

PLP - Práctica 3: Inferencia de tipos

Zamboni, Gianfranco

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3.1. Ejercicio 1

I)

$$S(\{x : t \rightarrow Bool\}) = \{x : Nat \rightarrow Bool\}$$

II)

$$\begin{aligned} & S(\{x : t \rightarrow Bool\} \triangleright \lambda x : t_1 \rightarrow Bool. x : Nat \rightarrow t_2) \\ &= S\{x : t \rightarrow Bool\} \triangleright S(\lambda x : t_1 \rightarrow Bool. x) : S(Nat \rightarrow t_2) \\ &= \{x : Bool \rightarrow Bool\} \triangleright \lambda x : t_2 \rightarrow t_3 \rightarrow Bool. x : Nat \rightarrow t_2 \end{aligned}$$

3.2. Ejercicio 2

I)

$$MGU(\{t_1 \rightarrow t_2 \doteq Nat \rightarrow Bool\}) \xrightarrow{1} \{t_1 \doteq Nat, t_2 \doteq Bool\} \xrightarrow[4]{Nat/t_1} \{t_2 \doteq Bool\} \xrightarrow[4]{Bool/t_2} \emptyset$$

$$\text{Entonces } S = \{Bool/t_2\} \circ \{Nat/t_1\} = \{Bool/t_2, Nat/t_1\}$$

II) $MGU(\{t_1 \rightarrow t_2 \doteq t_3\}) \xrightarrow[t_1 \rightarrow t_2/t_3]{4} \emptyset$

$$\text{Entonces } S = \{t_1 \rightarrow t_2/t_3\}$$

III) $MGU(\{t_1 \rightarrow t_2 \doteq t_2\}) \xrightarrow{6} \text{falla}$ porque $t_2 \in FV(t_1 \rightarrow t_2)$ y $t_2 \neq t_1 \rightarrow t_2$

IV) $MGU(\{(t_2 \rightarrow t_1) \rightarrow Bool \doteq t_2 \rightarrow t_3\}) \xrightarrow{1} \{(t_2 \rightarrow t_1) \doteq t_2, Bool \doteq t_3\} \xrightarrow{6} \text{falla}$ porque $t_2 \in FV(t_2 \rightarrow t_1)$ y $t_2 \neq t_2 \rightarrow t_1$

V) $MGU(\{t_2 \rightarrow t_1 \rightarrow Bool \doteq t_2 \rightarrow t_3\}) \xrightarrow{1} \{(t_2 \doteq t_2, t_1 \rightarrow Bool \doteq t_3\} \xrightarrow{2} \{t_1 \rightarrow Bool \doteq t_3\} \xrightarrow[t_1 \rightarrow t_2/t_3]{4} \emptyset$

$$\text{Entonces } S = \{t_1 \rightarrow Bool/t_3\}$$

V) $MGU(\{t_2 \rightarrow t_1 \rightarrow Bool \doteq t_2 \rightarrow t_3\}) \xrightarrow{1} \{(t_2 \doteq t_2, t_1 \rightarrow Bool \doteq t_3\} \xrightarrow{2} \{t_1 \rightarrow Bool \doteq t_3\} \xrightarrow[t_1 \rightarrow t_2/t_3]{4} \emptyset$

$$\text{Entonces } S = \{t_1 \rightarrow Bool/t_3\}$$

VI)

$$MGU(\{t_1 \rightarrow Bool \doteq Nat \rightarrow Bool, t_1 \doteq t_2 \rightarrow t_3\}) \xrightarrow{1} \{t_1 \doteq Nat, Bool \doteq Bool, t_1 \doteq t_2 \rightarrow t_3\}$$

$$\xrightarrow[Nat/t_1]{4} \{Bool \doteq Bool, Nat \doteq t_2 \rightarrow t_3\} \xrightarrow{5} \text{falla}$$

VII)

$$MGU(\{t_1 \rightarrow Bool \doteq Nat \rightarrow Bool, t_2 \doteq t_1 \rightarrow t_1\}) \xrightarrow{1} \{t_1 \doteq Nat, Bool \doteq Bool, t_2 \doteq t_1 \rightarrow t_1\}$$

$$\xrightarrow[4]{Nat/t_1} \{Bool \doteq Bool, t_2 \doteq Nat \rightarrow Nat\} \xrightarrow{2} \{t_2 \doteq Nat \rightarrow Nat\} \xrightarrow[4]{Nat \rightarrow Nat/t_2} \emptyset$$

