The dynamic model and control algorithm for the Active Suspension System

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***Abstract*—The suspension system can be classified as a passive, semi-active, and active suspension, according to its ability to add or extract energy. Road surface roughness is the leading cause of vehicle oscillation and the suspension system is used to dampen these oscillations. The objectives of the active suspension system are to improve the ride quality and handling performance within a given suspension stroke limitation. This paper will analyse the aspects of passive and active suspensions for a quarter-car model and construct an active suspension control for a quarter-car model subject to excitation from a road profile. Algorithms like PID (proportional–integral–derivative) and LQR (linear quadratic regulator) will fit the linear model. In contrast, the nonlinear model's algorithms, such as SMC (sliding mode control), fuzzy, and ANN (artificial neural network), will perform better.**

I. Introduction

VEHICLE suspension main task is to separate passenger and vehicular body interactions from oscillations generated by road abnormalities whilst still maintaining continuous wheel-road contact. Generally, traditional suspension consisting in springs and dampers are referred to as passive suspension, while if the suspension is externally controlled it is known as a semi-active or active suspension. A good suspension system should provide good vibration isolation, i.e. small acceleration of the body mass, and a small "rattle space", which is the maximal allowable relative displacement between the vehicle body and various suspension components.

An active suspension system has the ability to store, dissipate and to introduce energy into the system. It may vary its parameters depending on operating conditions. The passive suspension system is an open loop control system. It is designs to achieve certain condition only. The characteristic of passive suspension are fix and cannot be adjusted by any mechanical part. The problem of passive suspension is that it cannot be designed heavily damped or too hard, otherwise it will transfer a lot of road input or it could throw around the car due to unevenness of the road. Additionally, if it is lightly damped or too soft, the suspension will reduce the vehicle's stability in turns, change lane or even swing the car. Therefore, the performance of the passive suspension depends on the road profile.

An active suspension system improves vehicle ride comfort, generating impact force that is transmitted to the sprung and unsprung masses. If the value of this force is small, the system's response will not be good. This means that the car's vibration has yet to be improved. If the impact force is bigger, the ride comfort can be further improved. However, a more significant impact force will cause a change in the dynamic load at the wheel. Once the value of the dynamic force at the wheel is reduced to zero, the wheel may be lifted off the road, and instability will occur. That's why, it is difficult to satisfy both conditions of smoothness and stability when studying control algorithms for active suspension.

The main content of this paper is to implement two different control systems, based on the dynamic equations of the quarter-car model. This paper, also compares the active suspension performances, given by different control systems, with the passive suspension when the car hits a sinusoidal and more realistic road profile (i.e. irregular bump).

II. Suspension performance

The suspension system in a vehicle aims at fulfilling the requirements of both comfort, for passengers, and road handling, for the driver. These two aspects, however, characterise in a general way what the suspension system should provide to the driver and the vehicle. Hence, they must be translated into physical quantities that can be observed, to evaluate the suspension performance. The most important parameters are:

- **Ride comfort**: is the ability of the vehicle suspension system of insulating passengers, and payloads, from vibrations caused by the road profile roughness.
- **Body motion**: which is known as bounce, pitch and roll of the sprung mass created primarily by cornering and braking manoeuvres.
- **Suspension travel**: refers to the relative displacement between the sprung and the unsprung masses.

III. Physical and mathematical modeling

A quarter-car dynamic model is commonly used in studies of oscillation control for suspension systems. This model, Figure 1, includes only two masses: the sprung mass, *ms*, and the unsprung mass, *mus*. The suspension system is modelled as a spring and a shock absorber. The tires are also modelled similarly to the suspension system.

For vehicles with an active suspension system, an actuator is located between the sprung mass and the unsprung mass, Figure 2.

