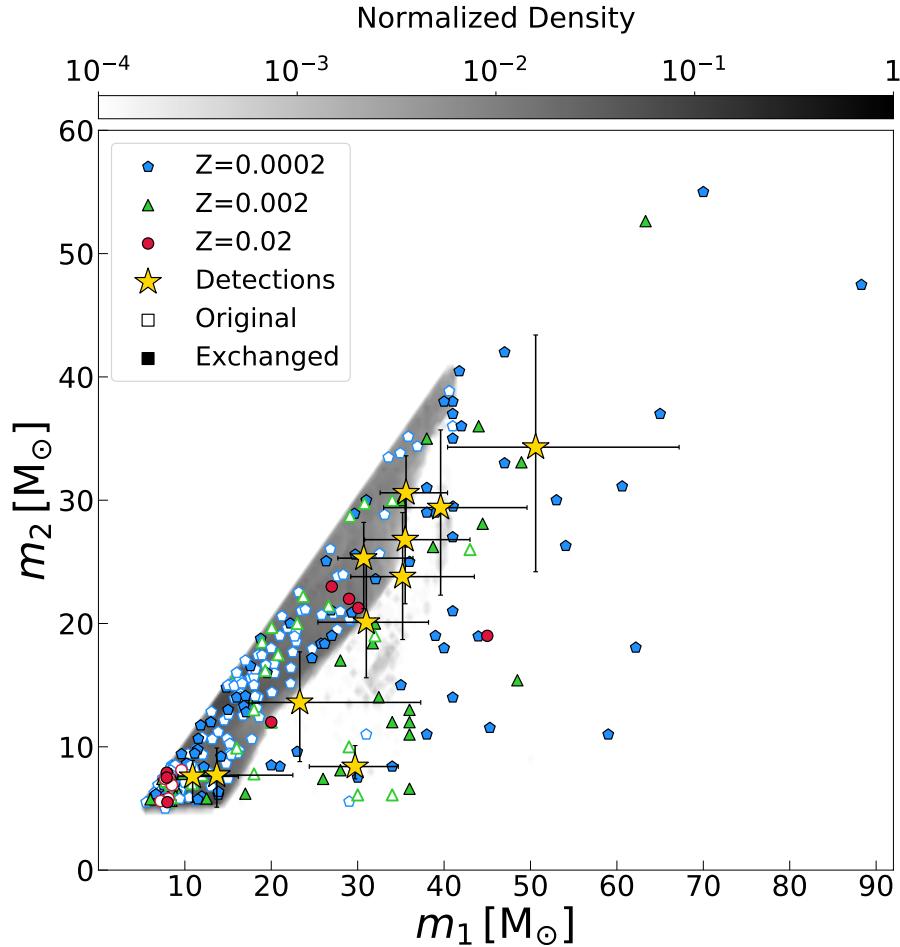


Fig. 1.9 Mass of the secondary BH versus mass of the primary BH in a set of BBH mergers. Yellow stars: O1 and O2 BBHs plus GW190412. Blue, green and red symbols: simulated BBHs in young star clusters with metallicity $Z = 0.0002, 0.002$ and 0.02 . Filled symbols: dynamical exchanges. Open symbols: original binaries. Gray contours: BBHs from isolated binary evolution. Simulations from [95]. Courtesy of Ugo N. Di Carlo.

lated binaries is $\sim 80 M_{\odot}$ (see Figure 1.6), BBH mergers in young star clusters have total masses up to $\sim 130 M_{\odot}$. This difference springs mainly from two facts:

- i) single stellar evolution at low metallicity can lead to the formation of single BHs with mass up to $\sim 70 M_{\odot}$, if the hydrogen envelope collapses to the final BH. In the field such massive BHs remain single, while in a star cluster they can acquire companions via dynamical exchanges and they can merge by dynamical hardening and GW emission;

- ii) in star clusters, stellar collisions are quite common and can lead to the build up of more massive BHs, even with mass inside the pair instability mass gap [93, 94, 95, 187, 290].

Spin misalignments are another possible feature to discriminate between field binaries and star cluster binaries (e.g. [117, 115]). We expect that an isolated binary system in which both the primary and the secondary component undergo direct collapse results in a BBH with nearly aligned spins. For dynamically formed BH binaries we expect misaligned, or even nearly isotropic spins, because any original spin alignment is completely reset by three-body encounters.

Currently, we have limited constraints on BH spins from GW detections. In a few events, such as GW151226 [4], GW170729 [8] and especially GW190412 [17], the measured value of χ_{eff} is significantly larger than zero, indicating at least partial alignment. The vast majority of events listed in GWTC-2 have χ_{eff} consistent with zero. This might be the result of either low spins or misaligned spins or a combination of both. On the other hand, the most recent population study by the LVC [15] shows that $\sim 12\%$ to 44% of BBH systems have spins tilted by more than 90° with respect to their orbital angular momentum, supporting a negative effective spin parameter.

Interestingly, GW190412 has a non-zero precessing spin χ_p to the 90% credible level [17] and GW190521 [20], the most massive BBH event to date, shows mild evidence for non-zero precessing spin. This might support a dynamical formation for these events (e.g., [17, 20, 302, 140, 126]).

1.4.6 Stellar mergers and BHs in the pair-instability mass gap

Binary-single encounters in young star clusters trigger collisions between massive stars. This process is sped up by dynamical friction, which can generate an excess of massive stars in the cluster core in less than a Myr. Stellar collisions can lead to the formation of very massive stars ($> 150 M_\odot$), blue straggler stars [226] and unusually massive BHs.

Figure 1.10 shows the dynamical formation of a BBH with primary mass $m_1 = 88 M_\odot$ and secondary mass $m_2 = 47.5 M_\odot$ [95]. The masses of the components of this BBH are similar to those of GW190521 [18, 20]. In particular, the primary BH has a mass in the pair-instability mass gap. Its formation is possible because its stellar progenitor is the result of the merger between a giant star with a well-developed He core and a main sequence companion (MS). The result of this stellar merger is a massive core helium burning (cHeB) star with an over-sized hydrogen envelope with respect to the He core [93, 94, 95, 187, 290]. Given the short timescale for He, C, O, Ne and Si burning with respect to H burning, the star collapses to a BH before the He core grows above the threshold for pair instability: the result is a $\sim 88 M_\odot$ BH. After its formation, this BH acquires a companion by dynamical exchanges and merges within a Hubble time.

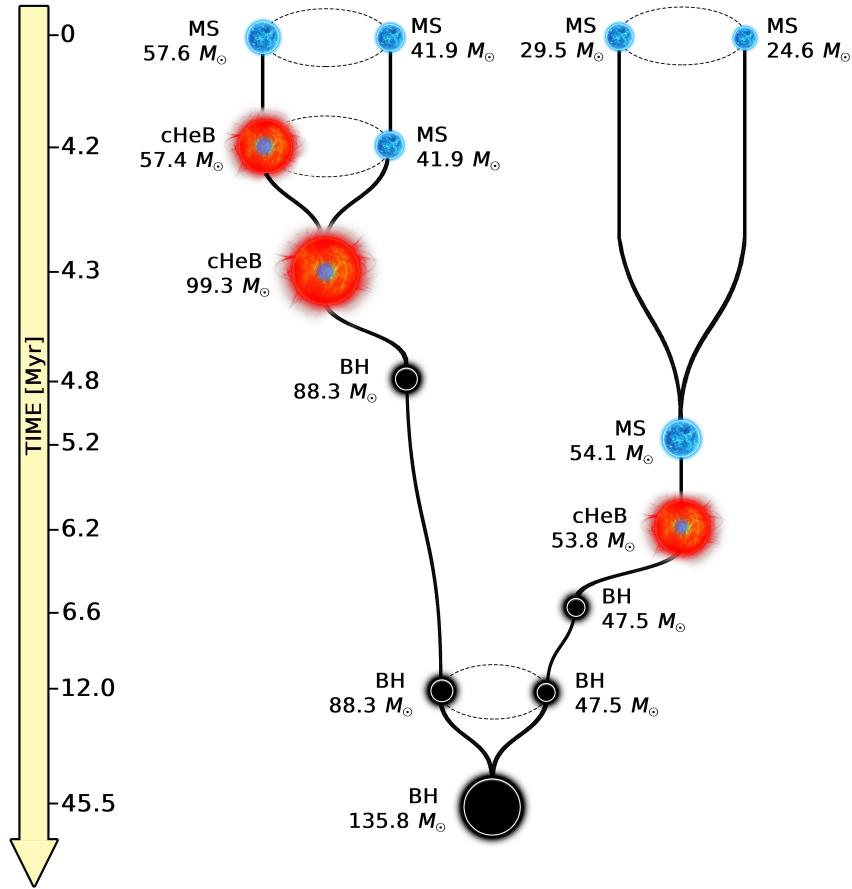


Fig. 1.10 Cartoon of the dynamical assembly of a GW190521-like BBH from the simulations of [95]. Courtesy of Ugo N. Di Carlo.

This is a viable scenario to produce systems like GW190521 not only because of the good match of the BBH mass, but also because a BH born from the direct collapse of a very massive star might form with a large spin (depending on the final spin of the massive progenitor) and because this dynamical formation leads to isotropically oriented spins. The main uncertainties of this model are mass loss during stellar collision [139] and the possible loss of a fraction of the envelope during the final collapse [118].

1.4.7 Direct three-body binary formation

In the most massive star clusters (globular clusters, nuclear star clusters), stellar velocities are so large that dynamical encounters can unbind most of the original binary stars (i.e., those binary stars that were already there at the formation of the star cluster). The minimum relative velocity v_c between a binary star and an intruder star needed to unbind a binary is in fact [317]:

$$v_c = \sqrt{\frac{G m_1 m_2 (m_1 + m_2 + m_3)}{m_3 (m_1 + m_2) a}}, \quad (1.24)$$

where m_1 , m_2 and m_3 are the masses of the two binary members and that of the intruder, while a is the binary semi-major axis.

In these extreme environments, most BBHs are expected to form by direct encounters of three single bodies [253, 304], during core collapse. This leads to the formation of extremely hard BBHs, which survive further ionization from intruders. The timescale for binary formation via three single body encounters is [199, 25]:

$$t_{3\text{bb}} = 0.1 \text{ Myr} \left(\frac{n}{10^6 \text{ pc}^{-3}} \right)^{-2} \left(\frac{\sigma}{30 \text{ km s}^{-1}} \right)^9 \left(\frac{m_{\text{BH}}}{30 M_\odot} \right)^{-5}, \quad (1.25)$$

where n is the local stellar density, σ is the local velocity dispersion of the star cluster and m_{BH} is the typical BH mass in the star cluster. The properties of BBHs born from three single body encounters are similar to those of BBHs born via dynamical exchanges: they tend to be more massive than isolated binaries, have high initial eccentricity and isotropically oriented spins (e.g. [27]).

Direct three-body encounters are likely the most common BBH formation channel in globular clusters and nuclear star clusters [253], while binary – single star exchanges are likely the most common formation channel of BBHs in young star clusters [386, 93]. In young star clusters, dynamical exchanges affect both already formed BHs and their stellar progenitors.

1.4.8 Dynamical ejections

During three-body encounters, a fraction of the internal energy of a hard binary is transferred into kinetic energy of the intruders and of the centre-of-mass of the binary. As a consequence, the binary and the intruder recoil. The recoil velocity is generally of the order of a few km s^{-1} , but can be up to several hundred km s^{-1} .

Since the escape velocity from a globular cluster is $\sim 30 \text{ km s}^{-1}$ and the escape velocity from a young star cluster or an open cluster is even lower, both the recoiling binary and the intruder can be ejected from the parent star cluster. If the binary and/or the intruder are ejected, they become field objects and cannot participate in the dynamics of the star cluster anymore. Thus, not only the ejected BBH stops

hardening, but also the ejected intruder, if it is another compact object, loses any chance of entering a new binary by dynamical exchange.

A general expression for the recoil velocity of the binary center of mass if $(m_1 + m_2) \gg \langle m \rangle$ (where $\langle m \rangle$ is the average mass of a star in a star cluster) is

$$v_{\text{rec}} \sim \frac{\langle m \rangle}{m_1 + m_2} \sqrt{\frac{2\xi}{(m_1 + m_2 + \langle m \rangle)} E_b}, \quad (1.26)$$

where $E_b = Gm_1 m_2 / (2a)$ is the binary binding energy. The above equation can help us deriving the minimum binding energy above which a binary star is ejected by a binary-single encounter $E_{b,\min}$ [246]:

$$E_{b,\min} \sim \frac{(m_1 + m_2)^3}{2\xi \langle m \rangle^2} v_{\text{esc}}^2, \quad (1.27)$$

where v_{esc} is the escape velocity from the star cluster. A BBH will merge inside the star cluster only if $E_{b,\min} > E_{b,\text{GW}}$, where $E_{b,\text{GW}}$ is the minimum binding energy to reach coalescence by GW emission.

Most BNSs, BHs and BH-NS systems in young star clusters are ejected before they merge [93, 289]. Dynamical ejections of BNS and BH-NS binaries were proposed to be one of the possible explanations for the host-less short gamma-ray bursts, i.e. gamma-ray bursts whose position in the sky appears to be outside any observed galaxy [122]. Host-less bursts may be $\sim 25\%$ all short gamma-ray bursts.

In general, ejections of compact objects and compact-object binaries from their parent star cluster can be the result of at least three different processes:

- dynamical ejections (as described above);
- SN kicks [169, 133];
- GW recoil [205, 69, 151].

GW recoil is a relativistic kick occurring when a BBH merges. It is the result of asymmetric linear momentum loss by GW emission, when the binary has asymmetric component masses and/or misaligned spins. It results in kick velocities up to thousands of km s^{-1} and usually of the order of hundreds of km s^{-1} .

Ejections by dynamics, SN kicks or GW recoil may be the main process at work against mergers of second-generation BHs, where for second-generation BHs we mean BHs which were born from the merger of two BHs rather than from the collapse of a star [143]. In globular clusters, open clusters and young star clusters, a BH has good chances of being ejected by three-body encounters before it merges [246] and a very high chance of being ejected by GW recoil after it merges [301, 234]. The only place where merging BHs can easily avoid ejection by GW recoil are nuclear star clusters, whose escape velocity can reach hundreds of km s^{-1} [25, 28, 127, 234].

1.4.9 Formation of intermediate-mass black holes by runaway collisions

In Section 1.2.4, we have mentioned that intermediate-mass black holes (IMBHs, i.e. BHs with mass $100 \lesssim m_{\text{BH}} \lesssim 10^4 M_{\odot}$) might form from the direct collapse of metal-poor extremely massive stars [321]. Other formation channels have been proposed for IMBHs and most of them involve the dynamics of star clusters. The formation of massive BHs by runaway collisions has been originally proposed about half a century ago [83, 308] and was then elaborated by several authors (e.g. [279, 281, 278, 160, 130, 226, 231, 149, 223, 93, 95, 297]).

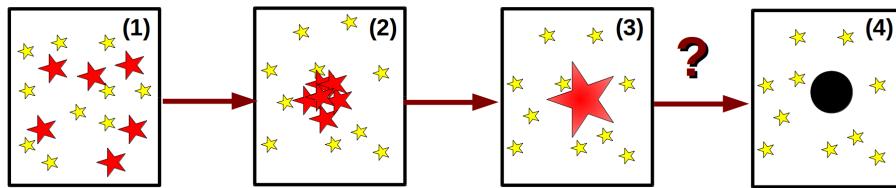


Fig. 1.11 Cartoon of the runaway collision scenario in dense young star clusters (see e.g. [281]). From left to right: (1) the massive stars (red big stars) and the low-mass stars (yellow small stars) follow the same initial spatial distribution; (2) dynamical friction leads the massive stars to sink to the core of the cluster, where they start colliding between each other; (3) a very massive star ($\gg 100 M_{\odot}$) forms as a consequence of the runaway collisions; (4) this massive star might be able to directly collapse into a BH.

The basic idea is the following (as summarized by the cartoon in Figure 1.11). In a dense star cluster, dynamical friction [74] makes massive stars to decelerate because of the drag exerted by lighter bodies, on a timescale $t_{\text{DF}}(M) \sim (\langle m \rangle/M) t_{\text{rlx}}$ (equation 1.18).

Since the two-body relaxation timescale in a young star cluster can be as short as $t_{\text{rlx}} \sim 10 - 100 M_{\odot}$ [282], for a star with mass $M \geq 40 M_{\odot}$ we estimate $t_{\text{DF}} \leq 2.5$ Myr: dynamical friction is very effective in dense massive young star clusters. Because of dynamical friction, massive stars segregate to the core of the cluster before they become BHs.

If the most massive stars in a dense young star cluster sink to the centre of the cluster by dynamical friction on a time shorter than their lifetime (i.e., before core-collapse SNe take place, removing a large fraction of their mass), then the density of massive stars in the cluster core becomes extremely high. This makes collisions between massive stars extremely likely. Actually, direct N-body simulations show that collisions between massive stars proceed in a runaway sense, leading to the formation of a very massive ($\gg 100 M_{\odot}$) star [281]. The main open question is: “What is the final mass of the collision product? Is the collision product going to collapse to an IMBH?”.

There are essentially two critical issues: i) how much mass is lost during the collisions? ii) how much mass does the very-massive star lose by stellar winds?

Hydrodynamical simulations of colliding stars [138, 139] show that massive star can lose $\approx 25\%$ of their mass during collisions. Even if we optimistically assume that no mass is lost during and immediately after the collision (when the collision product relaxes to a new equilibrium), the resulting very massive star will be strongly radiation-pressure dominated and is expected to lose a significant fraction of its mass by stellar winds. Recent studies including the effect of the Eddington factor on mass loss [223, 321] show that IMBHs cannot form from runaway collisions at solar metallicity. At lower metallicity ($Z \lesssim 0.1 Z_{\odot}$) approximately 10–30% of runaway collision products in young dense star clusters can become IMBHs by direct collapse (they also avoid being disrupted by pair-instability SNe).

The majority of runaway collision products do not become IMBHs but they end up as relatively massive BHs ($\sim 20 - 90 M_{\odot}$, [223]). If they remain inside their parent star cluster, such massive BHs are extremely efficient in acquiring companions by dynamical exchanges. Mapelli (2016, [223]) find that all stable binaries formed by the runaway collision product are BBHs and thus are possibly important sources of GWs in the LIGO–Virgo range.

1.4.10 Hierarchical BBH formation and IMBHs

The runaway collision scenario occurs only in the early stages of the evolution of a star cluster, when massive stars are still alive (the lifetime of a $\sim 30 M_{\odot}$ star is ~ 6 Myr). However, it has been proposed that IMBHs form even in old clusters (e.g., globular clusters) by repeated mergers of smaller BHs (e.g. [246, 149, 28, 127, 234, 235]).