Entonces $S = \{Nat/t_1\} \circ \{Nat \rightarrow Nat/t_2\} = \{Nat/t_1, Nat \rightarrow Nat/t_2\}$

VIII)

$$MGU(\{t_1 \rightarrow t_2 \doteq t_3 \rightarrow t_4, t_3 \doteq t_2 \rightarrow t_1\}) \xrightarrow{1} \{t_1 \doteq t_3, t_2 \doteq t_4, t_3 \doteq t_2 \rightarrow t_1\}$$

$$\xrightarrow[4]{t_3/t_1} \{t_2 \doteq t_4, t_3 \doteq t_2 \rightarrow t_3\} \xrightarrow{6} \text{falla}$$

3.3. Ejercicio 3

- $t \rightarrow u$ unifica con $Nat \rightarrow Bool$ y $(Nat \rightarrow u) \rightarrow Bool$,

$$MGU(\{t \rightarrow u \doteq Nat \rightarrow Bool\}) = \{Nat/t, Bool/u\}$$

$$MGU(\{t \rightarrow u \doteq (Nat \rightarrow u) \rightarrow Bool\}) = \{Nat \rightarrow Bool/t, Bool/u\}$$

- Nat unifica con t ,

$$MGU(\{Nat \doteq t\}) = \{Nat/t\}$$

- $u \rightarrow bool$ unifica con t y $Nat \rightarrow Bool$,

$$MGU(\{u \rightarrow Bool \doteq t\}) = \{u \rightarrow Bool/t\}$$

$$MGU(\{u \rightarrow Bool \doteq Nat \rightarrow Bool\}) = \{Nat/u\}$$

- $a \rightarrow b \rightarrow c$ unifica con t , y $Nat \rightarrow u \rightarrow Bool$

$$MGU(\{a \rightarrow b \rightarrow c \doteq t\}) = \{a \rightarrow b \rightarrow c/t\}$$

$$MGU(\{a \rightarrow b \rightarrow c \doteq Nat \rightarrow u \rightarrow Bool\}) = \{Nat/a, u/b, Bool/c\}$$

3.4. Ejercicio 4

I)

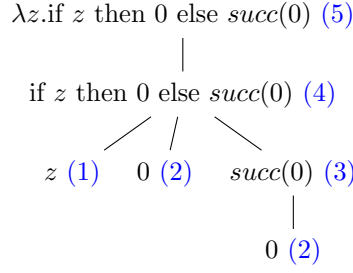
- $M_1 = Id_{Bool}$ y $M_2 = Id_{Nat}$
- $M_1 = (\lambda x : Nat \rightarrow Nat.0) (\lambda y : Nat.0)$ y $M_2 = (\lambda x : Bool \rightarrow Nat.0) (\lambda y : Bool.0)$

II)

- La identidad otra vez.
- No se me ocurrió

3.5. Ejercicio 5

I)



$$(1) \quad \mathbb{W}(z) \stackrel{def}{=} \{z : t_1\} \triangleright z : t_1$$

$$(2) \quad \mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$$

$$(3) \quad \mathbb{W}(\text{succ}(0)) \stackrel{def}{=} S\emptyset \triangleright S(\text{succ}(0)) : Nat = \emptyset \triangleright \text{succ}(0) : Nat$$

$$S = MGU(\{Nat \doteq Nat\}) \stackrel{2}{\rightsquigarrow} \emptyset$$

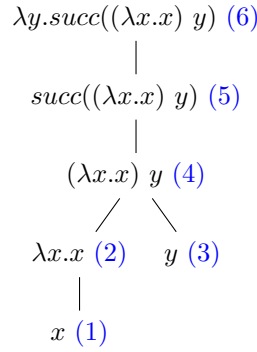
$$(4) \quad \mathbb{W}(\text{if } z \text{ then } 0 \text{ else } \text{succ}(0)) \stackrel{def}{=} S\{z : t_1\} \cup S\emptyset \cup S\emptyset \triangleright \text{if } z \text{ then } 0 \text{ else } \text{succ}(0) : SNat$$

$$= \{z : Bool\} \triangleright \text{if } z \text{ then } 0 \text{ else } \text{succ}(0) : Nat$$

$$S = MGU(\{t_1 \doteq Bool\}) \stackrel{4}{\rightsquigarrow}_{Bool/t_1} \emptyset \Rightarrow S = \{Bool/t_1\}$$

$$(5) \quad \mathbb{W}(\lambda z. \text{if } z \text{ then } 0 \text{ else } \text{succ}(0)) \stackrel{def}{=} \emptyset \triangleright \lambda z : Bool. \text{if } z \text{ then } 0 \text{ else } \text{succ}(0) : Nat$$

