Control of Mobile Robots - Laboratory 1
Kinematics and Dynamics of mobile robots
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Exercise 1

Create four different ROS packages to simulate the following robots:

1. a unicycle mobile robot through its kinematic representation;
2. a bicycle mobile robot through its simplified kinematic representation, assuming an instantaneous change in the steering position;
3. a unicycle mobile robot through its dynamic representation;
4. a bicycle mobile robot through its simplified dynamic representation (using β, r, ψ, x, and y as state variables, and the usual approximation for trigonometric functions), considering a linear tyre model, or a Fiala tyre model with and without saturation.

Each package should include two different nodes:

1. the robot simulator, that takes as input a *std_msgs::Float64MultiArray*, representing the actual time and robot commands, and publishes a *std_msgs::Float64MultiArray*, representing the actual time and robot state;
2. a test node that publishes robot inputs, used to test the simulator.

For the unicycle dynamic model consider a mass of $2\,Kg$ and an inertia of $0.03\,Kgm^2$.
For the bicycle dynamic model consider a mass of $2\,Kg$, a yaw inertia of $0.03\,Kgm^2$, a front and rear cornering stiffness of $50\,N/rad$ and $122\,N/rad$, respectively, a position of the center of gravity from the front and rear axle of $0.14\,m$ and $0.12\,m$, respectively, a friction coefficient of 0.385.
Use simple commands (e.g., constant velocities or constant torques) to verify the correctness of each model.

Exercise 2

An eight-shaped trajectory can be described in parametric form as follows

$$
\begin{aligned}
x &= a \sin\left(\frac{2\pi}{T}t\right) \\
y &= a \sin\left(\frac{2\pi}{T}t\right) \cos\left(\frac{2\pi}{T}t\right)
\end{aligned}
$$

where the trajectory extends from $-a$ to a, and T is the time duration of each lap.
Modify the test nodes created in Exercise 1 using the eight-shaped trajectory to compute the robot inputs through the flatness transformation.

The time derivatives of x and y, that are needed to set up the flatness transformation should be analytically computed, using the following expression:

$$
\begin{aligned}
\dot{x} &= \frac{2\pi}{T} a \cos\left(\frac{2\pi}{T}t\right) \\
\ddot{x} &= -\left(\frac{2\pi}{T}\right)^2 a \sin\left(\frac{2\pi}{T}t\right) \\
\dot{y} &= \frac{2\pi}{T} a \left(\cos^2\left(\frac{2\pi}{T}t\right) - \sin^2\left(\frac{2\pi}{T}t\right)\right) \\
\ddot{y} &= -4\left(\frac{2\pi}{T}\right)^2 a \sin\left(\frac{2\pi}{T}t\right) \cos\left(\frac{2\pi}{T}t\right)
\end{aligned}
$$

The flatness transformation, using MATLAB notation, is instead given by

```
v     = sqrt(dx^2+dy^2);
omega = (dx*ddy-dy*ddx)/(dx^2+dy^2);
```

for the unicycle kinematic model, and

```
v     = sqrt(dx^2+dy^2);
phi   = atan(L*(dxref*ddyref-dyref*ddxref)/(dxref^2+dyref^2)^1.5);
```

for the bicycle kinematic model, with straightforward meaning of symbols.