

PROBLEM:	30
CLASSIFICATION:	QQR-T1-8
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 6$
OBJECTIVE FUNCTION:	
	$f(x) = x_1^2 + x_2^2 + x_3^2$
CONSTRAINTS:	
	$x_1^2 + x_2^2 - 1 \geq 0$
	$1 \leq x_1 \leq 10$
	$-10 \leq x_2 \leq 10$
	$-10 \leq x_3 \leq 10$
START:	$x_0 = (1, 1, 1)$ (feasible) $f(x_0) = 3$
SOLUTION:	$x^* = (1, 0, 0)$ $f(x^*) = 1$ $r(x^*) = 0$ $e(x^*) = 0$ $u = 2$ $I(x^*) = (1, 2)$ $u_{\max}^*/u_{\min}^* = 1/0$ $\lambda_{\max}^*/\lambda_{\min}^* = 2/2 = 1$

PROBLEM:	31
CLASSIFICATION:	QQR-T1-9
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 6$
OBJECTIVE FUNCTION:	
	$f(x) = 9x_1^2 + x_2^2 + 9x_3^2$
CONSTRAINTS:	
	$x_1 x_2 - 1 \geq 0$
	$-10 \leq x_1 \leq 10$
	$1 \leq x_2 \leq 10$
	$-10 \leq x_3 \leq 1$
START:	$x_0 = (1, 1, 1)$ (feasible) $f(x_0) = 19$
SOLUTION:	$x^* = (1/\sqrt{3}, \sqrt{3}, 0)$ $f(x^*) = 6$ $r(x^*) = 0$ $e(x^*) = .57E-10$ $\mu = 1$ $I(x^*) = (1)$ $u_{\max}^*/u_{\min}^* = 6/6 = 1$ $\lambda_{\max}^*/\lambda_{\min}^* = 18/7.2 = 2.5$

PROBLEM:	32
CLASSIFICATION:	QPR-T1-2
SOURCE:	Evtushenko [25]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 1(1)$, $b = 3$
OBJECTIVE FUNCTION:	
	$f(x) = (x_1 + 3x_2 + x_3)^2 + 4(x_1 - x_2)^2$
CONSTRAINTS:	
	$6x_2 + 4x_3 - x_1^3 - 3 \geq 0$
	$1 - x_1 - x_2 - x_3 = 0$
	$0 \leq x_i$, $i = 1, 2, 3$
START:	$x_0 = (.1, .7, .2)$ (feasible) $f(x_0) = 7.2$
SOLUTION:	$x^* = (0, 0, 1)$ $f(x^*) = 1$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 2$ $I(x^*) = (2, 3)$ $u_{\max}^*/u_{\min}^* = 4/0$ $\lambda_{\max}^*/\lambda_{\min}^* = -$

PROBLEM:	33
CLASSIFICATION:	PQR-T1-8
SOURCE:	Beltrami [6], Hartmann [28]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 4$
OBJECTIVE FUNCTION:	
	$f(x) = (x_1 - 1)(x_1 - 2)(x_1 - 3) + x_3$
CONSTRAINTS:	
	$x_3^2 - x_2^2 - x_1^2 \geq 0$
	$x_1^2 + x_2^2 + x_3^2 - 4 \geq 0$
	$0 \leq x_1$
	$0 \leq x_2$
	$0 \leq x_3 \leq 5$
START:	$x_0 = (0, 0, 3)$ (feasible) $f(x_0) = -3$
SOLUTION:	$x^* = (0, \sqrt{2}, \sqrt{2})$ $f(x^*) = \sqrt{2} - 6$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 3$ $I(x^*) = (1, 2, 3)$ $u_{\max}^*/u_{\min}^* = 11/.17678 = 62.23$ $\lambda_{\max}^*/\lambda_{\min}^* = -$

PROBLEM:	34
CLASSIFICATION:	LGR-T1-1
SOURCE:	Eckhardt [24]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 6$
OBJECTIVE FUNCTION:	
	$f(x) = -x_1$
CONSTRAINTS:	
	$x_2 - \exp(x_1) \geq 0$
	$x_3 - \exp(x_2) \geq 0$
	$0 \leq x_1 \leq 100$
	$0 \leq x_2 \leq 100$
	$0 \leq x_3 \leq 10$
START:	$x_0 = (0, 1.05, 2.9)$ (feasible) $f(x_0) = 0$
SOLUTION:	$x^* = (\ln(\ln 10), \ln 10, 10)$ $f(x^*) = -\ln(\ln 10)$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 3$ $I(x^*) = (1, 2, 8)$ $u_{\max}^*/u_{\min}^* = .4343/.04343 = 10$ $\lambda_{\max}^*/\lambda_{\min}^* = -$

