GiacomoMenegatti_HW4

April 23, 2024

1 Homework 4

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[ ]:
```python
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.cm as cmap
import pandas as pd

%config InlineBackend.print_figure_kwargs = {'bbox_inches':None}
```

1.2 Finding the eccentricity e to have the maximum angular deceleration at $f = \frac{\pi}{4}$

The equations used in this problem are

$$r^2 \dot{f} = h$$

$$r = \frac{p}{1 + e\cos f} \implies \dot{r} = \frac{he}{p}\sin f$$

To find the maximum angular deceleration $-\ddot{f}$ I derive $\dot{f}$ twice (with h, p and e constants) obtaining

$$\dot{f} = \frac{h}{r^2} = \frac{h}{p^2}(1 + e\cos f)^2$$

$$\ddot{f} = -2\frac{h^2 e}{p^4}(1 + e\cos f)^3 \sin f$$

$$\frac{d^3 f}{dt^3} = -2\frac{h^2 e}{p^4}(1 + e\cos f)^2[-3e\sin^2 f + (1 + e\cos f)\cos f]\dot{f}$$

Imposing $[...] = 0$, I obtain the equation

$$e = \frac{\cos f}{3 - 4\cos^2 f}$$

Using $f = \frac{\pi}{4}$, I get $e = 0.7071$

1.3 Laplace to Heliocentric RF transformation

1.3.1 Constant of the system

[ ]:
```python
AU = 149597870707 #Astronomical Unit in m
G = 6.6743e-11
M_sun = 1.9891e30
mu = G*M_sun  #Gravitational parameter of the Sun
R_Earth = 6.378137e6  #Radius of the spherical Earth

#Latitude and longitude of Padua
long_PD = np.deg2rad(11 +52.3/60)
lat_PD = np.deg2rad(45 +24/60)
```

1.3.2 Read the orbital elements from the file

[ ]:
```python
# Read the file skipping the comments and the last column (;)
df = pd.read_csv('Elementi Orbitali (Meeus - MEE of date).m', delimiter='\s+', comment='%', header=None, usecols=[0,1,2,3])

planet_names = ['Mercury', 'Venus', 'Earth', 'Mars', 'Jupiter', 'Saturn', 'Uranus', 'Neptune']
planet = {}
for i, name in enumerate(planet_names):
    #Save the orbital elements in a dictionary. I pick blocks of 7 rows at every time because the last one is blank
    orbital_elements = {}
    orbital_elements['L'] = np.deg2rad(np.array(df.iloc[7*i]))
    orbital_elements['a'] = np.array(df.iloc[7*i+1])*AU
    orbital_elements['e'] = np.array(df.iloc[7*i+2])
    orbital_elements['i'] = np.deg2rad(np.array(df.iloc[7*i+3]))
    orbital_elements['Omega'] = np.deg2rad(np.array(df.iloc[7*i+4]))
    orbital_elements['pi'] = np.deg2rad(np.array(df.iloc[7*i+5]))

    #Save the dictionary with the planet name
    planet[name]=orbital_elements
```

1.3.3 Get the Julian time at a specific date

For a given date Day-Month-Year hour the Julian time is given by

$$JD_0 = D – 32075 + 1461 \times (Y + 4800 + (M – 14)/12)/4 + 367 \times (M – 2 – (M – 14)/12 \times 12)/12 – 3 \times ((Y + 4900 + (M – 14)/12)/1$$

$$JD = JD_0 + F – 0.5$$

where JD_0 is the Julian date at noon and JD the date at any fraction of day F from the midnight.

[ ]:
```python
def JD(Y=2000, M=1, D=1, h=0, m=0, s=0, TZ=0):
    '''Calculate the Julian Time from the Gregorian date'''
```

```
    # In python trunc keeps only the integer part. //, round and floor do not␣
 ↪work with negative nubers
    C = np.trunc((M-14)/12)
    JD0 = D-32075 + np.trunc(1461*(Y+4800+C)/4) + np.trunc(367*(M-2-C*12)/12) -␣
 ↪np.trunc(3*np.trunc((Y+4900+C)/100)/4)
    # Add the hour information and the Time Zone
    JD = JD0 - 0.5 - TZ/24 + (h+m/60+s/3600)/24
    return JD
```

