Heat Exchanger Free Design

MASTER'S DEGREE IN THERMAL ENGINEERING

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Definition of the Problem

An air-to-water heat exchanger is given with all lateral walls well insulated and considered adiabatic. The geometry of the device is illustrated in the following figure:

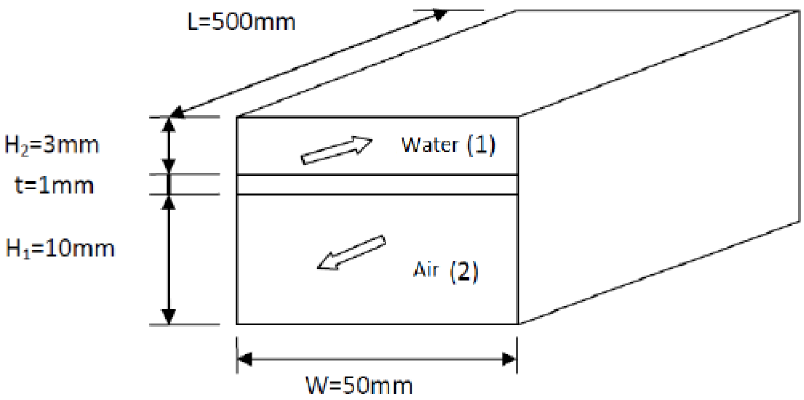

Figure 1. Geometry of the given heat exchanger.

Table 1. Inlet temperature and frontal velocity of the two fluids.

	T_i $[°C]$	v_i $[m/s]$
Water (1)	95	1
Air (2)	15	5

Additionally, the inlet conditions of the two fluids are given and summarized in *Table 1*, the intermediate wall is made of steel (thermal conductivity assumed $\lambda_w = 43$ $[W/m \cdot K]$).

It is asked to study the optimal design of the device through the implementation of extended surfaces made of steel, likewise the intermediate wall.

Rating of Initial Design

A step-by-step procedure is suggested to achieve the best results. As a starting point, the heat transfer capacity and pressure drop along the heat exchanger are obtained conducing a rating study of the device. In fact, knowing the geometry and the inlet conditions it is possible to evaluate the outlet conditions of the two fluids using the $\epsilon - NTU$ analytical method. The given problem is a counter-flow configuration, consequently the following set of equations has been used:

$$\dot{Q} = \dot{m}_1 c_{p_1}\left(T_{1_i} - T_{1_o}\right) \qquad \dot{Q} = \dot{m}_2 c_{p_2}\left(T_{2_o} - T_{2_i}\right) \qquad \dot{Q} = \epsilon\left(\dot{m}c_p\right)_{min}\left(T_{1_i} - T_{2_i}\right)$$

$$\epsilon = \frac{1 - e^{-NTU(1-Z)}}{1 - Ze^{-NTU(1-Z)}} \qquad NTU = \frac{U_o A_o}{C_{min}} \qquad Z = \frac{C_{min}}{C_{max}}$$

U_o is the overall heat transfer coefficient, it is a key parameter in this problem since it highlights how the heat transfer changes with the implementation of fins or a change in geometry.

$$U_o = \left[\frac{1}{\alpha_{water}} + \frac{t}{\lambda_w} + \frac{1}{\alpha_{air}}\right]^{-1}$$

All the thermophysical properties of the two fluids have been evaluated thanks to the scientific library of CoolProp [1] and updated through an iterative process to assure precise results. In particular, to evaluate the convective heat transfer coefficients of the two fluids it was necessary to first identify the flow regime. Then the Reynolds number has been calculated:

$$Re = \frac{\rho \bar{v} D_h}{\mu} \qquad D_h = \frac{4S}{P_{wet}}$$

$$Re_{water} = 18048 > 4000 \rightarrow Turbulent \qquad Re_{air} = 5634 \rightarrow Turbulent$$

Both flows turned out to be in turbulent regime, consequently it was possible to evaluate the mean Nusselt number $\overline{Nu}$:

$$\overline{Nu} = C Re^m Pr^n K \qquad Pr = \frac{\mu c_p}{\lambda}$$

Where Pr is the Prandtl number and C, m, n, K are constants that depends on the fluid and flow regime. In this case:

		C	m	n	K
Air	Turbulent flow of gasses Re > 2000	0.023	0.8	0.4	1
Water	Turbulent flow of liquids Re > 2000, 0.6 < Pr < 100	0.027	0.8	0.33	$\left(\frac{\mu}{\mu_w}\right)^{0.14}$

Finally, the mean heat transfer coefficient α has been obtained.

$$\bar{\alpha} = \frac{\overline{Nu} \cdot \lambda_{fluid}}{D_h}$$

Moreover, the pressure drops inside both ducts have been evaluated with the following equation, integrating the shear stress $\bar{\tau}_w$ calculated with the Swamee - Jain correlation, considering the duct smooth ($\varepsilon_r \approx 0$) :

$$\Delta p_d = \frac{\dot{m}(v_o - v_i) + \bar{\tau}_w A}{S} \qquad \bar{\tau}_w = f \cdot \frac{\rho \bar{v}^2}{2} \qquad f = \frac{0.0625}{\left[\log_{10}\left(\frac{\varepsilon_r}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

All the correlations used to evaluate properties were taken from Section B2 (Forced convection inside ducts; liquids and gases at low Mach number) and Section B7 (Friction factors for flows inside ducts) of [2] and adapted to the study case.

Finally, the results obtained are resumed in Table 2.

Table 2. Prediction study results of the given heat exchanger.

	$\dot{m}\ [kg/s]$	$T_o\ [°C]$	$\dot{Q}\ [W]$	$\Delta p_d\ [Pa]$
Water (1)	0.144	94.91	- 53.44	528.92
Air (2)	0.003	32.87	53.44	6.87

Design Improvement

To improve the performance of the given device it is suggested to add extended surfaces. The procedure starts with the design of a suitable single fin for the study problem, keeping the same overall size and length of the heat exchanger.

An important point to consider when adding fins is in which fluid install them. The answer has been deducted analyzing the overall heat transfer coefficient U_o.

Table 3. Summary of the resistance of the system.

$U_0\ [W/m^2K]$	$R_1\ [m^2 \cdot K/W]$	$R_w\ [m^2 \cdot K/W]$	$R_2\ [m^2 \cdot K/W]$
30.25	$9.85 * 10^{-5}$	$2.33 * 10^{-5}$	$3293.03 * 10^{-5}$

In *Table 3* it is shown clearly that the dominant thermal resistance is the one of air, for this reason fins have been studied to be implemented in the air channel. Adding fins to a heat exchanger enhances its thermal performances by increasing the heat transfer surface area without significantly impact the device. This is particularly beneficial for fluids with low thermal conductivity or low convective heat transfer coefficient, such as air. Fins improve the overall heat transfer coefficient; however, they can increase pressure drop or negatively impact on convection and add complexity and cost to manufacturing.

Single Fin Design

As a first approach, a single fin has been designed and its effect on the heat exchanger has been evaluated. The geometry chosen for the fin is rectangular and, for simplicity of calculations, extended all along the duct. The thermal conductivity of the fin is the same as the intermediate wall as they're both made of steel and the size of the HX is kept constant.

Table 4. Geometry of a single fin.

$e_f\ [m]$	$h_f\ [m]$	$\lambda_f\ [W/m \cdot K]$
0.0005	0.006	43

To evaluate the effect of this single fin, the new overall heat transfer coefficient has been calculated according to the following equation:

$$U_{o_f} = \left(\frac{1}{\eta_o \alpha_{air}} + \frac{e}{\lambda_w} + \frac{1}{\alpha_{water}}\right)^{-1}$$

Where η_o is the overall surface efficiency, which considers the added surface of the fin.

$$\eta_o = 1 - \frac{A_f}{A_{o_{new}}} \cdot (1 - \eta_f) \qquad A_{o_{new}} = A_{o_{eff}} + A_f \qquad A_f = 2 \cdot N \cdot h_f \cdot L$$

$$\eta_f = \frac{\tanh(m \cdot h_f)}{m \cdot h_f} \qquad m = \sqrt{\frac{2 \cdot \alpha_{air}}{\lambda_f \cdot e_f}} \qquad NTU_f = \frac{U_{o_f} \cdot A_{o_{new}}}{C_{min}}$$

Moreover, the Reynolds number and pressure drop equation have been computed updating the Hydraulic Diameter with the added surface of the fin.

Table 5. Summary of the results obtained adding 1 single fin.

