Outline

- Vector space models
- Advantages
- Applications

Why learn vector space models?

Where are you heading?

Where are you from?

Different meaning

What is your age?

How old are you?

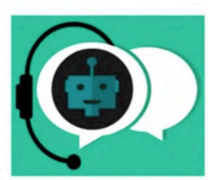
Same Meaning

Vector space models applications

- You eat *cereal* from a *bowl*
- You *buy* something and someone else *sells* it

Information Extraction

Machine Translation



Chatbots

Fundamental concept

"You shall know a word by the company it keeps"

Firth, 1957

(Firth, J. R. 1957:11)

Summary

- Represent words and documents as vectors
- Representation that captures relative meaning

Outline

- Co-occurrence $\longrightarrow$ Vector representation
- Relationships between words/documents

Similarity

Word by Word Design

Number of times they *occur together within a certain distance k*

I like simple data

I prefer simple raw data

k=2

	simple	raw	like	I
data	2	1	1	0

n

for word "data"

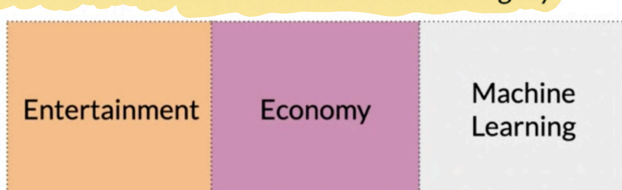
Word by Document Design

Number of times a word *occurs within a certain category*

Corpus

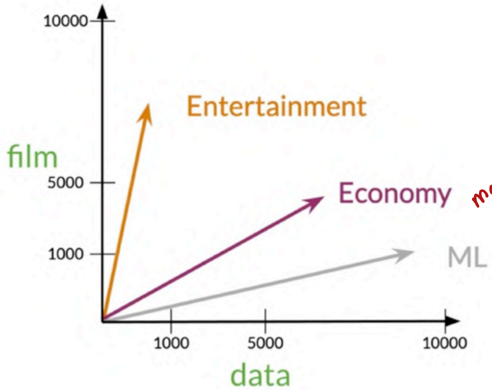
Word by Document Design

Number of times a word *occurs within a certain category*



	Entertainment	Economy	Machine Learning
data	500	6620	9320
film	7000	4000	1000

Vector Space



most similar

	Entertainment	Economy	ML
data	500	6620	9320
film	7000	4000	1000

Measures of "similarity:"
Angle
Distance

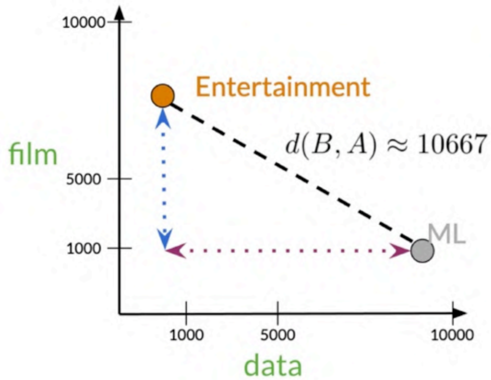
Summary

- W/W (-word) and W/D (-document), *counts* of occurrence
- Vector Spaces ⟶ Similarity between words/documents

Outline

- Euclidean distance
- N-dimension vector representations comparison

Euclidean distance



Corpus A: (500,7000)

Corpus B: (9320,1000)

$$d(B, A) = \sqrt{\underline{(B_1 - A_1)}^2 + \underline{(B_2 - A_2)}^2}$$

$$c^2 = a^2 + b^2$$

$$d(B, A) = \sqrt{(8820)^2 + (-6000)^2}$$

Euclidean distance for n-dimensional vectors

	data	boba ($\vec{w}$)	ice-cream ($\vec{v}$)
AI	6	0	1
drinks	0	4	6
food	0	6	8

$$= \sqrt{(1-0)^2 + (6-4)^2 + (8-6)^2}$$

$$= \sqrt{1+4+4} = \sqrt{9} = 3$$

distance for Boba & ice cream

$$d(\vec{v}, \vec{w}) = \sqrt{\sum_{i=1}^{n} (v_i - w_i)^2} \longrightarrow \text{Norm of } (\vec{v} - \vec{w})$$

