# Time Series Analysis & Recurrent Neural Networks

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## Exercise 4

To be uploaded before the exercise group on November 20th, 2019

**Task 1. Granger Causality.**

1. Create a 2-variate AR(2) time series $x_t = \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix}$ with $T = 1000$ time steps and the following parameters: $a_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $A_1 = \begin{pmatrix} 0.2 & -.2 \\ 0 & 0.1 \end{pmatrix}$, and $A_2 = \begin{pmatrix} 0.1 & -.1 \\ 0 & 0.1 \end{pmatrix}$, diagonal Gaussian noise matrix $\Sigma = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix}$, and initial condition $x_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.
2. Given your knowledge of model parameters above, does $x_1$ Granger-cause $x_2$, or does $x_2$ Granger-cause $x_1$ according to Granger's definition? Why?
3. Assuming that the order of the model ($p = 2$) is known, use the log-likelihood-ratio test statistic to confirm your results from 2 (see eq. 7.35 in script; command 'fcdf' in MatLab, and 'scipy.stats.f.cdf' in Python give the cumulative F-distribution).

**Task 2. M-Step in a linear Gaussian state space model**

Consider a linear Gaussian state space model,

$$\begin{aligned} z_t &= A z_{t-1} + \epsilon, && \epsilon \sim N(0, \Sigma) \\ x_t &= B z_t + \eta, && \eta \sim N(0, \Gamma) \end{aligned}$$

In the lecture we derived the M-step to determine the transition matrix $A$. Derive the M-step for the latent state noise $\Sigma$ and the observation matrix $B$ by maximizing the expected log-likelihood $\mathrm{E}[\log p(X, Z)]$, with respect to $\Sigma$ and $B$, where $X = \{x_t \mid t \in 1 \ldots T\}$ and $Z = \{z_t \mid t \in 1 \ldots T\}$ are the sets of all latent states and observations from time 1 to $T$.
(Clue: The matrix cookbook by Petersen and Pedersen (2012)
`https://www.math.uwaterloo.ca/~hwolkowi/matrixcookbook.pdf`
provides a helpful summary for matrix algebra).