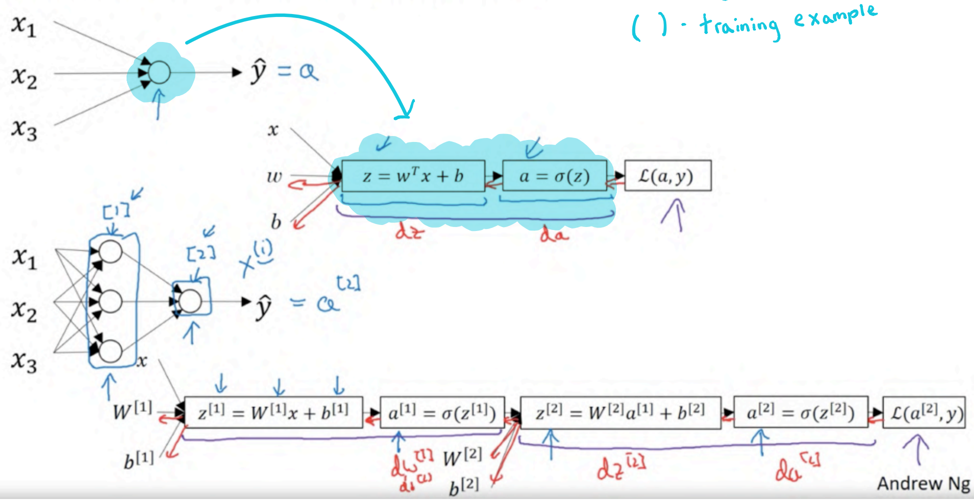
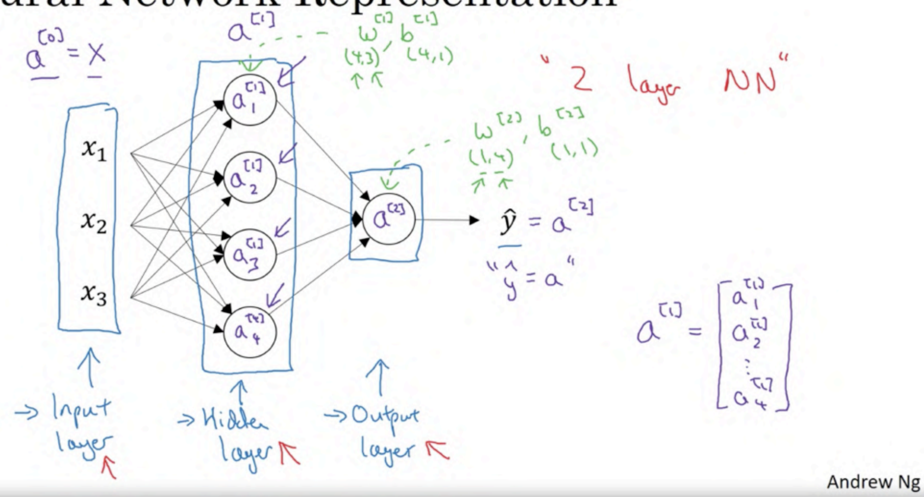
What is a Neural Network?

