MIET2449 – Stress Analysis

Mini-Project: Bridge Design

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What's next...

Table of Contents

I. Introduction & Theory **3**

1. Theoretical Principles: 3
2. Potential Errors: 7
3. Project Constraints: 7
4. Solutions: 8

II. Design Proposal and Analysis **9**

1. Design Overview: 9
2. Design Load Cases: 13
3. Maximum Stress Analysis: 13
4. Deflection Analysis: 13

III. Conclusion **19**

IV. References **20**

V. Appendix **21**

I. Introduction & Theory

In this project, students are required to work in a group of three to design a bridge made of acrylic material that can bear a loading passing on. By applying the knowledge from Stress Analysis, the students need to calculate and determine the parameters and structure of the bridge in order to meet certain requirements from this project:

- The group must design a bridge from a given bridge deck (1000 X 100 X 3 mm) made of PMMA, aka Acrylic that be able to bear a maximum 8kg load on 2 PAScars traveling through multiple times from one end of the bridge to the other end without breaking or exceeding 7mm of deflection. There is a slot (425 X 20 mm) in the middle section of the bridge.

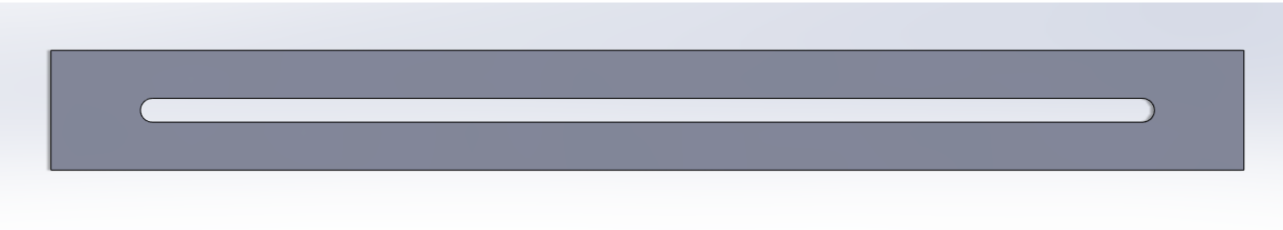

Figure 1: The provided bridge deck

- The first 50mm of the bridge will be fixed-supported by 2-hand clamps while the other end will be simple-supported (resting on top of a table), making the bridge chasm 900mm long. The chams also has guard rails to allow one-way traffic for 2 PAScars traveling through smoothly.
- The group permanent markings should be included on the bridge for identification.
- The bridge's extra parts are designed using SolidWorks software.

There is some equipment can be used for the demonstration:

- 2 PAScars to carry the masses and travel on the bridge.
- PASCO Hook Masses and PASCO 250g Bar Masses to increase the PAScars weight and test the bridge's strength.
- 1.2m PASCO Aluminium Tracks connect seamlessly to the bridge so as the PAScars can move freely.
- Electronic Digital Caliper to make measurements of the PAScars dimensions.

1. Theoretical Principles:

In order to design a perfect bridge that can meet the requirements above, the following principles must be followed:

- **Shear Stress:**
  Shear stress occurs when an external force is acting parallel to the cross-sectional area of an object.

$$\tau = \frac{V}{A}$$

where: τ = shear stress at the cross-sectional area
V = internal resultant force determined from equilibrium equations
A = area of the cross-sectional area where shear stress is determined

- **Centroid of Beam:**

Centroid or Center of Mass is one factor to calculate the moment of inertia for the shear, bending stress and deflection of beam, which can be presented as the formula below [1]:

$$\overline{y} = \frac{\sum A_i \, y_i}{\sum A_i}$$

where: A_i = The individual segment’s area

y_i = The individual segment's centroid distance from the reference line

- **Deflection of Beam:**

  The deflection of beam must be limited to provide integrity and strength to the structure and prevent it from breaking. There are multiple deflecting cases, however, in this project, the bridge deflection behavior can be predicted and described by the figure below when the moment diagram is known:

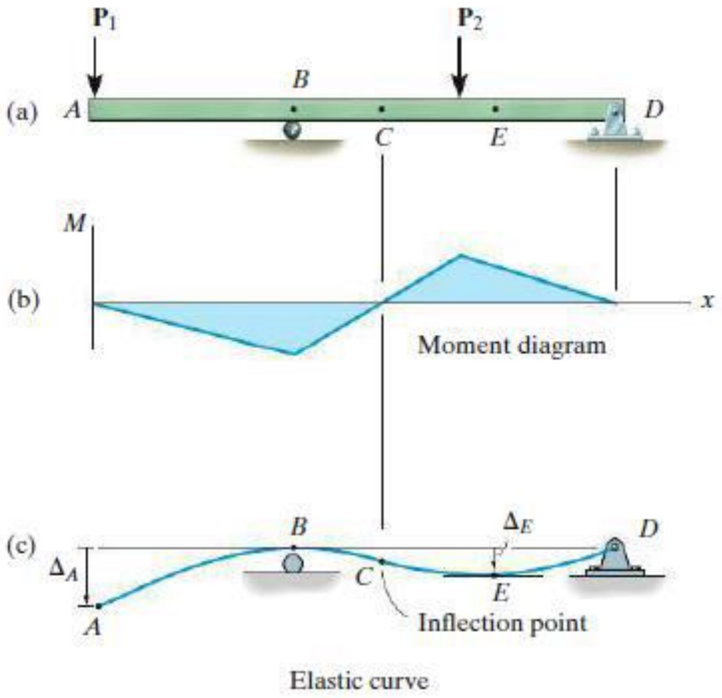

Figure 2: Elastic curve

- **Shear and Moment Diagram:**

  Shear and Moment diagram helps to identify the bridge’s bending behavior when under the loads, therefore helps to determine the shear force and bending moment at each section of the bridge [2]:

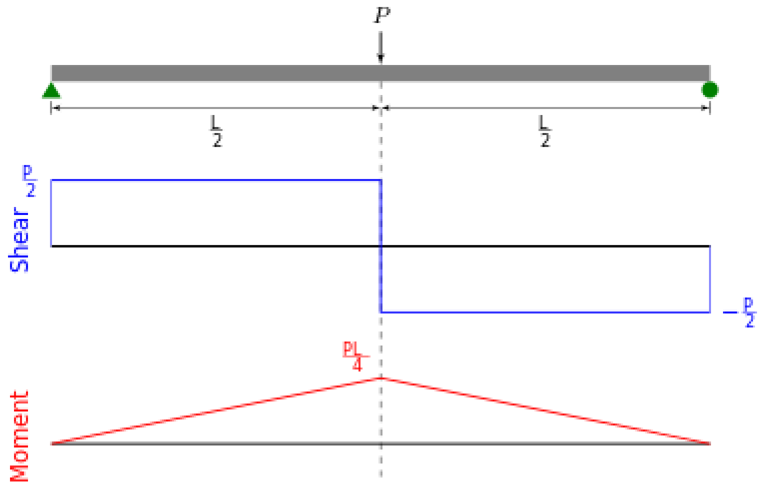

Figure 3: Shear and Moment diagram

The relationship between distributed loads and shear/moment diagrams can be described as below:

$$\Delta M = \int V(x)dx$$

- **Neutral Axis:**
  The neutral axis of the surface passes through the centroid of the section, which will serve as a reference line to determine the Second moment of Inertia of that section.
- **Second Moment of Inertia:**
  Second Moment of Inertia of area is an important geometric property as it is related to the amount of material strength the section has. In this project, the bridge supports are rectangular shape, therefore the formula for Second moment of Inertia of a rectangle is used, combined with Parallel Axis Theorem:

$$I_y = \frac{1}{12}bh^3 + Ad^2$$

where: b = base of the section
h = height of the section
A = area of the section
d = distance between the centroid of the section and the neutral axis

- **Bending Stress:**
  Bending stress occurs when there are stressed on one object acting on another that cause deformation. The maximum stress acting on one object can be described with the following formula:

$$\sigma_{max} = \frac{Mc}{I}$$

where: σ_{max} = the maximum normal stress in the member which occurs at a point on the cross-sectional area farthest away from the neutral axis
M = the resultant internal moment, determined from section method and equations of equilibrium
c = the perpendicular distance from the neutral axis to the farthest point from the neutral axis
I = the moment of inertia of the cross-sectional area about the neutral axis

- **Method of Superposition:**
  Superposition method is used for the deflection of elastic curve, in this project when the bridge is subjected under several loads. Hence, the whole structure can be separated to calculate the slope and deflection of each load and then summing them together.

