

# PLP - Práctica 3: Inferencia de tipos

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## 3.1. Ejercicio 1

I)

$$S(\{x : t \rightarrow Bool\}) = \{x : Nat \rightarrow Bool\}$$

II)

$$\begin{aligned} & S(\{x : t \rightarrow Bool\} \triangleright \lambda x : t_1 \rightarrow Bool. x : Nat \rightarrow t_2) \\ &= S(\{x : t \rightarrow Bool\} \triangleright S(\lambda x : t_1 \rightarrow Bool. x) : S(Nat \rightarrow t_2)) \\ &= \{x : Bool \rightarrow Bool\} \triangleright \lambda x : t_2 \rightarrow t_3 \rightarrow Bool. x : Nat \rightarrow t_2 \end{aligned}$$

## 3.2. Ejercicio 2

I)

$$MGU(\{t_1 \rightarrow t_2 \doteq Nat \rightarrow Bool\}) \xrightarrow{1} \{t_1 \doteq Nat, t_2 \doteq Bool\} \xrightarrow[4]{Nat/t_1} \{t_2 \doteq Bool\} \xrightarrow[4]{Bool/t_2} \emptyset$$

$$\text{Entonces } S = \{Bool/t_2\} \circ \{Nat/t_1\} = \{Bool/t_2, Nat/t_1\}$$

II)  $MGU(\{t_1 \rightarrow t_2 \doteq t_3\}) \xrightarrow[t_1 \rightarrow t_2/t_3]{4} \emptyset$

$$\text{Entonces } S = \{t_1 \rightarrow t_2/t_3\}$$

III)  $MGU(\{t_1 \rightarrow t_2 \doteq t_2\}) \xrightarrow{6} \text{falla}$  porque  $t_2 \in FV(t_1 \rightarrow t_2)$  y  $t_2 \neq t_1 \rightarrow t_2$

IV)  $MGU(\{(t_2 \rightarrow t_1) \rightarrow Bool \doteq t_2 \rightarrow t_3\}) \xrightarrow{1} \{(t_2 \rightarrow t_1) \doteq t_2, Bool \doteq t_3\} \xrightarrow{6} \text{falla}$  porque  $t_2 \in FV(t_2 \rightarrow t_1)$  y  $t_2 \neq t_2 \rightarrow t_1$

V)  $MGU(\{t_2 \rightarrow t_1 \rightarrow Bool \doteq t_2 \rightarrow t_3\}) \xrightarrow{1} \{(t_2 \doteq t_2, t_1 \rightarrow Bool \doteq t_3\} \xrightarrow{2} \{t_1 \rightarrow Bool \doteq t_3\} \xrightarrow[t_1 \rightarrow t_2/t_3]{4} \emptyset$

$$\text{Entonces } S = \{t_1 \rightarrow Bool/t_3\}$$

V)  $MGU(\{t_2 \rightarrow t_1 \rightarrow Bool \doteq t_2 \rightarrow t_3\}) \xrightarrow{1} \{(t_2 \doteq t_2, t_1 \rightarrow Bool \doteq t_3\} \xrightarrow{2} \{t_1 \rightarrow Bool \doteq t_3\} \xrightarrow[t_1 \rightarrow t_2/t_3]{4} \emptyset$

$$\text{Entonces } S = \{t_1 \rightarrow Bool/t_3\}$$

VI)

$$MGU(\{t_1 \rightarrow Bool \doteq Nat \rightarrow Bool, t_1 \doteq t_2 \rightarrow t_3\}) \xrightarrow{1} \{t_1 \doteq Nat, Bool \doteq Bool, t_1 \doteq t_2 \rightarrow t_3\}$$

$$\xrightarrow[Nat/t_1]{4} \{Bool \doteq Bool, Nat \doteq t_2 \rightarrow t_3\} \xrightarrow{5} \text{falla}$$

VII)

$$MGU(\{t_1 \rightarrow Bool \doteq Nat \rightarrow Bool, t_2 \doteq t_1 \rightarrow t_1\}) \xrightarrow{1} \{t_1 \doteq Nat, Bool \doteq Bool, t_2 \doteq t_1 \rightarrow t_1\}$$

$$\xrightarrow[4]{Nat/t_1} \{Bool \doteq Bool, t_2 \doteq Nat \rightarrow Nat\} \xrightarrow{2} \{t_2 \doteq Nat \rightarrow Nat\} \xrightarrow[4]{Nat \rightarrow Nat/t_2} \emptyset$$

Entonces  $S = \{Nat/t_1\} \circ \{Nat \rightarrow Nat/t_2\} = \{Nat/t_1, Nat \rightarrow Nat/t_2\}$

VIII)

$$MGU(\{t_1 \rightarrow t_2 \doteq t_3 \rightarrow t_4, t_3 \doteq t_2 \rightarrow t_1\}) \xrightarrow{1} \{t_1 \doteq t_3, t_2 \doteq t_4, t_3 \doteq t_2 \rightarrow t_1\}$$

$$\xrightarrow[4]{t_3/t_1} \{t_2 \doteq t_4, t_3 \doteq t_2 \rightarrow t_3\} \xrightarrow{6} \text{falla}$$

