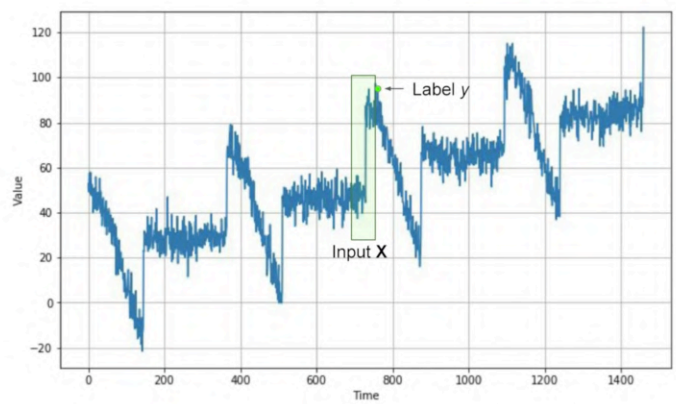
Machine Learning on Time Windows



As with any other ML problem, we have to divide our data into features and labels. In this case our feature is effectively a number of values in the series, with our label being the next value. We'll call that number of values that will treat as our feature, the window size, where we're taking a window of the data and training an ML model to predict the next value. So for example, if we take our time series data, say, 30 days at a time, we'll use 30 values as the feature and the next value is the label. Then over time, we'll train a neural network to match the 30 features to the single label.

```
dataset = tf.data.Dataset.range(10)
for val in dataset:
   print(val.numpy())


    0
    1
    2
    3
    4
    5
    6
    7
    8
    9
```

```
dataset = tf.data.Dataset.range(10)
dataset = dataset.window(5, shift=1)
for window_dataset in dataset:
  for val in window_dataset:
    print(val.numpy(), end=" ")
  print()

   0 1 2 3 4
   1 2 3 4 5
   2 3 4 5 6
   3 4 5 6 7
   4 5 6 7 8
   5 6 7 8 9
   6 7 8 9
   7 8 9
   8 9
   9
```

We'll use the dataset.window to expand our data set using windowing.

Its parameters are the size of the window and how much we want to shift by each time. So if we set a window size of 5 with a shift of 1 when we print it we'll see something like this, 01234, which just stops there because it's five values, then we see 12345 etc, etc,.

Once we get towards the end of the data set we'll have less values because they just don't exist. So we'll get 6789, and then 789, etc, etc,.

```python
dataset = tf.data.Dataset.range(10)
dataset = dataset.window(5, shift=1, drop_remainder=True)
for window_dataset in dataset:
  for val in window_dataset:
    print(val.numpy(), end=" ")
  print()
```

```
0 1 2 3 4
1 2 3 4 5
2 3 4 5 6
3 4 5 6 7
4 5 6 7 8
5 6 7 8 9
```

Let's edit our window a little bit, so that we have regularly sized data. We can do that with an additional parameter on the window called drop_remainder. And if we set this to true, it will truncate the data by dropping all of the remainders.

Namely, this means it will only give us windows of five items. So when we print it, it will now look like this, starting at 01234 and ending at 56789.

```python
dataset = tf.data.Dataset.range(10)
dataset = dataset.window(5, shift=1, drop_remainder=True)
dataset = dataset.flat_map(lambda window: window.batch(5))
for window in dataset:
  print(window.numpy())
```

```
[0 1 2 3 4]
[1 2 3 4 5]
[2 3 4 5 6]
[3 4 5 6 7]
[4 5 6 7 8]
[5 6 7 8 9]
```

Put these into numpy lists so that we can start using them with machine learning.

```python
dataset = tf.data.Dataset.range(10)
dataset = dataset.window(5, shift=1, drop_remainder=True)
dataset = dataset.flat_map(lambda window: window.batch(5))
dataset = dataset.map(lambda window: (window[:-1], window[-1:]))
for x,y in dataset:
  print(x.numpy(), y.numpy())
```

```
[0 1 2 3] [4]
[1 2 3 4] [5]
[2 3 4 5] [6]
[3 4 5 6] [7]
[4 5 6 7] [8]
[5 6 7 8] [9]
```

Next up is to split the data into features and labels.

For each item in the list it kind of makes sense to have all of the values but the last one to be the feature, and then the last one can be the label. And this can be achieved with mapping, like this, where we split into everything but the last one with :-1, and then just the last one itself with -1:. Which gives us this output when we print, which now looks like a nice set of features and labels.

```
dataset = tf.data.Dataset.range(10)
dataset = dataset.window(5, shift=1, drop_remainder=True)
dataset = dataset.flat_map(lambda window: window.batch(5))
dataset = dataset.map(lambda window: (window[:-1], window[-1:]))
dataset = dataset.shuffle(buffer_size=10)
for x,y in dataset:
  print(x.numpy(), y.numpy())

    [3 4 5 6] [7]
    [4 5 6 7] [8]
    [1 2 3 4] [5]
    [2 3 4 5] [6]
    [5 6 7 8] [9]
    [0 1 2 3] [4]
```