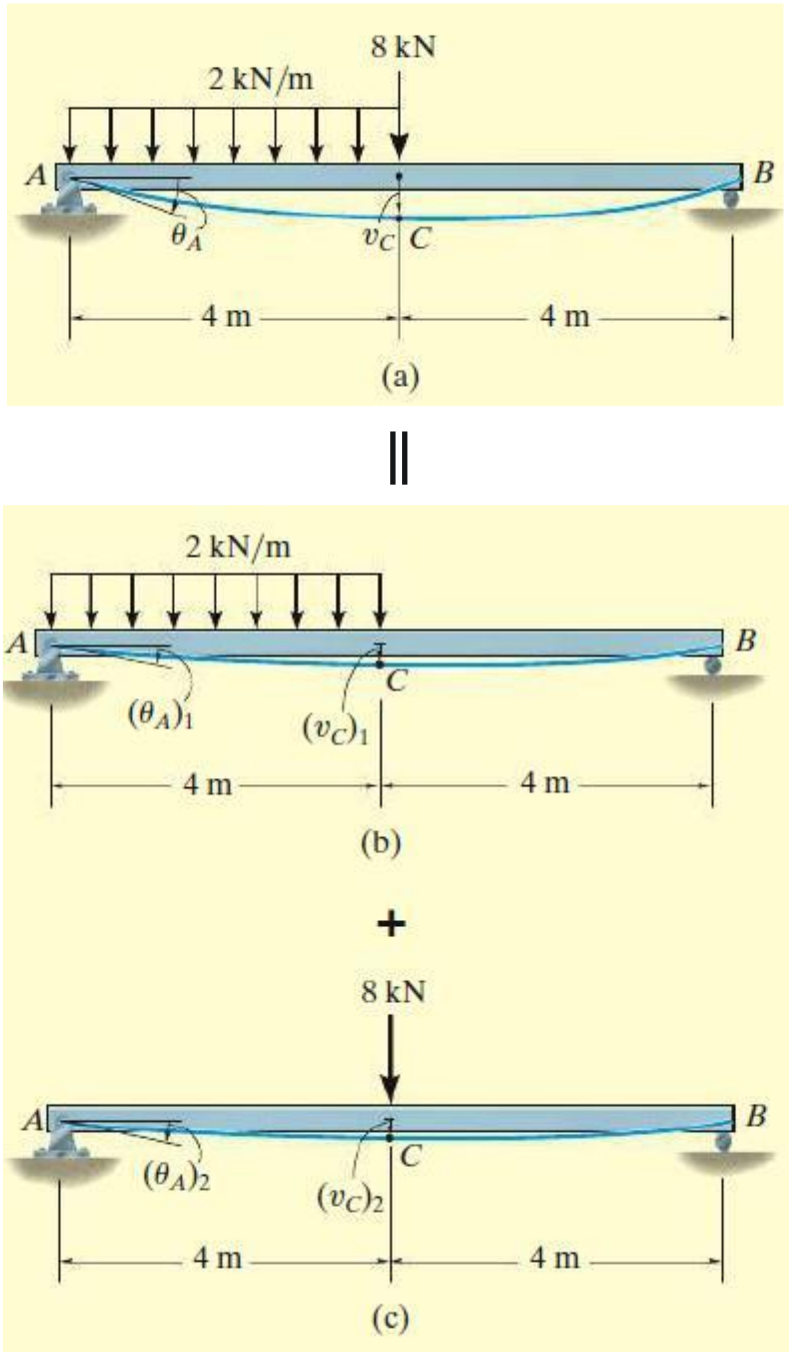

Figure 4: Method of Superposition

- **Factor of Safety:**
  Factor of Safety is an essential parameter of the material, which is a ratio of failure load to allowable load acting on the object, that indicates the maximum force or stress the material can handle before breaking. From these, the bridge's structure can be analysed, and the required parameters are identified. It can be described as the formula below:

$$F.S = \frac{F_{yield}}{F_{allowable}} = \frac{\sigma_{yield}}{\sigma_{allowable}} = \frac{\tau_{yield}}{\tau_{allowable}}$$

In this project, the bridge's allowable stress is recommended to have the factor of safety of 1.5 when under 8kg load.

- **Maximum moment of moving load:**
  The maximum load of a system of moving load is located in between the resultant load and the chosen one. The location of maximum moment is located at the centreline of the beam.
- **Maximum shear of moving load:**
  The maximum reaction of the load is placed over one support.

2. Potential Errors:

- Although the bridge's parts are manufactured using laser cutting with high accuracy, there still exists tolerance in these parts, around ± 3mm. After receiving the manufactured components, they will be measured before assembly.
- The theoretical calculations might not represent the exact results of deflection value of the bridge compared to the real results recorded from real experiment.
- Complications might occur during the assembly process such as bridge damage or displacement, especially when gluing the parts together due to human errors. Thus, the assemble process should be done carefully and handled with care.
- The glue used for sticking the parts together might not be strong enough and the bridge connection is not stable.

3. Project Constraints:

There are some constraints should be aware of in this project:

- All parts of the bridge must be made entirely of Acrylic, 3mm thick and held together with glue. Fasteners are not allowed. This is to prevent the students from using stronger materials to strengthen the bridge structure, exceeding the project limitation. The entire bridge structure is made from the same material also simplify the calculation process.
- The extra parts of the bridge should not exceed 3000 cm^2 of material. Therefore, the extra components' parameters must be carefully and accurately calculated to both meet the deflection requirements and utilize the limited material.
- The slot in the middle of the bridge should be kept clear without any obstruction by the extra parts. The purpose of this is to create a gap between the PAScars and the space below the bridge to hang extra masses to the PAScars. This does not significantly cause many troubles in the calculation process; however, students are required to think more about designing the bridge strengthening components.
- No human contact on the simple-supported end of the bridge and the PASCO Tracks during the demonstration. Therefore, the bridge must be kept secured on table before the experiment is performed.
- The PAScars should travel seamlessly to and from the PASCO Tracks without hindrance. So, students must measure the exact thickness of the PASCO Tracks in order to design the bridge with the same thickness for the smoothest run, also

limiting the material thickness and area of the bridge support structure, leading to more attentive calculations and analysis.

4. Solutions:

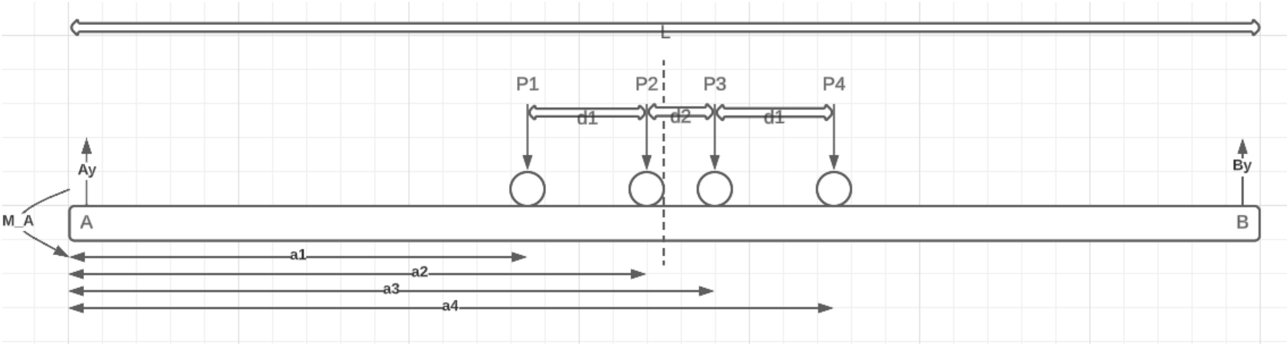

Figure 5: Free body Diagram

Specification of the free body diagram:

- $d1 = 0.1m$
- $d2 = 0.07\ m$
- $L = 0.9m$
- $P1 = P2 = P3 = P4\ = 21.25\ N$
- $S.F. = 1.5$
- $E = 1.1GPa$
- $\sigma_y = 24.4MPa$
- $a1, a2, a3, a4$ are the distance from the fix support to each of the point load location on the beam

II. Design Proposal and Analysis

1. Design Overview:

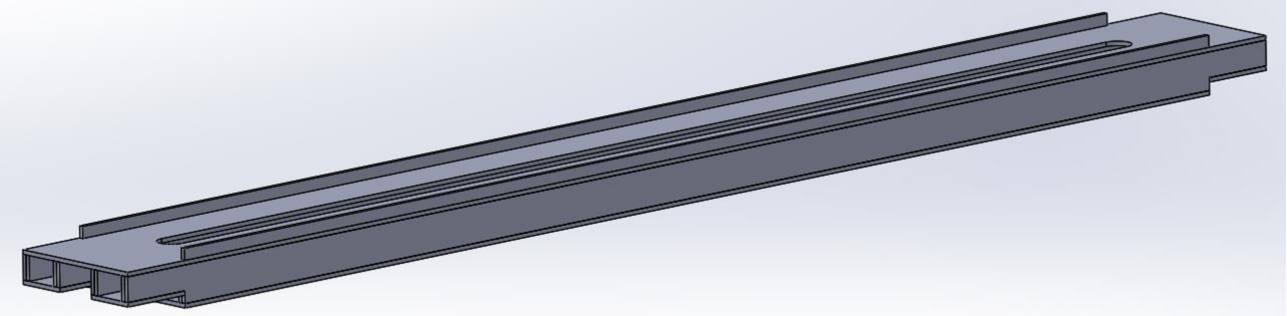

Figure 6: Full assembly of the bridge

The new bridge is design with additional rails, supports at the bottom and the flat surface parallel to the bridge deck. The bridge includes 5 main parts: bridge deck, rail, support, bottom plate 1 (bot_1) and bottom plate 2 (bot_2).

Table 1: Bill of Material

Parts	Dimensions (mm)	Surface Area (cm^2)	Quantity	Description
Rail	900 x 8	144	2	Place on top of the bridge flat on both sides
Support	1000 x 25.5	1972	8	Main pieces at the bottom to support the bridge
Bot_1	50 x 35	70	4	Flat surface parallel to the bridge on 2 sides
Bot_2	900 x 35	630	2	Flat surface parallel to the bridge on 2 sides
	Total	2816		