### 3.3. Ejercicio 3

- $t \rightarrow u$  unifica con  $Nat \rightarrow Bool$  y  $(Nat \rightarrow u) \rightarrow Bool$ ,

$$MGU(\{t \rightarrow u \doteq Nat \rightarrow Bool\}) = \{Nat/t, Bool/u\}$$

$$MGU(\{t \rightarrow u \doteq (Nat \rightarrow u) \rightarrow Bool\}) = \{Nat \rightarrow Bool/t, Bool/u\}$$

- $Nat$  unifica con  $t$ ,

$$MGU(\{Nat \doteq t\}) = \{Nat/t\}$$

- $u \rightarrow bool$  unifica con  $t$  y  $Nat \rightarrow Bool$ ,

$$MGU(\{u \rightarrow Bool \doteq t\}) = \{u \rightarrow Bool/t\}$$

$$MGU(\{u \rightarrow Bool \doteq Nat \rightarrow Bool\}) = \{Nat/u\}$$

- $a \rightarrow b \rightarrow c$  unifica con  $t$ , y  $Nat \rightarrow u \rightarrow Bool$

$$MGU(\{a \rightarrow b \rightarrow c \doteq t\}) = \{a \rightarrow b \rightarrow c/t\}$$

$$MGU(\{a \rightarrow b \rightarrow c \doteq Nat \rightarrow u \rightarrow Bool\}) = \{Nat/a, u/b, Bool/c\}$$

### 3.4. Ejercicio 4

I)

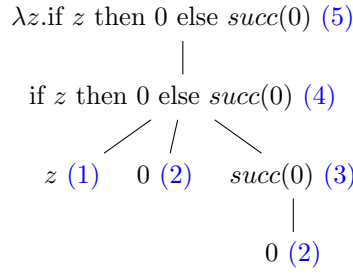
- $M_1 = Id_{Bool}$  y  $M_2 = Id_{Nat}$
- $M_1 = (\lambda x : Nat \rightarrow Nat.0) (\lambda y : Nat.0)$  y  $M_2 = (\lambda x : Bool \rightarrow Nat.0) (\lambda y : Bool.0)$

II)

- La identidad otra vez.
- No se me ocurrió

## 3.5. Ejercicio 5

I)



$$(1) \quad \mathbb{W}(z) \stackrel{def}{=} \{z : t_1\} \triangleright z : t_1$$

$$(2) \quad \mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$$

$$(3) \quad \mathbb{W}(\text{succ}(0)) \stackrel{def}{=} S\emptyset \triangleright S(\text{succ}(0)) : Nat = \emptyset \triangleright \text{succ}(0) : Nat$$

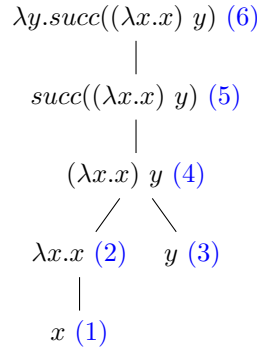
$$S = MGU(\{Nat \doteq Nat\}) \stackrel{2}{\rightsquigarrow} \emptyset$$

$$(4) \quad \mathbb{W}(\text{if } z \text{ then } 0 \text{ else } \text{succ}(0)) \stackrel{def}{=} S\{z : t_1\} \cup S\emptyset \cup S\emptyset \triangleright \text{if } z \text{ then } 0 \text{ else } \text{succ}(0) : SNat \\ = \{z : Bool\} \triangleright \text{if } z \text{ then } 0 \text{ else } \text{succ}(0) : Nat$$

$$S = MGU(\{t_1 \doteq Bool\}) \stackrel{4}{\rightsquigarrow}_{Bool/t_1} \emptyset \Rightarrow S = \{Bool/t_1\}$$

$$(5) \quad \mathbb{W}(\lambda z. \text{if } z \text{ then } 0 \text{ else } \text{succ}(0)) \stackrel{def}{=} \emptyset \triangleright \lambda z : Bool. \text{if } z \text{ then } 0 \text{ else } \text{succ}(0) : Nat$$

II)



$$(1) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_1\} \triangleright x : t_1$$

$$(2) \quad \mathbb{W}(\lambda x. x) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_1. x : t_1 \rightarrow t_1$$

$$(3) \quad \mathbb{W}(y) \stackrel{def}{=} \{y : t_2\} \triangleright y : t_2$$

$$(4) \quad \mathbb{W}((\lambda x. x) y) \stackrel{def}{=} S\emptyset \cup S\{y : t_2\} \triangleright S(\lambda x : t_1. x) Sy : St_3 = \{y : t_1\} \triangleright (\lambda x : t_1. x) y : t_1$$