PROBLEM:	35 (Beale's problem)
CLASSIFICATION:	QLR-T1-3
SOURCE: Asaadi [1], Charalambous [18], Dimitru [23], Sheela [57]	
NUMBER OF VARIABLES: $n = 3$	
NUMBER OF CONSTRAINTS: $m_1 = 1(1)$, $m-m_1 = 0$, $b = 3$	
OBJECTIVE FUNCTION:	
$f(x) = 9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2$ $+ 2x_1x_2 + 2x_1x_3$	
CONSTRAINTS:	
$3 - x_1 - x_2 - 2x_3 \geq 0$ $0 \leq x_i, i=1,2,3$	
START:	$x_0 = (.5, .5, .5)$ (feasible) $f(x_0) = 2.25$
SOLUTION:	$x^* = (4/3, 7/9, 4/9)$ $f(x^*) = 1/9$ $r(x^*) = 0$ $e(x^*) = .49E-10$ $\mu = 1$ $I(x^*) = (1)$ $u_{\max}^*/u_{\min}^* = .2222/.2222 = 1$ $\lambda_{\max}^*/\lambda_{\min}^* = 3.72/1.61 = 2.31$

PROBLEM:	36
CLASSIFICATION:	PLR-T1-2
SOURCE:	Biggs [10]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1(1)$, $m-m_1 = 0$, $b = 6$
OBJECTIVE FUNCTION:	
	$f(x) = -x_1 x_2 x_3$
CONSTRAINTS:	
	$72 - x_1 - 2x_2 - 2x_3 \geq 0$
	$0 \leq x_1 \leq 20$
	$0 \leq x_2 \leq 11$
	$0 \leq x_3 \leq 42$
START:	$x_0 = (10, 10, 10)$ (feasible) $f(x_0) = -1000$
SOLUTION:	$x^* = (20, 11, 15)$ $f(x^*) = -3300$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 3$ $I(x^*) = (1, 5, 6)$ $u_{\max}^*/u_{\min}^* = 110/55 = 2$ $\lambda_{\max}^*/\lambda_{\min}^* = -$

PROBLEM:	37
CLASSIFICATION:	PLR-T1-3
SOURCE:	Betts [8], Box [12]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 2(2)$, $m-m_1 = 0$, $b = 6$
OBJECTIVE FUNCTION:	
	$f(x) = -x_1 x_2 x_3$
CONSTRAINTS:	
	$72 - x_1 - 2x_2 - 2x_3 \geq 0$
	$x_1 + 2x_2 + 2x_3 \geq 0$
	$0 \leq x_i \leq 42, i=1,2,3$
START:	$x_0 = (10, 10, 10)$ (feasible)
	$f(x_0) = -1000$
SOLUTION:	$x^* = (24, 12, 12)$
	$f(x^*) = -3456$
	$r(x^*) = 0$
	$e(x^*) = 0$
	$\mu = 1$
	$I(x^*) = (1)$
	$u_{\max}^*/u_{\min}^* = 144/144 = 1$
	$\lambda_{\max}^*/\lambda_{\min}^* = 24/8 = 3$

PROBLEM:	38 (Colville No.4)
CLASSIFICATION:	PBR-T1-4
SOURCE:	Colville [20], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1-x_3)^2 \\ + 10.1((x_2 - 1)^2 + (x_4 - 1)^2) + 19.8(x_2 - 1)(x_4 - 1)$$

CONSTRAINTS:

$$-10 \leq x_i \leq 10 , i=1, \dots, 4$$

START:	x_0	=	(-3, -1, -3, -1)	(feasible)
	$f(x_0)$	=	19192	

SOLUTION:	x^*	=	(1, 1, 1, 1)
	$f(x^*)$	=	0
	$r(x^*)$	=	0
	$e(x^*)$	=	-
	μ	=	0
	$I(x^*)$	=	-
	u_{\max}^*/u_{\min}^*	=	-
	$\lambda_{\max}^*/\lambda_{\min}^*$	=	.10E4/.72 = .14E4

PROBLEM:	39
CLASSIFICATION:	LPR-T1-1
SOURCE:	Miele e.al. [44,45]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2$, $b = 0$
OBJECTIVE FUNCTION:	
	$f(x) = -x_1$
CONSTRAINTS:	
	$x_2 - x_1^3 - x_3^2 = 0$
	$x_1^2 - x_2 - x_4^2 = 0$
START:	$x_0 = (2, 2, 2, 2)$ (not feasible) $f(x_0) = -2$
SOLUTION:	$x^* = (1, 1, 0, 0)$ $f(x^*) = -1$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = 1/1 = 1$ $\lambda_{\max}^*/\lambda_{\min}^* = 2/2 = 1$

