Quantum Information with atoms and photons
Final report

Cavity Blockade

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1 Introduction

In this report, we aim to investigate the cavity blockade effect, a phenomenon in which a cavity containing a two level atom prevents the absorption of a second photon once the first one is inside. Our goal is to identify the optimal conditions for this effect to occur. To do so, we start with a driven cavity model based on the Jaynes-Cummings Hamiltonian, which describes the interaction between a two level atom and a cavity mode.

The Hamiltonian used to treat this problem is the following:

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_a |e\rangle\langle e| + \hbar\tilde{g}(\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger) + \hat{H}_d \tag{1}$$

where $\hat{H}_d = \varepsilon e^{i\omega t}\hat{a}$+h.c. is a driving pumping filed. This driving term corresponds to a monochromatic driving of the cavity photon field at resonance frequency ω with strength $\varepsilon > 0$.

2 Rotating wave approximation

To work handily with the problem, it is useful to move to the rotating frame applying the rotating wave approximation to the previous Hamiltonian (Eq. 1). To do so we have to apply the following unitary transformation:

$$H' = R(t)H(t)R^\dagger(t) - R(t)\frac{d}{dt}R^\dagger(t) \tag{2}$$

where $\hat{R}(t) = e^{i\omega t(\hat{a}^\dagger \hat{a} + |e\rangle\langle e|)} = e^{i\hat{K}(t)}$ is the unitary operator that performs the transformation.

Once applied, with the help of Hadamard's rule: $\hat{R}\hat{H}\hat{R}^\dagger = \hat{H} + \left[\hat{K}, \hat{H}\right] + \frac{1}{2}\left[\hat{K}, \left[\hat{K}, \hat{H}\right]\right] + ...$ the resulting Hamiltonian has the form ($\hbar = 1$):

$$H = \delta_c \hat{a}^\dagger \hat{a} + \delta_a |e\rangle\langle e| + \tilde{g}\left(\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger\right) + \varepsilon \hat{a} + \varepsilon^* \hat{a}^\dagger \tag{3}$$

Where δ_c and δ_a are the cavity and atomic detuning, respectively.

In order to simulate the dynamics of the system we are going to solve numerically the Lindblad Master equation in the steady state, whose general equation is:

$$\dot{\rho}(t) = \frac{i}{\hbar}\left[\rho_s, H\right] + \sum_j (L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\}) \tag{4}$$

where L_j are the Lindblad operators, that for our specific problem can be written as:

$$L_\Gamma = \sqrt{\Gamma}\hat{\sigma}_- \tag{5}$$

$$L_\kappa = \sqrt{\kappa}\hat{a} \tag{6}$$

In these operators appear two decay rates: Γ which is a small decay rate of the atom outside the cavity and κ which represent the fact that the cavity is not perfect and can loose photons.

3 No coupling between atom and cavity

For a first analysis we have considered a situation in which all the frequencies are resonant, $\omega = \omega_a = \omega_c$ and no coupling between atom and cavity, $\tilde{g} = 0$. We started from the state $|g, 0\rangle$ corresponding to an initially empty cavity with the atom in the ground state, and with the help of the QuTiP library we have numerically solved the master equation to understand which is the mean photon number $\langle \hat{a}^\dagger \hat{a} \rangle \equiv \langle \hat{N} \rangle$ in the steady state.

Under these conditions, the total Hamiltonian (Eq. 3) reads:

$$H = \varepsilon \hat{a} + \varepsilon^* \hat{a}^\dagger \tag{7}$$

As a choice of parameters we decided to keep the photon Hilbert space small, with a value of $N = 6$, the decay rates $\kappa/2\pi = 2.0$ MHz and $\Gamma/2\pi = 3.0$ MHz with the same order of magnitude and the driving amplitude $\varepsilon/2\pi = 0.55$ MHz one order of magnitude lower than the decay rates. [2]

The result we get is a mean photon number of $\langle \hat{N} \rangle = 0.30$. If we look at the distribution that we get in (Fig. 1) we can observe the fact that we have a high probability (around 74%) of not having photons in the steady state, which means that they are not able to stay in the cavity and there is an high probability that they go outside. To understand if we are facing a coherent state distribution, we have plotted a red line corresponding to a Poisson distribution, which fits perfectly our histogram, and from it we have extrapolated its mean value which is exactly equal to the mean photon number (0.30) and so we can state that the photon number distribution is coherent.