Typically, you would shuffle their data before training. And this is possible using the shuffle method. We call it with the buffer size of ten, because that's the amount of data items that we have.

```
dataset = tf.data.Dataset.range(10)
dataset = dataset.window(5, shift=1, drop_remainder=True)
dataset = dataset.flat_map(lambda window: window.batch(5))
dataset = dataset.map(lambda window: (window[:-1], window[-1:]))
dataset = dataset.shuffle(buffer_size=10)
dataset = dataset.batch(2).prefetch(1)
for x,y in dataset:
  print("x = ", x.numpy())
  print("y = ", y.numpy())


    x =  [[4 5 6 7] [1 2 3 4]]
    y =  [[8] [5]]
    x =  [[3 4 5 6] [2 3 4 5]]
    y =  [[7] [6]]
    x =  [[5 6 7 8] [0 1 2 3]]
    y =  [[9] [4]]
```

Finally, we can look at batching the data, and this is done with the batch method.

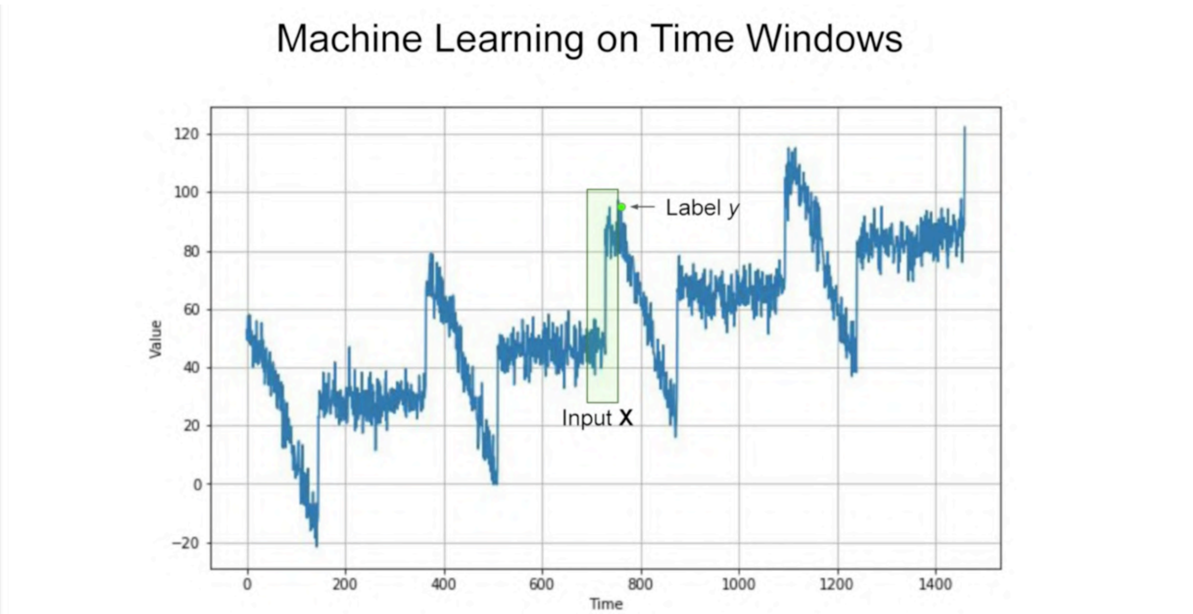
```
dataset = tf.data.Dataset.range(10)
dataset = dataset.window(5, shift=1, drop_remainder=True)
dataset = dataset.flat_map(lambda window: window.batch(5))
dataset = dataset.map(lambda window: (window[:-1], window[-1:]))
dataset = dataset.shuffle(buffer_size=10)
dataset = dataset.batch(2).prefetch(1)
for x,y in dataset:
  print("x = ", x.numpy())
  print("y = ", y.numpy())


    x =  [[4 5 6 7] [1 2 3 4]]
    y =  [[8] [5]]
    x =  [[3 4 5 6] [2 3 4 5]]
    y =  [[7] [6]]
    x =  [[5 6 7 8] [0 1 2 3]]
    y =  [[9] [4]]
```

It'll take a size parameter, and in this case it's 2. So what we'll do is we'll batch the data into sets of two, and if we print them out, we'll see this. We now have three batches of two data items each. And if you look at the first set, you'll see the corresponding x and y. So when x is four, five, six and seven, our y is eight, or when x is zero, one, two, three, you'll see our y is four.

Sequence bias

Sequence bias is when the order of things can impact the selection of things. For example, if I were to ask you your favorite TV show, and listed "Game of Thrones", "Killing Eve", "Travellers" and "Doctor Who" in that order, you're probably more likely to select 'Game of Thrones' as you are familiar with it, and it's the first thing you see. Even if it is equal to the other TV shows. So, when training data in a dataset, we don't want the sequence to impact the training in a similar way, so it's good to shuffle them up.