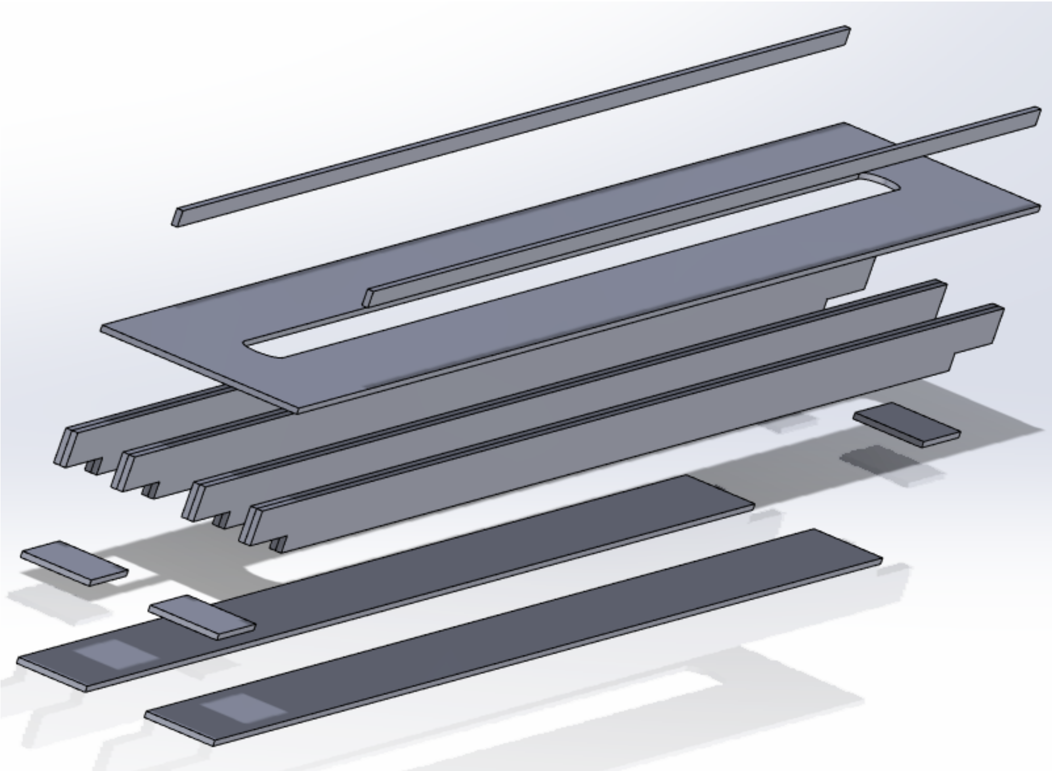

Figure 7: Exploded view of the bridge

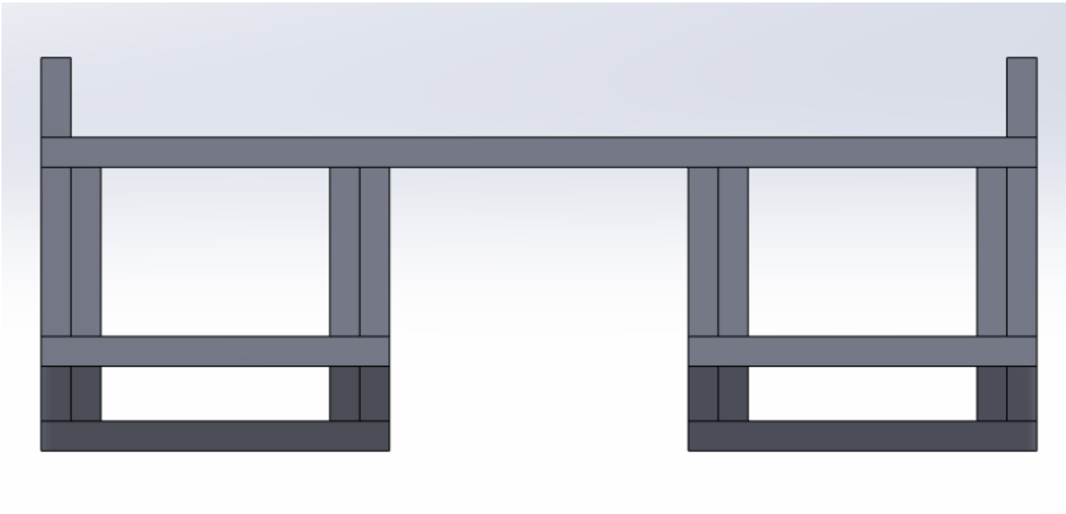

Figure 8: Front view of the bridge

The main structure design that helps support the bridge to withstand the load of the moving car is placing under the bridge deck.

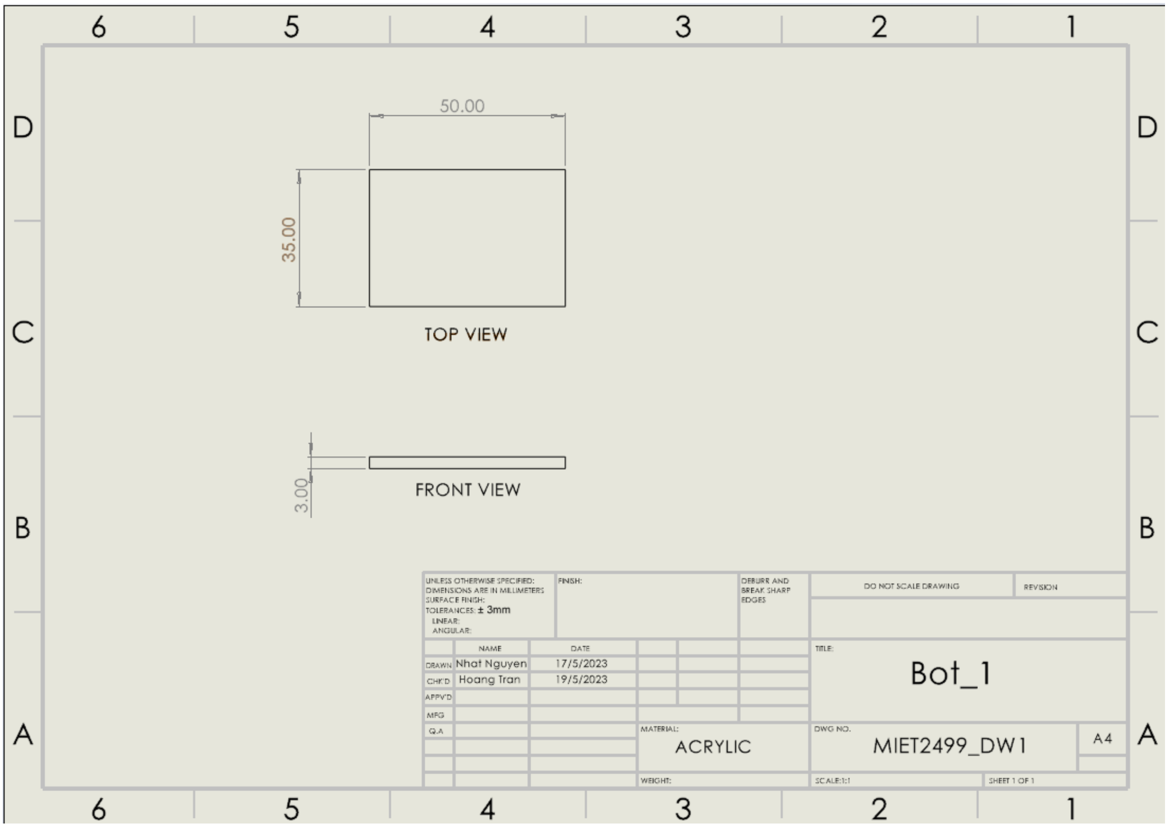

Figure 9: Bot_1 drawing and dimensions.

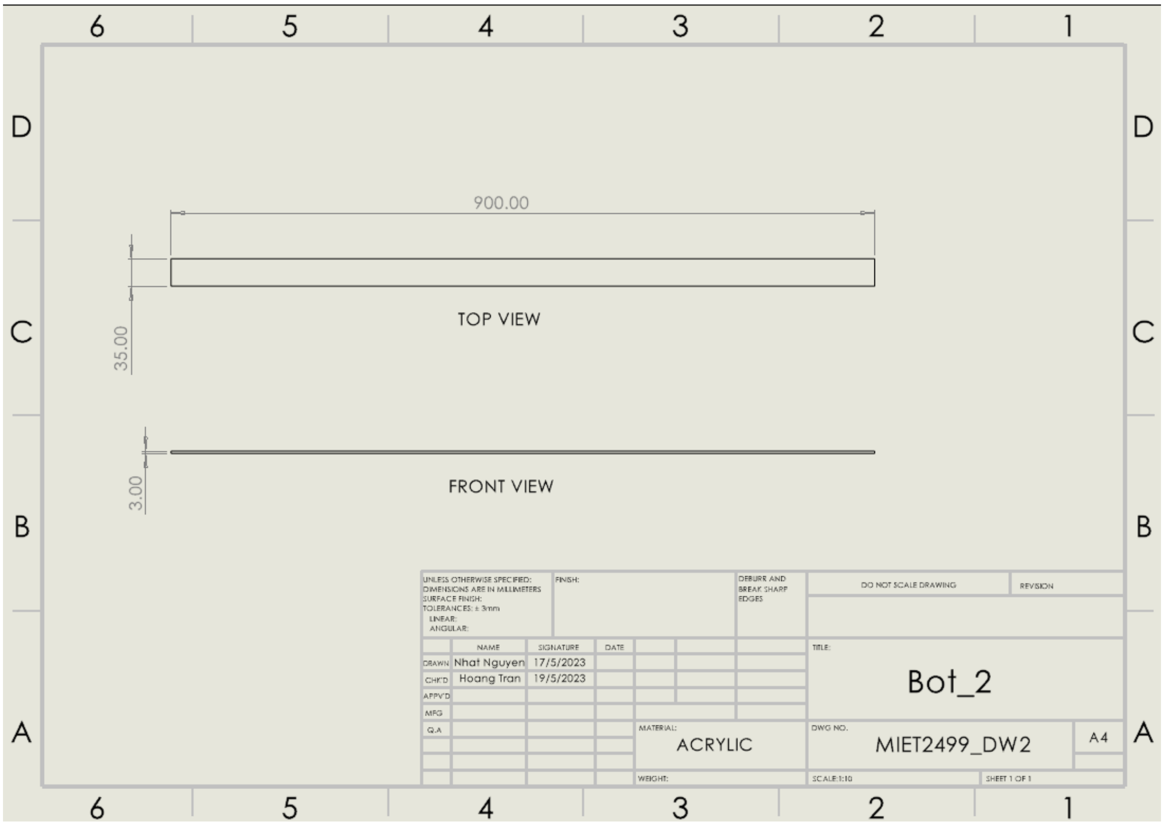

Figure 10: Bot_2 drawing and dimensions

The two bottom plates are put underneath the supports to provide their strength and stability under stresses.