The simple idea is illustrated in Figure 1.12. A stellar BBH in a star cluster is usually a rather hard binary. Thus, it shrinks by dynamical hardening till it may enter the regime where GW emission is effective. In this case, the BBH merges leaving a single more massive BH. Given its relatively large mass, the new BH has good chances to acquire a new companion by exchange. Then, the new BBH starts hardening again by three-body encounters and the story may repeat several times, till the main BH becomes an IMBH.

This scenario has one big advantage: it does not depend on stellar evolution, so we are confident that the BH will grow in mass by mergers, if it remains inside the cluster. However, there are several issues. First, the BBH may be ejected by dynamical recoil, received as an effect of three-body encounters. Recoils get stronger and stronger, as the orbital separation decreases (equation 1.27). The BBH will avoid ejection by dynamical recoil only if it is sufficiently massive ($\gtrsim 50 M_{\odot}$ for a dense globular cluster, [85]). If the BBH is ejected, the loop breaks and no IMBH is formed.

Second and even more important, the merger of two BHs involves a relativistic kick. This kick may be as large as hundreds of km s^{-1} [205], leading to the ejection

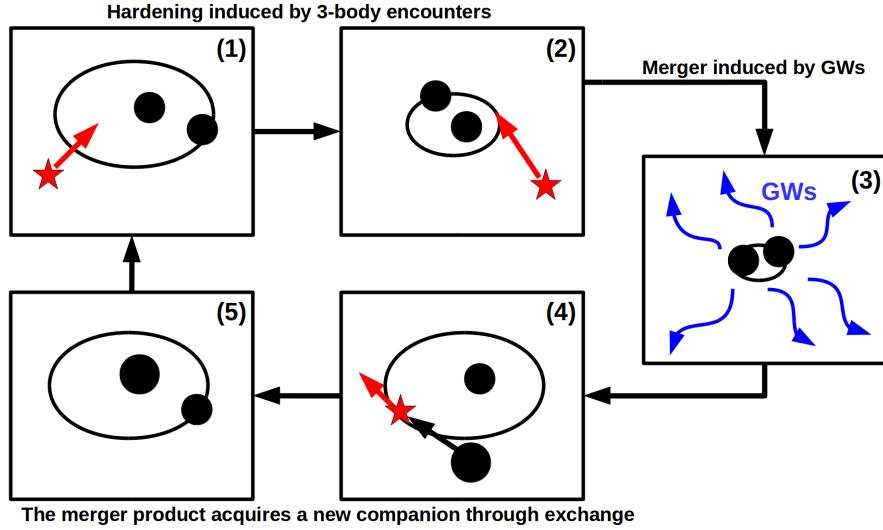


Fig. 1.12 Cartoon of the repeated merger scenario in old star clusters (see, e.g., [246, 149]). From top to bottom and from left to right: (1) a BBH undergoes three-body encounters in a star cluster; (2) three-body encounters harden the BBH, shrinking its semi-major axis; (3) the BBH hardens by three-body encounters till it enters the regime where GW emission is efficient: the BBH semi-major axis decays by GW emission and the binary merges; (4) a single bigger BH forms as result of the merger and may acquire a new companion by dynamical exchange (if it is not ejected by GW recoil); (5) the new BBH containing the bigger BH starts shrinking again by three-body encounters (1). This loop may be repeated several times till the main BH becomes an IMBH.

of the BH from the parent star cluster [170]. Also in this case, the loop breaks and no IMBH is formed. Finally, even if the BH binary is not ejected, this scenario is relatively inefficient: if the seed BH is $\sim 50 M_{\odot}$, several Gyr are required to form an IMBH with mass $\sim 500 M_{\odot}$ [246].

Monte Carlo simulations by Giersz et al. (2015, [149]) show that both the runaway collision scenario and the repeated merger scenario can be at work in star clusters: runaway collision IMBHs form in the first few Myr of the life of a star cluster and grow in mass very efficiently, while repeated-merger IMBHs start forming much later ($\gtrsim 5$ Gyr) and their growth is less efficient.

1.4.11 Alternative models for massive BHs and IMBH formation in galactic nuclei

Several additional models predict the formation of IMBHs in galactic nuclei. For example, Miller & Davies (2012, [247]) propose that IMBHs can efficiently grow in galactic nuclei from runaway tidal capture of stars, provided that the velocity

dispersion in the nuclear star cluster is $\gtrsim 40 \text{ km s}^{-1}$. Below this critical velocity, binary stars can support the system against core collapse, quenching the growth of the central density and leading to the ejection of the most massive BHs. Above this velocity threshold, the stellar density can grow sufficiently fast to enhance tidal captures and BH–star collisions. Tidal captures are more efficient than BH–star collisions in building up IMBHs, because the mass growth rate of the former scales as $\dot{m}_{\text{IMBH}} \propto m_{\text{IMBH}}^{4/3}$ (where m_{IMBH} is the IMBH seed mass), whereas the mass growth of the latter scales as $\dot{m}_{\text{IMBH}} \propto m_{\text{IMBH}}$ [332].

Furthermore, McKernan et al. (2012, [243], see also [242, 244, 44, 380, 381]) suggest that IMBHs could efficiently grow in the accretion disc of a SMBH. Nuclear star cluster members trapped in the accretion disk are subject to two competing effects: orbital excitation due to dynamical heating by encounters with other stars and orbital damping due to gas drag. Gas damping is expected to be more effective than orbital excitation, quenching the relative velocity between nuclear cluster members and enhancing the collision rate. This favours the growth of IMBHs via both gas accretion and multiple stellar collisions. This mechanism might be considered as a gas-aided hierarchical merger scenario.

As a final remark, it is worth mentioning that all the IMBH formation scenarios we have discussed here – i)runaway collisions of massive stars, ii) hierarchical merger of BHs and iii) BH trapping in AGN discs– can also work as possible formation mechanisms for GW190521-like events: they can lead to the formation of BHs with mass in the pair instability mass gap and with large spins. These BHs are born in dense stellar environments, where they can acquire companions dynamically and form BBHs with isotropically oriented spins [20].

1.4.12 Kozai-Lidov resonance

Unlike the other dynamical processes discussed so far, Kozai-Lidov (KL) resonance [184, 201] can occur both in the field and in star clusters. KL resonance appears whenever we have a stable hierarchical triple system (i.e., a triple composed of an inner binary and an outer body orbiting the inner binary), in which the orbital plane of the outer body is inclined with respect to the orbital plane of the inner binary. Periodic perturbations induced by the outer body on the inner binary cause i) the eccentricity of the inner binary and ii) the inclination between the orbital plane of the inner binary and that of the outer body to oscillate. The semi-major axis of the binary star is not affected, because KL resonance does not imply an energy exchange between inner and outer binary. KL oscillations may enhance BBH mergers, because the timescale for merger by GW emission strongly depends on the eccentricity e of the binary $t_{\text{GW}} \propto (1 - e^2)^{7/2}$ (see equation 1.22, [274]).

It might seem that hierarchical triples are rather exotic systems. This is not the case. In fact, $\sim 10\%$ of low-mass stars are in triple systems [345, 346, 287]. This fraction gradually increases for more massive stars [100], up to $\sim 50\%$ for B-type

stars [306, 252, 347]. In star clusters, stable hierarchical triple systems may form dynamically, via four-body or multiple-body encounters.

Kimpson et al. (2016, [181]) find that KL resonance may enhance the BBH merger rate by $\approx 40\%$ in young star clusters and open clusters. According to Fragione et al. (2019, [125]), the merger fraction of BBHs in galactic nuclei can be up to $\sim 5 - 8$ times higher for triples than for binaries. On the other hand, Antonini et al. (2017, [26]) find that KL resonance in field triples can account for $\lesssim 3$ mergers $\text{Gpc}^{-3} \text{ yr}^{-1}$. The main signature of the merger of a KL system is the non-zero eccentricity until very few seconds before the merger. Eccentricity might be significantly non-zero even when the system enters the LIGO–Virgo frequency range.

KL resonances have an intriguing application in nuclear star clusters. If the stellar BH binary is gravitationally bound to the super-massive BH (SMBH) at the centre of the galaxy, then we have a peculiar triple system where the inner binary is composed of the stellar BH binary and the outer body is the SMBH [29]. Also in this case, the merging BBH has good chances of retaining a non-zero eccentricity till it emits GWs in the LIGO–Virgo frequency range.

1.4.13 Summary of dynamics and open issues

In this Section, we have seen that dynamics is a crucial ingredient to understand BBH demography. Dynamical interactions (three- and few-body close encounters) can favour the coalescence of BBHs through dynamical hardening. New BBHs can form via dynamical exchanges and via direct three-body encounters or GW captures. Hierarchical mergers of BHs in dense star clusters and in AGN disks can trigger the formation of unusually massive BHs (like GW190521) and even IMBHs. Stellar mergers might also lead to the formation of unusually massive BHs (like GW190521) and IMBHs in dense star clusters. All these dynamical processes suggest a boost of the BBH merger rate in dynamically active environments.

Overall, dynamically formed BBHs are expected to be more massive than BBHs from isolated binary evolution, with higher initial eccentricity and with misaligned spins. Also, KL resonances favour the coalescence of BBHs with extremely high eccentricity, even close to the last stable orbit. On the other hand, three-body encounters might trigger the ejection of binary compact objects from their natal environment, inducing a significant displacement between the birth place of the binary and the location of its merger.

Figure 1.13 summarizes one of the possible evolutionary pathways of merging BBHs which originate from dynamics (the variety of this formation channel is too large to account for all dynamical channels mentioned above in a single cartoon). As in the isolated binary case, we start from a binary star. In the dynamical scenario, it is not important that this binary evolves through Roche lobe or CE (although this may happen). After the primary has turned into a BH, the binary undergoes a dynamical exchange: the secondary is replaced by a massive BH and a new BBH forms. The new binary system is not ejected from the star cluster and undergoes further three-

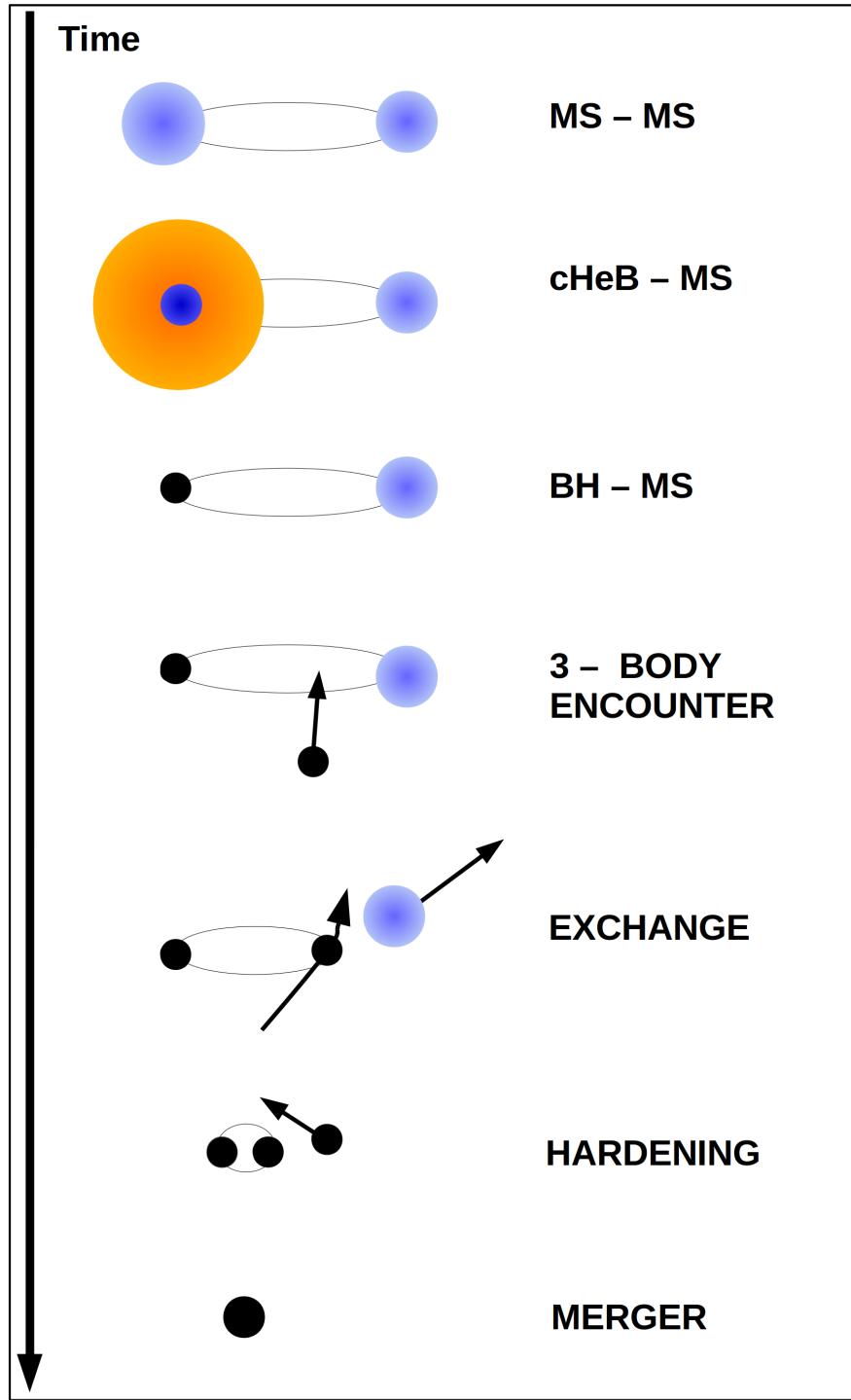


Fig. 1.13 Schematic evolution of a merging BBH formed by dynamical exchange (see, e.g., [98, 99, 386, 223, 300, 299, 36]).

body encounters. As an effect of these three-body encounters, the binary system hardens enough to enter the regime in which GW emission is efficient: the BBH merges by GW decay.

We expect dynamics to be important for BBHs also because massive stars (which are the progenitors of BHs) form preferentially in young star clusters [282], which are dynamically active places. It is reasonable to expect that most BHs participate in the dynamics of their parent star cluster before being ejected by dynamical recoil or relativistic kicks.

1.5 BBHs in the cosmological context

At design sensitivity, LIGO and Virgo will observe BBHs up to redshift $z \sim 1$. The farthest event observed to date, GW190521, is at $z \sim 0.8$, when the Universe was only ~ 6.8 Gyr old. Next generation ground-based GW detectors, such as the Einstein Telescope [284], will observe BBH mergers up to redshift $z \gtrsim 10$, when the Universe was $\lesssim 500$ Myr old. Almost the entire Universe will be transparent to BBH mergers and it will be possible to investigate the cosmic evolution of stars and galaxies through the observations of BBH mergers. In other words, we will do cosmology through BBH mergers.

Accounting for the cosmological context of BBH mergers might appear as a desperate challenge, because of the humongous dynamical range: the orbital separations of BBHs are of the order of tens of solar radii, while cosmic structures are several hundreds of Mpc. Several theoretical studies have addressed this challenge, adopting two different methodologies.

1.5.1 Data-driven semi-analytic models

Some authors (e.g. [96, 97, 50, 197, 102, 107, 103, 145, 59, 256, 108, 37, 363, 337, 309, 310]) combine the outputs of population synthesis codes with analytic prescriptions. The main ingredients are the cosmic star formation rate density and the evolution of metallicity with redshift [214, 215, 81, 80, 79]. In some previous work (e.g. [96, 197]) a Press-Schechter like formalism is adopted, to include the mass of the host galaxy in the general picture. Lamberts et al. (2016, [197]) even include a redshift-dependent description for the mass-metallicity relation (hereafter MZR), to account for the fact that the mass of a galaxy and its observed metallicity are deeply connected. The main advantage of this procedure is that the star formation rate and the metallicity evolution can be derived more straightforwardly from the data. The main drawback is that it is more difficult to trace the evolution of the host galaxy of the BBH, through its galaxy merger tree.

In the semi-analytic description, the merger rate density evolution can be described as [309]:

$$\mathcal{R}(z) = \int_{z_{\min}}^z \psi(z') \frac{dt(z')}{dz'} \left[\int_{Z_{\min}}^{Z_{\max}} \eta(Z) \mathcal{F}(z', z, Z) dZ \right] dz', \quad (1.28)$$

where $dt(z')/dz' = (1+z')^{-1} H(z')^{-1}$, with $H(z') = H_0 [(1+z')^3 \Omega_M + \Omega_\Lambda]^{1/2}$. Furthermore, Z_{\min} and Z_{\max} are the minimum and maximum metallicity, $\psi(z')$ is the cosmic star formation rate (SFR) at redshift z' , $\mathcal{F}(z', z, Z)$ is the rate of compact binaries that form at redshift z' from stars with metallicity Z and merge at redshift z , and $\eta(Z)$ is the merger efficiency, namely the ratio between the total number $\mathcal{N}_{\text{TOT}}(Z)$ of compact binaries (formed from a coeval population) that merge within an Hubble time ($t_{H_0} \lesssim 14$ Gyr) and the total initial mass $M_*(Z)$ of the simulation with metallicity Z . The value of $\mathcal{F}(z', z, Z)$ and that of $\eta(Z)$ can be calculated either from catalogues of population-synthesis simulations or from phenomenological models.

This formalism yields an evolution of the merger rate density with redshift as shown in Fig. 1.14. From the upper panel, we see that the cosmic merger rate density evolution is sensitive to the choice of the CE parameter α . From the lower panel, it is apparent that the BBH merger rate density is tremendously affected by the metallicity evolution and by the observational uncertainties on metallicity evolution. Most models agree on this result (e.g., [59, 256, 108, 37, 337, 309, 310]). The reason for this trend is twofold. On the one hand, according to current models of BBH formation, the merger efficiency η is 2–4 orders of magnitude higher at low metallicity ($Z < 0.0002$) than at high metallicity. Hence, the merger rate is extremely sensitive to the underlying metallicity evolution. On the other hand, the uncertainties on metallicity evolution from observational data are large, as discussed by, e.g., [214, 218, 81, 80, 309, 79].

The trend of the cosmic merger rate in Fig. 1.14 is similar to the trend of the cosmic star formation rate density curve, modulated by both metallicity evolution and time delay⁴. The peak of the BBH merger rate density curve ($z \sim 3 - 4$, depending on α) is at a higher redshift than the peak of the cosmic star formation rate density ($z = 2$). This happens because the lower the metallicity is, the higher is the efficiency of BBH mergers, and metal-poor stars are more common at high redshift than at low redshift. With this approach, it will also be possible to put constraints, in a statistical sense, on the fraction of BBH mergers from the dynamical channel with respect to BBH mergers from the isolated evolutionary channel (e.g., [298, 62, 309, 373, 385]).

1.5.2 Cosmological simulations

The alternative approach to reconstruct the BBH merger history feeds the outputs of population-synthesis simulations into cosmological simulations [265, 314, 228, 227, 230, 229, 70, 237, 159, 33, 32, 31], through a Monte Carlo approach. This has the

⁴ We define as time delay the time elapsed between the formation of the progenitor binary and the merger of the two BHs.