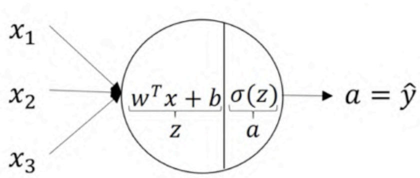
[ ] - layer
( ) - training example



Neural Network Representation

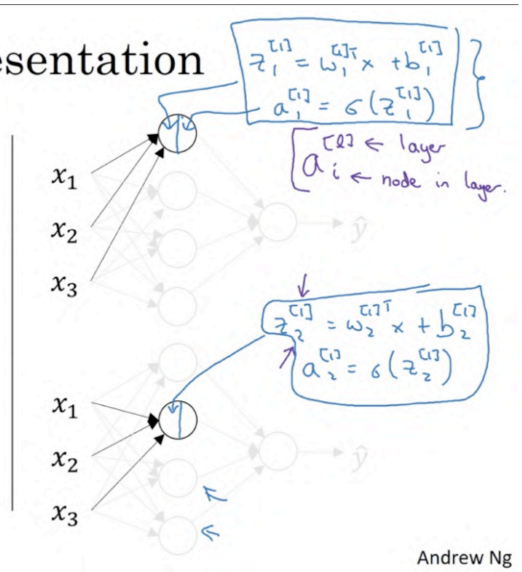


Neural Network Representation

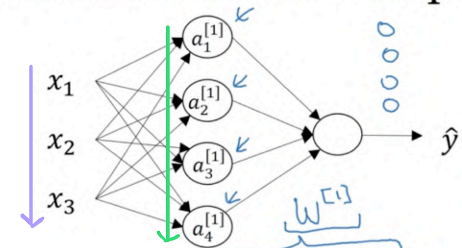


$z = w^T x + b$

$a = \sigma(z)$



Neural Network Representation



$(w_1^{[1]})^T x$ $\quad a^{[1]}$

$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}, \quad a_1^{[1]} = \sigma(z_1^{[1]})$$
$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}, \quad a_2^{[1]} = \sigma(z_2^{[1]})$$
$$z_3^{[1]} = w_3^{[1]T} x + b_3^{[1]}, \quad a_3^{[1]} = \sigma(z_3^{[1]})$$
$$z_4^{[1]} = w_4^{[1]T} x + b_4^{[1]}, \quad a_4^{[1]} = \sigma(z_4^{[1]})$$

$$z^{[1]} = \underbrace{\begin{bmatrix} — w_1^{[1]T} — \\ — w_2^{[1]T} — \\ — w_3^{[1]T} — \\ — w_4^{[1]T} — \end{bmatrix}}_{4 \times 3}^{W^{[1]}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}}_{b^{[1]}\ (4,1)} = \begin{bmatrix} w_1^{[1]T} x + b_1^{[1]} \\ w_2^{[1]T} x + b_2^{[1]} \\ w_3^{[1]T} x + b_3^{[1]} \\ w_4^{[1]T} x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix} \ (4,1)$$

$$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]}) \quad (4,1)$$

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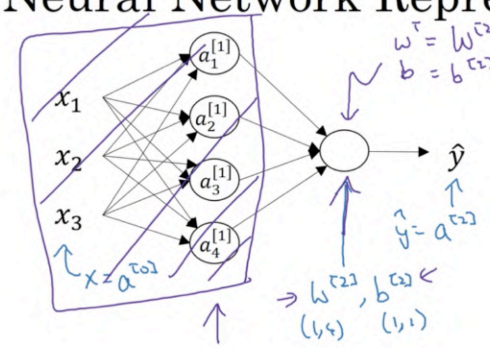
Neural Network Representation learning



$z = w^T x + b$

$\hat{y} = a = \sigma(z)$

Given input x:

$$\underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}} \underset{(3,1)}{\cancel{x}\, a^{[0]}} + \underset{(4,1)}{b^{[1]}}$$

$$\underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\underset{(1,1)}{z^{[2]}} = \underset{(1,4)}{W^{[2]}} \underset{(4,1)}{a^{[1]}} + \underset{(1,1)}{b^{[2]}}$$

$$\underset{(1,1)}{a^{[2]}} = \sigma(\underset{(1,1)}{z^{[2]}})$$

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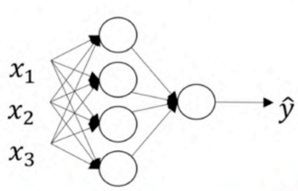
Vectorizing across multiple examples



$$z^{[1]} = W^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$a^{[2]} = \sigma(z^{[2]})$$

