Assignment 3

Quantum Information and Computing

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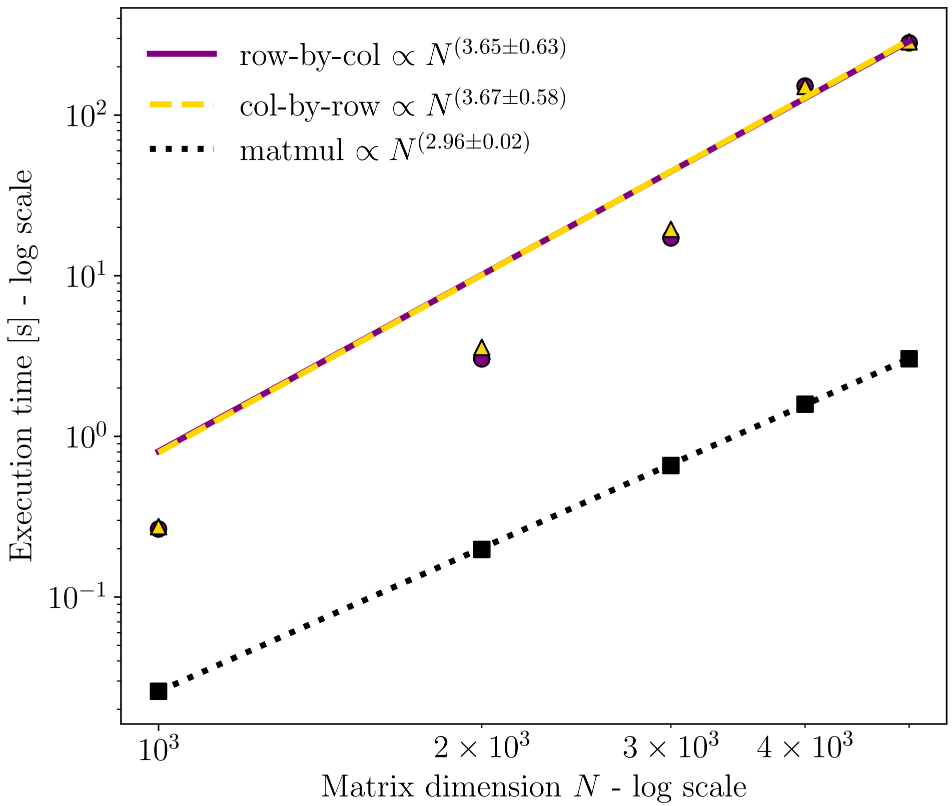
25 November 2025, A.Y. 2025/2026

Scaling of the matrix-matrix multiplication

Automatisation of the compiling and the execution of the matrix-matrix multiplication Fortran file.

- Matrix dimensions tested: *N = [500, 1000, 1500, …, 5000]*
- Computing performances in term of *CPUtime*
- Fit the resulting point with a polynomial: expect complexity $O(N^3)$



Eigenproblem

Two functions:

- `generate_hermitian_matrix`

```
def generate_hermitian_matrix(dim, distr="uniform", low=0.0, high=1.0, mean=0.0, std=1.0, complex=False):
    . . .

    # Make the matrix hermitian
    A = A + A.T
```

- `compute_eigenvals_spacing`

```
def compute_eigenvals_spacing(A, k=None):

    . . .

    eigenval = np.sort(eigsh(A, k=k, which="LM")[0])

    # compute the space between the eigenvalues
    Lambda_i = np.diff(eigenval)
    Lambda_i = Lambda_i[:-1] # discard the largest eigenvalue
    Lambda_mean = np.mean(Lambda_i)

    # normalize space between the eigenvalues
    s = Lambda_i/Lambda_mean

    return s
```