Let's start with this function that will call a windows dataset. It will take in a data series along with the parameters for the size of the window that we want. The size of the batches to use when training, and the size of the shuffle buffer, which determines how the data will be shuffled.

```
def windowed_dataset(series, window_size, batch_size, shuffle_buffer):
  dataset = tf.data.Dataset.from_tensor_slices(series)
  dataset = dataset.window(window_size + 1, shift=1, drop_remainder=True)
  dataset = dataset.flat_map(lambda window: window.batch(window_size + 1))
  dataset = dataset.shuffle(shuffle_buffer)
                   .map(lambda window: (window[:-1], window[-1]))
  dataset = dataset.batch(batch_size).prefetch(1)
  return dataset
```

The first step will be to create a dataset from the series using a tf.data dataset. And we'll pass the series to it using its from_tensor_slices method.

We will then use the window method of the dataset based on our window_size to slice the data up into the appropriate windows. Each one being shifted by one time set. We'll keep them all the same size by setting drop remainder to true.

```
def windowed_dataset(series, window_size, batch_size, shuffle_buffer):
  dataset = tf.data.Dataset.from_tensor_slices(series)
  dataset = dataset.window(window_size + 1, shift=1, drop_remainder=True)
  dataset = dataset.flat_map(lambda window: window.batch(window_size + 1))
  dataset = dataset.shuffle(shuffle_buffer)
                   .map(lambda window: (window[:-1], window[-1]))
  dataset = dataset.batch(batch_size).prefetch(1)
  return dataset
```

We then flatten the data out to make it easier to work with. And it will be flattened into chunks in the size of our window_size + 1.

```
def windowed_dataset(series, window_size, batch_size, shuffle_buffer):
  dataset = tf.data.Dataset.from_tensor_slices(series)
  dataset = dataset.window(window_size + 1, shift=1, drop_remainder=True)
  dataset = dataset.flat_map(lambda window: window.batch(window_size + 1))
  dataset = dataset.shuffle(shuffle_buffer)
                   .map(lambda window: (window[:-1], window[-1]))
  dataset = dataset.batch(batch_size).prefetch(1)
  return dataset
```

Once it's flattened, it's easy to shuffle it. You call a shuffle and you pass it the shuffle buffer. Using a shuffle buffer speeds things up a bit. So for example, if you have 100,000 items in your dataset, but you set the buffer to a thousand. It will just fill the buffer with the first thousand elements, pick one of them at random. And then it will replace that with the 1,000 and first element before randomly picking again, and so on. This way with super large datasets, the random element choosing can choose from a smaller number which effectively speeds things up.

```
def windowed_dataset(series, window_size, batch_size, shuffle_buffer):
  dataset = tf.data.Dataset.from_tensor_slices(series)
  dataset = dataset.window(window_size + 1, shift=1, drop_remainder=True)
  dataset = dataset.flat_map(lambda window: window.batch(window_size + 1))
  dataset = dataset.shuffle(shuffle_buffer)
                   .map(lambda window: (window[:-1], window[-1]))
  dataset = dataset.batch(batch_size).prefetch(1)
  return dataset
```

The shuffled dataset is then split into the xs, which is all of the elements except the last, and the y which is the last element.

```
def windowed_dataset(series, window_size, batch_size, shuffle_buffer):
  dataset = tf.data.Dataset.from_tensor_slices(series)
  dataset = dataset.window(window_size + 1, shift=1, drop_remainder=True)
  dataset = dataset.flat_map(lambda window: window.batch(window_size + 1))
  dataset = dataset.shuffle(shuffle_buffer)
                   .map(lambda window: (window[:-1], window[-1]))
  dataset = dataset.batch(batch_size).prefetch(1)
  return dataset
```

It's then batched into the selected batch size and returned.

```
def windowed_dataset(series, window_size, batch_size, shuffle_buffer):
  dataset = tf.data.Dataset.from_tensor_slices(series)
  dataset = dataset.window(window_size + 1, shift=1, drop_remainder=True)
  dataset = dataset.flat_map(lambda window: window.batch(window_size + 1))
  dataset = dataset.shuffle(shuffle_buffer)
                   .map(lambda window: (window[:-1], window[-1]))
  dataset = dataset.batch(batch_size).prefetch(1)
  return dataset
```

```
split_time = 1000
time_train = time[:split_time]
x_train = series[:split_time]
time_valid = time[split_time:]
x_valid = series[split_time:]
```

Before we can do a training, we have to split our dataset into training and validation sets.

Here's the code to do that at time step 1000. We can see that the training data is the subset of the series called x train up to the split time.

Here's the code to do a simple linear regression.

