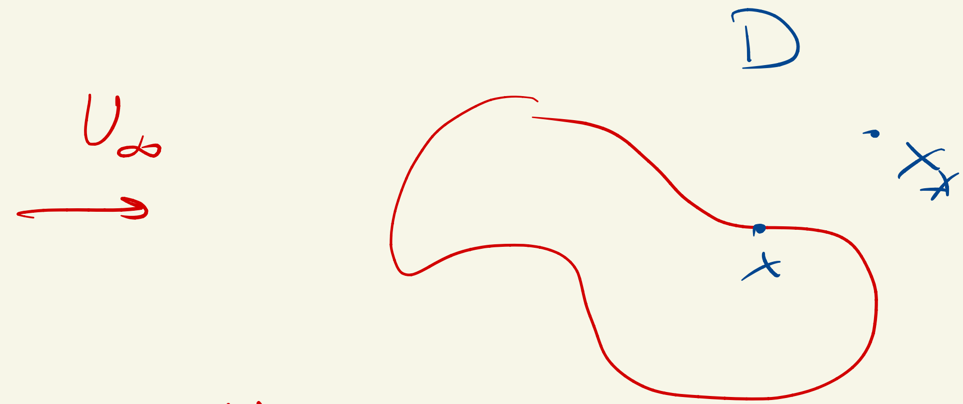
Lezione 3/03/2020

$$\nabla^2\varphi = 0$$

$$\left.\frac{\partial\varphi}{\partial n}\right|_{\partial B} = -U_\infty\cdot n$$

$$|\varphi|\to 0 \qquad |x|\to\infty$$



$$u = U_\infty + v$$

$$v = \nabla\varphi \qquad \nabla\cdot v = 0 \to \nabla^2\varphi = 0$$

$$\varphi\nabla^2\psi - \psi\nabla^2\varphi = \nabla\cdot\left(\varphi\nabla\psi - \psi\nabla\varphi\right)$$

$$\int_D \left(\varphi\nabla^2\psi - \psi\nabla^2\varphi\right)dV = \int_D \nabla\cdot\left(\varphi\nabla\psi - \psi\nabla\varphi\right)dS =$$

$$= \oint_{\partial D}\left(\varphi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\varphi}{\partial n}\right)dS$$

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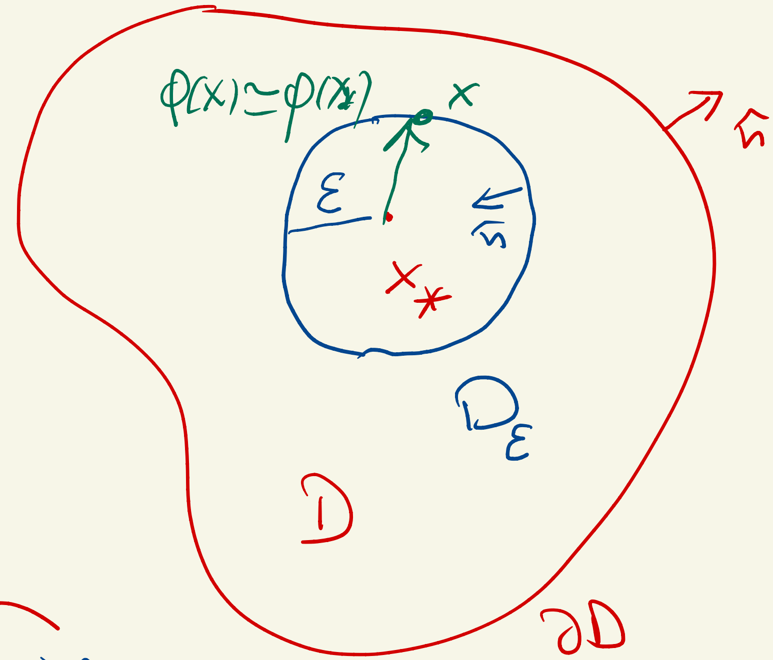
Green

$$\psi \equiv g \qquad \nabla^2 g = \delta(x - x_*)$$

$$\phi_* = \oint_{\partial B} \left[\phi \frac{\partial g(x, x_*)}{\partial n_x} - g(x, x_*) \frac{\partial \phi}{\partial n_x} \right] dS$$

$$g = -\frac{1}{4\pi} \frac{1}{|x - x_*|}$$

$$\int_{\partial D_\varepsilon} \left(\varphi \frac{\partial g}{\partial n_x} - g \frac{\partial \varphi}{\partial n_x} \right) dS \equiv 0$$