Euclidean distance in Python

```
# Create numpy vectors v and w
v = np.array([1, 6, 8])
w = np.array([0, 4, 6])

# Calculate the Euclidean distance d
d = np.linalg.norm(v-w)
# Print the result
print("The Euclidean distance between v and w is: ", d)
```

```
The Euclidean distance between v and w is: 3
```

Summary

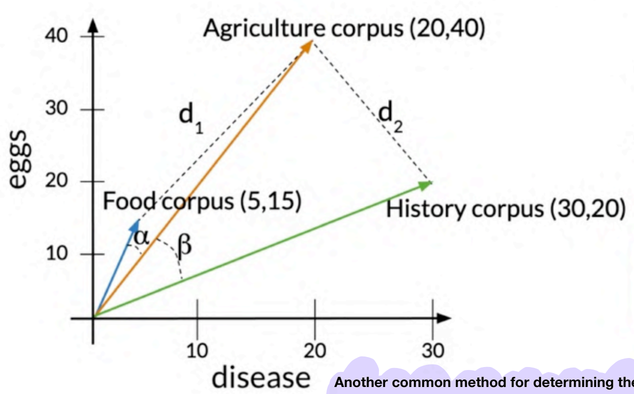
- Straight line between points
- Norm of the difference between vectors

Outline

- Problems with Euclidean Distance
- Cosine similarity

Euclidean distance vs Cosine similarity

The distance d2 is smaller than the distance d1, which would suggest that the agriculture and history corpora are more similar than the agriculture and food corpora.



Euclidean distance: $d_2 < d_1$

Angles comparison: $\beta > \alpha$

$\boxed{\text{The cosine of the angle between the vectors}}$

Another common method for determining the similarity between vectors is computing the cosine of their inner angle. If the angle is small, the cosine would be close to one. And as the angle approaches 90 degrees, the cosine approaches zero. As you can see here, the angle alpha between food and agriculture is smaller than the angle beta between agriculture and history. In this particular case, the cosine of those angles is a better proxy of similarity between these vector representations than their euclidean distance.

Summary

- Cosine similarity when corpora are different sizes

Outline

- How to get the cosine of the angle between two vectors
- Relation of this metric to similarity

Previous definitions

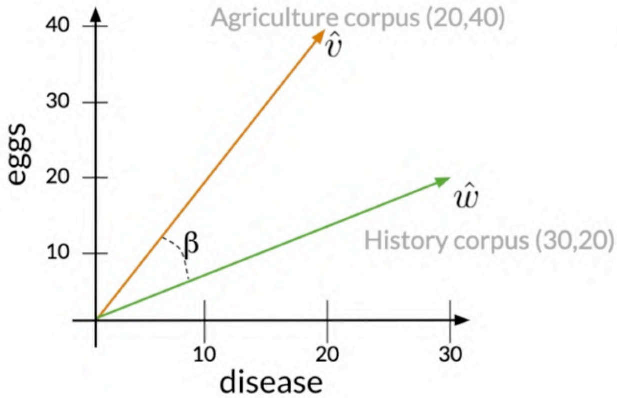
Vector norm

$$\|\vec{v}\| = \sqrt{\sum_{i=1}^{n} v_i^2}$$

Dot product

$$\vec{v}.\vec{w} = \sum_{i=1}^{n} v_i.w_i$$

Cosine Similarity



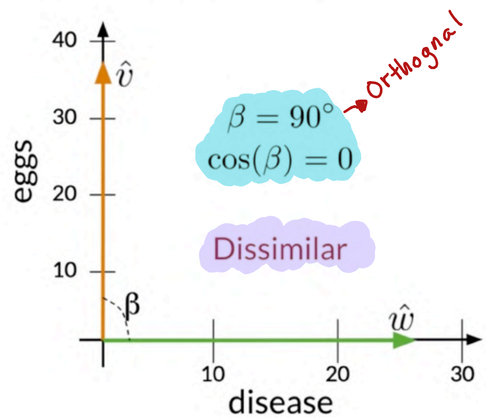
$$\hat{v} \cdot \hat{w} = ||\hat{v}|| \, ||\hat{w}|| \cos(\beta)$$

