PLP - Práctica 3: Inferencia de tipos

Zamboni, Gianfranco

19 de febrero de 2018

3.1. Ejercicio 1

I)

$$S(\{x : t \to Bool\}) = \{x : Nat \to Bool\}$$

II)

$$\begin{aligned} &S(\{x : t \to Bool\} \triangleright \lambda x : t_1 \to Bool.x : Nat \to t_2) \\ &= S\{x : t \to Bool\} \triangleright S(\lambda x : t_1 \to Bool.x) : S(Nat \to t_2) \\ &= \{x : Bool \to Bool\} \triangleright \lambda x : t_2 \to t_3 \to Bool.x : Nat \to t_2 \end{aligned}$$

3.2. Ejercicio 2

I)

$$MGU(\{t_1 \to t_2 \doteq Nat \to Bool\}) \overset{1}{\rightsquigarrow} \{t_1 \doteq Nat,\ t_2 \doteq Bool\} \underset{Nat/t_1}{\overset{4}{\rightsquigarrow}} \{t_2 \doteq Bool\} \underset{Bool/t_2}{\overset{4}{\rightsquigarrow}} \emptyset$$

Entonces $S = \{Bool/t_2\} \circ \{Nat/t_1\} = \{Bool/t_2,\ Nat/t_1\}$

II) $MGU(\{t_1 \to t_2 \doteq t_3\}) \underset{t_1 \to t_2/t_3}{\overset{4}{\rightsquigarrow}} \emptyset$

Entonces $S = \{t_1 \to t_2/t_3\}$

III) $MGU(\{t_1 \to t_2 \doteq t_2\}) \overset{6}{\rightsquigarrow}$ `falla` porque $t_2 \in FV(t_1 \to t_2)$ y $t_2 \neq t_1 \to t_2$

IV) $MGU(\{(t_2 \to t_1) \to Bool \doteq t_2 \to t_3\}) \overset{1}{\rightsquigarrow} \{(t_2 \to t_1) \doteq t_2,\ Bool \doteq t_3\} \overset{6}{\rightsquigarrow}$ `falla` porque $t_2 \in FV(t_2 \to t_1)$ y $t_2 \neq t_2 \to t_1$

V) $MGU(\{t_2 \to t_1 \to Bool \doteq t_2 \to t_3\}) \overset{1}{\rightsquigarrow} \{(t_2 \doteq t_2,\ t_1 \to Bool \doteq t_3\} \overset{2}{\rightsquigarrow} \{t_1 \to Bool \doteq t_3\} \underset{t_1 \to t_2/t_3}{\overset{4}{\rightsquigarrow}} \emptyset$

Entonces $S = \{t_1 \to Bool/t_3\}$

V) $MGU(\{t_2 \to t_1 \to Bool \doteq t_2 \to t_3\}) \overset{1}{\rightsquigarrow} \{(t_2 \doteq t_2,\ t_1 \to Bool \doteq t_3\} \overset{2}{\rightsquigarrow} \{t_1 \to Bool \doteq t_3\} \underset{t_1 \to t_2/t_3}{\overset{4}{\rightsquigarrow}} \emptyset$

Entonces $S = \{t_1 \to Bool/t_3\}$

VI)

$$MGU(\{t_1 \to Bool \doteq Nat \to Bool,\ t_1 \doteq t_2 \to t_3\}) \overset{1}{\rightsquigarrow} \{t_1 \doteq Nat,\ Bool \doteq Bool,\ t_1 \doteq t_2 \to t_3\}$$
$$\underset{Nat/t_1}{\overset{4}{\rightsquigarrow}} \{Bool \doteq Bool,\ Nat \doteq t_2 \to t_3\} \overset{5}{\rightsquigarrow} \texttt{falla}$$

VII)

$$MGU(\{t_1 \to Bool \doteq Nat \to Bool,\ t_2 \doteq t_1 \to t_1\}) \overset{1}{\rightsquigarrow} \{t_1 \doteq Nat,\ Bool \doteq Bool,\ t_2 \doteq t_1 \to t_1\}$$
$$\underset{Nat/t_1}{\overset{4}{\rightsquigarrow}} \{Bool \doteq Bool,\ t_2 \doteq Nat \to Nat\} \overset{2}{\rightsquigarrow} \{t_2 \doteq Nat \to Nat\} \underset{Nat \to Nat/t_2}{\overset{4}{\rightsquigarrow}} \emptyset$$