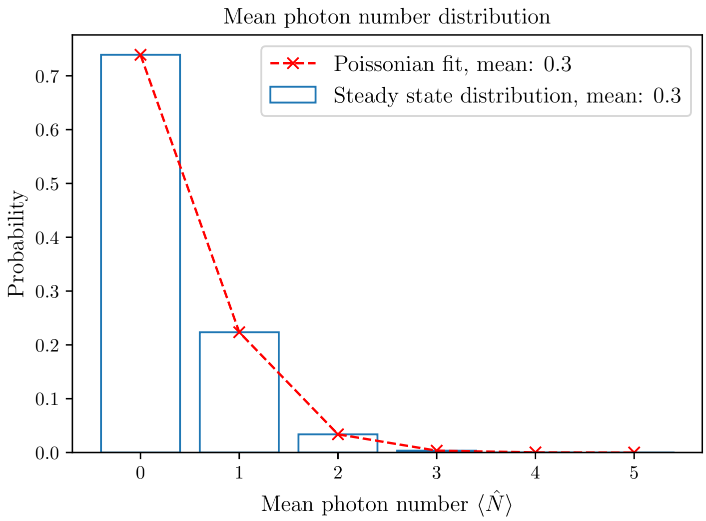

Figure 1: Mean photon number distribution

From the master equation (Eq. 4) we can also extrapolate the temporal evolution of the mean photon number which reaches the value of the steady state in a short time and directly, without any oscillations because of the absence of the coupling $\tilde{g}$ that generates the Rabi frequency. The result is plotted in (Fig. 2).

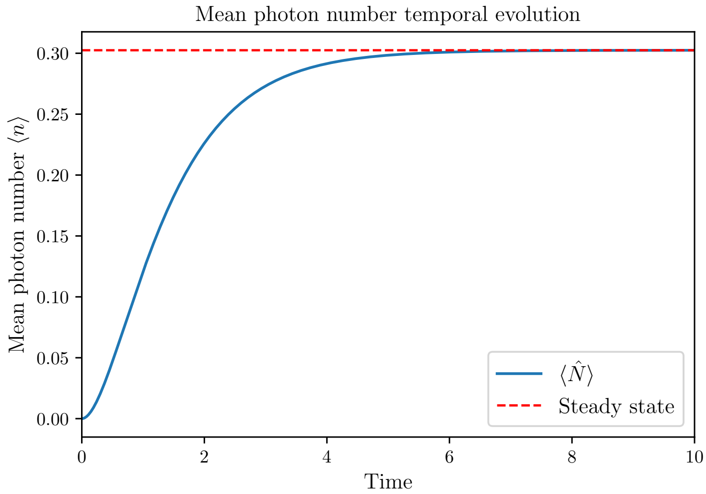

Figure 2: Temporal evolution

4 Coupling between atom and cavity

Now we want to focus our analysis on checking the effect of the coupling $\tilde{g}$ in the system and perform the same steps than before under the strong coupling condition $\tilde{g} \gg \Gamma, \kappa$ and weak drive $\varepsilon \ll \tilde{g}$.

The effect of the coupling $\tilde{g}$ is to split the energy levels in the so-called dressed states, and so we have to put the drive frequency in resonance with the lowest of them. By tuning the frequency on the lowest dressed state, the system efficiently absorbs the first photon resonantly suppressing the possibility to absorb a second one because the next dressed state is off-resonant. This anharmonicity in the energy levels induced by $\tilde{g}$, creates the blockade effect.

According to [2], the choice of parameters is now $\tilde{g}/2\pi = 10.0$ MHz, $\omega_a/2\pi = \omega_c/2\pi = 9.0$ MHz, while the others are kept unchanged.

Therefore, we have computed analytically the frequency of the lowest dressed state diagonalizing the matrix corresponding to the Hamiltonian:

$$H_n = \begin{pmatrix} n\hbar\omega_c & \hbar\tilde{g}\sqrt{n} \\ \hbar\tilde{g}\sqrt{n} & \hbar\omega_a + (n-1)\hbar\omega_c \end{pmatrix} \tag{8}$$

whose eigenvalues are:

$$E_{\pm}^{(n)} = n\hbar\omega_c + \hbar\frac{\delta}{2} \pm \hbar\sqrt{\frac{\delta^2}{4} + \tilde{g}^2 n} \tag{9}$$