$$\partial D_\varepsilon = \partial D + \partial B_\varepsilon(x_*)$$

$$\oint_{\partial D} \left(\varphi \frac{\partial g}{\partial n_x} - g \frac{\partial \varphi}{\partial n_x} \right) dS + \overbrace{\oint_{\partial B_\varepsilon(x_*)} \left(\varphi \frac{\partial g}{\partial n_x} - g \frac{\partial \varphi}{\partial n_x} \right) dS}^{-\varphi_*} \equiv 0$$

$$x_* \in \mathring{D}$$

$$\varphi(x) = \varphi(x_*) + (x - x_*) \cdot \nabla \varphi \Big|_{x_*} + O(|x - x_*|^2)$$

$$g = -\frac{1}{4\pi}\frac{1}{|x-x_0|} \qquad \frac{\partial g}{\partial n_x} = \hat{n}\cdot\nabla g \qquad r = |x-x_0|$$

$$\nabla g = \frac{dg}{dr}\nabla r \qquad \nabla r = \hat{r} = -\hat{n}$$

$$r = \sqrt{(x_k - x_k^*)(x_k - x_k^*)} \qquad \frac{\partial r}{\partial x_i} = \frac{1}{\cancel{2}}\frac{\cancel{2}(x_k - x_k^*)}{\sqrt{(\ )(\ )}}\frac{\partial}{\partial x_i}(x_k - x_k^*)$$

$$= \frac{x_k - x_k^*}{r}\delta_{ki} = \frac{x_i - x_i^*}{r} = \frac{r_i}{r} = \hat{r}_i$$

$$\nabla r = \hat{r} = -\hat{n} \qquad g = -\frac{1}{4\pi r} \qquad \frac{dg}{dr} = \frac{1}{4\pi r^2}$$

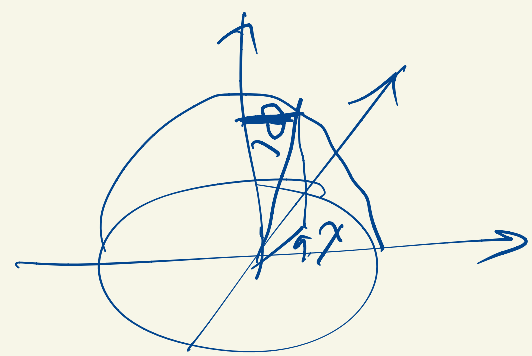
$$\nabla g = -\frac{1}{4\pi r^2}\hat{n} \qquad r = \varepsilon \qquad \nabla g = -\frac{1}{4\pi\varepsilon^2}\hat{n}$$

$$\left.\frac{\partial g}{\partial n_x}\right|_{x \in \partial B_\varepsilon(x_*)} = -\frac{1}{4\pi\varepsilon^2}$$

$$dS = r^2\, d\Omega$$

$$\oint_{\partial B_\varepsilon(x_*)} \varphi \frac{\partial g}{\partial n_x}\, dS = \oint_{\partial B_\varepsilon(x_*)} \left[\varphi_* + \varepsilon\, \hat{r}\cdot\nabla\varphi\big|_* + O(\varepsilon^2)\right]\left(-\frac{1}{4\pi\cancel{\varepsilon^2}}\right)\cancel{\varepsilon^2}\, d\Omega$$

$$= -\varphi_*$$



$$dS = r^2 \underbrace{\sin\vartheta\, d\vartheta\, d\chi}_{d\Omega}$$

$$dS = \sin\vartheta\, r\, d\chi\, r\, d\vartheta$$

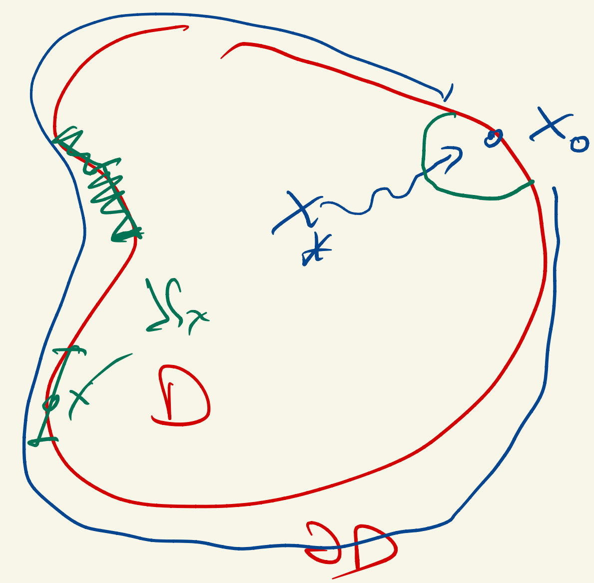
$$\lim_{\varepsilon \to 0} \oint_{\partial B_\varepsilon(x_*)} \left(\varphi \frac{\partial g}{\partial n_x} - g \frac{\partial \varphi}{\partial n_x} \right) dS = -\varphi_*$$