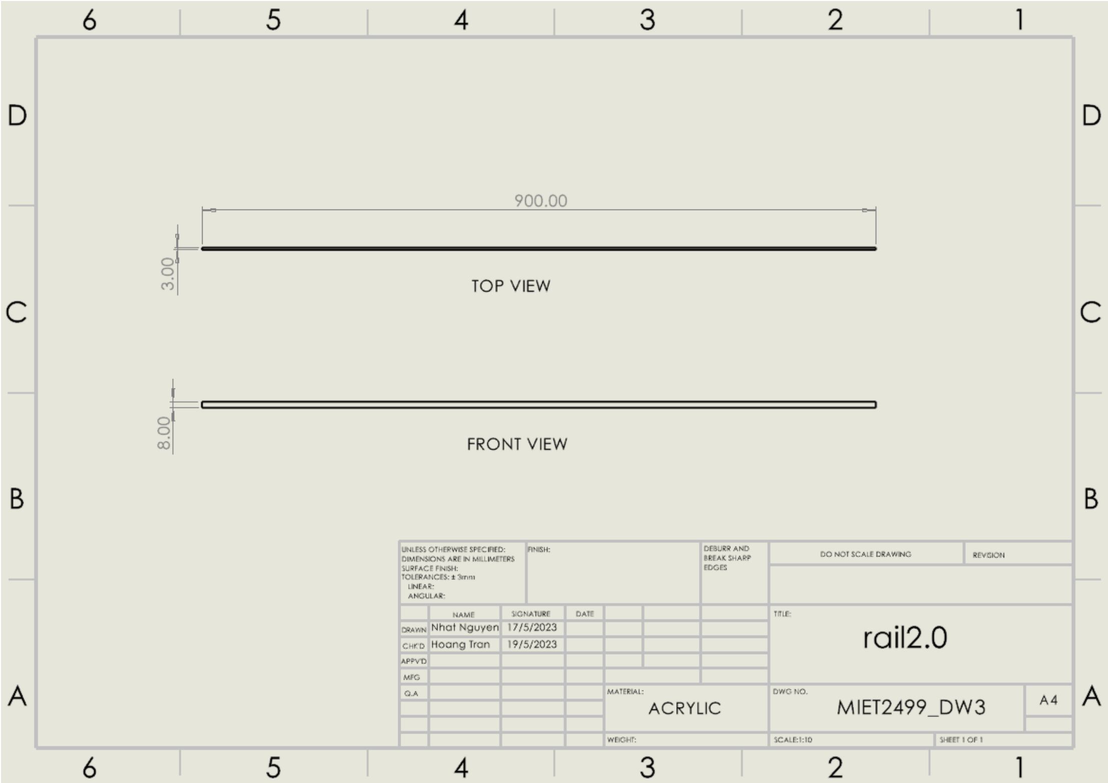

Figure 11: Rail drawing and dimensions

The rail of the bridge is designed with the length of 900mm and the height of 8mm and placed on two side across the bridge, acts as the safeguard so that when the car travel over the bridge, it would not fall off out of the bridge.

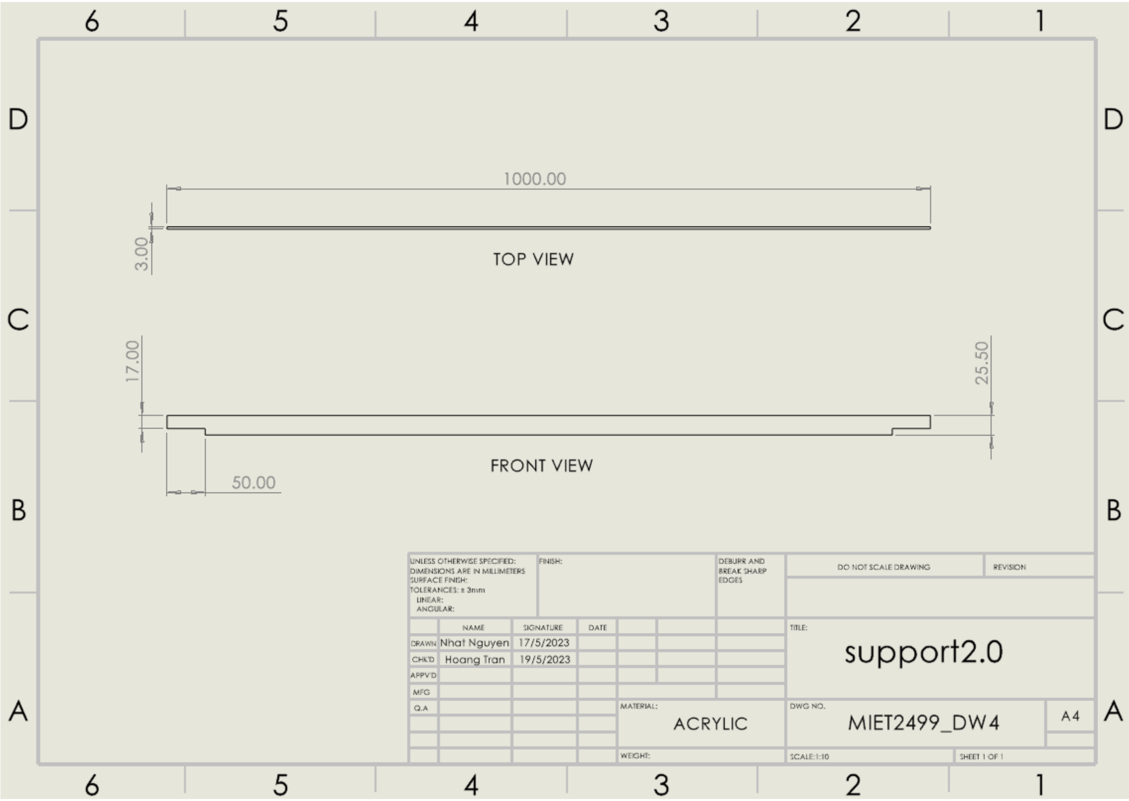

Figure 12: Support drawing and dimensions

2. Design Load Cases:

Each PASCAR mass is 250g, 10cm apart between its wheels and 7.5cm between 2 cars' wheels when they are stick together. PASCO tracks thickness is 2.5cm.

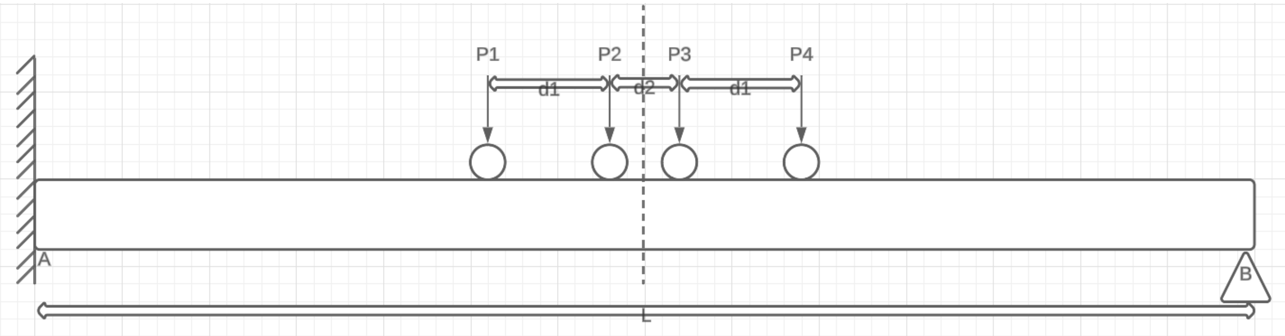

Figure 13: Maximum moment

For the design of the bridge, we consider the location of the car along the bridge to find about the maximum shear stress and the maximum bending moment and deflection. Because the bridge is fixed support at one end and simple support at the other, we calculated about the maximum shear stress would occur at the location of the fixed support. And the maximum bending moment and deflection happen when the car is located near the middle of the bridge.

3. Maximum Stress Analysis:

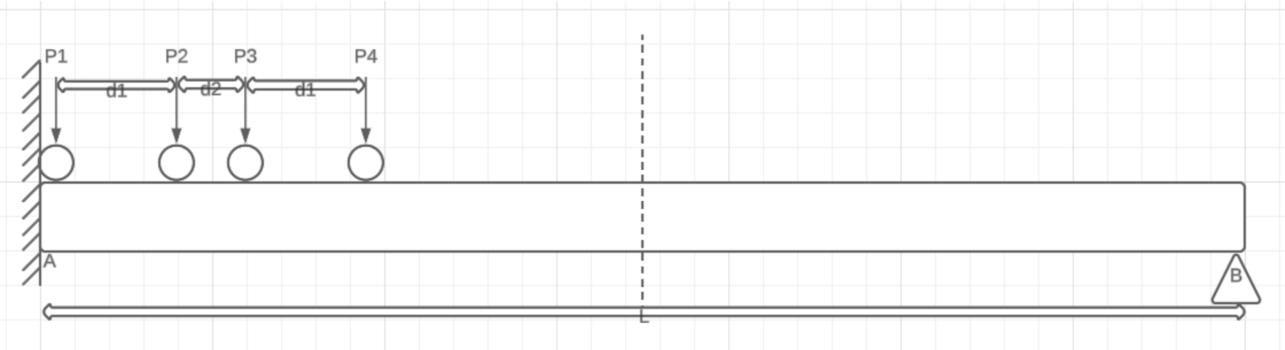

Figure 5: Maximum shear stress

4. Deflection Analysis:

The bridge has 1 fixed support for one end and a simple support for the other while subject underload of 2 PAScars and the distributed load of the bridge weight, the bridge become statically indeterminate. To solve this problem, we applied the superposition method and divided the problem into 3 cases. The first case would be the distributed load only on the bridge. The second case would be the bridge under the load from the PAScars. The third case would be the bridge reaction for apply at the simple support.

