# INTEGRALES INDEFINIDAS

## REGLAS GENERALES

A continuación $u$, $v$, $w$ son funciones de $x$; $a$, $b$, $p$, $q$, $n$, son constantes

$$\int a\,dx = ax$$

$$\int af(x)\,dx = a\int f(x)\,dx$$

$$\int (u \pm v \pm w \pm \cdots)\,dx = \int u\,dx \pm \int v\,dx \pm \int w\,dx \pm \cdots$$

$$\int u\,dv = uv - \int v\,du \qquad \text{[Integración por partes]}$$

$$\int f(ax)\,dx = \frac{1}{a}\int f(u)\,du$$

$$\int F\{f(x)\}\,dx = \int F(u)\frac{dx}{du}\,du = \int \frac{F(u)}{f'(x)}\,du$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1}, \quad n \neq -1$$

$$\int \frac{du}{u} = \ln u \quad \text{si } u > 0 \text{ o } \ln(-u) \text{ si } u < 0$$
$$= \ln|u|$$

$$\int e^u\,du = e^u$$

$$\int a^u\,du = \int e^{u\ln a}\,du = \frac{e^{u\ln a}}{\ln a} = \frac{a^u}{\ln a}, \quad a > 0,\ a \neq 1$$

$$\int \operatorname{sen} u\,du = -\cos u$$

$$\int \cos u\,du = \operatorname{sen} u$$

$$\int \tan u\,du = \ln\sec u = -\ln\cos u$$

$$\int \cot u\,du = \ln\operatorname{sen} u$$

$$\int \sec u\,du = \ln(\sec u + \tan u) = \ln\tan\left(\frac{u}{2}+\frac{\pi}{4}\right)$$

$$\int \csc u\, du = \ln(\csc u - \cot u) = \ln \tan \frac{u}{2}$$

$$\int \sec^2 u\, du = \tan u$$

$$\int \csc^2 u\, du = -\cot u$$

$$\int \tan^2 u\, du = \tan u - u$$

$$\int \cot^2 u\, du = -\cot u - u$$

$$\int \operatorname{sen}^2 u\, du = \frac{u}{2} - \frac{\operatorname{sen} 2u}{4} = \tfrac{1}{2}(u - \operatorname{sen} u \cos u)$$

$$\int \cos^2 u\, du = \frac{u}{2} + \frac{\operatorname{sen} 2u}{4} = \tfrac{1}{2}(u + \operatorname{sen} u \cos u)$$

$$\int \sec u \tan u\, du = \sec u$$

$$\int \csc u \cot u\, du = -\csc u$$

$$\int \operatorname{senh} u\, du = \cosh u$$

$$\int \cosh u\, du = \operatorname{senh} u$$

$$\int \tanh u\, du = \ln \cosh u$$

$$\int \coth u\, du = \ln \operatorname{senh} u$$

$$\int \operatorname{sech} u\, du = \operatorname{sen}^{-1}(\tanh u) \quad \text{o} \quad 2\tan^{-1} e^u$$

$$\int \operatorname{csch} u\, du = \ln \tanh \frac{u}{2} \quad \text{o} \quad -\coth^{-1} e^u$$

$$\int \operatorname{sech}^2 u\, du = \tanh u$$

$$\int \operatorname{csch}^2 u\, du = -\coth u$$

$$\int \tanh^2 u\, du = u - \tanh u$$

$$\int \coth^2 u\, du = u - \coth u$$

$$\int \operatorname{senh}^2 u\, du \qquad \frac{\operatorname{senh} 2u}{4} - \frac{u}{2} = \tfrac{1}{2}(\operatorname{senh} u \cosh u - u)$$

$$\int \cosh^2 u\, du = \frac{\operatorname{senh} 2u}{4} + \frac{u}{2} = \tfrac{1}{2}(\operatorname{senh} u \cosh u + u)$$

$$\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u$$

$$\int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u$$

$$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a}$$

$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln\left(\frac{u - a}{u + a}\right) = -\frac{1}{a} \coth^{-1} \frac{u}{a} \qquad u^2 > a^2$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln\left(\frac{a + u}{a - u}\right) = \frac{1}{a} \tanh^{-1} \frac{u}{a} \qquad u^2 < a^2$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{sen}^{-1} \frac{u}{a}$$

$$\int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2}) \quad \text{o} \quad \operatorname{senh}^{-1} \frac{u}{a}$$

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln(u + \sqrt{u^2 - a^2})$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1}\left|\frac{u}{a}\right|$$

$$\int \frac{du}{u\sqrt{u^2 + a^2}} = -\frac{1}{a} \ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right)$$

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$$\int f^{(n)} g\, dx = f^{(n-1)} g - f^{(n-2)} g' + f^{(n-3)} g'' - \cdots (-1)^n \int f g^{(n)}\, dx$$

Esta última es llamada *fórmula generalizada de integración* por partes.

## SUSTITUCIONES IMPORTANTES

$$\int F(ax + b)\, dx = \frac{1}{a} \int F(u)\, du \qquad \text{donde} \quad u = ax + b$$

$$\int F(\sqrt{ax + b})\, dx = \frac{2}{a} \int u F(u)\, du \qquad \text{donde} \quad u = \sqrt{ax + b}$$

$$\int F(\sqrt[n]{ax+b})\,dx = \frac{n}{a}\int u^{n-1}F(u)\,du \qquad \text{donde} \quad u = \sqrt[n]{ax+b}$$

$$\int F(\sqrt{a^2-x^2})\,dx = a\int F(a\cos u)\cos u\,du \qquad \text{donde} \quad x = a\,\text{sen}\,u$$

$$\int F(\sqrt{x^2+a^2})\,dx = a\int F(a\sec u)\sec^2 u\,du \qquad \text{donde} \quad x = a\tan u$$

$$\int F(\sqrt{x^2-a^2})\,dx = a\int F(a\tan u)\sec u\tan u\,du \qquad \text{donde} \quad x = a\sec u$$

$$\int F(e^{ax})\,dx = \frac{1}{a}\int \frac{F(u)}{u}\,du \qquad \text{donde} \quad u = e^{ax}$$

$$\int F(\ln x)\,dx = \int F(u)\,e^u\,du \qquad \text{donde} \quad u = \ln x$$

$$\int F\left(\text{sen}^{-1}\frac{x}{a}\right)dx = a\int F(u)\cos u\,du \qquad \text{donde} \quad u = \text{sen}^{-1}\frac{x}{a}$$

Resultados similares se aplican para otras funciones trigonométricas recíprocas

$$\int F(\text{sen}\,x, \cos x)\,dx = 2\int F\left(\frac{2u}{1+u^2}, \frac{1-u^2}{1+u^2}\right)\frac{du}{1+u^2} \qquad \text{donde} \quad u = \tan\frac{x}{2}$$

## INTEGRALES NOTABLES
## INTEGRALES QUE CONTIENEN $ax + b$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b)$$

$$\int \frac{x\,dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b)$$

$$\int \frac{x^2\,dx}{ax+b} = \frac{(ax+b)^2}{2a^3} - \frac{2b(ax+b)}{a^3} + \frac{b^2}{a^3}\ln(ax+b)$$

$$\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left(\frac{x}{ax+b}\right)$$

$$\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left(\frac{ax+b}{x}\right)$$

$$\int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3}\ln\left(\frac{x}{ax+b}\right)$$

$$\int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$$

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Resultados similares se aplican para otras funciones trigonométricas recíprocas

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$$\int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$$

$$\int \frac{x\,dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln(ax+b)$$

$$\int \frac{x^2\,dx}{(ax+b)^2} = \frac{ax+b}{a^3} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3}\ln(ax+b)$$

$$\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} + \frac{1}{b^2}\ln\left(\frac{x}{ax+b}\right)$$

$$\int \frac{dx}{x^2(ax+b)^2} = \frac{-a}{b^2(ax+b)} - \frac{1}{b^2x} + \frac{2a}{b^3}\ln\left(\frac{ax+b}{x}\right)$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)a}$$

$$\int x(ax+b)^n\,dx = \frac{(ax+b)^{n+2}}{(n+2)a^2} - \frac{b(ax+b)^{n+1}}{(n+1)a^2}, \quad n \neq -1, -2$$

$$\int x^2(ax+b)^n\,dx = \frac{(ax+b)^{n+3}}{(n+3)a^3} - \frac{2b(ax+b)^{n+2}}{(n+2)a^3} + \frac{b^2(ax+b)^{n+1}}{(n+1)a^3}$$

$$\int x^m(ax+b)^n\,dx = \begin{cases} \dfrac{x^{m+1}(ax+b)^n}{m+n+1} + \dfrac{nb}{m+n+1}\displaystyle\int x^m(ax+b)^{n-1}\,dx \\ \dfrac{x^m(ax+b)^{n+1}}{(m+n+1)a} - \dfrac{mb}{(m+n+1)a}\displaystyle\int x^{m-1}(ax+b)^n\,dx \\ \dfrac{-x^{m+1}(ax+b)^{n+1}}{(n+1)b} + \dfrac{m+n+2}{(n+1)b}\displaystyle\int x^m(ax+b)^{n+1}\,dx \end{cases}$$