Let's look at it line by line. We'll start by setting up all the constants that we want to pass to the window dataset function. These include the window size on the data, the batch size that we want for training, and the size of the shuffled buffer as we've just discussed.

```
window_size = 20
batch_size = 32
shuffle_buffer_size = 1000

dataset = windowed_dataset(series, window_size, batch_size, shuffle_buffer_size)
l0 = tf.keras.layers.Dense(1, input_shape=[window_size])
model = tf.keras.models.Sequential([l0])
```

Then we'll create our dataset. We'll do this by taking our series. You'll pass it your series along what your desired window size, batch size, and shuffled buffer size, and it will give you back a formatted datasets that you could use for training.

```
window_size = 20
batch_size = 32
shuffle_buffer_size = 1000

dataset = windowed_dataset(series, window_size, batch_size, shuffle_buffer_size)
l0 = tf.keras.layers.Dense(1, input_shape=[window_size])
model = tf.keras.models.Sequential([l0])
```

I'm then going to create a single dense layer with its input shape being the window size. For linear regression, that's all you need. I'm using this approach. By passing the layer to a variable called L0, because later I'm want to print out its learned weights, and it's a lot easier for me to do that if I have a variable to refer to the layer for that.

```
window_size = 20
batch_size = 32
shuffle_buffer_size = 1000

dataset = windowed_dataset(series, window_size, batch_size, shuffle_buffer_size)
l0 = tf.keras.layers.Dense(1, input_shape=[window_size])
model = tf.keras.models.Sequential([l0])
```

Then I simply define my model as a sequential containing the sole layer just like this.

```
window_size = 20
batch_size = 32
shuffle_buffer_size = 1000

dataset = windowed_dataset(series, window_size, batch_size, shuffle_buffer_size)
l0 = tf.keras.layers.Dense(1, input_shape=[window_size])
model = tf.keras.models.Sequential([l0])
```

```
model.compile(loss="mse", optimizer=tf.keras.optimizers.SGD(lr=1e-6, momentum=0.9))
model.fit(dataset,epochs=100,verbose=0)
```

My optimizer will use Stochastic Gradient Descent. I'd use this methodology instead of the raw string, so I can set parameters on it to initialize it such as the learning rate or LR and the momentum. Experiment with different values here to see if you can get your model to converge more quickly or more accurately

```
model.compile(loss="mse", optimizer=tf.keras.optimizers.SGD(lr=1e-6, momentum=0.9))
model.fit(dataset,epochs=100,verbose=0)
```

Next you can fit your model by just passing it the dataset, which has already been preformatted with the x and y values. I'm going to run for a 100 epochs here. Ignoring the epoch but epoch output by setting verbose to zero.

```
model.compile(loss="mse", optimizer=tf.keras.optimizers.SGD(lr=1e-6, momentum=0.9))
model.fit(dataset,epochs=100,verbose=0)
```

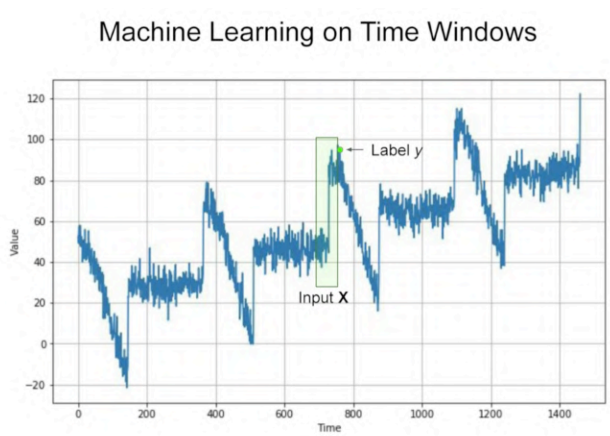
```
print("Layer weights {}".format(l0.get_weights()))

Layer weights [array([[ 0.01633573],
        [-0.02911791],
        [ 0.00845617],
        [-0.02175158],
        [ 0.04962169],
        [-0.03212642],
        [-0.02596855],
        [-0.00689476],
        [ 0.0616533 ],
        [-0.00668752],
        [-0.02735964],
        [ 0.0377918 ],
        [-0.02855931],
        [ 0.05299238],
        [-0.0121608 ],
        [ 0.00138755],
        [ 0.0905595 ],
        [ 0.19994621],
        [ 0.2556632 ],
        [ 0.41660047]], dtype=float32), array([0.01430958], dtype=float32)]
```

W_{t0} W_{t1} W_{t2} W_{t3} b

Once it's done training, you can actually inspect the different weights with this code. Remember earlier when we referred to the layer with a variable called L0? Well, here's where that's useful. The output will look like this. If you inspect it closely, you will see that the first array has 20 values in it, and the secondary has only one value. This is because the network has learned a linear regression to fit the values as best as they can.