1.3.4 Definition of T, GMST and ϵ

```
[ ]: def T(JD):
         '''Return the time in centuries from J2000.0'''
         return (JD-2451545.0)/36525

     def eps(JD):
         '''Inclination of the Earth axis in RADIANS'''
         t = T(JD)
         eps = 23.439291 - 0.0130042*t - 0.00059*t**2 + 0.001813*t**3
         return np.deg2rad(eps)

     def GMST(Y, M, D, h=0, m=0, s=0, TZ=0):
         '''Greenwich Mean Sidereal Time'''
         t = T(JD(Y,M,D))  #The Julian date is calculated from the midnight at UT1=0
         GMST0 = 24110.54841 + 8640184.812866*t + 0.093104*t**2 - 0.0000062*t**3 #GMST␣
      ↪at UT1=0

         F = (h - TZ + m/60 + s/3600)/24  #Time from UT1 in fraction of days
         GMST = GMST0 + F*24*3600*1.002737909350795 #GMST at the right time calculated␣
      ↪in seconds
         return GMST*2*np.pi/(24*3600) #GMST angle in RADIANS
```

1.3.5 Cartesian Position of a planet at a given date and time

The Inertial position is found by the Lagrangian position by reversing the transformation

$$\mathbf{r}_L = R_z(\pi - \Omega) R_x(i) R_z(\Omega) \mathbf{r}_I$$

in

$$\mathbf{r}_I = R_z(-\Omega) R_x(-i) R_z(\pi - \Omega) \mathbf{r}_L$$

```
[ ]: def Rx(theta, v):
         '''Rotation to align to a vector along the x-axis'''
         return np.array([v[0], v[1]*np.cos(theta)+v[2]*np.sin(theta), -v[1]*np.
      ↪sin(theta)+v[2]*np.cos(theta)])

     def Rz(theta, v):
```

```
    '''Rotation to align to a vector around the z-axis'''
    return np.array([v[0]*np.cos(theta)+v[1]*np.sin(theta), -v[0]*np.
    ↪sin(theta)+v[1]*np.cos(theta), v[2]])

def R_L2I(v, pi, i, Omega):
    '''Rotation from the Lagrange to the Inertial RF'''
    return Rz(-Omega, Rx( -i, Rz( Omega-pi, v )))

def orbital_elements(planet_name, JD):
    '''Returns the Orbital elements at a given Julian date'''
    t = T(JD)
    T_pol = np.array([1,t,t**2,t**3]) #Polynomial in T (1, T, T^2, T^3)
    orbital_elements = {}
    for x in planet[planet_name].keys():
        orbital_elements[x] = np.dot(planet[planet_name][x], T_pol)
    return orbital_elements
```

1.3.6 Lagrange Coordinates from orbital elements

The Orbital elements contained in the file are $a, e, L = M + \Omega + \omega, i, \Omega, \pi = \omega + \Omega$. The simplest orbital angle to obtain is the eccentric anomaly E by solving the Kepler's equation $M = E - e\sin E$, then the position and velocity of the body are given by

$$\mathbf{r} = a(\cos E - e)\hat{\mathbf{P}} + a\sqrt{1-e^2}\sin E\hat{\mathbf{Q}}$$

$$\dot{\mathbf{r}} = -\frac{na^2}{r}\sin E\hat{\mathbf{P}} + \frac{na^2}{r}\sqrt{1-e^2}\cos E\hat{\mathbf{Q}}$$