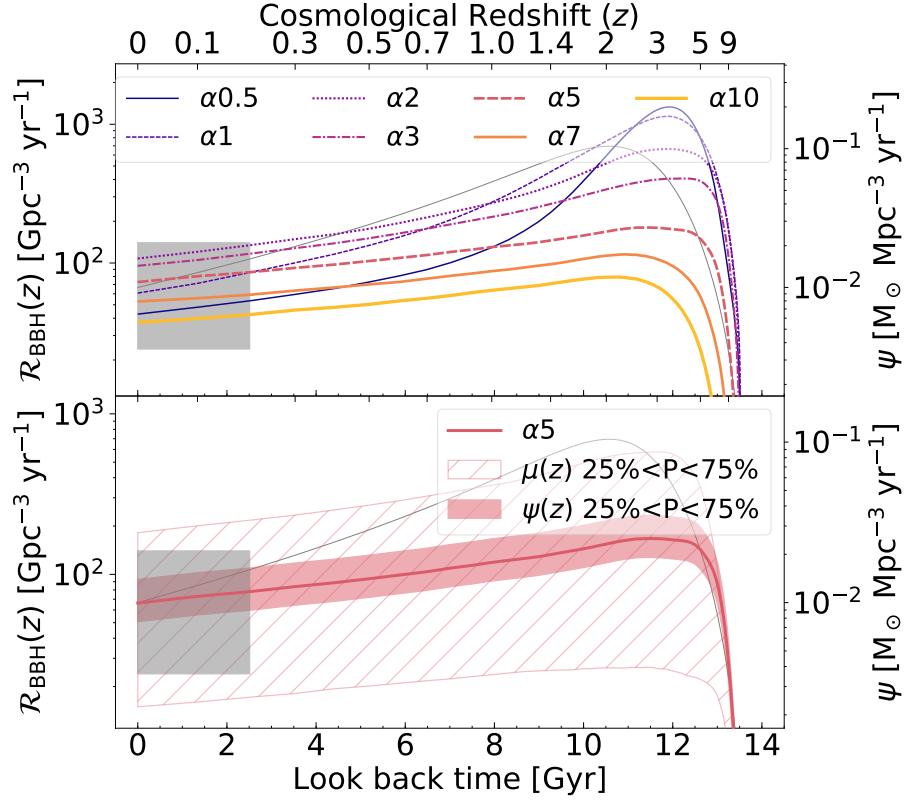


Fig. 1.14 Cosmic merger rate density of BBHs in the comoving frame (\mathcal{R}_{BBH}) as a function of the look-back time (t_{bb} , lower x -axis) and of the redshift (z , upper x -axis) from some of the models presented in [310], which are based on equation 1.28. We use catalogues of population synthesis simulations run with MOBSE [228, 147]. In both panels: thin gray line and right y -axis: cosmic star formation rate density as a function of redshift from [215]. Gray shaded box: 90% credible interval for the local merger rate density of BBHs, as inferred from the LVC data, considering the union of the rates obtained with model A, B and C in [7]. The width of the grey shaded area on the x -axis corresponds to the instrumental horizon obtained by assuming BBHs with total mass of $20 M_{\odot}$ and O2 sensitivity [9]. Upper panel: BBH merger rate density for different assumptions on the α parameter (i.e. the efficiency of CE ejection). Models for $\alpha = 0.5, 1, 2, 3, 5, 7$ and 10 are shown. Lower panel: the solid red line shows the median value of the BBH merger rate density for $\alpha = 5$, while the shaded areas show the 50% credible areas estimated from the uncertainties on metallicity evolution (hatched region) and star formation rate density evolution (shaded region). The cosmic star formation rate density evolution is modelled as in [215] and is obtained by fitting data. The metallicity evolution uses the fitting formula by [90], adapted as described in [309]. Courtesy of Filippo Santoliquido.

clear advantage that the properties of the host galaxies can be easily reconstructed across cosmic time. However, the ideal thing would be to have a high-resolution cosmological simulation (sufficient to resolve also small dwarf galaxies) with a box as large as the instrumental horizon of the GW detectors. This is obviously impossible. High-resolution simulations have usually a box of a few comoving Mpc³, while simulations with a larger box cannot resolve dwarf galaxies. Moreover, this procedure requires to use the cosmic star formation rate density and the redshift-dependent MZR which are intrinsic to the cosmological simulations. While most state-of-the-art cosmological simulations reproduce the cosmic star formation rate density reasonably well, the MZR is an elusive feature, creating more than a trouble even in the most advanced cosmological simulations.

For example, [230] investigate the main properties of the host galaxies of merging BHs, by combining their population-synthesis simulations [228, 227] with the ILLUSTRIS cosmological box [364, 365, 257]. The size of the ILLUSTRIS (length = 106.5 comoving Mpc) is sufficient to satisfy the cosmological principle, but galaxies with stellar mass $\lesssim 10^8 M_\odot$ are heavily under-resolved. Their results show that BHs merging in the local Universe ($z < 0.024$) have formed in galaxies with relatively small stellar mass ($< 10^{10} M_\odot$) and relatively low metallicity ($Z \leq 0.1 Z_\odot$). These BHs reach coalescence either in the galaxy where they formed or in larger galaxies (with stellar mass up to $\sim 10^{12} M_\odot$). In fact, most BHs reaching coalescence in the local Universe appear to have formed in the early Universe ($\gtrsim 9$ Gyr ago), when metal-poor galaxies were more common. A significant fraction of these high-redshift metal-poor galaxies merged within larger galaxies before the BBHs reached coalescence by GWs. Moreover, these models show that the mass spectrum and the other main properties of BBHs do not evolve significantly with redshift [229].

Schneider et al. (2017, [314], see also [237, 159]) adopt a complementary approach to study the importance of dwarf galaxies for GW detections. They use the GAMESH pipeline to produce a high-resolution simulation of the Local Group (length = 4 Mpc comoving). This means that the considered portion of the Universe is strongly biased, but the resolution is sufficient to investigate BH binaries in small ($\gtrsim 10^6 M_\odot$) dwarf galaxies. One of their main conclusions is that GW150914-like events originate mostly from small metal-poor galaxies.

Similarly, Cao et al. (2018, [70]) investigate the host galaxies of BBHs by applying a semi-analytic model to the Millennium-II N-body simulation [64]. The Millennium-II N-body simulation is a large-box (length = 137 comoving Mpc) dark-matter only simulation. The physics of baryons is implemented through a semi-analytic model. Using a dark-matter only simulation coupled with a semi-analytic approach offers the possibility of improving the resolution significantly, but baryons are added only in post-processing. With this approach, Cao et al. (2018, [70]) find that BBHs merging at redshift $z \lesssim 0.3$ are located mostly in massive galaxies (stellar mass $\gtrsim 2 \times 10^{10} M_\odot$).

Finally, [33, 32, 31] combine population-synthesis simulations with the EAGLE cosmological simulation [312]. They show that the merger rate per galaxy strongly correlates with the stellar mass of a galaxy for both BBHs, BHNSs and BNSs. They

also find a dependence of the merger rate per galaxy on the star formation rate and a weaker dependence on the metallicity, but the correlation with the stellar mass is definitely the strongest one. This result has been used to optimize electromagnetic follow-up strategies, weighting the galaxies in the LVC uncertainty box by their stellar mass [101, 22]. These studies show that the combination of population-synthesis tools with cosmological simulations is an effective approach to understand the cosmic evolution of the merger rate and the properties of the host galaxies of BBH mergers.

1.6 Summary and outlook

We reviewed our current understanding of the formation channels of BBHs. The last five years have witnessed considerable progress in this field, thanks to the ground-breaking results of the LVC and to a renewed interest in BH astrophysics, mostly triggered by GW data.

The final stages of massive star evolution deserve particular attention to understand the mass and spins of BHs. GW data have recently challenged the concept of pair-instability mass gap, i.e. the existence of a mass range (between $\sim 65 M_{\odot}$ and $\sim 120 M_{\odot}$) in which we do not expect to find BHs as an effect of (pulsational) pair instability.

The process of mass transfer between two massive stars in a binary system is another key aspect shaping the demography of BBHs. The efficiency of mass accretion via Roche lobe overflow and the stability of mass transfer are possibly the main unknowns affecting the mass and the delay time of a BBH. In addition, the dynamical formation channel with its richness and multifaceted processes (hardening, exchanges, ejections, runaway collisions, hierarchical mergers, etc) adds to the complexity of the general picture.

Current astrophysical models of BBH formation face a number of essential questions:

- impact of core overshooting, rotation and possibly magnetic fields on massive star evolution and BH formation;
- angular momentum transfer in massive stars and its link to BH spins;
- explodability of massive stars;
- stability and efficiency of mass transfer;
- evolution of common-envelope systems;
- physics of stellar collisions and mergers;
- impact of the environment (e.g., metallicity, star clusters versus field) on the formation of BBHs across cosmic time.

Overall, we have a better understanding of the main dynamical processes with respect to both massive star evolution and SN physics. But our understanding of BBH dynamics is mainly hampered by two problems: i) we cannot properly model

the dynamics of BBHs if we do not know BH masses and spins from stellar evolution; ii) dynamical simulations are still too computationally expensive to allow us to investigate the relevant parameter space.

Next-generation ground-based detectors will observe BBH mergers up to redshift $z \gtrsim 10$, beyond the epoch of cosmic reionization [179]. This will open a completely new scenario for the study of BBHs across cosmic time and for the characterization of their evolutionary pathways.

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References

- [1] J Aasi, B P Abbott, R Abbott, T Abbott, M R Abernathy, K Ackley, C Adams, T Adams, P Addesso, R X Adhikari, V Adya, C Affeldt, N Aggarwal, O D Aguiar, A Ain, P Ajith, and et al. Advanced ligo. *Classical and Quantum Gravity*, 32(7):074001, 2015.
- [2] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al. Astrophysical Implications of the Binary Black-hole Merger GW150914. *Astrophysical Journal Letter*, 818:L22, February 2016.
- [3] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al. Binary Black Hole Mergers in the First Advanced LIGO Observing Run. *Physics Review X*, 6(4):041015, October 2016.
- [4] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al. GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence. *Physics Review Letters*, 116(24):241103, June 2016.
- [5] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al. Observation of gravitational waves from a binary black hole merger. *Phys. Rev. Lett.*, 116:061102, Feb 2016.
- [6] B. P. Abbott, R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Agathos, K. Agatsuma, and et al. GW190425: Observation of a Compact Binary Coalescence with Total Mass $\sim 3.4 M_{\odot}$. *Astrophysical Journal Letter*, 892(1):L3, March 2020.
- [7] B. P. Abbott, R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Agathos, and et al. Binary Black Hole Population Properties Inferred from the First and Sec-

- ond Observing Runs of Advanced LIGO and Advanced Virgo. *Astrophysical Journal Letter*, 882(2):L24, Sep 2019.
- [8] B. P. Abbott, R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Agathos, and et al. GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs. *Physics Review X*, 9(3):031040, Jul 2019.
 - [9] B. P. Abbott, R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, V. B. Adya, C. Affeldt, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, and et al. Prospects for observing and localizing gravitational-wave transients with Advanced LIGO, Advanced Virgo and KAGRA. *Living Reviews in Relativity*, 23(1):3, September 2020.
 - [10] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, and et al. GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2. *Physics Review Letters*, 118(22):221101, June 2017.
 - [11] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, and et al. Multi-messenger Observations of a Binary Neutron Star Merger. *Astrophysical Journal Letter*, 848:L12, October 2017.
 - [12] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al. Gravitational waves and gamma-rays from a binary neutron star merger: Gw170817 and grb 170817a. *The Astrophysical Journal Letters*, 848(2):L13, 2017.
 - [13] D. C. Abbott. The theory of radiatively driven stellar winds. II. The line acceleration. *Astrophysical Journal*, 259:282–301, August 1982.
 - [14] R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, A. Adams, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, S. Akcay, G. Allen, A. Allocata, P. A. Altin, A. Amato, S. Anand, A. Ananyeva, S. B. Anderson, W. G. Anderson, S. V. Angelova, S. Ansoldi, J. M. Antelis, S. Antier, S. Appert, K. Arai, M. C. Araya, J. S. Areeda, M. Arène, N. Arnaud, S. M. Aronson, K. G. Arun, Y. Asali, S. Ascenzi, G. Ashton, S. M. Aston, P. Astone, F. Aubin, P. Aufmuth, K. AultONeal, C. Austin, V. Avendano, S. Babak, F. Badaracco, M. K. M. Bader, S. Bae, A. M. Baer, S. Bagnasco, J. Baird, M. Ball, G. Ballardin, S. W. Ballmer, A. Bals, A. Balsamo, G. Baltus, S. Banagiri, D. Bankar, R. S. Bankar, J. C. Barayoga, C. Barbieri, B. C. Barish, D. Barker, P. Barneo, S. Barnum, F. Barone, B. Barr, L. Barsotti, M. Barsuglia, D. Barta, J. Bartlett, I. Bartos, R. Bassiri, A. Basti, M. Bawaj, J. C. Bayley, M. Bazzan, B. R. Becher, B. Bécsy, V. M. Bedakihale, M. Bejger, I. Belahcene, D. Beniwal, M. G. Benjamin, T. F. Bennett, and et al. GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo during the First Half of the Third Observing Run. *Physical Review X*, 11(2):021053, April 2021.

- [15] R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, A. Adams, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, and et al. Population Properties of Compact Objects from the Second LIGO-Virgo Gravitational-Wave Transient Catalog. *Astrophysical Journal Letter*, 913(1):L7, May 2021.
- [16] R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, A. Adams, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, and et al. Tests of general relativity with binary black holes from the second LIGO-Virgo gravitational-wave transient catalog. *Physics Review D*, 103(12):122002, June 2021.
- [17] R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, A. Aich, L. Aiello, A. Ain, P. Ajith, S. Akcay, G. Allen, and et al. GW190412: Observation of a binary-black-hole coalescence with asymmetric masses. *Physics Review D*, 102(4):043015, August 2020.
- [18] R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, A. Aich, L. Aiello, A. Ain, P. Ajith, and et al. GW190521: A Binary Black Hole Merger with a Total Mass of $150 M_{\odot}$. *Physics Review Letters*, 125(10):101102, September 2020.
- [19] R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, A. Aich, L. Aiello, A. Ain, P. Ajith, and et al. GW190814: Gravitational Waves from the Coalescence of a 23 Solar Mass Black Hole with a 2.6 Solar Mass Compact Object. *Astrophysical Journal Letter*, 896(2):L44, June 2020.
- [20] R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, A. Aich, L. Aiello, A. Ain, P. Ajith, and et al. Properties and Astrophysical Implications of the $150 M_{\odot}$ Binary Black Hole Merger GW190521. *Astrophysical Journal Letter*, 900(1):L13, September 2020.
- [21] F. Acernese, M. Agathos, K. Agatsuma, D. Aisa, N. Allemandou, A. Allocata, J. Amarni, P. Astone, G. Balestri, G. Ballardin, and et al. Advanced Virgo: a second-generation interferometric gravitational wave detector. *Classical and Quantum Gravity*, 32(2):024001, January 2015.
- [22] K. Ackley, L. Amati, C. Barbieri, F. E. Bauer, S. Benetti, M. G. Bernardini, K. Bhirombhakdi, M. T. Botticella, M. Branchesi, E. Brocato, S. H. Bruun, M. Bulla, S. Campana, E. Cappellaro, A. J. Castro-Tirado, K. C. Chambers, S. Chaty, T. W. Chen, R. Ciolfi, A. Coleiro, C. M. Copperwheat, S. Covino, R. Cutter, F. D'Ammando, P. D'Avanzo, G. De Cesare, V. D'Elia, M. Della Valle, L. Denneau, M. De Pasquale, V. S. Dhillon, M. J. Dyer, N. Elias-Rosa, P. A. Evans, R. A. J. Eyles-Ferris, A. Fiore, M. Fraser, A. S. Fruchter, J. P. U. Fynbo, L. Galbany, C. Gall, D. K. Galloway, F. I. Getman, G. Ghirlanda, J. H. Gillanders, A. Gomboc, B. P. Gompertz, C. González-

- Fernández, S. González-Gaitán, A. Grado, G. Greco, M. Gromadzki, P. J. Groot, C. P. Gutiérrez, T. Heikkilä, K. E. Heintz, J. Hjorth, Y. D. Hu, M. E. Huber, C. Inserra, L. Izzo, J. Japelj, A. Jerkstrand, Z. P. Jin, P. G. Jonker, E. Kankare, D. A. Kann, M. Kennedy, S. Kim, S. Klose, E. C. Kool, R. Kotak, H. Kuncarayakti, G. P. Lamb, G. Leloudas, A. J. Levan, F. Longo, T. B. Lowe, J. D. Lyman, E. Magnier, K. Maguire, E. Maiorano, I. Mandel, M. Mapelli, S. Mattila, O. R. McBrien, A. Melandri, M. J. Michałowski, B. Milvang-Jensen, S. Moran, L. Nicastro, M. Nicholl, A. Nicuesa Guelbenzu, L. Nuttal, S. R. Oates, P. T. O’Brien, F. Onori, E. Palazzi, B. Patricelli, A. Perego, M. A. P. Torres, D. A. Perley, E. Pian, G. Pignata, S. Piranomonte, S. Poshyachinda, A. Possenti, M. L. Pumo, J. Quirola-Vásquez, F. Ragosta, G. Ramsay, A. Rau, A. Rest, T. M. Reynolds, S. S. Rosetti, A. Rossi, S. Rosswoog, N. B. Sabha, A. Sagués Carracedo, O. S. Salafia, L. Salmon, R. Salvaterra, S. Savaglio, L. Sbordone, P. Schady, P. Schipani, A. S. B. Schultz, T. Schweyer, S. J. Smartt, K. W. Smith, M. Smith, J. Sollerman, S. Srivastav, E. R. Stanway, R. L. C. Starling, D. Steeghs, G. Stratta, C. W. Stubbs, N. R. Tanvir, V. Testa, E. Thrane, J. L. Tonry, M. Turatto, K. Ulaczyk, A. J. van der Horst, S. D. Vergani, N. A. Walton, D. Watson, K. Wiersema, K. Wiik, Ł. Wyrzykowski, S. Yang, S. X. Yi, and D. R. Young. Observational constraints on the optical and near-infrared emission from the neutron star-black hole binary merger candidate S190814bv. *Astronomy & Astrophysics*, 643:A113, November 2020.
- [23] S. M. Adams, C. S. Kochanek, J. R. Gerke, K. Z. Stanek, and X. Dai. The search for failed supernovae with the Large Binocular Telescope: confirmation of a disappearing star. *MNRAS*, 468(4):4968–4981, July 2017.
 - [24] K. D. Alexander, E. Berger, W. Fong, P. K. G. Williams, C. Guidorzi, R. Margutti, B. D. Metzger, J. Annis, P. K. Blanchard, D. Brout, D. A. Brown, H.-Y. Chen, R. Chornock, P. S. Cowperthwaite, M. Drout, T. Eftekhar, J. Frieman, D. E. Holz, M. Nicholl, A. Rest, M. Sako, M. Soares-Santos, and V. A. Villar. The electromagnetic counterpart of the binary neutron star merger ligo/virgo gw170817. vi. radio constraints on a relativistic jet and predictions for late-time emission from the kilonova ejecta. *The Astrophysical Journal Letters*, 848(2):L21, 2017.
 - [25] F. Antonini and F. A. Rasio. Merging Black Hole Binaries in Galactic Nuclei: Implications for Advanced-LIGO Detections. *Astrophysical Journal*, 831:187, November 2016.
 - [26] F. Antonini, S. Toonen, and A. S. Hamers. Binary Black Hole Mergers from Field Triples: Properties, Rates, and the Impact of Stellar Evolution. *Astrophysical Journal*, 841:77, June 2017.
 - [27] Fabio Antonini and Mark Gieles. Population synthesis of black hole binary mergers from star clusters. *MNRAS*, 492(2):2936–2954, February 2020.
 - [28] Fabio Antonini, Mark Gieles, and Alessia Gualandris. Black hole growth through hierarchical black hole mergers in dense star clusters: implications for gravitational wave detections. *MNRAS*, 486(4):5008–5021, July 2019.