So each of the values in the first array can be seen as the weights for the 20 values in x, and the value for the second array is the b value



So if you think back to this diagram and you consider the input window to be 20 values wide, then let's call them x0, x1, x2, etc, all the way up to x19. But let's be clear. That's not the value on the horizontal axis which is commonly called the x-axis, it's the value of the time series at that point on the horizontal axis. So the value at time t0, which is 20 steps before the current value is called x0, and t1 is called x1, etc. Similarly, for the output, which we would then consider to be the value at the current time to be the y.

```
print("Layer weights {}".format(l0.get_weights()))

Layer weights [array([[ 0.01633573],
       [-0.02911791],
       [ 0.00845617],
       [-0.02175158],
       [ 0.04962169],
       [-0.03212642],
       [-0.02596855],
       [-0.00689476],
       [ 0.0616533 ],
       [-0.00668752],
       [-0.02735964],
       [ 0.0377918 ],
       [-0.02855931],
       [ 0.05299238],
       [-0.0121608 ],
       [ 0.00138755],
       [ 0.0905595 ],
       [ 0.19994621],
       [ 0.2556632 ],
       [ 0.41660047]], dtype=float32), array([0.01430958], dtype=float32)]
```

W_{t0} W_{t1} W_{t2} W_{t3} b


```
print("Layer weights {}".format(l0.get_weights()))

Layer weights [array([[ 0.01633573],
       [-0.02911791],
       [ 0.00845617],
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       [-0.02596855],
       [-0.00689476],
       [ 0.0616533 ],
       [-0.00668752],
       [-0.02735964],
       [ 0.0377918 ],
       [-0.02855931],
       [ 0.05299238],
       [-0.0121608 ],
       [ 0.00138755],
       [ 0.0905595 ],
       [ 0.19994621],
       [ 0.2556632 ],
       [ 0.41660047]], dtype=float32), array([0.01430958], dtype=float32)]
```


$$Y = W_{t0}X_0 + W_{t1}X_1 + W_{t2}X_2 + \ldots\ldots + W_{t19}X_{19} + b$$

```
print(series[1:21])
model.predict(series[1:21][np.newaxis])

[49.35275  53.314735 57.711823 48.934444 48.931244 57.982895 53.897125
 47.67393  52.68371  47.591717 47.506374 50.959415 40.086178 40.919415
 46.612473 44.228207 50.720642 44.454983 41.76799  55.980938]

array([[49.08478]], dtype=float32)
```

If I take 20 items in my series and print them out, I can see the 20x values. If I want to predict them, I can pass that series into my model to get a prediction.

The NumPy new axis then just reshapes it to the input dimension that's used by the model.

The top array is the 20 values that provide the input to our model and the bottom is the predicted value back from the model. So we've trained our model to say that when it sees 20 values like this, the predicted next value is 49.08478.

We create an empty list of forecasts

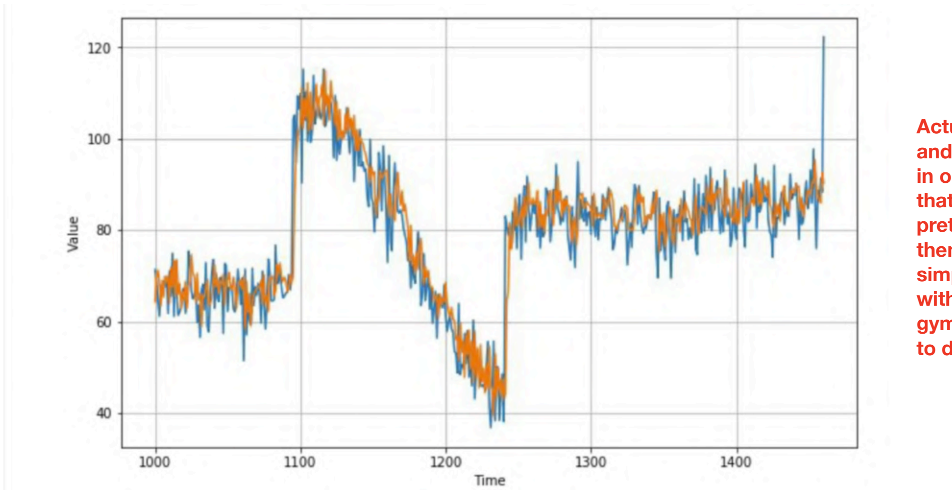
```
forecast = []
for time in range(len(series) - window_size):
  forecast.append(model.predict(series[time:time + window_size][np.newaxis]))

forecast = forecast[split_time-window_size:]
results = np.array(forecast)[:, 0, 0]
```

Iterate over the series taking slices and window size,
predicting them, and adding the results to the forecast list.

```
forecast = []
for time in range(len(series) - window_size):
  forecast.append(model.predict(series[time:time + window_size][np.newaxis]))

forecast = forecast[split_time-window_size:]
results = np.array(forecast)[:, 0, 0]
```

Split our time series into training and testing sense taking
everything before a certain time is training and the rest is
validation. So we'll just take the forecasts after the split
time and load them into a NuimPy array for charting

```
forecast = []
for time in range(len(series) - window_size):
  forecast.append(model.predict(series[time:time + window_size][np.newaxis]))

forecast = forecast[split_time-window_size:]
results = np.array(forecast)[:, 0, 0]
```



Actual values in blue and the predicted ones in orange. You can see that our predictions look pretty good and getting them was relatively simple in comparison with all the statistical gymnastics that we had to do above

```
tf.keras.metrics.mean_absolute_error(x_valid, results).numpy()

    4.9526777
```

Let's measure the mean absolute error as we've done before, and we can see that we're in a similar ballpark to where we were with a complex analysis that we did previously. Now that's just using a single layer in a neural network to calculate a linear regression. Let's see if we could do better with a fully-connected DNN next.

