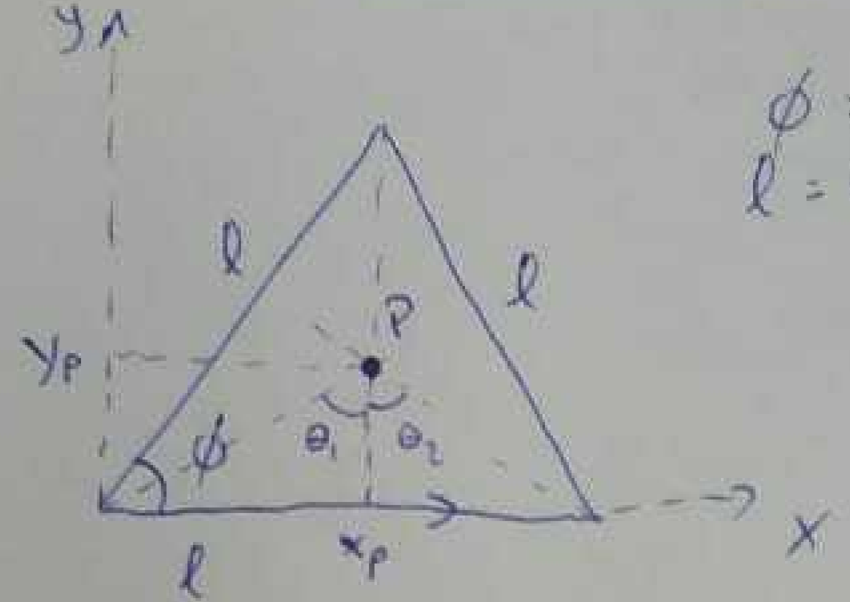
P2



$\phi = 60°$
$l = 0.1$

$\therefore \quad tg\left(\frac{\phi}{2}\right) = \frac{y_P}{x_P}$

$$\Rightarrow \boxed{y_P = \frac{l}{2}\, tg(30°)} \quad \boxed{x_P = \frac{l}{2}}$$

Forma 1

$$\bar{B}(\bar{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\bar{l} \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3}$$

con $\left.\begin{array}{l} \bar{r} = x_P\,\hat{i} + y_P\,\hat{j} + 0\,\hat{k} \\ \bar{r}' = x\,\hat{i} + 0\,\hat{j} + 0\,\hat{k} \end{array}\right\}$ $\begin{array}{l} \bar{r} - \bar{r}' = (x_P - x)\,\hat{i} + y_P\,\hat{j} + 0\,\hat{k} \\ |\bar{r} - \bar{r}'| = \left[(x_P - x)^2 + y_P^2\right]^{1/2} \end{array}$

$$d\bar{l} = dx\,\hat{i} + 0\,\hat{j} + 0\,\hat{k}$$

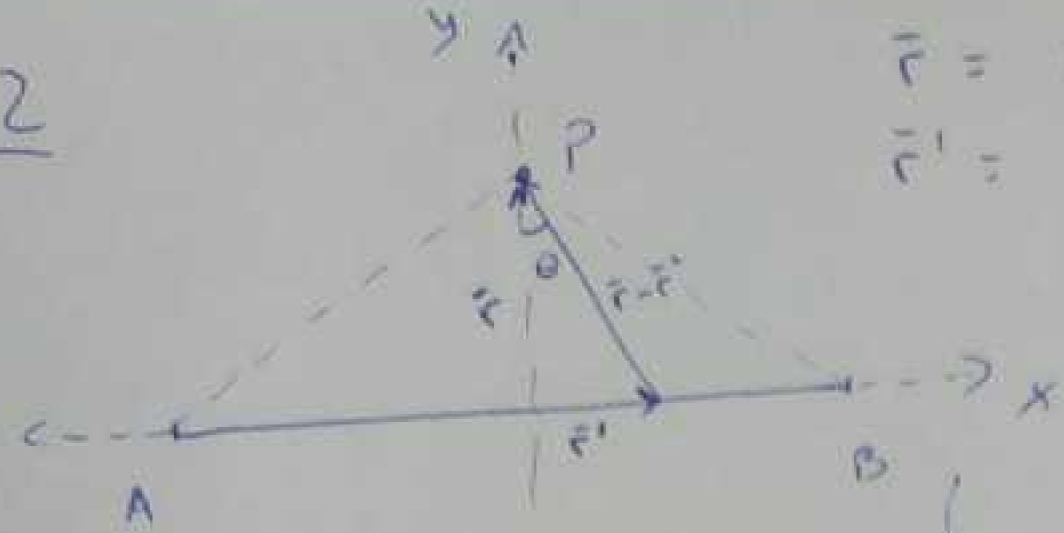
$$d\bar{l} \times (\bar{r} - \bar{r}') = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & 0 & 0 \\ x_P - x & y_P & 0 \end{vmatrix} = y_P\, dx\, \hat{k}$$

$$\therefore \quad \bar{B}(P) = \frac{\mu_0 I}{4\pi}\, y_P \int \frac{dx}{\left[(x_P - x)^2 + y_P^2\right]^{3/2}}\, \hat{k}$$

$$\bar{B}(P) = \frac{\mu_0 I}{4\pi}\, \frac{l}{2}\, tg(30°) \int_0^l \frac{dx}{\left[\left(\frac{l}{2} - x\right)^2 + \left(\frac{l}{2}\, tg(30°)\right)^2\right]^{3/2}}\, \hat{k}$$

$$\boxed{\bar{B}(P) = \frac{\mu_0 I}{4\pi} \cdot \frac{6}{l}\, \hat{k}}$$

Forma 2



$\bar{r} = 0\,\hat{i} + y_P\,\hat{j} + 0\,\hat{k}$

$\bar{r}' = x\,\hat{i} + 0\,\hat{j} + 0\,\hat{k}$

$y_P = \frac{l}{2}\,tg(30^\circ)$

$$\bar{B}(P) = \frac{\mu_0 I}{4\pi}\int \frac{d\bar{l} \times (\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$$