- [29] Fabio Antonini and Hagai B. Perets. Secular Evolution of Compact Binaries near Massive Black Holes: Gravitational Wave Sources and Other Exotica. *Astrophysical Journal*, 757(1):27, September 2012.
- [30] Manuel Arca Sedda, Abbas Askar, and Mirek Giersz. MOCCA-Survey Database - I. Unravelling black hole subsystems in globular clusters. *MNRAS*, 479(4):4652–4664, October 2018.
- [31] M. Celeste Artale, Yann Bouffanais, Michela Mapelli, Nicola Giacobbo, Nadeen B. Sabha, Filippo Santoliquido, Mario Pasquato, and Mario Spera. An astrophysically motivated ranking criterion for low-latency electromagnetic follow-up of gravitational wave events. *MNRAS*, 495(2):1841–1852, May 2020.
- [32] M. Celeste Artale, Michela Mapelli, Yann Bouffanais, Nicola Giacobbo, Mario Pasquato, and Mario Spera. Mass and star formation rate of the host galaxies of compact binary mergers across cosmic time. *MNRAS*, 491(3):3419–3434, January 2020.
- [33] M. Celeste Artale, Michela Mapelli, Nicola Giacobbo, Nadeen B. Sabha, Mario Spera, Filippo Santoliquido, and Alessandro Bressan. Host galaxies of merging compact objects: mass, star formation rate, metallicity, and colours. *MNRAS*, 487(2):1675–1688, August 2019.
- [34] Z. Arzoumanian, D. F. Chernoff, and J. M. Cordes. The Velocity Distribution of Isolated Radio Pulsars. *Astrophysical Journal*, 568:289–301, March 2002.
- [35] A. Askar, M. Arca Sedda, and M. Giersz. MOCCA-SURVEY Database I: Galactic globular clusters harbouring a black hole subsystem. *MNRAS*, 478:1844–1854, August 2018.
- [36] A. Askar, M. Szkudlarek, D. Gondek-Rosińska, M. Giersz, and T. Bulik. MOCCA-SURVEY Database - I. Coalescing binary black holes originating from globular clusters. *MNRAS*, 464:L36–L40, January 2017.
- [37] Vishal Baibhav, Emanuele Berti, Davide Gerosa, Michela Mapelli, Nicola Giacobbo, Yann Bouffanais, and Ugo N. Di Carlo. Gravitational-wave detection rates for compact binaries formed in isolation: LIGO/Virgo O3 and beyond. *Physics Review D*, 100(6):064060, September 2019.
- [38] S. Banerjee. Stellar-mass black holes in young massive and open stellar clusters and their role in gravitational-wave generation. *MNRAS*, 467:524–539, May 2017.
- [39] S. Banerjee. Stellar-mass black holes in young massive and open stellar clusters and their role in gravitational-wave generation - II. *MNRAS*, 473:909–926, January 2018.
- [40] S. Banerjee, H. Baumgardt, and P. Kroupa. Stellar-mass black holes in star clusters: implications for gravitational wave radiation. *MNRAS*, 402:371–380, February 2010.
- [41] Sambaran Banerjee. Stellar-mass black holes in young massive and open stellar clusters - IV. Updated stellar-evolutionary and black hole spin models and comparisons with the LIGO-Virgo O1/O2 merger-event data. *MNRAS*, 500(3):3002–3026, January 2021.

- [42] Z. Barkat, G. Rakavy, and N. Sack. Dynamics of Supernova Explosion Resulting from Pair Formation. *Physics Review Letters*, 18(10):379–381, March 1967.
- [43] Jim W. Barrett, Sebastian M. Gaebel, Coenraad J. Neijssel, Alejandro Vigna-Gómez, Simon Stevenson, Christopher P. L. Berry, Will M. Farr, and Ilya Mandel. Accuracy of inference on the physics of binary evolution from gravitational-wave observations. *MNRAS*, 477(4):4685–4695, July 2018.
- [44] Imre Bartos, Bence Kocsis, Zoltán Haiman, and Szabolcs Márka. Rapid and Bright Stellar-mass Binary Black Hole Mergers in Active Galactic Nuclei. *Astrophysical Journal*, 835(2):165, February 2017.
- [45] Simone S. Bavera, Tassos Fragos, Ying Qin, Emmanouil Zapartas, Coenraad J. Neijssel, Ilya Mandel, Aldo Batta, Sebastian M. Gaebel, Chase Kimball, and Simon Stevenson. The origin of spin in binary black holes. Predicting the distributions of the main observables of Advanced LIGO. *Astronomy & Astrophysics*, 635:A97, March 2020.
- [46] Simone S. Bavera, Tassos Fragos, Michael Zevin, Christopher P. L. Berry, Pablo Marchant, Jeff J. Andrews, Scott Coughlin, Aaron Dotter, Konstantinos Kovlakas, Devina Misra, Juan G. Serra-Perez, Ying Qin, Kyle A. Rocha, Jaime Román-Garza, Nam H. Tran, and Emmanouil Zapartas. The impact of mass-transfer physics on the observable properties of field binary black hole populations. *Astronomy & Astrophysics*, 647:A153, March 2021.
- [47] Simone S. Bavera, Michael Zevin, and Tassos Fragos. Approximations to the spin of close Black-hole-Wolf-Rayet binaries. *arXiv e-prints*, page arXiv:2105.09077, May 2021.
- [48] K. Belczynski, T. Bulik, C. L. Fryer, A. Ruiter, F. Valsecchi, J. S. Vink, and J. R. Hurley. On the Maximum Mass of Stellar Black Holes. *Astrophysical Journal*, 714:1217–1226, May 2010.
- [49] K. Belczynski, A. Heger, W. Gladysz, A. J. Ruiter, S. Woosley, G. Wiktorowicz, H.-Y. Chen, T. Bulik, R. O’Shaughnessy, D. E. Holz, C. L. Fryer, and E. Berti. The effect of pair-instability mass loss on black-hole mergers. *Astronomy & Astrophysics*, 594:A97, October 2016.
- [50] K. Belczynski, D. E. Holz, T. Bulik, and R. O’Shaughnessy. The first gravitational-wave source from the isolated evolution of two stars in the 40–100 solar mass range. *Nature*, 534:512–515, June 2016.
- [51] K. Belczynski, V. Kalogera, F. A. Rasio, R. E. Taam, A. Zezas, T. Bulik, T. J. Maccarone, and N. Ivanova. Compact Object Modeling with the StarTrack Population Synthesis Code. *Astrophysical Journal Supplement*, 174:223–260, January 2008.
- [52] K. Belczynski, J. Klencki, C. E. Fields, A. Olejak, E. Berti, G. Meynet, C. L. Fryer, D. E. Holz, R. O’Shaughnessy, D. A. Brown, T. Bulik, S. C. Leung, K. Nomoto, P. Madau, R. Hirschi, E. Kaiser, S. Jones, S. Mondal, M. Chruslinska, P. Drozda, D. Gerosa, Z. Doctor, M. Giersz, S. Ekstrom, C. Georgy, A. Askar, V. Baibhav, D. Wysocki, T. Natan, W. M. Farr, G. Wiktorowicz, M. Coleman Miller, B. Farr, and J. P. Lasota. Evolutionary roads leading to low effective spins, high black hole masses, and O1/O2 rates for

- LIGO/Virgo binary black holes. *Astronomy & Astrophysics*, 636:A104, April 2020.
- [53] K. Belczynski, A. Sadowski, and F. A. Rasio. A Comprehensive Study of Young Black Hole Populations. *Astrophysical Journal*, 611:1068–1079, August 2004.
 - [54] Matthew J. Benacquista and Jonathan M. B. Downing. Relativistic Binaries in Globular Clusters. *Living Reviews in Relativity*, 16(1):4, March 2013.
 - [55] P. Beniamini and T. Piran. Formation of double neutron star systems as implied by observations. *MNRAS*, 456:4089–4099, March 2016.
 - [56] H. A. Bethe and G. E. Brown. Evolution of Binary Compact Objects That Merge. *Astrophysical Journal*, 506:780–789, October 1998.
 - [57] H. A. Bethe and J. R. Wilson. Revival of a stalled supernova shock by neutrino heating. *Astrophysical Journal*, 295:14–23, August 1985.
 - [58] S. Bird, I. Cholis, J. B. Muñoz, Y. Ali-Haïmoud, M. Kamionkowski, E. D. Kovetz, A. Raccanelli, and A. G. Riess. Did LIGO Detect Dark Matter? *Physics Review Letters*, 116(20):201301, May 2016.
 - [59] L. Boco, A. Lapi, S. Goswami, F. Perrotta, C. Baccigalupi, and L. Danese. Merging Rates of Compact Binaries in Galaxies: Perspectives for Gravitational Wave Detections. *Astrophysical Journal*, 881(2):157, August 2019.
 - [60] Torsten Böker, Seppo Laine, Roeland P. van der Marel, Marc Sarzi, Hans-Walter Rix, Luis C. Ho, and Joseph C. Shields. A Hubble Space Telescope Census of Nuclear Star Clusters in Late-Type Spiral Galaxies. I. Observations and Image Analysis. *Astronomical Journal*, 123(3):1389–1410, March 2002.
 - [61] H. Bondi and F. Hoyle. On the mechanism of accretion by stars. *MNRAS*, 104:273, January 1944.
 - [62] Yann Bouffanais, Michela Mapelli, Davide Gerosa, Ugo N. Di Carlo, Nicola Giacobbo, Emanuele Berti, and Vishal Baibhav. Constraining the Fraction of Binary Black Holes Formed in Isolation and Young Star Clusters with Gravitational-wave Data. *Astrophysical Journal*, 886(1):25, November 2019.
 - [63] Yann Bouffanais, Michela Mapelli, Filippo Santoliquido, Nicola Giacobbo, Giuliano Iorio, and Guglielmo Costa. Constraining accretion efficiency in massive binary stars with LIGO -Virgo black holes. *MNRAS*, 505(3):3873–3882, August 2021.
 - [64] M. Boylan-Kolchin, V. Springel, S. D. M. White, A. Jenkins, and G. Lemson. Resolving cosmic structure formation with the Millennium-II Simulation. *MNRAS*, 398:1150–1164, September 2009.
 - [65] J. C. Bray and J. J. Eldridge. Neutron star kicks and their relationship to supernovae ejecta mass. *MNRAS*, 461(4):3747–3759, October 2016.
 - [66] J. C. Bray and J. J. Eldridge. Neutron star kicks - II. Revision and further testing of the conservation of momentum ‘kick’ model. *MNRAS*, 480(4):5657–5672, November 2018.
 - [67] Philip G. Breen and Douglas C. Heggie. Dynamical evolution of black hole subsystems in idealized star clusters. *MNRAS*, 432(4):2779–2797, July 2013.

- [68] A. Burrows, D. Vartanyan, J. C. Dolence, M. A. Skinner, and D. Radice. Crucial Physical Dependencies of the Core-Collapse Supernova Mechanism. *Space Science Reviews*, 214(1):33, February 2018.
- [69] Manuela Campanelli, Carlos Lousto, Yosef Zlochower, and David Merritt. Large Merger Recoils and Spin Flips from Generic Black Hole Binaries. *Astrophysical Journal Letter*, 659(1):L5–L8, April 2007.
- [70] L. Cao, Y. Lu, and Y. Zhao. Host galaxy properties of mergers of stellar binary black holes and their implications for advanced LIGO gravitational wave sources. *MNRAS*, 474:4997–5007, March 2018.
- [71] B. Carr, F. Kühnel, and M. Sandstad. Primordial black holes as dark matter. *Physics Review D*, 94(8):083504, October 2016.
- [72] B. J. Carr and S. W. Hawking. Black holes in the early Universe. *MNRAS*, 168:399–416, Aug 1974.
- [73] S. Chandrasekhar. The Maximum Mass of Ideal White Dwarfs. *Astrophysical Journal*, 74:81, July 1931.
- [74] S. Chandrasekhar. Dynamical Friction. I. General Considerations: the Coefficient of Dynamical Friction. *Astrophysical Journal*, 97:255, March 1943.
- [75] Y. Chen, A. Bressan, L. Girardi, P. Marigo, X. Kong, and A. Lanza. PARSEC evolutionary tracks of massive stars up to $350 M_{\odot}$ at metallicities $0.0001 \leq Z \leq 0.04$. *MNRAS*, 452:1068–1080, September 2015.
- [76] A. Chieffi and M. Limongi. Pre-supernova Evolution of Rotating Solar Metallicity Stars in the Mass Range $13\text{--}120 M_{\odot}$ and their Explosive Yields. *Astrophysical Journal*, 764:21, February 2013.
- [77] Alessandro Chieffi and Marco Limongi. The Presupernova Core Mass-Radius Relation of Massive Stars: Understanding Its Formation and Evolution. *Astrophysical Journal*, 890(1):43, February 2020.
- [78] R. Chornock, E. Berger, D. Kasen, P. S. Cowperthwaite, M. Nicholl, V. A. Villar, K. D. Alexander, P. K. Blanchard, T. Eftekhari, W. Fong, R. Margutti, and et al. The electromagnetic counterpart of the binary neutron star merger ligo/virgo gw170817. iv. detection of near-infrared signatures of r-process nucleosynthesis with gemini-south. *The Astrophysical Journal Letters*, 848(2):L19, 2017.
- [79] M. Chruścińska, T. Jeřábková, G. Nelemans, and Z. Yan. The effect of the environment-dependent IMF on the formation and metallicities of stars over the cosmic history. *Astronomy & Astrophysics*, 636:A10, April 2020.
- [80] Martyna Chruslinska and Gijs Nelemans. Metallicity of stars formed throughout the cosmic history based on the observational properties of star-forming galaxies. *MNRAS*, 488(4):5300–5326, October 2019.
- [81] Martyna Chruslinska, Gijs Nelemans, and Krzysztof Belczynski. The influence of the distribution of cosmic star formation at different metallicities on the properties of merging double compact objects. *MNRAS*, 482(4):5012–5017, February 2019.
- [82] Drew Clausen, Anthony L. Piro, and Christian D. Ott. The Black Hole Formation Probability. *Astrophysical Journal*, 799(2):190, February 2015.

- [83] S. A. Colgate. Stellar Coalescence and the Multiple Supernova Interpretation of Quasi-Stellar Sources. *Astrophysical Journal*, 150:163, October 1967.
- [84] Stirling A. Colgate and Richard H. White. The Hydrodynamic Behavior of Supernovae Explosions. *Astrophysical Journal*, 143:626, March 1966.
- [85] M. Colpi, M. Mapelli, and A. Possenti. Probing the Presence of a Single or Binary Black Hole in the Globular Cluster NGC 6752 with Pulsar Dynamics. *Astrophysical Journal*, 599:1260–1271, December 2003.
- [86] Guglielmo Costa, Alessandro Bressan, Michela Mapelli, Paola Marigo, Giuliano Iorio, and Mario Spera. Formation of GW190521 from stellar evolution: the impact of the hydrogen-rich envelope, dredge-up, and $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate on the pair-instability black hole mass gap. *MNRAS*, 501(3):4514–4533, March 2021.
- [87] D. A. Coulter, R. J. Foley, C. D. Kilpatrick, M. R. Drout, A. L. Piro, B. J. Shappee, M. R. Siebert, J. D. Simon, N. Ulloa, D. Kasen, B. F. Madore, A. Murguia-Berthier, Y.-C. Pan, J. X. Prochaska, E. Ramirez-Ruiz, A. Rest, and C. Rojas-Bravo. Swope Supernova Survey 2017a (SSS17a), the optical counterpart to a gravitational wave source. *Science*, 358:1556–1558, December 2017.
- [88] P. S. Cowperthwaite, E. Berger, V. A. Villar, B. D. Metzger, M. Nicholl, R. Chornock, P. K. Blanchard, W. Fong, R. Margutti, M. Soares-Santos, K. D. Alexander, S. Allam, J. Annis, D. Brout, D. A. Brown, R. E. Butler, H.-Y. Chen, H. T. Diehl, Z. Doctor, and et al. The electromagnetic counterpart of the binary neutron star merger ligo/virgo gw170817. ii. uv, optical, and near-infrared light curves and comparison to kilonova models. *The Astrophysical Journal Letters*, 848(2):L17, 2017.
- [89] Soumi De, Morgan MacLeod, Rosa Wallace Everson, Andrea Antoni, Ilya Mandel, and Enrico Ramirez-Ruiz. Common Envelope Wind Tunnel: The Effects of Binary Mass Ratio and Implications for the Accretion-driven Growth of LIGO Binary Black Holes. *Astrophysical Journal*, 897(2):130, July 2020.
- [90] Annalisa De Cia, Cédric Ledoux, Patrick Petitjean, and Sandra Savaglio. The cosmic evolution of dust-corrected metallicity in the neutral gas. *Astronomy & Astrophysics*, 611:A76, April 2018.
- [91] S. E. de Mink and I. Mandel. The chemically homogeneous evolutionary channel for binary black hole mergers: rates and properties of gravitational-wave events detectable by advanced LIGO. *MNRAS*, 460:3545–3553, August 2016.
- [92] R. J. deBoer, J. Görres, M. Wiescher, R. E. Azuma, A. Best, C. R. Brune, C. E. Fields, S. Jones, M. Pignatari, D. Sayre, K. Smith, F. X. Timmes, and E. Uberseder. The $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction and its implications for stellar helium burning. *Reviews of Modern Physics*, 89(3):035007, July 2017.
- [93] Ugo N. Di Carlo, Nicola Giacobbo, Michela Mapelli, Mario Pasquato, Mario Spera, Long Wang, and Francesco Haardt. Merging black holes in young star clusters. *MNRAS*, 487(2):2947–2960, Aug 2019.
- [94] Ugo N. Di Carlo, Michela Mapelli, Yann Bouffanais, Nicola Giacobbo, Filippo Santoliquido, Alessandro Bressan, Mario Spera, and Francesco Haardt.

