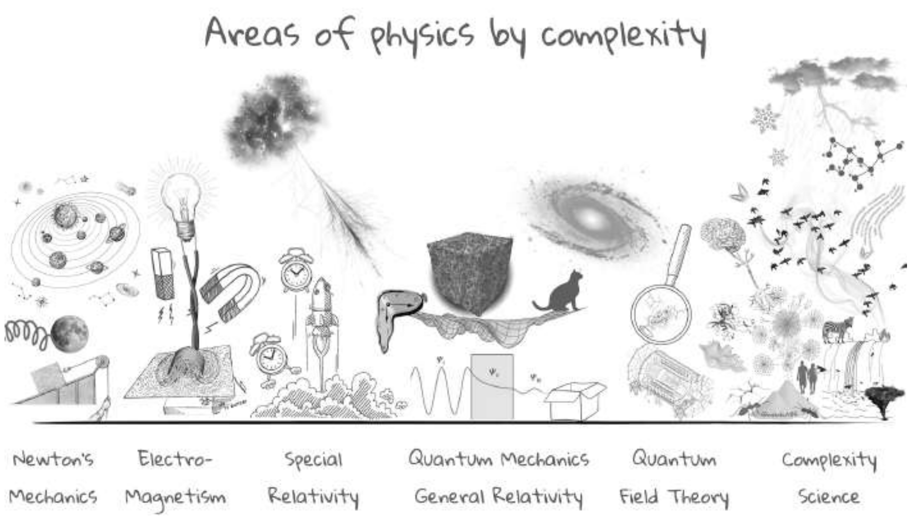
Università degli Studi di Padova



Projects # 40 & 24

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Final Report

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1.1 | Task definition

The objective of this task is to extract meaningful information from a dataset of raw data describing the topology of the subways of different big cities: Barcelona, Beijin, Berlin, Chicago, Hong Kong, London, Madrid, Mexico City. The final goal is to create two files one for nodes including the following information: *nodeID, nodeLabel, latitude, longitude, year* and one for the edges with: *nodeIDfrom, nodeIDto, line, year*.
The whole process can be defined as data cleaning and extraction, taking this into account it has been choosen a python framework to manipulate and extract files.

1.2 | Raw Data structure

Here is an example of the structure of the data provided

```
raw data
└─city
   ├─stations dataset (txt)
   ├─secondary station dataset with different infos (txt)
   ├─dedicated line files (txt)
   ├─extra files...  (txt/dat/csv)
   └─Topologies
      └─subway topology over the years (txt/net/mat)
```

To pursue the final task, it was sufficient to identify the minimal set of files and extract the necessary information from them. This initial step was carried out simply by opening the files. The information about the nodes was entirely contained within the one/two station datasets, while the information about the edges was found in the topology folder, with a few exceptions that will be discussed later.

1.2.1 Data issues

For the second file, the main challenge was to determine, for each station, the subway line it belonged to across different years. Regarding the line information, we can distinguish two cases: in Beijing and Barcelona, the line data were stored within the secondary station dataset, while in the second case, all line information was contained in the dedicated line files. and finally the Chicago data where totally lacking of the topology, so the line information across the years.

1.3 Data extraction

Once defined, the set of files needed was copied, renamed, and moved into another folder with a more defined structure to easily extract information from them, the new structure is the following:

```
data
└─city
  ├─stations_dataset(s) (.txt)
  ├─lines
  │ ├─line_i (.txt)
  │ └─...
  └─topologies
    ├─topology_year_XXXX (.txt/.net/.mat)
    └─...
```

After this reassessment of the files with some python functions the folders where esplored end the data where extracted in dictionaries as follow:

- Nodes file informations:
  - *nodeLabel, latitude, longitude, year* extracted from `station_dataset.txt`
  - *nodeID* defined by python function
- Edges file informations:
  - *nodeIDfrom, nodeIDto, year* extracted from `city-yearXXXX-adjecency-number.txt`
  - *line* defined by matching stations in `line_i` files

1.4 final files

The final files with all the information are in JSON format obtained from the dictionary previusly, with the following structure:

City_nodes.json

```
id :{
      "label" : ...,
      "lat"   : ...,
      "lon"   : ...,
      "year"  : ...,
      "ID"    : ...
    }
```

City_edges.json

```
year :[
    {
      "id_from" : ...,
      "id_to"   : ...,
      "lines": [...]
    },
    ...
    ]
```

`id` and `year` can be use as a key for extracting information. Above an example of graphs obtained from this two files for Madrid