II)



$$(1) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_1\} \triangleright x : t_1$$

$$(2) \quad \mathbb{W}(\lambda x. x) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_1. x : t_1 \rightarrow t_1$$

$$(3) \quad \mathbb{W}(y) \stackrel{def}{=} \{y : t_2\} \triangleright y : t_2$$

$$(4) \quad \mathbb{W}((\lambda x. x) y) \stackrel{def}{=} S\emptyset \cup S\{y : t_2\} \triangleright S(\lambda x : t_1. x) Sy : St_3 = \{y : t_1\} \triangleright (\lambda x : t_1. x) y : t_1$$

$$S = MGU(\{t_1 \rightarrow t_1 \doteq t_2 \rightarrow t_3\}) \stackrel{1}{\rightsquigarrow} \{t_1 \doteq t_2, t_1 \doteq t_3\} \stackrel{4}{\rightsquigarrow}_{t_1/t_2} \{t_1 \doteq t_3\} \stackrel{4}{\rightsquigarrow}_{t_1/t_3} \emptyset$$

$$\Rightarrow S = \{t_1/t_2, t_1/t_3\}$$

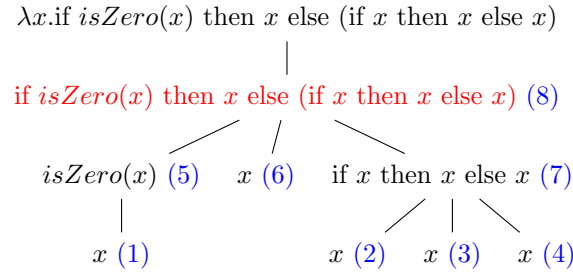
$$(5) \quad \mathbb{W}(\text{succ}((\lambda x. x) y)) \stackrel{def}{=} S\{y : t_1\} \triangleright S(\text{succ}((\lambda x : t_1. x) y)) : Nat$$

$$= \{y : Nat\} \triangleright \text{succ}((\lambda x : Nat. x) y) : Nat$$

$$S = MGU(\{t_1 \doteq Nat\}) \stackrel{4}{\rightsquigarrow}_{Nat/t_1} \emptyset \Rightarrow S = \{Nat/t_1\}$$

$$(6) \quad \mathbb{W}(\lambda y. succ((\lambda x. x) y)) \stackrel{def}{=} \emptyset \triangleright \lambda y : Nat. succ((\lambda x : Nat. x) y) : Nat \rightarrow Nat$$

III)



$$(1) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_1\} \triangleright x : t_1$$

$$(3) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_3\} \triangleright x : t_3$$

$$(2) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_2\} \triangleright x : t_2$$

$$(4) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_4\} \triangleright x : t_4$$

$$(5) \quad \mathbb{W}(isZero(x)) \stackrel{def}{=} S\{x : t_1\} \triangleright S(isZero(x)) : Bool = \{x : Nat\} \triangleright isZero(x) : Bool$$

$$S = MGU(\{t_1 \doteq Nat\}) \xrightarrow[Nat/t_1]{4} \emptyset \Rightarrow S = \{Nat/t_1\}$$

$$(6) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_5\} \triangleright x : t_5$$

(7)

$$\begin{aligned}
 \mathbb{W}(\text{if } x \text{ then } x \text{ else } x) &\stackrel{def}{=} S\{x : t_2\} \cup S\{x : t_3\} \cup S\{x : t_4\} \triangleright \text{if } x \text{ then } x \text{ else } x : t_3 \\
 &= \{x : Bool\} \triangleright \text{if } x \text{ then } x \text{ else } x : Bool
 \end{aligned}$$

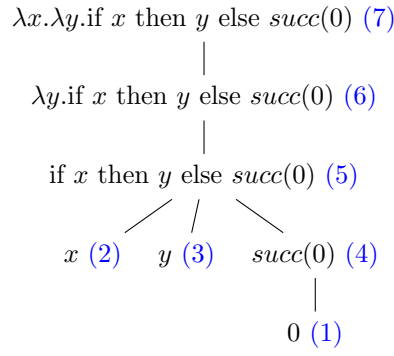
$$\begin{aligned}
 S &= MGU(\{t_2 \doteq t_3, t_2 \doteq t_4, t_3 \doteq t_4, t_2 \doteq Bool\}) \xrightarrow[t_2/t_3]{4} \{t_2 \doteq t_4, t_2 \doteq t_4, t_2 \doteq Bool\} \\
 &\xrightarrow[t_2/t_4]{4} \{t_2 \doteq t_2, t_2 \doteq Bool\} \xrightarrow{2} \{t_2 \doteq Bool\} \xrightarrow[Bool/t_2]{4} \emptyset \\
 &\Rightarrow S = \{Bool/t_2, Bool/t_4, Bool/t_3\}
 \end{aligned}$$