- Binary black holes in the pair instability mass gap. *MNRAS*, 497(1):1043–1049, July 2020.
- [95] Ugo N. Di Carlo, Michela Mapelli, Nicola Giacobbo, Mario Spera, Yann Bouffanais, Sara Rastello, Filippo Santoliquido, Mario Pasquato, Alessandro Ballone, Alessandro A. Trani, Stefano Torniamenti, and Francesco Haardt. Binary black holes in young star clusters: the impact of metallicity. *MNRAS*, 498(1):495–506, August 2020.
- [96] M. Dominik, K. Belczynski, C. Fryer, D. E. Holz, E. Berti, T. Bulik, I. Mandel, and R. O’Shaughnessy. Double Compact Objects. II. Cosmological Merger Rates. *Astrophysical Journal*, 779:72, December 2013.
- [97] M. Dominik, E. Berti, R. O’Shaughnessy, I. Mandel, K. Belczynski, C. Fryer, D. E. Holz, T. Bulik, and F. Pannarale. Double Compact Objects III: Gravitational-wave Detection Rates. *Astrophysical Journal*, 806:263, June 2015.
- [98] J. M. B. Downing, M. J. Benacquista, M. Giersz, and R. Spurzem. Compact binaries in star clusters - I. Black hole binaries inside globular clusters. *MNRAS*, 407:1946–1962, September 2010.
- [99] J. M. B. Downing, M. J. Benacquista, M. Giersz, and R. Spurzem. Compact binaries in star clusters - II. Escapers and detection rates. *MNRAS*, 416:133–147, September 2011.
- [100] Gaspard Duchêne and Adam Kraus. Stellar Multiplicity. *Annual Review of Astronomy and Astrophysics*, 51(1):269–310, August 2013.
- [101] J. G. Ducoin, D. Corre, N. Leroy, and E. Le Floch. Optimizing gravitational waves follow-up using galaxies stellar mass. *MNRAS*, 492(4):4768–4779, March 2020.
- [102] I. Dvorkin, E. Vangioni, J. Silk, J.-P. Uzan, and K. A. Olive. Metallicity-constrained merger rates of binary black holes and the stochastic gravitational wave background. *MNRAS*, 461:3877–3885, October 2016.
- [103] Irina Dvorkin, Jean-Philippe Uzan, Elisabeth Vangioni, and Joseph Silk. Exploring stellar evolution with gravitational-wave observations. *MNRAS*, 479(1):121–129, September 2018.
- [104] P. P. Eggleton. Approximations to the radii of Roche lobes. *Astrophysical Journal*, 268:368–369, May 1983.
- [105] Peter Eggleton. *Evolutionary Processes in Binary and Multiple Stars*. Cambridge University Press, 2006.
- [106] S. Ekström, C. Georgy, P. Eggenberger, G. Meynet, N. Mowlavi, A. Wyttenbach, A. Granada, T. Decressin, R. Hirschi, U. Frischknecht, C. Charbonnel, and A. Maeder. Grids of stellar models with rotation. I. Models from 0.8 to 120 M_{\odot} at solar metallicity ($Z = 0.014$). *Astronomy & Astrophysics*, 537:A146, January 2012.
- [107] J. J. Eldridge and E. R. Stanway. BPASS predictions for binary black hole mergers. *MNRAS*, 462(3):3302–3313, November 2016.
- [108] J. J. Eldridge, E. R. Stanway, and P. N. Tang. A consistent estimate for gravitational wave and electromagnetic transient rates. *MNRAS*, 482:870–880, January 2019.

- [109] J. J. Eldridge, E. R. Stanway, L. Xiao, L. A. S. McClelland , G. Taylor, M. Ng, S. M. L. Greis, and J. C. Bray. Binary Population and Spectral Synthesis Version 2.1: Construction, Observational Verification, and New Results. *Publications of the Astronomical Society of Australia*, 34:e058, November 2017.
- [110] T. Ertl, H. Th. Janka, S. E. Woosley, T. Sukhbold, and M. Ugliano. A Two-parameter Criterion for Classifying the Explodability of Massive Stars by the Neutrino-driven Mechanism. *Astrophysical Journal*, 818(2):124, February 2016.
- [111] T. Ertl, S. E. Woosley, Tuguldur Sukhbold, and H. T. Janka. The Explosion of Helium Stars Evolved with Mass Loss. *Astrophysical Journal*, 890(1):51, February 2020.
- [112] P. Esposito, G. L. Israel, D. Milisavljevic, M. Mapelli, L. Zampieri, L. Sidoli, G. Fabbiano, and G. A. Rodríguez Castillo. Periodic signals from the Circinus region: two new cataclysmic variables and the ultraluminous X-ray source candidate GC X-1. *MNRAS*, 452(2):1112–1127, September 2015.
- [113] R. Farmer, M. Renzo, S. E. de Mink, M. Fishbach, and S. Justham. Constraints from Gravitational-wave Detections of Binary Black Hole Mergers on the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ Rate. *Astrophysical Journal Letter*, 902(2):L36, October 2020.
- [114] R. Farmer, M. Renzo, S. E. de Mink, P. Marchant, and S. Justham. Mind the Gap: The Location of the Lower Edge of the Pair-instability Supernova Black Hole Mass Gap. *Astrophysical Journal*, 887(1):53, Dec 2019.
- [115] Ben Farr, Daniel E. Holz, and Will M. Farr. Using Spin to Understand the Formation of LIGO and Virgo’s Black Holes. *Astrophysical Journal Letter*, 854(1):L9, February 2018.
- [116] W. M. Farr, N. Sravan, A. Cantrell, L. Kreidberg, C. D. Bailyn, I. Mandel, and V. Kalogera. The Mass Distribution of Stellar-mass Black Holes. *Astrophysical Journal*, 741:103, November 2011.
- [117] Will M. Farr, Simon Stevenson, M. Coleman Miller, Ilya Mand el, Ben Farr, and Alberto Vecchio. Distinguishing spin-aligned and isotropic black hole populations with gravitational waves. *Nature*, 548(7667):426–429, August 2017.
- [118] Rodrigo Fernández, Eliot Quataert, Kazumi Kashiyama, and Eric R. Coughlin. Mass ejection in failed supernovae: variation with stellar progenitor. *MNRAS*, 476(2):2366–2383, May 2018.
- [119] Laura Ferrarese, Patrick Côté, Elena Dalla Bontà, Eric W. Peng, David Merritt, Andrés Jordán, John P. Blakeslee, Monica Haşegan, Simona Mei, Sławomir Piatek, John L. Tonry, and Michael J. West. A Fundamental Relation between Compact Stellar Nuclei, Supermassive Black Holes, and Their Host Galaxies. *Astrophysical Journal Letter*, 644(1):L21–L24, June 2006.
- [120] Maya Fishbach and Daniel E. Holz. Minding the Gap: GW190521 as a Straddling Binary. *Astrophysical Journal Letter*, 904(2):L26, December 2020.
- [121] Thierry Foglizzo, Rémi Kazeroni, Jérôme Guilet, Frédéric Masset, Matthias González, Brendan K. Krueger, Jérôme Novak, Micaela Oertel, Jérôme Margueron, Julien Faure, Noël Martin, Patrick Blottiau, Bruno Peres, and Gilles

- Durand. The Explosion Mechanism of Core-Collapse Supernovae: Progress in Supernova Theory and Experiments. *Publications of the Astronomical Society of Australia*, 32:e009, March 2015.
- [122] W. Fong and E. Berger. The Locations of Short Gamma-Ray Bursts as Evidence for Compact Object Binary Progenitors. *Astrophysical Journal*, 776:18, October 2013.
 - [123] William A. Fowler and F. Hoyle. Neutrino Processes and Pair Formation in Massive Stars and Supernovae. *Astrophysical Journal Supplement*, 9:201, December 1964.
 - [124] Giacomo Fragione and Bence Kocsis. Black Hole Mergers from an Evolving Population of Globular Clusters. *Physics Review Letters*, 121(16):161103, October 2018.
 - [125] Giacomo Fragione, Nathan W. C. Leigh, and Rosalba Perna. Black hole and neutron star mergers in galactic nuclei: the role of triples. *MNRAS*, 488(2):2825–2835, September 2019.
 - [126] Giacomo Fragione, Abraham Loeb, and Frederic A. Rasio. On the Origin of GW190521-like Events from Repeated Black Hole Mergers in Star Clusters. *Astrophysical Journal Letter*, 902(1):L26, October 2020.
 - [127] Giacomo Fragione and Joseph Silk. Repeated mergers and ejection of black holes within nuclear star clusters. *MNRAS*, 498(4):4591–4604, November 2020.
 - [128] T. Fragos, B. Willems, V. Kalogera, N. Ivanova, G. Rockefeller, C. L. Fryer, and P. A. Young. Understanding Compact Object Formation and Natal Kicks. II. The Case of XTE J1118 + 480. *Astrophysical Journal*, 697(2):1057–1070, June 2009.
 - [129] Tassos Fragos, Jeff J. Andrews, Enrico Ramirez-Ruiz, Georges Meynet, Vicky Kalogera, Ronald E. Taam, and Andreas Zezas. The Complete Evolution of a Neutron-star Binary through a Common Envelope Phase Using 1D Hydrodynamic Simulations. *Astrophysical Journal Letter*, 883(2):L45, October 2019.
 - [130] Marc Freitag, M. Atakan Gürkan, and Frederic A. Rasio. Runaway collisions in young star clusters - II. Numerical results. *MNRAS*, 368(1):141–161, May 2006.
 - [131] C. Fryer, A. Burrows, and W. Benz. Population Syntheses for Neutron Star Systems with Intrinsic Kicks. *Astrophysical Journal*, 496:333–351, March 1998.
 - [132] C. L. Fryer. Mass Limits For Black Hole Formation. *Astrophysical Journal*, 522:413–418, September 1999.
 - [133] C. L. Fryer, K. Belczynski, G. Wiktorowicz, M. Dominik, V. Kalogera, and D. E. Holz. Compact Remnant Mass Function: Dependence on the Explosion Mechanism and Metallicity. *Astrophysical Journal*, 749:91, April 2012.
 - [134] C. L. Fryer and V. Kalogera. Theoretical Black Hole Mass Distributions. *Astrophysical Journal*, 554:548–560, June 2001.

- [135] M. S. Fujii, A. Tanikawa, and J. Makino. The detection rates of merging binary black holes originating from star clusters and their mass function. *Publications of the Astronomical Society of Japan*, 69:94, December 2017.
- [136] Jim Fuller and Linhao Ma. Most Black Holes Are Born Very Slowly Rotating. *Astrophysical Journal Letter*, 881(1):L1, August 2019.
- [137] Jim Fuller, Anthony L. Piro, and Adam S. Jermyn. Slowing the spins of stellar cores. *MNRAS*, 485(3):3661–3680, May 2019.
- [138] E. Gaburov, J. C. Lombardi, and S. Portegies Zwart. Mixing in massive stellar mergers. *MNRAS*, 383(1):L5–L9, January 2008.
- [139] E. Gaburov, J. C. Lombardi, Jr., and S. Portegies Zwart. On the onset of runaway stellar collisions in dense star clusters - II. Hydrodynamics of three-body interactions. *MNRAS*, 402:105–126, February 2010.
- [140] V. Gayathri, J. Healy, J. Lange, B. O’Brien, M. Szczepanczyk, I. Bartos, M. Campanelli, S. Klimenko, C. Lousto, and R. O’Shaughnessy. GW190521 as a Highly Eccentric Black Hole Merger. *arXiv e-prints*, page arXiv:2009.05461, September 2020.
- [141] Cyril Georgy, Georges Meynet, Sylvia Ekström, Gregg A. Wade, Véronique Petit, Zsolt Keszthelyi, and Raphael Hirschi. Possible pair-instability supernovae at solar metallicity from magnetic stellar progenitors. *Astronomy & Astrophysics*, 599:L5, March 2017.
- [142] J. R. Gerke, C. S. Kochanek, and K. Z. Stanek. The search for failed supernovae with the Large Binocular Telescope: first candidates. *MNRAS*, 450(3):3289–3305, July 2015.
- [143] D. Gerosa and E. Berti. Are merging black holes born from stellar collapse or previous mergers? *Physics Review D*, 95(12):124046, June 2017.
- [144] Davide Gerosa, Michael Kesden, Emanuele Berti, Richard O’Shaughnessy, and Ulrich Sperhake. Resonant-plane locking and spin alignment in stellar-mass black-hole binaries: A diagnostic of compact-binary formation. *Physics Review D*, 87(10):104028, May 2013.
- [145] N. Giacobbo and M. Mapelli. The progenitors of compact-object binaries: impact of metallicity, common envelope and natal kicks. *MNRAS*, 480:2011–2030, October 2018.
- [146] N. Giacobbo and M. Mapelli. The impact of electron-capture supernovae on merging double neutron stars. *MNRAS*, 482:2234–2243, January 2019.
- [147] N. Giacobbo, M. Mapelli, and M. Spera. Merging black hole binaries: the effects of progenitor’s metallicity, mass-loss rate and Eddington factor. *MNRAS*, 474:2959–2974, March 2018.
- [148] Nicola Giacobbo and Michela Mapelli. Revising Natal Kick Prescriptions in Population Synthesis Simulations. *Astrophysical Journal*, 891(2):141, March 2020.
- [149] M. Giersz, N. Leigh, A. Hypki, N. Lützgendorf, and A. Askar. MOCCA code for star cluster simulations - IV. A new scenario for intermediate mass black hole formation in globular clusters. *MNRAS*, 454:3150–3165, December 2015.

- [150] A. Goldstein, P. Veres, E. Burns, M. S. Briggs, R. Hamburg, D. Kocevski, C. A. Wilson-Hodge, R. D. Preece, S. Poolakkil, O. J. Roberts, C. M. Hui, V. Connaughton, J. Racusin, A. von Kienlin, T. Dal Canton, N. Christensen, and et al. An Ordinary Short Gamma-Ray Burst with Extraordinary Implications: Fermi-GBM Detection of GRB 170817A. *Astrophysical Journal Letter*, 848:L14, October 2017.
- [151] José A. González, Ulrich Sperhake, Bernd Brügmann, Mark Hannam, and Sascha Husa. Maximum Kick from Nonspinning Black-Hole Binary Inspiral. *Physics Review Letters*, 98(9):091101, March 2007.
- [152] Lijun Gou, Jeffrey E. McClintock, Jifeng Liu, Ramesh Narayan, James F. Steiner, Ronald A. Remillard, Jerome A. Orosz, Shane W. Davis, Ken Ebisawa, and Eric M. Schlegel. A Determination of the Spin of the Black Hole Primary in LMC X-1. *Astrophysical Journal*, 701(2):1076–1090, August 2009.
- [153] Lijun Gou, Jeffrey E. McClintock, Ronald A. Remillard, James F. Steiner, Mark J. Reid, Jerome A. Orosz, Ramesh Narayan, Manfred Hanke, and Javier García. Confirmation via the Continuum-fitting Method that the Spin of the Black Hole in Cygnus X-1 Is Extreme. *Astrophysical Journal*, 790(1):29, July 2014.
- [154] G. Gräfener and W.-R. Hamann. Mass loss from late-type WN stars and its Z-dependence. Very massive stars approaching the Eddington limit. *Astronomy & Astrophysics*, 482:945–960, May 2008.
- [155] G. Gräfener, J. S. Vink, A. de Koter, and N. Langer. The Eddington factor as the key to understand the winds of the most massive stars. Evidence for a Γ -dependence of Wolf-Rayet type mass loss. *Astronomy & Astrophysics*, 535:A56, November 2011.
- [156] Alister W. Graham and Lee R. Spitler. Quantifying the coexistence of massive black holes and dense nuclear star clusters. *MNRAS*, 397(4):2148–2162, August 2009.
- [157] M. J. Graham, K. E. S. Ford, B. McKernan, N. P. Ross, D. Stern, K. Burdge, M. Coughlin, S. G. Djorgovski, A. J. Drake, D. Duev, M. Kasliwal, A. A. Mahabal, S. van Velzen, J. Belecki, E. C. Bellm, R. Burruss, S. B. Cenko, V. Cunningham, G. Helou, S. R. Kulkarni, F. J. Masci, T. Prince, D. Reiley, H. Rodriguez, B. Rusholme, R. M. Smith, and M. T. Soumagnac. Candidate Electromagnetic Counterpart to the Binary Black Hole Merger Gravitational-Wave Event S190521g*. *Physics Review Letters*, 124(25):251102, June 2020.
- [158] Raffaele Gratton, Angela Bragaglia, Eugenio Carretta, Valentina D’Orazi, Sara Lucatello, and Antonio Sollima. What is a globular cluster? An observational perspective. *The Astronomy and Astrophysics Review*, 27(1):8, November 2019.
- [159] L. Graziani, R. Schneider, S. Marassi, W. Del Pozzo, M. Mapelli, and N. Giacobbo. Cosmic archaeology with massive stellar black hole binaries. *MNRAS*, 495(1):L81–L85, April 2020.