PROBLEM:	40
CLASSIFICATION:	PPR-T1-2
SOURCE:	Beltrami [6], Indusi [35]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m - m_1 = 3$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = -x_1 x_2 x_3 x_4$$

CONSTRAINTS:

$$x_1^3 + x_2^2 - 1 = 0$$

$$x_1^2 x_4 - x_3 = 0$$

$$x_4^2 - x_2 = 0$$

START: . $x_0 = (.8, .8, .8, .8)$ (not feasible)
 $f(x_0) = -.4096$

SOLUTION: $x^* = (2^a, 2^{2b}, (-1)^i 2^c, (-1)^i 2^b)$

$$f(x^*) = -.25 \quad i=1,2$$

$$r(x^*) = 0 \quad a = -1/3$$

$$e(x^*) = .80E-11 \quad b = -1/4$$

$$\mu = 0 \quad c = -11/12$$

$$I(x^*) = -$$

$$u_{\max}^*/u_{\min}^* = .5/.3536 = 1.41$$

$$\lambda_{\max}^*/\lambda_{\min}^* = 1.74/1.74 = 1$$

PROBLEM:	41
CLASSIFICATION:	PLR-T1-4
SOURCE:	Betts [8], Miele e.al. [42]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1(1)$, $b = 8$
OBJECTIVE FUNCTION:	
	$f(x) = 2 - x_1 x_2 x_3$
CONSTRAINTS:	
	$x_1 + 2x_2 + 2x_3 - x_4 = 0$
	$0 \leq x_i \leq 1, i=1,2,3$
	$0 \leq x_4 \leq 2$
START:	$x_0 = (2, 2, 2, 2)$ (not feasible) $f(x_0) = -6$
SOLUTION:	$x^* = (2/3, 1/3, 1/3, 2)$ $f(x^*) = 52/27$ $r(x^*) = 0$ $e(x^*) = .13E-10$ $\mu = 1$ $I(x^*) = (8)$ $u_{\max}^*/u_{\min}^* = .1111/.1111 = 1$ $\lambda_{\max}^*/\lambda_{\min}^* = .67/.22 = 3$

PROBLEM:	42
CLASSIFICATION:	QQR-T1-10
SOURCE:	Brusch [14]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2(1)$, $b = 0$
OBJECTIVE FUNCTION:	
	$f(x) = (x_1 - 1)^2 + (x_2 - 2)^2 + (x_3 - 3)^2$ $+ (x_4 - 4)^2$
CONSTRAINTS:	
	$x_1 - 2 = 0$ $x_3^2 + x_4^2 - 2 = 0$
START:	$x_0 = (1, 1, 1, 1)$ (not feasible) $f(x_0) = 14$
SOLUTION:	$x^* = (2, 2, .6\sqrt{2}, .8\sqrt{2})$ $f(x^*) = 28 - 10\sqrt{2}$ $r(x^*) = 0$ $e(x^*) = .2E-23$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = 2.5355/2.0000 = 1.26$ $\lambda_{\max}^*/\lambda_{\min}^* = 7.07/2.00 = 3.54$

PROBLEM:	43 (Rosen-Suzuki)
CLASSIFICATION:	QQR-T1-11
SOURCE: Betts [8], Charalambous [18], Gould [27], Sheela [57]	
NUMBER OF VARIABLES: $n = 4$	
NUMBER OF CONSTRAINTS: $m_1 = 3$, $m-m_1 = 0$, $b = 0$	
OBJECTIVE FUNCTION:	
$f(x) = x_1^2 + x_2^2 + 2x_3^2 + x_4^2 - 5x_1 - 5x_2 - 21x_3$ $+ 7x_4$	
CONSTRAINTS:	
$8 - x_1^2 - x_2^2 - x_3^2 - x_4^2 - x_1 + x_2 - x_3 + x_4 \geq 0$	
$10 - x_1^2 - 2x_2^2 - x_3^2 - 2x_4^2 + x_1 + x_4 \geq 0$	
$5 - 2x_1^2 - x_2^2 - x_3^2 - 2x_1 + x_2 + x_4 \geq 0$	
START:	
x_0	$= (0, 0, 0, 0)$ (feasible)
$f(x_0)$	$= 0$
SOLUTION: x^*	
x^*	$= (0, 1, 2, -1)$
$f(x^*)$	$= -44$
$r(x^*)$	$= 0$
$e(x^*)$	$= .21E-9$
μ	$= 2$
$I(x^*)$	$= (1, 3)$
u_{\max}^*/u_{\min}^*	$= 2/1 = 2$
$\lambda_{\max}^*/\lambda_{\min}^*$	$= 9/8.07 = 1.12$