`scipy.sparse.linalg.`**`eigsh`**

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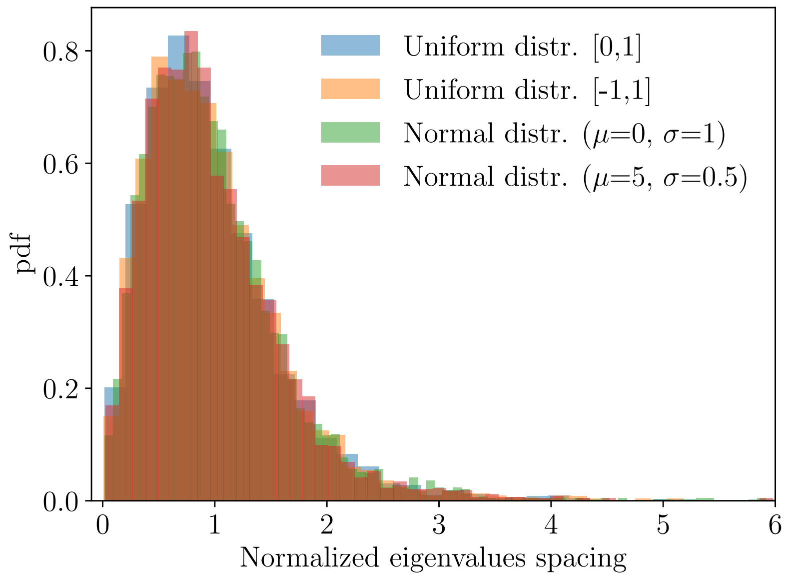
ARPACK

↓

Implicit Restarted Lanczos Method



Same pdfs for each distribution due to normalisation.

Random matrix Theory

Theory

Compute the normalised eigenvalue spacing for 4 different cases:

1. **Real hermitian matrix**
2. **Real diagonal matrix**
3. **Complex hermitian matrix**
4. **Sparse matrix**

Statistics based on generating *50 1000x1000* matrices for each type.

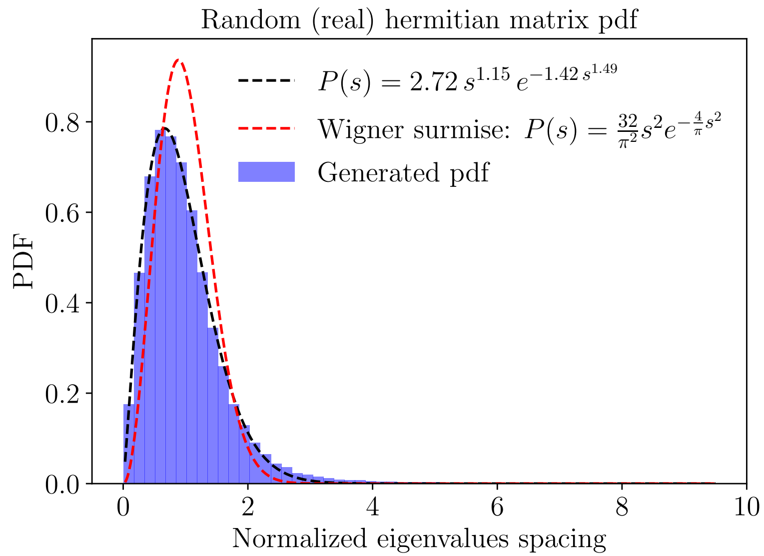
Overlapped with:

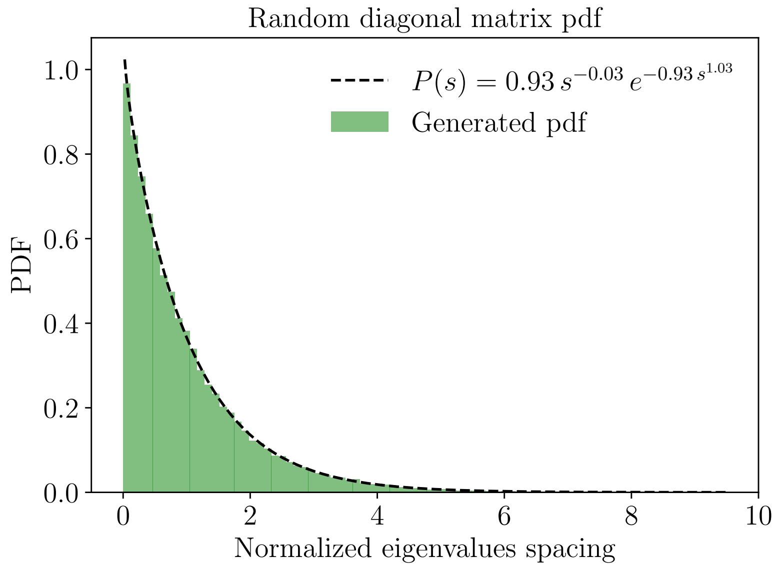
- **Generic 4-parameters fit**: $P(s) = as^{\alpha}e^{-bs^{\beta}}$