- [160] M. A. Gürkan, J. M. Fregeau, and F. A. Rasio. Massive Black Hole Binaries from Collisional Runaways. *Astrophysical Journal Letter*, 640:L39–L42, March 2006.
- [161] Z. Han, P. Podsiadlowski, and P. P. Eggleton. A possible criterion for envelope ejection in asymptotic giant branch or first giant branch stars. *MNRAS*, 270:121–130, September 1994.
- [162] William E. Harris, Gretchen L. H. Harris, and Matthew Alessi. A Catalog of Globular Cluster Systems: What Determines the Size of a Galaxy’s Globular Cluster Population? *Astrophysical Journal*, 772(2):82, August 2013.
- [163] A. Heger, C. L. Fryer, S. E. Woosley, N. Langer, and D. H. Hartmann. How Massive Single Stars End Their Life. *Astrophysical Journal*, 591:288–300, July 2003.
- [164] A. Heger and S. E. Woosley. The Nucleosynthetic Signature of Population III. *Astrophysical Journal*, 567(1):532–543, March 2002.
- [165] D. C. Heggie. Binary evolution in stellar dynamics. *MNRAS*, 173:729–787, December 1975.
- [166] Jason W. T. Hessels, Scott M. Ransom, Ingrid H. Stairs, Paulo C. C. Freire, Victoria M. Kaspi, and Fernando Camilo. A Radio Pulsar Spinning at 716 Hz. *Science*, 311(5769):1901–1904, March 2006.
- [167] J. G. Hills. The effect of low-velocity, low-mass intruders (collisionless gas) on the dynamical evolution of a binary system. *Astronomical Journal*, 88:1269–1283, August 1983.
- [168] J. G. Hills and L. W. Fullerton. Computer simulations of close encounters between single stars and hard binaries. *Astronomical Journal*, 85:1281–1291, September 1980.
- [169] G. Hobbs, D. R. Lorimer, A. G. Lyne, and M. Kramer. A statistical study of 233 pulsar proper motions. *MNRAS*, 360:974–992, July 2005.
- [170] Kelly Holley-Bockelmann, Kayhan Gültekin, Deirdre Shoemaker, and Nicolas Yunes. Gravitational Wave Recoil and the Retention of Intermediate-Mass Black Holes. *Astrophysical Journal*, 686(2):829–837, October 2008.
- [171] S. Horiuchi, K. Nakamura, T. Takiwaki, K. Kotake, and M. Tanaka. The red supergiant and supernova rate problems: implications for core-collapse supernova physics. *MNRAS*, 445:L99–L103, November 2014.
- [172] J. R. Hurley, C. A. Tout, and O. R. Pols. Evolution of binary stars and the effect of tides on binary populations. *MNRAS*, 329:897–928, February 2002.
- [173] K. Inomata, M. Kawasaki, K. Mukaida, Y. Tada, and T. T. Yanagida. Inflationary primordial black holes for the LIGO gravitational wave events and pulsar timing array experiments. *Physics Review D*, 95(12):123510, June 2017.
- [174] N. Ivanova and S. Chaichenets. Common Envelope: Enthalpy Consideration. *Astrophysical Journal Letter*, 731(2):L36, April 2011.
- [175] N. Ivanova, S. Justham, X. Chen, O. De Marco, C. L. Fryer, E. Gaburov, H. Ge, E. Glebbeek, Z. Han, X.-D. Li, G. Lu, T. Marsh, P. Podsiadlowski, A. Potter, N. Soker, R. Taam, T. M. Tauris, E. P. J. van den Heuvel, and R. F. Webbink. Common envelope evolution: where we stand and how we

- can move forward. *The Astronomy and Astrophysics Review*, 21:59, February 2013.
- [176] N. Ivanova, Ph. Podsiadlowski, and H. Spruit. Hydrodynamical simulations of the stream-core interaction in the slow merger of massive stars. *MNRAS*, 334(4):819–832, August 2002.
 - [177] H.-T. Janka. Neutron Star Kicks by the Gravitational Tug-boat Mechanism in Asymmetric Supernova Explosions: Progenitor and Explosion Dependence. *Astrophysical Journal*, 837:84, March 2017.
 - [178] Hans-Thomas Janka. Explosion Mechanisms of Core-Collapse Supernovae. *Annual Review of Nuclear and Particle Science*, 62(1):407–451, November 2012.
 - [179] Vicky Kalogera, Christopher P. L. Berry, Monica Colpi, Steve Fairhurst, Stephen Justham, Ilya Mandel, Alberto Mangiagli, Michela Mapelli, Cameron Mills, B. S. Sathyaprakash, Raffaella Schneider, Thomas Tauris, and Rosa Valiante. Deeper, Wider, Sharper: Next-Generation Ground-based Gravitational-Wave Observations of Binary Black Holes. *Bulletin of the American Astronomical Society*, 51(3):242, May 2019.
 - [180] Z. Keszthelyi, G. Meynet, C. Georgy, G. A. Wade, V. Petit, and A. David-Uraz. The effects of surface fossil magnetic fields on massive star evolution: I. Magnetic field evolution, mass-loss quenching, and magnetic braking. *MNRAS*, 485(4):5843–5860, June 2019.
 - [181] T. O. Kimpson, M. Spera, M. Mapelli, and B. M. Ziosi. Hierarchical black hole triples in young star clusters: impact of Kozai-Lidov resonance on mergers. *MNRAS*, 463:2443–2452, December 2016.
 - [182] C. S. Kochanek. Failed Supernovae Explain the Compact Remnant Mass Function. *Astrophysical Journal*, 785(1):28, April 2014.
 - [183] Christopher S. Kochanek, John F. Beacom, Matthew D. Kistler, José L. Prieto, Krzysztof Z. Stanek, Todd A. Thompson, and Hasan Yüksel. A Survey About Nothing: Monitoring a Million Supergiants for Failed Supernovae. *Astrophysical Journal*, 684(2):1336–1342, September 2008.
 - [184] Y. Kozai. Secular perturbations of asteroids with high inclination and eccentricity. *Astronomical Journal*, 67:591, November 1962.
 - [185] M. Kramer, J. F. Bell, R. N. Manchester, A. G. Lyne, F. Camilo, I. H. Stairs, N. D’Amico, V. M. Kaspi, G. Hobbs, D. J. Morris, F. Crawford, A. Possenti, B. C. Joshi, M. A. McLaughlin, D. R. Lorimer, and A. J. Faulkner. The Parkes Multibeam Pulsar Survey - III. Young pulsars and the discovery and timing of 200 pulsars. *MNRAS*, 342(4):1299–1324, July 2003.
 - [186] Kyle Kremer, Carl L. Rodriguez, Pau Amaro-Seoane, Katelyn Breivik, Sourav Chatterjee, Michael L. Katz, Shane L. Larson, Frederic A. Rasio, Johan Samsing, Claire S. Ye, and Michael Zevin. Post-Newtonian dynamics in dense star clusters: Binary black holes in the LISA band. *Physics Review D*, 99(6):063003, March 2019.
 - [187] Kyle Kremer, Mario Spera, Devin Becker, Sourav Chatterjee, Ugo N. Di Carlo, Giacomo Fragione, Carl L. Rodriguez, Claire S. Ye, and Frederic A.

- Rasio. Populating the Upper Black Hole Mass Gap through Stellar Collisions in Young Star Clusters. *Astrophysical Journal*, 903(1):45, November 2020.
- [188] M. U. Kruckow, T. M. Tauris, N. Langer, M. Kramer, and R. G. Izzard. Progenitors of gravitational wave mergers: binary evolution with the stellar grid-based code COMBINE. *MNRAS*, 481:1908–1949, December 2018.
- [189] M. U. Kruckow, T. M. Tauris, N. Langer, D. Szécsi, P. Marchant, and Ph. Podsiadlowski. Common-envelope ejection in massive binary stars. Implications for the progenitors of GW150914 and GW151226. *Astronomy & Astrophysics*, 596:A58, November 2016.
- [190] R. P. Kudritzki, A. Pauldrach, and J. Puls. Radiation driven winds of hot luminous stars. II - Wind models for O-stars in the Magellanic Clouds. *Astronomy & Astrophysics*, 173:293–298, February 1987.
- [191] Rolf-Peter Kudritzki and Joachim Puls. Winds from Hot Stars. *Annual Review of Astronomy and Astrophysics*, 38:613–666, January 2000.
- [192] S. R. Kulkarni, P. Hut, and S. McMillan. Stellar black holes in globular clusters. *Nature*, 364:421–423, July 1993.
- [193] Jun Kumamoto, Michiko S. Fujii, and Ataru Tanikawa. Gravitational-wave emission from binary black holes formed in open clusters. *MNRAS*, 486(3):3942–3950, Jul 2019.
- [194] Jun Kumamoto, Michiko S. Fujii, and Ataru Tanikawa. Merger rate density of binary black holes formed in open clusters. *MNRAS*, 495(4):4268–4278, July 2020.
- [195] Doron Kushnir, Matias Zaldarriaga, Juna A. Kollmeier, and Roni Waldman. GW150914: spin-based constraints on the merger time of the progenitor system. *MNRAS*, 462(1):844–849, October 2016.
- [196] C. J. Lada and E. A. Lada. Embedded Clusters in Molecular Clouds. *Annual Review of Astronomy and Astrophysics*, 41:57–115, 2003.
- [197] A. Lamberts, S. Garrison-Kimmel, D. R. Clausen, and P. F. Hopkins. When and where did GW150914 form? *MNRAS*, 463:L31–L35, November 2016.
- [198] Jamie A. P. Law-Smith, Rosa Wallace Everson, Enrico Ramirez-Ruiz, Selma E. de Mink, Lieke A. C. van Son, Ylva Götberg, Stefan Zellmann, Alejandro Vigna-Gómez, Mathieu Renzo, Samantha Wu, Sophie L. Schröder, Ryan J. Foley, and Tenley Hutchinson-Smith. Successful Common Envelope Ejection and Binary Neutron Star Formation in 3D Hydrodynamics. *arXiv e-prints*, page arXiv:2011.06630, November 2020.
- [199] Hyung Mok Lee. Evolution of galactic nuclei with 10-M_⊙ black holes. *MNRAS*, 272(3):605–617, February 1995.
- [200] Claus Leitherer, Carmelle Robert, and Laurent Drissen. Deposition of Mass, Momentum, and Energy by Massive Stars into the Interstellar Medium. *Astrophysical Journal*, 401:596, December 1992.
- [201] M. L. Lidov. The evolution of orbits of artificial satellites of planets under the action of gravitational perturbations of external bodies. *Planetary and Space Science*, 9:719–759, October 1962.

- [202] Marco Limongi. Supernovae from Massive Stars. In Athem W. Alsabti and Paul Murdin, editors, *Handbook of Supernovae*, page 513. Springer International Publishing AG, 2017.
- [203] Marco Limongi and Alessandro Chieffi. Presupernova Evolution and Explosive Nucleosynthesis of Rotating Massive Stars in the Metallicity Range $-3 \leq [\text{Fe}/\text{H}] \leq 0$. *Astrophysical Journal Supplement*, 237(1):13, Jul 2018.
- [204] Jifeng Liu, Jeffrey E. McClintock, Ramesh Narayan, Shane W. Davis, and Jerome A. Orosz. Precise Measurement of the Spin Parameter of the Stellar-Mass Black Hole M33 X-7. *Astrophysical Journal Letter*, 679(1):L37, May 2008.
- [205] Carlos O. Lousto and Yosef Zlochower. Modeling gravitational recoil from precessing highly spinning unequal-mass black-hole binaries. *Physics Review D*, 79(6):064018, March 2009.
- [206] A. J. Loveridge, M. V. van der Sluys, and V. Kalogera. Analytical Expressions for the Envelope Binding Energy of Giants as a Function of Basic Stellar Parameters. *Astrophysical Journal*, 743:49, December 2011.
- [207] Morgan MacLeod, Andrea Antoni, Ariadna Murguia-Berthier, Phillip Macias, and Enrico Ramirez-Ruiz. Common Envelope Wind Tunnel: Coefficients of Drag and Accretion in a Simplified Context for Studying Flows around Objects Embedded within Stellar Envelopes. *Astrophysical Journal*, 838(1):56, March 2017.
- [208] Morgan MacLeod and Abraham Loeb. Pre-common-envelope Mass Loss from Coalescing Binary Systems. *Astrophysical Journal*, 895(1):29, May 2020.
- [209] Morgan MacLeod and Abraham Loeb. Runaway Coalescence of Pre-common-envelope Stellar Binaries. *Astrophysical Journal*, 893(2):106, April 2020.
- [210] Morgan MacLeod, Phillip Macias, Enrico Ramirez-Ruiz, Jonathan Grindlay, Aldo Batta, and Gabriela Montes. Lessons from the Onset of a Common Envelope Episode: the Remarkable M31 2015 Luminous Red Nova Outburst. *Astrophysical Journal*, 835(2):282, February 2017.
- [211] Morgan MacLeod, Eve C. Ostriker, and James M. Stone. Runaway Coalescence at the Onset of Common Envelope Episodes. *Astrophysical Journal*, 863(1):5, August 2018.
- [212] Morgan MacLeod and Enrico Ramirez-Ruiz. Asymmetric Accretion Flows within a Common Envelope. *Astrophysical Journal*, 803(1):41, April 2015.
- [213] Morgan MacLeod and Enrico Ramirez-Ruiz. On the Accretion-fed Growth of Neutron Stars during Common Envelope. *Astrophysical Journal Letter*, 798(1):L19, January 2015.
- [214] Piero Madau and Mark Dickinson. Cosmic Star-Formation History. *Annual Review of Astronomy and Astrophysics*, 52:415–486, August 2014.
- [215] Piero Madau and Tassos Fragos. Radiation Backgrounds at Cosmic Dawn: X-Rays from Compact Binaries. *Astrophysical Journal*, 840(1):39, May 2017.
- [216] A. Maeder. Evidences for a bifurcation in massive star evolution. The ON-blue stragglers. *Astronomy & Astrophysics*, 178:159–169, May 1987.

- [217] André Maeder. *Physics, Formation and Evolution of Rotating Stars*. Springer Berlin Heidelberg, 2009.
- [218] R. Maiolino and F. Mannucci. De re metallica: the cosmic chemical evolution of galaxies. *The Astronomy and Astrophysics Review*, 27(1):3, February 2019.
- [219] R. N. Manchester, G. B. Hobbs, A. Teoh, and M. Hobbs. The Australia Telescope National Facility Pulsar Catalogue. *Astronomical Journal*, 129(4):1993–2006, April 2005.
- [220] I. Mandel and S. E. de Mink. Merging binary black holes formed through chemically homogeneous evolution in short-period stellar binaries. *MNRAS*, 458:2634–2647, May 2016.
- [221] Ilya Mandel and Bernhard Müller. Simple recipes for compact remnant masses and natal kicks. *MNRAS*, October 2020.
- [222] Ilya Mandel, Bernhard Müller, Jeff Riley, Selma E. de Mink, Alejandro Vigna-Gómez, and Debatri Chattopadhyay. Binary population synthesis with probabilistic remnant mass and kick prescriptions. *MNRAS*, 500(1):1380–1384, January 2021.
- [223] M. Mapelli. Massive black hole binaries from runaway collisions: the impact of metallicity. *MNRAS*, 459:3432–3446, July 2016.
- [224] M. Mapelli, M. Colpi, A. Possenti, and S. Sigurdsson. The fingerprint of binary intermediate-mass black holes in globular clusters: suprathermal stars and angular momentum alignment. *MNRAS*, 364(4):1315–1326, December 2005.
- [225] M. Mapelli, M. Colpi, and L. Zampieri. Low metallicity and ultra-luminous X-ray sources in the Cartwheel galaxy. *MNRAS*, 395:L71–L75, May 2009.
- [226] M. Mapelli, A. Ferrara, and N. Rea. Constraints on Galactic intermediate mass black holes. *MNRAS*, 368:1340–1350, May 2006.
- [227] M. Mapelli and N. Giacobbo. The cosmic merger rate of neutron stars and black holes. *MNRAS*, 479:4391–4398, October 2018.
- [228] M. Mapelli, N. Giacobbo, E. Ripamonti, and M. Spera. The cosmic merger rate of stellar black hole binaries from the Illustris simulation. *MNRAS*, 472:2422–2435, December 2017.
- [229] M. Mapelli, N. Giacobbo, F. Santoliquido, and M. C. Artale. The properties of merging black holes and neutron stars across cosmic time. *MNRAS*, April 2019.
- [230] M. Mapelli, N. Giacobbo, M. Toffano, E. Ripamonti, A. Bressan, M. Spera, and M. Branchesi. The host galaxies of double compact objects merging in the local Universe. *MNRAS*, 481:5324–5330, December 2018.
- [231] M. Mapelli, B. Moore, L. Giordano, L. Mayer, M. Colpi, E. Ripamonti, and S. Callegari. Intermediate-mass black holes and ultraluminous X-ray sources in the Cartwheel ring galaxy. *MNRAS*, 383(1):230–246, January 2008.
- [232] M. Mapelli, E. Ripamonti, L. Zampieri, M. Colpi, and A. Bressan. Ultraluminous X-ray sources and remnants of massive metal-poor stars. *MNRAS*, 408:234–253, October 2010.

- [233] M. Mapelli, L. Zampieri, E. Ripamonti, and A. Bressan. Dynamics of stellar black holes in young star clusters with different metallicities - I. Implications for X-ray binaries. *MNRAS*, 429:2298–2314, March 2013.
- [234] Michela Mapelli, Marco Dall’Amico, Yann Bouffanais, Nicola Giacobbo, Manuel Arca Sedda, M. Celeste Artale, Alessandro Ballone, Ugo N. Di Carlo, Giuliano Iorio, Filippo Santoliquido, and Stefano Torniamenti. Hierarchical black hole mergers in young, globular and nuclear star clusters: the effect of metallicity, spin and cluster properties. *MNRAS*, May 2021.
- [235] Michela Mapelli, Marco Dall’Amico, Yann Bouffanais, Nicola Giacobbo, Manuel Arca Sedda, M. Celeste Artale, Alessandro Ballone, Ugo N. Di Carlo, Giuliano Iorio, Filippo Santoliquido, and Stefano Torniamenti. Hierarchical black hole mergers in young, globular and nuclear star clusters: the effect of metallicity, spin and cluster properties. *MNRAS*, 505(1):339–358, July 2021.
- [236] Michela Mapelli, Mario Spera, Enrico Montanari, Marco Limongi, Alessandro Chieffi, Nicola Giacobbo, Alessandro Bressan, and Yann Bouffanais. Impact of the Rotation and Compactness of Progenitors on the Mass of Black Holes. *Astrophysical Journal*, 888(2):76, January 2020.
- [237] S. Marassi, L. Graziani, M. Ginolfi, R. Schneider, M. Mapelli, M. Spera, and M. Alparone. Evolution of dwarf galaxies hosting GW150914-like events. *MNRAS*, 484(3):3219–3232, April 2019.
- [238] P. Marchant, N. Langer, P. Podsiadlowski, T. M. Tauris, and T. J. Moriya. A new route towards merging massive black holes. *Astronomy & Astrophysics*, 588:A50, April 2016.
- [239] Pablo Marchant and Takashi J. Moriya. The impact of stellar rotation on the black hole mass-gap from pair-instability supernovae. *Astronomy & Astrophysics*, 640:L18, August 2020.
- [240] Pablo Marchant, Mathieu Renzo, Robert Farmer, Kaliroe M. W. Pappas, Ronald E. Taam, Selma E. de Mink, and Vassiliki Kalogera. Pulsational Pair-instability Supernovae in Very Close Binaries. *Astrophysical Journal*, 882(1):36, September 2019.
- [241] R. Margutti, E. Berger, W. Fong, C. Guidorzi, K. D. Alexander, B. D. Metzger, P. K. Blanchard, P. S. Cowperthwaite, R. Chornock, T. Eftekhari, M. Nicholl, V. A. Villar, P. K. G. Williams, J. Annis, D. A. Brown, H. Chen, Z. Doctor, J. A. Frieman, D. E. Holz, M. Sako, and M. Soares-Santos. The electromagnetic counterpart of the binary neutron star merger ligo/virgo gw170817. v. rising x-ray emission from an off-axis jet. *The Astrophysical Journal Letters*, 848(2):L20, 2017.
- [242] B. McKernan, K. E. S. Ford, B. Kocsis, W. Lyra, and L. M. Winter. Intermediate-mass black holes in AGN discs - II. Model predictions and observational constraints. *MNRAS*, 441(1):900–909, June 2014.
- [243] B. McKernan, K. E. S. Ford, W. Lyra, and H. B. Perets. Intermediate mass black holes in AGN discs - I. Production and growth. *MNRAS*, 425(1):460–469, Sep 2012.