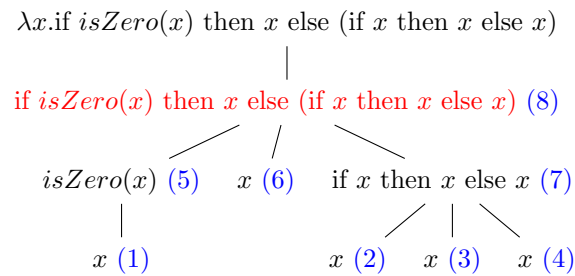
$$S = MGU(\{t_1 \rightarrow t_1 \doteq t_2 \rightarrow t_3\}) \stackrel{1}{\rightsquigarrow} \{t_1 \doteq t_2, t_1 \doteq t_3\} \stackrel{4}{\rightsquigarrow}_{t_1/t_2} \{t_1 \doteq t_3\} \stackrel{4}{\rightsquigarrow}_{t_1/t_3} \emptyset \\ \Rightarrow S = \{t_1/t_2, t_1/t_3\}$$

$$(5) \quad \mathbb{W}(\text{succ}((\lambda x. x) y)) \stackrel{def}{=} S\{y : t_1\} \triangleright S(\text{succ}((\lambda x : t_1. x) y)) : Nat \\ = \{y : Nat\} \triangleright \text{succ}((\lambda x : Nat. x) y) : Nat$$

$$S = MGU(\{t_1 \doteq Nat\}) \stackrel{4}{\rightsquigarrow}_{Nat/t_1} \emptyset \Rightarrow S = \{Nat/t_1\}$$

$$(6) \quad \mathbb{W}(\lambda y. succ((\lambda x. x) y)) \stackrel{def}{=} \emptyset \triangleright \lambda y : Nat. succ((\lambda x : Nat. x) y) : Nat \rightarrow Nat$$

III)


$$(1) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_1\} \triangleright x : t_1$$
$$(2) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_2\} \triangleright x : t_2$$
$$(3) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_3\} \triangleright x : t_3$$
$$(4) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_4\} \triangleright x : t_4$$
$$\begin{aligned} \text{(5)} \quad & \mathbb{W}(isZero(x)) \stackrel{def}{=} S\{x : t_1\} \triangleright S(isZero(x)) : Bool = \{x : Nat\} \triangleright isZero(x) : Bool \\ & S = MGU(\{t_1 \doteq Nat\}) \xrightarrow[Nat/t_1]{4} \emptyset \Rightarrow S = \{Nat/t_1\} \end{aligned}$$
$$(6) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_5\} \triangleright x : t_5$$

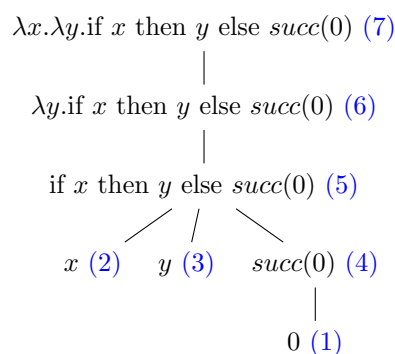
(7)

$$\begin{aligned}\mathbb{W}(\text{if } x \text{ then } x \text{ else } x) &\stackrel{\text{def}}{=} S\{x : t_2\} \cup S\{x : t_3\} \cup S\{x : t_4\} \triangleright \text{if } x \text{ then } x \text{ else } x : t_3 \\ &= \{x : Bool\} \triangleright \text{if } x \text{ then } x \text{ else } x : Bool\end{aligned}$$
$$\begin{aligned} S &= MGU(\{t_2 \doteq t_3, t_2 \doteq t_4, t_3 \doteq t_4, t_2 \doteq Bool\}) \xrightarrow[t_2/t_3]{4} \{t_2 \doteq t_4, t_2 \doteq t_4, t_2 \doteq Bool\} \\ &\xrightarrow[t_2/t_4]{4} \{t_2 \doteq t_2, t_2 \doteq Bool\} \xrightarrow{2} \{t_2 \doteq Bool\} \xrightarrow[Bool/t_2]{4} \emptyset \\ &\Rightarrow S = \{Bool/t_2, Bool/t_4, Bool/t_3\} \end{aligned}$$

(8)

$$\mathbb{W}(\text{if } isZero(x) \text{ then } x \text{ else } (\text{if } x \text{ then } x \text{ else } x))$$
$$\stackrel{def}{=} S\{x : Nat\} \cup S\{x : t_5\} \cup S\{x : Bool\} \triangleright \text{if } isZero(x) \text{ then } x \text{ else (if } x \text{ then } x \text{ else } x) : t_5$$
$$S = MGU(\{Nat \dot{=} t_5, Nat \dot{=} Bool, t_5 \dot{=} Bool, \textcolor{red}{Nat} \dot{=} \textcolor{red}{Bool}\}) \overset{6}{\rightsquigarrow} \textcolor{red}{falla}$$

IV)



$$(1) \quad \mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$$

$$(2) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_1\} \triangleright x : t_1$$

$$(3) \quad \mathbb{W}(x) \stackrel{def}{=} \{y : t_2\} \triangleright x : t_2$$

$$(4) \quad \mathbb{W}(succ(0)) \stackrel{def}{=} S\emptyset \triangleright S(succ(0)) : Nat = \emptyset \triangleright (succ(0)) : Nat \text{ y } S = \emptyset$$

