assignment-4

September 29, 2024

1 Problem 1: Split the data

Prepare the data by encoding the non-numeric variables and applying a train-test split:

[78]:
```
import pandas as pd
from sklearn.model_selection import \
        train_test_split
from sklearn.preprocessing import LabelEncoder

data = pd.read_csv('data/life_expectancy.csv')

label_encoder = LabelEncoder()
data['Country'] = (
        label_encoder.fit_transform(data['Country'])
)

data_train, data_test = train_test_split(
        data,
        test_size=0.2,
        random_state=42
)
```

2 Problem 2: Single variable linear regression model

2.0.1 Find a candidate variable

Identify the variables with the strongest relationship with the target variable `Life Expectancy at Birth, both sexes (years)` using the Pearson correlation coefficient:

[79]:
```
target_variable = \
        'Life Expectancy at Birth, both sexes (years)'

correlations = (
        data_train
        .corr(method='pearson')[target_variable]
        .sort_values(ascending=False)
)
```

```python
candidate_variable = correlations.index[1]
print(
        f'Candidate variable: "{candidate_variable}"'
)
```

```
Candidate variable: "Human Development Index (value)"
```

2.0.2 Constructing the model

Construct a linear regression model using the variable with the strongest relationship with the target variable:

[65]:
```python
from sklearn.linear_model import LinearRegression

X_train = data_train[[candidate_variable]]
y_train = data_train[target_variable]

model = LinearRegression()
model.fit(X_train, y_train)
```

[65]:
```
LinearRegression()
```

Computing the metrics of the model:

[66]:
```python
r_squared = model.score(X_train, y_train)
coefficients = model.coef_
intercept = model.intercept_

print(f'R-squared: {r_squared}')
print(f'Coefficients: {coefficients}')
print(f'Intercept: {intercept}')
```

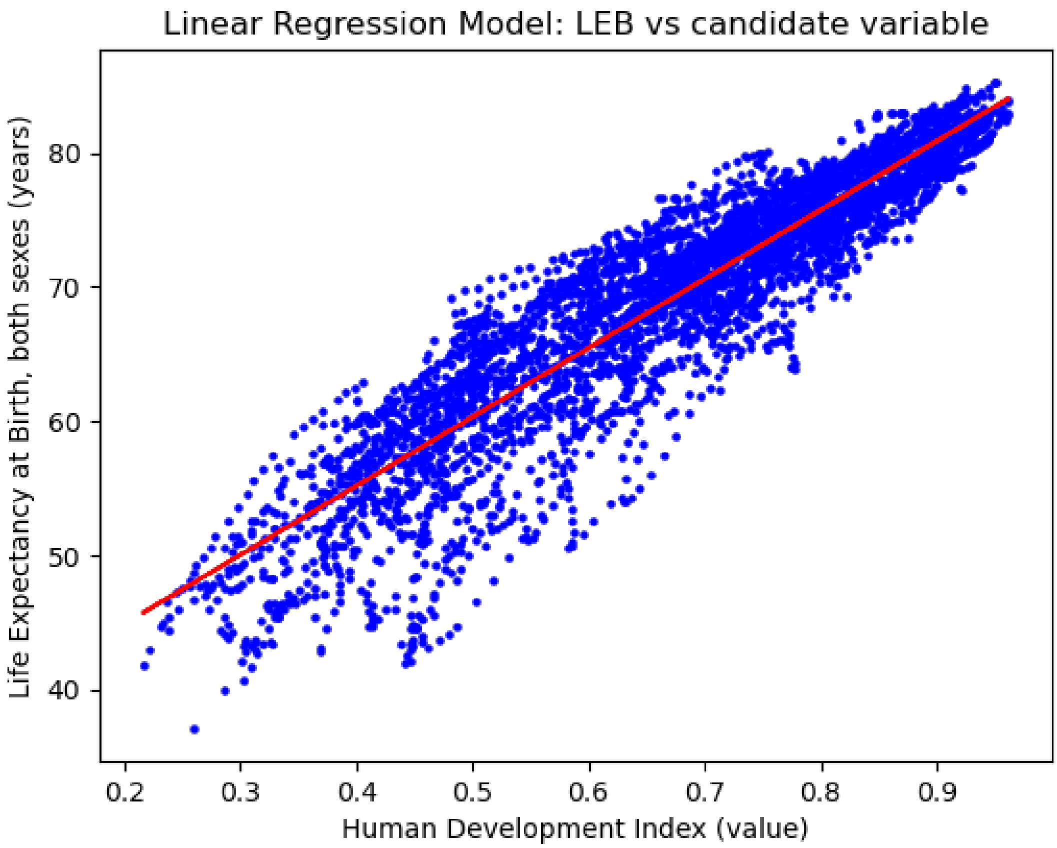
```
R-squared: 0.8433493090941087
Coefficients: [51.42339338]
Intercept: 34.60462419807184
```