Entonces $S = \{Nat/t_1\} \circ \{Nat \to Nat/t_2\} = \{Nat/t_1,\ Nat \to Nat/t_2\}$

VIII)

$$MGU(\{t_1 \to t_2 \doteq t_3 \to t_4,\ t_3 \doteq t_2 \to t_1\}) \overset{1}{\rightsquigarrow} \{t_1 \doteq t_3,\ t_2 \doteq t_4,\ t_3 \doteq t_2 \to t_1\}$$
$$\underset{t_3/t_1}{\overset{4}{\rightsquigarrow}} \{t_2 \doteq t_4,\ t_3 \doteq t_2 \to t_3\} \overset{6}{\rightsquigarrow} \texttt{falla}$$

3.3. Ejercicio 3

- $t \to u$ unifica con $Nat \to Bool$ y $(Nat \to u) \to Bool$,

$$MGU(\{t \to u \doteq Nat \to Bool\}) = \{Nat/t,\ Bool/u\}$$

$$MGU(\{t \to u \doteq (Nat \to u) \to Bool\}) = \{Nat \to Bool/t,\ Bool/u\}$$

- Nat unifica con t,

$$MGU(\{Nat \doteq t\}) = \{Nat/t\}$$

- $u \to bool$ unifica con t y $Nat \to Bool$,

$$MGU(\{u \to Bool \doteq t\}) = \{u \to Bool/t\}$$

$$MGU(\{u \to Bool \doteq Nat \to Bool\}) = \{Nat/u\}$$

- $a \to b \to c$ unifica con t, y $Nat \to u \to Bool$

$$MGU(\{a \to b \to c \doteq t\}) = \{a \to b \to c/t\}$$

$$MGU(\{a \to b \to c \doteq Nat \to u \to Bool\}) = \{Nat/a,\ u/b\ , Bool/c\}$$

3.4. Ejercicio 4

I)

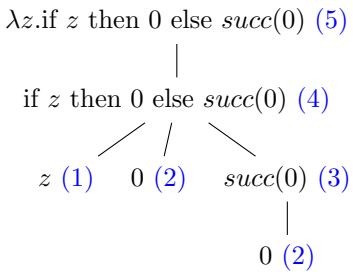
- $M_1 = Id_{Bool}$ y $M_2 = Id_{Nat}$
- $M_1 = (\lambda x : Nat \to Nat.0)\ (\lambda y : Nat.0)$ y $M_2 = (\lambda x : Bool \to Nat.0)\ (\lambda y : Bool.0)$

II)

- La identidad otra vez.
- No se me ocurrió

3.5. Ejercicio 5

I)



(1) $\mathbb{W}(z) \stackrel{def}{=} \{z : t_1\} \triangleright z : t_1$

(2) $\mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$

(3) $\mathbb{W}(succ(0)) \stackrel{def}{=} S\emptyset \triangleright S(succ(0)) : Nat = \emptyset \triangleright succ(0) : Nat$

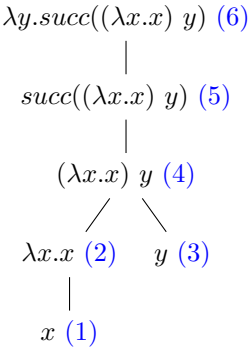
$S = MGU(\{Nat \doteq Nat\}) \stackrel{2}{\rightsquigarrow} \emptyset$

(4) $\mathbb{W}(\text{if } z \text{ then } 0 \text{ else } succ(0)) \stackrel{def}{=} S\{z : t_1\} \cup S\emptyset \cup S\emptyset \triangleright \text{if } z \text{ then } 0 \text{ else } succ(0) : SNat$
$= \{z : Bool\} \triangleright \text{if } z \text{ then } 0 \text{ else } succ(0) : Nat$

$S = MGU(\{t_1 \doteq Bool\}) \underset{Bool/t1}{\overset{4}{\rightsquigarrow}} \emptyset \Rightarrow S = \{Bool/t1\}$

(5) $\mathbb{W}(\lambda z.\text{if } z \text{ then } 0 \text{ else } succ(0)) \stackrel{def}{=} \emptyset \triangleright \lambda z : Bool.\text{if } z \text{ then } 0 \text{ else } succ(0) : Nat$

