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State pricing, effectively complete markets, and corporate finance☆

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ABSTRACT

Event study, panel regression, and difference-in-difference techniques are among the most prominent research methodologies in corporate finance. However, these techniques are inappropriate if corporate events are anticipated to some degree, as most events are. This paper proposes options as an additional model-free source of information to identify the likelihood and impact of corporate events. We show how to quantify event impact in a simple example and assert that few restrictions on the state space are required for the approach to work in more complex settings.

1. Introduction

Finance is traditionally divided into two broad disciplines: corporate finance and asset pricing. Corporate finance analyzes the decisions made by firms and assesses their merits. Corporate finance issues are extensive, but a major branch of corporate finance studies the impact of events, typically tied to corporate actions, on firm value. Asset pricing is highly related to this research, as the valuation of securities—the central focus of asset pricing—plays a key role in the understanding of these corporate actions.

Despite all the reasons for these two subdisciplines to be intertwined, corporate finance researchers seldom pay close attention to new developments in asset pricing, and vice versa. In recent decades, the symphony of Eugene Fama and Merton Miller in Fama and Miller (1972) has rarely been replayed. One reason is that corporate finance and asset pricing have become increasingly specialized. The required specialized skills and technical hurdles in each discipline have hindered the exchange of ideas and techniques across the subdisciplines.

To illustrate, “natural experiments,” implied natural experiments from regression discontinuity techniques, event study, panel regression, and difference-in-difference techniques are predominantly used to analyze corporate finance problems. However, these tools rely on a key assumption: that corporate events are unanticipated by the market. The adjustment in stock prices to announcements can be used to infer the impact of corporate events. However, many real-world corporate events are (partially or fully) anticipated prior to announcement. Thus, depending on the event's perceived likelihood, stock prices have already adjusted to varying degrees before its announcement.

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Since event outcome anticipation tends to weaken observed stock price movements, any research technique that quantifies event impact from stock price changes will likely understate the event's true impact. This includes difference-in-difference techniques because treated and control groups may be subject to differing degrees of anticipation.

To mitigate the event outcome anticipation problem—and illustrate the benefits from cross-pollination of the two subfields—we propose a modern asset pricing approach to analyze the valuation impact of corporate events. Specifically, we suggest the use of option prices to identify hidden state prices and payoffs, allowing model-free estimation of the impact and (risk-neutral) likelihood of corporate events. Multiple pre-event options with different strike prices contain information that can filter out the impact of market anticipation, allowing identification of the event's full impact. The identification is feasible even if corporate events are anticipated because market anticipation has different effects for options with different strike prices, depending on the future event's impact on the stock price. Consider a collection of options that will expire after the event has either occurred or not occurred. At expiration, sufficiently deep in-the-money options are identically affected by the occurrence of the event. Likewise, sufficiently out-of-the-money options are identically unaffected by the event's occurrence of lack thereof. However, the event affects options with intermediate strike prices to differing degrees. Event anticipation tends to inflates the pre-event prices of all options if confounding non-event factors can have unbounded valuation effects. However, options can identify and isolate the impact of these non-event factors. For each possible realization of a non-event factor, event impact can then be identified by locating the boundaries of the intermediate strike prices from the pre-event pricing of options.

Our approach is model free in that options alone, at different strikes, can complete markets and simultaneously identify state prices and the event's valuation impact, thus offering richer and more unique information about corporate events than stock prices alone. It is the nonlinearity of option pricing that generates additional information from its higher moments that is not contained in stock prices.

The use of options to study corporate events is not new. For example, Barraclough et al. (2013) and Borochin (2014) use pre-event option prices at different strike prices to identify the likelihood and synergies of mergers and acquisitions. Both papers place strong and unorthodox assumptions on the structure of the option pricing model—specifically, that all options are priced as linear combinations of two Black-Scholes prices. Bester et al. (2019) value cash mergers with stochastic probabilities of success and Black-Scholes processes for the fallback price of the target. The latter paper's primary objective is to show that options are good predictors of merger success and that the standard Black-Scholes model will have an implied volatility smile and a kink. However, they note that that their technique is generalizable (but hard to implement) on more general diffusions of the fallback price.