PROBLEM:	44
CLASSIFICATION:	QLR-T1-4
SOURCE:	Konno [37]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 6(6)$, $m-m_1 = 0$, $b = 4$
OBJECTIVE FUNCTION:	
	$f(x) = x_1 - x_2 - x_3 - x_1x_3 + x_1x_4 + x_2x_3 - x_2x_4$
CONSTRAINTS:	
	$8 - x_1 - 2x_2 \geq 0$
	$12 - 4x_1 - x_2 \geq 0$
	$12 - 3x_1 - 4x_2 \geq 0$
	$8 - 2x_3 - x_4 \geq 0$
	$8 - x_3 - 2x_4 \geq 0$
	$5 - x_3 - x_4 \geq 0$, $0 \leq x_i$, $i=1, \dots, 4$
START:	$x_0 = (0, 0, 0, 0)$ (feasible) $f(x_0) = 0$
SOLUTION:	$x^* = (0, 3, 0, 4)$ $f(x^*) = -15$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 4$ $I(x^*) = (3, 5, 7, 9)$ $u_{\max}^*/u_{\min}^* = 8.75/1.25 = 7$ $\lambda_{\max}^*/\lambda_{\min}^* = -$

PROBLEM:	45
CLASSIFICATION:	PBR-T1-5
SOURCE:	Betts [8], Miele e.al. [42]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 10$
OBJECTIVE FUNCTION:	
	$f(x) = 2 - \frac{1}{120} x_1 x_2 x_3 x_4 x_5$
CONSTRAINTS:	
	$0 \leq x_i \leq i, i=1, \dots, 5$
START:	$x_0 = (2, 2, 2, 2)$ (not feasible) $f(x_0) = 26/15$
SOLUTION:	$x^* = (1, 2, 3, 4, 5)$ $f(x^*) = 1$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 5$ $I(x^*) = (6, 7, 8, 9, 10)$ $u_{\max}^*/u_{\min}^* = 1/.2 = 5$ $\lambda_{\max}^*/\lambda_{\min}^* = -$

PROBLEM:	47
CLASSIFICATION:	PPR-T1-3
SOURCE:	Huang, Aggerwal [34], Miele e.al. [43]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 0$
OBJECTIVE FUNCTION:	
	$f(x) = (x_1 - x_2)^2 + (x_2 - x_3)^3 + (x_3 - x_4)^4 + (x_4 - x_5)^4$
CONSTRAINTS:	
	$x_1 + x_2^2 + x_3^3 - 3 = 0$
	$x_2 - x_3^2 + x_4 - 1 = 0$
	$x_1 x_5 - 1 = 0$
START:	$x_0 = (2, \sqrt{2}, -1, 2-\sqrt{2}, .5)$ (feasible) $f(x_0) = 12.4954368$
SOLUTION:	$x^* = (1, 1, 1, 1, 1)$ $f(x^*) = 0$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = 0/0$ $\lambda_{\max}^*/\lambda_{\min}^* = 2.08/.53 = 3.92$

PROBLEM:	48
CLASSIFICATION:	QLR-T1-5
SOURCE:	Huang, Aggerwal [34], Miele e.al. [43]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2(2)$, $b = 0$
OBJECTIVE FUNCTION:	
	$f(x) = (x_1 - 1)^2 + (x_2 - x_3)^2 + (x_4 - x_5)^2$
CONSTRAINTS:	
	$x_1 + x_2 + x_3 + x_4 + x_5 - 5 = 0$
	$x_3 - 2(x_4 + x_5) + 3 = 0$
START:	$x_0 = (3, 5, -3, 2, -2)$ (feasible) $f(x_0) = 84$
SOLUTION:	$x^* = (1, 1, 1, 1, 1)$ $f(x^*) = 0$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = 0/0$ $\lambda_{\max}^*/\lambda_{\min}^* = 4/1.49 = 2.69$

PROBLEM:	49
CLASSIFICATION:	PLR-T1-5
SOURCE:	Huang, Aggerwal [34]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2(2)$, $b = 0$
OBJECTIVE FUNCTION:	$f(x) = (x_1 - x_2)^2 + (x_3 - 1)^2 + (x_4 - 1)^4 + (x_5 - 1)^6$
CONSTRAINTS:	$x_1 + x_2 + x_3 + 4x_4 - 7 = 0$ $x_3 + 5x_5 - 6 = 0$
START:	$x_0 = (10, 7, 2, -3, .8)$ (feasible) $f(x_0) = 266$
SOLUTION:	$x^* = (1, 1, 1, 1, 1)$ $f(x^*) = 0$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = 0/0$ $\lambda_{\max}^*/\lambda_{\min}^* = 4/.70E-10 = .57E11$