The equations describing the dynamics of a quarter-car model are shown as follows:

$$\begin{aligned} ms\ddot{Z}s &= bsZ\dot{u}s - bs\dot{Z}s - ks(Zs - Zus) + Fa \\ mus\ddot{Z}s &= -bsZ\dot{u}s - bus\dot{Z}s + bs\dot{Z}s + bus\dot{Z}r \\ &\quad -ks(Zus - Zs) - kus(Zus - Zr) - Fa \end{aligned} \tag{1}$$

A state-space model describing the active suspension system will be created using the two equations of motion found in (1). The state variable representing the system are:

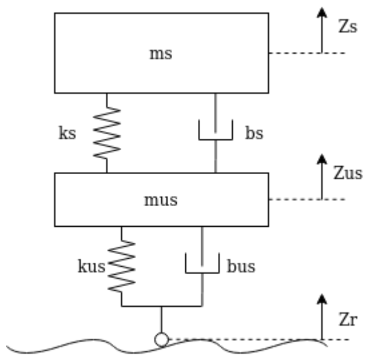

Fig. 1. Passive Suspension Quarter Car Model

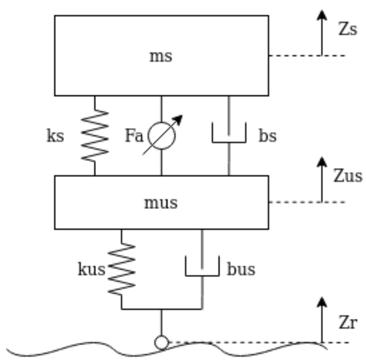

Fig. 2. Quarter car model active configuration

$x1 = Zs - Zus$: suspension travel
$x2 = \dot{Z}_s$: sprung mass velocity
$x3 = Zus - Zr$: wheel's deflection
$x4 = \dot{Zus}$: wheel's vertical velocity

The state-space model can easily be written in the matrix form shown below:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-ks}{ms} & \frac{-bs}{ms} & 0 & \frac{bs}{ms} \\ 0 & 0 & 0 & 1 \\ \frac{ks}{mus} & \frac{bs}{mus} & \frac{-kus}{mus} & \frac{-(bs+bus)}{mus} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{ms} \\ -1 & 0 \\ \frac{bus}{mus} & -\frac{1}{mus} \end{bmatrix} \begin{bmatrix} \dot{Zr} \\ Fa \end{bmatrix} \quad (2)$$

To perform the simulations the dynamic system described in (2) has been modelled using *simulink* as shown in Figure 3, where the inputs are the derivative of the road profile and the actuator force, while the outputs are the state space variable and the sprung mass acceleration.

IV. Controller design

A. LQR

The linear time-invariant system (LTI), is described by equation (2). Consider a state variable feedback regulator:

$$u = -Kx \quad (3)$$

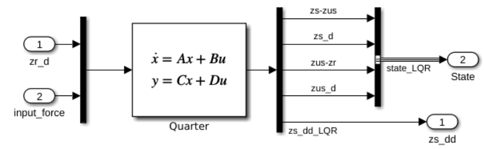

Fig. 3. Active suspension control system

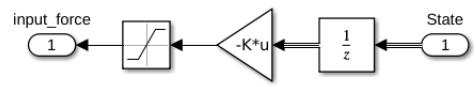

Fig. 4. LQR controller

where *K* is the state feedback gain matrix. The optimisation procedure consists of determining the control input *u*, which minimises the performance index. The performance index *J* represents the performance characteristic requirements, as well as the controller input limitation. The optimal controller of the given system is defined as controller design, which minimises the following performance index:

$$J = \int_0^\infty (x'Qx + u'Ru)\ dt \quad (4)$$

The gain matrix *K* is represented by:

$$K = R^{-1}B'P \quad (5)$$

The matrix *P* must satisfy the Riccati's reduced matrix equation:

$$A'P + PA - PBR^{-1}B'P + Q = 0 \quad (6)$$

Then the feedback regulator is:

$$u = -(R^{-1}B'P)x \quad (7)$$

The *simulink* model of the LQR controller described in (7) is shown in Figure 4.