(5)

$$\begin{aligned} \mathbb{W}(\text{if } x \text{ then } y \text{ else } succ(0)) &\stackrel{def}{=} S\{x : t_1\} \cup S\{y : t_2\} \cup S\emptyset \triangleright S(\text{if } x \text{ then } y \text{ else } succ(0)) : t_2 \\ &= \{x : Bool, y : Nat\} \triangleright \text{if } x \text{ then } y \text{ else } succ(0) : Nat \end{aligned}$$

$$\begin{aligned} S = MGU(\{t_1 \doteq Bool, t_2 \doteq Nat\}) &\stackrel{4}{\rightsquigarrow}_{Bool/t_1} \{t_2 \doteq Nat\} \stackrel{4}{\rightsquigarrow}_{Nat/t_2} \emptyset \\ \Rightarrow S &= \{Bool/t_1, Nat/t_2\} \end{aligned}$$

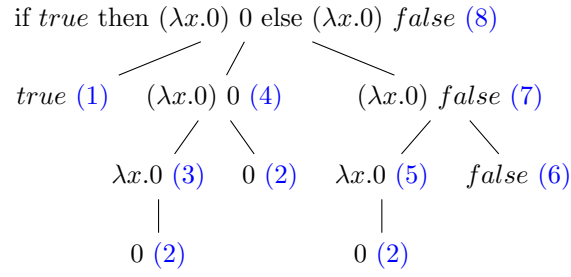
(6)

$$\mathbb{W}(\lambda y. \text{if } x \text{ then } y \text{ else } succ(0)) \stackrel{def}{=} \{x : Bool\} \triangleright \lambda y : Nat. \text{if } x \text{ then } y \text{ else } succ(0) : Nat \rightarrow Nat$$

(7)

$$\begin{aligned} \mathbb{W}(\lambda x. \lambda y. \text{if } x \text{ then } y \text{ else } succ(0)) \\ \stackrel{def}{=} \emptyset \triangleright \lambda x : Bool. \lambda y : Nat. \text{if } x \text{ then } y \text{ else } succ(0) : Bool \rightarrow Nat \rightarrow Nat \end{aligned}$$

V)



$$(1) \quad \mathbb{W}(true) \stackrel{def}{=} \emptyset \triangleright true : Bool$$

$$(2) \quad \mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$$

$$(3) \quad \mathbb{W}(\lambda x.0) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_1.0 : t_1 \rightarrow Nat$$

$$(4) \quad \mathbb{W}((\lambda x.0) \ 0) \stackrel{def}{=} S\emptyset \triangleright S(\lambda x : t_1.0) \ S0 : St_2 = \emptyset \triangleright (\lambda x : Nat.0) \ 0 : Nat$$

$$\begin{aligned}
S &= MGU(\{t_1 \rightarrow Nat \doteq Nat \rightarrow t_2\}) \xrightarrow{1} \{t_1 \doteq Nat, \ Nat \rightarrow t_4\} \xrightarrow[ Nat/t_1]{4} \{Nat \rightarrow t_4\} \\
&\xrightarrow[ Nat/t_4]{4} \emptyset \\
&\Rightarrow S = \{Nat/t_1, Nat/t_2\}
\end{aligned}$$

$$(5) \quad \mathbb{W}(\lambda x.0) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_3.0 : t_3 \rightarrow Nat$$

$$(6) \quad \mathbb{W}(false) \stackrel{def}{=} \emptyset \triangleright false : Bool$$

$$(7) \quad \mathbb{W}((\lambda x.0) \ false) \stackrel{def}{=} S\emptyset \triangleright S(\lambda x : t_3.0) \ Sfalse : St_4 = \emptyset \triangleright (\lambda x : Bool.0) \ false : Nat$$

$$\begin{aligned}
S &= MGU(\{t_3 \rightarrow Nat \doteq Bool \rightarrow t_4\}) \xrightarrow{1} \{t_3 \doteq Bool, \ Nat \rightarrow t_4\} \xrightarrow[ Bool/t_3]{4} \\
&\{Nat \rightarrow t_4\} \xrightarrow[ Nat/t_4]{4} \emptyset \\
&\Rightarrow S = \{Bool/t_3, Nat/t_4\}
\end{aligned}$$

(8)

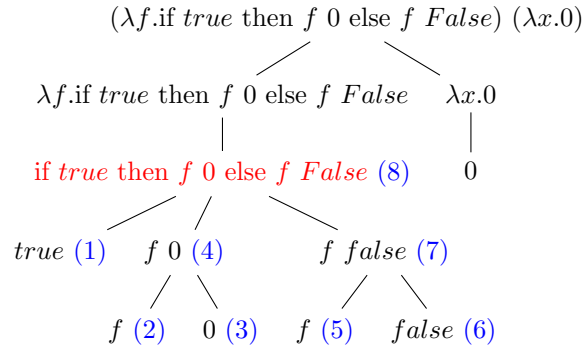
$$\mathbb{W}(\text{if } true \text{ then } (\lambda x.0) \ 0 \text{ else } (\lambda x.0) \ false)$$

$$\stackrel{def}{=} S\emptyset \triangleright S(\text{if } true \text{ then } (\lambda x : Nat.0) \ 0 \text{ else } (\lambda x : Bool.0) \ false) : SNat$$

$$= \emptyset \triangleright \text{if } true \text{ then } (\lambda x : Nat.0) \ 0 \text{ else } (\lambda x : Bool.0) \ false : Nat$$

$$\begin{aligned}
S &= MGU(\{Bool \doteq Bool, \ Nat \doteq Nat\}) \xrightarrow[\times 2]{2} \emptyset \\
&\Rightarrow S = \emptyset
\end{aligned}$$