Plot the linear regression model:

[67]:
```python
import matplotlib.pyplot as plt

plt.scatter(X_train, y_train, color='blue', s=5)
plt.plot(
        X_train, model.predict(X_train),
        color='red'
)
plt.title(
        'Linear Regression Model: LEB vs candidate variable'
)
plt.xlabel(candidate_variable)
plt.ylabel(target_variable)
```

```
plt.savefig(
        "scatter_plot_single_linear_model.png"
)
```



2.0.3 Predict the test set

Predict the target variable using the test data and computing the **mean squared error** and **correlation coefficient**:

[68]:
```
from scipy.stats import pearsonr
from sklearn.metrics import mean_squared_error

X_test = data_test[[candidate_variable]]
y_test = data_test[target_variable]
y_pred = model.predict(X_test)

mse = mean_squared_error(y_test, y_pred)
correlation, _ = pearsonr(y_pred, y_test)
```

```
print(f'Mean squared error: {mse}')
print(f'Correlation: {correlation}')
```

```
Mean squared error: 12.519251362188522
Correlation: 0.920387001630666
```

3 Problem 3: Non-linear relationship

3.0.1 Find a second candidate

Identify the variables with the strongest relationship with the target variable `Life Expectancy at Birth, both sexes (years)` using the Spearman correlation coefficient:

[69]:
```
data_train = data_train.drop(
        columns=[candidate_variable]
)

correlations = (
        data_train
        .corr(method='spearman')[target_variable]
        .abs().sort_values(ascending=False)
)
correlations
```

[69]:
```
Life Expectancy at Birth, both sexes (years)
1.000000
Gross National Income Per Capita (2017 PPP$)
0.864828
Median Age, as of 1 July (years)
0.863765
Crude Birth Rate (births per 1,000 population)
0.848640
Expected Years of Schooling, female (years)
0.834567
Coefficient of human inequality
0.828904
Expected Years of Schooling (years)
0.819759
Total Fertility Rate (live births per woman)
0.816936
Adolescent Birth Rate (births per 1,000 women ages 15-19)
0.811920
Expected Years of Schooling, male (years)
0.806563
Material footprint per capita (tonnes)
0.789033
Net Reproduction Rate (surviving daughters per woman)
0.784511
```