Centroid of the bridge cross-section area:

Part	Area (mm^2)	Quantity	Total (mm^2)	Dimension			total	X_avg				X_total	Y_avg
				h	b	A							
support2.0	153	4	612	25.5	6	153	612	3	32	68	97	200	15.75
bot_2	105	2	210	3	35	105	210	17.5	82.5			100	1.5
bridge_flat	120	2	240	3	40	120	240	21.295	78.705			100	30
rail	24	2	48	8	3	24	48	1.5	98.5			100	35.5
		Total	1110										

	A*x / A*y	A_sum	result	
X	55500	1110	50	mm
Y	18858	1110	16.9891892	mm

Figure 6: Centroid of each component on the bridge

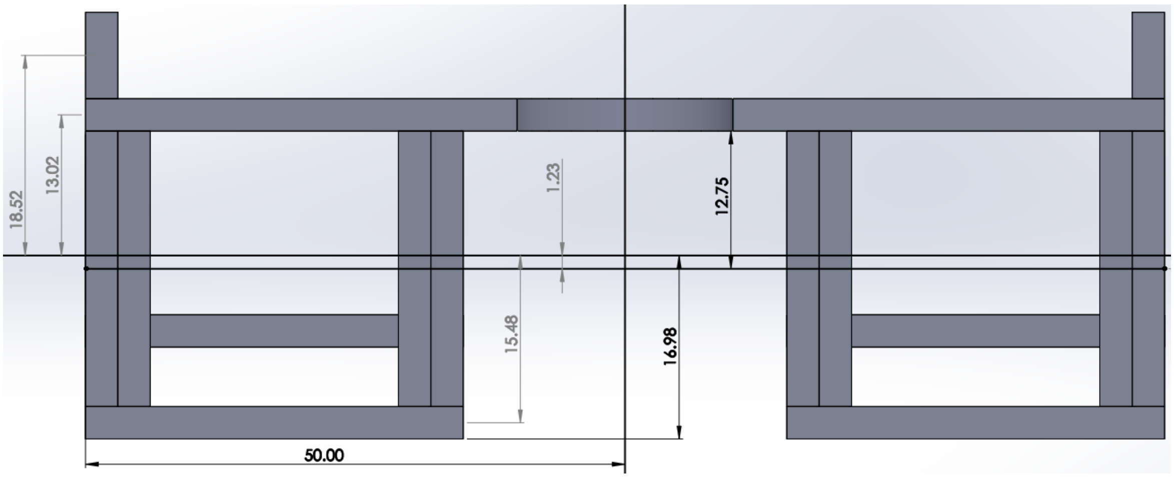

Figure 7: Cross-section area of the bridge

The moment of inertia is located on the centroid of the bridge cross sectional area and its value can be find by the sum of all moment of inertia (MOI) of the rectangle shape about the centroidal axis using the Parallel Axis Theorem.

Total moment of Inertia of the bridge:

$$I = (I_1 + A_1 d_1^2) + (I_2 + A_2 d_2^2) + (I_3 + A_3 d_3^2) + (I_4 + A_4 d_4^2)$$

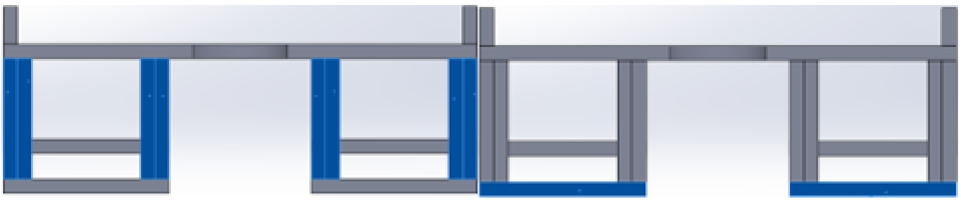

Figure 8: MOI of support part (left) and bot part (right)

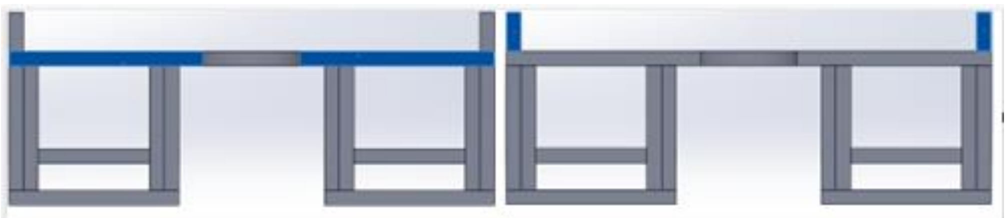

Figure 9:MOI of bridge flat part (left) and rail part (right)

								Parallel axis theorem				
						I_x	I_y	A*d^2		I_x abt ref(mm^4)	I_y abt ref(mm^4)	
No.	h(mm)	b(mm)	A(mm2)	quantity	d (mm)							
support	25.5	3	76.5	8	1.23	4145.34	57.375	115.73685		4261.0806	173.11185	
bot2	3	35	105	2	15.48	78.75	10718.8	25161.192		25239.942	35879.942	
flat	3	40	120	2	13.02	90	16000	20342.448		20432.448	36342.448	
rail	8	3	24	2	18.52	128	18	8231.7696		8359.7696	8249.7696	
									Final	142152.964	162329.214	mm^4

Figure 10: Moment of Inertia about the centroidal axis of bridge's cross section area

Applying the superposition method, we divided into two smaller case of the bridge

Case 1: Bridge only under one reaction force at the support B

Apply static equilibrium equation:

$$\sum F_y = -Ay + V = 0$$

⇨ $V = A_y$

$$\sum M_1 = M_1 + A_y x - M_A = 0$$

⇨ $M_1 = -A_y x + M_A$

Integrate the equation of moment, we can obtain the equation of the slope of the elastic curve and the equation describe the curve as well:

➔ $\frac{1}{EI}\theta = \int M_1 dx = \frac{-A_y}{2}x^2 + M_A x + C_1$

➔ $y = f(x) = \frac{1}{EI}[\frac{-A_y}{6}x^3 + \frac{M_A}{2}x^2 + C_1 x + C_2]$

Case 2:

- **Section 4c – Maximum Stress Analysis:**

Physical Background for 4 point loads:

The distance a in this section is counting from the fixed end to the first wheel seeing from the left

From the Free Body Diagram (Figure 13), without considering the bridge weight, the 2 equations of equilibrium are formed:

$$\Sigma F_x = 0 = A_x$$

$$\Sigma F_y = A_y + B_y - P = 0 \ (1)$$

$$\Sigma M_{at\ A} = M_a - \frac{P}{4}(a_1 + a_2 + a_3 + a_4) + B_y * L = 0 \ (2)$$

With $P = 85N, a_1 = a, a_2 = a + 0.1, a_3 = a + 0.1 + 0.07, a_4 = a + 2 * 0.1 + 0.07$ and $L = 0.9$ (a and L is measured in meter):

$$\Sigma F_y = A_y + B_y - 85 = 0$$

$$\Sigma M_{at\ A} = M_a - \frac{85}{4}(4a + 0.54) + B_y * L = 0$$

With the fact that a is a parameter defining the position of the car on the bridge, the set of equations is statistically indeterminate (2 equations for 3 variables A_x, B_y, M_a).

=> **Superposition method** is chosen to find A_x, B_y, M_a with respect to a.

Consider a case when one beam is exerted to P force with distance a from the fixed end.

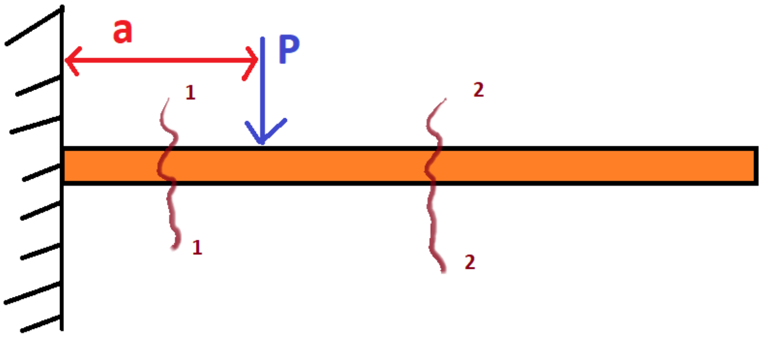

Figure 11: FBD when P apply at a distance a

There are 2 different formulas for the deflection of 2 sections which are proved in the tutorial: Section 1-1: $-\frac{Px^2(3a-x)}{6EI}$ and Section 2-2: $-\frac{Pa^2(3x-a)}{6EI}$. However, to keep things simple and straightforward to manipulate, the **minus sign** of the formula will be removed (making the numerical result of the deflection **positive**).

For each section in the 4 point-load model, we have the formula of deflection:

Section 1-1: y1= $\frac{Fx^2(3a_1-x)}{6EI} + \frac{Fx^2(3a_2-x)}{6EI} + \frac{Fx^2(3a_3-x)}{6EI} + \frac{Fx^2(3a_4-x)}{6EI} - \frac{B_yx^2(3L-x)}{6EI}$

Section 2-2: y2 = $\frac{Fa_1^2(3x-a_1)}{6EI} + \frac{Fx^2(3a_2-x)}{6EI} + \frac{Fx^2(3a_3-x)}{6EI} + \frac{Fx^2(3a_4-x)}{6EI} - \frac{B_yx^2(3L-x)}{6EI}$

Section 3-3: y3 = $\frac{Fa_1^2(3x-a_1)}{6EI} + \frac{Fa_2^2(3x-a_2)}{6EI} + \frac{Fx^2(3a_3-x)}{6EI} + \frac{Fx^2(3a_4-x)}{6EI} - \frac{B_yx^2(3L-x)}{6EI}$

Section 4-4: y4 = $\frac{Fa_1^2(3x-a_1)}{6EI} + \frac{Fa_2^2(3x-a_2)}{6EI} + \frac{Fa_3^2(3x-a_3)}{6EI} + \frac{Fx^2(3a_4-x)}{6EI} - \frac{B_yx^2(3L-x)}{6EI}$

Section 5-5: y5 = $\frac{Fa_1^2(3x-a_1)}{6EI} + \frac{Fa_2^2(3x-a_2)}{6EI} + \frac{Fa_3^2(3x-a_3)}{6EI} + \frac{Fa_4^2(3x-a_4)}{6EI} - \frac{B_yx^2(3L-x)}{6EI}$

Due to the simple support at the end, deflection at this point must be 0 => y5(when x = L) = 0
=> The function of By with respect to a only (after substituting F, E, I, L, a1, a2, a3, a4) is found.
=> Substituting By into equation (1) and (2) brings up the formula of Ma and Ay with respect to a.

1. Find function of Shear force: V (x, a)

From the FBD (Figure …), by going to the left, shear force in each section can be analyzed as below:

Section 1-1: $V_1 = A_y$

Section 2-2: $V_2 = A_y - \frac{P}{4}$

Section 3-3: $V_3 = A_y - \frac{P}{4} - \frac{P}{4} = A_y - \frac{P}{2}$

Section 4-4: $V_4 = A_y - \frac{P}{4} - \frac{P}{4} - \frac{P}{4} = A_y - \frac{3P}{4}$

Section 5-5: $V_5 = A_y - \frac{P}{4} - \frac{P}{4} - \frac{P}{4} - \frac{P}{4} = A_y - P$

Here, there are 2 ways to find the maximum Shear Force and its location on the beam – Analytical and Numerical Method.