II)



(1) $\mathbb{W}(x) \stackrel{def}{=} \{x : t_1\} \triangleright x : t_1$

(2) $\mathbb{W}(\lambda x.x) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_1.x : t_1 \to t_1$

(3) $\mathbb{W}(y) \stackrel{def}{=} \{y : t_2\} \triangleright y : t_2$

(4) $\mathbb{W}((\lambda x.x)\ y) \stackrel{def}{=} S\emptyset \cup S\{y : t_2\} \triangleright S(\lambda x : t_1.x)\ Sy : St_3 = \{y : t_1\} \triangleright (\lambda x : t_1.x)\ y : t_1$

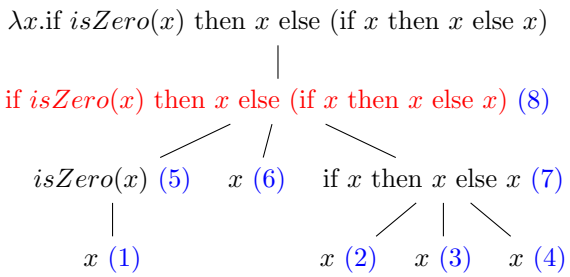
$S = MGU(\{t_1 \to t_1 \doteq t_2 \to t_3\}) \stackrel{1}{\rightsquigarrow} \{t_1 \doteq t_2,\ t_1 \doteq t_3\} \underset{t_1/t_2}{\overset{4}{\rightsquigarrow}} \{t_1 \doteq t_3\} \underset{t_1/t_3}{\overset{4}{\rightsquigarrow}} \emptyset$
$\Rightarrow S = \{t_1/t_2,\ t_1/t_3\}$

(5) $\mathbb{W}(succ((\lambda x.x)\ y)) \stackrel{def}{=} S\{y : t_1\} \triangleright S(succ((\lambda x : t_1.x)\ y)) : Nat$
$= \{y : Nat\} \triangleright succ((\lambda x : Nat.x)\ y) : Nat$

$S = MGU(\{t_1 \doteq Nat\}) \underset{Nat/t_1}{\overset{4}{\rightsquigarrow}} \emptyset \Rightarrow S = \{Nat/t_1\}$

(6) $\mathbb{W}(\lambda y.succ((\lambda x.x)\ y)) \stackrel{def}{=} \emptyset \triangleright \lambda y : Nat.succ((\lambda x : Nat.x)\ y) : Nat \to Nat$

III)



(1) $\mathbb{W}(x) \stackrel{def}{=} \{x : t_1\} \triangleright x : t_1$

(2) $\mathbb{W}(x) \stackrel{def}{=} \{x : t_2\} \triangleright x : t_2$

(3) $\mathbb{W}(x) \stackrel{def}{=} \{x : t_3\} \triangleright x : t_3$

(4) $\mathbb{W}(x) \stackrel{def}{=} \{x : t_4\} \triangleright x : t_4$

(5) $\mathbb{W}(isZero(x)) \stackrel{def}{=} S\{x : t_1\} \triangleright S(isZero(x)) : Bool = \{x : Nat\} \triangleright isZero(x) : Bool$

$S = MGU(\{t_1 \doteq Nat\}) \underset{Nat/t_1}{\overset{4}{\rightsquigarrow}} \emptyset \Rightarrow\ S = \{Nat/t_1\}$

(6) $\mathbb{W}(x) \stackrel{def}{=} \{x : t_5\} \triangleright x : t_5$

(7)

$$\begin{aligned}\mathbb{W}(\text{if } x \text{ then } x \text{ else } x) &\stackrel{def}{=} S\{x : t_2\} \cup S\{x : t_3\} \cup S\{x : t_4\} \triangleright \text{if } x \text{ then } x \text{ else } x : t_3 \\ &= \{x : Bool\} \triangleright \text{if } x \text{ then } x \text{ else } x : Bool\end{aligned}$$

$S = MGU(\{t_2 \doteq t_3,\ t_2 \doteq t_4, t_3 \doteq t_4,\ t_2 \doteq Bool\}) \underset{t_2/t_3}{\overset{4}{\rightsquigarrow}} \{t_2 \doteq t_4, t_2 \doteq t_4,\ t_2 \doteq Bool\}$