Carbon dioxide emissions per capita (production) (tonnes)
0.762387
Rate of Natural Change (per 1,000 population)
0.746721
Mean Years of Schooling, female (years)
0.745441
Mean Years of Schooling (years)
0.742560
Mean Years of Schooling, male (years)
0.731290
Population with at least some secondary education, female (% ages 25 and older)
0.694487
Inequality in eduation
0.654956
Population with at least some secondary education, male (% ages 25 and older)
0.653522
Gender Development Index (value)
0.605025
Population Annual Doubling Time (years)
0.490936
Births by women aged 15 to 19 (thousands)
0.488655
Population Growth Rate (percentage)
0.485993
Sex Ratio at Birth (males per 100 female births)
0.437146
Natural Change, Births minus Deaths (thousands)
0.428493
Inequality in income
0.408609
Crude Death Rate (deaths per 1,000 population)
0.395167
Net Migration Rate (per 1,000 population)
0.378329
Net Number of Migrants (thousands)
0.371957
Share of seats in parliament, male (% held by men)
0.327508
Share of seats in parliament, female (% held by women)
0.327508
Population Density, as of 1 July (persons per square km)
0.297218
Population Change (thousands)
0.285238
Births (thousands)
0.274848
Live births Surviving to Age 1 (thousands)

```
0.264739
Labour force participation rate, male (% ages 15 and older)
0.231725
Year
0.220283
Labour force participation rate, female (% ages 15 and older)
0.193560
Female Deaths (thousands)
0.139715
Total Deaths (thousands)
0.138426
Male Deaths (thousands)
0.137720
Mean Age Childbearing (years)
0.108880
Female Population, as of 1 July (thousands)
0.051371
Total Population, as of 1 July (thousands)
0.047173
Total Population, as of 1 January (thousands)
0.045615
Male Population, as of 1 July (thousands)
0.043831
Population Sex Ratio, as of 1 July (males per 100 females)
0.043340
Country
0.014885
Name: Life Expectancy at Birth, both sexes (years), dtype: float64
```

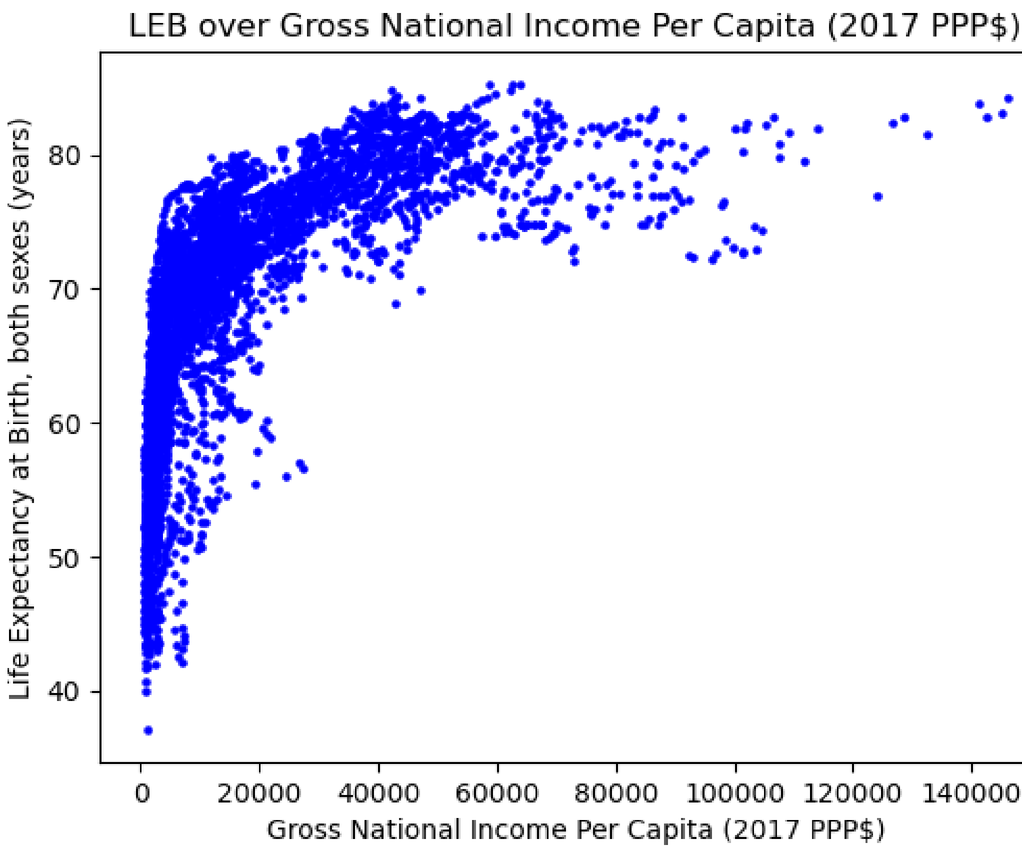
After choosing `Gross National Income Per Capita (2017 PPP$)` as candidate, we plot the relationship:

