Master SMT

Sustainable and Entrepreneurial Finance

Assignment 1
Due date: March 11, 2024 at the beginning of the class

Objectives:

The objectives of this homework are the following:

- Evaluate the impact of diversification in portfolio construction
- Build portfolios of stocks based on the mean-variance criterion (efficient frontier)
- Evaluate the effect of imposing restrictions on the set of assets (positivity)

Instructions:

- Assignments should be done in groups of 4 students.
- You should work with the same group through the entire course.
- Submit on Moodle only one copy of the solution per group, with the code in an independent file.
- For each assignment, you can get a maximum of 100 points.
- All assignments turned in late will not be graded (zero point).

Additional important information:

- The data is collected from 2000 to 2022 for prices and market values (monthly frequency), from 2002 to 2022 for ESG scores (annual frequency), and from 2005 to 2021 for carbon data (annual frequency). Given the availability of data, the allocation will be run annually from the end of 2007 (for 2008) up the the end of 2021 (for 2022). Ex-post portfolio returns will be available from January 2008 to December 2022.

- Price data are collected at the monthly frequency. Prices should be converted into returns using the simple return definition. Returns observed between month t and month $t+1$ are collected in the vector $R_{t+1} = \{R_{1,t+1}, ..., R_{N,t+1}\}$, where N denotes the number of firms. The first 8 years of data (2000 to 2007) are used to initialize the calculation of expected returns and the covariance matrix.

- Revenues and CO_2 emissions are updated annually. Some data might be missing. In such instances, interpolate the missing number using the average of the previous and next year. If the missing number corresponds to 2005, take the number of 2006. The allocation is based on information available at the end of the year (score, carbon and return data) for the next year. The portfolio is rebalanced only once per year. However, the performance of the portfolio is calculated month by month.

- It is important to distinguish the frequency of returns (monthly) and the frequency of rebalancing (annual). The investment decision is made at the end of a given year for the next year. However, expected returns and the covariance must be calculated based on monthly returns. So, to avoid confusion, I use the index t when the variable is monthly and the index Y when the variable is annual. The reason for computing monthly return is that this allows us to compute some important characteristics of the portfolio, such as its volatility and Sharpe ratio.

All data will be available on Moodle. Once you have selected your set of firms, answer the following questions:

Portfolio Allocation based on Financial Performance

1. Compute the annualized average return and annualized volatility for all individual assets over the period 2008–2022. Plot and comment the histogram of these distributions. Compute and comment on the correlation between individual average returns and volatilities in the cross section. (10 points)

2. Form the equally-weighted portfolio and the market-cap (or value) weighted portfolio with monthly rebalancing over the period 2008–2022. The performance of the equally-weighted portfolio at date $t+1$ is $R_{t+1}^{(ew)} = \frac{1}{N}\sum_{i=1}^{N} R_{i,t+1}$. The performance of the value-weighted portfolio at date $t+1$ is $R_{t+1}^{(vw)} = \sum_{i=1}^{N} w_{i,t} R_{i,t+1}$, where $w_{i,t} = Cap_{i,t}/\sum_{j=1}^{N} Cap_{j,t}$ denotes the relative market capitalization of firm i at date t. We call this portfolio "$P^{(vw)}$".

   Report the following statistics for both portfolios: annualized average return, annualized volatility, Sharpe ratio, minimum return, and maximum return. Plot the time series of cumulated returns for both portfolios. (15 points)

3. Take all firms in your dataset for which you have an E/S/G score or carbon emissions from 2007 on. The objective is to build the efficient frontier over the sample period. Compute the average (or expected) return and the covariance matrix matrix for the 2008–2021 period, using the following formulas: $\hat{\mu} = \frac{1}{T}\sum_{k=0}^{T-1} R_{t+1}$ for the expected return and $\hat{\Sigma} = \frac{1}{T}\sum_{k=0}^{T-1}(R_{t+1} - \hat{\mu})'(R_{t+1} - \hat{\mu})$ for the covariance matrix, where T is the number of months.

   Now, you compute the minimum variance portfolio and the maximum return portfolio, using the optimizer. Often, the covariance matrix $\hat{\Sigma}$ is not invertible, so using the optimal weight formula will not work. Instead, you use the following optimization problems, while restricting the optimal weights to be positive:

   Minimum variance portfolio:

$$\begin{aligned} &\min_{\{\alpha\}} && \sigma_p^2 = \alpha'\hat{\Sigma}\alpha \\ &s.t. && \alpha' e = 1 && e = (1, ..., 1)' \\ &s.t. && \alpha_i \geq 0 \quad \text{for all } i \end{aligned}$$

   Maximum return portfolio:

$$\begin{aligned} &\max_{\{\alpha\}} && \mu_p = \alpha'\hat{\mu} \\ &s.t. && \alpha' e = 1 \\ &s.t. && \alpha_i \geq 0 \quad \text{for all } i \end{aligned}$$