Inspired by this research, we illustrate how a model-free option pricing approach can replace traditional techniques to understand certain corporate finance problems. This approach is especially desirable in corporate finance problems that are likely to be anticipated by the market, and for which a predictable resolution of uncertainty associated with that anticipation is likely to occur prior to option expiration. As noted above, current application of this approach is limited to the estimation of synergies and the likelihood of mergers and acquisitions in a highly structured model. We believe that the option pricing approach is relevant to a broader set of issues and can be applied with minimal structure on the valuation model.

Our paper is organized as follows: Section 2 describes the common framework of asset pricing theory. Section 3 discusses the traditional approach to address corporate finance problems, whereas Section 4 describes the option pricing approach. Section 5 discusses some caveats about finite state spaces under the option pricing approach and offers some final thoughts.

2. Asset pricing primer

Asset pricing theory uses a common framework to derive all asset prices, including the prices of options. There are three equivalent versions of a fundamental and highly general valuation formula: pricing kernel, risk-neutral pricing, and state prices. A simple setting, where we value a risky asset's end-of-period stochastic payoff at the beginning of the period, illustrates each version. Each approach denotes the asset's state s future payoff as $\mathrm{X}(s)$ and its beginning-of-period value as PV(X), where X is stochastic because it depends on the state realization. Expectations are denoted by $E[\ ]$ and are taken over probabilities assigned to each state at the time of valuation.

The first version of the valuation equation, which uses a pricing kernel ($m(s)$) to derive the current price of the risky asset is:

$$\mathrm{PV}(\mathrm{X}) = E[m(s)\mathrm{X}(s)] \tag{1}$$

Here, the current price of the risky asset is the expected value of the product of the pricing kernel and the future payoff. The pricing kernel m is also known as the stochastic discount factor because it represents a different discount factor for payoffs promised in different states. When the state s pricing kernel is multiplied by the states contingent payoff, it gives a present value for the payoff that occurs with a specific probability, $\pi(s)$.

The second version uses risk neutral pricing to determine the current price of the risky asset. Here, there exists a risk neutral probability for every state. The current price of the asset can be computed by using the risk-free rate to discount an "expected value" of the asset at date 1 based on the risk neutral probability measure of the corresponding payoff in each state of the world. Using algebra, the risk neutral probability measure, denoted as $E^*[\ ]$, values the payoff as follows:

$$\mathrm{PV}(\mathrm{X}) = E^*[\mathrm{X}(s)]/(1 + r_f) \tag{2}$$

where r_f is the risk free rate. Transforming the physical probability of state s, $\pi(s)$, used for expectations in Eq. (1) into a risk-neutral probability $q(s)$ used for the $E^*[\ ]$ expectation in Eq. (2)

$$q(s) = (1 + r_f)m(s)\pi(s)$$

and recognizing that $E[m(s)] = 1/(1 + r_f)$ illustrates that Eqs. (1) and (2) are equivalent. $q(s)$ is like a probability in that it sums (or integrates) to 1, as is obvious from the expression for $q(s)$ above. This expression is the present value of the state contingent payoff of $(1 + r_f)$, which summed (or integrated) over all states is the one dollar present value of a risk-free bond paying $(1 + r_f)$.

The third version uses state prices based on Arrow and Debreu (1954) to derive the current prices of risky assets. Here, the current price of the risky asset is:

$$\mathrm{PV(X)} = \sum_s p(s)\mathrm{X}(s) \tag{3a}$$

for a countable state space or

$$\mathrm{PV(X)} = \int_s p(s)X(s)ds \tag{3b}$$

for a continuous state space, where *p(s)* is the "state price," the present value of a \$1 payoff in state *s*. Eqs. (3a) and (3b) are equivalent to Eqs. (1) and (2) once we recognize that *p(s)* is just the pricing kernel times the physical probability of the state, $m(s)\pi(s)$, or equivalently, the risk neutral probability discounted at the risk-free rate, $q(s)/(1 + r_f)$.