- [244] Barry McKernan, K. E. Saavik Ford, J. Bellovary, N. W. C. Leigh, Z. Haiman, B. Kocsis, W. Lyra, M. M. Mac Low, B. Metzger, M. O'Dowd, S. Endlich, and D. J. Rosen. Constraining Stellar-mass Black Hole Mergers in AGN Disks Detectable with LIGO. *Astrophysical Journal*, 866(1):66, Oct 2018.
- [245] N. Mennekens and D. Vanbeveren. Massive double compact object mergers: gravitational wave sources and r-process element production sites. *Astronomy & Astrophysics*, 564:A134, April 2014.
- [246] M. C. Miller and D. P. Hamilton. Production of intermediate-mass black holes in globular clusters. *MNRAS*, 330:232–240, February 2002.
- [247] M. Coleman Miller and Melvyn B. Davies. An Upper Limit to the Velocity Dispersion of Relaxed Stellar Systems without Massive Black Holes. *Astrophysical Journal*, 755(1):81, August 2012.
- [248] M. Coleman Miller and Jon M. Miller. The masses and spins of neutron stars and stellar-mass black holes. *Physics Reports*, 548:1–34, January 2015.
- [249] James C. A. Miller-Jones, Arash Bahramian, Jerome A. Orosz, Ilya Mandel, Lijun Gou, Thomas J. Maccarone, Coenraad J. Neijssel, Xueshan Zhao, Janusz Ziolkowski, Mark J. Reid, Phil Uttley, Xueying Zheng, Do-Young Byun, Richard Dodson, Victoria Grinberg, Taehyun Jung, Jeong-Sook Kim, Benito Marcote, Sera Markoff, María J. Rioja, Anthony P. Rushton, David M. Russell, Gregory R. Sivakoff, Alexandra J. Tetarenko, Valeriu Tudose, and Joern Wilms. Cygnus X-1 contains a 21-solar mass black hole—Implications for massive star winds. *Science*, 371(6533):1046–1049, March 2021.
- [250] Félix Mirabel. The formation of stellar black holes. *New Astronomy Review*, 78:1–15, August 2017.
- [251] I. F. Mirabel, V. Dhawan, R. P. Mignani, I. Rodrigues, and F. Guglielmetti. A high-velocity black hole on a Galactic-halo orbit in the solar neighbourhood. *Nature*, 413(6852):139–141, September 2001.
- [252] Maxwell Moe and Rosanne Di Stefano. Mind Your Ps and Qs: The Interrelation between Period (P) and Mass-ratio (Q) Distributions of Binary Stars. *Astrophysical Journal Supplement*, 230(2):15, June 2017.
- [253] Meagan Morscher, Bharath Pattabiraman, Carl Rodriguez, Frederic A. Rasio, and Stefan Umbreit. The Dynamical Evolution of Stellar Black Holes in Globular Clusters. *Astrophysical Journal*, 800(1):9, February 2015.
- [254] Bernhard Müller, Hans-Thomas Janka, and Alexander Heger. New Two-dimensional Models of Supernova Explosions by the Neutrino-heating Mechanism: Evidence for Different Instability Regimes in Collapsing Stellar Cores. *Astrophysical Journal*, 761(1):72, December 2012.
- [255] Bernhard Müller, Hans-Thomas Janka, and Andreas Marek. A New Multi-dimensional General Relativistic Neutrino Hydrodynamics Code for Core-collapse Supernovae. II. Relativistic Explosion Models of Core-collapse Supernovae. *Astrophysical Journal*, 756(1):84, September 2012.
- [256] Coenraad J. Neijssel, Alejandro Vigna-Gómez, Simon Stevenson, Jim W. Barrett, Sebastian M. Gaebel, Floor S. Broekgaarden, Selma E. de Mink, Dorottya Szécsi, Serena Vinciguerra, and Ilya Mandel. The effect of the

- metallicity-specific star formation history on double compact object mergers. *MNRAS*, 490(3):3740–3759, Dec 2019.
- [257] D. Nelson, A. Pillepich, S. Genel, M. Vogelsberger, V. Springel, P. Torrey, V. Rodriguez-Gomez, D. Sijacki, G. F. Snyder, B. Griffen, F. Marinacci, L. Blecha, L. Sales, D. Xu, and L. Hernquist. The *Illustris* simulation: Public data release. *Astronomy and Computing*, 13:12–37, November 2015.
- [258] Nadine Neumayer, Anil Seth, and Torsten Böker. Nuclear star clusters. *The Astronomy and Astrophysics Review*, 28(1):4, July 2020.
- [259] M. Nicholl, E. Berger, D. Kasen, B. D. Metzger, J. Elias, C. Briceño, K. D. Alexander, P. K. Blanchard, R. Chornock, P. S. Cowperthwaite, T. Eftekhar, W. Fong, R. Margutti, V. A. Villar, P. K. G. Williams, and et al. The electromagnetic counterpart of the binary neutron star merger ligo/virgo gw170817. iii. optical and uv spectra of a blue kilonova from fast polar ejecta. *The Astrophysical Journal Letters*, 848(2):L18, 2017.
- [260] Alexander H. Nitz and Collin D. Capano. GW190521 May Be an Intermediate-mass Ratio Inspiral. *Astrophysical Journal Letter*, 907(1):L9, January 2021.
- [261] Alexander H. Nitz, Thomas Dent, Gareth S. Davies, and Ian Harry. A Search for Gravitational Waves from Binary Mergers with a Single Observatory. *Astrophysical Journal*, 897(2):169, July 2020.
- [262] Evan O’Connor and Christian D. Ott. Black Hole Formation in Failing Core-Collapse Supernovae. *Astrophysical Journal*, 730(2):70, April 2011.
- [263] Sebastian T. Ohlmann, Friedrich K. Röpke, Rüdiger Pakmor, and Volker Springel. Hydrodynamic Moving-mesh Simulations of the Common Envelope Phase in Binary Stellar Systems. *Astrophysical Journal Letter*, 816(1):L9, January 2016.
- [264] J. A. Orosz. Inventory of black hole binaries. In K. van der Hucht, A. Herrero, and C. Esteban, editors, *A Massive Star Odyssey: From Main Sequence to Supernova*, volume 212 of *IAU Symposium*, page 365, 2003.
- [265] R. O’Shaughnessy, J. M. Bellovary, A. Brooks, S. Shen, F. Governato, and C. R. Christensen. The effects of host galaxy properties on merging compact binaries detectable by LIGO. *MNRAS*, 464:2831–2839, January 2017.
- [266] F. Özel, D. Psaltis, R. Narayan, and J. E. McClintock. The Black Hole Mass Distribution in the Galaxy. *Astrophysical Journal*, 725:1918–1927, December 2010.
- [267] Feryal Özel and Paulo Freire. Masses, Radii, and the Equation of State of Neutron Stars. *Annual Review of Astronomy and Astrophysics*, 54:401–440, September 2016.
- [268] Jean-Claude Passy, Orsola De Marco, Chris L. Fryer, Falk Herwig, Steven Diehl, Jeffrey S. Oishi, Mordecai-Mark Mac Low, Greg L. Bryan, and Gabriel Rockefeller. Simulating the Common Envelope Phase of a Red Giant Using Smoothed-particle Hydrodynamics and Uniform-grid Codes. *Astrophysical Journal*, 744(1):52, January 2012.
- [269] Rachel A. Patton and Tuguldur Sukhbold. Towards a Realistic Explosion Landscape for Binary Population Synthesis. *MNRAS*, October 2020.

- [270] Bill Paxton, Lars Bildsten, Aaron Dotter, Falk Herwig, Pierre Lesaffre, and Frank Timmes. Modules for Experiments in Stellar Astrophysics (MESA). *Astrophysical Journal Supplement*, 192(1):3, January 2011.
- [271] Bill Paxton, Matteo Cantiello, Phil Arras, Lars Bildsten, Edward F. Brown, Aaron Dotter, Christopher Mankovich, M. H. Montgomery, Dennis Stello, F. X. Timmes, and Richard Townsend. Modules for Experiments in Stellar Astrophysics (MESA): Planets, Oscillations, Rotation, and Massive Stars. *Astrophysical Journal Supplement*, 208(1):4, September 2013.
- [272] Bill Paxton, Pablo Marchant, Josiah Schwab, Evan B. Bauer, Lars Bildsten, Matteo Cantiello, Luc Dessart, R. Farmer, H. Hu, N. Langer, R. H. D. Townsend, Dean M. Townsley, and F. X. Timmes. Modules for Experiments in Stellar Astrophysics (MESA): Binaries, Pulsations, and Explosions. *Astrophysical Journal Supplement*, 220(1):15, September 2015.
- [273] Ondřej Pejcha and Todd A. Thompson. The Landscape of the Neutrino Mechanism of Core-collapse Supernovae: Neutron Star and Black Hole Mass Functions, Explosion Energies, and Nickel Yields. *Astrophysical Journal*, 801(2):90, March 2015.
- [274] P. C. Peters. Gravitational Radiation and the Motion of Two Point Masses. *Physics Review*, 136:1224–1232, November 1964.
- [275] V. Petit, Z. Keszthelyi, R. MacInnis, D. H. Cohen, R. H. D. Townsend, G. A. Wade, S. L. Thomas, S. P. Owocki, J. Puls, and A. ud-Doula. Magnetic massive stars as progenitors of ‘heavy’ stellar-mass black holes. *MNRAS*, 466(1):1052–1060, April 2017.
- [276] E. Pian, P. D’Avanzo, S. Benetti, M. Branchesi, E. Brocato, S. Campana, E. Cappellaro, S. Covino, V. D’Elia, J. P. U. Fynbo, F. Getman, G. Ghirlanda, G. Ghisellini, and et al. Spectroscopic identification of r-process nucleosynthesis in a double neutron-star merger. *Nature*, 551:67–70, November 2017.
- [277] Ph. Podsiadlowski, S. Rappaport, and Z. Han. On the formation and evolution of black hole binaries. *MNRAS*, 341(2):385–404, May 2003.
- [278] S. F. Portegies Zwart, H. Baumgardt, P. Hut, J. Makino, and S. L. W. McMillan. Formation of massive black holes through runaway collisions in dense young star clusters. *Nature*, 428:724–726, April 2004.
- [279] S. F. Portegies Zwart, J. Makino, S. L. W. McMillan, and P. Hut. Star cluster ecology. III. Runaway collisions in young compact star clusters. *Astronomy & Astrophysics*, 348:117–126, August 1999.
- [280] S. F. Portegies Zwart and S. L. W. McMillan. Black Hole Mergers in the Universe. *Astrophysical Journal Letter*, 528:L17–L20, January 2000.
- [281] S. F. Portegies Zwart and S. L. W. McMillan. The Runaway Growth of Intermediate-Mass Black Holes in Dense Star Clusters. *Astrophysical Journal*, 576:899–907, September 2002.
- [282] S. F. Portegies Zwart, S. L. W. McMillan, and M. Gieles. Young Massive Star Clusters. *Annual Review of Astronomy and Astrophysics*, 48:431–493, September 2010.
- [283] S. F. Portegies Zwart and F. Verbunt. Population synthesis of high-mass binaries. *Astronomy & Astrophysics*, 309:179–196, May 1996.

- [284] M. Punturo, M. Abernathy, F. Acernese, B. Allen, N. Andersson, K. Arun, F. Barone, B. Barr, M. Barsuglia, M. Beker, N. Beveridge, S. Birindelli, S. Bose, L. Bosi, S. Braccini, C. Bradaschia, T. Bulik, E. Calloni, G. Celli, E. Chassande Mottin, and et al. The Einstein Telescope: a third-generation gravitational wave observatory. *Classical and Quantum Gravity*, 27(19):194002, October 2010.
- [285] Y. Qin, T. Fragos, G. Meynet, J. Andrews, M. Sørensen, and H. F. Song. The spin of the second-born black hole in coalescing binary black holes. *Astronomy & Astrophysics*, 616:A28, August 2018.
- [286] Gerald D. Quinlan. The dynamical evolution of massive black hole binaries I. Hardening in a fixed stellar background. *New Astronomy*, 1(1):35–56, July 1996.
- [287] Deepak Raghavan, Harold A. McAlister, Todd J. Henry, David W. Latham, Geoffrey W. Marcy, Brian D. Mason, Douglas R. Gies, Russel J. White, and Theo A. ten Brummelaar. A Survey of Stellar Families: Multiplicity of Solar-type Stars. *Astrophysical Journal Supplement*, 190(1):1–42, September 2010.
- [288] G. Rakavy and G. Shaviv. Instabilities in Highly Evolved Stellar Models. *Astrophysical Journal*, 148:803, June 1967.
- [289] Sara Rastello, Michela Mapelli, Ugo N. Di Carlo, Nicola Giacobbo, Filippo Santoliquido, Mario Spera, Alessandro Ballone, and Giuliano Iorio. Dynamics of black hole-neutron star binaries in young star clusters. *MNRAS*, 497(2):1563–1570, July 2020.
- [290] M. Renzo, R. J. Farmer, S. Justham, S. E. de Mink, Y. Götberg, and P. Marchant. Sensitivity of the lower edge of the pair-instability black hole mass gap to the treatment of time-dependent convection. *MNRAS*, 493(3):4333–4341, April 2020.
- [291] S. Repetto, M. B. Davies, and S. Sigurdsson. Investigating stellar-mass black hole kicks. *MNRAS*, 425:2799–2809, October 2012.
- [292] S. Repetto, A. P. Igoshev, and G. Nelemans. The Galactic distribution of X-ray binaries and its implications for compact object formation and natal kicks. *MNRAS*, 467:298–310, May 2017.
- [293] Christopher S. Reynolds. Observational Constraints on Black Hole Spin. *arXiv e-prints*, page arXiv:2011.08948, November 2020.
- [294] Paul M. Ricker and Ronald E. Taam. The Interaction of Stellar Objects within a Common Envelope. *Astrophysical Journal Letter*, 672(1):L41, January 2008.
- [295] Paul M. Ricker and Ronald E. Taam. An AMR Study of the Common-envelope Phase of Binary Evolution. *Astrophysical Journal*, 746(1):74, February 2012.
- [296] Jeff Riley, Ilya Mandel, Pablo Marchant, Ellen Butler, Kaila Nathaniel, Coenraad Neijssel, Spencer Shortt, and Alejandro Vigna-Gómez. Chemically Homogeneous Evolution: A rapid population synthesis approach. *MNRAS*, May 2021.
- [297] Francesco Paolo Rizzuto, Thorsten Naab, Rainer Spurzem, Mirek Giersz, J. P. Ostriker, N. C. Stone, Long Wang, Peter Berczik, and M. Rampp. Intermedi-

- ate mass black hole formation in compact young massive star clusters. *MNRAS*, 501(4):5257–5273, March 2021.
- [298] C. L. Rodriguez, P. Amaro-Seoane, S. Chatterjee, K. Kremer, F. A. Rasio, J. Samsing, C. S. Ye, and M. Zevin. Post-Newtonian dynamics in dense star clusters: Formation, masses, and merger rates of highly-eccentric black hole binaries. *Physics Review D*, 98(12):123005, December 2018.
- [299] C. L. Rodriguez, S. Chatterjee, and F. A. Rasio. Binary black hole mergers from globular clusters: Masses, merger rates, and the impact of stellar evolution. *Physics Review D*, 93(8):084029, April 2016.
- [300] C. L. Rodriguez, M. Morscher, B. Pattabiraman, S. Chatterjee, C.-J. Haster, and F. A. Rasio. Binary Black Hole Mergers from Globular Clusters: Implications for Advanced LIGO. *Physics Review Letters*, 115(5):051101, July 2015.
- [301] Carl L. Rodriguez, Michael Zevin, Pau Amaro-Seoane, Sourav Chatterjee, Kyle Kremer, Frederic A. Rasio, and Claire S. Ye. Black holes: The next generation—repeated mergers in dense star clusters and their gravitational-wave properties. *Physics Review D*, 100(4):043027, August 2019.
- [302] Isobel Romero-Shaw, Paul D. Lasky, Eric Thrane, and Juan Calderón Bustillo. GW190521: Orbital Eccentricity and Signatures of Dynamical Formation in a Binary Black Hole Merger Signal. *Astrophysical Journal Letter*, 903(1):L5, November 2020.
- [303] J. Samsing. Eccentric black hole mergers forming in globular clusters. *Physics Review D*, 97(10):103014, May 2018.
- [304] J. Samsing, M. MacLeod, and E. Ramirez-Ruiz. Formation of Tidal Captures and Gravitational Wave Inspirals in Binary-single Interactions. *Astrophysical Journal*, 846:36, September 2017.
- [305] Johan Samsing, Morgan MacLeod, and Enrico Ramirez-Ruiz. The Formation of Eccentric Compact Binary Inspirals and the Role of Gravitational Wave Emission in Binary-Single Stellar Encounters. *Astrophysical Journal*, 784(1):71, March 2014.
- [306] H. Sana, J. B. Le Bouquin, S. Lacour, J. P. Berger, G. Duvert, L. Gauchet, B. Norris, J. Olofsson, D. Pickel, G. Zins, O. Absil, A. de Koter, K. Kratter, O. Schnurr, and H. Zinnecker. Southern Massive Stars at High Angular Resolution: Observational Campaign and Companion Detection. *Astrophysical Journal Supplement*, 215(1):15, November 2014.
- [307] A. A. C. Sander, W. R. Hamann, H. Todt, R. Hainich, T. Shenar, V. Ramachandran, and L. M. Osokinova. The Galactic WC and WO stars. The impact of revised distances from Gaia DR2 and their role as massive black hole progenitors. *Astronomy & Astrophysics*, 621:A92, January 2019.
- [308] R. H. Sanders. The Effects of Stellar Collisions in Dense Stellar Systems. *Astrophysical Journal*, 162:791, December 1970.
- [309] Filippo Santoliquido, Michela Mapelli, Yann Bouffanais, Nicola Giacobbo, Ugo N. Di Carlo, Sara Rastello, M. Celeste Artale, and Alessandro Ballone. The Cosmic Merger Rate Density Evolution of Compact Binaries Formed

