

Chapter 6 Association Rule

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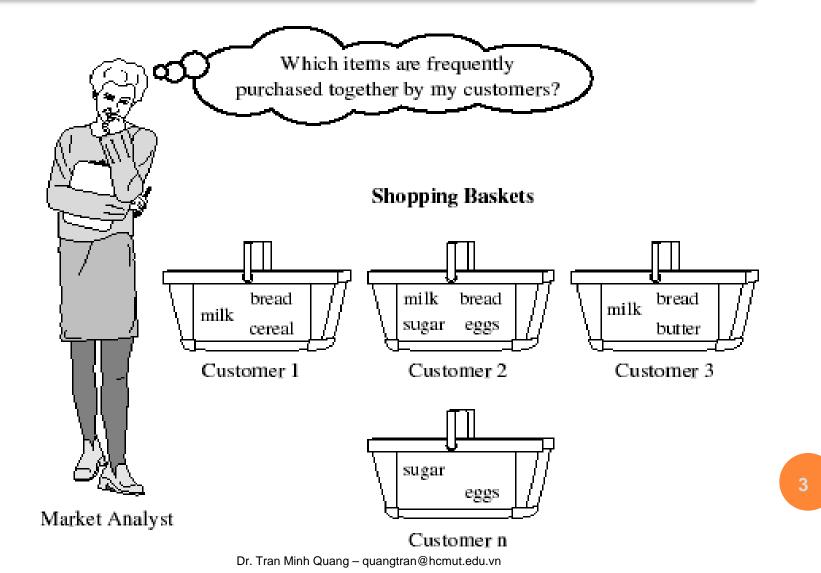
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CONTENT

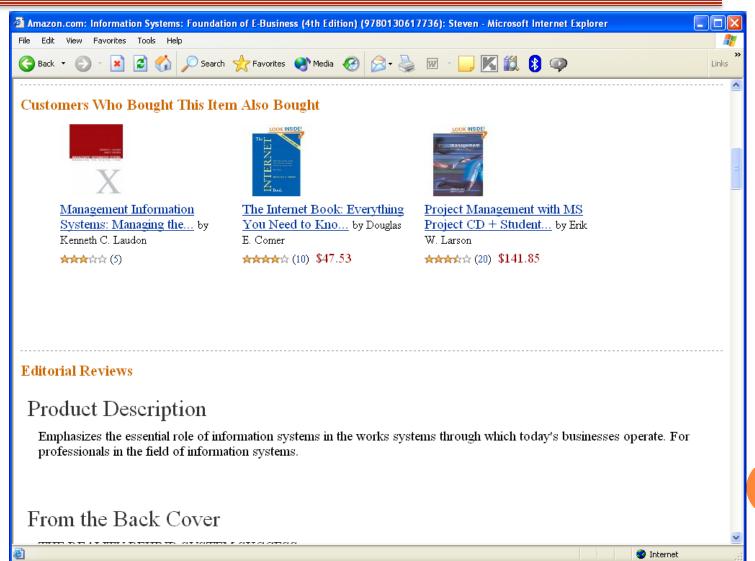
- 1. Situations
- 2. Overview on association rules
- 3. Association rule representation
- 4. Frequent itemset mining
 - 1. Apriori
 - 2. FP-Growth
- 5. Mining association rules from frequent itemsets
- 6. Mining association rules based on constraints
- 7. Correlation analysis
- 8. Summary

1. SITUATION 1 - BASKET ANALYSIS



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1. SITUATION 2 - RECOMMENDATION



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1. SITUATION 2 - RECOMMENDATION

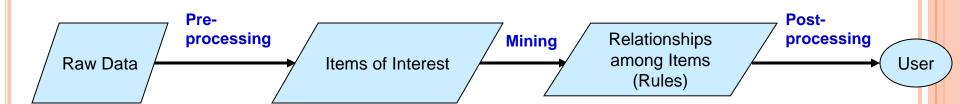


1. SITUATION ...

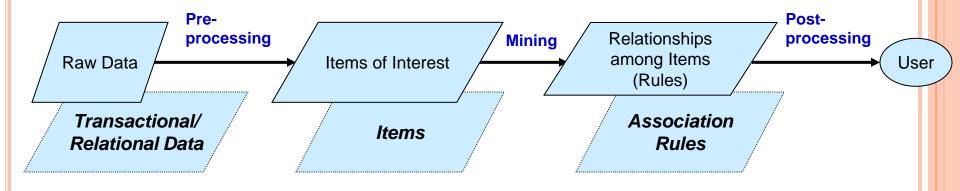
- Basket data analysis
- Cross-marketing, recommendation,...
- Catalog design
- Classification and clustering with frequent patterns

O ...

Association rule mining process



Association rule mining process



Transaction	Items_bought
2000 1000 4000 5000	A, B, C A, C A, D B, E, F
•••	

A, B, C, D, F, ... $A \to C$ (50%, 66.6%) ...