$\underset{t_2/t_4}{\overset{4}{\rightsquigarrow}} \{t_2 \doteq t_2,\ t_2 \doteq Bool\} \overset{2}{\rightsquigarrow} \{t_2 \doteq Bool\} \underset{Bool/t_2}{\overset{4}{\rightsquigarrow}} \emptyset$

$\Rightarrow\ S = \{Bool/t_2,\ Bool/t_4,\ Bool/t_3\}$

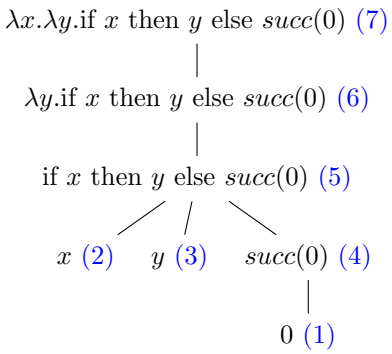
(8)

$\mathbb{W}(\text{if } isZero(x) \text{ then } x \text{ else } (\text{if } x \text{ then } x \text{ else } x))$

$\stackrel{def}{=} S\{x : Nat\} \cup S\{x : t_5\} \cup S\{x : Bool\} \triangleright \text{if } isZero(x) \text{ then } x \text{ else } (\text{if } x \text{ then } x \text{ else } x) : t_5$

$S = MGU(\{Nat \doteq t_5,\ Nat \doteq Bool, t_5 \doteq Bool,\ Nat \doteq Bool\}) \overset{6}{\rightsquigarrow}$ `falla`

IV)



(1) $\mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$

(2) $\mathbb{W}(x) \stackrel{def}{=} \{x : t_1\} \triangleright x : t_1$

(3) $\mathbb{W}(x) \stackrel{def}{=} \{y : t_2\} \triangleright x : t_2$

(4) $\mathbb{W}(succ(0)) \stackrel{def}{=} S\emptyset \triangleright S(succ(0)) : Nat = \emptyset \triangleright (succ(0)) : Nat$ y $S = \emptyset$

(5)

$$\begin{aligned}\mathbb{W}(\text{if } x \text{ then } y \text{ else } succ(0)) &\stackrel{def}{=} S\{x : t_1\} \cup S\{y : t_2\} \cup S\emptyset \triangleright S(\text{if } x \text{ then } y \text{ else } succ(0)) : t_2 \\ &= \{x : Bool, y : Nat\} \triangleright \text{if } x \text{ then } y \text{ else } succ(0) : Nat\end{aligned}$$

$$S = MGU(\{t_1 \doteq Bool,\ t_2 \doteq Nat\}) \underset{Bool/t_1}{\overset{4}{\rightsquigarrow}} \{t_2 \doteq Nat\} \underset{Nat/t_2}{\overset{4}{\rightsquigarrow}} \emptyset$$
$$\Rightarrow S = \{Bool/t_1,\ Nat/t_2\}$$

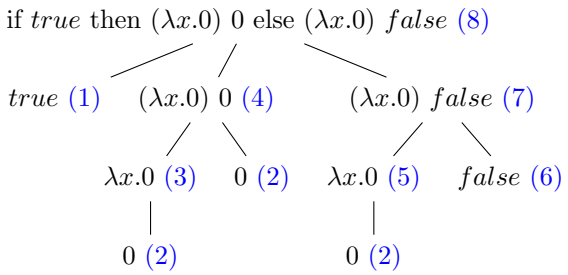
(6)

$$\mathbb{W}(\lambda y.\text{if } x \text{ then } y \text{ else } succ(0)) \stackrel{def}{=} \{x : Bool\} \triangleright \lambda y : Nat.\text{if } x \text{ then } y \text{ else } succ(0) : Nat \to Nat$$

(7)

$$\begin{aligned}&\mathbb{W}(\lambda x.\lambda y.\text{if } x \text{ then } y \text{ else } succ(0)) \\ &\stackrel{def}{=} \emptyset \triangleright \lambda x : Bool.\lambda y : Nat.\text{if } x \text{ then } y \text{ else } succ(0) : Bool \to Nat \to Nat\end{aligned}$$

V)