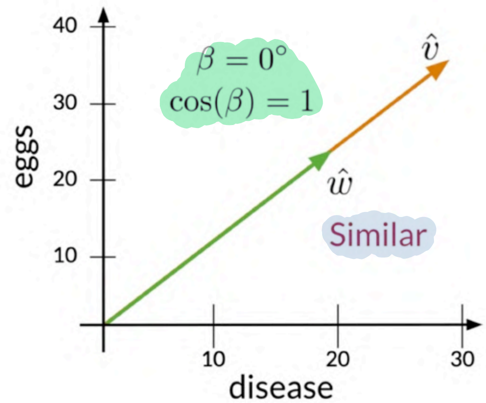
$$\boxed{\cos(\beta) = \frac{\hat{v} \cdot \hat{w}}{||\hat{v}|| \, ||\hat{w}||}}$$

$$= \frac{(20 \times 30) + (40 \times 20)}{\sqrt{20^2 + 40^2} \times \sqrt{30^2 + 20^2}}$$

$$= 0.87$$

Cosine Similarity





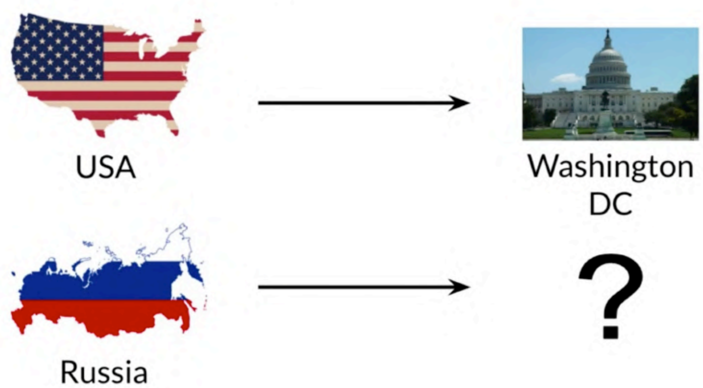
Summary

- Cosine $\propto$ Similarity
- Cosine Similarity gives values between 0 and 1

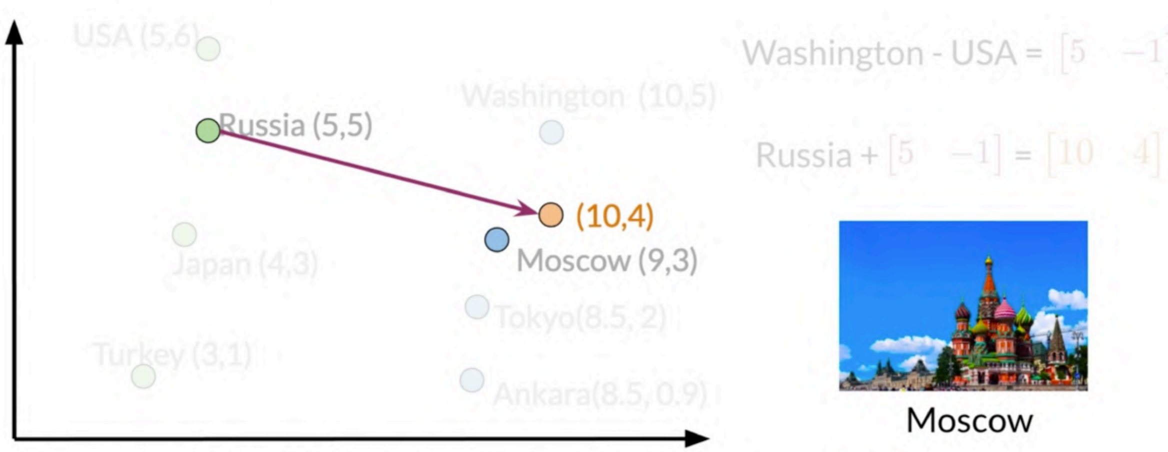
Outline

- How to use vector representations

Manipulating word vectors



Manipulating word vectors



Summary

- Use known relationships to make predictions

Outline

- Some motivation for visualization
- Principal Component Analysis

Visualization of word vectors

$\overbrace{\quad\quad\quad\quad\quad\quad}^{d > 2}$ dimension space

oil	0.20	...	0.10
gas	2.10	...	3.40
city	9.30	...	52.1
town	6.20	...	34.3

How can you visualize if your representation captures these relationships?