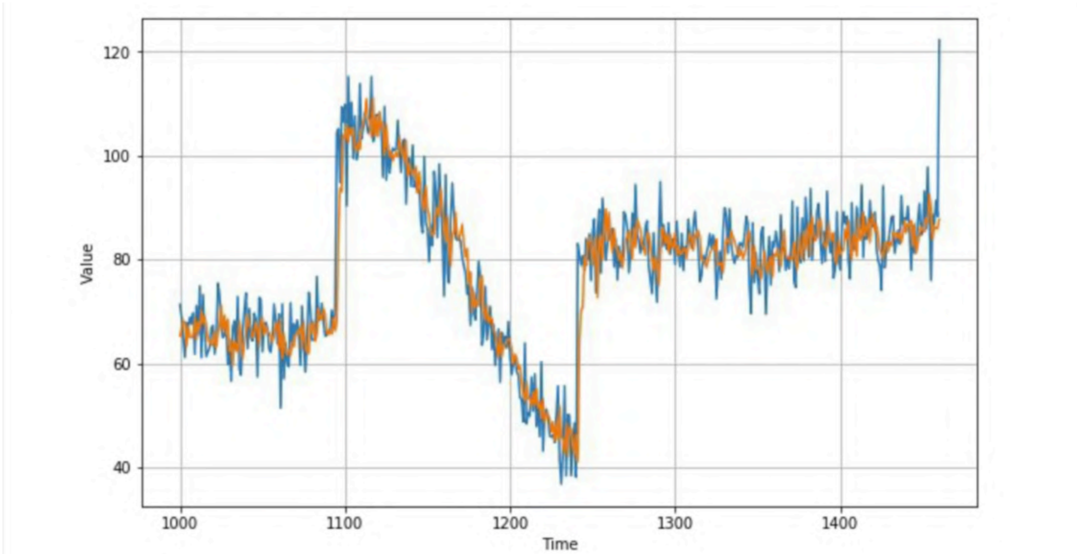
Deep Neural Network

```
dataset = windowed_dataset(x_train, window_size, batch_size, shuffle_buffer_size)

model = tf.keras.models.Sequential([
    tf.keras.layers.Dense(10, input_shape=[window_size], activation="relu"),
    tf.keras.layers.Dense(10, activation="relu"),
    tf.keras.layers.Dense(1)
])

model.compile(loss="mse", optimizer=tf.keras.optimizers.SGD(lr=1e-6, momentum=0.9))
model.fit(dataset,epochs=100,verbose=0)
```

```
dataset = windowed_dataset(x_train, window_size, batch_size, shuffle_buffer_size)

model = tf.keras.models.Sequential([
    tf.keras.layers.Dense(10, input_shape=[window_size], activation="relu"),
    tf.keras.layers.Dense(10, activation="relu"),
    tf.keras.layers.Dense(1)
])

model.compile(loss="mse", optimizer=tf.keras.optimizers.SGD(lr=1e-6, momentum=0.9))
model.fit(dataset,epochs=100,verbose=0)
```

```
dataset = windowed_dataset(x_train, window_size, batch_size, shuffle_buffer_size)

model = tf.keras.models.Sequential([
    tf.keras.layers.Dense(10, input_shape=[window_size], activation="relu"),
    tf.keras.layers.Dense(10, activation="relu"),
    tf.keras.layers.Dense(1)
])

model.compile(loss="mse", optimizer=tf.keras.optimizers.SGD(lr=1e-6, momentum=0.9))
model.fit(dataset,epochs=100,verbose=0)
```

```
dataset = windowed_dataset(x_train, window_size, batch_size, shuffle_buffer_size)

model = tf.keras.models.Sequential([
    tf.keras.layers.Dense(10, input_shape=[window_size], activation="relu"),
    tf.keras.layers.Dense(10, activation="relu"),
    tf.keras.layers.Dense(1)
])

model.compile(loss="mse", optimizer=tf.keras.optimizers.SGD(lr=1e-6, momentum=0.9))
model.fit(dataset,epochs=100,verbose=0)
```




```
tf.keras.metrics.mean_absolute_error(x_valid, results).numpy()

4.9833784
```

```
dataset = windowed_dataset(x_train, window_size, batch_size, shuffle_buffer_size)

model = tf.keras.models.Sequential([
    tf.keras.layers.Dense(10, input_shape=[window_size], activation="relu"),
    tf.keras.layers.Dense(10, activation="relu"),
    tf.keras.layers.Dense(1)
])

lr_schedule = tf.keras.callbacks.LearningRateScheduler(
    lambda epoch: 1e-8 * 10**(epoch / 20))

optimizer = tf.keras.optimizers.SGD(lr=1e-8, momentum=0.9)

model.compile(loss="mse", optimizer=optimizer)

history = model.fit(dataset, epochs=100, callbacks=[lr_schedule])
```

Wouldn't it be nice if we could pick the optimal learning rate instead of the one that we chose? We might learn more efficiently and build a better model.