## $\sqrt{ax+b}$

$$\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$$

$$\int \frac{x\,dx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}$$

$$\int \frac{x^2\,dx}{\sqrt{ax+b}} = \frac{2(3a^2x^2-4abx+8b^2)}{15a^3}\sqrt{ax+b}$$

$$\int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \dfrac{1}{\sqrt{b}}\ln\left(\dfrac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right) \\ \dfrac{2}{\sqrt{-b}}\tan^{-1}\sqrt{\dfrac{ax+b}{-b}} \end{cases}$$

$$\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}}$$

$$\int \sqrt{ax+b}\,dx = \frac{2\sqrt{(ax+b)^3}}{3a}$$

$$\int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)}{15a^2}\sqrt{(ax+b)^3}$$

$$\int x^2\sqrt{ax+b}\,dx = \frac{2(15a^2x^2-12abx+8b^2)}{105a^3}\sqrt{(a+bx)^3}$$

$$\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int\frac{dx}{x\sqrt{ax+b}}$$

$$\int \frac{\sqrt{ax+b}}{x^2}\,dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int\frac{dx}{x\sqrt{ax+b}}$$

$$\int \frac{x^m}{\sqrt{ax+b}}\,dx = \frac{2x^m\sqrt{ax+b}}{(2m+1)a} - \frac{2mb}{(2m+1)a}\int\frac{x^{m-1}}{\sqrt{ax+b}}\,dx$$

$$\int \frac{dx}{x^m\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(m-1)bx^{m-1}} - \frac{(2m-3)a}{(2m-2)b}\int\frac{dx}{x^{m-1}\sqrt{ax+b}}$$

$$\int x^m\sqrt{ax+b}\,dx = \frac{2x^m}{(2m+3)a}(ax+b)^{3/2} - \frac{2mb}{(2m+3)a}\int x^{m-1}\sqrt{ax+b}\,dx$$

$$\int \frac{\sqrt{ax+b}}{x^m}\,dx = -\frac{\sqrt{ax+b}}{(m-1)x^{m-1}} + \frac{a}{2(m-1)}\int\frac{dx}{x^{m-1}\sqrt{ax+b}}$$

$$\int \frac{\sqrt{ax+b}}{x^m}\,dx = \frac{-(ax+b)^{3/2}}{(m-1)bx^{m-1}} - \frac{(2m-5)a}{(2m-2)b}\int\frac{\sqrt{ax+b}}{x^{m-1}}\,dx$$

$$\int (ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+2)/2}}{a(m+2)}$$

$$\int x(ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+4)/2}}{a^2(m+4)} - \frac{2b(ax+b)^{(m+2)/2}}{a^2(m+2)}$$

$$\int x^2(ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+6)/2}}{a^3(m+6)} - \frac{4b(ax+b)^{(m+4)/2}}{a^3(m+4)} + \frac{2b^2(ax+b)^{(m+2)/2}}{a^3(m+2)}$$

$$\int \frac{(ax+b)^{m/2}}{x}\,dx = \frac{2(ax+b)^{m/2}}{m} + b\int\frac{(ax+b)^{(m-2)/2}}{x}\,dx$$

$$\int \frac{(ax+b)^{m/2}}{x^2}\,dx = -\frac{(ax+b)^{(m+2)/2}}{bx} + \frac{ma}{2b}\int\frac{(ax+b)^{m/2}}{x}\,dx$$

$$\int \frac{dx}{x(ax+b)^{m/2}} = \frac{2}{(m-2)b(ax+b)^{(m-2)/2}} + \frac{1}{b}\int\frac{dx}{x(ax+b)^{(m-2)/2}}$$

## $ax+b$ Y $px+q$

$$\int \frac{dx}{(ax+b)(px+q)} = \frac{1}{bp-aq}\ln\left(\frac{px+q}{ax+b}\right)$$

$$\int \frac{x\,dx}{(ax+b)(px+q)} = \frac{1}{bp-aq}\left\{\frac{b}{a}\ln(ax+b) - \frac{q}{p}\ln(px+q)\right\}$$

$$\int \frac{dx}{(ax+b)^2(px+q)} = \frac{1}{bp-aq}\left\{\frac{1}{ax+b} + \frac{p}{bp-aq}\ln\left(\frac{px+q}{ax+b}\right)\right\}$$

$$\int \frac{x\,dx}{(ax+b)^2(px+q)} = \frac{1}{bp-aq}\left\{\frac{q}{bp-aq}\ln\left(\frac{ax+b}{px+q}\right) - \frac{b}{a(ax+b)}\right\}$$

$$\int \frac{x^2\,dx}{(ax+b)^2(px+q)} = \frac{b^2}{(bp-aq)a^2(ax+b)} + \frac{1}{(bp-aq)^2}\left\{\frac{q^2}{p}\ln(px+q) + \frac{b(bp-2aq)}{a^2}\ln(ax+b)\right\}$$

$$\int \frac{dx}{(ax+b)^m(px+q)^n} = \frac{-1}{(n-1)(bp-aq)}\left\{\frac{1}{(ax+b)^{m-1}(px+q)^{n-1}} + a(m+n-2)\int\frac{dx}{(ax+b)^m(px+q)^{n-1}}\right\}$$

$$\int \frac{ax+b}{px+q}dx = \frac{ax}{p} + \frac{bp-aq}{p^2}\ln(px+q)$$

$$\int \frac{(ax+b)^m}{(px+q)^n}dx = \begin{cases} \dfrac{-1}{(n-1)(bp-aq)}\left\{\dfrac{(ax+b)^{m+1}}{(px+q)^{n-1}} + (n-m-2)a\displaystyle\int\frac{(ax+b)^m}{(px+q)^{n-1}}dx\right\} \\[2ex] \dfrac{-1}{(n-m-1)p}\left\{\dfrac{(ax+b)^m}{(px+q)^{n-1}} + m(bp-aq)\displaystyle\int\frac{(ax+b)^{m-1}}{(px+q)^n}dx\right\} \\[2ex] \dfrac{-1}{(n-1)p}\left\{\dfrac{(ax+b)^m}{(px+q)^{n-1}} - ma\displaystyle\int\frac{(ax+b)^{m-1}}{(px+q)^{n-1}}dx\right\} \end{cases}$$

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## $\sqrt{ax+b}$ Y $px+q$

$$\int \frac{px+q}{\sqrt{ax+b}}dx = \frac{2(apx+3aq-2bp)}{3a^2}\sqrt{ax+b}$$

$$\int \frac{dx}{(px+q)\sqrt{ax+b}} = \begin{cases} \dfrac{1}{\sqrt{bp-aq}\sqrt{p}}\ln\left(\dfrac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\[2ex] \dfrac{2}{\sqrt{aq-bp}\sqrt{p}}\tan^{-1}\sqrt{\dfrac{p(ax+b)}{aq-bp}} \end{cases}$$

$$\int \frac{\sqrt{ax+b}}{px+q}dx = \begin{cases} \dfrac{2\sqrt{ax+b}}{p} + \dfrac{\sqrt{bp-aq}}{p\sqrt{p}}\ln\left(\dfrac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\[2ex] \dfrac{2\sqrt{ax+b}}{p} - \dfrac{2\sqrt{aq-bp}}{p\sqrt{p}}\tan^{-1}\sqrt{\dfrac{p(ax+b)}{aq-bp}} \end{cases}$$

$$\int (px+q)^n \sqrt{ax+b}\,dx = \frac{2(px+q)^{n+1}\sqrt{ax+b}}{(2n+3)p} + \frac{bp-aq}{(2n+3)p}\int \frac{(px+q)^n}{\sqrt{ax+b}}\,dx$$

$$\int \frac{dx}{(px+q)^n\sqrt{ax+b}} = \frac{\sqrt{ax+b}}{(n-1)(aq-bp)(px+q)^{n-1}} + \frac{(2n-3)a}{2(n-1)(aq-bp)}\int \frac{dx}{(px+q)^{n-1}\sqrt{ax+b}}$$

$$\int \frac{(px+q)^n}{\sqrt{ax+b}}\,dx = \frac{2(px+q)^n\sqrt{ax+b}}{(2n+1)a} + \frac{2n(aq-bp)}{(2n+1)a}\int \frac{(px+q)^{n-1}\,dx}{\sqrt{ax+b}}$$

$$\int \frac{\sqrt{ax+b}}{(px+q)^n}\,dx = \frac{-\sqrt{ax+b}}{(n-1)p(px+q)^{n-1}} + \frac{a}{2(n-1)p}\int \frac{dx}{(px+q)^{n-1}\sqrt{ax+b}}$$