$x \longrightarrow a^{[2]} = \hat{y}$

$x^{(1)} \longrightarrow a^{[2](1)} = \hat{y}^{(1)}$

$x^{(2)} \longrightarrow a^{[2](2)} = \hat{y}^{(2)}$

$\vdots$

$x^{(m)} \longrightarrow a^{[2](m)} = \hat{y}^{(m)}$

$a^{[2](i)}$ ← example i, ← layer 2

for $i = 1$ to m,

$\quad z^{[1](i)} = W^{[1]} x^{(i)} + b^{[1]}$

$\quad a^{[1](i)} = \sigma(z^{[1](i)})$

$\quad z^{[2](i)} = W^{[2]} a^{[1](i)} + b^{[2]}$

$\quad a^{[2](i)} = \sigma(z^{[2](i)})$

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Vectorizing across multiple examples

```
for i = 1 to m:
```

$\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$

$\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$

$\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$

$\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$

$$\boxed{\begin{aligned} Z^{[1]} &= W^{[1]}X + b^{[1]} \\ A^{[1]} &= \sigma(Z^{[1]}) \\ Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\ A^{[2]} &= \sigma(Z^{[2]}) \end{aligned}}$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \quad (n_x, m)$$

training examples

hidden units

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

hidden units

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Justification for vectorized implementation

$z^{[1](1)} = W^{[1]}x^{(1)} + \cancel{b^{[1]}} \to 0, \quad z^{[1](2)} = W^{[1]}x^{(2)} + \cancel{b^{[1]}} \to 0, \quad z^{[1](3)} = W^{[1]}x^{(3)} + \cancel{b^{[1]}} \to 0$

$W^{[1]} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \qquad W^{[1]}x^{(1)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(2)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(3)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$

$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \end{bmatrix} = Z^{[1]}$$

$+b^{[1]} \quad +b^{[1]} \quad +b^{[1]}$

$Z^{[1]} = W^{[1]}X + b^{[1]}$

$W^{[1]}x^{(i)} = z^{[1](i)}$

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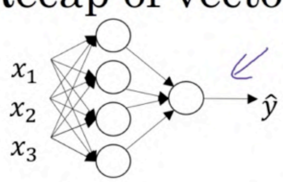
Recap of vectorizing across multiple examples



