Chapter 31

Elements of the Planetary Orbits

Although Appendix III mentions the principal periodic terms needed to calculate the heliocentric positions of the planets (with explanations given in Chapter 32), it may be of interest to have information about the *mean* orbits of these bodies.

The orbital elements of the major planets can be expressed as polynomials of the form

$$a_0 + a_1 T + a_2 T^2 + a_3 T^3$$

where T is the time measured in Julian centuries of 36525 ephemeris days from the epoch J2000.0 = 2000 January 1.5 TD = JDE 2451 545.0.

In other words,

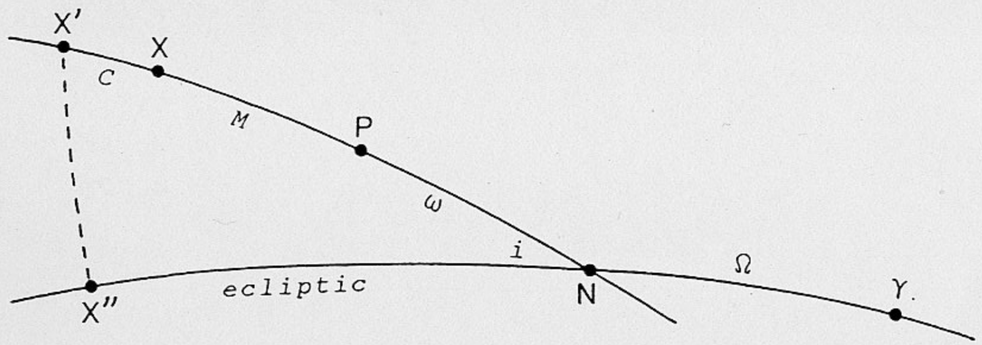
$$T = \frac{\text{JDE} - 2451\,545.0}{36525} \tag{31.1}$$

This quantity is negative before the beginning of the year 2000, positive afterwards. The orbital elements are:

L = mean longitude of the planet;
a = semimajor axis of the orbit;
e = eccentricity of the orbit;
i = inclination on the plane of the ecliptic;
Ω = longitude of the ascending node;
π = longitude of the perihelion.

Many authors denote the longitude of the perihelion by ϖ, which is a modified form of π. But this may be confusing because the *argument* of the perihelion has the symbol ω. For this reason, we prefer the symbol π for the longitude of the perihelion, and we have $\pi = \Omega + \omega$. (But don't confuse π with the parallax or with the number 3.14159...!)

Note that the angles L and π are measured in two different planes, namely from the vernal equinox along the ecliptic to the orbit's ascending node, and then from this node along the orbit. See the Figure on next page.



The arc γNX" is a part of the ecliptic as seen from the Sun, and NPXX' *is a part of the orbit of the planet (the intersection of the orbital plane with the celestial sphere).* γ *is the vernal equinox (longitude 0°),* N *the ascending node of the orbit,* P *the planet's perihelion. At a given instant, the mean planet is at* X, *the true planet at* X'. *Then we have*

Ω = *arc* γN = *longitude of the ascending node,*
ω = *arc* NP = *argument of the perihelion,*
π = *arc* γN + *arc* NP = $\Omega + \omega$ = *longitude of the perihelion,*
L = *arc* γN + *arc* NX = $\Omega + \omega + M$ = *mean longitude of the planet,*
M = *arc* PX = *planet's mean anomaly,*
C = *arc* XX' = *equation of the center,*
v = *arc* PX' = $M + C$ = *planet's true anomaly,*
i = *inclination of the orbit = angle between arcs* NP *and* NX".

The planet's mean anomaly is given by

$$M = L - \pi$$

Table 31.A gives the coefficients a_0 to a_3 for the orbital elements of the planets Mercury to Neptune. The values for the semimajor axes are in astronomical units. Those for the angular quantities L, i, Ω, and π are expressed in degrees and decimals; they are referred to the ecliptic and mean equinox of the date.

The values have been deduced from a study by Simon *e.a.* [1]. However, in the case of the planets Mercury to Mars we added the correction $+0''.2766\,T$ to a_1 for the elements L, Ω, and π in order to bring them in accordance with the VSOP87 theory. The elements L, i, Ω, and π are actually referred to the mean *dynamical* ecliptic and equinox of the date, which differ very slightly from the FK5 system (see Chapter 25).

In some cases, it may be desirable to refer the elements L, i, Ω, and π to a standard equinox. This is the case, for instance, when one wishes to calculate the

least distance between the orbit of a comet and that of a major planet, when the elements of the first orbit are referred to a standard equinox.

By means of Table 31.B, it is possible to calculate these elements for the major planets, referred to the standard equinox of J2000.0. The elements a and e are not modified by a change of reference frame, of course. They should be calculated by means of Table 31.A.