Knowing the product of each state's pricing kernel and its physical probability, each risk neutral probability, or each state price effectively gives the value of any stochastic payoff. The fundamental theorem of asset pricing, which assumes only that there is no arbitrage, shows that these entities exist. They generate what is called a linear pricing operator (hence the expectations or scaling of them), implying that portfolios are priced as linear combinations of the values of their component assets.

Wonderful insights are generated in asset pricing from the linear pricing property. These insights are often found in settings where dynamically changing portfolio holdings and assumptions about functional forms of the pricing kernel (or of preferences that implicitly generate the kernel) relate asset prices to consumption dynamics. However, in the practice of corporate finance, the impact of an event is best understood when we estimate entities like state prices. They can be estimated from option prices and option prices alone can yield more information than just state prices.

The key insight here derives from Ross (1976), who showed that options expiring at the same time with different strike prices effectively complete the market for payoffs at option expiration. In a complete market, state prices (or their equivalent entities) are unique. Once state prices are known, researchers can use them to value anything in corporate finance. In practice, options are especially useful at identifying state prices that value a set of discrete outcomes.

In the corporate finance literature, there are numerous topics of interest associated with discrete and often binary outcomes. We refer to them as events. We have already discussed mergers and acquisitions that, once announced, have the discrete outcomes of successful vs. unsuccessful completion. However, there are also proxy contests involving votes over corporate policies and leadership including hostile takeovers, poison pills, board slates; institutional holdings that exceed some thresholds; increases versus decreases in investment over the next year; as well as corporate decisions about splits, spin offs, and share classes (DeAngelo and DeAngelo, 1989; Shivdasani, 1993; Mulherin and Poulsen, 1998). Other applications in corporate finance include corporate outcomes that are above or below the analyst's consensus forecast, internal vs. external CEO succession in response to resignation of incumbent CEOs, and long-term vs. short-term financing of new investment. These examples are featured because the most precisely estimated applications of the option pricing approach occur when the event has a near 50% likelihood of taking place and the outcome is fully known prior to option expiration. It is the degree to which uncertainty is resolved, as well as the time to expiration of the option that correlates with the statistical power of this approach. Large uncertainty resolution close to expiration enhances the technique's statistical power.

Perhaps the simplest way to understand the methodology's power is to look at an extreme case—call options of various strikes that are about to expire in an instant over which the event uncertainty will be fully resolved. If the event occurs, it increases the stock price by *V*, an unknown amount. We choose this example because it already isolates the impact of non-event factors. An instant before expiration, the impact of these factors is typically modeled as zero (as they are on the order of *dt* in the continuous-time diffusions generally used to model the factors). The only uncertainty remaining over the last instant is whether the event occurs. If it does occur, the event creates a stock price jump of finite size.

In this simplified setting, it is obvious that there are two classes of options: those that are useful to identify the event's full impact and those that are not. The latter consists of both sufficiently in-the-money and out-of-the-money options.[1] The sufficiently in-the-money options are identically impacted by the event; the sufficiently out of the money options are unaffected by the event. Hence, understanding how option prices with extreme strike prices are distorted by the event adds little power to identify the event's impact. However, options with strike prices between those of the sufficiently in-the-money and out-of-the-money options have values that depend on the event and its (risk-neutral) likelihood to varying degrees. For such call options with intermediate strike prices, the derivative of their value with respect to the strike price is not -1 or 0 ($+1$ and 0 for puts), as is typically the case, and is the case here for the unaffected options with more extreme strike prices. Rather, the derivative is the negative of the risk-neutral probability of the event (positive risk-neutral probability for puts). Thus, the call option value vs. strike function—just prior to expiration and before the event uncertainty is resolved—has three slopes and two kinks, as illustrated in Fig. 1. The horizontal distance between the kinks

[1] They are useless to identify the event's full impact because changes in option prices for sufficiently in-the-money and sufficiently out-of-money options are linear. If the event occurs prices of sufficiently in-the-money options increase linearly by a constant (or *V*) whereas those of sufficiently out-of-money options remain unchanged at zero.