- With Analytical approach, the steps are:

Substituting A_y (function of a) into these 5 sections' formulas -> take derivative in each section with respect to a and the range of a in each section (e.g. with section 3-3, the range of a is from a2 to a3) -> solve those derivatives when their values equal 0 to find a and max shear force in each section -> compare them to each other to find the maximum shear force value.

- With the Numerical method, the steps are:

Set the range of a with small enough steps for each section (in this case a is from 0 to L – 0.1*2-0.07 meter since the examining cases in this project is when all 4 wheels are within the track). -> Compare the values in all 5 sections at the numerical value of with the "Finding the maximum" algorithm -> Get the biggest Shear Force and its corresponding a value.

2. Find function of Bending Moment: M (x, a)

With the formula of Shear Force, it is straightforward to find that Moment at an arbitrary point of the bridge (at x) as the car moves.

Section 1-1: $M_1 = V_1 x - M_a$

Section 2-2: $M_2 = V_2 x - \frac{P}{4} * a_1 - M_a$

Section 3-3: $M_3 = V_3 x - \frac{P}{4} * a_1 - \frac{P}{4} * a_2 - M_a$

Section 4-4: $M_4 = V_4 x - \frac{P}{4} * a_1 - \frac{P}{4} * a_2 - \frac{P}{4} * a_3 - M_a$

Section 5-5: $M_5 = V_5 x - \frac{P}{4} * a_1 - \frac{P}{4} * a_2 - \frac{P}{4} * a_3 - \frac{P}{4} * a_4 - M_a$

Since the Moment at A is always the maximum bending moment during the car moves, we only need to care about value of maximum of M_a with respect to a when the car moves to decide boundary conditions of size and cross section of the bridge. There are 2 possible ways to solve this problem: Analytical and Numerical Method.

- With Analytical Approach, the steps are:

Taking the derivative of M_a with respect to a

-> find a when the derivative is 0 -> select the a value in range from 0 to L – 0.1*2-0.07

-> subtitute a into the M_a equation to get the maximum value of M_a.

- With Numerical Approach, the steps are:

Setting the range of a value from 0 to L – 0.1*2-0.07 with small step

-> Compare M_a of each a value to find the biggest M_a.

3. Find function of Deflection: y(x, a)

Via substituting By into the formula of deflection in each section, y1, y2, y3, y4, y5 become formulas of x and a solely. To save time, the numerical method should be used to find the maximum deflection of the bridge:

- Initiate an x vector from 0 -> 0.9m and an a vector from 0 -> 0.9 – 0.1*2-0.07 m
- For each value of x and a, substitute into each section formula to get the results of the deflection at every position of the car and every point on the bridge
- Save the largest deflection data with its a and x value.

Physical Background for 1 point load:

Converting 4 point loads to 1 point load by finding the centroid of this combination makes it less complicated to examine deflection, bending moment and shear force at bridge design.

The centroid of the converted applied force is in the middle of the 2 nearest wheels of the 2 cars and its magnitude is the sum of 4 point loads (which is P=85N).

Since the scope for 1 point load analysis is just to find the maximum bending moment at A as the car moves as discussed in the tutorials.

- Step 1: Find B_y

Recall the 2 provided formulas of deflection in the case that a point load P applyied at a distance a on a fixed beam:

Section 1-1: $-\frac{Px^2(3a-x)}{6EI}$ and Section 2-2: $-\frac{Pa^2(3x-a)}{6EI}$

Using superposition for the beam, at the simple support, the formula is:

$\frac{Pa^2(3L-a)}{6EI}$ - $\frac{B_y L^2(3L-L)}{6EI}$ = 0 => $B_y = \frac{Pa^2(3L-a)}{2L^3}$

<u>- Step 2: Find M_a</u>

At A, the moment is: $M_a = Pa - B_y L = Pa - \frac{Pa^2(3L-a)}{2L^3} L = \frac{Pa^3}{2L^2} - \frac{3Pa^2}{2L} + Pa$

Take the derivative of M_a and set it to 0: $\frac{3Pa^2}{2L^2} - \frac{3Pa}{L} + P = 0$

Substitute P = 85N, L = 0.9m, the equation becomes: $157.407a^2 - 283.33a + 85 = 0$

Solve the above equation: a = 0.38, a =1.42

The condition of a is 0.07/2 + 0.1< a < 0.9 - 0.07/2 - 0.1 ⇔0.135< a <0.765

=> a = 0.38m is the right answer

With a = 0.38, the value of Moment at A is: $\frac{85*0.38^3}{2*0.9^2} - \frac{3*85*0.38^2}{2*0.9} + 85 * 0.38 = 14.7224$ (Nm)

ANSWERS for the main questions of section 4c

Load Case for Maximum Shear Force

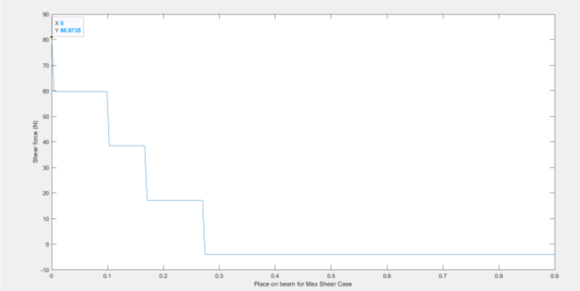

Figure 21: Shear force Diagram when the shear is largest

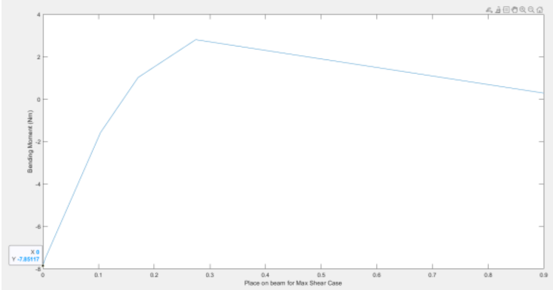

Figure 22: Moment Diagram when the shear is largest

With numerical method, it is discovered that the shear force is largest at the a = 0m – when the 2 cars have merely entered entirely the the track (the last wheel is on the fixed end). 80.9735N is the value of the maximum shear force that the fixed end of the bridge deals with as it can be seen in the figure.

Load Case for Maximum Bending Moment

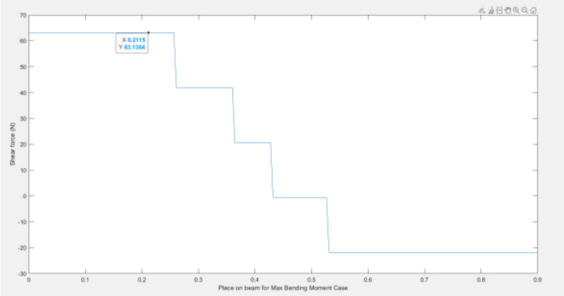

Figure 23: Shear force Diagram when the Bending Moment is largest

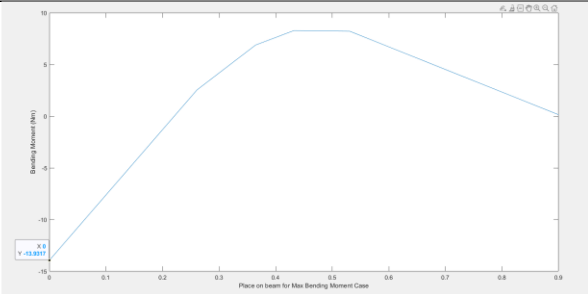

Figure 24: Moment Diagram when the Bending Moment is largest

With numerical method, it is discovered that the bending moment is largest at the a = 0.2604 m and x = 0.4909m. The maximum value calculated is 13.9317 Nm.

Load Case for Maximum Deflection

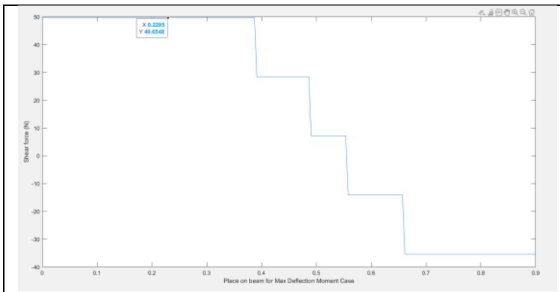

Figure 25: Shear force Diagram when the Deflection is largest

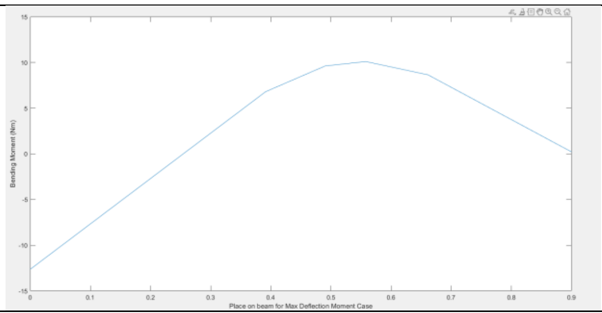

Figure 26: Moment Diagram when the Deflection is largest

In the 2 figures, it can be easily witnessed that with a different value pair of a and x, the value of Shear force and Moment is changed and lower than the above discovered Maximum Shear Force an Bending Moment.

- About Maximum Stresses:

Maximum Shear Stress = $\frac{V_{maximum}}{A} = \frac{81N}{345mm^2} = 0.246MPa$ < 16,27 Mpa (This is an average value of shear stress on the plane)

With: 16.27 = $\frac{\tau_{yield}}{F.S} = \frac{24.4MPa}{1.5}$ and 345 mm^2 is the area that the force applied on.

=> The bridge will not break due to shear stress

Maximum Bending Moment = $\frac{Mc}{I} = \frac{81N*16mm}{142152.96mm^4} = 9.12MPa$ < 16,27 Mpa

This maximum value is **tensile** as the bending stress happen underneath the bridge

=> The bridge will not break due to bending stress

- **Section 4d – Deflection Analysis:**

Below, X* is the location of the maximum deflection of the bridge in each case and a* locate where the car is to have Maximum Bending Moment, Maximum Shear Force, Maximum Deflection.