The lowest dressed state corresponds to E_-^1 and as soon as we choose to put $\omega_a = \omega_c$, the physical quantity $\delta = \omega_a - \omega_c$ cancel out, leading to the following final expression for the lowest dressed state:

$$E_-^{(1)} = \hbar\omega_c - \hbar\tilde{g} \tag{10}$$

from which we can compute the corresponding frequency ($\omega = E/\hbar$) which is the one of the lowest dressed state that we have used for the drive:

$$\omega_{drive} = \omega_c - \tilde{g} \tag{11}$$

With this setup we have calculated again the mean photon number distribution (Fig. 3) by numerically solving the master equation (Eq. 4) in the steady state, which brings to a trend pretty similar than before. The result that we get now for the mean photon number in the steady state is $\langle \hat{N} \rangle = 0.04$ which is the same that we have extrapolated from the Poissonian fit (red dashed line in the plot). There is an high probability, now around 95% not to see photons in the cavity, and so, even in presence of the coupling $\tilde{g}$ there is no chance that they stay inside the cavity. Even in that case, the drive strength ε is too weak to let photons stay inside the cavity because of the decay rates.

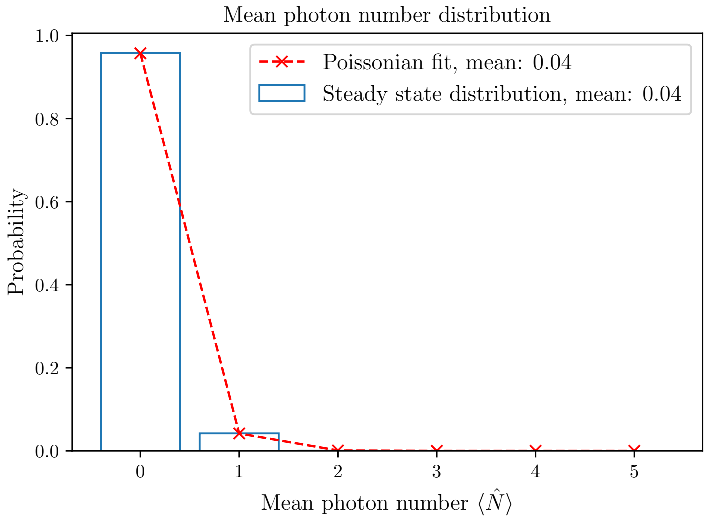

Figure 3: Mean photon number distribution

Looking at the evolution of the mean photon number in time (Fig. 4) we can observe the typical Rabi oscillations. In the strong coupling regime, they are clearly visible because $\tilde{g}$ is much larger than the decay rates Γ and κ. These oscillations represent the cyclic exchange of energy between the atom and the cavity. However, dissipative processes gradually damp the oscillations, and eventually, the system reaches a steady state where the energy supplied by the drive balances the losses.

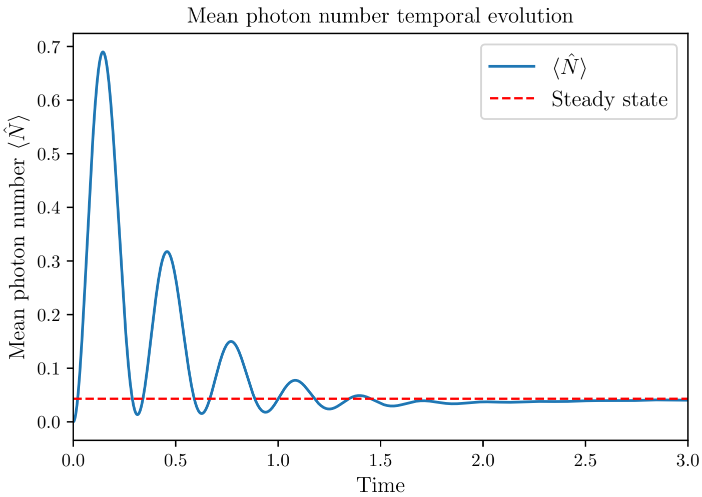

Figure 4: Temporal evolution

5 Parameters tuning

A further analysis can be done in order to understand if the choice of parameters we did was the best one, or if changing some of them we could have obtain different behaviors. To verify that we tried to understand which is the relation between the drive strength ε and the two decay rates Γ and κ to determine the mean photon number.

In the following plots (Figs. 5, 6, 7) there are the graphical results that we have obtained. In all of them we have scanned the values of the parameter from 0 to 40 and we have marked two lines. The yellow dashed line represent the level $\langle \hat{N} \rangle = 1$ while the other marks $\langle \hat{N} \rangle = 2$. In fact from these three plots one can understand which are the right combinations of parameters in order to expect a mean occupation number of one or two photons. As we can see, these plot are in agreement with our choice of parameters from which we expect $\langle \hat{N} \rangle = 0.04$.