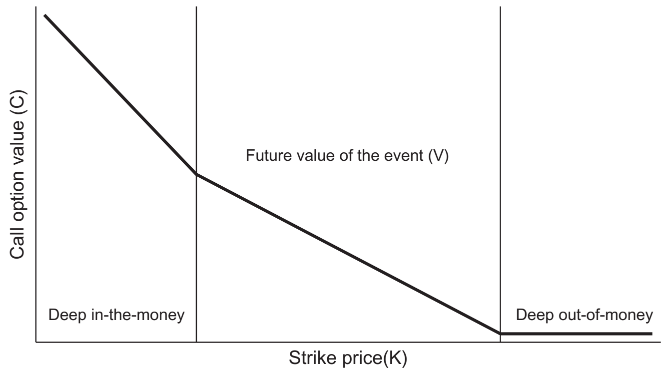

Fig. 1. Call option payoffs vs. different strikes in event study.
This figure plots the relationship between call option payoffs at different strike prices with the same maturity in an event study.

represents the value of the event, *V*, while the kinks' vertical distance is the product of *V* and it risk neutral probability. The latter product represents the anticipatory value that the event embeds in the stock price at the instant before event uncertainty is resolved and the options have expired. To separate the value of the event from its risk neutral probability, we only need to view the distance between the two kinks in the vertical and horizontal domains.

Note that all of the options to the left of the leftmost kink are affected by the event. However, there is no way to isolate the risk-neutral probability or the event impact with these options alone, even if we look at changes in the value of these in-the-money options over the instant before expiration. We need to identify the rightmost kink in Fig. 1 to isolate the risk-neutral probability of the event as the slope between the two kinks. The horizontal distance then provides the event impact—all before expiration occurs.

If we know the state prices at times prior to this instant before uncertainty resolution, we can still perform the same exercise. It is econometrically more challenging because the kinks start to smooth out due to the impact of non-event factors. However, their second derivatives never cease to appear a bit abnormal at strike price distances tied to the value of the event. All we have to do is guess the value of the event and its risk neutral probability, draw the payoff diagram at expiration for each option (which will inversely mirror Fig. 1), and see if our guess about the two values matches option prices at various strikes. If it does not match, we iteratively regress the risk neutral probability and event future value until we obtain theoretical prices that match the data.

Of course, we don't know the state prices without closer examination of the data, but here, Ross's (1976) insight tells us that these state prices are unique if we have prices for enough options with distinct strike prices. Under mild conditions assuring non-degeneracy, this approach generalizes, even to continuous state spaces. While empirical applications are in progress and beyond this paper's scope, we use the more pedagogical theme of this paper, and instead, illustrate the technique on a simple 2×2 example with as little technical complexity as possible. Before introducing that example, we review the traditional approach to the study of events.

3. Traditional approach: time-series changes in stock prices

In efficient markets, the current price of a security fully reflects available information, implying price changes occur when new information arrives (Fama, 1970). The efficient markets hypothesis is the foundation of many corporate finance methodologies. One such methodology is the event study, which uses the reaction of stock prices—mostly around announcements—to measure the impact of corporate events. Event studies have been widely used in corporate finance to study management turnover, mergers and acquisitions, financial reporting, and securities issues (Beaver, 1968; Worrell et al., 1986; Warner et al., 1988; Ritter, 1991; Jensen and Ruback, 1983). Event studies have also quantified the impact of stock split and stock dividend announcements (Fama et al., 1969; Grinblatt et al., 1984).