(8)

$$\begin{aligned}
 &\mathbb{W}(\text{if } isZero(x) \text{ then } x \text{ else } (\text{if } x \text{ then } x \text{ else } x)) \\
 &\stackrel{def}{=} S\{x : Nat\} \cup S\{x : t_5\} \cup S\{x : Bool\} \triangleright \text{if } isZero(x) \text{ then } x \text{ else } (\text{if } x \text{ then } x \text{ else } x) : t_5
 \end{aligned}$$

$$S = MGU(\{Nat \doteq t_5, Nat \doteq Bool, t_5 \doteq Bool, \text{Nat} \doteq Bool\}) \xrightarrow{6} \text{falla}$$

IV)



$$(1) \quad \mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$$

$$(2) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_1\} \triangleright x : t_1$$

$$(3) \quad \mathbb{W}(x) \stackrel{def}{=} \{y : t_2\} \triangleright x : t_2$$

$$(4) \quad \mathbb{W}(\text{succ}(0)) \stackrel{def}{=} S\emptyset \triangleright S(\text{succ}(0)) : Nat = \emptyset \triangleright (\text{succ}(0)) : Nat \text{ y } S = \emptyset$$

(5)

$$\begin{aligned}
 \mathbb{W}(\text{if } x \text{ then } y \text{ else } \text{succ}(0)) &\stackrel{def}{=} S\{x : t_1\} \cup S\{y : t_2\} \cup S\emptyset \triangleright S(\text{if } x \text{ then } y \text{ else } \text{succ}(0)) : t_2 \\
 &= \{x : Bool, y : Nat\} \triangleright \text{if } x \text{ then } y \text{ else } \text{succ}(0) : Nat
 \end{aligned}$$

$$\begin{aligned}
 S &= MGU(\{t_1 \doteq Bool, t_2 \doteq Nat\}) \xrightarrow[\text{Bool}/t_1]{4} \{t_2 \doteq Nat\} \xrightarrow[\text{Nat}/t_2]{4} \emptyset \\
 &\Rightarrow S = \{Bool/t_1, Nat/t_2\}
 \end{aligned}$$

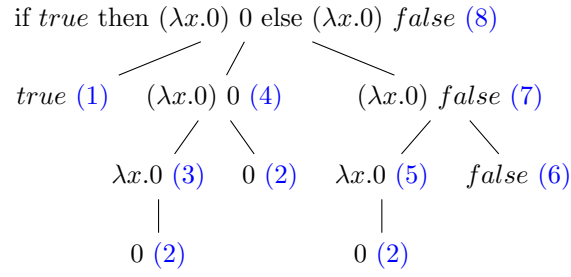
(6)

$$\mathbb{W}(\lambda y. \text{if } x \text{ then } y \text{ else } \text{succ}(0)) \stackrel{def}{=} \{x : Bool\} \triangleright \lambda y : Nat. \text{if } x \text{ then } y \text{ else } \text{succ}(0) : Nat \rightarrow Nat$$

(7)

$$\begin{aligned}
 &\mathbb{W}(\lambda x. \lambda y. \text{if } x \text{ then } y \text{ else } \text{succ}(0)) \\
 &\stackrel{def}{=} \emptyset \triangleright \lambda x : Bool. \lambda y : Nat. \text{if } x \text{ then } y \text{ else } \text{succ}(0) : Bool \rightarrow Nat \rightarrow Nat
 \end{aligned}$$