## $\sqrt{ax+b}$ Y $\sqrt{px+q}$

$$\int \frac{dx}{\sqrt{(ax+b)(px+q)}} = \begin{cases} \dfrac{2}{\sqrt{ap}}\ln\left(\sqrt{a(px+q)}+\sqrt{p(ax+b)}\right) \\ \dfrac{2}{\sqrt{-ap}}\tan^{-1}\sqrt{\dfrac{-p(ax+b)}{a(px+q)}} \end{cases}$$

$$\int \frac{x\,dx}{\sqrt{(ax+b)(px+q)}} = \frac{\sqrt{(ax+b)(px+q)}}{ap} - \frac{bp+aq}{2ap}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$\int \sqrt{(ax+b)(px+q)}\,dx = \frac{2apx+bp+aq}{4ap}\sqrt{(ax+b)(px+q)} - \frac{(bp-aq)^2}{8ap}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$\int \sqrt{\frac{px+q}{ax+b}}\,dx = \frac{\sqrt{(ax+b)(px+q)}}{a} + \frac{aq-bp}{2a}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$\int \frac{dx}{(px+q)\sqrt{(ax+b)(px+q)}} = \frac{2\sqrt{ax+b}}{(aq-bp)\sqrt{px+q}}$$

## $x^2+a^2$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2 (x^2+a^2)^{1/2}}$$

$$\int \frac{x\,dx}{x^2+a^2} = \frac{1}{2} \ln (x^2+a^2)$$

$$\int \frac{x^2\,dx}{x^2+a^2} = x - a \tan^{-1}\frac{x}{a}$$

$$\int \frac{x^3\,dx}{x^2+a^2} = \frac{x^2}{2} - \frac{a^2}{2} \ln (x^2+a^2)$$

$$\int \frac{dx}{x(x^2+a^2)} = \frac{1}{2a^2} \ln \left(\frac{x^2}{x^2+a^2}\right)$$

$$\int \frac{dx}{x^2(x^2+a^2)} = -\frac{1}{a^2x} - \frac{1}{a^3} \tan^{-1}\frac{x}{a}$$

$$\int \frac{dx}{x^3(x^2+a^2)} = -\frac{1}{2a^2x^2} - \frac{1}{2a^4} \ln \left(\frac{x^2}{x^2+a^2}\right)$$

$$\int \frac{dx}{(x^2+a^2)^2} = \frac{x}{2a^2(x^2+a^2)} + \frac{1}{2a^3} \tan^{-1}\frac{x}{a}$$

$$\int \frac{x\,dx}{(x^2+a^2)^2} = \frac{-1}{2(x^2+a^2)}$$

$$\int \frac{x^2\,dx}{(x^2+a^2)^2} = \frac{-x}{2(x^2+a^2)} + \frac{1}{2a} \tan^{-1}\frac{x}{a}$$

$$\int \frac{x^3\,dx}{(x^2+a^2)^2} = \frac{a^2}{2(x^2+a^2)} + \frac{1}{2} \ln (x^2+a^2)$$

$$\int \frac{dx}{x(x^2+a^2)^2} = \frac{1}{2a^2(x^2+a^2)} + \frac{1}{2a^4} \ln \left(\frac{x^2}{x^2+a^2}\right)$$

$$\int \frac{dx}{x^2(x^2+a^2)^2} = -\frac{1}{a^4x} - \frac{x}{2a^4(x^2+a^2)} - \frac{3}{2a^5} \tan^{-1}\frac{x}{a}$$

$$\int \frac{dx}{x^3(x^2+a^2)^2} = -\frac{1}{2a^4x^2} - \frac{1}{2a^4(x^2+a^2)} - \frac{1}{a^6} \ln \left(\frac{x^2}{x^2+a^2}\right)$$

$$\int \frac{dx}{(x^2+a^2)^n} = \frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2} \int \frac{dx}{(x^2+a^2)^{n-1}}$$

$$\int \frac{x\,dx}{(x^2+a^2)^n} = \frac{-1}{2(n-1)(x^2+a^2)^{n-1}}$$

$$\int \frac{dx}{x(x^2+a^2)^n} = \frac{1}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{1}{a^2} \int \frac{dx}{x(x^2+a^2)^{n-1}}$$

$$\int \frac{x^m\,dx}{(x^2+a^2)^n} = \int \frac{x^{m-2}\,dx}{(x^2+a^2)^{n-1}} - a^2 \int \frac{x^{m-2}\,dx}{(x^2+a^2)^n}$$

$$\int \frac{dx}{x^m(x^2+a^2)^n} = \frac{1}{a^2} \int \frac{dx}{x^m(x^2+a^2)^{n-1}} - \frac{1}{a^2} \int \frac{dx}{x^{m-2}(x^2+a^2)^n}$$

## $x^2 - a^2,\ x^2 > a^2$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left( \frac{x-a}{x+a} \right) \quad \text{o} \quad -\frac{1}{a} \coth^{-1} \frac{x}{a}$$

$$\int \frac{x\,dx}{x^2 - a^2} = \frac{1}{2} \ln (x^2 - a^2)$$

$$\int \frac{x^2\,dx}{x^2 - a^2} = x + \frac{a}{2} \ln \left( \frac{x-a}{x+a} \right)$$

$$\int \frac{x^3\,dx}{x^2 - a^2} = \frac{x^2}{2} + \frac{a^2}{2} \ln (x^2 - a^2)$$

$$\int \frac{dx}{x(x^2 - a^2)} = \frac{1}{2a^2} \ln \left( \frac{x^2 - a^2}{x^2} \right)$$

$$\int \frac{dx}{x^2(x^2 - a^2)} = \frac{1}{a^2 x} + \frac{1}{2a^3} \ln \left( \frac{x-a}{x+a} \right)$$

$$\int \frac{dx}{x^3(x^2 - a^2)} = \frac{1}{2a^2 x^2} - \frac{1}{2a^4} \ln \left( \frac{x^2}{x^2 - a^2} \right)$$

$$\int \frac{dx}{(x^2 - a^2)^n} = \frac{-x}{2(n-1)a^2(x^2 - a^2)^{n-1}} - \frac{2n-3}{(2n-2)a^2} \int \frac{dx}{(x^2 - a^2)^{n-1}}$$

$$\int \frac{x\,dx}{(x^2 - a^2)^n} = \frac{-1}{2(n-1)(x^2 - a^2)^{n-1}}$$

$$\int \frac{dx}{x(x^2 - a^2)^n} = \frac{-1}{2(n-1)a^2(x^2 - a^2)^{n-1}} - \frac{1}{a^2} \int \frac{dx}{x(x^2 - a^2)^{n-1}}$$

$$\int \frac{x^m\,dx}{(x^2 - a^2)^n} = \int \frac{x^{m-2}\,dx}{(x^2 - a^2)^{n-1}} + a^2 \int \frac{x^{m-2}\,dx}{(x^2 - a^2)^n}$$

$$\int \frac{dx}{x^m(x^2 - a^2)^n} = \frac{1}{a^2} \int \frac{dx}{x^{m-2}(x^2 - a^2)^n} - \frac{1}{a^2} \int \frac{dx}{x^m(x^2 - a^2)^{n-1}}$$

## $a^2 - x^2,\ x^2 < a^2$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left( \frac{a+x}{a-x} \right) \quad \text{o} \quad \frac{1}{a} \tanh^{-1} \frac{x}{a}$$

$$\int \frac{x\,dx}{a^2 - x^2} = -\frac{1}{2} \ln (a^2 - x^2)$$

$$\int \frac{x^2\,dx}{a^2 - x^2} = -x + \frac{a}{2} \ln \left( \frac{a+x}{a-x} \right)$$

$$\int \frac{x^3\,dx}{a^2-x^2} = -\frac{x^2}{2} - \frac{a^2}{2}\ln(a^2-x^2)$$

$$\int \frac{dx}{x(a^2-x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2-x^2}\right)$$

$$\int \frac{dx}{x^2(a^2-x^2)} = -\frac{1}{a^2x} + \frac{1}{2a^3}\ln\left(\frac{a+x}{a-x}\right)$$

$$\int \frac{dx}{x^3(a^2-x^2)} = -\frac{1}{2a^2x^2} + \frac{1}{2a^4}\ln\left(\frac{x^2}{a^2-x^2}\right)$$

$$\int \frac{dx}{(a^2-x^2)^n} = \frac{x}{2(n-1)a^2(a^2-x^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2}\int \frac{dx}{(a^2-x^2)^{n-1}}$$

$$\int \frac{x\,dx}{(a^2-x^2)^n} = \frac{1}{2(n-1)(a^2-x^2)^{n-1}}$$

$$\int \frac{dx}{x(a^2-x^2)^n} = \frac{1}{2(n-1)a^2(a^2-x^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x(a^2-x^2)^{n-1}}$$

$$\int \frac{x^m\,dx}{(a^2-x^2)^n} = a^2\int \frac{x^{m-2}dx}{(a^2-x^2)^n} - \int \frac{x^{m-2}dx}{(a^2-x^2)^{n-1}}$$

$$\int \frac{dx}{x^m(a^2-x^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^m(a^2-x^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x^{m-2}(a^2-x^2)^n}$$

