Time Series Analysis & Recurrent Neural Networks

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Exercise 1

To be uploaded before the exercise group on October 30th, 2019

Task 1: Return plots
The file 'sunspotData.xls' (or 'sunspotData.mat') contains data of sunspot counts, orignally obtained from:

http://www.sidc.be/silso/versionarchive.

To smooth out day-to-day fluctuations, average observations over yearly periods. Plot the first return-map from the resulting time series. What do you notice? What is the oscillation period in years?

Task 2: Detrending and autocorrelation
The file 'investment.xls' (or 'investment.mat') contains scaled quarterly United States private investment per capita rates over the years 1948-1989, downloaded from Prof. Mark Watson's webpage under

http://www.princeton.edu/~mwatson/publi.html.

1. By using linear regression, remove the trend from the data.
2. Examine (loosely) whether the residual time series is approximately stationary. Is the time series of first differences stationary? How about the time series of second-order differences?
3. Compute the autocorrelation function of the time series with the linear trend removed. Are business cycles (corresponding to peaks in the autocorrelation function) periodic? [Note: write the autocorrelation function yourself]

Task 3: AR models

1. Create your own AR time series of length $T = 200$ and order $p = 4$, with the following coefficients given: $a_0 = 0, a_1 = -.8, a_2 = 0, a_3 = 0$, and $a_4 = .4$, with $\epsilon_t \sim N(0, 1)$, i.e. the noise process drawn from a standard normal distribution, and with the initial value of the time series being $x_0 = 0$.
2. Plot the time series in time as well as the first return-map. What do you notice?

Task 4: Stationarity and AR coefficients
We have derived that the criterion for an AR(1) process, $x_t = a_0 + a_1 x_{t-1} + \epsilon_t, \epsilon_t \sim W(0, \sigma^2)$, to be stationary is $|a_1| < 1$. Show that for an AR(p) process, $p > 1$, the mean of the time series is stationary if $|\sum a_i| < 1$, by expanding the process in time.