PROBLEM:	50
CLASSIFICATION:	PLR-T1-6
SOURCE:	Huang, Aggerwal [34]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 0$
OBJECTIVE FUNCTION:	
	$f(x) = (x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_4)^4$ $+ (x_4 - x_5)^2$
CONSTRAINTS:	
	$x_1 + 2x_2 + 3x_3 - 6 = 0$
	$x_2 + 2x_3 + 3x_4 - 6 = 0$
	$x_3 + 2x_4 + 3x_5 - 6 = 0$
START:	$x_0 = (35, -31, 11, 5, -5)$ (feasible) $f(x_0) = 17416$
SOLUTION:	$x^* = (1, 1, 1, 1, 1)$ $f(x^*) = 0$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = 0/0$ $\lambda_{\max}^*/\lambda_{\min}^* = 5.89/1.64 = 3.6$

PROBLEM:	51
CLASSIFICATION:	QLR-T1-6
SOURCE:	Huang, Aggerwal [34]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 0$
OBJECTIVE FUNCTION:	
	$f(x) = (x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2$ $+ (x_5 - 1)^2$
CONSTRAINTS:	
	$x_1 + 3x_2 - 4 = 0$
	$x_3 + x_4 - 2x_5 = 0$
	$x_2 - x_5 = 0$
START:	$x_0 = (2.5, .5, 2, -1, .5)$ (feasible) $f(x_0) = 8.5$
SOLUTION:	$x^* = (1, 1, 1, 1, 1)$ $f(x^*) = 0$ $r(x^*) = 0$ $e(x^*) = 0$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = 0/0$ $\lambda_{\max}^*/\lambda_{\min}^* = 3.49/1.90 = 1.84$

PROBLEM:	52
CLASSIFICATION:	QLR-T1-7
SOURCE:	Miele e.al. [44,45]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 0$
OBJECTIVE FUNCTION:	
	$f(x) = (4x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2$ $+ (x_5 - 1)^2$
CONSTRAINTS:	
	$x_1 + 3x_2 = 0$
	$x_3 + x_4 - 2x_5 = 0$
	$x_2 - x_5 = 0$
START:	$x_0 = (2, 2, 2, 2, 2)$ (not feasible) $f(x_0) = 42$
SOLUTION:	$x^* = (-33, 11, 180, -158, 11)/349$ $f(x^*) = 1859/349$ $r(x^*) = 0$ $e(x^*) = .14E-9$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = 7.7479/2.9054 = 2.6667$ $\lambda_{\max}^*/\lambda_{\min}^* = 26.93/1.99 = 13.51$

PROBLEM:	53
CLASSIFICATION:	QLR-T1-8
SOURCE:	Betts [8], Miele e.al. [42,43]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 10$
OBJECTIVE FUNCTION:	$f(x) = (x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2$ $+ (x_5 - 1)^2$
CONSTRAINTS:	$x_1 + 3x_2 = 0$ $x_3 + x_4 - 2x_5 = 0$ $x_2 - x_5 = 0$ $-10 \leq x_i \leq 10, \quad i=1, \dots, 5$
START:	$x_0 = (2, 2, 2, 2, 2)$ (not feasible) $f(x_0) = 6$
SOLUTION:	$x^* = (-33, 11, 27, -5, 11)/43$ $f(x^*) = 176/43$ $r(x^*) = 0$ $e(x^*) = .28E-9$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = 5.9535/2.0465 = 1.84$ $\lambda_{\max}^*/\lambda_{\min}^* = 3.49/1.90 = 1.84$

PROBLEM:	65
CLASSIFICATION:	QQR-P1-3
SOURCE:	Murtagh, Sargent [47]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 6$
OBJECTIVE FUNCTION:	
	$f(x) = (x_1 - x_2)^2 + (x_1 + x_2 - 10)^2/9 + (x_3 - 5)^2$
CONSTRAINTS:	
	$48 - x_1^2 - x_2^2 - x_3^2 \geq 0$
	$-4.5 \leq x_i \leq 4.5, i=1,2$
	$-5 \leq x_3 \leq 5$
START:	$x_0 = (-5, 5, 0)$ (not feasible) $f(x_0) = 1225/9$
SOLUTION:	$x^* = (3.650461821, 3.65046168, 4.6204170507)$ $f(x^*) = .9535288567$ $r(x^*) = 0$ $e(x^*) = .40E-6$ $\mu = 1$ $I(x^*) = (1)$ $u_{\max}^*/u_{\min}^* = .08215/.08215 = 1$ $\lambda_{\max}^*/\lambda_{\min}^* = 1.95/1.68 = 1.16$