---

## $\sqrt{x^2+a^2}$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2}) \quad \text{o} \quad \operatorname{senh}^{-1}\frac{x}{a}$$

$$\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$$

$$\int \frac{x^2\,dx}{\sqrt{x^2+a^2}} = \frac{x\sqrt{x^2+a^2}}{2} - \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2})$$

$$\int \frac{x^3\,dx}{\sqrt{x^2+a^2}} = \frac{(x^2+a^2)^{3/2}}{3} - a^2\sqrt{x^2+a^2}$$

$$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2x}$$

$$\int \frac{dx}{x^3\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{2a^2x^2} + \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2})$$

$$\int x\sqrt{x^2+a^2}\,dx = \frac{(x^2+a^2)^{3/2}}{3}$$

$$\int x^2\sqrt{x^2+a^2}\,dx = \frac{x(x^2+a^2)^{3/2}}{4} - \frac{a^2x\sqrt{x^2+a^2}}{8} - \frac{a^4}{8}\ln(x+\sqrt{x^2+a^2})$$

$$\int x^3\sqrt{x^2+a^2}\,dx = \frac{(x^2+a^2)^{5/2}}{5} - \frac{a^2(x^2+a^2)^{3/2}}{3}$$

$$\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} - a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \frac{\sqrt{x^2+a^2}}{x^2}\,dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln(x+\sqrt{x^2+a^2})$$

$$\int \frac{\sqrt{x^2+a^2}}{x^3}\,dx = -\frac{\sqrt{x^2+a^2}}{2x^2} - \frac{1}{2a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

---

## $\sqrt{x^2 - a^2}$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln(x+\sqrt{x^2-a^2}), \qquad \int \frac{x\,dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2}$$

$$\int \frac{x^2\,dx}{\sqrt{x^2-a^2}} = \frac{x\sqrt{x^2-a^2}}{2} + \frac{a^2}{2}\ln(x+\sqrt{x^2-a^2})$$

$$\int \frac{x^3\,dx}{\sqrt{x^2-a^2}} = \frac{(x^2-a^2)^{3/2}}{3} + a^2\sqrt{x^2-a^2}$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|$$

$$\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x}$$

$$\int \frac{dx}{x^3\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{2a^2x^2} + \frac{1}{2a^3}\sec^{-1}\left|\frac{x}{a}\right|$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\ln(x+\sqrt{x^2-a^2})$$

$$\int x\sqrt{x^2-a^2}\,dx = \frac{(x^2-a^2)^{3/2}}{3}$$

$$\int x^2\sqrt{x^2-a^2}\,dx = \frac{x(x^2-a^2)^{3/2}}{4} + \frac{a^2x\sqrt{x^2-a^2}}{8} - \frac{a^4}{8}\ln(x+\sqrt{x^2-a^2})$$

$$\int x^3\sqrt{x^2-a^2}\,dx = \frac{(x^2-a^2)^{5/2}}{5} + \frac{a^2(x^2-a^2)^{3/2}}{3}$$

$$\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} - a\sec^{-1}\left|\frac{x}{a}\right|$$

$$\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln(x+\sqrt{x^2-a^2})$$

$$\int \frac{\sqrt{x^2-a^2}}{x^3}\,dx = -\frac{\sqrt{x^2-a^2}}{2x^2} + \frac{1}{2a}\sec^{-1}\left|\frac{x}{a}\right|$$

---

## $\sqrt{a - x^2}$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{sen}^{-1}\frac{x}{a}$$

$$\int \frac{x\,dx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2}$$

$$\int \frac{x^2\,dx}{\sqrt{a^2-x^2}} = -\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{x}{a}$$

$$\int \frac{x^3\,dx}{\sqrt{a^2-x^2}} = \frac{(a^2-x^2)^{3/2}}{3} - a^2\sqrt{a^2-x^2}$$

$$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

$$\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x}$$

$$\int \frac{dx}{x^3\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{2a^2x^2} - \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{x}{a}$$

$$\int x\sqrt{a^2-x^2}\,dx = -\frac{(a^2-x^2)^{3/2}}{3}$$

$$\int x^2\sqrt{a^2-x^2}\,dx = -\frac{x(a^2-x^2)^{3/2}}{4} + \frac{a^2x\sqrt{a^2-x^2}}{8} + \frac{a^4}{8}\operatorname{sen}^{-1}\frac{x}{a}$$

$$\int x^3\sqrt{a^2-x^2}\,dx = \frac{(a^2-x^2)^{5/2}}{5} - \frac{a^2(a^2-x^2)^{3/2}}{3}$$

$$\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

$$\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{\sqrt{a^2-x^2}}{x} - \operatorname{sen}^{-1}\frac{x}{a}$$

$$\int \frac{\sqrt{a^2-x^2}}{x^3}\,dx = -\frac{\sqrt{a^2-x^2}}{2x^2} + \frac{1}{2a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

## $ax^2 + bx + c$

$$\int \frac{dx}{ax^2+bx+c} = \begin{cases} \dfrac{2}{\sqrt{4ac-b^2}}\tan^{-1}\dfrac{2ax+b}{\sqrt{4ac-b^2}} \\ \dfrac{1}{\sqrt{b^2-4ac}}\ln\left(\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right) \end{cases}$$

$$\int \frac{x\,dx}{ax^2+bx+c} = \frac{1}{2a}\ln(ax^2+bx+c) - \frac{b}{2a}\int\frac{dx}{ax^2+bx+c}$$

$$\int \frac{x^2\,dx}{ax^2+bx+c} = \frac{x}{a} - \frac{b}{2a^2}\ln(ax^2+bx+c) + \frac{b^2-2ac}{2a^2}\int\frac{dx}{ax^2+bx+c}$$

$$\int \frac{x^m\,dx}{ax^2+bx+c} = \frac{x^{m-1}}{(m-1)a} - \frac{c}{a}\int\frac{x^{m-2}\,dx}{ax^2+bx+c} - \frac{b}{a}\int\frac{x^{m-1}\,dx}{ax^2+bx+c}$$

$$\int \frac{dx}{x^n(ax^2+bx+c)} = -\frac{1}{(n-1)cx^{n-1}} - \frac{b}{c}\int\frac{dx}{x^{n-1}(ax^2+bx+c)} - \frac{a}{c}\int\frac{dx}{x^{n-2}(ax^2+bx+c)}$$

$$\int \frac{x^m\,dx}{(ax^2+bx+c)^n} = -\frac{x^{m-1}}{(2n-m-1)a(ax^2+bx+c)^{n-1}} + \frac{(m-1)c}{(2n-m-1)a}\int\frac{x^{m-2}\,dx}{(ax^2+bx+c)^n} - \frac{(n-m)b}{(2n-m-1)a}\int\frac{x^{m-1}\,dx}{(ax^2+bx+c)^n}$$

$$\int \frac{dx}{x^m(ax^2+bx+c)^n} = -\frac{1}{(m-1)cx^{m-1}(ax^2+bx+c)^{n-1}} - \frac{(m+2n-3)a}{(m-1)c}\int\frac{dx}{x^{m-2}(ax^2+bx+c)^n} - \frac{(m+n-2)b}{(m-1)c}\int\frac{dx}{x^{m-1}(ax^2+bx+c)^n}$$

## $x^n \pm a^n$

$$\int \frac{dx}{x(x^n+a^n)} = \frac{1}{na^n}\ln\frac{x^n}{x^n+a^n}$$

$$\int \frac{x^{n-1}\,dx}{x^n + a^n} = \frac{1}{n}\ln(x^n + a^n)$$

$$\int \frac{x^m\,dx}{(x^n + a^n)^r} = \int \frac{x^{m-n}\,dx}{(x^n + a^n)^{r-1}} - a^n \int \frac{x^{m-n}\,dx}{(x^n + a^n)^r}$$

$$\int \frac{dx}{x^m(x^n + a^n)^r} = \frac{1}{a^n}\int \frac{dx}{x^m(x^n + a^n)^{r-1}} - \frac{1}{a^n}\int \frac{dx}{x^{m-n}(x^n + a^n)^r}$$

$$\int \frac{dx}{x\sqrt{x^n + a^n}} = \frac{1}{n\sqrt{a^n}}\ln\left(\frac{\sqrt{x^n + a^n} - \sqrt{a^n}}{\sqrt{x^n + a^n} + \sqrt{a^n}}\right)$$

$$\int \frac{dx}{x(x^n - a^n)} = \frac{1}{na^n}\ln\left(\frac{x^n - a^n}{x^n}\right)$$

$$\int \frac{x^{n-1}\,dx}{x^n - a^n} = \frac{1}{n}\ln(x^n - a^n)$$

$$\int \frac{x^m\,dx}{(x^n - a^n)^r} = a^n \int \frac{x^{m-n}\,dx}{(x^n - a^n)^r} + \int \frac{x^{m-n}\,dx}{(x^n - a^n)^{r-1}}$$

$$\int \frac{dx}{x^m(x^n - a^n)^r} = \frac{1}{a^n}\int \frac{dx}{x^{m-n}(x^n - a^n)^r} - \frac{1}{a^n}\int \frac{dx}{x^m(x^n - a^n)^{r-1}}$$

$$\int \frac{dx}{x\sqrt{x^n - a^n}} = \frac{2}{n\sqrt{a^n}}\cos^{-1}\sqrt{\frac{a^n}{x^n}}$$