$$\varphi_* = \oint_{\partial D} \left(\varphi \frac{\partial g}{\partial n_x} - g \frac{\partial \varphi}{\partial n_x} \right) dS$$

$$x_* \in \mathring{D} \longrightarrow x_0 \in \partial D \qquad \tfrac{1}{2} \varphi_0 = \oint_{\partial D} \left(\varphi \frac{\partial g}{\partial n_x} - g \frac{\partial \varphi}{\partial n_x} \right) dS$$

$$\lim_{x_* \to x_0} \varphi_* = \varphi_0 \qquad \lim_{x_* \to x_0} \oint_{\partial D} \varphi \frac{\partial g}{\partial n_x} dS = \tfrac{1}{2} \varphi_0 + \cancel{\oint} \;\text{— —}$$

$$\cancel{\oint_{\partial D / B_\varepsilon(x_0)}} + \overbrace{\int_{\partial B_\varepsilon(x_0) \cap D}}^{-1/2\,\varphi_0}$$



$$\tfrac{1}{2}\varphi_0 - \cancel{\oint_{\partial D}} \varphi(x) \frac{\partial g}{\partial n_x}(x, x_0)\, dS = -\oint_{\partial D} \frac{\partial \varphi}{\partial n_x}(x)\, g(x, x_0)\, dS_x$$

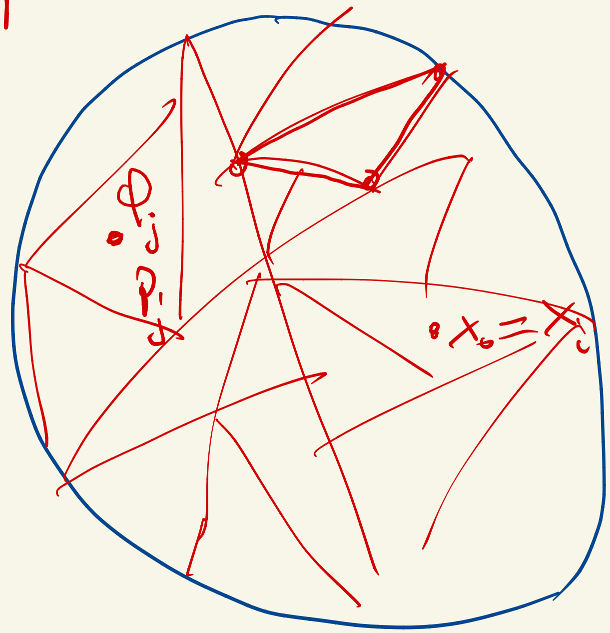
$$\forall\, x_0 \in \partial D$$

$$\nabla^2 \varphi = 0$$

$$\frac{\partial \varphi}{\partial n_x} = -U_\infty \cdot \hat{n}$$

$P_j \quad j=1, \ , N_P$

φ_j



$$\oint_{\partial D} \varphi(x) \frac{\partial g}{\partial n_x}(x, x_0) = \sum_{j=1}^{N_P} \varphi_j \overbrace{\int_{P_j} \frac{\partial g}{\partial n_x}(x, x_i)\, dS_x}^{A_{ij}} =$$

$$x_0 \equiv x_i$$

$$= \sum_{j=1}^{N_P} A_{ij} \varphi_j$$

$$\frac{1}{2}\varphi_i - \sum_{j=1}^{N_p} A_{ij}\varphi_j = \sum_{j=1}^{N_p} B_{ij}\left.\frac{\partial\varphi}{\partial n}\right|_j = \qquad i = 1, N_p$$

$$= -\sum_{j=1}^{N_p} B_{ij}\left(U_\infty \cdot \hat{n}_j\right) = t_i \qquad \{\varphi\} = (\varphi_1 \, - - \, \varphi_{N_p})$$

$$\frac{1}{2} I \{\varphi\} - A\{\varphi\} = \{t\}$$

$$M = \frac{1}{2} I - A \qquad M\{\varphi\} = \{t\}$$

$$Mx = b \qquad \sum_{j=1}^{N} M_{ij} x_j = b_i \qquad i = 1, \dots, N$$

$$M_{11} x_1 + M_{12} x_2 \cdots M_{1N} x_N = b_1$$

$$\vdots$$