PROBLEM:	71
CLASSIFICATION:	PPR-P1-3
SOURCE:	Bartholomew-Biggs [4]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 1$, $b = 8$
OBJECTIVE FUNCTION:	
	$f(x) = x_1 x_4 (x_1 + x_2 + x_3) + x_3$
CONSTRAINTS:	
	$x_1 x_2 x_3 x_4 - 25 \geq 0$
	$x_1^2 + x_2^2 + x_3^2 + x_4^2 - 40 = 0$
	$1 \leq x_i \leq 5, i=1, \dots, 4$
START:	$x_0 = (1, 5, 5, 1)$ (feasible) $f(x_0) = 16$
SOLUTION:	$x^* = (1, 4.7429994, 3.8211503, 1.3794082)$ $f(x^*) = 17.0140173$ $r(x^*) = 0$ $e(x^*) = .51E-6$ $\mu = 2$ $I(x^*) = (1, 2)$ $u_{\max}^*/u_{\min}^* = 1.0879/.1615 = 6.74$ $\lambda_{\max}^*/\lambda_{\min}^* = 1.18/1.18 = 1$

PROBLEM:	76
CLASSIFICATION:	QLR-P1-1
SOURCE:	Murtagh, Sargent [47]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 3(3)$, $m-m_1 = 0$, $b = 4$
OBJECTIVE FUNCTION:	
	$f(x) = x_1^2 + .5x_2^2 + x_3^2 + .5x_4^2 - x_1x_3 + x_3x_4$ $- x_1 - 3x_2 + x_3 - x_4$
CONSTRAINTS:	
	$5 - x_1 - 2x_2 - x_3 - x_4 \geq 0$
	$4 - 3x_1 - x_2 - 2x_3 + x_4 \geq 0$
	$x_2 + 4x_3 - 1.5 \geq 0$
	$0 \leq x_i, i=1, \dots, 4$
START:	$x_0 = (.5, .5, .5, .5)$ (feasible)
	$f(x_0) = -1.25$
SOLUTION:	$x^* = (.2727273, 2.090909, -.26E-10, .5454545)$
	$f(x^*) = -4.681818181$
	$r(x^*) = .84E-10$
	$e(x^*) = .15E-10$
	$\mu = 2$
	$I(x^*) = (1, 6)$
	$u_{\max}^*/u_{\min}^* = 1.7272/.4545 = 3.8$
	$\lambda_{\max}^*/\lambda_{\min}^* = 1.83/1 = 1.83$

PROBLEM:	77
CLASSIFICATION:	PGR-P1-3
SOURCE:	Betts [8], Miele e.al. [42,44,45]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2$, $b = 0$
OBJECTIVE FUNCTION:	
	$f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_3 - 1)^2$ $+ (x_4 - 1)^4 + (x_5 - 1)^6$
CONSTRAINTS:	
	$x_1^2 x_4 + \sin(x_4 - x_5) - 2\sqrt{2} = 0$ $x_2 + x_3^4 x_4^2 - 8 - \sqrt{2} = 0$
START:	$x_0 = (2, 2, 2, 2, 2)$ $f(x_0) = 4$ (not feasible)
SOLUTION:	$x^* = (1.166172, 1.182111, 1.380257, 1.506036,$ $f(x^*) = .24150513$ $r(x^*) = .12E-9$ $e(x^*) = .53E-7$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = .08554/.03188 = 2.68$ $\lambda_{\max}^*/\lambda_{\min}^* = 3.92/.75 = 5.25$

PROBLEM:	78
CLASSIFICATION:	PPR-P1-4
SOURCE:	Asaadi [1], Powell [51]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 0$
OBJECTIVE FUNCTION:	
	$f(x) = x_1 x_2 x_3 x_4 x_5$
CONSTRAINTS:	
	$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0$
	$x_2 x_3 - 5x_4 x_5 = 0$
	$x_1^3 + x_2^3 + 1 = 0$
START:	$x_0 = (-2, 1.5, 2, -1, -1)$ $f(x_0) = -6$ (not feasible)
SOLUTION:	$x^* = (-1.717142, 1.595708, 1.827248, -.7636429, -.7636435)$ $f(x^*) = -2.91970041$ $r(x^*) = .35E-9$ $e(x^*) = .91E-5$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = .7444/.09681 = 7.69$ $\lambda_{\max}^*/\lambda_{\min}^* = 3.04/2.98 = 1.02$