---

## sen $ax$ Y cos $ax$

$$\int x^m \operatorname{sen} ax\,dx = -\frac{x^m \cos ax}{a} + \frac{mx^{m-1}\operatorname{sen} ax}{a^2} - \frac{m(m-1)}{a^2}\int x^{m-2}\operatorname{sen} ax\,dx$$

$$\int \frac{\operatorname{sen} ax}{x^n}\,dx = -\frac{\operatorname{sen} ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\cos ax}{x^{n-1}}\,dx$$

$$\int \operatorname{sen}^n ax\,dx = -\frac{\operatorname{sen}^{n-1} ax \cos ax}{an} + \frac{n-1}{n}\int \operatorname{sen}^{n-2} ax\,dx$$

$$\int \frac{dx}{\operatorname{sen}^n ax} = \frac{-\cos ax}{a(n-1)\operatorname{sen}^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\operatorname{sen}^{n-2} ax}$$

$$\int \frac{x\,dx}{\operatorname{sen}^n ax} = \frac{-x\cos ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\operatorname{sen}^{n-2} ax} + \frac{n-2}{n-1}\int \frac{x\,dx}{\operatorname{sen}^{n-2} ax}$$

$$\int x^m \cos ax\,dx = \frac{x^m \operatorname{sen} ax}{a} + \frac{mx^{m-1}}{a^2}\cos ax - \frac{m(m-1)}{a^2}\int x^{m-2}\cos ax\,dx$$

$$\int \frac{\cos ax}{x^n}\,dx = -\frac{\cos ax}{(n-1)x^{n-1}} - \frac{a}{n-1}\int \frac{\operatorname{sen} ax}{x^{n-1}}\,dx$$

$$\int \cos^n ax\, dx = \frac{\operatorname{sen} ax \cos^{n-1} ax}{an} + \frac{n-1}{n}\int \cos^{n-2} ax\, dx$$

$$\int \frac{dx}{\cos^n ax} = \frac{\operatorname{sen} ax}{a(n-1)\cos^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} ax}$$

$$\int \frac{x\, dx}{\cos^n ax} = \frac{x \operatorname{sen} ax}{a(n-1)\cos^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\cos^{n-2} ax} + \frac{n-2}{n-1}\int \frac{x\, dx}{\cos^{n-2} ax}$$

$$\int \operatorname{sen} ax \cos ax\, dx = \frac{\operatorname{sen}^2 ax}{2a}$$

$$\int \operatorname{sen} px \cos qx\, dx = -\frac{\cos (p-q)x}{2(p-q)} - \frac{\cos (p+q)x}{2(p+q)}$$

$$\int \operatorname{sen}^n ax \cos ax\, dx = \frac{\operatorname{sen}^{n+1} ax}{(n+1)a}$$

$$\int \cos^n ax \operatorname{sen} ax\, dx = -\frac{\cos^{n+1} ax}{(n+1)a}$$

$$\int \operatorname{sen}^2 ax \cos^2 ax\, dx = \frac{x}{8} - \frac{\operatorname{sen} 4ax}{32a}$$

$$\int \frac{dx}{\operatorname{sen} ax \cos ax} = \frac{1}{a}\ln \tan ax$$

$$\int \frac{dx}{\operatorname{sen}^2 ax \cos ax} = \frac{1}{a}\ln \tan\left(\frac{\pi}{4} + \frac{ax}{2}\right) - \frac{1}{a \operatorname{sen} ax}$$

$$\int \frac{dx}{\operatorname{sen} ax \cos^2 ax} = \frac{1}{a}\ln \tan \frac{ax}{2} + \frac{1}{a \cos ax}$$

$$\int \frac{dx}{\operatorname{sen}^2 ax \cos^2 ax} = -\frac{2 \cot 2ax}{a}$$

---

## $\tan ax$

$$\int \tan^n ax \sec^2 ax\, dx = \frac{\tan^{n+1} ax}{(n+1)a}$$

$$\int x \tan^2 ax\, dx = \frac{x \tan ax}{a} + \frac{1}{a^2}\ln \cos ax - \frac{x^2}{2}$$

$$\int \frac{dx}{p \div q \tan ax} = \frac{px}{p^2+q^2} + \frac{q}{a(p^2+q^2)}\ln (q \operatorname{sen} ax + p \cos ax)$$

$$\int \tan^n ax\, dx = \frac{\tan^{n-1} ax}{(n-1)a} - \int \tan^{n-2} ax\, dx$$

$$\int \cot^n ax \csc^2 ax\, dx = -\frac{\cot^{n+1} ax}{(n+1)a}$$

$$\int x \cot^2 ax\, dx = -\frac{x \cot ax}{a} + \frac{1}{a^2} \ln \operatorname{sen} ax - \frac{x^2}{2}$$

$$\int \frac{dx}{p + q \cot ax} = \frac{px}{p^2 + q^2} - \frac{q}{a(p^2 + q^2)} \ln (p \operatorname{sen} ax + q \cos ax)$$

$$\int \cot^n ax\, dx = -\frac{\cot^{n-1} ax}{(n-1)a} - \int \cot^{n-2} ax\, dx$$

$$\int x \cot ax\, dx = \frac{1}{a^2}\left\{ax - \frac{(ax)^3}{9} - \frac{(ax)^5}{225} - \cdots - \frac{2^{2n} B_n (ax)^{2n+1}}{(2n+1)!} - \cdots\right\}$$

$$\int \frac{\cot ax}{x}\, dx = -\frac{1}{ax} - \frac{ax}{3} - \frac{(ax)^3}{135} - \cdots - \frac{2^{2n} B_n (ax)^{2n-1}}{(2n-1)(2n)!} - \cdots$$

## sec $ax$

$$\int \sec^n ax \tan ax\, dx = \frac{\sec^n ax}{na}$$

$$\int \frac{dx}{\sec ax} = \frac{\operatorname{sen} ax}{a}$$

$$\int x \sec ax\, dx = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots + \frac{E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$$

$$\int \frac{\sec ax}{x}\, dx = \ln x + \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} + \frac{61(ax)^6}{4320} + \cdots + \frac{E_n (ax)^{2n}}{2n(2n)!} + \cdots$$

$$\int x \sec^2 ax\, dx = \frac{x}{a} \tan ax + \frac{1}{a^2} \ln \cos ax$$

$$\int \frac{dx}{q + p \sec ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{p + q \cos ax}$$

$$\int \sec^n ax\, dx = \frac{\sec^{n-2} ax \tan ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax\, dx$$

## csc $ax$

$$\int \csc^n ax \cot ax\,dx = -\frac{\csc^n ax}{na}$$

$$\int \frac{dx}{\csc ax} = -\frac{\cos ax}{a}$$

$$\int x\csc ax\,dx = \frac{1}{a^2}\left\{ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \cdots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$$

$$\int \frac{\csc ax}{x}\,dx = -\frac{1}{ax} + \frac{ax}{6} + \frac{7(ax)^3}{1080} + \cdots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$$

$$\int x\csc^2 ax\,dx = -\frac{x\cot ax}{a} + \frac{1}{a^2}\ln \operatorname{sen} ax$$

$$\int \frac{dx}{q+p\csc ax} = \frac{x}{q} - \frac{p}{q}\int \frac{dx}{p+q\operatorname{sen} ax}$$

$$\int \csc^n ax\,dx = -\frac{\csc^{n-2} ax\cot ax}{a(n-1)} + \frac{n-2}{n-1}\int \csc^{n-2} ax\,dx$$