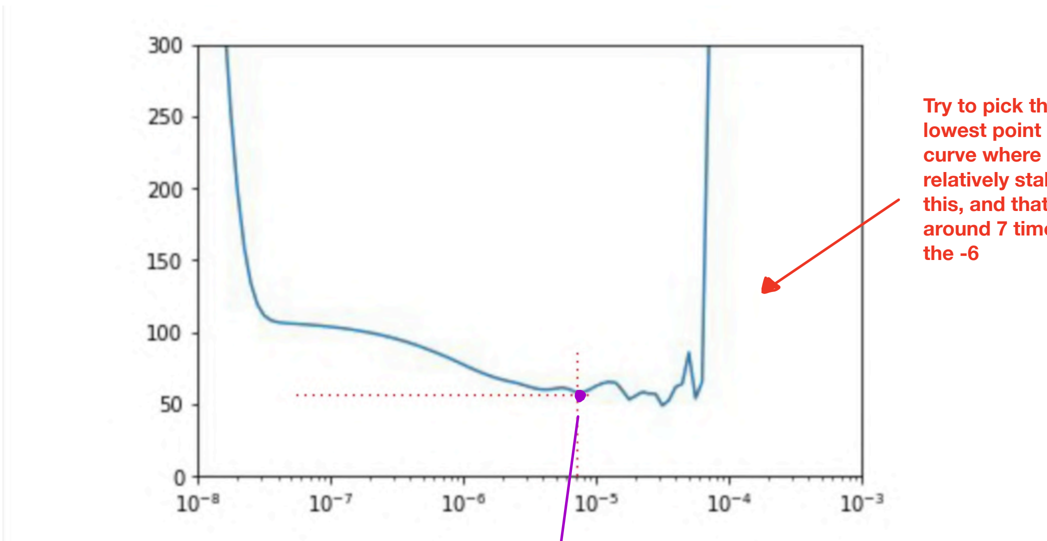

```
dataset = windowed_dataset(x_train, window_size, batch_size, shuffle_buffer_size)

model = tf.keras.models.Sequential([
    tf.keras.layers.Dense(10, input_shape=[window_size], activation="relu"),
    tf.keras.layers.Dense(10, activation="relu"),
    tf.keras.layers.Dense(1)
])

lr_schedule = tf.keras.callbacks.LearningRateScheduler(
    lambda epoch: 1e-8 * 10**(epoch / 20))

optimizer = tf.keras.optimizers.SGD(lr=1e-8, momentum=0.9)

model.compile(loss="mse", optimizer=optimizer)

history = model.fit(dataset, epochs=100, callbacks=[lr_schedule])
```


So here's a code for the previous neural network. But I've added a callback to tweak the learning rate using a learning rate scheduler. You can see that code here. This will be called at the callback at the end of each epoch. What it will do is change the learning rates to a value based on the epoch number. So in epoch 1, it is 1 times 10 to the -8 times 10 to the power of 1 over 20. And by the time we reach the 100 epoch, it'll be 1 times 10 to the -8 times 10 to the power of 5, and that's 100 over 20.

This will happen on each callback because we set it in the callbacks parameter of modeled outfit.

```
lrs = 1e-8 * (10 ** (np.arange(100) / 20))
plt.semilogx(lrs, history.history["loss"])
plt.axis([1e-8, 1e-3, 0, 300])
```

After training with this, we can then plot the last per epoch against the learning rate per epoch by using this code, and we'll see a chart like this.

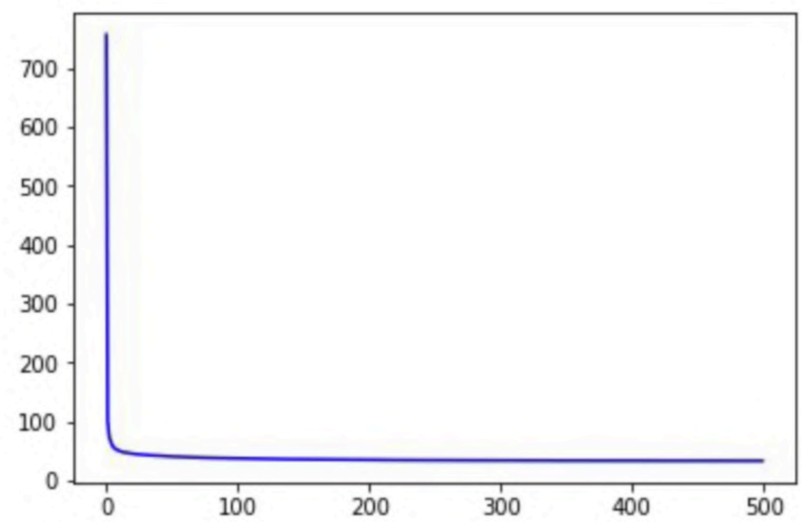


Try to pick the lowest point of the curve where it's still relatively stable like this, and that's right around 7 times 10 to the -6

```
window_size = 30
dataset = windowed_dataset(x_train, window_size, batch_size, shuffle_buffer_size)

model = tf.keras.models.Sequential([
  tf.keras.layers.Dense(10, activation="relu", input_shape=[window_size]),
  tf.keras.layers.Dense(10, activation="relu"),
  tf.keras.layers.Dense(1)
])

optimizer = tf.keras.optimizers.SGD(lr=7e-6, momentum=0.9)
model.compile(loss="mse", optimizer=optimizer)
history = model.fit(dataset, epochs=500)
```