However, event study quantification is compromised by event outcome anticipation. If the market somehow anticipates the announcements or the information is leaked in advance, stock prices would adjust to the market's anticipation prior to the announcements. Thus, changes in stock price in response to announcements reflect only the partial (or the unanticipated) impact of corporate events. For example, if the market has anticipated firms may split their shares before stock split announcements, changes in stock prices to announcements underestimate the valuation impact of stock splits. In addition, event outcome anticipation may trigger corporate events themselves. If the market anticipates that some firms may split their shares, managers who disappoint the market by not splitting may endure lower share prices (Grinblatt et al., 1984).

Event outcome anticipation is particularly relevant in scenarios for which information is likely to leak in advance. The anticipation problem is also serious when insider trading is likely to occur, e.g., merger and acquisition transactions. This is because negotiations of M&A transactions involve multiple parties and take weeks (or even months) to consummate (Wan and Wong, 2009; Mulherin and Womack, 2015). As merger and acquisition transactions are anticipated before announcements, almost any firm with the potential for synergies will have the potential for the synergy embedded in its stock price, even long before the merger announcement (Cao et al., 2005; Becher, 2009; Cai et al., 2011).

Event outcome anticipation poses challenges for any methodology that uses the announcement day adjustment in stock prices to quantify the impact of the event. This includes the difference-in-difference methodology as well as cross-sectional and panel regressions that predict the stock price impact around corporate events. The difference-in-difference technique compares the difference in impact after and before an event for the treatment group to the same difference for the control group. Theoretically, the difference-in-difference technique is regarded as a gold standard because it eliminates confounding factors unrelated to the event that affect both the treatment and the control groups. However, anticipation is a confounding condition that persists and is tied to the likelihood of being a treatment group. Hence, if anticipation differs between the treatment and control groups, as seems likely except in randomized treatment assignment, even this gold standard falls short.

This anticipation limitation of the difference-in-difference methodology extends to variables besides stock price reactions and can lead to downward and even upwardly biased estimates of impact. As one timely example, if a country's auto manufacturers anticipate an import tariff on foreign auto parts, they might stock up on auto parts inventory prior to the tariff date. The treatment group (the country imposing the tariff) might appear to experience a relatively greater reduction in imports of foreign auto parts after the imposition of the tariff than warranted by the tariff per se. This example also points to the futility of using announcement dates for the difference. Even if the difference-in-difference benchmark draws comparisons to imports long before the announcement date to avoid the contamination described above, greater anticipation of the announcement by the treatment group will lead to the same overstatement of the tariff impact.

4. Asset pricing approach: option prices at different strike prices

The prices of options at different strikes expiring after a major resolution of event uncertainty resolve the event outcome anticipation problem. Such option prices identify the impact and (risk neutral) likelihood of corporate events. While the stock price can be used in conjunction with option prices (as it is technically an option with a zero strike price), it is the nonlinearity of option payoffs that ultimately separates the impact of the event from its likelihood.

4.1. An illustrative example

Table 1 provides a simple example to illustrate how model-free option prices can be used to identify the likelihood and impact of a corporate event. Unlike Fig. 1's example, Table 1 allows a small degree of additional uncertainty from a non-event factor that is modeled as a binomial outcome. The setting of the example includes two dates (0 and 1); two event outcomes at date 1 (occur and does not occur); and two stock values at date 1 ($110 for high value and $100 for low value) that get added to the impact of the event if it occurs. The latter binomial outcome stems from the non-event factor. The risk neutral probability of the event occurring is π; and λ is the risk-neutral probability that the stock has the high value (before adding potential event impact). These probabilities could in principle (and generally will) depend on each other. In this example, because of its symmetry, we learn that they are independent.

This setting thus has four states at date 1: (1) low stock value and event does not occur; (2) low stock value and event occurs; (3) high stock value and event does not occur; (4) high stock value and event occurs. If the event occurs, the unknown value change of the stock is denoted as Δ. The objective is to use the date 0 prices of the stock, a riskless zero coupon bond, and options with 3 different strikes to identify Δ and the four state prices. The risk-neutral probability of the event, as well as the unconditional risk neutral probability of the stock taking a high value, is just the sum of two of the state prices multiplied by $1 + r_f$.