[70]:
```
second_candidate_variables = 'Gross National Income Per Capita (2017 PPP$)'

plt.scatter(
        data_train[second_candidate_variables],
        data_train[target_variable],
        color='blue', s=5
)
plt.title(
        f'LEB over {second_candidate_variables}'
)
plt.xlabel(second_candidate_variables)
plt.ylabel(target_variable)

plt.savefig(
        "scatter_plot_nonlinear_mono_model.png"
```

```
)
```



The relationship appears to be logarithmic.

3.0.2 Construct the model on the transformed scale

Applying the logarithmic transformation to the candidate variable:

[71]:
```
import numpy as np

X_train = data_train[[second_candidate_variables]]
y_train = data_train[target_variable]

log_X_train = np.log(X_train)

model = LinearRegression()
model.fit(log_X_train, y_train)
```

[71]:
```
LinearRegression()
```

Computing the metrics of the model:

[72]:

```
r_squared = model.score(log_X_train, y_train)
coefficients = model.coef_
intercept = model.intercept_

print(f'R-squared: {r_squared}')
print(f'Coefficients: {coefficients}')
print(f'Intercept: {intercept}')
```

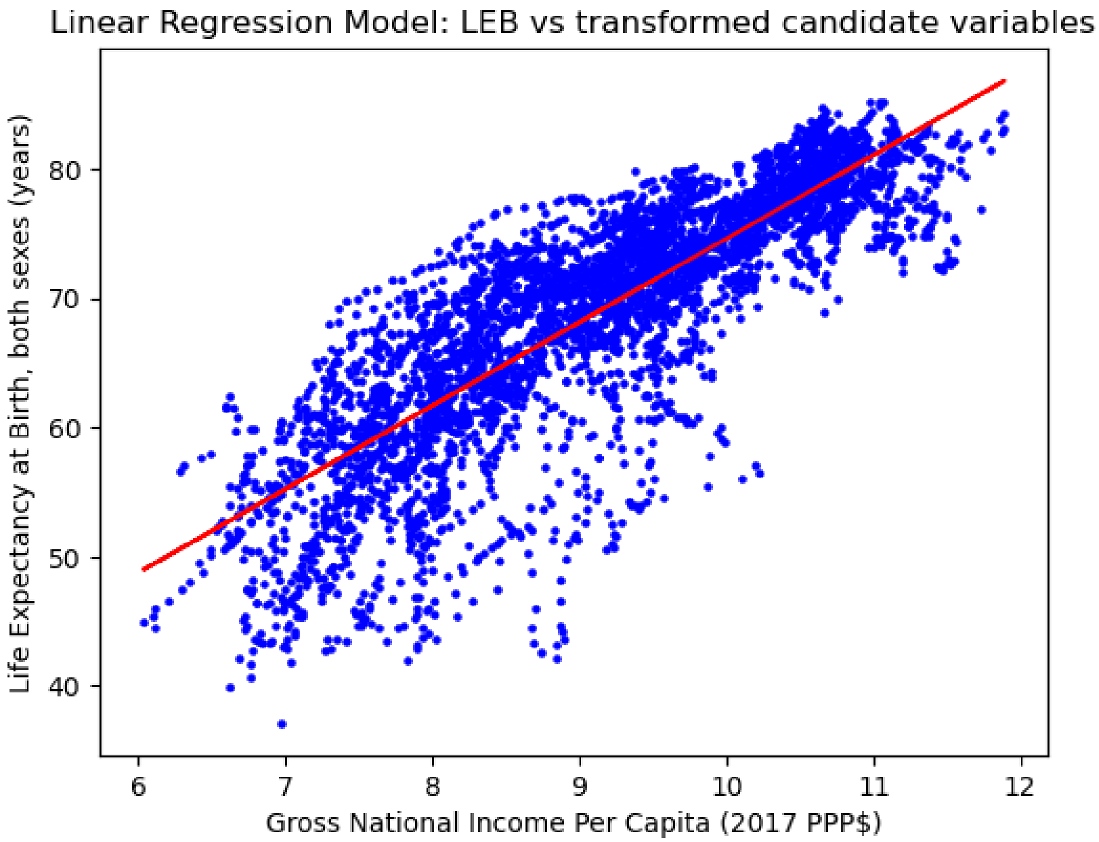
```
R-squared: 0.6939267719160521
Coefficients: [6.46720934]
Intercept: 9.942320920335249
```

