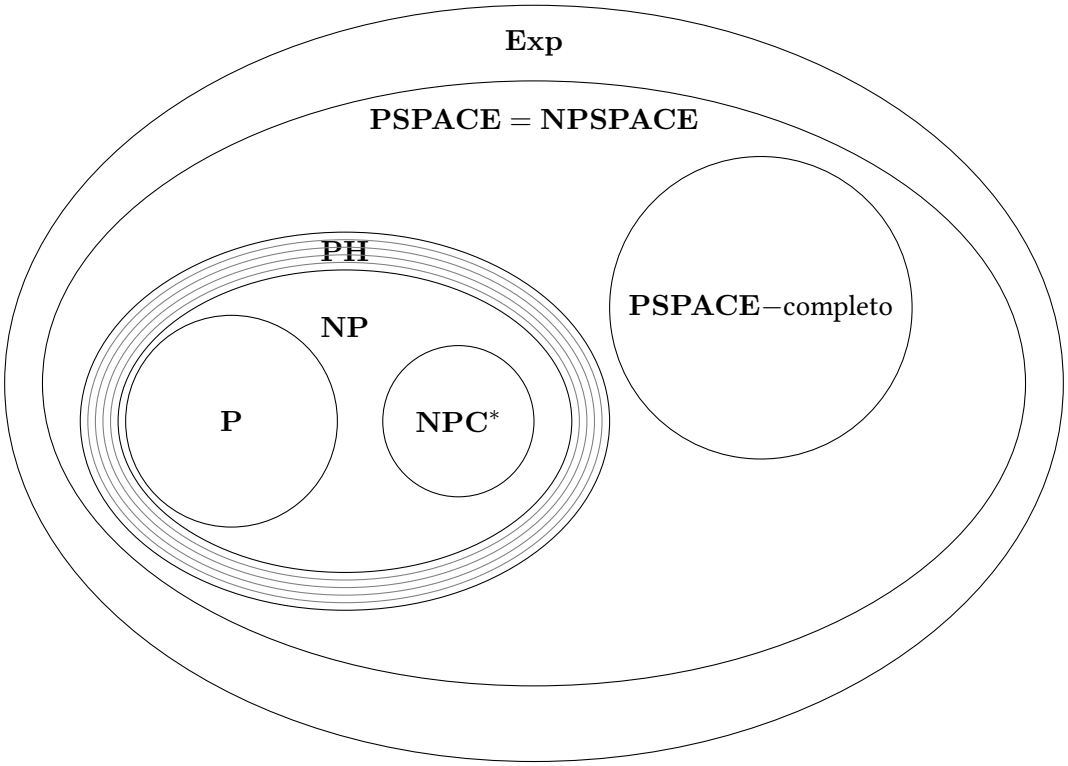
1 Riassunto delle classi di complessità



2 Classi di complessità in breve

2.1 Temporali

$$\mathbf{P} = \{\mathbb{A} \mid \exists B \; t.c. \; \forall x \in J(\mathbb{A}), B(x) = \mathbb{A}(x), B \in O(|x|^c)\}$$

$$\mathbf{NP} = \{\mathbb{A} \quad | \quad \exists \mathcal{B}(\overset{x}{\cdot}, \overset{w}{\cdot}) \quad t.c. \quad T_{\mathcal{B}}(|x| + |w|) = O((|x| + |w|)^c)$$
$$\forall x \in \mathcal{I}(\mathbb{A}) \quad \mathbb{A}(x) = yes \Leftrightarrow \exists w \; t.c. \quad |w| = O(|x|^d) \text{ e } \mathcal{B}(x, w) = yes\}$$

$$\mathbf{Exp} = \{\mathbb{A} \mid \exists \mathcal{A} \; t.c. \; \forall x \in \mathcal{I}(\mathbb{A}), \; \mathcal{A}(x) = \mathbb{A}(x) \text{ e } T_{\mathcal{A}}(|x|) \leq 2^{|x|^c}\}$$

$$\mathbf{NP\text{-}completo} = \{\exists p(x) = x^k, \; \exists V(\cdot, \cdot) \quad t.c. \quad T_V(a, b) = \mathcal{O}\big(p(|a| + |b|)\big)$$
$$\text{e } \forall x \in \mathcal{I}(\mathbb{A}), \; \mathbb{A}(x) = yes \Leftrightarrow \exists w \in \{0,1\}^{p(|x|)}, \; V(x, w) = yes\}$$