We use $X(s)$ to denote the payoff in state *s,* where $s = 1, 2, 3$, and 4; and p_1, p_2, p_3 and p_4 to denote their corresponding state prices, or scaled risk-neural probabilities. Table 1 presents payoffs and state prices of these scenarios at date 1.

To identify the value change of the event (Δ), we solve a system of equations with five unknown parameters: p_1, p_2, p_3 p_4, and Δ. Let us assume the stock price is $110 at date 0. We use the no arbitrage condition to derive the current stock price as follows:

$$\$110 = p_1 \times \$100 + p_2 \times (\$100 + \Delta) + p_3 \times \$110 + p_4 \times (\$110 + \Delta) \tag{4}$$

As indicated in Eq. (4), the $110 stock price alone cannot identify the five unknowns. To find a unique solution of these unknowns, we need to introduce four more securities. In this example, we include one riskless zero coupon bond with a face value of $100 maturing at date 1; and three call options maturing at date 1 with different strike prices as follows: $119, $112, and $105. The riskless zero coupon bond has a risk free rate (r_f) equal to $\frac{1}{44}$ implying that the no arbitrage bond price of a $100 payoff at date 0 is assumed to be $97.77778, i.e., $\frac{\$100}{(1+r_f)}$. As the payoff for the risk free zero coupon bond is always $100 at date 1 regardless of the state of the world, the no arbitrage bond price provides another unique piece of information to identify four of the five unknowns as

Table 1
Payoffs and state prices.

		Occurrence of the event	
		Does not occur (Prob. = 1-π)	Occurs (Prob. = π)
Stock value	Low ($100) (Prob. = 1-$\lambda$)	State 1: X(s) = $100; state price = p_1	State 2: X(s) = $100 + Δ; state price = p_2
	High ($110) (Prob. = λ)	State 3: X(s) = $110; state price = p_3	State 4: X(s) = $110 + Δ; state price = p_4

follows:

$$\$97.77778 = [p_1 + p_2 + p_3 + p_4\ ] \times \$100 \tag{5}$$

The remaining security type is a European call option with a strike price of K.[2] The no arbitrage call option price, C(K), can be derived as follows:

$$C(K) = p_1 \times \text{MAX}(\$100 - K, 0) + p_2 \times \text{MAX}(\$100 + \Delta - K, 0) + p_3 \times \text{MAX}(\$110 - K, 0) + p_4 \times \text{MAX}(\$110 + \Delta - K, 0) \tag{6}$$

Our identification strategy relies on each of the five securities adding unique information that aids in solving a system of equations with five unknowns. For this to be the case in the four-state setting, some call options have to be out of the money in different states than others. With call option prices of \$8.5556, \$3.9111, and \$1.4667 for strike prices at \$105, \$112, and \$119, respectively, this will be the case if the value change parameter (Δ) is greater than 13. Let's see if we got lucky in guessing this lower bound on Δ. We are going to assume that this lower bound holds in writing out Eq. (6) and solving the system of linear equations. Imposing the no arbitrage condition, namely, that such state prices exist in all state-contingent claims, means that we use prices of the five securities to form the following system of equations:

$$\text{Call}(\text{K} = 119)\text{: } \$1.4667 = p_4 \times [\Delta - 9] \tag{7a}$$

$$\text{Call}(\text{K} = 112)\text{: } \$3.9111 = p_2 \times [\Delta - 12] + p_4 \times [\Delta - 2] \tag{7b}$$

$$\text{Call}(\text{K} = 105)\text{: } \$8.5556 = p_2 \times [\Delta - 5] + 5p_3 + p_4 \times [\Delta + 5] \tag{7c}$$