oil & gas

town & city

Visualization of word vectors

$d > 2$

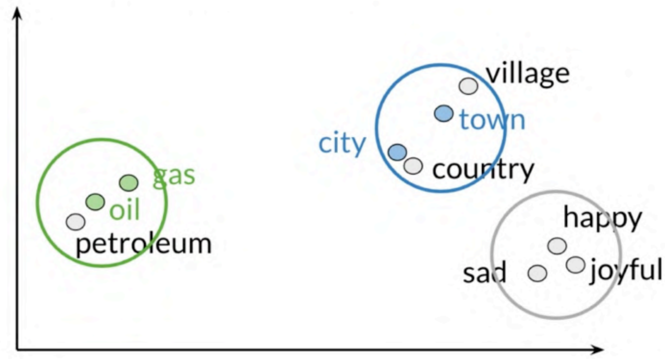
oil	0.20	...	0.10
gas	2.10	...	3.40
city	9.30	...	52.1
town	6.20	...	34.3

$\xrightarrow{\text{PCA}}$

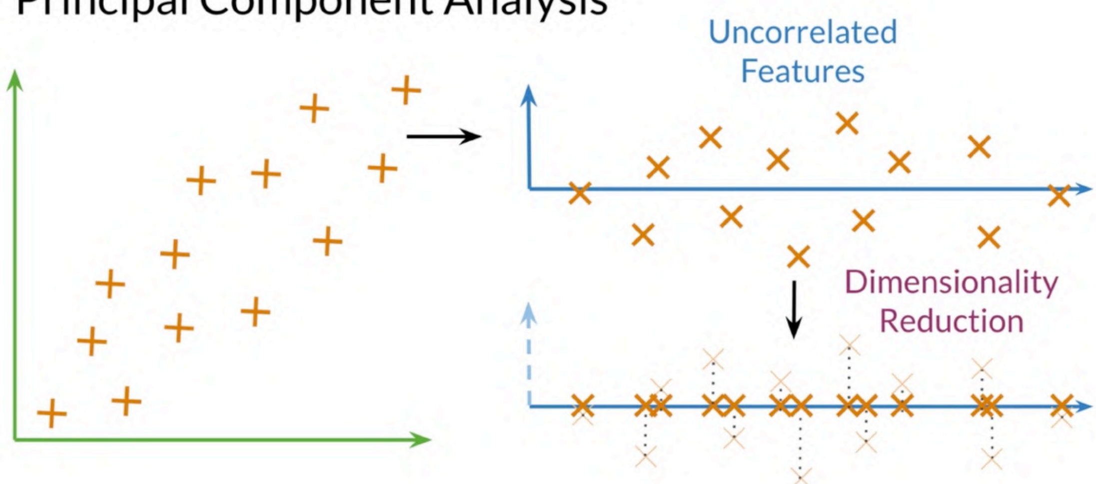
$d = 2$

oil	2.30	21.2
gas	1.56	19.3
city	13.4	34.1
town	15.6	29.8

Visualization of word vectors



Principal Component Analysis



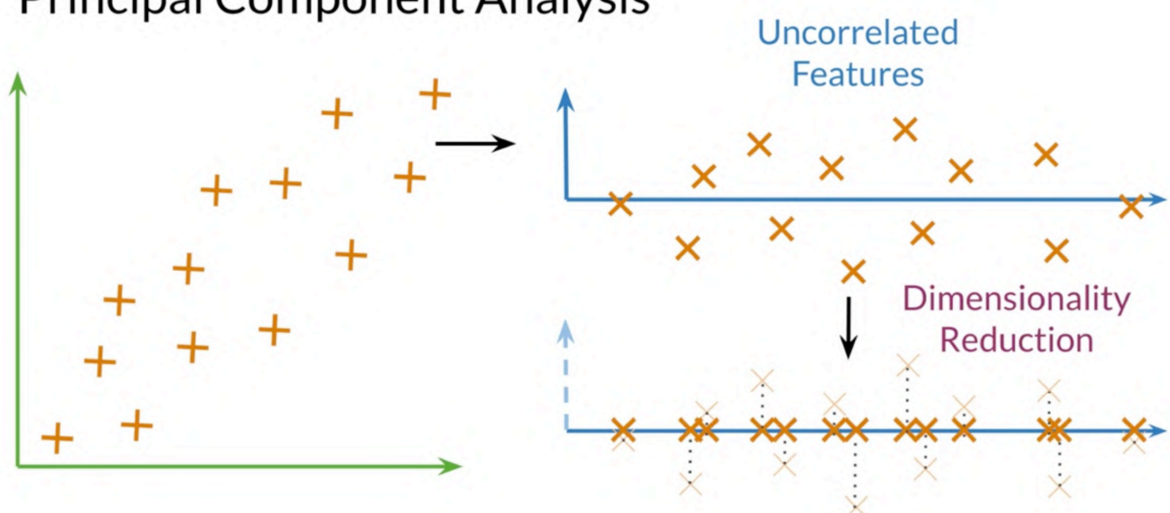
Summary

- Original Space → Uncorrelated features → Dimension reduction
- Visualization to see words relationships in the vector space

Outline

- How to get uncorrelated features
- How to reduce dimensions while retaining as much information as possible

Principal Component Analysis



PCA algorithm

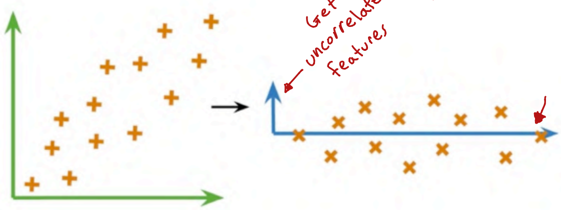
Eigenvector: Uncorrelated features for your data

Eigenvalue: the amount of information retained by each feature