## FUNCIONES TRIGONOMETRICAS RECIPROCAS

$$\int \operatorname{sen}^{-1}\frac{x}{a}\,dx = x\operatorname{sen}^{-1}\frac{x}{a} + \sqrt{a^2-x^2}$$

$$\int x\operatorname{sen}^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2}-\frac{a^2}{4}\right)\operatorname{sen}^{-1}\frac{x}{a} + \frac{x\sqrt{a^2-x^2}}{4}$$

$$\int x^2\operatorname{sen}^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\operatorname{sen}^{-1}\frac{x}{a} + \frac{(x^2+2a^2)\sqrt{a^2-x^2}}{9}$$

$$\int \frac{\operatorname{sen}^{-1}(x/a)}{x}\,dx = \frac{x}{a} + \frac{(x/a)^3}{2\cdot 3\cdot 3} + \frac{1\cdot 3(x/a)^5}{2\cdot 4\cdot 5\cdot 5} + \frac{1\cdot 3\cdot 5(x/a)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots$$

$$\int \frac{\operatorname{sen}^{-1}(x/a)}{x^2}\,dx = -\frac{\operatorname{sen}^{-1}(x/a)}{x} - \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

$$\int \left(\operatorname{sen}^{-1}\frac{x}{a}\right)^2 dx = x\left(\operatorname{sen}^{-1}\frac{x}{a}\right)^2 - 2x + 2\sqrt{a^2-x^2}\operatorname{sen}^{-1}\frac{x}{a}$$

## $e^{ax}$

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a}\left(x-\frac{1}{a}\right)$$

$$\int \csc^n ax \cot ax\, dx = -\frac{\csc^n ax}{na}$$

$$\int \frac{dx}{\csc ax} = -\frac{\cos ax}{a}$$

$$\int x \csc ax\, dx = \frac{1}{a^2}\left\{ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \cdots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$$

$$\int \frac{\csc ax}{x}\, dx = -\frac{1}{ax} + \frac{ax}{6} + \frac{7(ax)^3}{1080} + \cdots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$$

$$\int x \csc^2 ax\, dx = -\frac{x \cot ax}{a} + \frac{1}{a^2}\ln \operatorname{sen} ax$$

$$\int \frac{dx}{q + p \csc ax} = \frac{x}{q} - \frac{p}{q}\int \frac{dx}{p + q \operatorname{sen} ax}$$

$$\int \csc^n ax\, dx = -\frac{\csc^{n-2} ax \cot ax}{a(n-1)} + \frac{n-2}{n-1}\int \csc^{n-2} ax\, dx$$

## FUNCIONES TRIGONOMETRICAS RECIPROCAS

$$\int \operatorname{sen}^{-1}\frac{x}{a}\, dx = x \operatorname{sen}^{-1}\frac{x}{a} + \sqrt{a^2 - x^2}$$

$$\int x \operatorname{sen}^{-1}\frac{x}{a}\, dx = \left(\frac{x^2}{2} - \frac{a^2}{4}\right)\operatorname{sen}^{-1}\frac{x}{a} + \frac{x\sqrt{a^2 - x^2}}{4}$$

$$\int x^2 \operatorname{sen}^{-1}\frac{x}{a}\, dx = \frac{x^3}{3}\operatorname{sen}^{-1}\frac{x}{a} + \frac{(x^2 + 2a^2)\sqrt{a^2 - x^2}}{9}$$

$$\int \frac{\operatorname{sen}^{-1}(x/a)}{x}\, dx = \frac{x}{a} + \frac{(x/a)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 (x/a)^5}{2 \cdot 4 \cdot 5 \cdot 5} + \frac{1 \cdot 3 \cdot 5 (x/a)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \cdots$$

$$\int \frac{\operatorname{sen}^{-1}(x/a)}{x^2}\, dx = -\frac{\operatorname{sen}^{-1}(x/a)}{x} - \frac{1}{a}\ln\left(\frac{a + \sqrt{a^2 - x^2}}{x}\right)$$

$$\int \left(\operatorname{sen}^{-1}\frac{x}{a}\right)^2 dx = x\left(\operatorname{sen}^{-1}\frac{x}{a}\right)^2 - 2x + 2\sqrt{a^2 - x^2}\operatorname{sen}^{-1}\frac{x}{a}$$

## $e^{ax}$

$$\int e^{ax}\, dx = \frac{e^{ax}}{a}$$

$$\int x e^{ax}\, dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$$

$$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$$

$$= \frac{e^{ax}}{a}\left(x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} - \cdots \frac{(-1)^n n!}{a^n}\right)$$

si $n$ = entero positivo

$$\int \frac{e^{ax}}{x}\,dx = \ln x + \frac{ax}{1\cdot 1!} + \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^3}{3\cdot 3!} + \cdots$$

$$\int \frac{e^{ax}}{x^n}\,dx = \frac{-e^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{e^{ax}}{x^{n-1}}\,dx$$

$$\int \frac{dx}{p+qe^{ax}} = \frac{x}{p} - \frac{1}{ap}\ln(p+qe^{ax})$$

$$\int \frac{dx}{(p+qe^{ax})^2} = \frac{x}{p^2} + \frac{1}{ap(p+qe^{ax})} - \frac{1}{ap^2}\ln(p+qe^{ax})$$

$$\int \frac{dx}{pe^{ax}+qe^{-ax}} = \begin{cases} \dfrac{1}{a\sqrt{pq}}\tan^{-1}\left(\sqrt{\dfrac{p}{q}}\,e^{ax}\right) \\[2ex] \dfrac{1}{2a\sqrt{-pq}}\ln\left(\dfrac{e^{ax}-\sqrt{-q/p}}{e^{ax}+\sqrt{-q/p}}\right) \end{cases}$$

$$\int e^{ax}\operatorname{sen} bx\,dx = \frac{e^{ax}(a\operatorname{sen} bx - b\cos bx)}{a^2+b^2}$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b\operatorname{sen} bx)}{a^2+b^2}$$

$$\int xe^{ax}\operatorname{sen} bx\,dx = \frac{xe^{ax}(a\operatorname{sen} bx - b\cos bx)}{a^2+b^2} - \frac{e^{ax}\{(a^2-b^2)\operatorname{sen} bx - 2ab\cos bx\}}{(a^2+b^2)^2}$$

$$\int xe^{ax}\cos bx\,dx = \frac{xe^{ax}(a\cos bx + b\operatorname{sen} bx)}{a^2+b^2} - \frac{e^{ax}\{(a^2-b^2)\cos bx + 2ab\operatorname{sen} bx\}}{(a^2+b^2)^2}$$

$$\int e^{ax}\ln x\,dx = \frac{e^{ax}\ln x}{a} - \frac{1}{a}\int \frac{e^{ax}}{x}\,dx$$

$$\int e^{ax}\operatorname{sen}^n bx\,dx = \frac{e^{ax}\operatorname{sen}^{n-1} bx}{a^2+n^2b^2}(a\operatorname{sen} bx - nb\cos bx) + \frac{n(n-1)b^2}{a^2+n^2b^2}\int e^{ax}\operatorname{sen}^{n-2} bx\,dx$$

$$\int e^{ax}\cos^n bx\,dx = \frac{e^{ax}\cos^{n-1} bx}{a^2+n^2b^2}(a\cos bx + nb\operatorname{sen} bx) + \frac{n(n-1)b^2}{a^2+n^2b^2}\int e^{ax}\cos^{n-2} bx\,dx$$

## $\ln x$

$$\int \ln x\,dx = x\ln x - x$$

$$\int x\ln x\,dx = \frac{x^2}{2}(\ln x - \tfrac{1}{2})$$

$$\int x^m \ln x\,dx = \frac{x^{m+1}}{m+1}\left(\ln x - \frac{1}{m+1}\right)$$

$$\int \frac{\ln x}{x}\,dx = \frac{1}{2}\ln^2 x$$

$$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x}$$

$$\int \ln^2 x\,dx = x\ln^2 x - 2x\ln x + 2x$$

$$\int \frac{\ln^n x\,dx}{x} = \frac{\ln^{n+1} x}{n+1}$$

$$\int \frac{dx}{x\ln x} = \ln(\ln x)$$

$$\int \frac{dx}{\ln x} = \ln(\ln x) + \ln x + \frac{\ln^2 x}{2\cdot 2!} + \frac{\ln^3 x}{3\cdot 3!} + \cdots$$

$$\int \frac{x^m\,dx}{\ln x} = \ln(\ln x) + (m+1)\ln x + \frac{(m+1)^2\ln^2 x}{2\cdot 2!} + \frac{(m+1)^3\ln^3 x}{3\cdot 3!} + \cdots$$

$$\int \ln^n x\,dx = x\ln^n x - n\int \ln^{n-1} x\,dx$$

$$\int x^m \ln^n x\,dx = \frac{x^{m+1}\ln^n x}{m+1} - \frac{n}{m+1}\int x^m \ln^{n-1} x\,dx$$

$$\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) - 2x + 2a\tan^{-1}\frac{x}{a}$$

$$\int \ln(x^2-a^2)\,dx = x\ln(x^2-a^2) - 2x + a\ln\left(\frac{x+a}{x-a}\right)$$