B. PID

The Proportional-Integral-Derivative controller (PID) is the most-used feedback control design. PID shows the three terms operating on the error signal in order to produce a control signal. If *u(t)* is the control signal sent to the system, *y(t)* is the actual output and *r(t)* is the desired output. The tracking error can be defined as:

$$e(t) = r(t) - y(t) \quad (8)$$

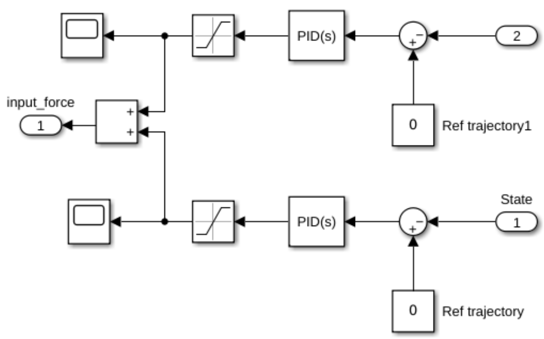

Fig. 5. PID controller

The PID controller can be written in the following form:

$$u(t) = Kp\ e(t) + Ki \int e(t)\ dt + Kd\ \frac{d}{dt}\ e(t) \tag{9}$$

The desired closed loop dynamics can be obtained by adjusting the three parameters *Kp*, *Ki* and *Kd*, often iteratively with ”tuning” and without specific knowledge of a plant model. Stability can often be obtained using only the proportional term. The integral term permits the rejection of a step disturbance. Meanwhile, the derivative term provides damping or shaping of the response.

The *simulink* model of the PID controller described in (9) is shown in Figure 5.

V. Simulation results and discussion

In order to test the effectiveness of the proposed schemes, a series of simulations have been carried out. The parameters considered for such simulations are:

$ms = 234[kg]; mus = 43[kg]; ks = 26000[N/m]; bs = 1544[Nsec/m]; kus = 10^5[N/m]; bus = 0[Nsec/m];$

which represent the parameters of E-Agle Trento Racing Team’s vehicle, *Fenice*.

The response of the system is tested considering two different road profiles:

- a sinusoidal profile:

$$1\ sin(2\ \pi\ t)$$

- a bumpy road:

$$zr = \begin{cases} 0.5\ (1 - sin(0.8\ \pi\ t)) & 0.5 \le t \le 0.75 \\ 0 & otherwise \end{cases}$$

The control variables used in the two controllers are different, for the PID controller only the suspension travel and the sprung mass acceleration are controlled, while for the LQR controller all the state space variables are controlled by the feedback regulator.

The tuning of the parameters has been done differently for the two controllers:

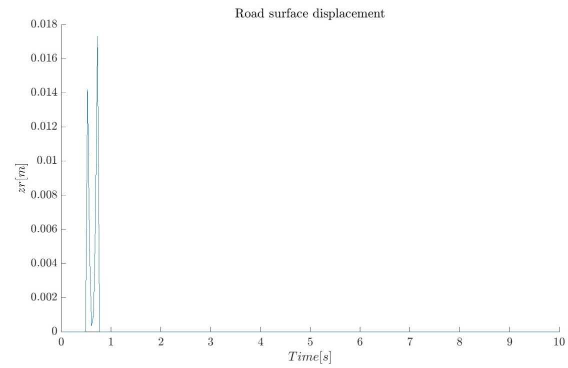

Fig. 6. Bumpy road profile

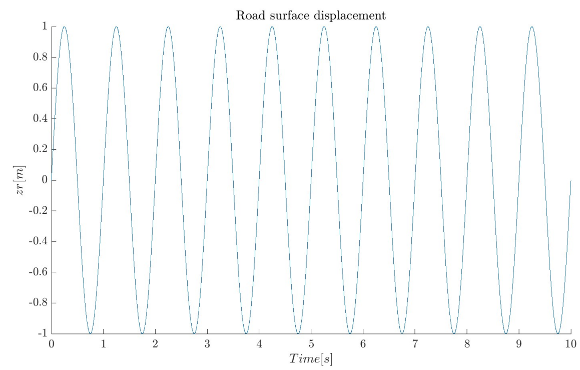

Fig. 7. Sinusoidal road profile

For the PID controller, the MATLAB function *pidtune* has been used, which given the transfer function and the type of controller (PID in this case) returns the correct values of the parameters (*Kp, Ki* and *Kd*). The parameters obtained are described in the following table.