(1) $\mathbb{W}(true) \stackrel{def}{=} \emptyset \triangleright true : Bool$

(2) $\mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$

(3) $\mathbb{W}(\lambda x.0) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_1.0 : t_1 \to Nat$

(4) $\mathbb{W}((\lambda x.0)\ 0) \stackrel{def}{=} S\emptyset \triangleright S(\lambda x : t_1.0)\ S0 : St_2 = \emptyset \triangleright (\lambda x : Nat.0)\ 0 : Nat$

$$S = MGU(\{t_1 \to Nat \doteq Nat \to t_2\}) \overset{1}{\rightsquigarrow} \{t_1 \doteq Nat,\ Nat \to t_4\} \underset{Nat/t_1}{\overset{4}{\rightsquigarrow}} \{Nat \to t_4\} \underset{Nat/t_4}{\overset{4}{\rightsquigarrow}} \emptyset$$
$$\Rightarrow S = \{Nat/t_1, Nat/t_2\}$$

(5) $\mathbb{W}(\lambda x.0) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_3.0 : t_3 \to Nat$

(6) $\mathbb{W}(false) \stackrel{def}{=} \emptyset \triangleright false : Bool$

(7) $\mathbb{W}((\lambda x.0)\ false) \stackrel{def}{=} S\emptyset \triangleright S(\lambda x : t_3.0)\ Sfalse : St_4 = \emptyset \triangleright (\lambda x : Bool.0)\ false : Nat$

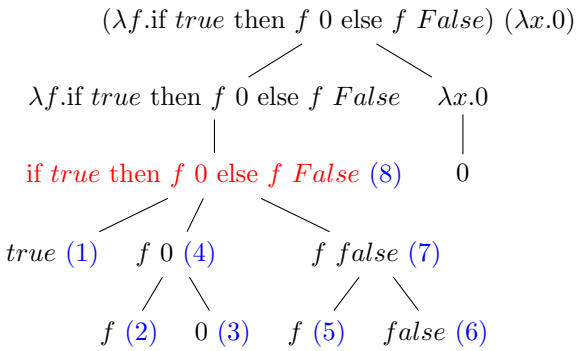
$$S = MGU(\{t_3 \to Nat \doteq Bool \to t_4\}) \overset{1}{\rightsquigarrow} \{t_3 \doteq Bool,\ Nat \to t_4\} \underset{Bool/t_3}{\overset{4}{\rightsquigarrow}} \{Nat \to t_4\} \underset{Nat/t_4}{\overset{4}{\rightsquigarrow}} \emptyset$$
$$\Rightarrow S = \{Bool/t_3, Nat/t_4\}$$

(8)

$$\mathbb{W}(\text{if } true \text{ then } (\lambda x.0)\ 0 \text{ else } (\lambda x.0)\ false)$$
$$\stackrel{def}{=} S\emptyset \triangleright S(\text{if } true \text{ then } (\lambda x : Nat.0)\ 0 \text{ else } (\lambda x : Bool.0)\ false) : SNat$$
$$= \emptyset \triangleright \text{if } true \text{ then } (\lambda x : Nat.0)\ 0 \text{ else } (\lambda x : Bool.0)\ false : Nat$$

$$S = MGU(\{Bool \doteq Bool,\ Nat \doteq Nat\}) \underset{\times 2}{\overset{2}{\rightsquigarrow}} \emptyset$$
$$\Rightarrow S = \emptyset$$

VI)



(1) $\mathbb{W}(true) \stackrel{def}{=} \emptyset \triangleright true : Bool$

(3) $\mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$

(2) $\mathbb{W}(f) \stackrel{def}{=} \{f : t_1\} \triangleright f : t_1$

(4) $\mathbb{W}(f\ 0) \stackrel{def}{=} S\{f : t_1\} \triangleright Sf\ S0 : St_2 = \{f : Nat \to t_2\} \triangleright f\ 0 : t_2$

$$S = MGU(\{t_1 \doteq Nat \to t_2\}) \underset{Nat \to t_2/t1}{\overset{4}{\rightsquigarrow}} \emptyset$$
$$\Rightarrow S = \{Nat \to t_2/t_1\}$$

(5) $\mathbb{W}(f) \stackrel{def}{=} \{f : t_3\} \triangleright f : t_3$

(6) $\mathbb{W}(false) \stackrel{def}{=} \emptyset \triangleright false : Bool$

(7) $\mathbb{W}(f\ false) \stackrel{def}{=} S\{f : t_3\} \triangleright Sf\ Sfalse : St_4 = \{f : Bool \to t_4\} \triangleright f\ false : t_4$