$$\int x^m \ln(x^2\pm a^2)\,dx = \frac{x^{m+1}\ln(x^2\pm a^2)}{m+1} - \frac{2}{m+1}\int \frac{x^{m+2}}{x^2\pm a^2}\,dx$$

$$\int \cos^{-1}\frac{x}{a}\,dx = x\cos^{-1}\frac{x}{a} - \sqrt{a^2-x^2}$$

$$\int x\cos^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2}-\frac{a^2}{4}\right)\cos^{-1}\frac{x}{a} - \frac{x\sqrt{a^2-x^2}}{4}$$

$$\int x^2\cos^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\cos^{-1}\frac{x}{a} - \frac{(x^2+2a^2)\sqrt{a^2-x^2}}{9}$$

$$\int \frac{\cos^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x - \int \frac{\operatorname{sen}^{-1}(x/a)}{x}\,dx$$

$$\int \frac{\cos^{-1}(x/a)}{x^2}\,dx = -\frac{\cos^{-1}(x/a)}{x} + \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

$$\int \left(\cos^{-1}\frac{x}{a}\right)^2 dx = x\left(\cos^{-1}\frac{x}{a}\right)^2 - 2x - 2\sqrt{a^2-x^2}\cos^{-1}\frac{x}{a}$$

$$\int \tan^{-1}\frac{x}{a}\,dx = x\tan^{-1}\frac{x}{a} - \frac{a}{2}\ln(x^2+a^2)$$

$$\int x\tan^{-1}\frac{x}{a}\,dx = \tfrac{1}{2}(x^2+a^2)\tan^{-1}\frac{x}{a} - \frac{ax}{2}$$

$$\int x^2\tan^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\tan^{-1}\frac{x}{a} - \frac{ax^2}{6} + \frac{a^3}{6}\ln(x^2+a^2)$$

$$\int \frac{\tan^{-1}(x/a)}{x}\,dx = \frac{x}{a} - \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} - \frac{(x/a)^7}{7^2} + \cdots$$

$$\int \frac{\tan^{-1}(x/a)}{x^2}\,dx = -\frac{1}{x}\tan^{-1}\frac{x}{a} - \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$$

$$\int \cot^{-1}\frac{x}{a}\,dx = x\cot^{-1}\frac{x}{a} + \frac{a}{2}\ln(x^2+a^2)$$

$$\int x\cot^{-1}\frac{x}{a}\,dx = \tfrac{1}{2}(x^2+a^2)\cot^{-1}\frac{x}{a} + \frac{ax}{2}$$

$$\int x^2\cot^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\cot^{-1}\frac{x}{a} + \frac{ax^2}{6} - \frac{a^3}{6}\ln(x^2+a^2)$$

$$\int \frac{\cot^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x - \int \frac{\tan^{-1}(x/a)}{x}\,dx$$

$$\int \frac{\cot^{-1}(x/a)}{x^2}\,dx = -\frac{\cot^{-1}(x/a)}{x} + \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$$

$$\int \sec^{-1}\frac{x}{a}\,dx = \begin{cases} x\sec^{-1}\dfrac{x}{a} - a\ln\left(x+\sqrt{x^2-a^2}\right) & 0 < \sec^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\[2ex] x\sec^{-1}\dfrac{x}{a} + a\ln\left(x+\sqrt{x^2-a^2}\right) & \dfrac{\pi}{2} < \sec^{-1}\dfrac{x}{a} < \pi \end{cases}$$

$$\int \csc^{-1}\frac{x}{a}\,dx = \begin{cases} x\csc^{-1}\dfrac{x}{a} + a\ln\left(x+\sqrt{x^2-a^2}\right) & 0 < \csc^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ x\csc^{-1}\dfrac{x}{a} - a\ln\left(x+\sqrt{x^2-a^2}\right) & -\dfrac{\pi}{2} < \csc^{-1}\dfrac{x}{a} < 0 \end{cases}$$

$$\int x\csc^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^2}{2}\csc^{-1}\dfrac{x}{a} + \dfrac{a\sqrt{x^2-a^2}}{2} & 0 < \csc^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ \dfrac{x^2}{2}\csc^{-1}\dfrac{x}{a} - \dfrac{a\sqrt{x^2-a^2}}{2} & -\dfrac{\pi}{2} < \csc^{-1}\dfrac{x}{a} < 0 \end{cases}$$

$$\int x^m \operatorname{sen}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{sen}^{-1}\frac{x}{a} - \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{a^2-x^2}}\,dx$$

$$\int x^m \cos^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\cos^{-1}\frac{x}{a} + \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{a^2-x^2}}\,dx$$

$$\int x^m \tan^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\tan^{-1}\frac{x}{a} - \frac{a}{m+1}\int \frac{x^{m+1}}{x^2+a^2}\,dx$$

$$\int x^m \cot^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\cot^{-1}\frac{x}{a} + \frac{a}{m+1}\int \frac{x^{m+1}}{x^2+a^2}\,dx$$

$$\begin{cases} \dfrac{x^{m+1}\sec^{-1}(x/a)}{m+1} - \dfrac{a}{m+1}\displaystyle\int \frac{x^m\,dx}{\sqrt{x^2-a^2}} & 0 < \sec^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ \dfrac{x^{m+1}\sec^{-1}(x/a)}{m+1} + \dfrac{a}{m+1}\displaystyle\int \frac{x^m\,dx}{\sqrt{x^2-a^2}} & \dfrac{\pi}{2} < \sec^{-1}\dfrac{x}{a} < \pi \end{cases}$$

$$\int x^m \csc^{-1}\frac{x}{a}\,dx =$$

$$\begin{cases} \dfrac{x^{m+1}\csc^{-1}(x/a)}{m+1} + \dfrac{a}{m+1}\displaystyle\int \frac{x^m\,dx}{\sqrt{x^2-a^2}} & 0 < \csc^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ \dfrac{x^{m+1}\csc^{-1}(x/a)}{m+1} - \dfrac{a}{m+1}\displaystyle\int \frac{x^m\,dx}{\sqrt{x^2-a^2}} & -\dfrac{\pi}{2} < \csc^{-1}\dfrac{x}{a} < 0 \end{cases}$$

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$$\int x\operatorname{senh} ax\,dx = \frac{x\cosh ax}{a} - \frac{\operatorname{senh} ax}{a^2}$$

$$\int \frac{\operatorname{senh} ax}{x}\,dx = ax + \frac{(ax)^3}{3\cdot 3!} + \frac{(ax)^5}{5\cdot 5!} + \cdots$$

$$\int \operatorname{senh} ax \operatorname{senh} px\,dx = \frac{\operatorname{senh}(a+p)x}{2(a+p)} - \frac{\operatorname{senh}(a-p)x}{2(a-p)}$$

$$\int \operatorname{senh} ax \operatorname{sen} px\,dx = \frac{a \cosh ax \operatorname{sen} px - p \operatorname{senh} ax \cos px}{a^2+p^2}$$

$$\int \operatorname{senh} ax \cos px\,dx = \frac{a \cosh ax \cos px + p \operatorname{senh} ax \operatorname{sen} px}{a^2+p^2}$$

$$\int \frac{dx}{p+q\operatorname{senh} ax} = \frac{1}{a\sqrt{p^2+q^2}} \ln\left(\frac{qe^{ax}+p-\sqrt{p^2+q^2}}{qe^{ax}+p+\sqrt{p^2+q^2}}\right)$$

$$\int \frac{dx}{(p+q\operatorname{senh} ax)^2} = \frac{-q\cosh ax}{a(p^2+q^2)(p+q\operatorname{senh} ax)} + \frac{p}{p^2+q^2}\int \frac{dx}{p+q\operatorname{senh} ax}$$

$$\int \frac{dx}{p^2+q^2\operatorname{senh}^2 ax} = \begin{cases} \dfrac{1}{ap\sqrt{q^2-p^2}} \tan^{-1} \dfrac{\sqrt{q^2-p^2}\tanh ax}{p} \\[2ex] \dfrac{1}{2ap\sqrt{p^2-q^2}} \ln\left(\dfrac{p+\sqrt{p^2-q^2}\tanh ax}{p-\sqrt{p^2-q^2}\tanh ax}\right) \end{cases}$$

$$\int \frac{dx}{p^2-q^2\operatorname{senh}^2 ax} = \frac{1}{2ap\sqrt{p^2+q^2}} \ln\left(\frac{p+\sqrt{p^2+q^2}\tanh ax}{p-\sqrt{p^2+q^2}\tanh ax}\right)$$

$$\int x^m \operatorname{senh} ax\,dx = \frac{x^m\cosh ax}{a} - \frac{m}{a}\int x^{m-1}\cosh ax\,dx$$

$$\int \operatorname{senh}^n ax\,dx = \frac{\operatorname{senh}^{n-1} ax \cosh ax}{an} - \frac{n-1}{n}\int \operatorname{senh}^{n-2} ax\,dx$$

$$\int \frac{\operatorname{senh} ax}{x^n}\,dx = \frac{-\operatorname{senh} ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\cosh ax}{x^{n-1}}\,dx$$

$$\int \frac{dx}{\operatorname{senh}^n ax} = \frac{-\cosh ax}{a(n-1)\operatorname{senh}^{n-1} ax} - \frac{n-2}{n-1}\int \frac{dx}{\operatorname{senh}^{n-2} ax}$$

$$\int \frac{x\,dx}{\operatorname{senh}^n ax} = \frac{-x\cosh ax}{a(n-1)\operatorname{senh}^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\operatorname{senh}^{n-2} ax} - \frac{n-2}{n-1}\int \frac{x\,dx}{\operatorname{senh}^{n-2} ax}$$