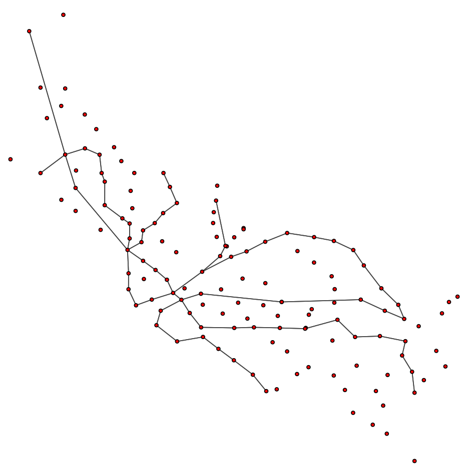

Figure 1.1: Madrid subways 1940

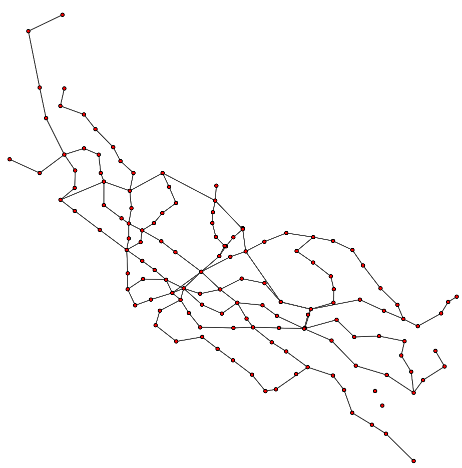

Figure 1.2: Madrid subways 2010

2 | Entanglement percolation

The study of a quantum networks can be an exciting topic for the future development of new technologies. In this report are shown some interesting results regarding entanglement percolation presented in [3] on some classical topologies and on a real geometric network.

2.1 | Theorical background

We can define a quantum complex network (QCN) as a set of nodes storing some qbits connected by edges that encode the entanglement between the qbits of two nodes [2]. In this framework each partially entangled edge is defined by the two state vector

$$|\psi\rangle = \sqrt{\lambda_0}|00\rangle + \sqrt{\lambda_1}|11\rangle$$

Each partially entangled state $|\psi\rangle$ can be converted into a maximally entangled by LOCC, with the singlet conversion probability (SCP) $p = \min\{1, 2(1-\lambda_0)\}$, this probability can change depending on the use of quantum distillation which is the shares of two identical copies of qbits, in this case the SCP becomes $p_2 = 2p - p^2$. This intrinsic undeterministic nature of a QCN is the perfect framework to study percolation behaviour.

Let us start with an initial graph whose edges represent partially entangled edges. In entanglement percolation, the goal is to study how the size of the giant connected component (GCC) changes as the singlet conversion probability (SCP) varies, where an edge is considered present only when a maximally entangled state is obtained. The percolation can be significantly altered in his critical probability p_c by LOCC performing "q-swaps" i.e. a swap in entanglment from a "star-tipe" into "ring" tipe, as shown in fig 2.1. Note that the q-swap can only be made between non adiacent nodes, and once performed the new partially entangled edges has an SPC of p on avarage. By Targeting some specific nodes, it is possible to alter the topology of the initial graph and reduce the critical probability p_c.

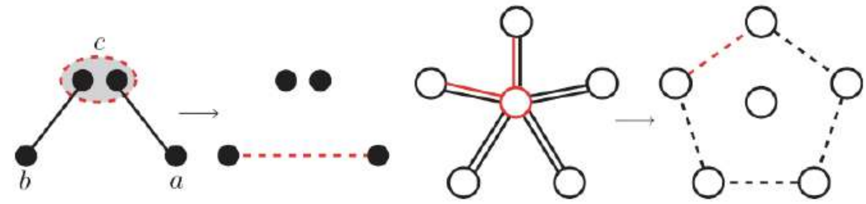

Figure 2.1: (*left*) entanglement swap from $(b-c, a-c)$ to $a-b$, (*right*) entanglement swap performed on an hub: the edges of the hub are disconnected and the neighbors are connected in a ring-like structure

2.2 Studied models and metodology

The percolation process can be investigated by first constructing either a synthetic or a real graph, and then performing edge selection using the SPC as the probability. We can then compare two cases: one in which the graph has been modified through the application of qswap, and the other in which it remains unmodified. The node targeting strategy can be very different, in general in this report it has been choosen to target the node as in the reference paper, it would be interesting to study the best strategy for different topology. All the analisis are performed with python.

2.2.1 Synthetic models

The models considered are the Erdős–Rényi and the Watts–Strogatz networks. In both cases, is applied q-swap strategy with different replicas of the same network, thus always increases the probability of forming a giant connected component (GCC) 2.2, 2.3. For the Erdős–Rényi model, an analytical solution can be derived with some approximation. In general, an effective approach appears to be targeting low-degree nodes. It would be interesting to further investigate the optimal strategy as a function of network characteristic and node centrality.