V)



$$(1) \quad \mathbb{W}(true) \stackrel{def}{=} \emptyset \triangleright true : Bool$$

$$(2) \quad \mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$$

$$(3) \quad \mathbb{W}(\lambda x.0) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_1.0 : t_1 \rightarrow Nat$$

$$(4) \quad \mathbb{W}((\lambda x.0) \ 0) \stackrel{def}{=} S\emptyset \triangleright S(\lambda x : t_1.0) \ S0 : St_2 = \emptyset \triangleright (\lambda x : Nat.0) \ 0 : Nat$$

$$\begin{aligned}
 S &= MGU(\{t_1 \rightarrow Nat \dot{=} Nat \rightarrow t_2\}) \xrightarrow{1} \{t_1 \dot{=} Nat, Nat \rightarrow t_4\} \xrightarrow[Nat/t_1]{4} \{Nat \rightarrow t_4\} \\
 &\xrightarrow[Nat/t_4]{4} \emptyset \\
 &\Rightarrow S = \{Nat/t_1, Nat/t_2\}
 \end{aligned}$$

$$(5) \quad \mathbb{W}(\lambda x.0) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_3.0 : t_3 \rightarrow Nat$$

$$(6) \quad \mathbb{W}(false) \stackrel{def}{=} \emptyset \triangleright false : Bool$$

$$(7) \quad \mathbb{W}((\lambda x.0) \ false) \stackrel{def}{=} S\emptyset \triangleright S(\lambda x : t_3.0) \ Sfalse : St_4 = \emptyset \triangleright (\lambda x : Bool.0) \ false : Nat$$

$$\begin{aligned}
 S &= MGU(\{t_3 \rightarrow Nat \dot{=} Bool \rightarrow t_4\}) \xrightarrow{1} \{t_3 \dot{=} Bool, Nat \rightarrow t_4\} \xrightarrow[Bool/t_3]{4} \{Nat \rightarrow t_4\} \\
 &\xrightarrow[Nat/t_4]{4} \emptyset \\
 &\Rightarrow S = \{Bool/t_3, Nat/t_4\}
 \end{aligned}$$

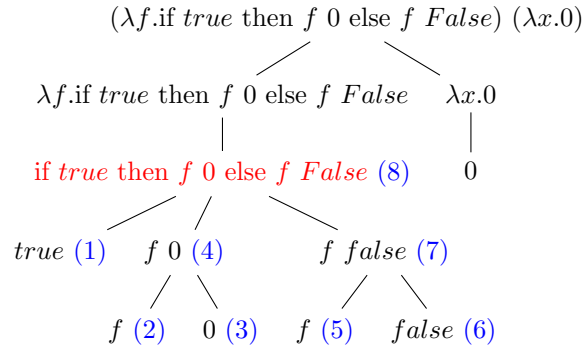
(8)

$$\mathbb{W}(\text{if } true \text{ then } (\lambda x.0) \ 0 \text{ else } (\lambda x.0) \ false)$$

$$\begin{aligned}
 &\stackrel{def}{=} S\emptyset \triangleright S(\text{if } true \text{ then } (\lambda x : Nat.0) \ 0 \text{ else } (\lambda x : Bool.0) \ false) : SNat \\
 &= \emptyset \triangleright \text{if } true \text{ then } (\lambda x : Nat.0) \ 0 \text{ else } (\lambda x : Bool.0) \ false : Nat
 \end{aligned}$$

$$\begin{aligned}
 S &= MGU(\{Bool \dot{=} Bool, Nat \dot{=} Nat\}) \xrightarrow[\times 2]{2} \emptyset \\
 &\Rightarrow S = \emptyset
 \end{aligned}$$

VI)



$$(1) \quad \mathbb{W}(true) \stackrel{def}{=} \emptyset \triangleright true : Bool$$

$$(3) \quad \mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$$

$$(2) \quad \mathbb{W}(f) \stackrel{def}{=} \{f : t_1\} \triangleright f : t_1$$

$$(4) \quad \mathbb{W}(f \ 0) \stackrel{def}{=} S\{f : t_1\} \triangleright Sf \ S0 : St_2 = \{f : Nat \rightarrow t_2\} \triangleright f \ 0 : t_2$$

$$\begin{aligned} S &= MGU(\{t_1 \doteq Nat \rightarrow t_2\}) \xrightarrow[Nat \rightarrow t_2/t_1]{4} \emptyset \\ &\Rightarrow S = \{Nat \rightarrow t_2/t_1\} \end{aligned}$$

$$(5) \quad \mathbb{W}(f) \stackrel{def}{=} \{f : t_3\} \triangleright f : t_3$$

$$(6) \quad \mathbb{W}(false) \stackrel{def}{=} \emptyset \triangleright false : Bool$$

$$(7) \quad \mathbb{W}(f \ false) \stackrel{def}{=} S\{f : t_3\} \triangleright Sf \ Sfalse : St_4 = \{f : Bool \rightarrow t_4\} \triangleright f \ false : t_4$$

$$\begin{aligned} S &= MGU(\{t_1 \doteq Nat \rightarrow t_2\}) \xrightarrow[Nat \rightarrow t_2/t_1]{4} \emptyset \\ &\Rightarrow S = \{Nat \rightarrow t_2/t_1\} \end{aligned}$$