PCA algorithm

Get uncorrelated features

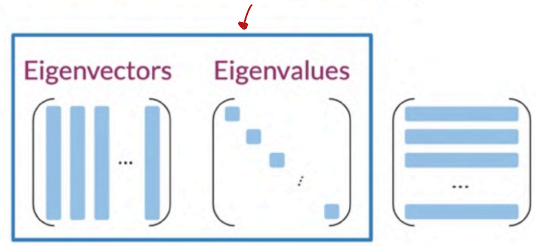
so



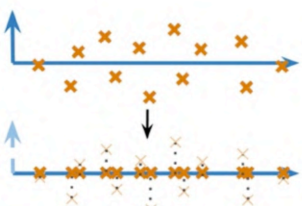
Mean Normalize Data $x_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}}$

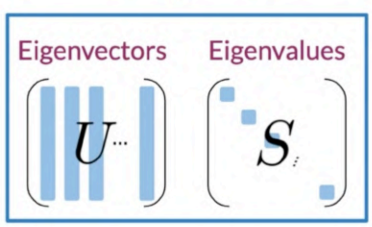
Get Covariance Matrix Σ

Perform SVD $\mathrm{SVD}(\Sigma)$



PCA algorithm





Dot Product to Project Data $X' = XU[:, 0:2]$

Percentage of Retained Variance $\frac{\sum_{i=0}^{1} S_{ii}}{\sum_{j=0}^{d} S_{jj}}$

Your eigenvectors and eigenvalues should be organized according to the eigenvalues in descending order. This condition will ensure that you retain as much information as possible from your original embedding.

Summary

- Eigenvectors give the direction of uncorrelated features
- Eigenvalues are the variance of the new features
- Dot product gives the projection on uncorrelated features