Time Series Analysis & Recurrent Neural Networks

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WS2019/2020

Exercise 11

To be uploaded before the exercise group on January 22nd, 2020

1. *Extended Kalman filter*

In file *par.mat* (or corresponding *par.pkl*), you are given matrices $\mathbf{A}$, $\mathbf{W}$, $\mathbf{B}$, Γ, Σ, and vector $\boldsymbol{\mu}_0$, which specify the parameters of the following non-linear recurrent neural network:

$$\mathbf{z}_t = f(\mathbf{z}_{t-1}, \boldsymbol{\eta}_t) = \mathbf{A}\mathbf{z}_{t-1} + \mathbf{W}\phi(\mathbf{z}_{t-1}) + \mathbf{h} + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim N(0, \Sigma)\ ,$$

where $\phi(\mathbf{z}_t) = \max(\mathbf{z}_t, 0)$ is an (element-wise) piece-wise linear transfer function which returns each element of $\mathbf{z}_t$ if it crosses the threshold 0, or 0 otherwise. We will assume that observations drawn from this (latent) dynamical system are just a linear transformation of the latent states $\mathbf{z}_t$:

$$\mathbf{x}_t = g(\mathbf{z}_t, \boldsymbol{\eta}_t) = \mathbf{B}\mathbf{z}_t + \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim N(0, \Gamma)\ .$$

Both our evolution and observation process are noisy (that is, the noise follows a normal distribution with 0 mean and covariance Σ or Γ, respectively). The initial state estimate is $\boldsymbol{\mu}_0$.

(a) Assuming your latent state dimension to be equal to $M = 3$, and the observation dimension to be $N = 10$, let this system run for $T = 500$ time steps, and create "true" latent states $\{\mathbf{z}_t\}_{1:T}$, and observations $\{\mathbf{x}_t\}_{1:T}$, based on this system and the given parameters.

(b) Run the Kalman filter that we have previously implemented (tutorial 5) to obtain latent state estimates based on your created observations (see also the uploaded Kalman-filter code `KalmanFilter.m` or `kalmanfilter.py` in this weeks' folder if you have not implemented it yourself so far).

(c) Implement the extended Kalman filter (EKF), as described in (9.46) of the script. Use it to obtain latent state estimates based on your created observations.

(d) Compare the obtained estimate of $\{\mathbf{z}_t\}_{1:T}$ between the Kalman filter and the EKF by quantifying the mean squared error (MSE) between true and estimated latent states for both methods.