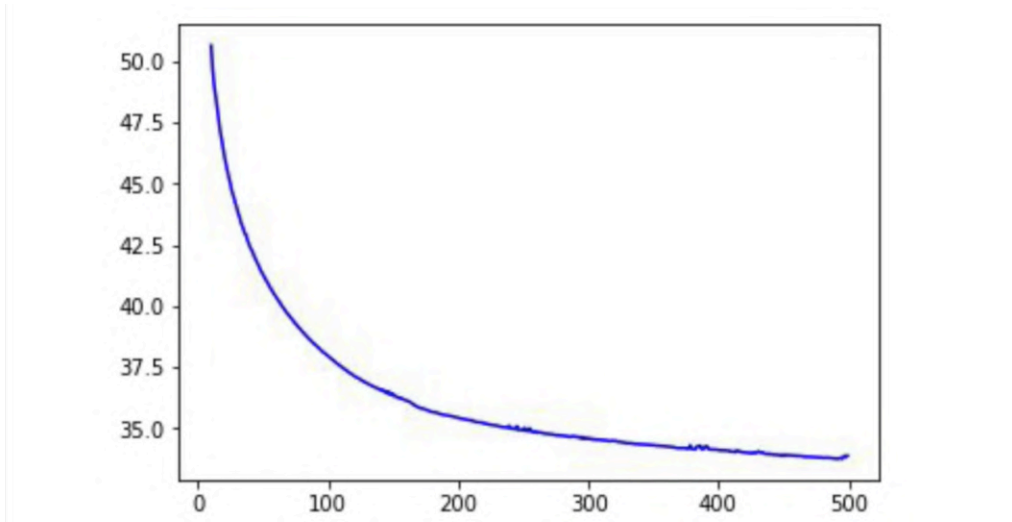
Upon first inspection looks like we're probably wasting our time training beyond maybe only 10 epochs, but it's somewhat skewed by the fact that the earlier losses were so high.

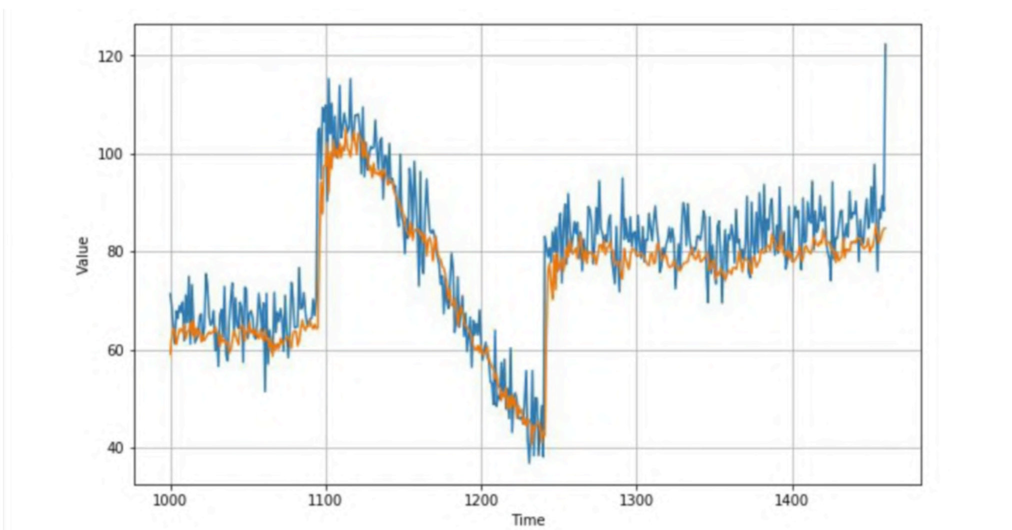
```
loss = history.history['loss']
epochs = range(len(acc))
plt.plot(epochs, loss, 'b', label='Training Loss')
plt.show()
```



If we cropped them off and plot the loss for epochs after number 10 with code like this, then the chart will tell us a different story.

```
# Plot all but the first 10
loss = history.history['loss']
epochs = range(10, len(acc))
plot_loss = loss[10:]
print(plot_loss)
plt.plot(epochs, plot_loss, 'b', label='Training Loss')
plt.show()
```






```
tf.keras.metrics.mean_absolute_error(x_valid, results).numpy()

4.4847784
```