For the Earth, in order to avoid a discontinuity in the variation of the inclination and a jump of 180° in the longitude of the ascending node at the epoch J2000.0, the inclination on the ecliptic of 2000.0 is considered as negative before A.D. 2000.

Example 31.a — Calculate the mean orbital elements of Mercury on 2065 June 24 at 0^h TD.

We have (see Chapter 7)

2065 June 24.0 = JDE 2475 460.5

whence, by formula (31.1),

$T = +0.654\,770\,704\,997$

Consequently, from Table 31.A we find:

$$\begin{aligned} L &= 252°.250\,906 + (149\,474°.072\,2491 \times 0.654\,770\,704\,997) \\ &\quad + (0.000\,303\,50)\,(0.654\,770\,704\,997)^2 \\ &\quad + (0.000\,000\,018)\,(0.654\,770\,704\,997)^3 \\ &= 98\,123°.494\,701 = 203°.494\,701 \end{aligned}$$

a	= 0.387 098 310	π	= 78°.475 382
e	= 0.205 645 10	from which we deduce	
i	= 7°.006 171	$M = L - \pi$	= 125°.019 319
Ω	= 49°.107 650	$\omega = \pi - \Omega$	= 29°.367 732

From Tables 31.A and 31.B it appears that the inclination of the orbit of Mercury on the ecliptic of the date is increasing, but that it is decreasing with respect to the fixed ecliptic of 2000.0. The opposite occurs for Saturn and Neptune.

Between $T = -30$ and $T = +30$, Venus' orbital inclination on the ecliptic of the date is continuously increasing, but with respect to the fixed ecliptic of 2000.0 Venus' inclination reached a maximum about the year +690.

Uranus' orbital inclination on the ecliptic of the date reached a minimum about the year +1000, but with respect to the fixed ecliptic of 2000.0 its value is continuously decreasing during the time period considered here.

The longitudes of the nodes, referred to the equinox of the date, are increasing for all planets. But with respect to the fixed equinox of 2000.0 these longitudes are decreasing, except for Jupiter and Uranus.

TABLE 31.A

Orbital Elements for the mean equinox of the date

	a_0	a_1	a_2	a_3
MERCURY				
L	252.250 906	+149 474.072 2491	+0.000 303 50	+0.000 000 018
a	0.387 098 310			
e	0.205 631 75	+0.000 020 407	−0.000 000 0283	−0.000 000 000 18
i	7.004 986	+0.001 8215	−0.000 018 10	+0.000 000 056
Ω	48.330 893	+1.186 1883	+0.000 175 42	+0.000 000 215
π	77.456 119	+1.556 4776	+0.000 295 44	+0.000 000 009
VENUS				
L	181.979 801	+58 519.213 0302	+0.000 310 14	+0.000 000 015
a	0.723 329 820			
e	0.006 771 92	−0.000 047 765	+0.000 000 0981	+0.000 000 000 46
i	3.394 662	+0.001 0037	−0.000 000 88	−0.000 000 007
Ω	76.679 920	+0.901 1206	+0.000 406 18	−0.000 000 093
π	131.563 703	+1.402 2288	−0.001 076 18	−0.000 005 678
EARTH				
L	100.466 457	+36 000.769 8278	+0.000 303 22	+0.000 000 020
a	1.000 001 018			
e	0.016 708 63	−0.000 042 037	−0.000 000 1267	+0.000 000 000 14
i	0			
π	102.937 348	+1.719 5366	+0.000 456 88	−0.000 000 018
MARS				
L	355.433 000	+19 141.696 4471	+0.000 310 52	+0.000 000 016
a	1.523 679 342			
e	0.093 400 65	+0.000 090 484	−0.000 000 0806	−0.000 000 000 25
i	1.849 726	−0.000 6011	+0.000 012 76	−0.000 000 007
Ω	49.558 093	+0.772 0959	+0.000 015 57	+0.000 002 267
π	336.060 234	+1.841 0449	+0.000 134 77	+0.000 000 536

TABLE 31.A (cont.)