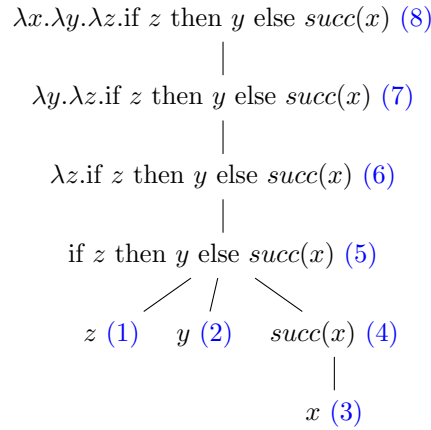
(8)

$$\mathbb{W}(\text{if } true \text{ then } f \ 0 \text{ else } f \ false)$$

$$\stackrel{def}{=} S\{f : Nat \rightarrow t_2\} \cup S\{f : Bool \rightarrow t_4\} \triangleright S(\text{if } true \text{ then } f \ 0 \text{ else } f \ false) : St_2$$

$$S = MGU(\{Bool \doteq Bool, \ Nat \rightarrow t_2 \doteq Bool \rightarrow t_4\}) \xrightarrow{1} \{Bool \doteq Bool, \ \text{Nat} \doteq Bool, \ t_2 \doteq t_4\} \xrightarrow{5} \text{falla}$$

VII)



$$(1) \quad \mathbb{W}(z) \stackrel{def}{=} \{z : t_1\} \triangleright z : t_1$$

$$(3) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_3\} \triangleright x : t_3$$

$$(2) \quad \mathbb{W}(y) \stackrel{def}{=} \{y : t_2\} \triangleright y : t_2$$

$$(4) \quad \mathbb{W}(\text{succ}(x)) \stackrel{def}{=} S\{x : t_3\} \triangleright S\text{succ}(x) : \text{Nat} = \{x : \text{Nat}\} \triangleright \text{succ}(x) : \text{Nat}$$

$$S = \text{MGU}(\{t_3 \doteq \text{Nat}\}) \xrightarrow[\text{Nat}/t_3]{4} \emptyset \Rightarrow S = \{\text{Nat}/t_3\}$$

(5)

$$\begin{aligned}
 & \mathbb{W}(\text{if } z \text{ then } y \text{ else } \text{succ}(x)) \\
 & \stackrel{def}{=} S\{z : t_1\} \cup S\{y : t_2\} \cup S\{x : \text{Nat}\} \triangleright S(\text{if } z \text{ then } y \text{ else } \text{succ}(x)) : St_2 \\
 & = \{z : \text{Bool}, y : \text{Nat}, x : \text{Nat}\} \triangleright \text{if } z \text{ then } y \text{ else } \text{succ}(x) : \text{Nat}
 \end{aligned}$$

$$S = \text{MGU}(\{t_1 \doteq \text{Bool}, \text{Nat} \doteq t_3\}) = \{\text{Bool}/t_1, \text{Nat}/t_3\}$$

(6)

$$\mathbb{W}(\lambda z. \text{if } z \text{ then } y \text{ else } \text{succ}(x)) \stackrel{def}{=} \{y : \text{Nat}, x : \text{Nat}\} \triangleright \lambda z : \text{Bool}. \text{if } z \text{ then } y \text{ else } \text{succ}(x) : \text{Nat}$$

(7)

$$\mathbb{W}(\lambda y. \lambda z. \text{if } z \text{ then } y \text{ else } \text{succ}(x)) \stackrel{def}{=} \{x : \text{Nat}\} \triangleright \lambda y : \text{Nat}. \lambda z : \text{Bool}. \text{if } z \text{ then } y \text{ else } \text{succ}(x) : \text{Nat}$$

(8)

$$\begin{aligned}
 & \mathbb{W}(\lambda x. \lambda y. \lambda z. \text{if } z \text{ then } y \text{ else } \text{succ}(x)) \\
 & \stackrel{def}{=} \emptyset \triangleright \lambda x : \text{Nat}. \lambda y : \text{Nat}. \lambda z : \text{Bool}. \text{if } z \text{ then } y \text{ else } \text{succ}(x) : \text{Nat}
 \end{aligned}$$