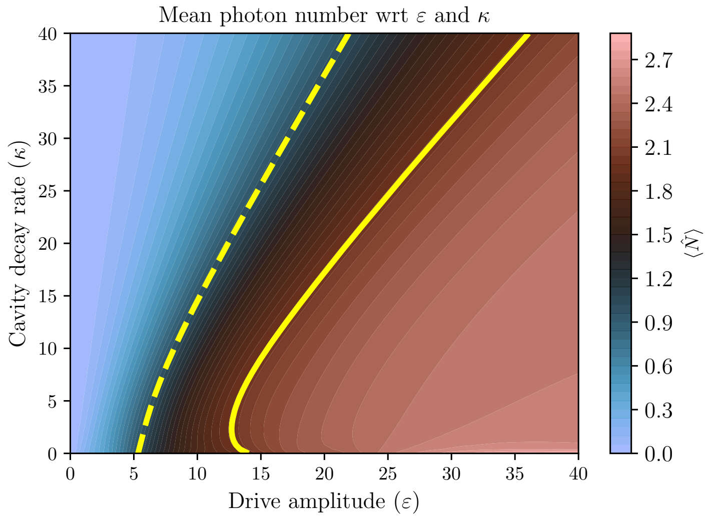

Figure 5: Tuning of κ and ε.

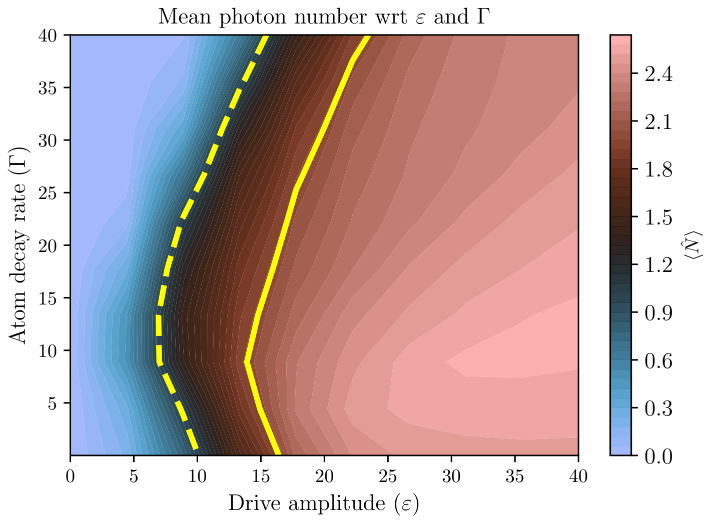

Figure 6: Tuning of Γ and ε.

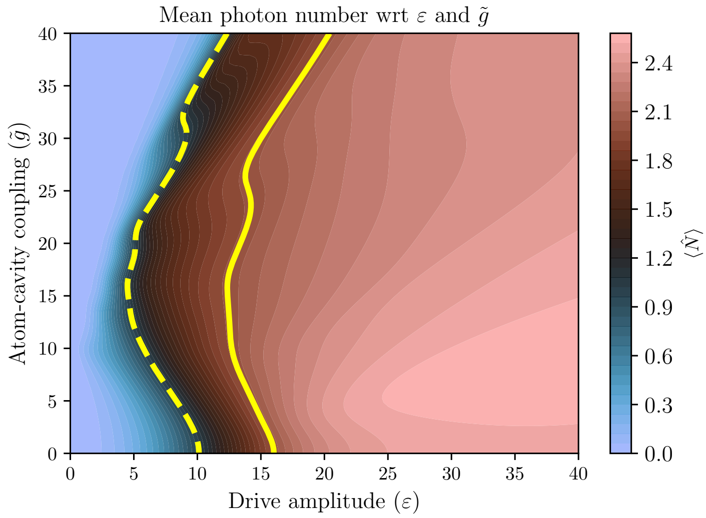

Figure 7: Tuning of $\tilde{g}$ and ε.

In the plots of the photon mean number distributions, we do not observe the occupation of a single photon, likely because we have set the drive strength one order of magnitude lower than the decay rates. From these three plots, it seems that if we set the value of ε to approximately the same order of magnitude as Γ and κ, we should be able to observe the occupation of a single photon.

Regarding the value of $\tilde{g}$, we might have increased it slightly, but its order of magnitude appears to be appropriate relative to the other parameters to stay in a cavity blockade regime.

If we further increase the drive amplitude to around 15.0 in all three cases, we could even observe the occupation of two photons.

