Cascade Problem

MASTER'S DEGREE IN THERMAL ENGINEERING

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Objectives

In this second exercise about the use of CoolProp for thermodynamic calculations in vapor compression cycles the main idea was to play with the influence of parameters on the cycle and to observe impact. Initially, a fixed problem is proposed as a starting point to understand the working principles of a cascade cycle, followed by further investigations into key parameters.

Starting problem

The object of this analysis is a cascade refrigeration cycle for a cold storage unit, characterized by an ammonia top cycle and carbon dioxide bottom cycle. The starting data given are the following:

Table 1. Data given for primary fluids and efficiencies.

	Refrigerants	T_e [°C]	T_c [°C]	T_{sh} [°C]	T_{sc} [°C]	η_s	η_{me}
Top Cycle	NH_3	10	50	15	40	0.8	0.75
Bottom Cycle	CO_2	-25	20	-20	15	0.9	0.85

Table 2. Data given for secondary fluids.

	Secondary fluids	$\dot{m}$ [kg/s]	T_{in} [°C]	T_{out} [°C]
Top Cycle	Air	1	30	?
Bottom Cycle	Air	1	-18	-23

It is asked to analyze mass flow rate of the refrigerants, vapor mass fraction at the inlet of each evaporator, air outlet temperature of the top cycle, electric work spent on each compressor and to plot both cycles in the T-s diagram.

Methodology

To solve the problem, the same equations have been used for both cycles calculations but starting from the bottom cycle, with secondary fluid properties all known (see points numeration in Figure 1).

$$Q_e = \dot{m}_{air} \cdot c_{p_{air}}\left(T_{air_{in}} - T_{air_{out}}\right) \qquad \dot{m}_{ref} = Q_e(h_1 - h_4) \qquad h_2 = \frac{\left(h_{2_{iso}} - h_1 + h_1 \cdot \eta_s\right)}{\eta_s}$$

$$Q_c = \dot{m}_{ref}(h_2 - h_3) \qquad X_4 = \frac{\left(h_4 - h_{4_{sl}}\right)}{\left(h_{4_{sv}} - h_{4_{sl}}\right)} \qquad Q_{e_{Top}} = Q_{c_{Bottom}}$$

$$T_{air_{out}} = \frac{Q_c}{\dot{m}_{air} \cdot c_{p_{air}}} + T_{air_{in}} \qquad W_c = \dot{m}_{ref}(h_2 - h_1) \qquad W_e = \frac{W_c}{\eta_{me}}$$

$$COP = \frac{Q_{e_{Bottom}}}{W_{e_{Top}} + W_{e_{Bottom}}}$$

Both the expansion valves have been considered isenthalpic and, most important, the heat delivered by the condenser of the bottom cycle has been considered equal to the heat absorbed by the evaporator of the top cycle. In fact, the working principle of cascade refrigeration systems involves splitting the cooling process into two distinct cycles, each operating with a different refrigerant optimized for its temperature range. This design enables the system to efficiently achieve very low temperatures that would be impractical for a single-stage vapor-compression cycle, while distributing the work required for compression across both stages to maintain better performance.

Results obtained

The results obtained are aligned with the ones given.

Table 3. Starting problem's results obtained.

	$\dot{m}_{ref}$ $[kg/s]$	$X_{e_{in}}$	$T_{air_{out}}$ $[°C]$	W_e $[W]$	COP
Top Cycle	$5.979 \cdot 10^{-3}$	0.118	37.82	1763.55	3.71
Bottom Cycle	$2.452 \cdot 10^{-2}$	0.321	-	1788.93	2.81
Cascade					1.42