$$r = a(1 - e\cos E)$$

$$\mu = n^2a^3$$

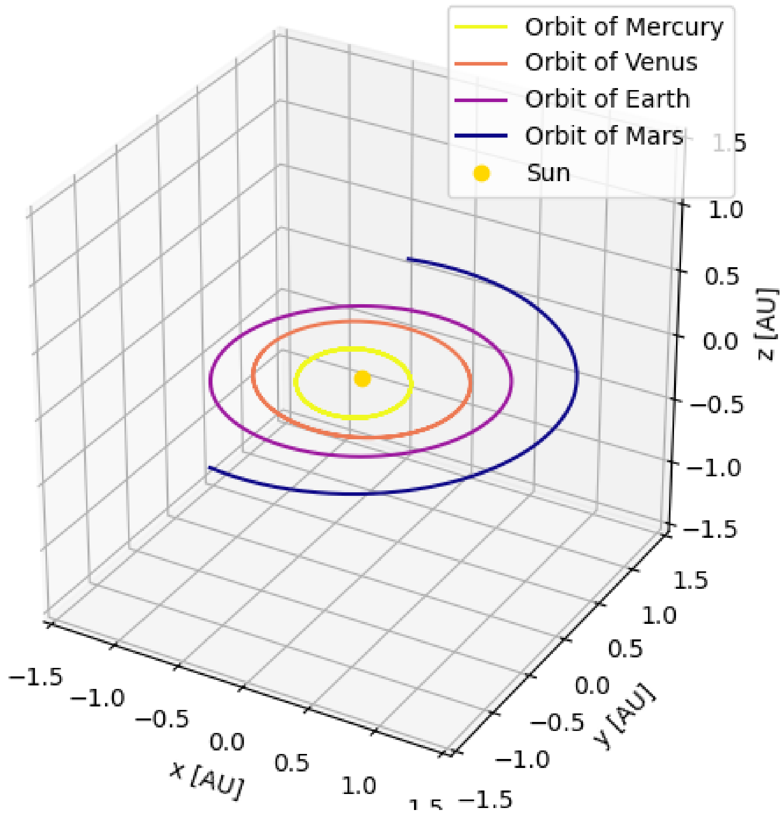
[ ]:
```
def SolveKeplerEq(M,e, atol=1e-14):
    '''Solver of the Kepler's equation'''
    # This works only if M is between 0 and 2pi, so I have to rescalate it
    k = np.floor(M/(2*np.pi))
    M = M-k*2*np.pi            #Subtract all the complete orbits
    E = M
    Delta = 1e2
    while(Delta > atol):
        Delta = (E - e*np.sin(E) - M)/(1-e*np.cos(E))
        E = E - Delta
    return E

def Lagrange_coord(planet_name, JD):
    '''Coordinates of the planet in the Lagrange RF'''
    OE = orbital_elements(planet_name, JD)
    a = OE['a']
    e = OE['e']
```

```
    M = OE['L']-OE['pi']

    E = SolveKeplerEq(M,e)
    n = (mu*a**-3)**0.5
    r = a*(1-e*np.cos(E))

    X = np.array([a*(np.cos(E)-e), a*(1-e**2)**0.5*np.sin(E), 0])
    V = np.array([-n*a**2/r*np.sin(E), n*a*2/r*(1-e**2)**0.5*np.cos(E), 0])
    return X,V

def Inertial_coord(planet_name, JD):
    '''Inertial coordinates of a planet at a given Julian Date'''
    OE = orbital_elements(planet_name, JD)
    X, _ = Lagrange_coord(planet_name, JD)
    return R_L2I(X, OE['pi'], OE['i'], OE['Omega'])
```

1.3.7 Orbits of the planets during 2024

Here I plotted the orbits of the inner planets from 1-1-2024 to 31-12-2024 in the Inertial RF

[ ]:
```
fig = plt.figure()
ax = fig.add_subplot(projection='3d')
colors = cmap.plasma_r(np.linspace(0,1,4))
for c, name in zip(colors, ['Mercury','Venus', 'Earth', 'Mars']):
    trace = np.array([Inertial_coord(name,JD(2024,1,i)) for i in range(1,366)])
    ax.plot(trace[:,0]/AU, trace[:,1]/AU, trace[:,2]/AU, color=c, label = f'Orbit␣
    ↪of {name}')
ax.plot(0,0,0, 'o', color='gold', label='Sun')
ax.set_box_aspect([1,1,1])
ax.set_xlim([-1.5,1.5])
ax.set_xlabel('x [AU]')
ax.set_ylim([-1.5,1.5])
ax.set_ylabel('y [AU]')
ax.set_zlim([-1.5,1.5])
ax.set_zlabel('z [AU]')
ax.legend()
plt.tight_layout()
plt.show()
```