VI)



$$(1) \quad \mathbb{W}(true) \stackrel{def}{=} \emptyset \triangleright true : Bool$$

$$(2) \quad \mathbb{W}(f) \stackrel{def}{=} \{f : t_1\} \triangleright f : t_1$$

$$(3) \quad \mathbb{W}(0) \stackrel{def}{=} \emptyset \triangleright 0 : Nat$$

$$(4) \quad \mathbb{W}(f \ 0) \stackrel{def}{=} S\{f : t_1\} \triangleright Sf \ S0 : St_2 = \{f : Nat \rightarrow t_2\} \triangleright f \ 0 : t_2$$

$$\begin{aligned}
 S &= MGU(\{t_1 \doteq Nat \rightarrow t_2\}) \xrightarrow[ Nat \rightarrow t_2 / t_1 ]{4} \emptyset \\
 &\Rightarrow S = \{Nat \rightarrow t_2 / t_1\}
 \end{aligned}$$

$$(5) \quad \mathbb{W}(f) \stackrel{def}{=} \{f : t_3\} \triangleright f : t_3$$

$$(6) \quad \mathbb{W}(false) \stackrel{def}{=} \emptyset \triangleright false : Bool$$

$$(7) \quad \mathbb{W}(f \ false) \stackrel{def}{=} S\{f : t_3\} \triangleright Sf \ Sfalse : St_4 = \{f : Bool \rightarrow t_4\} \triangleright f \ false : t_4$$

$$\begin{aligned}
 S &= MGU(\{t_1 \doteq Nat \rightarrow t_2\}) \xrightarrow[ Nat \rightarrow t_2 / t_1 ]{4} \emptyset \\
 &\Rightarrow S = \{Nat \rightarrow t_2 / t_1\}
 \end{aligned}$$

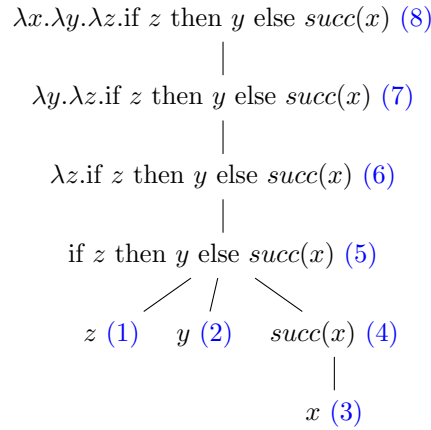
(8)

$$\mathbb{W}(\text{if } true \text{ then } f \ 0 \text{ else } f \ false)$$

$$\stackrel{def}{=} S\{f : Nat \rightarrow t_2\} \cup S\{f : Bool \rightarrow t_4\} \triangleright S(\text{if } true \text{ then } f \ 0 \text{ else } f \ false) : St_2$$

$$S = MGU(\{Bool \doteq Bool, \ Nat \rightarrow t_2 \doteq Bool \rightarrow t_4\}) \xrightarrow{1} \{Bool \doteq Bool, \ \text{Nat} \doteq Bool, \ t_2 \doteq t_4\} \xrightarrow{5} \text{falla}$$

VII)



$$(1) \quad \mathbb{W}(z) \stackrel{def}{=} \{z : t_1\} \triangleright z : t_1$$

$$(2) \quad \mathbb{W}(y) \stackrel{def}{=} \{y : t_2\} \triangleright y : t_2$$

$$(3) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_3\} \triangleright x : t_3$$

$$(4) \quad \mathbb{W}(\text{succ}(x)) \stackrel{def}{=} S\{x : t_3\} \triangleright S\text{succ}(x) : \text{Nat} = \{x : \text{Nat}\} \triangleright \text{succ}(x) : \text{Nat}$$

$$S = \text{MGU}(\{t_3 \doteq \text{Nat}\}) \xrightarrow[\text{Nat}/t_3]{4} \emptyset \Rightarrow S = \{\text{Nat}/t_3\}$$

(5)

$$\begin{aligned}
& \mathbb{W}(\text{if } z \text{ then } y \text{ else } \text{succ}(x)) \\
& \stackrel{def}{=} S\{z : t_1\} \cup S\{y : t_2\} \cup S\{x : \text{Nat}\} \triangleright S(\text{if } z \text{ then } y \text{ else } \text{succ}(x)) : St_2 \\
& = \{z : \text{Bool}, y : \text{Nat}, x : \text{Nat}\} \triangleright \text{if } z \text{ then } y \text{ else } \text{succ}(x) : \text{Nat}
\end{aligned}$$

$$S = \text{MGU}(\{t_1 \doteq \text{Bool}, \text{Nat} \doteq t_3\}) = \{\text{Bool}/t_1, \text{Nat}/t_3\}$$