Control variable	P	I	D
$zs - zus$	1.3701e+05	3.1131e+05	1.5075e+04
$\ddot{zs}$	1e+03	1.5665e+03	0

Meanwhile, for the LQR controller, a series of simulations have been made in order to obtain the proper coefficients. The performance index, or quadratic cost function, J must be minimised by adjusting both the weighting Q and R matrices, where Q is a diagonal positive definite and R is a positive constant. The desired closed-loop performance is obtained by tuning the weighting matrices, penalising bad performance by adjusting the Q matrix or penalising actuator effort by adjusting the R matrix until suitable results, regarding the cost function, are reached for the plant. The chosen parameters are:

$$Q = \begin{bmatrix} 1760 * 10^6 & 0 & 0 & 0 \\ 0 & 11.6 * 10^6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

$$R = 0.01 \tag{11}$$

The *simulink scheme* used to perform the overall simulation is reported in Appendix B.

Appendix A reports all the results obtained in the simulation for both of the road profiles studied.

Figure 8 and 9 illustrate the time response of the suspension travel. As we can see from the plots, both the peak and the settling time have been reduced by the active suspension system, compared to the passive system.

Figure 16 and 17 illustrate the time response of the sprung mass acceleration. As we can see from the plots, we have two different results depending on which road profile is being considered. For the bumpy road, we have a slight increase of the peak, but a reduced settling time, while, for the sinusoidal road profile both the peak and the settling time have been reduced.

For the other parameters reported in Appendix A, we can see that in general, we have better performance using an active suspension system.

VI. Conclusion

The simulation results show that an active suspension system gives a better performance in terms of comfort ride, compared to the passive one. Active suspensions also increase tire-to-road contact, making the vehicle more stable. In conclusion, from the simulation results, active suspension with an LQR or PID controller can be considered one of the valid solutions, to get excellent comfort ride and good handling of cars. A future implementation could be to introduce the effect of the actuator dynamics and its impact on the performance.