	a_0	a_1	a_2	a_3
JUPITER				
L	34.351 519	+3036.302 7748	+0.000 223 30	+0.000 000 037
a	5.202 603 209	+0.000 000 1913		
e	0.048 497 93	+0.000 163 225	−0.000 000 4714	−0.000 000 002 01
i	1.303 267	−0.005 4965	+0.000 004 66	−0.000 000 002
Ω	100.464 407	+1.020 9774	+0.000 403 15	+0.000 000 404
π	14.331 207	+1.612 6352	+0.001 030 42	−0.000 004 464
SATURN				
L	50.077 444	+1223.511 0686	+0.000 519 08	−0.000 000 030
a	9.554 909 192	−0.000 002 1390	+0.000 000 004	
e	0.055 548 14	−0.000 346 641	−0.000 000 6436	+0.000 000 003 40
i	2.488 879	−0.003 7362	−0.000 015 19	+0.000 000 087
Ω	113.665 503	+0.877 0880	−0.000 121 76	−0.000 002 249
π	93.057 237	+1.963 7613	+0.000 837 53	+0.000 004 928
URANUS				
L	314.055 005	+429.864 0561	+0.000 303 90	+0.000 000 026
a	19.218 446 062	−0.000 000 0372	+0.000 000 000 98	
e	0.046 381 22	−0.000 027 293	+0.000 000 0789	+0.000 000 000 24
i	0.773 197	+0.000 7744	+0.000 037 49	−0.000 000 092
Ω	74.005 957	+0.521 1278	+0.001 339 47	+0.000 018 484
π	173.005 291	+1.486 3790	+0.000 214 06	+0.000 000 434
NEPTUNE				
L	304.348 665	+219.883 3092	+0.000 308 82	+0.000 000 018
a	30.110 386 869	−0.000 000 1663	+0.000 000 000 69	
e	0.009 455 75	+0.000 006 033	+0.000 000 0000	−0.000 000 000 05
i	1.769 953	−0.009 3082	−0.000 007 08	+0.000 000 027
Ω	131.784 057	+1.102 2039	+0.000 259 52	−0.000 000 637
π	48.120 276	+1.426 2957	+0.000 384 34	+0.000 000 020

TABLE 31.B

Orbital Elements for the standard equinox J2000.0

	a_0	a_1	a_2	a_3
MERCURY				
L	252.250 906	+149 472.674 6358	−0.000 005 36	+0.000 000 002
i	7.004 986	−0.005 9516	+0.000 000 80	+0.000 000 043
Ω	48.330 893	−0.125 4227	−0.000 088 33	−0.000 000 200
π	77.456 119	+0.158 8643	−0.000 013 42	−0.000 000 007
VENUS				
L	181.979 801	+58 517.815 6760	+0.000 001 65	−0.000 000 002
i	3.394 662	−0.000 8568	−0.000 032 44	+0.000 000 009
Ω	76.679 920	−0.278 0134	−0.000 142 57	−0.000 000 164
π	131.563 703	+0.004 8746	−0.001 384 67	−0.000 005 695
EARTH				
L	100.466 457	+35 999.372 8565	−0.000 005 68	−0.000 000 001
i	0	+0.013 0548	−0.000 009 31	−0.000 000 034
Ω	174.873 176	−0.241 0908	+0.000 042 62	+0.000 000 001
π	102.937 348	+0.322 5654	+0.000 147 99	−0.000 000 039
MARS				
L	355.433 000	+19 140.299 3039	+0.000 002 62	−0.000 000 003
i	1.849 726	−0.008 1477	−0.000 022 55	−0.000 000 029
Ω	49.558 093	−0.295 0250	−0.000 640 48	−0.000 001 964
π	336.060 234	+0.443 9016	−0.000 173 13	+0.000 000 518

TABLE 31.B (cont.)

	a_0	a_1	a_2	a_3
JUPITER				
L	34.351 519	+3034.905 6606	−0.000 085 01	+0.000 000 016
i	1.303 267	−0.001 9877	+0.000 033 20	+0.000 000 097
Ω	100.464 407	+0.176 7232	+0.000 907 00	−0.000 007 272
π	14.331 207	+0.215 5209	+0.000 722 11	−0.000 004 485
SATURN				
L	50.077 444	+1222.113 8488	+0.000 210 04	−0.000 000 046
i	2.488 879	+0.002 5514	−0.000 049 06	+0.000 000 017
Ω	113.665 503	−0.256 6722	−0.000 183 99	+0.000 000 480
π	93.057 237	+0.566 5415	+0.000 528 50	+0.000 004 912
URANUS				
L	314.055 005	+428.466 9983	−0.000 004 86	+0.000 000 006
i	0.773 197	−0.001 6869	+0.000 003 49	+0.000 000 016
Ω	74.005 957	+0.074 1431	+0.000 405 39	+0.000 000 119
π	173.005 291	+0.089 3212	−0.000 094 70	+0.000 000 414
NEPTUNE				
L	304.348 665	+218.486 2002	+0.000 000 59	−0.000 000 002
i	1.769 953	+0.000 2256	+0.000 000 23	−0.000 000 000
Ω	131.784 057	−0.006 1651	−0.000 002 19	−0.000 000 078
π	48.120 276	+0.029 1866	+0.000 076 10	+0.000 000 000