1.3.8 Right Ascension and declination

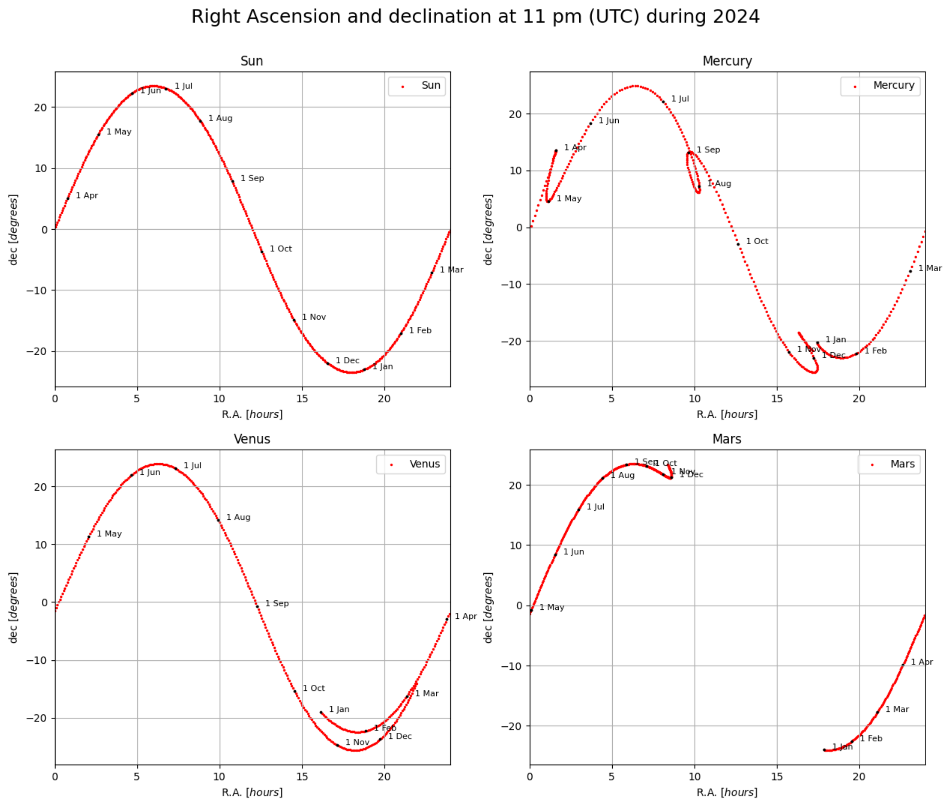
The Right ascension and delcination is found by considering the polar coordiantes of a body in the Geocentric RF and rotating the RF to align it with the Earth equator.

$$\mathbf{r}_{GEO} = \mathbf{r}_I - \mathbf{r}_{Earth}$$
$$\mathbf{r}_{EQ} = R_x(-\epsilon)\mathbf{r}_{GEO}$$

[ ]:
```
def Geo_coord(planet_name, JD):
  '''Coordinates in the Geocentric RF'''
  if planet_name == 'Sun': #The Sun has coordinates 0,0,0 in its RF
    X =  - Inertial_coord('Earth', JD)
  else:
    X = Inertial_coord(planet_name, JD)- Inertial_coord('Earth', JD)
  return X

def polar_coord(r):
  '''Polar Coordinates from cartesian ones'''
  theta = np.arctan2(r[2],(r[0]**2+r[1]**2)**0.5) #Theta is the latitude
  phi = np.arctan2(r[1],r[0]) #Phi is the longitude
  return np.rad2deg(phi), np.rad2deg(theta)
```

```python
def RA_dec(planet_name, JD):
  '''Right ascension and declination'''
  r_GEO = Geo_coord(planet_name, JD)
  r_EQ = Rx(-eps(JD), r_GEO)
  RA, dec = polar_coord(r_EQ)
  RA = RA/360*24 #R.A. is usually given in hours from 0 to 24
  RA = RA+24 if RA<0 else RA  #The values are remapped from -12 12 to 0 24
  return RA, dec
```