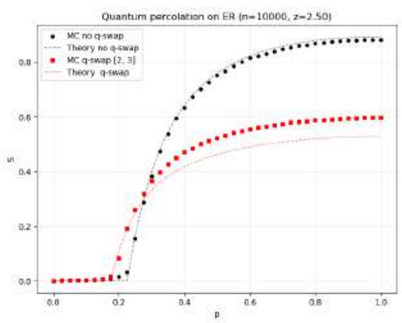

Figure 2.2: Entanglement percolation on an Erdős–Rényi graph by targeting nodes with degree 2 and 3

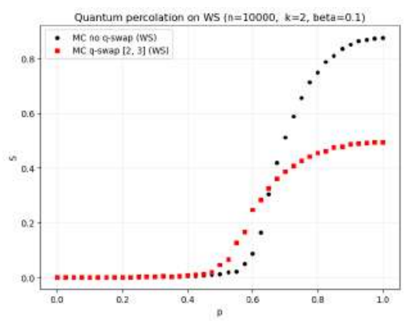

Figure 2.3: Entanglement percolation on a Watts–Strogatz graph by targeting nodes with degree 2 and 3

2.2.2 Real graph

The same methodology is applied to simulate an Italian quantum network. A dataset of Italian cities was obtained from [1], and for the analysis we selected all cities with more

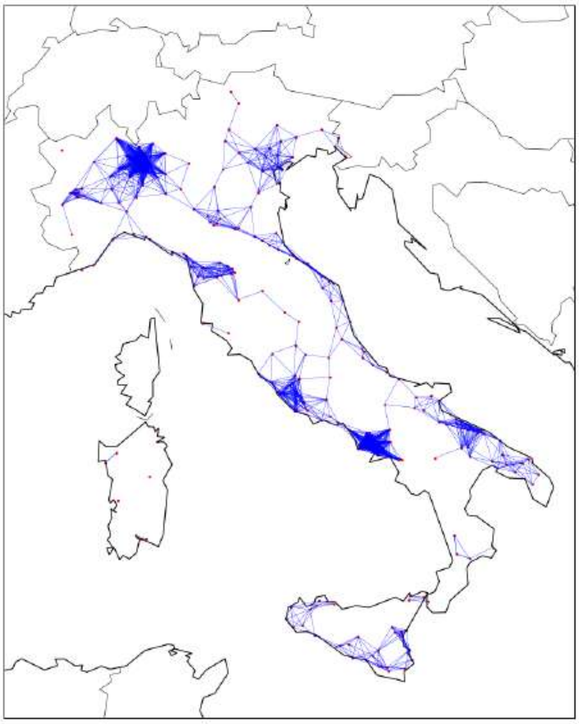

Figure 2.4: graph of italy with $r_{max} = 80$km

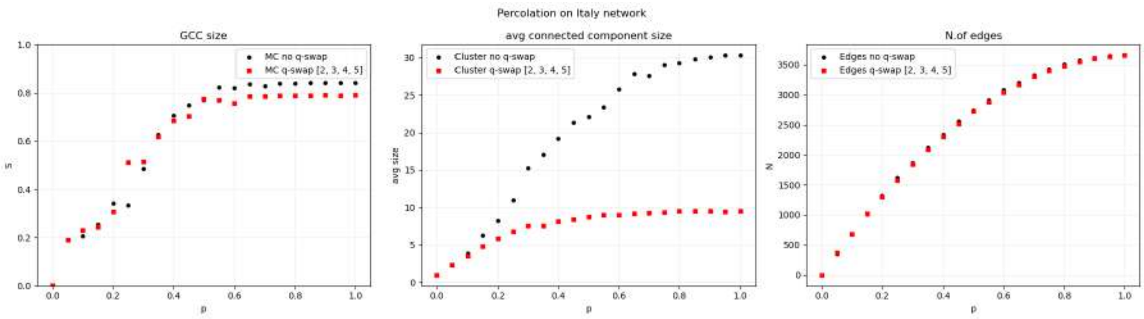

Figure 2.5: Percolation analysis in the Italian graph

than 25,000 inhabitants. The network graph was then constructed by connecting nodes whose mutual distance is less than 80 km. This threshold accounts for information loss over fiber links: while practical limits can extend up to 100 km, we adopt a conservative estimate and set the maximum distance to 80 km.

The results do not indicate any improvement in the size of the largest component; however, they do reveal a significant decrease in the average cluster size. A more detailed investigation using real network data would be a valuable direction for further assessing the effectiveness of this methodology.

3 | Bibliography

[1] Geonames - all cities with a population > 1000. https://public.opendatasoft.com/explore/dataset/geonames-all-cities-with-a-population-1000/export/?flg=en-us&utm_source=chatgpt.com&disjunctive.cou_name_en&sort=name. [Accessed sep-2025].

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[3] Martí Cuquet and John Calsamiglia. Entanglement percolation in quantum complex networks. *Phys. Rev. Lett.*, 103:240503, Dec 2009. doi: 10.1103/PhysRevLett.103.240503. URL https://link.aps.org/doi/10.1103/PhysRevLett.103.240503.