$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

```
for i = 1 to m
```

$\boxed{z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}}$

$a^{[1](i)} = \sigma(z^{[1](i)})$

$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$

$a^{[2](i)} = \sigma(z^{[2](i)})$

$A^{[0]} \qquad X = a^{[0]} \qquad x^{(i)} = a^{[0](i)}$

$Z^{[1]} = W^{[1]}X + b^{[1]} \leftarrow W^{[1]}A^{[0]} + b^{[1]}$

$A^{[1]} = \sigma(Z^{[1]})$

$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$

$A^{[2]} = \sigma(Z^{[2]})$

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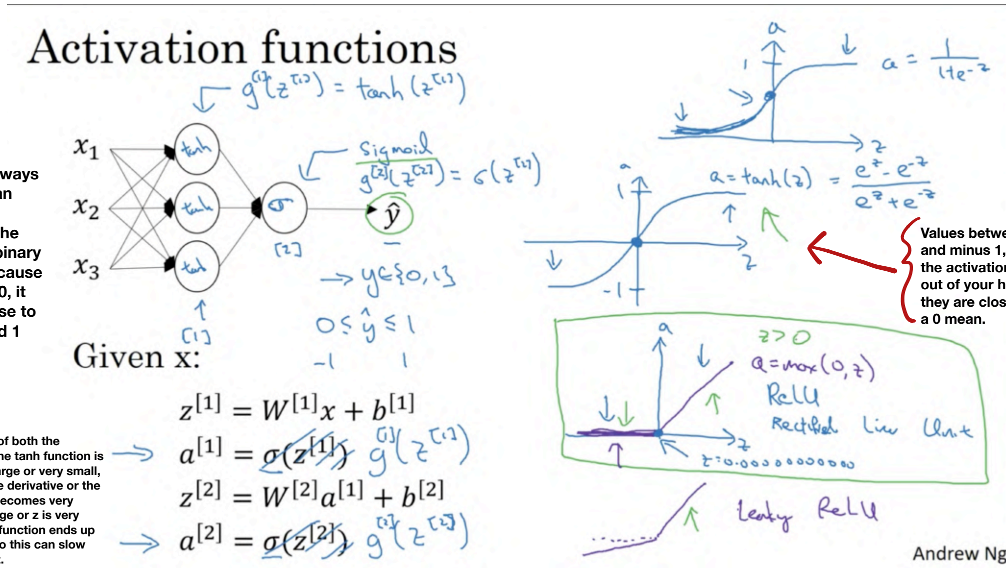
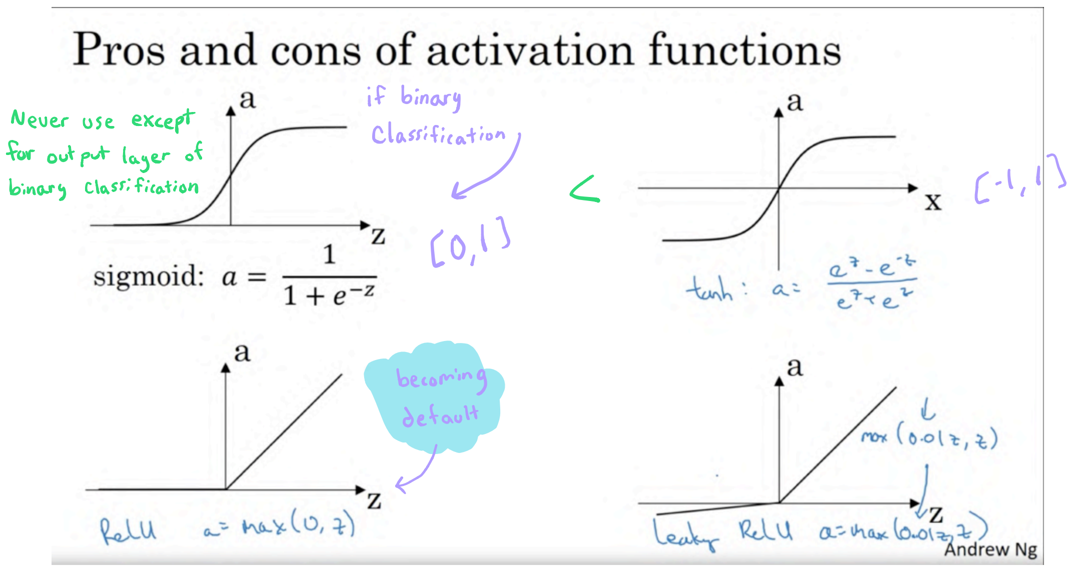
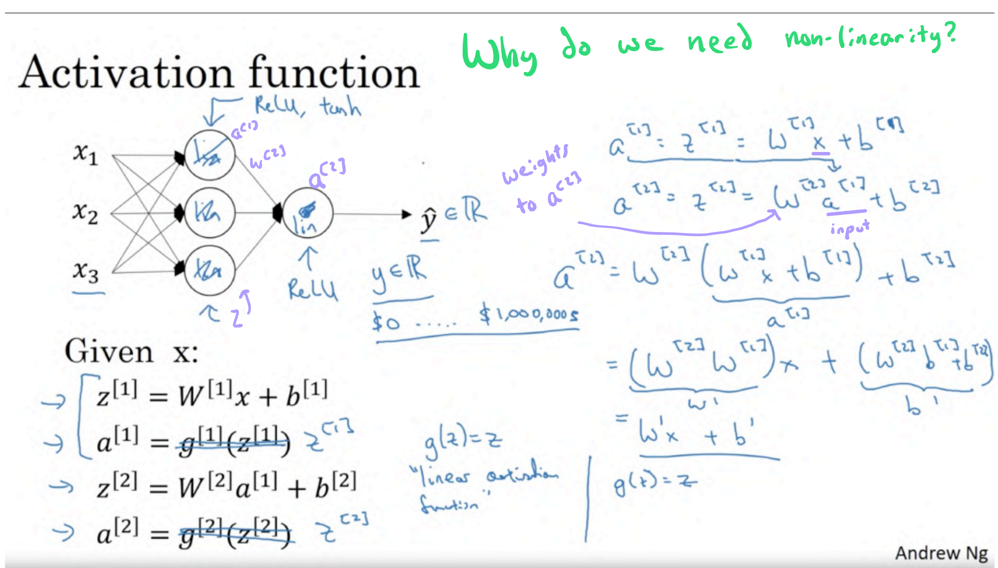
Tanh is almost always strictly better than sigmoid. Only exception is for the output layer for binary classification because if y is either 1 or 0, it makes more sense to be between 0 and 1 than 1 and -1.