$$\text{Stock:} 110 = p_1 \times 100 + p_2 \times (100 + \Delta) + p_3 \times 110 + p_4 \times (110 + \Delta) \tag{7d}$$

$$\text{Bond:} 97.77778 = [p_1 + p_2 + p_3 + p_4\ ] \times 100 \tag{7e}$$

In this example, the choice of strike prices makes each of these call options informationally unique, in that each option is out of the money in different states from the other securities. Specifically, the call option with the strike price at \$119 is out of the money in three states (1, 2, and 3); that with the strike price at \$112 is out of the money in two states (1 and 3); and that with the strike price at \$105 is out of the money in one state (1). On the other hand, the stock and risk free zero coupon bond are in the money in all states (1, 2, 3, and 4), but differ in their payoffs across the four states. The unique solution of this system of equations is

$\Delta = 15$ and $p_1 = p_2 = p_3 = p_4 = \frac{11}{45}$, implying

$1 + r_f = \frac{45}{44}$ and

$\pi = \lambda = (\frac{11}{45} + \frac{11}{45}) \times (\frac{45}{44}) = \frac{1}{2}$.

The set of option strikes used here turns out to be consistent with the independent information requirement and a unique event impact of 15. In this sense, we rigged the example's numbers to simplify the process of finding the solution. In more general finite state spaces, our choice of option strikes may have to iterate to find a set which is consistent with the event impact and there is no reason for the probabilities to be independent. They clearly are independent here because the solutions for π and λ satisfy

$$p_2 = \frac{\pi\lambda}{(1 + r_f)} \tag{8}$$

$$p_4 = \frac{\pi(1 - \lambda)}{(1 + r_f)} \tag{9}$$

This example demonstrates how option prices can be used to identify the risk-neutral probability and value change of corporate events. One period prior to the realization of the event, the stock price was \$110, reflecting event anticipation. An instant prior to the event, the stock price is at 112.50 (the \$110 stock's forward price). If the event occurs, the stock price would have jumped from \$112.50 to either \$115 or \$125 over the next instant, each with equal (risk-neutral) likelihood. If the risk neutral and physical probabilities were the same, an event study would thus measure the event as having a value of \$7.50 per share (the average of the \$2.50 and \$12.50 appreciations that differ because of the non-event factor). This is smaller than the \$15 that our approach measures as event impact from the pricing of securities before the event. Put another way, in the absence of any likelihood that the event would have occurred, the stock would be trading at a \$105 forward price, rather than at the \$112.50 forward price it actually trades at just prior to the event. Event anticipation inflates the share price by \$7.50 just prior to the event and by \$7.333 one period before the event outcome if the one-period risk-free rate is 1/44.

This approach remains feasible only if researchers can find or interpolate the prices of sufficient number of options with distinct strike prices to identify the unknown state prices and event impact. Our approach can accommodate changes in state prices due to

[2] The option pricing approach is general and appropriate for any option types (e.g., European put options, American options, and exotic options) as long as they satisfy the non-collinear pricing condition. This generally means that there is some possibility that the American options used for estimation will survive to the expiration date. Violating that condition makes an option at a particular strike redundant for identifying state prices and event impact. Without enough non-redundant options, the option payoff space may not complete the market. In practice, the hidden parameters are more accurately estimated if the chosen option type is very liquid with many distinct strike prices (to be discussed in Section 5).

revisions of market anticipation of the event. To identify hidden state prices, our approach relies on the selection of strike prices that avoid perfect collinearity in option payoffs with respect to the event—what we refer to as a non-collinear pricing condition.