PROBLEM:	79
CLASSIFICATION:	PPR-P1-5
SOURCE:	Betts [8], Miele e.al. [42,44,45]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 0$
OBJECTIVE FUNCTION:	
	$f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_2 - x_3)^2$ $+ (x_3 - x_4)^4 + (x_4 - x_5)^4$
CONSTRAINTS:	
	$x_1 + x_2^2 + x_3^3 - 2 - 3\sqrt{2} = 0$ $x_2 - x_3^2 + x_4 + 2 - 2\sqrt{2} = 0$ $x_1 x_5 - 2 = 0$
START:	$x_0 = (2, 2, 2, 2, 2)$ (not feasible) $f(x_0) = 1$
SOLUTION:	$x^* = (1.191127, 1.362603, 1.472818, 1.635017,$ $f(x^*) = .0787768209 \quad 1.679081)$ $r(x^*) = .58E-9$ $e(x^*) = .71E-10$ $\mu = 0$ $I(x^*) = -$ $u_{\max}^*/u_{\min}^* = .3882E-1/.2873E-3 = 135.1$ $\lambda_{\max}^*/\lambda_{\min}^* = 2.03/.70 = 2.88$

PROBLEM:	100
CLASSIFICATION:	PPR-P1-7
SOURCE:	Asaadi [1], Charalambous [18], Wong [59]
NUMBER OF VARIABLES:	$n = 7$
NUMBER OF CONSTRAINTS:	$m_1 = 4$, $m-m_1 = 0$, $b = 0$
OBJECTIVE FUNCTION:	
	$f(x) = (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2$ $+ 10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7$
CONSTRAINTS:	
	$127 - 2x_1^2 - 3x_2^4 - x_3 - 4x_4^2 - 5x_5 \geq 0$ $282 - 7x_1 - 3x_2 - 10x_3^2 - x_4 + x_5 \geq 0$ $196 - 23x_1 - x_2^2 - 6x_6^2 + 8x_7 \geq 0$ $-4x_1^2 - x_2^2 + 3x_1x_2 - 2x_3^2 - 5x_6 + 11x_7 \geq 0$
START:	$x_0 = (1, 2, 0, 4, 0, 1, 1)$ $f(x_0) = 714$ (feasible)
SOLUTION:	$x^* = (2.330499, 1.951372, -.4775414, 4.365726,$ $- .6244870, 1.038131, 1.594227)$ $f(x^*) = 680.6300573$ $r(x^*) = .90E-7$ $e(x^*) = .36E-8$ $\mu = 2$ $I(x^*) = (1, 4)$ $u_{\max}^*/u_{\min}^* = 1.140/.3686 = 3.09$ $\lambda_{\max}^*/\lambda_{\min}^* = 46.6/4.34 = 10.7$

PROBLEM:	106 (heat exchanger design)
CLASSIFICATION:	LQR-P1-5
SOURCE:	Avriel, Williams [2], Dembo [22]
NUMBER OF VARIABLES:	$n = 8$
NUMBER OF CONSTRAINTS:	$m_1 = 6(3)$, $m-m_1 = 0$, $b = 16$
OBJECTIVE FUNCTION:	$f(x) = x_1 + x_2 + x_3$
CONSTRAINTS:	$1 - .0025(x_4 + x_6) \geq 0$ $1 - .0025(x_5 + x_7 - x_4) \geq 0$ $1 - .01(x_8 - x_5) \geq 0$ $x_1x_6 - 833.33252x_4 - 100x_1 + 83333.333 \geq 0$ $x_2x_7 - 1250x_5 - x_2x_4 + 1250x_4 \geq 0$ $x_3x_8 - 1250000 - x_3x_5 + 2500x_5 \geq 0$ $100 \leq x_1 \leq 10000$ $1000 \leq x_i \leq 10000, i=2,3$ $10 \leq x_i \leq 1000, i=4,\dots,8$
START:	$x_0 = (5000, 5000, 5000, 200, 350, 150, 225, 425)$ $f(x_0) = 15000$ (not feasible)
SOLUTION:	$x^* = (579.3167, 1359.943, 5110.071, 182, 0174,$ $295.5985, 217.9799, 286.4162, 395, 5979)$ $f(x^*) = 7049.330923$ $r(x^*) = 0$ $e(x^*) = .19E-4$ $\mu = 6$ $I(x^*) = (1, 2, 3, 4, 5, 6)$ $u_{\max}^*/u_{\min}^* = 5210/.00848 = .61E6$ $\lambda_{\max}^*/\lambda_{\min}^* = .81E-3/.38E-3 = 2.11$