One of the downsides of both the sigmoid function and the tanh function is that if z is either very large or very small, then the gradient or the derivative or the slope of this function becomes very small. So if z is very large or z is very small, the slope of the function ends up being close to 0. And so this can slow down gradient descent.



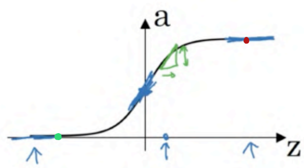
Values between plus 1 and minus 1, the mean of the activations that come out of your head, and they are closer to having a 0 mean.





Sigmoid activation function

$g'(z) = s(1-s)$



$$g(z) = \frac{1}{1+e^{-z}}$$

$a = g(z) = \frac{1}{1+e^{-z}}$

$g'(z) = \boxed{\frac{d}{dz} g(z)}$ = slope of $g(x)$ at z

$= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$

$= g(z)\,(1 - g(z))$

$= \boxed{a\,(1-a)}$

$g'(z) = a(1-a)$

- $z = 10$. $g(z) \approx 1$

  $\frac{d}{dz} g(z) \approx 1\,(1-1) \approx 0$

- $z = -10$ $g(z) \approx 0$

  $\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$

$z = 0$ $g(z) = \frac{1}{2}$

$\frac{d}{dz} g(z) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4}$

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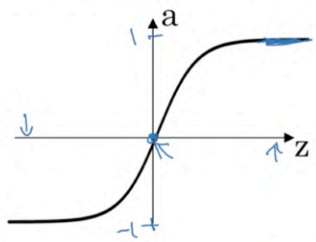
Tanh activation function



$$g(z) = \tanh(z)$$

$$= \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$g'(z) = \frac{d}{dz} g(z)$ = slope of $g(z)$ at z

$= 1 - (\tanh(z))^2$

$a = g(z)$, $g'(z) = 1 - a^2$

$z = 10$ $\tanh(z) \approx 1$

$g'(z) \approx 0$

$z = -10$ $\tanh(z) \approx -1$

$g'(z) \approx 0$

$z = 0$ $\tanh(z) = 0$

$g'(z) = 1$

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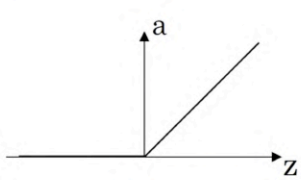
ReLU and Leaky ReLU

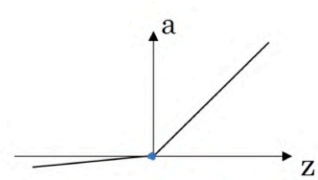


ReLU

$g(z) = \max(0, z)$

$$g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \\ \cancel{\text{undef}} & \cancel{\text{if } z = 0} \end{cases}$$

$z = 0.00000\ldots0$



Leaky ReLU

$g(z) = \max(0.01z, z)$

$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

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Gradient descent for neural networks

Parameters: $\underset{(n^{[1]}, n^{[0]})}{W^{[1]}}$, $\underset{(n^{[1]}, 1)}{b^{[1]}}$, $\underset{(n^{[2]}, n^{[1]})}{W^{[2]}}$, $\underset{(n^{[2]}, 1)}{b^{[2]}}$ $\quad n_x = n^{[0]}, \; n^{[1]}, \; \underline{n^{[2]} = 1}$