[ ]:
```python
plt.figure(figsize=(15, 12))
plt.suptitle("Right Ascension and declination at 11 pm (UTC) during 2024",␣
↪fontsize=18, y=0.95)
month_names = ['Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sep',␣
↪'Oct', 'Nov', 'Dec']

for n, planet_name in enumerate(['Sun', 'Mercury', 'Venus', 'Mars']):
  # add a new subplot iteratively
  ax = plt.subplot(2, 2, n + 1)
  ax.set_title(f'{planet_name}')
  coords = np.array([RA_dec(planet_name,  JD(2024,1,d, 23)) for d in np.
↪arange(1,366,1)])
  ax.scatter(coords[:,0], coords[:,1], color='r', s=2, label=f'{planet_name}')
  #Plotting the position at the beginning of each month
  for i, month in enumerate(month_names):
    coords = RA_dec(planet_name,  JD(2024,i+1,1, 23))
    ax.scatter(coords[0], coords[1], color='k', s=3)
    ax.text(coords[0]+0.5, coords[1], f'1 {month}', fontsize=8)

  ax.grid()
  ax.set_xlim(0,24)
  ax.set_xlabel('R.A. $[hours]$')
  ax.set_ylabel('dec $[degrees]$')
  ax.legend()
```



To be sure of the calculation, I compared this results with the ephemeridis calculated in the **Astronomical Almanac 2020** for Venus, 1st January 2020 at 0 UT1.

[ ]:
```
expected_RA = (21+9/60+44.587/3600)
expected_dec = -(18+15/60+56.18/3600)

RA, dec = RA_dec('Venus', JD(2020,1,1))
print('Expected values')
print(f'RA: {expected_RA}, dec: {expected_dec}')
print('Calculated values')
print(f'RA: {RA}, dec: {dec}')
print(f'Difference RA: {100*(RA/expected_RA-1):.2e}%, dec: {100*(dec/expected_dec-1):.2e}%')
```

```
Expected values
RA: 21.162385277777776, dec: -18.265605555555556
Calculated values
RA: 21.163271261633334, dec: -18.26182607847069
Difference RA: 4.19e-03%, dec: -2.07e-02%
```

1.3.9 Altitude and Azimuth

The Equatorial J2000.0 coordinates are converted in local Altitude and Azimuth by applying the transformation

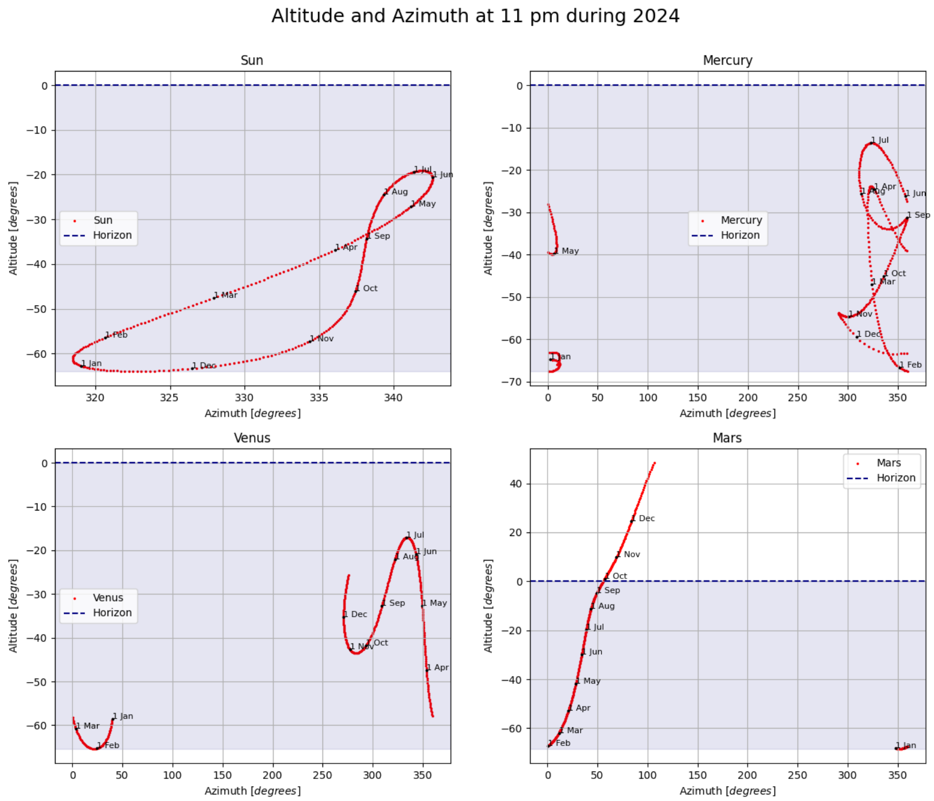
$$\mathbf{r}_{Loc} = R_x(\frac{\pi}{2} - \phi)R_z(\lambda + \frac{\pi}{2})R_z(\theta_{GMST})R_x(-\epsilon)\mathbf{r}_{GEO}$$