$\bar{r}-\bar{r}' = -x\,\hat{i} + y_P\,\hat{j} + 0\,\hat{k}$

$|\bar{r}-\bar{r}'| = (x^2 + y_P^2)^{1/2}$

$d\bar{l} = dx\,\hat{i} + 0\,\hat{j} + 0\,\hat{k}$

$$d\bar{l} \times (\bar{r}-\bar{r}') = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & 0 & 0 \\ -x & y_P & 0 \end{vmatrix} = y_P\,dx\,\hat{k}$$

$$\therefore\ \bar{B}(P) = \frac{\mu_0 I}{4\pi}\,y_P \int \frac{dx}{(x^2+y_P^2)^{3/2}}\,\hat{k}$$

con $x = y_P\,tg(\theta)$ $\quad \therefore\ dx = y_P \sec^2(\theta)\,d\theta$

$x^2 = y_P^2\,tg^2(\theta) \Rightarrow x^2 + y_P^2 = y_P^2\,tg^2(\theta) + y_P^2 = y_P^2\,(tg^2\theta + 1)$

$$x^2 + y_P^2 = y_P^2\left(\frac{sen^2\theta + \cos^2\theta}{\cos^2\theta}\right) = \frac{y_P^2}{\cos^2\theta}$$

$$x^2 + y_P^2 = y_P^2 \sec^2\theta$$

$$(x^2 + y_P^2)^{3/2} = y_P^3 \sec^3\theta$$

$$\therefore\ \bar{B}(P) = \frac{\mu_0 I}{4\pi}\,\cancel{y_P}\int_{\theta_1}^{\theta_2} \frac{\cancel{y_P}\,\cancel{\sec^2}\theta\,d\theta}{y_P^{\cancel{3}}\,\sec^{\cancel{3}}\theta}\,\hat{k}$$

$$\bar{B}(P) = \frac{\mu_0 I}{4\pi y_P}\int_{\theta_1=-60^\circ}^{\theta_2=60^\circ} \cos\theta\,d\theta\,\hat{k} = \frac{\mu_0 I}{4\pi y_P}\underbrace{\left[Sen(60^\circ) - sen(-60^\circ)\right]}_{\sqrt{3}}\hat{k}$$

$$\Rightarrow \quad \bar{B}(P) = \frac{\mu_0 I}{4\pi} \cdot \frac{1}{\frac{l}{2}\, tg(30^\circ)} \cdot \sqrt{3} \quad \hat{k}$$

$$\boxed{\bar{B}(P) = \frac{\mu_0 I}{4\pi} \frac{6}{l} \hat{k}}$$

(b) Por simetría, $\bar{B}(P)$ de la espira triangular es $\bar{B}(P) = 3 \cdot \bar{B}(P)\Big|_{AB}$ ya que el punto P es equidistante de cada tramo y simétrico respecto al centro

(c) $\int d\bar{F} = I \int d\bar{l} \times \bar{B}$

$d\bar{l} = dx\, \hat{i} + 0\, \hat{j} + 0\, \hat{k}$

$\bar{B} = 0\, \hat{i} + 0\, \hat{j} + 3\, \hat{k}$

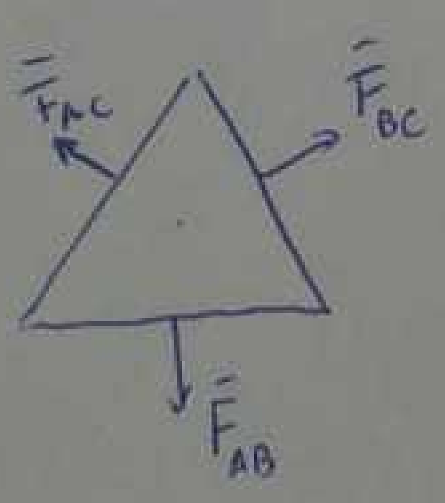
$$d\bar{l} \times \bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & 0 & 0 \\ 0 & 0 & 3 \end{vmatrix} = -3dx\, \hat{j}$$

$$\therefore \quad \bar{F}_{AB} = I \int_0^l -3\, dx\, \hat{j}$$

$$\boxed{\bar{F}_{AB} = -3 I l\, \hat{j}}$$

(d) $\sum \bar{F}$ a lo largo de la espira es "cero" ya que es una espira cerrada inmersa en un $\bar{B}$ uniforme.

(e)



$\bar{\tau} = 0$ ya que $\hat{n}$ de la espira es // a $\bar{B}$