(6)

$$\mathbb{W}(\lambda z. \text{if } z \text{ then } y \text{ else } \text{succ}(x)) \stackrel{def}{=} \{y : \text{Nat}, x : \text{Nat}\} \triangleright \lambda z : \text{Bool}. \text{if } z \text{ then } y \text{ else } \text{succ}(x) : \text{Nat}$$

(7)

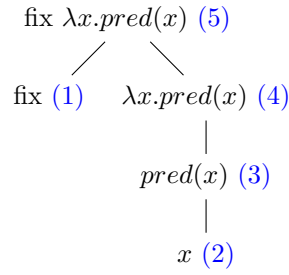
$$\mathbb{W}(\lambda y. \lambda z. \text{if } z \text{ then } y \text{ else } \text{succ}(x)) \stackrel{def}{=} \{x : \text{Nat}\} \triangleright \lambda y : \text{Nat}. \lambda z : \text{Bool}. \text{if } z \text{ then } y \text{ else } \text{succ}(x) : \text{Nat}$$

(8)

$$\begin{aligned}
& \mathbb{W}(\lambda x. \lambda y. \lambda z. \text{if } z \text{ then } y \text{ else } \text{succ}(x)) \\
& \stackrel{def}{=} \emptyset \triangleright \lambda x : \text{Nat}. \lambda y : \text{Nat}. \lambda z : \text{Bool}. \text{if } z \text{ then } y \text{ else } \text{succ}(x) : \text{Nat}
\end{aligned}$$



VIII)



$$(1) \quad \mathbb{W}(\text{fix } ) \stackrel{def}{=} \phi \triangleright \text{fix}_{t_1} : (t_1 \rightarrow t_1) \rightarrow t_1 \qquad (2) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_2\} \triangleright x : t_2$$

$$(3) \quad \mathbb{W}(\text{pred}(x)) \stackrel{def}{=} S\{x : t_2\} \triangleright S\text{succ}(x) : \text{Nat} = \{x : \text{Nat}\} \triangleright \text{succ}(x) : \text{Nat}$$

$$S = \text{MGU}(\{t_2 \doteq \text{Nat}\}) \xrightarrow[\text{Nat}/t_2]{4} \emptyset \Rightarrow S = \{\text{Nat}/t_2\}$$

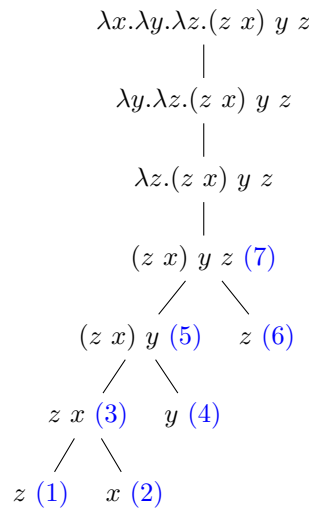
$$(4) \quad \mathbb{W}(\lambda x. \text{pred}(x)) \stackrel{def}{=} \emptyset \triangleright \lambda x : \text{Nat}. \text{pred}(x) : \text{Nat} \rightarrow \text{Nat}$$

$$(5) \quad \mathbb{W}(\text{fix } \lambda x. \text{pred}(x)) \stackrel{def}{=} \emptyset \triangleright S\text{fix}_{t_1} \lambda x : \text{Nat}. \text{pred}(x) : St_3 = \emptyset \triangleright \text{fix}_{\text{Nat}} \lambda x : \text{Nat}. \text{pred}(x) : \text{Nat}$$

$$\begin{aligned}
 S &= \text{MGU}(\{(t_1 \rightarrow t_1) \rightarrow t_1 \doteq (\text{Nat} \rightarrow \text{Nat}) \rightarrow t_3\}) \xrightarrow{1} \{(t_1 \rightarrow t_1) \doteq (\text{Nat} \rightarrow \text{Nat}), t_1 \doteq t_3\} \\
 &\xrightarrow{1} \{t_1 \doteq \text{Nat}, t_1 \doteq \text{Nat}, t_1 \doteq t_3\} \xrightarrow[\text{Nat}/t_1]{4} \{\text{Nat} \doteq \text{Nat}, \text{Nat} \doteq t_3\} \xrightarrow{2} \{\text{Nat} \doteq t_3\} \xrightarrow[\text{Nat}/t_3]{4} \emptyset \\
 &\Rightarrow S = \{\text{Nat}/t_1, \text{Nat}/t_3\}
 \end{aligned}$$

### 3.6. Ejercicio 6

I)



$$(1) \quad \mathbb{W}(z) \stackrel{def}{=} \{z : t_1\} \triangleright z : t_1 \qquad (2) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_2\} \triangleright x : t_2$$

$$(3) \quad \mathbb{W}(z \ x) \stackrel{def}{=} S\{z : t_1, x : t_2\} \triangleright S(z \ x) : St_3 = \{z : t_2 \rightarrow t_3, x : t_2\} \triangleright z \ x : t_3$$

$$S = \text{MGU}\{t_1 \doteq t_2 \rightarrow t_3\} = \{t_2 \rightarrow t_3/t_1\}$$

$$(4) \quad \mathbb{W}(y) \stackrel{def}{=} \{y : t_4\} \triangleright y : t_4$$

(5)