5. Some caveats and final thoughts

Event studies, panel regression, and difference-in-difference methodology—as well as related techniques in corporate finance—are incapable of fully capturing what we typically want to measure in corporate finance due to event anticipation. In theory, the option pricing approach can overcome this obstacle by identifying distinct sources of information about corporate events hidden in the nonlinear pricing of these securities. In practice, the identification may fail in a discrete state space if some options are redundant. This occurs when payoffs of one or more options are linearly related to each other. This implies that the system of equations is under-identified, i.e., the number of independent equations is less than the number of unknown parameters in a system of simultaneous equations. This always happens with the payoffs of future and forward contracts because they are linear functions of payoffs of their underlying stocks. Just as prices of future/forward contracts provide no incremental information to identify distinct sources of information about corporate events, this might be the case with options when the researcher arbitrarily selects the strike prices of the options to be studied.

In our illustrative example which is an exactly identified system of equations, the identification fails if one or more options have close strike prices. To prevent degeneracies, researchers have to choose option strikes judiciously so that they provide sufficiently distinct information to identify the system of equations. Options need to have a wide variety of strike prices. To be non-redundant, an option must be out of the money in different states of the world than the others used for estimation. In practice, it is difficult to know which strike prices to choose without knowing the approximate range of the value change of the event.

The system of equations is over-identified if the number of "distinct" securities in the system exceeds the number of unknown parameters. Over-identification is a desirable property because it allows researchers to select the options that provide the most informative data to identify unknown state prices. In an over-identified system of equations, researchers can also put more weight on options that are more informative and less on those that are less informative. Option pricing theory suggests that options with high gamma are more informative because they are more sensitive to changes in the event's value (Bester et al., 2019) and it is the local peaks in gamma, as illustrated earlier in Fig. 1's kinks, that determine the event impact. Similarly, options that are heavily traded are also informative and should be weighted more heavily in the estimation (Borochin, 2014).

Overall, the option pricing approach to event study valuation has some key properties that researchers have to keep in mind. First, to avoid degeneracies, each option must provide a distinct piece of information to help complete the market. This condition is particularly relevant in an exactly identified system of equations. Second, the event should have a finite number of outcomes so that we can more confidently identify hidden parameters of corporate events even if such events are anticipated by the market. Third, the estimation should be more accurate if the upper bound on the number of outcomes is known. Last, the market has to know that the options will expire after a major degree of uncertainty about the event has been resolved—typically, the event has occurred or did not occur.

The identification is aided by imposing structure on the valuation model. For example, to estimate merger synergy, researchers have assumed that options are a weighted average of two Black-Scholes values. Continuity in the density function of S with a single peak may also be necessary for there to be a solution to the system of nonlinear equations in a continuous time setting.

Theoretically, the model-free option pricing approach can be generalized to infinite and even continuous state spaces. We only need sufficient pieces of unique information from the options to uniquely identify the number of unknowns. Interpolated mid-quotes, using polynomial of high degree for interpolation, may be one approach that successfully matches the data without rigging the answer. This is a subject for future research. While approximate estimation of infinite state prices is therefore feasible, it requires substantial computing power and time.

The big elephant in the room is whether the methodology has any power given the infrequent trading of many options and the field's lack of experience with this approach to event impact estimation. While the power of the estimate is likely to be low for any one single firm's event, event studies are often conducted over hundreds if not thousands of event outcomes across many firms on different dates. Here, just like traditional event studies, the law of large numbers may greatly expand the method's econometrical ability to identify an event's common impact.

Our approach estimates event impact from a set of option prices with different strikes at a single point in time before the event. The estimation may also be more accurate if we can compute differences in option prices over time, comparing the change in probability of the event and value of the options both before and after major uncertainty resolution has occurred. This is because we can use the difference to pin down the role of other factors that cause the value change. This is a subject for future research.

Our approach also generalizes to cases where an event merely represents some, but not perfect resolution of uncertainty. In this case, the π represents a filtration—a reduction in the path of possible future states. Unfortunately, even when we combine options prices both before and after the filtration, if the resolution of uncertainty is small between the date of option pricing and option expiration, there is little econometric power to identify the state prices of the impact of the event. Thus, even in the case where the event occurs or does not occur, events or filtrations with probabilities closest to 0.5 are going to be the most fruitful applications—at least until computing power improves and sample sizes get far larger.

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