PROBLEM:	108	
CLASSIFICATION:	QQR-P1-6	
SOURCE:	Himmelblau [29], Pearson [49]	
NUMBER OF VARIABLES:	$n = 9$	
NUMBER OF CONSTRAINTS:	$m_1 = 13$, $m-m_1 = 0$, $b = 1$	
OBJECTIVE FUNCTION:		
$f(x) = -.5(x_1x_4 - x_2x_3 + x_3x_9 - x_5x_9 + x_5x_8 - x_6x_7)$		
CONSTRAINTS:		
$1 - x_3^2 - x_4^2 \geq 0$	$1 - x_9^2 \geq 0$	
$1 - x_5^2 - x_6^2 \geq 0$	$1 - x_1^2 - (x_2 - x_9)^2 \geq 0$	
$1 - (x_1 - x_5)^2 - (x_2 - x_6)^2 \geq 0$		
$1 - (x_1 - x_7)^2 - (x_2 - x_8)^2 \geq 0$		
$1 - (x_3 - x_5)^2 - (x_4 - x_6)^2 \geq 0$		
$1 - (x_3 - x_7)^2 - (x_4 - x_8)^2 \geq 0$		
$1 - x_7^2 - (x_8 - x_9)^2 \geq 0$	$x_1x_4 - x_2x_3 \geq 0$	
$x_3x_9 \geq 0$	$-x_5x_9 \geq 0$	
$x_5x_8 - x_6x_7 \geq 0$	$0 \leq x_9$	
START:	$x_0 = (1, 1, 1, 1, 1, 1, 1, 1, 1)$ (not feasible)	
	$f(x_0) = 0$	
SOLUTION:	$x^* = (.8841292, .4672425, .03742076, .9992996,$ $.8841292, .4672424, .03742076, .9992996,$ $.26E-19)$	
$f(x^*)$	$= -.8660254038$	
$r(x^*)$	$= .39E-9$	$e(x^*) = .33E-11$
u	$= 9$	$I(x^*) = (1, 3, 4, 6, 7, 9, 11,$ $12, 14)$
u_{\max}^*/u_{\min}^*	$= .1443/0$	
$\lambda_{\max}^*/\lambda_{\min}^*$	$= -$	

PROBLEM:	113 (Wong No.2)
CLASSIFICATION:	QQR-P1-7
SOURCE:	Asaadi [1], Charalambous [18], Wong [59]
NUMBER OF VARIABLES:	$n = 10$
NUMBER OF CONSTRAINTS:	$m_1 = 8(3)$, $m-m_1 = 0$, $b = 0$
OBJECTIVE FUNCTION:	$ \begin{aligned} f(x) = & x_1^2 + x_2^2 + x_1x_2 - 14x_1 - 16x_2 + (x_3 - 10)^2 \\ & + 4(x_4 - 5)^2 + (x_5 - 3)^2 + 2(x_6 - 1)^2 + 5x_7^2 \\ & + 7(x_8 - 11)^2 + 2(x_9 - 10)^2 + (x_{10} - 7)^2 + 45 \end{aligned} $
CONSTRAINTS:	$ \begin{aligned} 105 - 4x_1 - 5x_2 + 3x_7 - 9x_8 & \geq 0 \\ -10x_1 + 8x_2 + 17x_7 - 2x_8 & \geq 0 \\ 8x_1 - 2x_2 - 5x_9 + 2x_{10} + 12 & \geq 0 \\ -3(x_1 - 2)^2 - 4(x_2 - 3)^2 - 2x_3^2 + 7x_4 + 120 & \geq 0 \\ -5x_1^2 - 8x_2 - (x_3 - 6)^2 + 2x_4 + 40 & \geq 0 \\ -.5(x_1 - 8)^2 - 2(x_2 - 4)^2 - 3x_5^2 + x_6 + 30 & \geq 0 \\ -x_1^2 - 2(x_2 - 2)^2 + 2x_1x_2 - 14x_5 + 6x_6 & \geq 0 \\ 3x_1 - 6x_2 - 12(x_9 - 8)^2 + 7x_{10} & \geq 0 \end{aligned} $
START:	$x_0 = (2, 3, 5, 5, 1, 2, 7, 3, 6, 10)$ (feasible) $f(x_0) = 753$
SOLUTION:	$x^* = (2.171996, 2.363683, 8.773926, 5.095984,$ $.9906548, 1.430574, 1.321644, 9.828726,$ $8.280092, 8.375927)$ $f(x^*) = 24.3062091$ $r(x^*) = .12E-8$ $e(x^*) = .46E-9$ $u = 6$ $I(x^*) = (1, 2, 3, 4, 5, 7)$ $u_{\max}^*/u_{\min}^* = 1.717/.02055 = 83.5$ $\lambda_{\max}^*/\lambda_{\min}^* = 7.79/2.24 = 3.48$