$$\begin{aligned} \mathbb{W}((z \ x) \ y) &\stackrel{def}{=} S\{z : t_2 \rightarrow t_3, x : t_2\} \cup S\{y : t_4\} \triangleright S((z \ x) \ y) : St_5 \\ &= \{z : t_2 \rightarrow t_4 \rightarrow t_5, x : t_2, y : t_4\} \triangleright (z \ x) \ y : t_5 \end{aligned}$$

$$S = MGU(\{t_3 \doteq t_4 \rightarrow t_5\}) = \{t_4 \rightarrow t_5/t_3\}$$

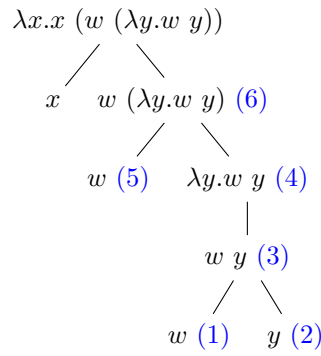
$$(6) \quad \mathbb{W}(z) \stackrel{def}{=} \{z : t_6\} \triangleright z : t_6$$

(7)

$$\mathbb{W}((z \ x) \ y) \stackrel{def}{=} S\{z : t_2 \rightarrow t_4 \rightarrow t_5, x : t_2, y : t_4\} \cup S\{z : t_6\} \triangleright S((z \ x) \ y) : St_7$$

$$S = MGU(\{t_6 \doteq t_2 \rightarrow t_4 \rightarrow t_5, t_5 \doteq t_6 \rightarrow t_7\}) \xrightarrow[t_2 \rightarrow t_4 \rightarrow t_5/t_6]{4} \{\textcolor{red}{t}_5 \doteq (t_2 \rightarrow t_4 \rightarrow \textcolor{red}{t}_5) \rightarrow t_7\} \xrightarrow{6} \textcolor{red}{falla}$$

II)



$$(1) \quad \mathbb{W}(w) \stackrel{def}{=} \{w : t_1\} \triangleright w : t_1$$

$$(2) \quad \mathbb{W}(y) \stackrel{def}{=} \{y : t_2\} \triangleright y : t_2$$

$$(3) \quad \mathbb{W}(w \ y) \stackrel{def}{=} S\{w : t_1\} \cup S\{y : t_2\} \triangleright S(w \ y) : St_3 = \{w : t_2 \rightarrow t_3, y : t_2\} \triangleright w \ y : t_3$$

$$S = MGU\{t_1 \doteq t_2 \rightarrow t_3\} = \{t_2 \rightarrow t_3/t_1\}$$

$$(4) \quad \mathbb{W}(\lambda y.w \ y) \stackrel{def}{=} \{w : t_2 \rightarrow t_3\} \triangleright \lambda y : t_2.w \ y : t_2 \rightarrow t_3$$

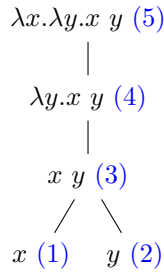
$$(5) \quad \mathbb{W}(w) \stackrel{def}{=} \{w : t_4\} \triangleright w : t_4$$

$$(6) \quad \mathbb{W}(w \ (\lambda y.w \ y)) \stackrel{def}{=} S\{w : t_4\} \cup S\{w : t_2 \rightarrow t_3\} \triangleright Sw \ S(\lambda y : t_2.w \ y) : St_5$$

$$S = MGU\{t_4 \doteq t_2 \rightarrow t_3, t_4 \doteq (t_2 \rightarrow t_3) \rightarrow t_5\} \xrightarrow[t_2 \rightarrow t_3/t_4]{4} \{t_2 \rightarrow t_3 \doteq (t_2 \rightarrow t_3) \rightarrow t_5\}$$

$$\xrightarrow{1} \{\textcolor{red}{t}_2 \doteq \textcolor{red}{t}_2 \rightarrow t_3, t_3 \doteq t_5\} \xrightarrow{6} \textcolor{red}{falla}$$

III)



$$(1) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_1\} \triangleright x : t_1 \qquad (2) \quad \mathbb{W}(y) \stackrel{def}{=} \{y : t_2\} \triangleright y : t_2$$

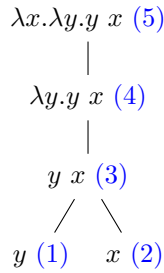
$$(3) \quad \mathbb{W}(x \ y) \stackrel{def}{=} S\{x : t_1\} \cup S\{y : t_2\} \triangleright S(x \ y) : St_3 = \{x : t_2 \rightarrow t_3, y : t_2\} \triangleright x \ y : t_3$$

$$S = MGU\{t_1 \doteq t_2 \rightarrow t_3\} = \{t_2 \rightarrow t_3/t_1\}$$

$$(4) \quad \mathbb{W}(\lambda y. x \ y) \stackrel{def}{=} \{x : t_2 \rightarrow t_3\} \triangleright \lambda y : t_2. x \ y : t_2 \rightarrow t_3$$

$$(5) \quad \mathbb{W}(\lambda x. \lambda y. x \ y) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_2 \rightarrow t_3. \lambda y : t_2. x \ y : (t_2 \rightarrow t_3) \rightarrow t_2 \rightarrow t_3$$