- Load case for Maximum Bending Moment:

X* = 0.4909 m; a* = 0.2604 m; Deflection = 1.6 mm; Maximum radius of Curvature = 20.4795 m

- Load case for Maximum Shear Force:

X* = 0.4273 m, a* = 0m, Deflection = 0.46 mm; Maximum radius of Curvature = 38.4028 m,

- Load case for Maximum Deflection:

X* = 0.5273 m, a* = 0.3879 m, Deflection = 1.83 mm; Maximum radius of Curvature = 23.8613 m

- About the Curvature

The less the radius of the curvature (ρ) is, the more bending moment the location has according to the formula: $\frac{1}{\rho} = \frac{M}{EI}$

Also according to the formula $\frac{1}{\rho} = \frac{M}{EI}$, the Curvature reach maximum when Moment get maximum

=> It is predicted that the location of maximum curvature is always at A (the fixed end)

- About how to find Maximum Deflection and Elastic Curve

With 4 loads, it is very complex to have the analytical load in a statistical indeterminate problem. As discussed in "**Physical Background for 4 point loads**", the numerical method was used to find the location and the magnitude of maximum deflection of the bridge. Also in the part "**Physical Background for 4 point loads**", the **sample** calculation and algorithm to find the location and the magnitude of maximum deflection of the bridge is presented in detail.

- Our design has 1.8mm maximum deflection, which can be shown in the graph below:

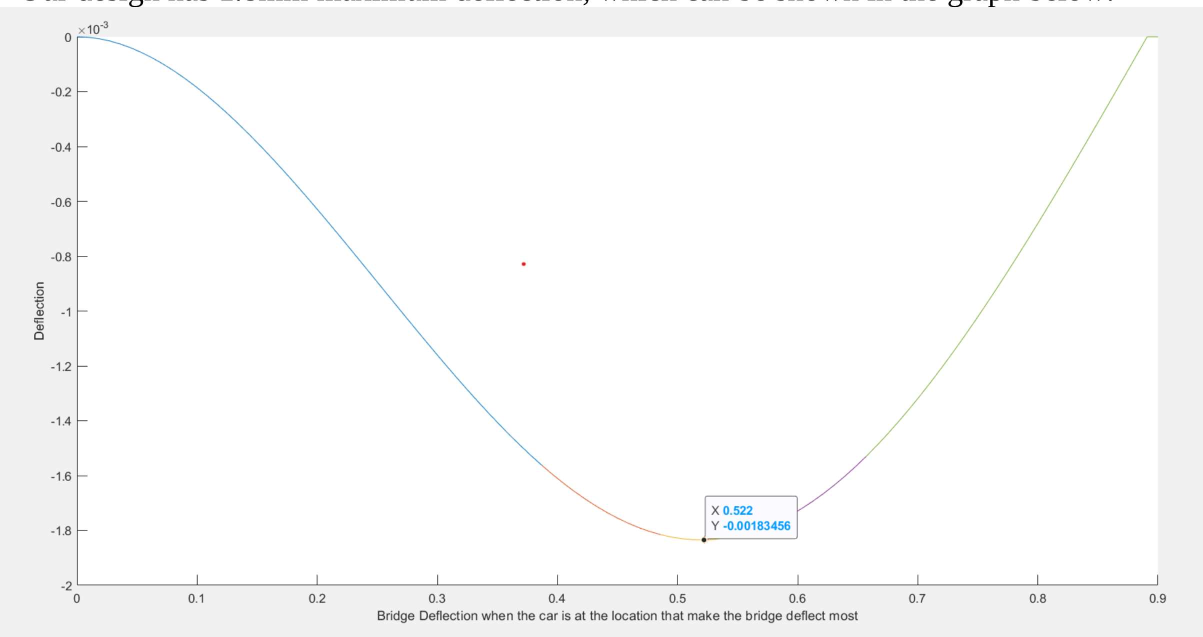

Figure 27: Bridge Deflection at the load case where the deflection is highest

From left to right in the Figure 27 is the direction from the fixed end to the simple support. The shape of the Deflection is matching with the theoretical calculation and assumption that are presented above:

- Both ends have 0 deflection
- The derivative of the deflection is 0, as the deflection change less when it get nearer to the fix end.

III. Conclusion

The shear stress will be max when the car is located at a = 0m. The maximum bending stress is collated at 0.2604m as the value is maximum. To prevent the maximum shear, happen at that location, we reenforce the bridge with extra material in hope of increase the moment of inertia of the cross-section of the bridge.

In the case of both end of the bridge only have simple support, the value of maximum shear stress, bending and deflection would be difference. The problem would be statically determinate and can be solve simply by section method. The bridge is symmetry about the middle of the bridge making A and B are identical to each other. Since there is no existing moment reaction at the fix support like before so the maximum moment would be allocated at the middle of the bridge. The maximum shear is equal at A and B when the car is located it at each end of the bridge. The maximum deflection of the bridge also happens at the middle of the bridge as the cars place there. The quantity of these value would be less than compared to the original case with one end have fix support because the reaction moment at A when have fix support does not exist in the equation and those three max value only rely on the location of the car and the load its carry.

IV. References

[1] "How to calculate centroid of a beam?: Skyciv Engineering," SkyCiv Cloud Structural Analysis Software | Cloud Structural Analysis Software and Calculators, https://skyciv.com/docs/tutorials/section-tutorials/calculate-the-centroid-of-a-beam-section/#:~:text=The%20centroid%20or%20center%20of,shear%2Fbending%20stress%20and%20deflection. (Accessed May 20, 2023).

[2] L. (Elizabeth) Osgood, G. Cameron, E. Christensen, A. Benny, and M. Hutchison, "6.2 shear/moment diagrams," Engineering Mechanics Statics, https://pressbooks.library.upei.ca/statics/chapter/shear-moment-diagrams/ (accessed May 20, 2023).

[3] W. Haynes and W. Haynes, "Engineering statics: Open and interactive," EngineeringStatics, https://engineeringstatics.org/Chapter_10-moment-of-inertia-of-composite-shapes.html#:~:text=For%20a%20composite%20shape%20made,total%20of%20the%20positive%20areas (accessed May 20, 2023).