- in Young Star Clusters and in Isolated Binaries. *Astrophysical Journal*, 898(2):152, August 2020.
- [310] Filippo Santoliquido, Michela Mapelli, Nicola Giacobbo, Yann Bouffanais, and M. Celeste Artale. The cosmic merger rate density of compact objects: impact of star formation, metallicity, initial mass function and binary evolution. *MNRAS*, February 2021.
- [311] V. Savchenko, C. Ferrigno, E. Kuulkers, A. Bazzano, E. Bozzo, S. Brandt, J. Chenevez, T. J.-L. Courvoisier, R. Diehl, A. Domingo, L. Hanlon, E. Jourdain, A. von Kienlin, P. Laurent, F. Lebrun, A. Lutovinov, A. Martin-Carrillo, S. Mereghetti, L. Natalucci, J. Rodi, J.-P. Roques, R. Sunyaev, and P. Ubertini. Integral detection of the first prompt gamma-ray signal coincident with the gravitational-wave event gw170817. *The Astrophysical Journal Letters*, 848(2):L15, 2017.
- [312] Joop Schaye, Robert A. Crain, Richard G. Bower, Michelle Furlong, Matthieu Schaller, Tom Theuns, Claudio Dalla Vecchia, Carlos S. Frenk, I. G. McCarthy, John C. Helly, Adrian Jenkins, Y. M. Rosas-Guevara, Simon D. M. White, Maarten Baes, C. M. Booth, Peter Camps, Julio F. Navarro, Yan Qu, Alireza Rahmati, Till Sawala, Peter A. Thomas, and James Trayford. The EAGLE project: simulating the evolution and assembly of galaxies and their environments. *MNRAS*, 446(1):521–554, January 2015.
- [313] Patricia Schmidt, Frank Ohme, and Mark Hannam. Towards models of gravitational waveforms from generic binaries: II. Modelling precession effects with a single effective precession parameter. *Physics Review D*, 91(2):024043, January 2015.
- [314] Raffaella Schneider, Luca Graziani, Stefania Marassi, Mario Spera, Michela Mapelli, Matteo Alparone, and Matteo de Bennassuti. The formation and coalescence sites of the first gravitational wave events. *MNRAS*, 471(1):L105–L109, October 2017.
- [315] B. F. Schutz. Gravitational wave sources and their detectability. *Classical and Quantum Gravity*, 6:1761–1780, December 1989.
- [316] S. Sigurdsson and L. Hernquist. Primordial black holes in globular clusters. *Nature*, 364:423–425, July 1993.
- [317] S. Sigurdsson and E. S. Phinney. Dynamics and Interactions of Binaries and Neutron Stars in Globular Clusters. *Astrophysical Journal Supplement*, 99:609, August 1995.
- [318] M. Soares-Santos, D. E. Holz, J. Annis, R. Chornock, K. Herner, E. Berger, D. Brout, H.-Y. Chen, R. Kessler, M. Sako, S. Allam, D. L. Tucker, R. E. Butler, A. Palmese, Z. Doctor, H. T. Diehl, J. Frieman, B. Yanny, H. Lin, D. Scolnic, P. Cowperthwaite, E. Neilson, J. Marriner, N. Kuropatkin, W. G. Hartley, F. Paz-Chinchón, K. D. Alexander, E. Balbinot, P. Blanchard, D. A. Brown, J. L. Carlin, C. Conselice, and et al. The electromagnetic counterpart of the binary neutron star merger ligo/virgo gw170817. i. discovery of the optical counterpart using the dark energy camera. *The Astrophysical Journal Letters*, 848(2):L16, 2017.

- [319] Noam Soker. Energy and angular momentum deposition during common envelope evolution. *New Astronomy*, 9(5):399–408, June 2004.
- [320] Noam Soker. The jet feedback mechanism (JFM) in stars, galaxies and clusters. *New Astronomy Review*, 75:1–23, December 2016.
- [321] M. Spera and M. Mapelli. Very massive stars, pair-instability supernovae and intermediate-mass black holes with the sevn code. *MNRAS*, 470:4739–4749, October 2017.
- [322] M. Spera, M. Mapelli, and A. Bressan. The mass spectrum of compact remnants from the PARSEC stellar evolution tracks. *MNRAS*, 451:4086–4103, August 2015.
- [323] Mario Spera, Michela Mapelli, Nicola Giacobbo, Alessandro A. Trani, Alessandro Bressan, and Guglielmo Costa. Merging black hole binaries with the SEVN code. *MNRAS*, 485(1):889–907, May 2019.
- [324] Jr. Spitzer, Lyman. Equipartition and the Formation of Compact Nuclei in Spherical Stellar Systems. *Astrophysical Journal Letter*, 158:L139, December 1969.
- [325] Jr. Spitzer, Lyman and Michael H. Hart. Random Gravitational Encounters and the Evolution of Spherical Systems. I. Method. *Astrophysical Journal*, 164:399, March 1971.
- [326] Lyman Spitzer. *Dynamical evolution of globular clusters*. Princeton University Press, 1987.
- [327] H. C. Spruit. Dynamo action by differential rotation in a stably stratified stellar interior. *Astronomy & Astrophysics*, 381:923–932, January 2002.
- [328] Jakob Stegmann and Fabio Antonini. Flipping spins in mass transferring binaries and origin of spin-orbit misalignment in binary black holes. *Physics Review D*, 103(6):063007, March 2021.
- [329] Simon Stevenson, Christopher P. L. Berry, and Ilya Mandel. Hierarchical analysis of gravitational-wave measurements of binary black hole spin-orbit misalignments. *MNRAS*, 471(3):2801–2811, November 2017.
- [330] Simon Stevenson, Matthew Sampson, Jade Powell, Alejandro Vigna-Gómez, Coenraad J. Neijssel, Dorottya Szécsi, and Ilya Mandel. The Impact of Pair-instability Mass Loss on the Binary Black Hole Mass Distribution. *Astrophysical Journal*, 882(2):121, Sep 2019.
- [331] Nicholas C. Stone and Brian D. Metzger. Rates of stellar tidal disruption as probes of the supermassive black hole mass function. *MNRAS*, 455(1):859–883, January 2016.
- [332] Nicholas C. Stone, Brian D. Metzger, and Zoltán Haiman. Assisted inspirals of stellar mass black holes embedded in AGN discs: solving the ‘final au problem’. *MNRAS*, 464(1):946–954, January 2017.
- [333] Tuguldur Sukhbold, T. Ertl, S. E. Woosley, Justin M. Brown, and H. T. Janka. Core-collapse Supernovae from 9 to 120 Solar Masses Based on Neutrino-powered Explosions. *Astrophysical Journal*, 821(1):38, Apr 2016.
- [334] Tuguldur Sukhbold and S. E. Woosley. The Compactness of Presupernova Stellar Cores. *Astrophysical Journal*, 783(1):10, March 2014.

- [335] Y. Suwa, T. Yoshida, M. Shibata, H. Umeda, and K. Takahashi. Neutrino-driven explosions of ultra-stripped Type Ic supernovae generating binary neutron stars. *MNRAS*, 454:3073–3081, December 2015.
- [336] J. Tang, A. Bressan, P. Rosenfield, A. Slemer, P. Marigo, L. Girardi, and L. Bianchi. New PARSEC evolutionary tracks of massive stars at low metallicity: testing canonical stellar evolution in nearby star-forming dwarf galaxies. *MNRAS*, 445:4287–4305, December 2014.
- [337] Petra N. Tang, J. J. Eldridge, Elizabeth R. Stanway, and J. C. Bray. Dependence of gravitational wave transient rates on cosmic star formation and metallicity evolution history. *MNRAS*, 493(1):L6–L10, March 2020.
- [338] A. Tanikawa. Dynamical evolution of stellar mass black holes in dense stellar clusters: estimate for merger rate of binary black holes originating from globular clusters. *MNRAS*, 435(2):1358–1375, October 2013.
- [339] Ataru Tanikawa, Tomoya Kinugawa, Takashi Yoshida, Kotaro Hijikawa, and Hideyuki Ueda. Population III binary black holes: effects of convective overshooting on formation of GW190521. *MNRAS*, 505(2):2170–2176, August 2021.
- [340] Ataru Tanikawa, Hajime Susa, Takashi Yoshida, Alessandro A. Trani, and Tomoya Kinugawa. Merger Rate Density of Population III Binary Black Holes Below, Above, and in the Pair-instability Mass Gap. *Astrophysical Journal*, 910(1):30, March 2021.
- [341] T. M. Tauris, M. Kramer, P. C. C. Freire, N. Wex, H.-T. Janka, N. Langer, P. Podsiadlowski, E. Bozzo, S. Chaty, M. U. Kruckow, E. P. J. van den Heuvel, J. Antoniadis, R. P. Breton, and D. J. Champion. Formation of Double Neutron Star Systems. *Astrophysical Journal*, 846:170, September 2017.
- [342] T. M. Tauris, N. Langer, T. J. Moriya, P. Podsiadlowski, S.-C. Yoon, and S. I. Blinnikov. Ultra-stripped Type Ic Supernovae from Close Binary Evolution. *Astrophysical Journal Letter*, 778:L23, December 2013.
- [343] T. M. Tauris, N. Langer, and P. Podsiadlowski. Ultra-stripped supernovae: progenitors and fate. *MNRAS*, 451:2123–2144, August 2015.
- [344] K. S. Thorne. Gravitational radiation. In S. W. Hawking and W. Israel, editors, *Three Hundred Years of Gravitation*, pages 330–458. Cambridge: Cambridge University Press, 1987.
- [345] A. Tokovinin. Comparative statistics and origin of triple and quadruple stars. *MNRAS*, 389(2):925–938, September 2008.
- [346] Andrei Tokovinin. From Binaries to Multiples. II. Hierarchical Multiplicity of F and G Dwarfs. *Astronomical Journal*, 147(4):87, April 2014.
- [347] Silvia Toonen, Adrian Hamers, and Simon Portegies Zwart. The evolution of hierarchical triple star-systems. *Computational Astrophysics and Cosmology*, 3(1):6, December 2016.
- [348] Christopher A. Tout, Sverre J. Aarseth, Onno R. Pols, and Peter P. Eggleton. Rapid binary star evolution for N-body simulations and population synthesis. *MNRAS*, 291(4):732–748, November 1997.
- [349] A. Tutukov and L. Yungelson. Evolution of massive close binaries. *Nauchnye Informatsii*, 27:70, 1973.

- [350] Richard Udall, Karan Jani, Jacob Lange, Richard O’Shaughnessy, James Clark, Laura Cadonati, Deirdre Shoemaker, and Kelly Holley-Bockelmann. Inferring Parameters of GW170502: The Loudest Intermediate-mass Black Hole Trigger in LIGO’s O1/O2 data. *Astrophysical Journal*, 900(1):80, September 2020.
- [351] Marcella Ugliano, Hans-Thomas Janka, Andreas Marek, and Almudena Arcones. Progenitor-explosion Connection and Remnant Birth Masses for Neutrino-driven Supernovae of Iron-core Progenitors. *Astrophysical Journal*, 757(1):69, September 2012.
- [352] E. P. J. van den Heuvel. Late Stages of Close Binary Systems. In Peter Eggleton, Simon Mitton, and John Whelan, editors, *Structure and Evolution of Close Binary Systems*, volume 73, page 35, January 1976.
- [353] Edward P. J. van den Heuvel. High-Mass X-ray Binaries: progenitors of double compact objects. *IAU Symposium*, 346:1–13, December 2019.
- [354] J. Th. van Loon, M. R. L. Cioni, A. A. Zijlstra, and C. Loup. An empirical formula for the mass-loss rates of dust-enshrouded red supergiants and oxygen-rich Asymptotic Giant Branch stars. *Astronomy & Astrophysics*, 438(1):273–289, July 2005.
- [355] L. A. C. van Son, S. E. De Mink, F. S. Broekgaarden, M. Renzo, S. Justham, E. Laplace, J. Morán-Fraile, D. D. Hendriks, and R. Farmer. Polluting the Pair-instability Mass Gap for Binary Black Holes through Super-Eddington Accretion in Isolated Binaries. *Astrophysical Journal*, 897(1):100, July 2020.
- [356] Tejaswi Venumadhav, Barak Zackay, Javier Roulet, Liang Dai, and Matias Zaldarriaga. New binary black hole mergers in the second observing run of Advanced LIGO and Advanced Virgo. *Physics Review D*, 101(8):083030, April 2020.
- [357] F. Verbunt, A. Igoshev, and E. Cator. The observed velocity distribution of young pulsars. *Astronomy & Astrophysics*, 608:A57, December 2017.
- [358] Michelle Vick, Morgan MacLeod, Dong Lai, and Abraham Loeb. Tidal dissipation impact on the eccentric onset of common envelope phases in massive binary star systems. *MNRAS*, 503(4):5569–5582, June 2021.
- [359] J. S. Vink. Mass loss and stellar superwinds. *ArXiv e-prints*, October 2016.
- [360] J. S. Vink, A. de Koter, and H. J. G. L. M. Lamers. Mass-loss predictions for O and B stars as a function of metallicity. *Astronomy & Astrophysics*, 369:574–588, April 2001.
- [361] J. S. Vink, L. E. Muijres, B. Anthonisse, A. de Koter, G. Gräfener, and N. Langer. Wind modelling of very massive stars up to 300 solar masses. *Astronomy & Astrophysics*, 531:A132, July 2011.
- [362] Jorick S. Vink and A. de Koter. On the metallicity dependence of Wolf-Rayet winds. *Astronomy & Astrophysics*, 442(2):587–596, November 2005.
- [363] Salvatore Vitale, Will M. Farr, Ken K. Y. Ng, and Carl L. Rodriguez. Measuring the Star Formation Rate with Gravitational Waves from Binary Black Holes. *Astrophysical Journal Letter*, 886(1):L1, November 2019.

- [364] M. Vogelsberger, S. Genel, V. Springel, P. Torrey, D. Sijacki, D. Xu, G. Snyder, S. Bird, D. Nelson, and L. Hernquist. Properties of galaxies reproduced by a hydrodynamic simulation. *Nature*, 509:177–182, May 2014.
- [365] M. Vogelsberger, S. Genel, V. Springel, P. Torrey, D. Sijacki, D. Xu, G. Snyder, D. Nelson, and L. Hernquist. Introducing the Illustris Project: simulating the coevolution of dark and visible matter in the Universe. *MNRAS*, 444:1518–1547, October 2014.
- [366] G. A. Wade, C. Neiner, E. Alecian, J. H. Grunhut, V. Petit, B. de Batz, D. A. Bohlander, D. H. Cohen, H. F. Henrichs, O. Kochukhov, J. D. Land street, N. Manset, F. Martins, S. Mathis, and et al. The MiMeS survey of magnetism in massive stars: introduction and overview. *MNRAS*, 456(1):2–22, February 2016.
- [367] Jacob L. Ward, J. M. Diederik Kruijssen, and Hans-Walter Rix. Not all stars form in clusters - Gaia-DR2 uncovers the origin of OB associations. *MNRAS*, 495(1):663–685, June 2020.
- [368] R. F. Webbbink. Double white dwarfs as progenitors of R Coronae Borealis stars and Type I supernovae. *Astrophysical Journal*, 277:355–360, February 1984.
- [369] R. F. Webbbink. Stellar evolution and binaries. In J. E. Pringle and R. A. Wade, editors, *Interacting Binary Stars*, page 39. Cambridge: Cambridge University Press, 1985.
- [370] C. Weidner and P. Kroupa. The maximum stellar mass, star-cluster formation and composite stellar populations. *MNRAS*, 365:1333–1347, February 2006.
- [371] C. Weidner, P. Kroupa, and I. A. D. Bonnell. The relation between the most-massive star and its parental star cluster mass. *MNRAS*, 401(1):275–293, January 2010.
- [372] B. Willems, M. Henninger, T. Levin, N. Ivanova, V. Kalogera, K. McGhee, F. X. Timmes, and C. L. Fryer. Understanding Compact Object Formation and Natal Kicks. I. Calculation Methods and the Case of GRO J1655-40. *Astrophysical Journal*, 625(1):324–346, May 2005.
- [373] Kaze W. K. Wong, Katelyn Breivik, Kyle Kremer, and Thomas Callister. Joint constraints on the field-cluster mixing fraction, common envelope efficiency, and globular cluster radii from a population of binary hole mergers via deep learning. *Physics Review D*, 103(8):083021, April 2021.
- [374] Tsing-Wai Wong, Francesca Valsecchi, Tassos Fragos, and Vassiliki Kalogera. Understanding Compact Object Formation and Natal Kicks. III. The Case of Cygnus X-1. *Astrophysical Journal*, 747(2):111, March 2012.
- [375] S. E. Woosley. Pulsational Pair-instability Supernovae. *Astrophysical Journal*, 836:244, February 2017.
- [376] S. E. Woosley. The Evolution of Massive Helium Stars, Including Mass Loss. *Astrophysical Journal*, 878(1):49, Jun 2019.
- [377] S. E. Woosley and A. Heger. The Progenitor Stars of Gamma-Ray Bursts. *Astrophysical Journal*, 637(2):914–921, February 2006.

- [378] S. E. Woosley, A. Heger, and T. A. Weaver. The evolution and explosion of massive stars. *Reviews of Modern Physics*, 74(4):1015–1071, November 2002.
- [379] X.-J. Xu and X.-D. Li. On the Binding Energy Parameter λ of Common Envelope Evolution. *Astrophysical Journal*, 716:114–121, June 2010.
- [380] Y. Yang, I. Bartos, V. Gayathri, K. E. S. Ford, Z. Haiman, S. Klimenko, B. Kocsis, S. Márka, Z. Márka, B. McKernan, and R. O’Shaughnessy. Hierarchical Black Hole Mergers in Active Galactic Nuclei. *Physics Review Letters*, 123(18):181101, November 2019.
- [381] Y. Yang, I. Bartos, Z. Haiman, B. Kocsis, Z. Márka, N. C. Stone, and S. Márka. AGN Disks Harden the Mass Distribution of Stellar-mass Binary Black Hole Mergers. *Astrophysical Journal*, 876(2):122, May 2019.
- [382] S. C. Yoon, N. Langer, and C. Norman. Single star progenitors of long gamma-ray bursts. I. Model grids and redshift dependent GRB rate. *Astronomy & Astrophysics*, 460(1):199–208, December 2006.
- [383] Barak Zackay, Tejaswi Venumadhav, Liang Dai, Javier Roulet, and Matias Zaldarriaga. Highly spinning and aligned binary black hole merger in the Advanced LIGO first observing run. *Physics Review D*, 100(2):023007, July 2019.
- [384] M. Zevin, C. Pankow, C. L. Rodriguez, L. Sampson, E. Chase, V. Kalogera, and F. A. Rasio. Constraining Formation Models of Binary Black Holes with Gravitational-wave Observations. *Astrophysical Journal*, 846:82, September 2017.
- [385] Michael Zevin, Luke Zoltan Kelley, Anya Nugent, Wen-fai Fong, Christopher P. L. Berry, and Vicky Kalogera. Forward Modeling of Double Neutron Stars: Insights from Highly Offset Short Gamma-Ray Bursts. *Astrophysical Journal*, 904(2):190, December 2020.
- [386] B. M. Ziosi, M. Mapelli, M. Branchesi, and G. Tormen. Dynamics of stellar black holes in young star clusters with different metallicities - II. Black hole-black hole binaries. *MNRAS*, 441:3703–3717, July 2014.