## cosh ax

$$\int x\cosh ax\,dx = \frac{x\operatorname{senh} ax}{a} - \frac{\cosh ax}{a^2}$$

$$\int \frac{\cosh ax}{x}\,dx = \ln x + \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^4}{4\cdot 4!} + \frac{(ax)^6}{6\cdot 6!} + \cdots$$

$$\int \cosh ax \cosh px\,dx = \frac{\operatorname{senh}(a-p)x}{2(a-p)} + \frac{\operatorname{senh}(a+p)x}{2(a+p)}$$

$$\int \cosh ax \operatorname{sen} px\,dx = \frac{a \operatorname{senh} ax \operatorname{sen} px - p\cosh ax \cos px}{a^2+p^2}$$

$$\int \cosh ax \cos px\,dx = \frac{a \operatorname{senh} ax \cos px + p\cosh ax \operatorname{sen} px}{a^2+p^2}$$

$$\int \frac{dx}{p+q\cosh ax} = \begin{cases} \dfrac{2}{a\sqrt{q^2-p^2}}\tan^{-1}\dfrac{qe^{ax}+p}{\sqrt{q^2-p^2}} \\[2ex] \dfrac{1}{a\sqrt{p^2-q^2}}\ln\left(\dfrac{qe^{ax}+p-\sqrt{p^2-q^2}}{qe^{ax}+p+\sqrt{p^2-q^2}}\right) \end{cases}$$

$$\int \frac{dx}{(p+q\cosh ax)^2} = \frac{q\operatorname{senh} ax}{a(q^2-p^2)(p+q\cosh ax)} - \frac{p}{q^2-p^2}\int\frac{dx}{p+q\cosh ax}$$

$$\int \frac{dx}{p^2-q^2\cosh^2 ax} = \begin{cases} \dfrac{1}{2ap\sqrt{p^2-q^2}}\ln\left(\dfrac{p\tanh ax+\sqrt{p^2-q^2}}{p\tanh ax-\sqrt{p^2-q^2}}\right) \\[2ex] \dfrac{-1}{ap\sqrt{q^2-p^2}}\tan^{-1}\dfrac{p\tanh ax}{\sqrt{q^2-p^2}} \end{cases}$$

$$\int \frac{dx}{p^2+q^2\cosh^2 ax} = \begin{cases} \dfrac{1}{2ap\sqrt{p^2+q^2}}\ln\left(\dfrac{p\tanh ax+\sqrt{p^2+q^2}}{p\tanh ax-\sqrt{p^2+q^2}}\right) \\[2ex] \dfrac{1}{ap\sqrt{p^2+q^2}}\tan^{-1}\dfrac{p\tanh ax}{\sqrt{p^2+q^2}} \end{cases}$$

$$\int x^m\cosh ax\,dx = \frac{x^m\operatorname{senh} ax}{a} - \frac{m}{a}\int x^{m-1}\operatorname{senh} ax\,dx$$

$$\int \cosh^n ax\,dx = \frac{\cosh^{n-1} ax \operatorname{senh} ax}{an} + \frac{n-1}{n}\int \cosh^{n-2} ax\,dx$$

$$\int \frac{\cosh ax}{x^n}\,dx = \frac{-\cosh ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int\frac{\operatorname{senh} ax}{x^{n-1}}\,dx$$

$$\int \frac{dx}{\cosh^n ax} = \frac{\operatorname{senh} ax}{a(n-1)\cosh^{n-1} ax} + \frac{n-2}{n-1}\int\frac{dx}{\cosh^{n-2} ax}$$

$$\int \frac{x\,dx}{\cosh^n ax} = \frac{x\operatorname{senh} ax}{a(n-1)\cosh^{n-1} ax} + \frac{1}{(n-1)(n-2)a^2\cosh^{n-2} ax} + \frac{n-2}{n-1}\int\frac{x\,dx}{\cosh^{n-2} ax}$$

## senh $ax$ Y cosh $ax$

$$\int \operatorname{senh} ax \cosh ax \, dx = \frac{\operatorname{senh}^2 ax}{2a}$$

$$\int \operatorname{senh} px \cosh qx \, dx = \frac{\cosh (p+q)x}{2(p+q)} + \frac{\cosh (p-q)x}{2(p-q)}$$

$$\int \operatorname{senh}^n ax \cosh ax \, dx = \frac{\operatorname{senh}^{n+1} ax}{(n+1)a}$$

$$\int \cosh^n ax \operatorname{senh} ax \, dx = \frac{\cosh^{n+1} ax}{(n+1)a}$$

$$\int \operatorname{senh}^2 ax \cosh^2 ax \, dx = \frac{\operatorname{senh} 4ax}{32a} - \frac{x}{8}$$

$$\int \frac{dx}{\operatorname{senh} ax \cosh ax} = \frac{1}{a} \ln \tanh ax$$

## $\tanh ax$

$$\int \tanh^n ax \operatorname{sech}^2 ax \, dx = \frac{\tanh^{n+1} ax}{(n+1)a}$$

$$\int \frac{\operatorname{sech}^2 ax}{\tanh ax} dx = \frac{1}{a} \ln \tanh ax$$

$$\int \frac{dx}{\tanh ax} = \frac{1}{a} \ln \operatorname{senh} ax$$

$$\int \frac{dx}{p + q \tanh ax} = \frac{px}{p^2 - q^2} - \frac{q}{a(p^2 - q^2)} \ln (q \operatorname{senh} ax + p \cosh ax)$$

$$\int \tanh^n ax \, dx = \frac{-\tanh^{n-1} ax}{a(n-1)} + \int \tanh^{n-2} ax \, dx$$

## $\coth ax$

$$\int \coth^n ax \operatorname{csch}^2 ax \, dx = -\frac{\coth^{n+1} ax}{(n+1)a}$$

$$\int \frac{\operatorname{csch}^2 ax}{\coth ax} dx = -\frac{1}{a} \ln \coth ax$$

$$\int \frac{dx}{\coth ax} = \frac{1}{a} \ln \cosh ax$$

$$\int \frac{dx}{p + q \coth ax} = \frac{px}{p^2 - q^2} - \frac{q}{a(p^2 - q^2)} \ln (p \operatorname{senh} ax + q \cosh ax)$$

$$\int \coth^n ax \, dx = -\frac{\coth^{n-1} ax}{a(n-1)} + \int \coth^{n-1} ax \, dx$$

## sech $ax$

$$\int \operatorname{sech}^n ax \tanh ax \, dx = -\frac{\operatorname{sech}^n ax}{na}$$

$$\int \frac{dx}{\operatorname{sech} ax} = \frac{\operatorname{senh} ax}{a}$$

$$\int \frac{dx}{q + p \operatorname{sech} ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{p + q \cosh ax}$$

$$\int \operatorname{sech}^n ax \, dx = \frac{\operatorname{sech}^{n-2} ax \tanh ax}{a(n-1)} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} ax \, dx$$

## csch $ax$

$$\int \operatorname{csch}^n ax \coth ax \, dx = -\frac{\operatorname{csch}^n ax}{na}$$

$$\int \frac{dx}{q + p \operatorname{csch} ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{p + q \operatorname{senh} ax}$$

$$\int \operatorname{csch}^n ax \, dx = \frac{-\operatorname{csch}^{n-2} ax \coth ax}{a(n-1)} - \frac{n-2}{n-1} \int \operatorname{csch}^{n-2} ax \, dx$$

# LA FUNCION GAMMA $\Gamma(n)$ PARA $n > 0$

## DEFINICION

$$\Gamma(n) = \int_0^\infty t^{n-1} e^{-t} \, dt \qquad n > 0$$

## FORMULA DE RECURRENCIA

$$\Gamma(n+1) = n\,\Gamma(n)$$

$$\Gamma(n+1) = n! \qquad \text{si } n = 0, 1, 2, \ldots \text{ donde } 0! = 1$$

## FUNCION GAMMA PARA $n < 0$

$$\Gamma(n) = \frac{\Gamma(n+1)}{n}$$

## ALGUNOS VALORES DE LA FUNCION GAMMA