η_f	U_{o_f} $[W/m^2K]$	$T_{o_{air}}$ $[°C]$	$\dot{Q}$ $[W]$	Δp_{air} $[Pa]$
0.95	30.54	36.82	64.83	9.05

This predesigned fin has been created to provide adequate efficiency to the system. In fact, *Table 5* shows how the heat transfer is increased by 21%, maintaining the water outlet temperature nearly the same. However, as expected, the pressure drop inside the air duct is increased by 32%. Then, 12 fins have been implemented to see how the system reacts.

Table 6. Summary of results obtained adding 12 fins of the same design.

η_f	U_{o_f} $[W/m^2K]$	$T_{o_{air}}$ $[°C]$	$\dot{Q}$ $[W]$	Δp_{air} $[Pa]$
0.95	33.66	69.85	154.49	42.36

Increasing the fin number greatly enhanced the performance of the heat exchanger, still increasing the pressure drop. Anyway, this value of pressure loss on the air side is generally not critical in most applications, but it should be evaluated in terms of fan compatibility, energy consumption, and overall system performance.

Optimization study

An optimization strategy is proposed to conclude the analysis of the given heat exchanger by balancing key design objectives. The Pareto front is employed as a powerful tool in this process, allowing the identification of the best trade-offs between competing objectives. In this analysis, it was used to study the optimal design of the air-to-water heat exchanger, focusing on maximizing heat transfer while minimizing pressure drop on the air side. This approach was implemented by systematically varying fin geometry parameters—such as fin spacing, thickness, and height—while maintaining the overall dimensions of the heat exchanger. The results were visualized by plotting the heat exchanged versus the pressure drop for various configurations, highlighting the set of most efficient design solutions that form the Pareto front.

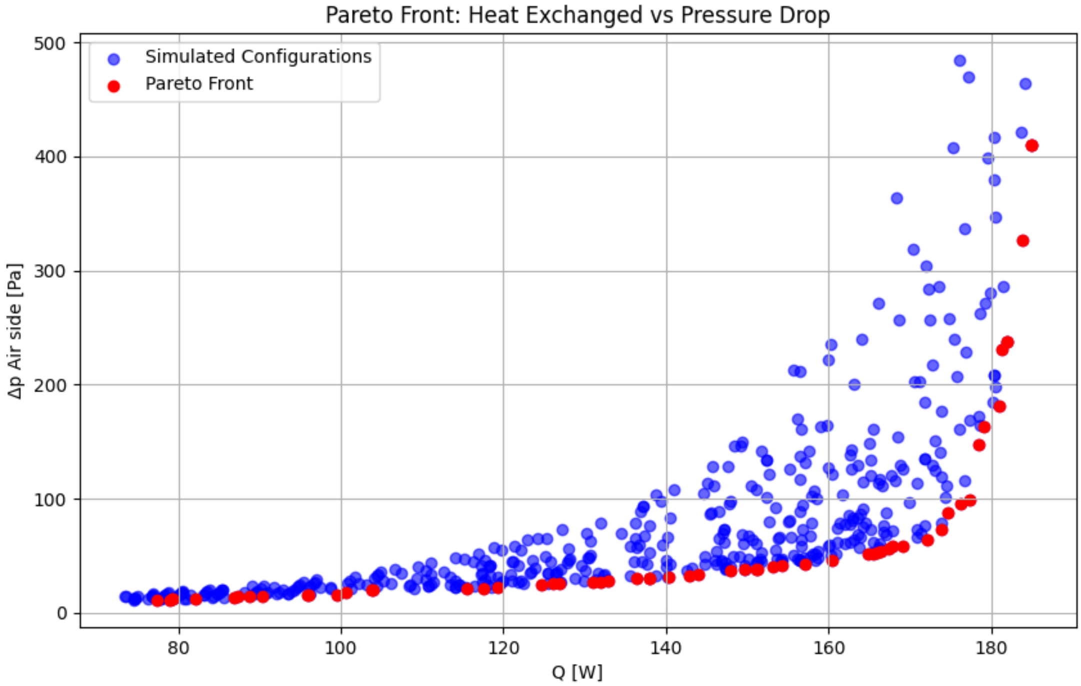

Figure 2. Pareto Front obtained implementing variable fins on the air duct of the heat exchanger.