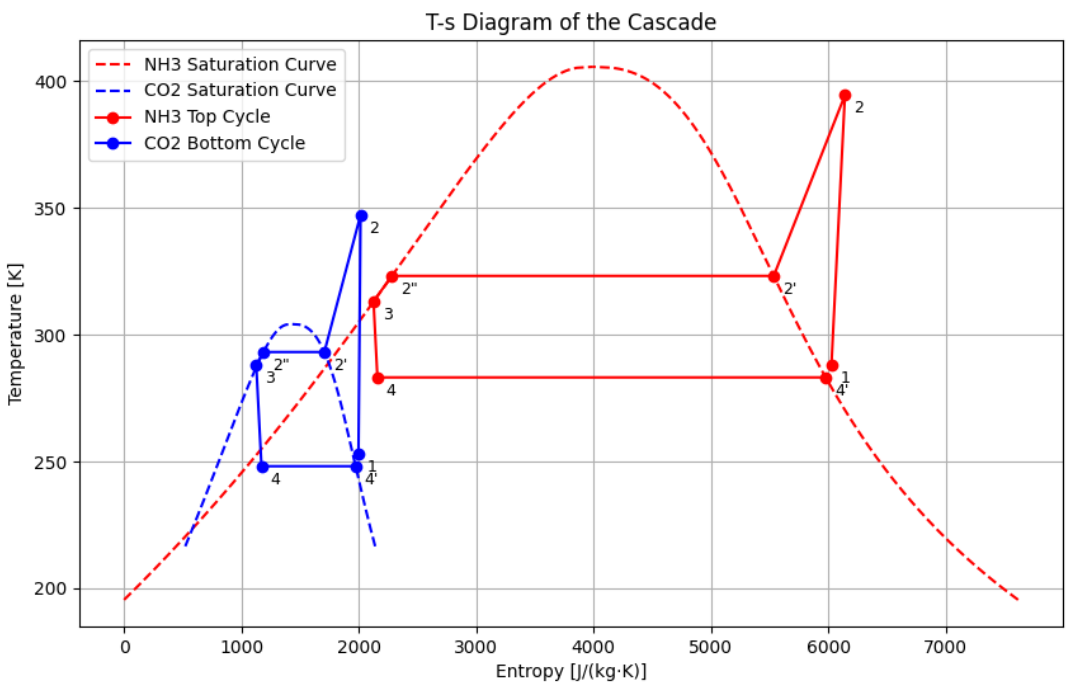

Figure 1. T-s Diagram of the Cascade System.

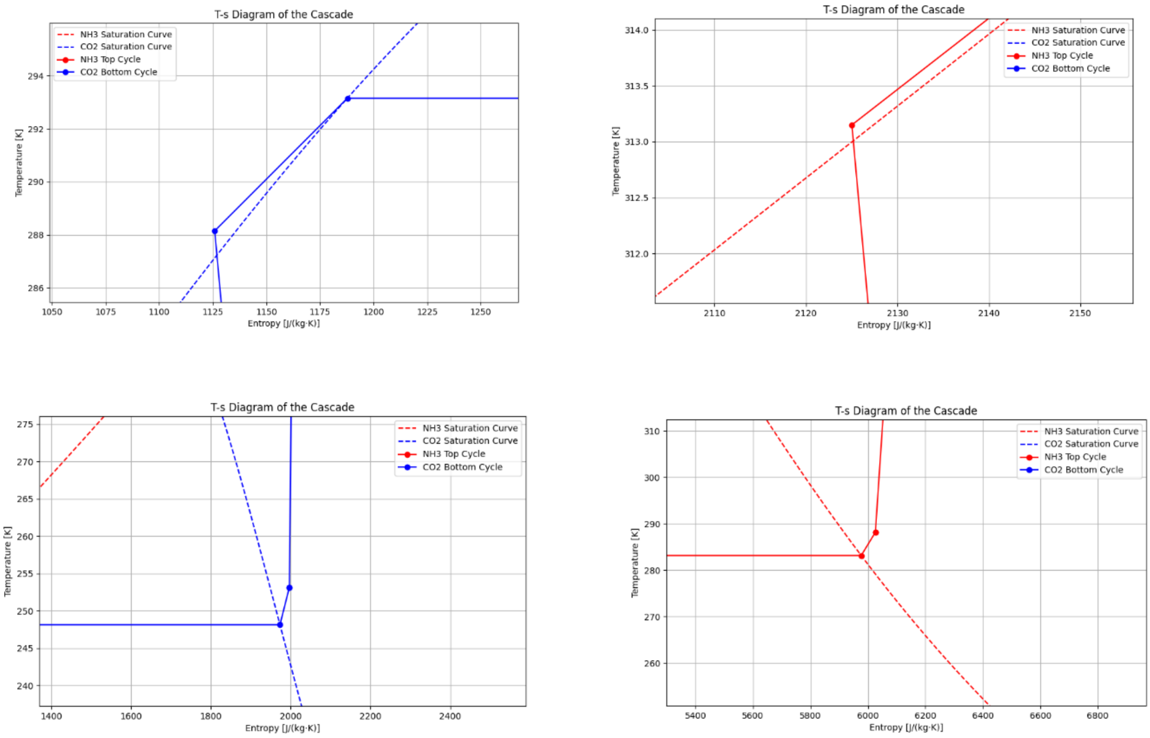

Figure 2. Zoom view of sub-cooling and super-heating of both Top and Bottom cycles.

Parametric study

The influence of key parameters has been analyzed to evaluate the system response to different environments, refrigerants or efficiencies.

Intermediate Condenser Temperatures

Firstly, the condensation temperature at the bottom and evaporation temperature at the top cycle have been changed according to the following table:

Table 4. Summary of the temperature changes inside the intermediate condenser.

	$T_{e_{Top}}$ [°C]	$T_{c_{Bottom}}$ [°C]	COP
Starting value	10	20	1.42
First change	10	15	1.57
Second change	15	20	1.53

The temperature changes have been done keeping constant the sub-cooling and super-heating temperature differences. In this way it was possible to avoid the over stretch of the cycle's diagram.