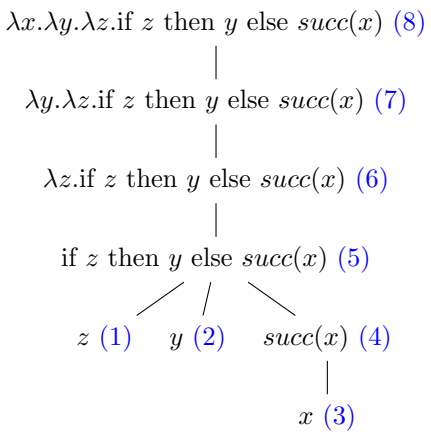
$$S = MGU(\{t_1 \doteq Nat \to t_2\}) \underset{Nat \to t_2/t1}{\overset{4}{\rightsquigarrow}} \emptyset$$
$$\Rightarrow S = \{Nat \to t_2/t_1\}$$

(8)

$$\mathbb{W}(\text{if } true \text{ then } f\ 0 \text{ else } f\ false)$$
$$\stackrel{def}{=} S\{f : Nat \to t_2\} \cup S\{f : Bool \to t_4\} \triangleright S(\text{if } true \text{ then } f\ 0 \text{ else } f\ false) : St_2$$

$$S = MGU(\{Bool \doteq Bool,\ Nat \to t_2 \doteq Bool \to t_4\}) \overset{1}{\rightsquigarrow} \{Bool \doteq Bool,\ Nat \doteq Bool,\ t_2 \doteq t_4\} \overset{5}{\rightsquigarrow} \texttt{falla}$$

VII)



(1) $\mathbb{W}(z) \stackrel{def}{=} \{z : t_1\} \triangleright z : t_1$

(2) $\mathbb{W}(y) \stackrel{def}{=} \{y : t_2\} \triangleright y : t_2$

(3) $\mathbb{W}(x) \stackrel{def}{=} \{x : t_3\} \triangleright x : t_3$

(4) $\mathbb{W}(succ(x)) \stackrel{def}{=} S\{x : t_3\} \triangleright Ssucc(x) : Nat = \{x : Nat\} \triangleright succ(x) : Nat$

$$S = MGU(\{t_3 \doteq Nat\}) \underset{Nat/t3}{\overset{4}{\rightsquigarrow}} \emptyset \Rightarrow S = \{Nat/t_3\}$$

(5)

$$\begin{aligned}&\mathbb{W}(\text{if } z \text{ then } y \text{ else } succ(x))\\ &\stackrel{def}{=} S\{z : t_1\} \cup S\{y : t_2\} \cup S\{x : Nat\} \triangleright S(\text{if } z \text{ then } y \text{ else } succ(x)) : St_2\\ &= \{z : Bool,\ y : Nat,\ x : Nat\} \triangleright \text{if } z \text{ then } y \text{ else } succ(x) : Nat\end{aligned}$$

$$S = MGU(\{t_1 \doteq Bool,\ Nat \doteq t_3\}) = \{Bool/t_1,\ Nat/t_3\}$$

(6)

$$\mathbb{W}(\lambda z.\text{if } z \text{ then } y \text{ else } succ(x)) \stackrel{def}{=} \{y : Nat,\ x : Nat\} \triangleright \lambda z : Bool.\text{if } z \text{ then } y \text{ else } succ(x) : Nat$$

(7)

$$\mathbb{W}(\lambda y.\lambda z.\text{if } z \text{ then } y \text{ else } succ(x)) \stackrel{def}{=} \{x : Nat\} \triangleright \lambda y : Nat.\lambda z : Bool.\text{if } z \text{ then } y \text{ else } succ(x) : Nat$$

(8)

$$\begin{aligned}&\mathbb{W}(\lambda x.\lambda y.\lambda z.\text{if } z \text{ then } y \text{ else } succ(x))\\ &\stackrel{def}{=} \emptyset \triangleright \lambda x : Nat.\lambda y : Nat.\lambda z : Bool.\text{if } z \text{ then } y \text{ else } succ(x) : Nat\end{aligned}$$