Cost function: $J(W^{[1]}, b^{[1]}, \underline{W^{[2]}}, \underline{b^{[2]}}) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$ $\quad \uparrow a^{[2]}$

Gradient descent:

→ Repeat {

→ Compute predicts $(\hat{y}^{(i)}, \; i = 1 \dots m)$

$\underline{dW^{[1]}} = \frac{\partial J}{\partial W^{[1]}}, \quad \underline{db^{[1]}} = \frac{\partial J}{\partial b^{[1]}}, \dots$

$W^{[1]} := W^{[1]} - \alpha \, dW^{[1]}$

$b^{[1]} := b^{[1]} - \alpha \, db^{[1]}$

Formulas for computing derivatives

Forward propagation:

$Z^{[1]} = W^{[1]} X + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$ ←

$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$

$A^{[2]} = g^{[2]}(Z^{[2]}) = \sigma(Z^{[2]})$

Back propagation:

$dZ^{[2]} = A^{[2]} - Y$ ← $\quad Y = [y^{(1)} \; y^{(2)} \dots y^{(m)}]$

$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$

Sum horizontally

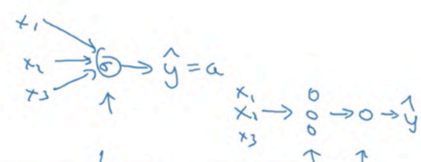
$db^{[2]} = \frac{1}{m} \text{np.sum}(dZ^{[2]}, \text{axis}=1, \text{keepdims}=\text{True})$ $\quad (n^{[2]}, 1)$ ← $\quad (n,) \leftarrow$ No $\quad \rightarrow (n,1)$

$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]}, m)}$ $\quad \uparrow$ element-wise product

$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$

$\underset{(n^{[1]}, 1)}{db^{[1]}} = \frac{1}{m} \underset{(n^{[1]},)}{\text{np.sum}(dZ^{[1]}, \text{axis}=1,} \; \text{keepdims}=\text{True})$

Computing gradients



Logistic regression

$x, w, b \rightarrow \boxed{z = w^T x + b} \rightarrow \boxed{a = \sigma(z)} \rightarrow \boxed{\mathcal{L}(a, y)}$

$dz = a - y$

$dw = dz \cdot x$

$db = dz$

$da = \frac{d}{da} \mathcal{L}(a, y) = \underline{-y \log a - (1-y) \log(1-a)}$

$= -\frac{y}{a} + \frac{1-y}{1-a}$

$\boxed{dz = da \cdot g'(z)}$

$g(z) = \sigma(z)$

$\boxed{\frac{\partial \mathcal{L}}{\partial z} = \frac{\partial \mathcal{L}}{\partial a} \cdot \boxed{\frac{da}{dz}}}$

"dz" = "da" $\quad \frac{d}{dz} g(z) = \underline{g'(z)}$

Neural network gradients

x_1, x_2, x_3 → o → $\hat{y}$

$n_x = n^{[0]}$ $\quad n^{[1]}$ $\quad n^{[2]} = 1$

x, $W^{[1]}$, $b^{[1]}$ → $z^{[1]} = W^{[1]}x + b^{[1]}$ → $a^{[1]} = \sigma(z^{[1]})$ → $z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$ → $a^{[2]} = \sigma(z^{[2]})$ → $\mathcal{L}(a^{[2]}, y)$

$W^{[2]}$ $\quad dW^{[2]}$

$b^{[2]}$ $\quad db^{[2]}$

$dW^{[1]}$ $\quad db^{[1]}$

$dz^{[1]} = W^{[2]T} dz^{[2]}$ $\quad da^{[1]} = \dots$ $\quad dz^{[2]} = a^{[2]} - y$ $\quad da^{[2]}$

$* \, g^{[1]\prime}(z^{[1]})$

element wise product

$dW^{[2]} = dz^{[2]} a^{[1]T}$ $\quad$ "$dw = dz \cdot x$"