Additionally, the intermediate condenser must have a sufficient temperature difference, typically 5-10°C, to ensure effective heat exchange. Decreasing $T_{c_{Bottom}}$ of 5°C means reducing the work of the compressor of the bottom cycle, increasing its the coefficient of performance of 20%. Similarly, increasing $T_{e_{Top}}$ of 5°C means enhancing the COP of the top cycle of 19%. Both changes lead to a higher COP of the system but not significantly.

Cooling Demand

Another parameter that has been studied is the influence of the outlet temperature of the air of the bottom cycle. Decreasing this value means an increase in the cooling demand, to do this the air outlet temperature of the bottom cycle has been reduced to – 40 °C. The results showed a brutal increase in the cooling demand, from 5 kW up to 22 kW. Consequently, the refrigerants mass flow rate and the work required to the compressors resulted in a value more than 4 times higher than the initial one. The table shows how this temperature change leads to higher quantity of refrigerants and higher electrical consumption, which obviously means higher costs.

Table 5. Summary of the effects produced by the increase of the cooling demand.

	$\dot{m}_{ref}$ $[kg/s]$	W_e $[W]$
Top Cycle	$2.631 \cdot 10^{-2}$	7760
Bottom Cycle	$1.079 \cdot 10^{-1}$	7871

Different Refrigerants

Switching to R-1234ze for the top cycle and keeping CO_2 for the bottom cycle offers multiple benefits in terms of safety, performance, and environmental impact.

- R-1234ze is a low-GWP refrigerant (GWP = 6) that is non-toxic, non-flammable, and eco-friendly, making it a safe alternative to ammonia. It provides efficient performance while reducing environmental harm, helping meet sustainability goals.
- CO_2 is a natural refrigerant with a GWP of 1, offering low environmental impact, non-toxicity, and non-flammability. It works well in low-temperature applications and is highly efficient in cascade systems, making it an ideal choice for the bottom cycle.

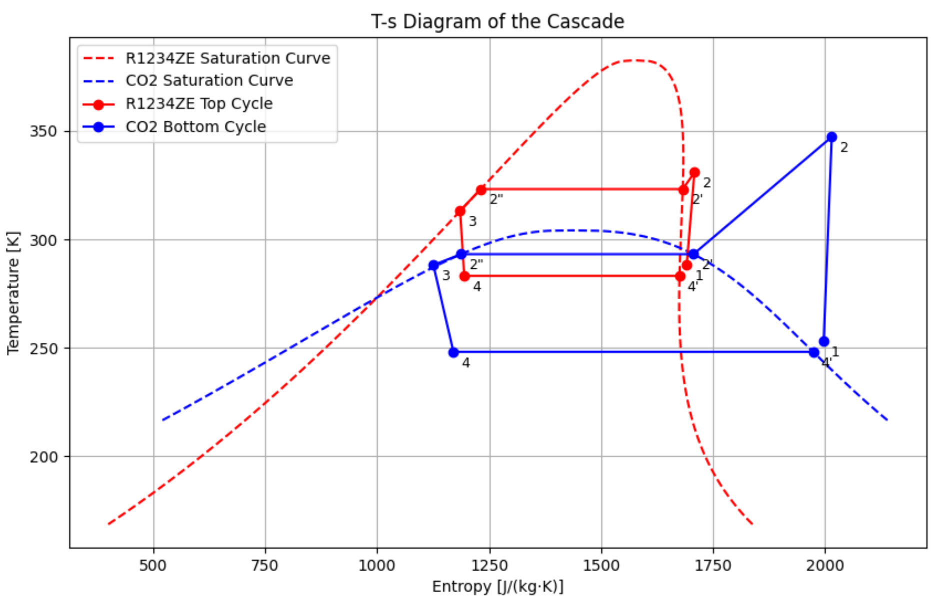

Figure 3. T-s Diagram of the Cascade System with R-1234ze(E) in the top cycle and CO_2 in the bottom cycle.

Compared to Ammonia, R-1234ze needs a lower pressure difference to achieve the same results, inducing a slightly higher COP, as shown in the following table:

Table 6. Comparison between NH_3 and R-1234ze in the Top Cycle.

	p_c $[bar]$	p_e $[bar]$	COP
NH_3	6.15	20.33	3.713
R-1234ze	9.97	3.10	3.729

Conclusions

The cascade refrigeration system effectively achieves low temperatures by splitting the cooling process into two optimized cycles, reducing compressor work and improving efficiency. Adjusting the intermediate condenser temperatures slightly enhances performance, with moderate gains in COP. Increasing cooling demand significantly raises refrigerant flow rates and energy consumption, highlighting cost implications. Replacing ammonia with R-1234ze in the top cycle improves safety, sustainability, and efficiency, while CO_2 remains an ideal choice for the bottom cycle due to its low environmental impact and high performance.