The time is calculated at 11 pm in the tocal timezone (+1 UTC) ant the radius of the Earth is subtracted to obtain before obtaining the final result (the Earth is considered a perfect sphere).

[ ]:
```
def Alt_Az(planet_name, JD, GMST, lat, long):
    '''Returns the altitude and azimuth in DEGREES'''
    r_GEO = Geo_coord(planet_name, JD)
    r_AltAz = Rx(np.pi/2-lat, Rz(long+np.pi/2+GMST, Rx(-eps(JD), r_GEO)))
    r_AltAz = r_AltAz - np.array([0,0,R_Earth]) #The RF is aligned with the␣
↪posistion on the surface, then traslated up to it

    # Now the RF is East-North-Up
    Alt = np.rad2deg(np.arctan2(r_AltAz[2],(r_AltAz[0]**2+r_AltAz[1]**2)**0.5))
    Az = np.rad2deg(np.arctan2(r_AltAz[0],r_AltAz[1]))
    Az = Az+360 if Az<0 else Az # The Azimuth is rescaled from -180 180 to 0 360
    return Alt, Az
```

[ ]:
```
plt.figure(figsize=(15, 12))
plt.suptitle("Altitude and Azimuth at 11 pm during 2024", fontsize=18, y=0.95)
month_names = ['Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sep',␣
↪'Oct', 'Nov', 'Dec']

for n, planet_name in enumerate(['Sun', 'Mercury', 'Venus', 'Mars']):
    # add a new subplot iteratively
    ax = plt.subplot(2, 2, n + 1)
    ax.set_title(f'{planet_name}')
    coords = np.array([Alt_Az(planet_name, JD(2024,1,d,23, TZ=1),␣
↪GMST(2024,1,d,23, TZ=1), lat_PD, long_PD) for d in range(1,366)])
    ax.scatter(coords[:,1], coords[:,0], color='r', s=2, label=f'{planet_name}')
    ax.axhline(0, color='navy', linestyle='dashed', label='Horizon')

    ax.axhspan(min(coords[:,0]), 0, color='navy', alpha=0.1)
    #Plotting the position at the beginning of each month
    for i, month in enumerate(month_names):
        coords = Alt_Az(planet_name, JD(2024,i+1,1,23, TZ=1), GMST(2024,i+1,1,23,␣
↪TZ=1), lat_PD, long_PD)
        ax.scatter(coords[1], coords[0], color='k', s=3)
        ax.text(coords[1], coords[0], f'1 {month}', fontsize=8)



    ax.grid()
    ax.set_xlabel('Azimuth $[degrees]$')
```

```
ax.set_ylabel('Altitude $[degrees]$')
ax.legend()
```



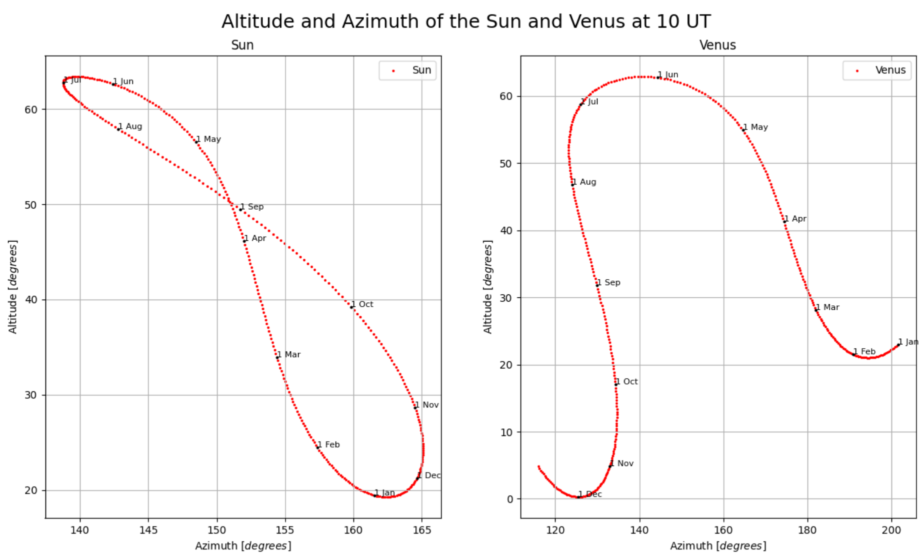
From the plot it appears that no planet is above the horizon at 11 pm except for Mars, which is visible from September 26.

