THE STRUCTURE OF THE NUMBER OF REPRESENTATIONS FUNCTION IN A BINARY QUADRATIC FORM[†]

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This paper contains, primarily, the extension to any integral, binary quadratic form of the results of a recent article[*] concerning positive, binary, quadratic forms. With suitable conventions almost all the results carry over without change, though some of the proofs need slight alterations. Incidentally, there are treated automorphs of binary quadratic forms, and (rather fully) properties of sets of representations (representations equivalent through automorphic transformations) in a binary quadratic form.

1. Dirichlet[†] has already in all essentials extended the notion of number of representations to indefinite forms. We shall utilize the following equivalent definition.

Two representations (x, y) and (x', y') of m in the form $f = [a, b, c]$,[‡] that is, two integral solutions of

$$ax^2 + bxy + cy^2 = m, \tag{1}$$

will be called *equivalent* if they are transformable one into the other by integral automorphs of f. The class of all representations equivalent to a given one will be called a *set* of representations. The number of sets of representations of m in f will be denoted by $f(m)$. (In MZ, $f(m)$ denoted the number of representations of m in f.)

This definition becomes more interesting when we observe that, if $d (= b^2 - 4ac) > 0$, the number of sets of representations of m in $[a, b, c]$ is equal to the actual number of solutions of (1) together with certain inequalities (cf. Theorem 9). The writer developed the theory of these inequalities before noticing that Dirichlet (§87, loc. cit.) obtains one such system. However, we shall obtain a substantial improvement on Dirichlet's inequalities and give a more complete discussion of the infinitely many alternative systems. The treatment in §3 is fairly comprehensive.