$db^{[2]} = dz^{[2]}$ $\quad W^{[2]} = [\text{———}]$

$W^{[2]}$ $\quad (n^{[2]}, n^{[1]})$

$z^{[2]}, dz^{[2]}$ $\quad (n^{[2]}, 1) \; - (1,1)$

$z^{[1]}, dz^{[1]}$ $\quad (n^{[1]}, 1)$

foo $\quad W^{[1]}$ $\quad dW^{[1]} = dz^{[1]} \cdot x^T$

dfoo $\quad dW^{[1]}$ $\quad db^{[1]} = dz^{[1]}$ $\quad a^{[0]T}$

$dz^{[1]} = \boxed{W^{[2]T} dz^{[2]}} * \boxed{g^{[1]\prime}(z^{[1]})}$

$(n^{[1]}, 1) = (n^{[1]}, n^{[2]}) \; (n^{[2]}, 1) * (n^{[1]}, 1)$

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Summary of gradient descent

$dz^{[2]} = a^{[2]} - y$

$dW^{[2]} = dz^{[2]} a^{[1]^T}$

$db^{[2]} = dz^{[2]}$

$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$

$dW^{[1]} = dz^{[1]} x^T$

$db^{[1]} = dz^{[1]}$

Vectorized Implementation:

$z^{[1]} = W^{[1]}x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$

$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \dots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$

$Z^{[1]} = W^{[1]}X + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$

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Summary of gradient descent

$dz^{[2]} = a^{[2]} - y$	$dZ^{[2]} = A^{[2]} - Y$
$dW^{[2]} = dz^{[2]} a^{[1]^T}$	$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]^T}$
$db^{[2]} = dz^{[2]}$	$db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]}, axis = 1, keepdims = True)$
$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$ $(n^{[1]}, 1)$	$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]}, m)}$ $(n^{[1]}, m)$
$dW^{[1]} = dz^{[1]} x^T$	$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$
$db^{[1]} = dz^{[1]}$	$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True)$

$J(\dots) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$

element-wise product

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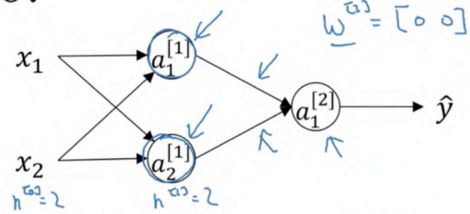
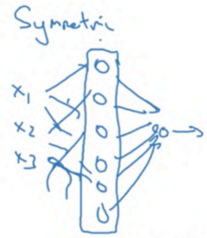
What happens if you initialize weights to zero?

$W^{[2]} = [0 \quad 0]$





If you initialize the weights to zero, then all of your hidden units are symmetric. And no matter how long you're upgrading the center, all continue to compute exactly the same function. So that's not helpful, because you want the different hidden units to compute different functions. The solution to this is to initialize your parameters randomly.

$$W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$W^{[1]} = \begin{bmatrix} - - - - \\ - - - - \end{bmatrix}$$

$$a_1^{[1]} = a_2^{[1]} \qquad dz_1^{[1]} = dz_2^{[1]}$$

$$dW = \begin{bmatrix} u & v \\ u & v \end{bmatrix} \qquad W^{[1]} = W^{[1]} - \alpha \, dW$$

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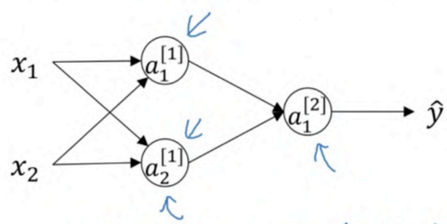
Random initialization



$\rightarrow W^{[1]} = $ np.random.randn((2,2)) $* 0.01$ ~~100?~~

$b^{[1]} = $ np.zeros((2,1))

$W^{[2]} = $ np.random.randn((1,2)) $* 0.01$

$b^{[2]} = 0$

$\rightarrow z^{[1]} = W^{[1]} x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$

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