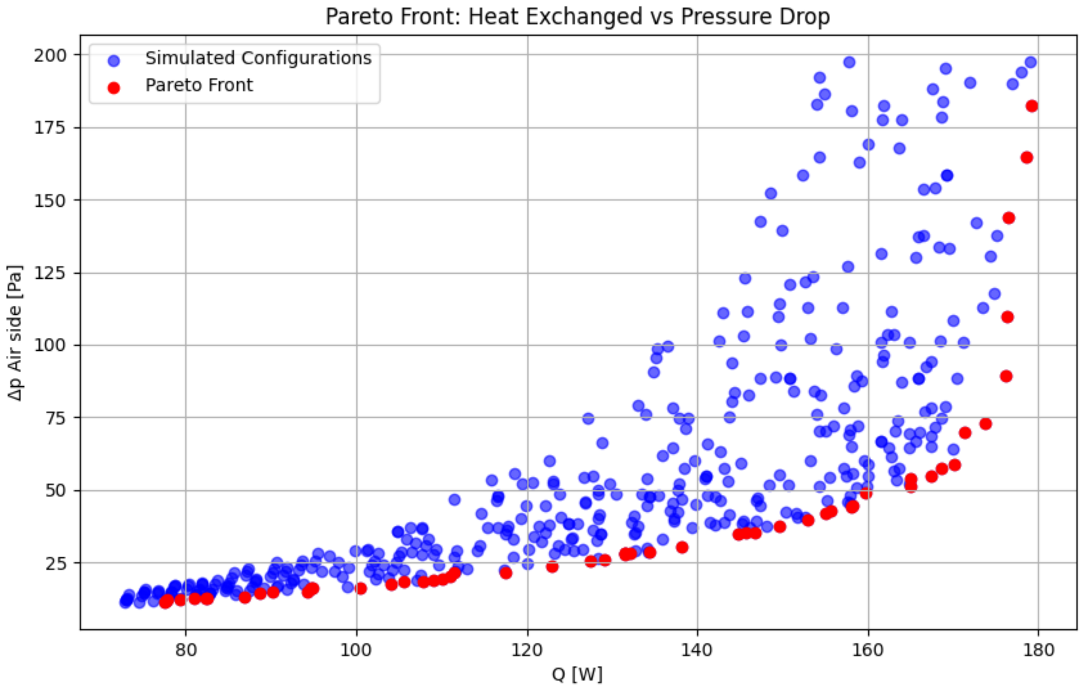

Figure 3. Zoom of Pareto Front for $\Delta p < 200\ Pa$.

The progression of red points in *Figure 3* shows the expected trade-off: as $\dot{Q}$ increases, Δp also tends to increase, indicating the aerodynamic penalty associated with higher heat transfer performance. The plot illustrates the impact of systematically varying the fin parameters: fin heights ranging from 1 mm to 10 mm, fin thicknesses from 0.1 mm to 4 mm, and the number of fins between 10 and 100.

As observed, configurations with lower pressure drops (e.g., below 50 Pa) tend to correspond to designs with fewer or thinner fins, as these reduce airflow resistance but limit the enhancement in heat transfer area. Conversely, designs achieving higher heat exchange rates (above 140 W) are likely associated with taller or more numerous fins, which maximize heat transfer by significantly increasing the available surface area, but this comes with a significant increase in pressure drop.

Conclusions

This study successfully analyzed and optimized the design of an air-to-water heat exchanger by adding fins to the air side. The analysis started with an evaluation of the initial design, which showed that the air-side resistance was the main factor limiting heat transfer. To improve performance, fins were added to increase the heat transfer area and coefficient. Different fin parameters, such as height, thickness, and number of fins, were tested to find the best balance between heat transfer and pressure drop.

The Pareto front was used to identify the optimal solutions, showing the trade-off between heat transfer and pressure drop on the air side. The results showed that increasing the heat exchanged also caused an increase in the pressure drop. Designs with smaller or fewer fins kept the pressure drop low but offered less improvement in heat transfer. On the other hand, taller or more numerous fins provided better heat transfer but caused higher pressure drops.

This study shows the importance of choosing the right design based on specific performance needs. The Pareto front makes it possible to select designs that offer the best balance between gains and losses, ensuring the efficiency of the system.

References

[1] C. Software, CoolProp (v6.4.1), 2024.

[2] H. a. M. T. T. C. (CTTC), "Formulae for the Resolution of Fluid Dynamics and Heat and Mass Transfer Problems," 2024.