IV)



$$(1) \quad \mathbb{W}(y) \stackrel{def}{=} \{y : t_1\} \triangleright y : t_1 \qquad (2) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_2\} \triangleright x : t_2$$

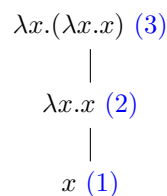
$$(3) \quad \mathbb{W}(y \ x) \stackrel{def}{=} S\{y : t_1\} \cup S\{x : t_2\} \triangleright S(y \ x) : St_3 = \{y : t_2 \rightarrow t_3, x : t_2\} \triangleright y \ x : t_3$$

$$S = MGU\{t_1 \doteq t_2 \rightarrow t_3\} = \{t_2 \rightarrow t_3/t_1\}$$

$$(4) \quad \mathbb{W}(\lambda y. y \ x) \stackrel{def}{=} \{x : t_2, \} \triangleright \lambda y : t_2 \rightarrow t_3. x \ y : (t_2 \rightarrow t_3) \rightarrow t_3$$

$$(5) \quad \mathbb{W}(\lambda x. \lambda y. y \ x) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_2. \lambda y : t_2 \rightarrow t_3. x \ y : t_2 \rightarrow (t_2 \rightarrow t_3) \rightarrow t_3$$

V)



$$(1) \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_1\} \triangleright x : t_1$$

$$(2) \quad \mathbb{W}(\lambda x. x) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_1. x : t_1 \rightarrow t_1$$

$$(3) \quad \mathbb{W}(\lambda x. \lambda x. x) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_2. (\lambda x : t_1. x) : t_2 \rightarrow t_1 \rightarrow t_1$$

VI)

$$\begin{array}{c}
 \lambda x.(\lambda y.y) x \text{ (5)} \\
 | \\
 (\lambda y.y) x \text{ (4)} \\
 / \quad \backslash \\
 \lambda y.y \text{ (2)} \quad x \text{ (3)} \\
 | \\
 y \text{ (1)}
 \end{array}
 \quad
 \begin{array}{l}
 \text{(1)} \quad \mathbb{W}(y) \stackrel{def}{=} \{y : t_1\} \triangleright y : t_1 \\
 \text{(2)} \quad \mathbb{W}(\lambda y.y) \stackrel{def}{=} \emptyset \triangleright \lambda y : t_1. y : t_1 \rightarrow t_1 \\
 \text{(3)} \quad \mathbb{W}(x) \stackrel{def}{=} \{x : t_2\} \triangleright x : t_2
 \end{array}$$

(4)  $\mathbb{W}((\lambda y.y) x) \stackrel{def}{=} S\{x : t_2\} \triangleright S((\lambda y : t_1.y) x) : St_3 = \{x : t_1\} \triangleright (\lambda y : t_1.y) x : t_1$

$S = MGU(\{t_1 \rightarrow t_1 \doteq t_2 \rightarrow t_3\}) \rightsquigarrow^1 \{t_1 \doteq t_2, \rightarrow t_1 \doteq t_3\} \rightsquigarrow^4_{t_1/t_2} \{t_1 \doteq t_3\} \rightsquigarrow^4_{t_1/t_3} \emptyset$   
 $\Rightarrow S = \{t_1/t_2, t_1/t_3\}$

(5)  $\mathbb{W}(\lambda x.(\lambda y.y) x) \stackrel{def}{=} \emptyset \triangleright \lambda x : t_1. (\lambda y : t_1.y) x : t_1 \rightarrow t_1$

VII)

$$\begin{array}{c}
 (\lambda z.\lambda x.x (z (\lambda y.z))) true \\
 / \quad \backslash \\
 \lambda z.\lambda x.x (z (\lambda y.z)) \quad true \\
 | \\
 \lambda x.x (z (\lambda y.z)) \\
 | \\
 x (z (\lambda y.z)) \\
 / \quad \backslash \\
 x \quad z (\lambda y.z) \text{ (4)} \\
 / \quad \backslash \\
 z \text{ (3)} \quad \lambda y.z \text{ (2)} \\
 | \\
 z \text{ (1)}
 \end{array}$$

(1)  $\mathbb{W}(z) \stackrel{def}{=} \{z : t_1\} \triangleright z : t_1$

(2)  $\mathbb{W}(\lambda y.z) \stackrel{def}{=} \{z : t_1\} \triangleright \lambda y : t_2. z : t_2 \rightarrow t_1$

(3)  $\mathbb{W}(z) \stackrel{def}{=} \{z : t_3\} \triangleright x : t_3$

(4)  $\mathbb{W}(z (\lambda y.z)) \stackrel{def}{=} S\{z : t_3\} \cup S\{z : t_1\} \triangleright S(z (\lambda y : t_2.z)) : St_4$

$S = MGU(\{t_3 \doteq t_1, t_3 \doteq (t_2 \rightarrow t_1) \rightarrow t_4\}) \rightsquigarrow^4_{t_3/t_1} \{\textcolor{red}{t_3} \doteq (t_2 \rightarrow \textcolor{red}{t_3}) \rightarrow t_4\} \rightsquigarrow^6 \textcolor{red}{falla}$