2.2 Spaziali

$$\mathbf{SPACE}(f(n)) = \Big\{\mathbb{A} \;\Big|\; \text{esiste un programma/algoritmo che risolve istanze di } \mathbb{A} \text{ usando al più } O(f(n)) \text{ bit di memoria di lavoro e accede all'istanza in sola lettura. } n \text{ è la taglia dell'istanza.}\Big\}$$

$$\mathbf{PSPACE} = \bigcup_{k>0} \mathbf{SPACE}(n^k)$$

$$\mathbf{NTIME} = \Big\{\mathbb{A} \;\Big|\; \text{esiste un verificatore } V_{\mathbb{A}}(\cdot,\cdot)\mathbb{A} = yes \;\Leftrightarrow\; V_{\mathbb{A}}(x,w) = yes \quad V_{\mathbb{A}} \text{ impiega tempo } O(f(|x|)) \text{ e } |w| = O(f(|x|)) \;\Big\}$$

$$\mathbf{NSPACE} = \Big\{\mathbb{A} \;\Big|\; \text{esiste un algoritmo/programma } \Pi \textit{ non deterministico} \text{ che risolve } x \in \mathcal{I}(\mathbb{A}) \text{ usando memoria di lavoro } O(f(|x|)) \quad \Pi(x) = \mathbb{A}(x) \;\Big\}$$

$$\mathbf{NPSPACE} = \bigcup_{k>0} \mathbf{NSPACE}(n^k)$$

$$\mathbf{PSPACE}\text{-completo} = \begin{cases} \mathbb{A} \in \mathbf{PSPACE} \\ \forall \mathbb{B} \in \mathbf{PSPACE} \quad \mathbb{B} \leq_K \mathbb{A} \text{ (hardness)} \end{cases}$$

3 Teoremi vari

Teorema 3.0.1 (Teorema di Savitch) *Per ogni funzione $f(n) \geq log(n)$ vale* $\mathbf{NSPACE}(f(n)) \leq \mathbf{SPACE}(f(n)^2)$

Teorema 3.0.2 (Teorema di Ladner) *Se $\mathbf{P} \neq \mathbf{NP}$ allora esiste un problema $\mathbb{A}$ tale che $\mathbb{A} \in \mathbf{NP} \setminus (\mathbf{P} \cup \mathbf{NPC})$*

4 Lista dei problemi visti e complessità

Problema	**P**	**NP**	**NPC**	**CO-NPC**	**PSPACE**	**PSPACE**-compl	**Riduzione da**
Eulerian Cycle	✓				✓		
K-Colouring (K=2)	✓				✓		
K-Colouring (K $>$ 2)		✓	✓		✓		($\leq_K$) (K+1)-Col
K-SAT (K=2)	✓				✓		
K-SAT (K $>$ 2)		✓	✓		✓		K-Colouring
Circuit-SAT		✓	✓		✓		($\leq_K$) SAT
Tautology				✓	✓		
Min-Circuit Bool[1]					✓		
Graph Isomorphism		✓			✓		
Clique		✓	✓		✓		3-SAT
Clique-no-Clique		✓	✓		✓		$\mathbb{A} \in \mathbf{DP}$
Independent Set		✓	✓		✓		Clique
Only Small IndSet				✓	✓		
Vertex Cover		✓	✓		✓		Independent Set
Hitting Set		✓	✓		✓		Vertex Cover
Max Cut		✓	✓		✓		NAE-3-SAT
Set Splitting		✓	✓		✓		NAE-3-SAT
Set Cover		✓	✓		✓		Vertex Cover
Hamiltonian Path		✓	✓		✓		
Q-SAT (= 2p-SAT)						✓	
Geography						✓	Q-SAT
Alternating Hampath						✓	Q-SAT
Reachability[2]	✓				✓		
Makespan-m		✓	✓		✓		Partition
SubsetSum		✓	✓		✓		
Partition		✓	✓		✓		
Traveling Salesman		✓	✓		✓		HamCycle
Knapsack		✓	✓		✓		Partition
Max-k-xor-SAT		✓	✓		✓		
Max-k-SAT		✓	✓		✓		Max Cut
Set Cover		✓	✓		✓		Vertex Cover