V. Appendix

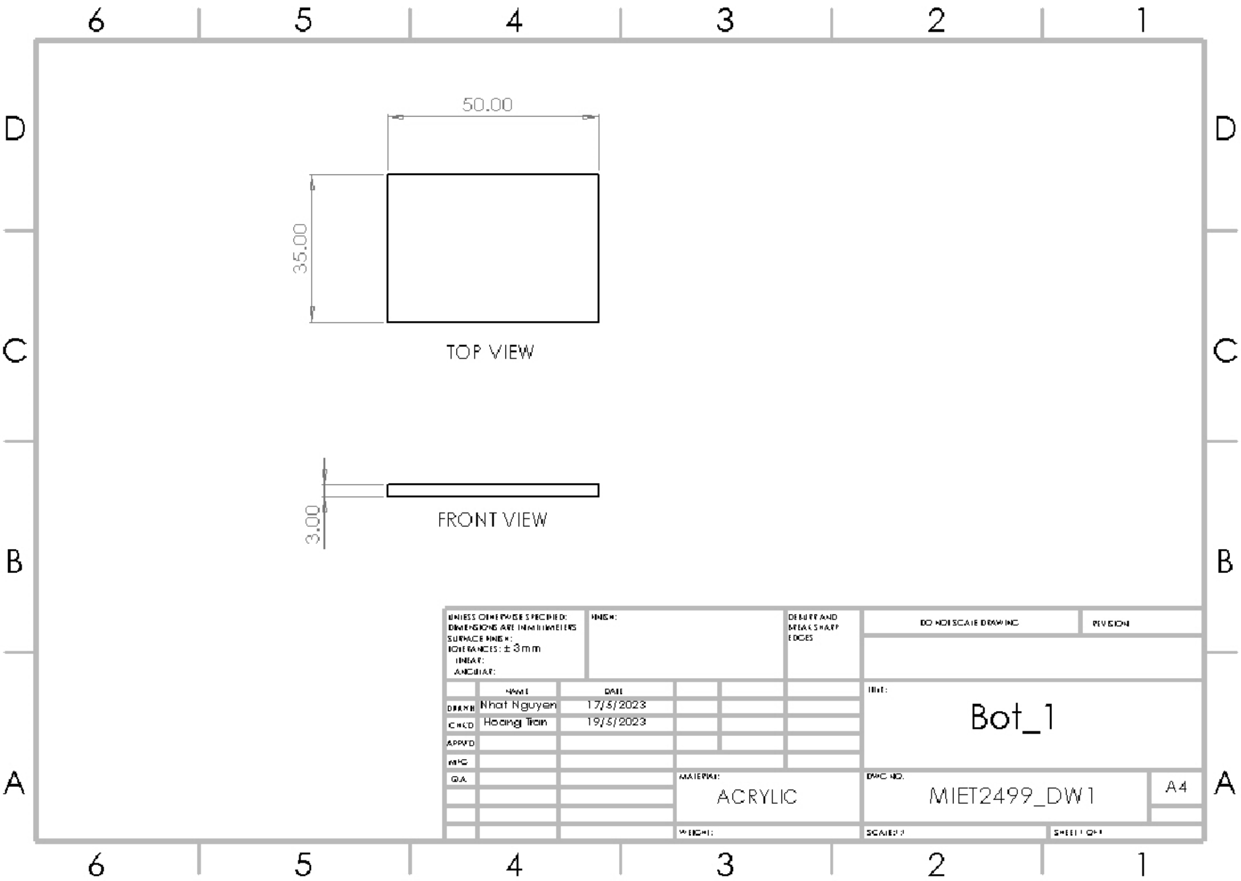

Figure 28: Bot_1 Drawing

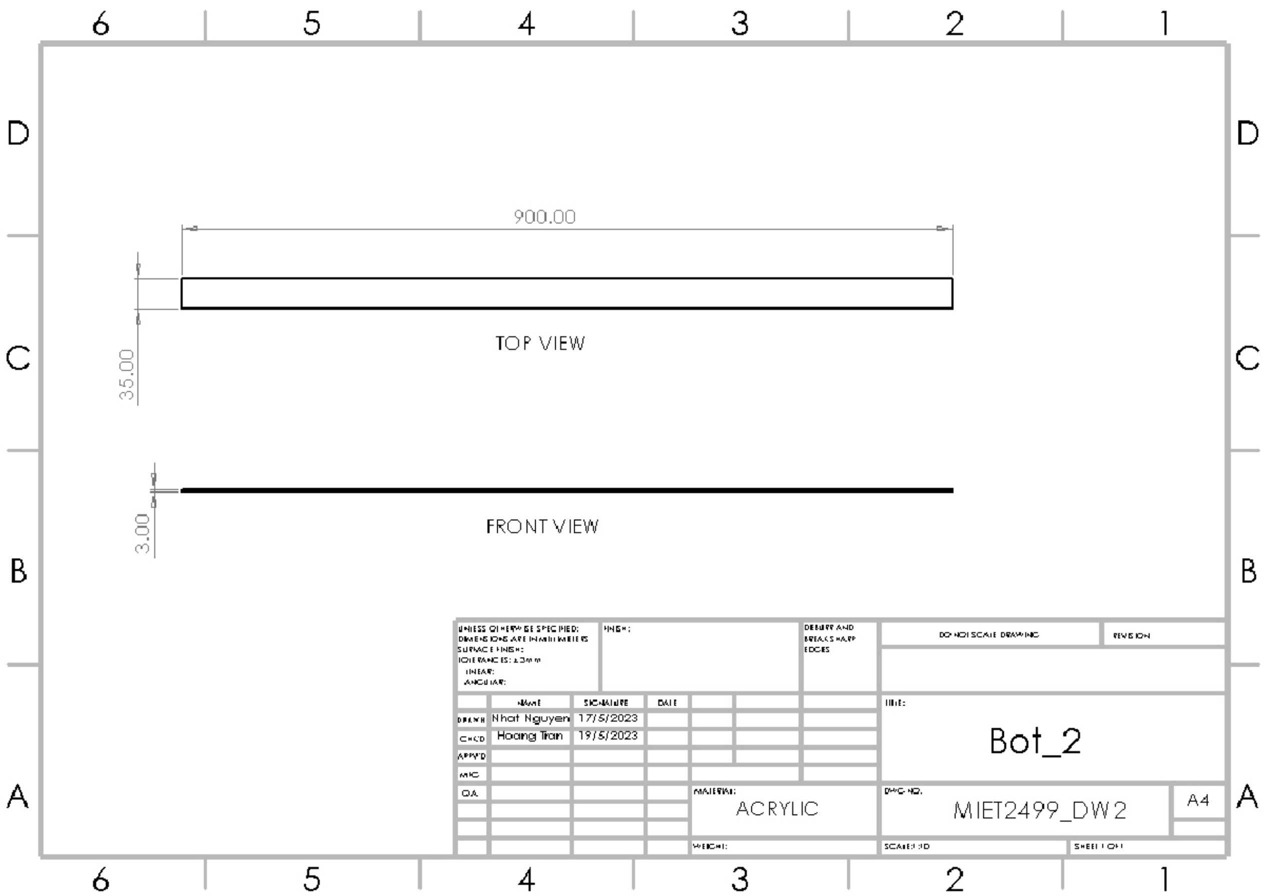

Figure 29: Bot_2 Drawing

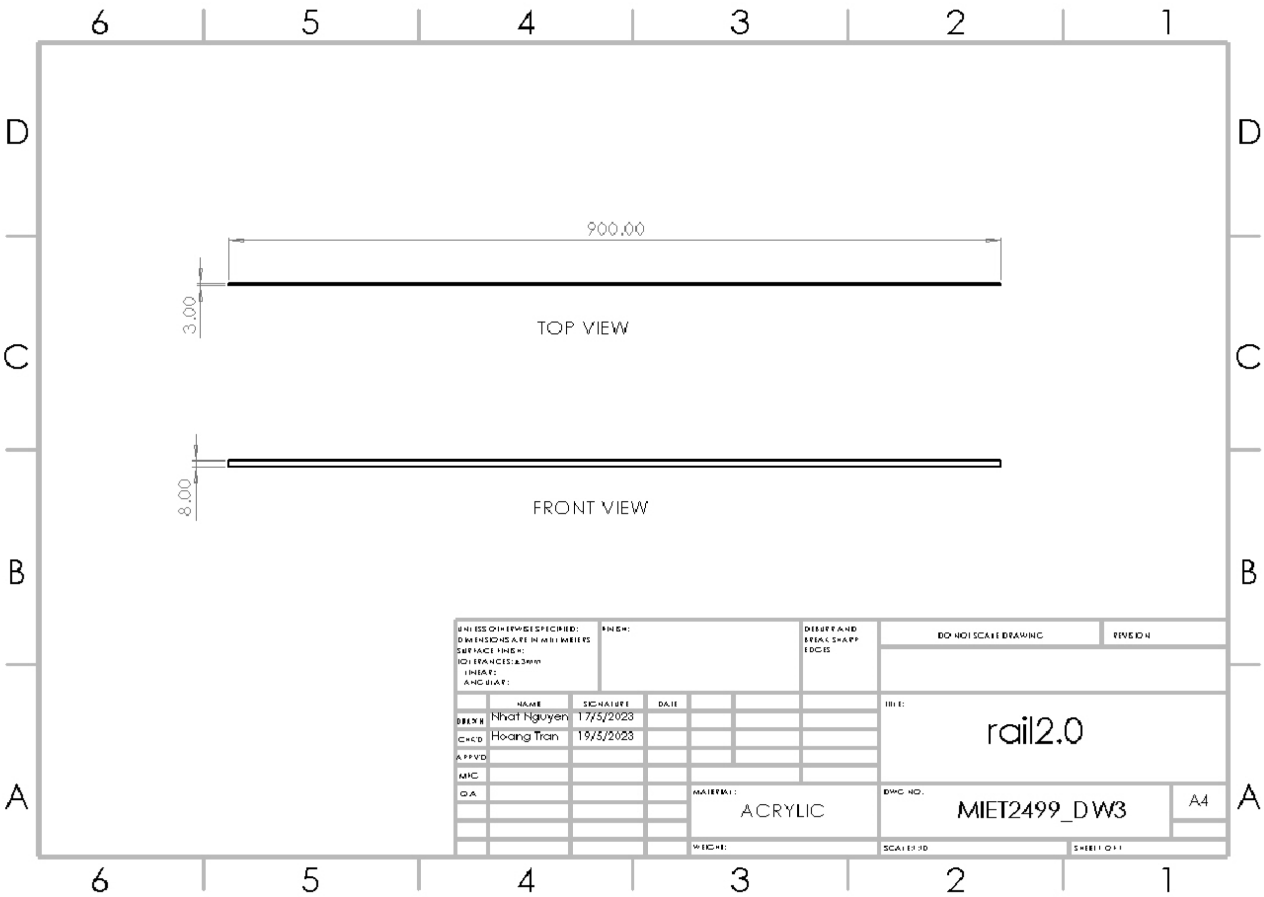

Figure 30: Rail Drawing

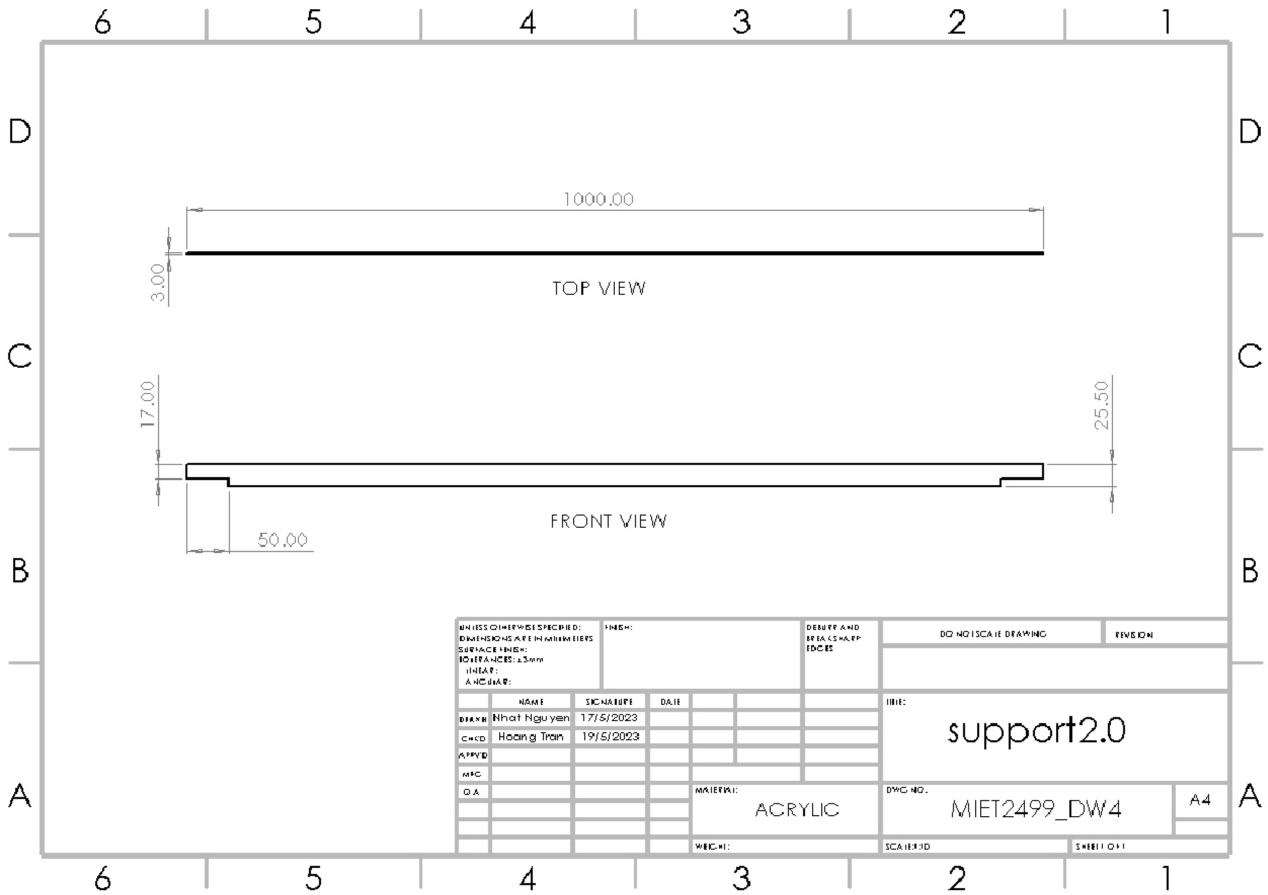

Figure 31: Support Drawing