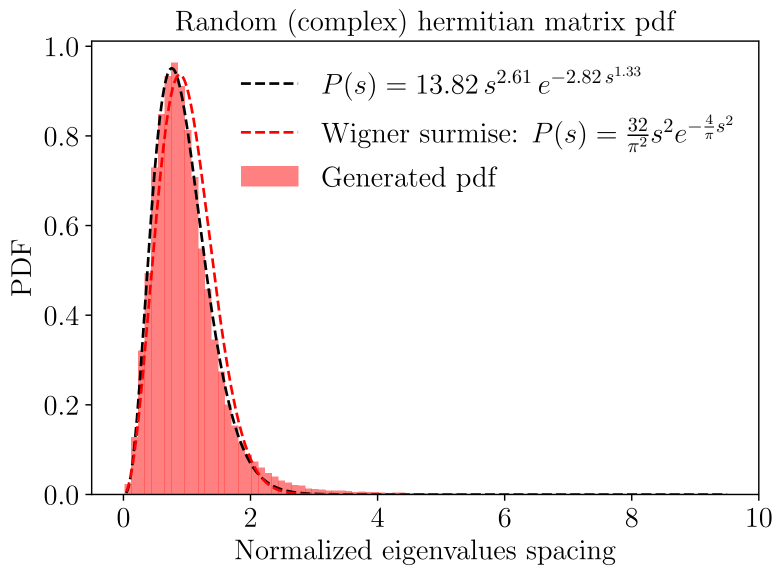
- **Wigner surmise distribution** - expected to be the theoretical distribution for hermitian matrices eigenvalues spacing (exact for 2x2 matrices):

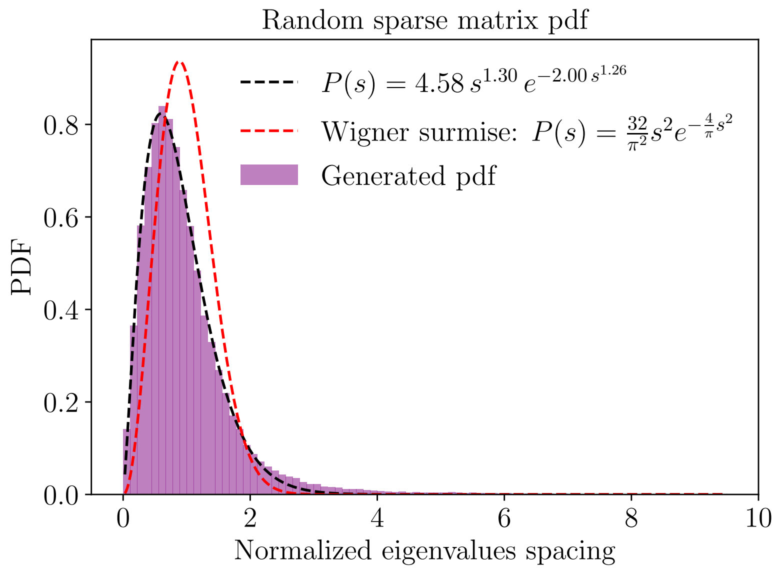
$$P(s) = \frac{32}{\pi^2}s^2e^{-\frac{4s^2}{\pi}}$$

Random matrix Theory









Random matrix Theory

Computational time increasing matrices dimensions

Diagonalization time comparison between different matrices.