6 Two photons correlation function

Numerically solving the master equation (Eq. 4), one can obtain the second-order photon-photon correlation function in the steady state:

$$g^{(2)}(0) = \frac{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle}{|\langle \hat{a}^\dagger \hat{a} \rangle|^2} \tag{12}$$

In general, the value of $g^2(0)$ compares the probability of detecting two simultaneous photons to the probability of detecting two independent photons at random. If $g^2(0) > 1$, two photons are more likely to be detected together. However, if $g^2(0) < 1$, the photon output exhibits antibunching behavior, meaning photons are detected one by one. Since we are in the blockade regime, we expect the latter case. Indeed, our numerical result of $g^2(0) = 0.53$ confirms a good photon blockade effect.

We have even tried to scan the values of ω around resonance in order to understand how the value of the two photons correlation function would have changed and which is the distribution of the mean photon number $\langle \hat{N} \rangle$ around ω. The findings are plotted in (Figs. 8, 9).

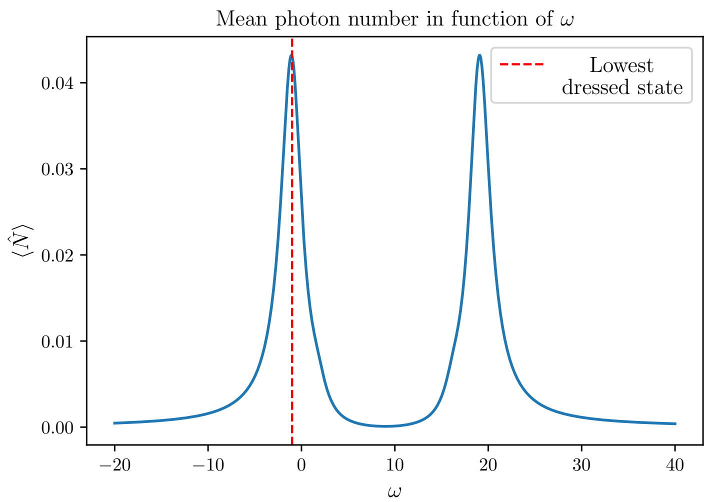

Figure 8: Distribution of the mean photon number in function of the frequency. The lowest dressed state is highlighted in red.

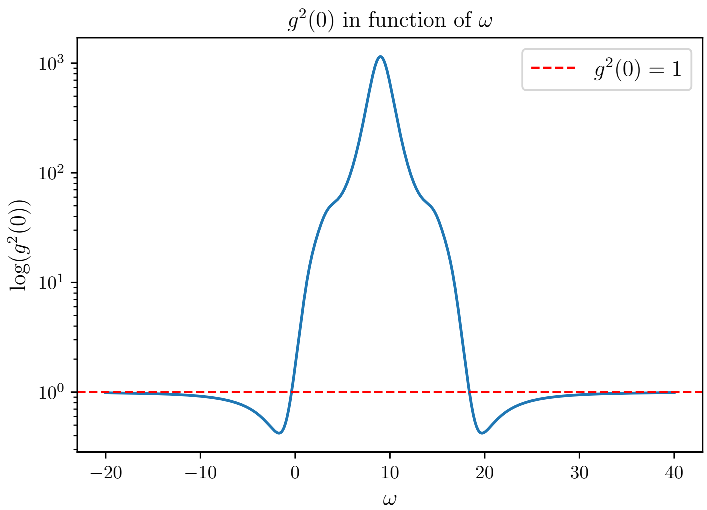

Figure 9: Distribution of the two-photons correlation function in function of the frequency (logarithmic scale for a better visualization).

In (Fig. 8) we can see two peaks around the values of $\omega_{dive} = \omega_c \pm \tilde{g}$ that are the frequencies of the two one-photon dressed states (the lowest one is highlighted with a dashed red line) in which the mean photon number reaches his greater value of 0.04, the one we found before.

In correspondence of these two peaks, if we look at (Fig. 9), we can see there are two minima which represent the lowest values under the red dashed threshold of $g^2(0) = 1$ around which we can state we have effectively reached the photon blockade condition. [1]

7 Conclusions

In this report, we have demonstrated how to achieve the cavity blockade regime. Specifically, we explored two different scenarios: one without coupling and one with coupling. We then adjusted the system parameters to better understand their impact and finally, we presented two plots that confirmed our hypotheses.

References

[1] Kevin M Birnbaum, Andreea Boca, Russell Miller, Allen D Boozer, Tracy E Northup, and H Jeff Kimble. Photon blockade in an optical cavity with one trapped atom. *Nature*, 436(7047):87–90, 2005.

[2] Christoph Hamsen, Karl Nicolas Tolazzi, Tatjana Wilk, and Gerhard Rempe. Two-photon blockade in an atom-driven cavity qed system. *Physical review letters*, 118(13):133604, 2017.