1.3.10 Trace of the Sun and Venus at 10 UT and 20 UT

In this part of the homework I used the function created before to trace the position of the Sun and of Venus in the sky at 10 UT and 20 UT. The Sun draws an 8-like figure called analemma.

[ ]:
```
plt.figure(figsize=(15, 8))
plt.suptitle("Altitude and Azimuth of the Sun and Venus at 10 UT", fontsize=18,␣
↪y=0.95)
month_names = ['Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sep',␣
↪'Oct', 'Nov', 'Dec']

h=10
for n, planet_name in enumerate(['Sun','Venus']):
    # add a new subplot iteratively
```

```
    ax = plt.subplot(1, 2, n + 1)
    ax.set_title(f'{planet_name}')
    coords = np.array([Alt_Az(planet_name, JD(2024,1,d,h, TZ=0), GMST(2024,1,d,h, TZ=0), lat_PD, long_PD) for d in range(1,366)])
    ax.scatter(coords[:,1], coords[:,0], color='r', s=2, label=f'{planet_name}')
    #ax.axhline(0, color='navy', linestyle='dashed', label='Horizon')

    #ax.axhspan(min(coords[:,0]), 0, color='navy', alpha=0.1)
    #Plotting the position at the beginning of each month
    for i, month in enumerate(month_names):
      coords = Alt_Az(planet_name, JD(2024,i+1,1,h, TZ=0), GMST(2024,i+1,1,h, TZ=0), lat_PD, long_PD)
      ax.scatter(coords[1], coords[0], color='k', s=3)
      ax.text(coords[1], coords[0], f'1 {month}', fontsize=8)



    ax.grid()
    ax.set_xlabel('Azimuth $[degrees]$')
    ax.set_ylabel('Altitude $[degrees]$')
    ax.legend()
```




```
[ ]: plt.figure(figsize=(15, 8))
     plt.suptitle("Altitude and Azimuth of the Sun and Venus at 20 UT", fontsize=18, y=0.95)
     month_names = ['Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sep', 'Oct', 'Nov', 'Dec']
```

```
h=20
for n, planet_name in enumerate(['Sun','Venus']):
  # add a new subplot iteratively
  ax = plt.subplot(1, 2, n + 1)
  ax.set_title(f'{planet_name}')
  coords = np.array([Alt_Az(planet_name, JD(2024,1,d,h, TZ=0), GMST(2024,1,d,h,␣
↪TZ=0), lat_PD, long_PD) for d in range(1,366)])
  ax.scatter(coords[:,1], coords[:,0], color='r', s=2, label=f'{planet_name}')
  ax.axhline(0, color='navy', linestyle='dashed', label='Horizon')

  ax.axhspan(min(coords[:,0]), 0, color='navy', alpha=0.1)
  #Plotting the position at the beginning of each month
  for i, month in enumerate(month_names):
    coords = Alt_Az(planet_name, JD(2024,i+1,1,h, TZ=0), GMST(2024,i+1,1,h,␣
↪TZ=0), lat_PD, long_PD)
    ax.scatter(coords[1], coords[0], color='k', s=3)
    ax.text(coords[1], coords[0], f'1 {month}', fontsize=8)



  ax.grid()
  ax.set_xlabel('Azimuth $[degrees]$')
  ax.set_ylabel('Altitude $[degrees]$')
  ax.legend()
```