$$[\text{notL}] \quad \frac{\Gamma \Longrightarrow \phi, \Delta}{\Gamma, \neg\phi \Longrightarrow \Delta}$$

$$[\text{notR}] \quad \frac{\Gamma, \phi \Longrightarrow \Delta}{\Gamma \Longrightarrow \neg\phi, \Delta}$$

$$[\text{andL}] \quad \frac{\Gamma, \phi, \psi \Longrightarrow \Delta}{\Gamma, \phi \wedge \psi \Longrightarrow \Delta}$$

$$[\text{andR}] \quad \frac{\Gamma \Longrightarrow \phi, \Delta \qquad \Gamma \Longrightarrow \psi, \Delta}{\Gamma \Longrightarrow \phi \wedge \psi, \Delta}$$

$$[\text{orL}] \quad \frac{\Gamma, \phi \Longrightarrow \Delta \qquad \Gamma, \psi \Longrightarrow \Delta}{\Gamma, \phi \vee \psi \Longrightarrow \Delta}$$

$$[\text{orR}] \quad \frac{\Gamma \Longrightarrow \phi, \psi, \Delta}{\Gamma \Longrightarrow \phi \vee \psi, \Delta}$$

$$[\text{impL}] \quad \frac{\Gamma \Longrightarrow \phi, \Delta \qquad \Gamma, \psi \Longrightarrow \Delta}{\Gamma, \phi \rightarrow \psi \Longrightarrow \Delta}$$

$$[\text{impR}] \quad \frac{\Gamma, \phi \Longrightarrow \psi, \Delta}{\Gamma \Longrightarrow \phi \rightarrow \psi, \Delta}$$

$$[\text{allL}] \quad \frac{\Gamma, \forall \tau\, x;\, \phi,\, [x/t']\phi \Longrightarrow \Delta}{\Gamma, \forall \tau\, x;\, \phi \Longrightarrow \Delta}$$

$$[\text{allR}] \quad \frac{\Gamma \Longrightarrow [x/c]\phi, \Delta}{\Gamma \Longrightarrow \forall \tau\, x;\, \phi, \Delta}$$

$$[\text{exL}] \quad \frac{\Gamma, [x/c]\phi \Longrightarrow \Delta}{\Gamma, \exists \tau\, x;\, \phi \Longrightarrow \Delta}$$

$$[\text{exR}] \quad \frac{\Gamma \Longrightarrow [x/t']\phi,\, \exists \tau\, x;\, \phi, \Delta}{\Gamma \Longrightarrow \exists \tau\, x;\, \phi, \Delta}$$

$$[\text{close}] \quad \frac{}{\Gamma, \phi \Longrightarrow \phi, \Delta}$$

$$[\text{closeT}] \quad \frac{}{\Gamma \Longrightarrow \text{true}, \Delta}$$

$$[\text{closeF}] \quad \frac{}{\Gamma, \text{false} \Longrightarrow \Delta}$$

- ▶ $[t/t']\phi$ is result of replacing each occurrence of t in ϕ with t'
- ▶ t, t' variable-free terms of type τ
- ▶ c **new** constant of type τ (occurs not on current proof branch)