   Then, you build the efficient frontier: the optimal portfolio corresponding to a given target portfolio return $\tilde{\mu}_p$ is obtained by solving:

$$\begin{aligned} &\min_{\{\alpha\}} && \sigma_p^2 = \alpha'\hat{\Sigma}\alpha \\ &s.t. && \mu_p = \alpha'\hat{\mu} \geq \tilde{\mu}_p \\ &s.t. && \alpha' e = 1 \\ &s.t. && \alpha_i \geq 0 \quad \text{for all } i \end{aligned}$$

   Target returns should be selected between the return of the minimum variance and maximum return portfolios, with an increment of 1% for instance.

   Compute the ex-post performance of each portfolio along the efficient frontier. The portfolio return can be computed every month using monthly stock returns R_{t+1} and optimal weights α^*. This gives: $R_{p,t+1} = \alpha^{*\prime} R_{t+1}$, for $t = 0, ..., T-1$. For a given portfolio on the efficient frontier, we now have a time series of ex-post returns: $\{R_{p,1}, ..., R_{p,T}\}$. For all these portfolios, compute the ex-post annualized average return, the ex-post annualized volatility, and the ex-post annualized Sharpe ratio. Which portfolio had the highest performance in terms of ex-ante Sharpe ratio? Which portfolio has produced the highest performance in terms of ex-post Sharpe ratio?

In the same figure, plot the efficient frontier (curve), the individual assets (dots), the capital allocation line, and the equally-weighted and value-weighted portfolios (crosses). Compare and comment the performance of the equally-weighted, value-weighted, and efficient portfolios over the same sample period. (35 points)

4. For this question, you take again all firms in your dataset for which you have an E/S/G score or carbon emissions. The objective is to build the minimum variance portfolio **out of sample** with an annual rebalancing. "Out of sample" means that you use only past data to compute the optimal portfolio for the next period. For instance, you use the first 8 years of monthly returns (from Jan. 2000 to Dec. 2007) to compute the vector of expected returns and the covariance matrix. Therefore, you have $\tau = 8 \times 12 = 96$ months of data available to compute the expected returns and the covariance matrix valid for the allocation at the end of Dec. 2007 for the allocation in 2008. Dec. 2007 corresponds to year $Y_0 = 2007$ for annual data and to $t_0 = \tau = 96$ for monthly data. Dec. 2008 corresponds to $Y_0 + 1 = 2008$ and $t_0 + 12 = 108$. You associate year Y to month t (Dec. of the year).

   Compute the expected returns as

$$\hat{\mu}_{Y+1} = \frac{1}{\tau} \sum_{k=0}^{\tau-1} R_{t-k}$$

   and the covariance matrix as

$$\hat{\Sigma}_{Y+1} = \frac{1}{\tau} \sum_{k=0}^{\tau-1} (R_{t-k} - \hat{\mu}_{Y+1})'(R_{t-k} - \hat{\mu}_{Y+1})$$

   The allocation is determined at the end of year Y for year $Y+1$. Compute the optimal allocation of the portfolio using Markowitz's approach, using the following minimization problem, while restricting the optimal weights to be positive:

$$\begin{aligned} \min_{\{\alpha_Y\}} \quad & \sigma^2_{p,Y+1} = \alpha_Y' \hat{\Sigma}_{Y+1} \alpha_Y \\ s.t. \quad & \alpha_Y' e = 1 \\ s.t. \quad & \alpha_{i,Y} \geq 0 \qquad \text{for all } i \end{aligned}$$

Then, roll the window by one year and iterate until the end of the sample, so that your portfolio is rebalanced every year from Dec. 2007 to Dec. 2020.

Compute the ex-post performance of the portfolio. The portfolio return can be computed every month of year $Y+1$ using monthly stock returns of year $Y+1$ and optimal weights calculated at the end of year Y. Be careful: every month, the weights should be updated to take the performance of the stocks of the previous month into account. This gives: $R_{p,t+k} = \alpha'_{t+k-1} R_{t+k}$, for $k = 1, ..., 12$, where $\alpha_{i,t+k-1} = \alpha_{i,t+k-2} \times (1 + R_{i,t+k-1})/(1 + R_{p,t+k-1})$, with $\alpha_t = \alpha_Y$. You now have a time series of ex post portfolio returns: $\{R_{p,\tau+1}, ..., R_{p,T}\}$, where T denotes the total number of months in the sample.

Compute the characteristics of this portfolio (denoted "$P_{oos}^{(gmv)}$") over the sample: annualized average return ($\bar{\mu}_p$), annualized volatility (σ_p), Sharpe ratio (SR_p), minimum return, and maximum return. Compare these properties to those of the value-weighted portfolio ($P^{(vw)}$) ("benchmark"). (40 points)