Plotting the linear regression model:

[73]:

```
plt.scatter(
        log_X_train, y_train,
        color='blue', s=5
)
plt.plot(
        log_X_train, model.predict(log_X_train),
        color='red'
)
plt.title(
        'Linear Regression Model: LEB vs transformed candidate variables'
)
plt.xlabel(second_candidate_variables)
plt.ylabel(target_variable)

plt.savefig(
        "scatter_plot_nonlinear_mono_transformed_model.png"
)
```



3.0.3 Comparing the transformation

Computing the correlation coefficient before and after the transformation

[74]:
```
original_correlation, _ = (
        pearsonr(X_train.values.flatten(), y_train)
)
transformed_correlation, _ = (
        pearsonr(log_X_train.values.flatten(), y_train)
)

print(f'''
Original correlation:    {original_correlation}
Transformed correlation: {transformed_correlation}
''')
```

```
Original correlation:    0.6514708331957302
Transformed correlation: 0.833022671909986
```

4 Problem 4: Multiple linear regression model

4.0.1 Systematically research the candidates

Apply a systematic search to identify the variables with the strongest relationship with the target variable `Life Expectancy at Birth, both sexes (years)` without using the most correlated variable.

For this task, it has been used the `SelectKBest` class with the regression score function from `sklearn`:

[75]:
```
from sklearn.feature_selection import SelectKBest, \
        f_regression

data_train = data_train.dropna()
data_test = data_test.dropna()

X_train = data_train.drop(
        columns=[target_variable]
)
y_train = data_train[target_variable]

selector = SelectKBest(f_regression, k=5)
selector.fit(X_train, y_train)

candidates = (
        X_train.columns[selector.get_support()]
)

candidates_list = "\n".join(candidates)
print(f'Selected features: \n{candidates_list}')
```

```
Selected features:
Coefficient of human inequality
Median Age, as of 1 July (years)
Rate of Natural Change (per 1,000 population)
Crude Birth Rate (births per 1,000 population)
Total Fertility Rate (live births per woman)
```

4.0.2 Construct the model

Construct a linear regression model using the found candidates with the target variable:

[76]:
```
X_train = data_train[candidates]

model = LinearRegression()
model.fit(X_train, y_train)

r_squared = model.score(X_train, y_train)
coefficients = model.coef_
```

```
intercept = model.intercept_

print(f'R-squared: {r_squared}')
print(f'Coefficients: {coefficients}')
print(f'Intercept: {intercept}')
```

```
R-squared: 0.9703153322838306
Coefficients: [-0.09895655  0.70923533  1.70270563 -2.13815957  2.98240358]
Intercept: 67.97955121326358
```

4.0.3 Predict the test set

Predict the target variable using the test data and computing the **mean squared error** and **correlation coefficient**:

[77]:
```
X_test = data_test[candidates]
y_test = data_test[target_variable]
y_pred = model.predict(X_test)

mse = mean_squared_error(y_test, y_pred)
correlation, _ = pearsonr(y_pred, y_test)

print(f'Mean squared error: {mse}')
print(f'Correlation: {correlation}')
```

```
Mean squared error: 2.2279402053130903
Correlation: 0.9854073119079798
```