deep se
dependable evolvable pervasive software engineering group

Exercise Session – Complex Numbers

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$$z = a + ib = \rho\left[\cos(\vartheta) + i\sin(\vartheta)\right]$$

Modulus: $\rho = \sqrt{a^2 + b^2}$

Phase: $$\vartheta = \begin{cases} \arctan\left(\frac{b}{a}\right) & \text{if } a > 0 \\ \arctan\left(\frac{b}{a}\right) + \pi & \text{if } a < 0 \text{ and } b \geq 0 \\ \arctan\left(\frac{b}{a}\right) - \pi & \text{if } a < 0 \text{ and } b < 0 \\ \frac{\pi}{2} & \text{if } a = 0 \text{ and } b > 0 \\ -\frac{\pi}{2} & \text{if } a = 0 \text{ and } b < 0 \\ \text{indeterminate} & \text{if } a = 0 \text{ and } b = 0 \end{cases}$$

Operations

$$s = l + r = (a_l + ib_l) + (a_r + ib_r) = (a_l + a_r) + i\,(b_l + b_r)$$

$$d = l - r = (a_l + ib_l) - (a_r + ib_r) = (a_l - a_r) + i\,(b_l - b_r)$$

$$p = l \cdot r = (a_l + ib_l)\,(a_r + ib_r) = (a_l a_r - b_l b_r) + i\,(b_l a_r + a_l b_r)$$

$$f = \frac{l}{r} = \frac{(a_l + ib_l)}{(a_r + ib_r)} = \frac{(a_l a_r + b_l b_r) + i(b_l a_r - a_l b_r)}{a_r^2 + b_r^2}$$

Operations in polar form

$$p = l \cdot r = \rho_l \rho_r \left[\cos(\vartheta_l + \vartheta_r) + i \sin(\vartheta_l + \vartheta_r)\right]$$

$$f = \frac{l}{r} = \frac{\rho_l}{\rho_r} \left[\cos(\vartheta_l - \vartheta_r) + i \sin(\vartheta_l - \vartheta_r)\right]$$

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