Appendix A
Simulation Plots

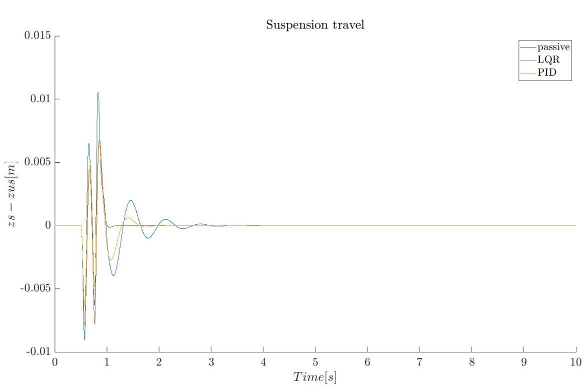

Fig. 8. Suspension travel with bumpy road

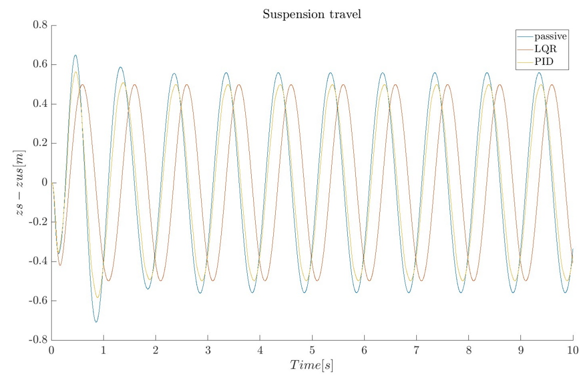

Fig. 9. Suspension travel with sinusoidal road

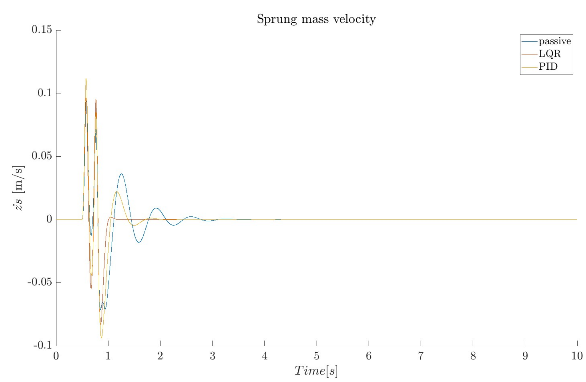

Fig. 10. Sprung mass velocity with bumpy road

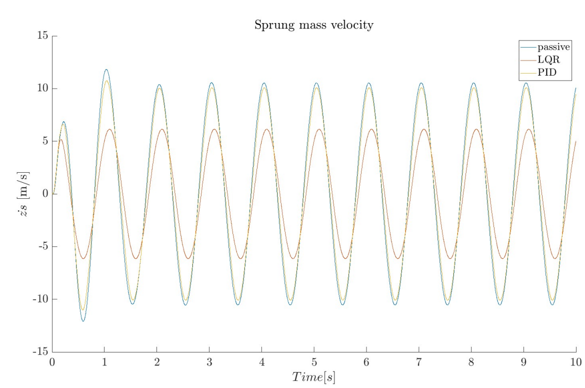

Fig. 11. Sprung mass velocity with sinusoidal road

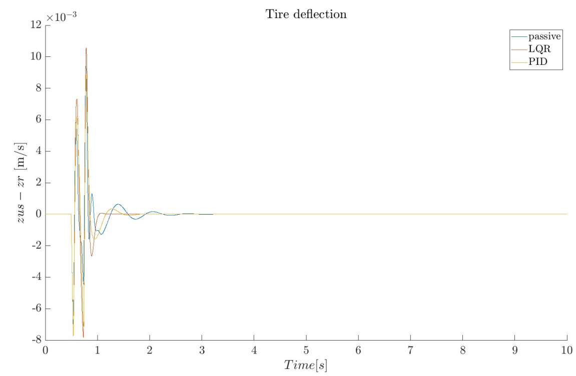

Fig. 12. Tire deflection with bumpy road

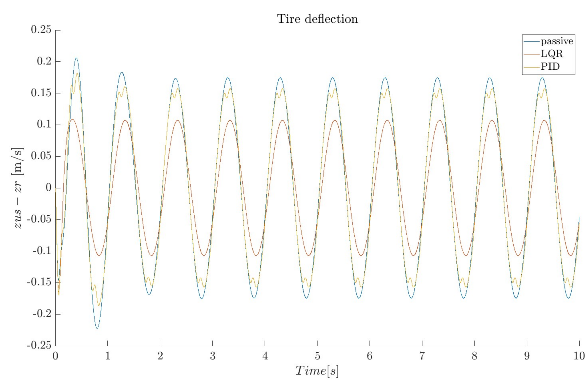

Fig. 13. Tire deflection with sinusoidal road

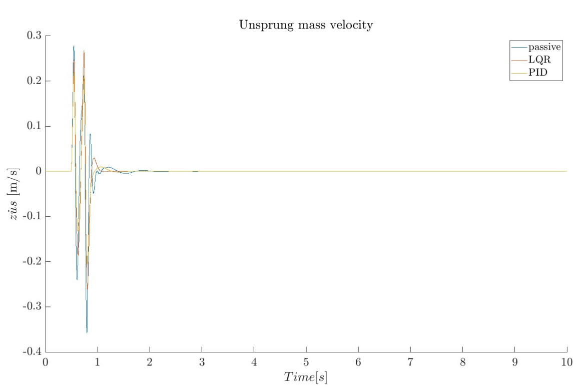

Fig. 14. Unsprung mass velocity with bumpy road

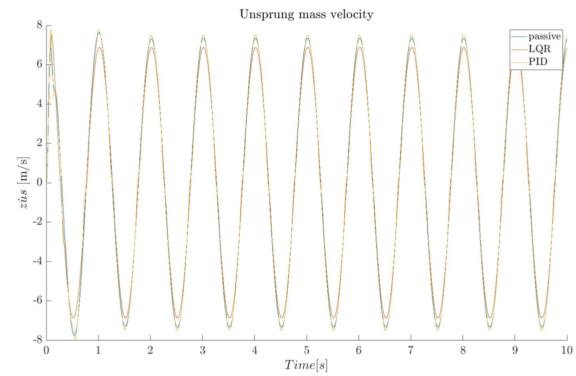

Fig. 15. Unsprung mass velocity with sinusoidal road

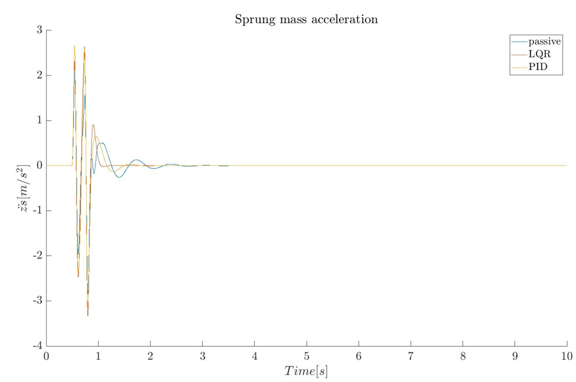

Fig. 16. Sprung mass acceleration with bumpy road

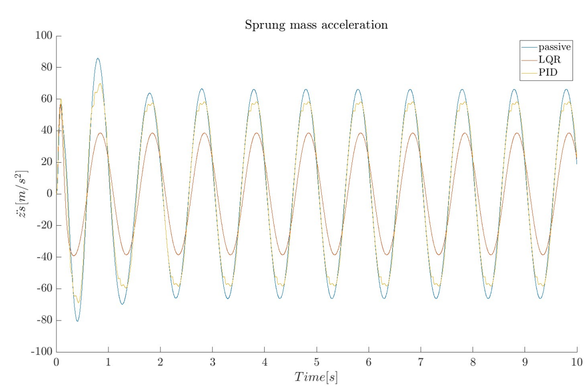

Fig. 17. Sprung mass acceleration with sinusoidal road

APPENDIX B
SIMULINK SCHEME

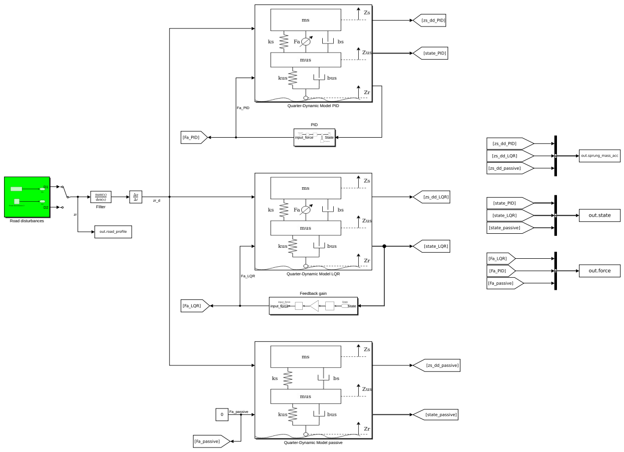

Fig. 18. Simulink scheme simulation