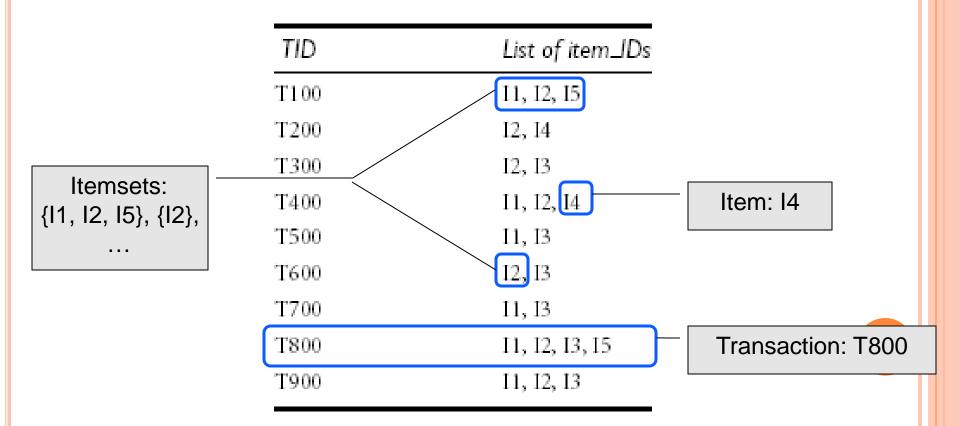
Basket-based analysis

- Basic concepts
 - Item
 - Itemset
 - Transaction
 - Association and association rules
 - Support
 - Confidence
 - Frequent itemset
 - Strong association rules

• AllElectronics dataset (after preprocessing)

TID	List of item_IDs
T100	11, 12, 15
T200	12, 14
T300	12, 13
T400	11, 12, 14
T500	I1, I3
T600	12, 13
T700	11, 13
T800	11, 12, 13, 15
T900	11, 12, 13

• AllElectronics dataset (after preprocessing)



- Basic concepts
 - Item: pattern, sample, object of interest
 - $J = \{I_1, I_2, ..., I_m\}$: a set of all m items in the dataset
 - Itemset
 - A set of items
 - k-itemset: itemset of k items
 - Transaction
 - •A record in a transactional dataset
 - A set of T items in the same transaction/record

• Basic concepts

- Association vs. association rule
 - Association: occurrence of items in the same transaction(s)
 - Represents the linkage/association between items/itemsets
- Association rule: a criteria/rule for the association between itemsets
 - Represents the linkage (with some conditions) between itemsets
 - Let A and B be itemsets, an association rule between A and B is $\mathbf{A} \rightarrow \mathbf{B}$ (B occurs in the condition that A occurs)

• Basic concept

- Support: measure the occurrence frequency of items/itemsets
- Minimum support threshold (MinSup): is the minimum support value defined by user to validate the "support" of a frequent itemset/association
- Confidence: to measure the occurrence frequency of an itemset upon the occurrence of another itemset
- Minimum confidence threshold (MinConf): is the minimum confidence value defined by user to validate the "confidence" of an association rule.

• Basic concepts

• Frequent itemset: is an itemset whose support satisfies the minimum support threshold

A is a frequent itemset iff

support(A) ≥ MinSup

- Strong association rule: is an association rule whose support and confidence satisfy MinSup and MinConf
 - Give $A \rightarrow B$, where A and B are itemsets
 - A \rightarrow B is a strong association rule iff support(A \rightarrow B) \geq MinSup and confidence(A \rightarrow B) \geq MinConf

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- Association rule categories
 - Boolean association rule vs. quantitative association rule
 - Single-dimensional association rule vs. multidimensional association rule
 - Single-level association rule vs. multilevel association rule
 - Association rule vs. correlation rule

- o Boolean vs. quantitative association rules
 - •Boolean association rule: represents the association of occurrence/absence of itemsets

```
Computer → Financial_management_software [support=50%, confidence=60%]
```

• Quantitative association rule: represents the association of between quantitative items/features

```
Age(X, "30..39") \land Income(X, "42K..48K") \rightarrow buys(X, high resolution TV)
```

[support=50%, confidence=60%]

- Single-dimensional vs. multidimensional association rules
 - Single-dimensional association rule: is the rule that relates to only one dimension of items

```
Buys(X, "computer") \rightarrow Buys(X, "financial_management_software")
```

• Multidimensional association rule: is the rule that relates to multiple dimensions of items

$$Age(X, "30..39") \rightarrow Buys(X, "computer")$$

- Single-level vs. multilevel association rules
 - Single-level association rule: is the rule that relates to items/features in one level of abstraction

```
Age(X, "30..39") \rightarrow Buys(X, "computer")

Age(X, "18..29") \rightarrow Buys(X, "camera")
```

• Multilevel association rule: is the rule that relates to items/features in multiples levels of abstraction

$$Age(X, "30..39") \rightarrow Buys(X, "laptop computer")$$

 $Age(X, "30..39") \rightarrow Buys(X, "computer")$

- Association rule vs. correlation rule
 - Association rule: strong association rule A→B is that satisfies minimum support threshold and minimum confidence threshold
 - Correlation rule: strong correlation rules A → B is that satisfies correlation conditions in statistics (using different statistical correlation measures)

3. ASSOCIATION RULE REPRESENTATION

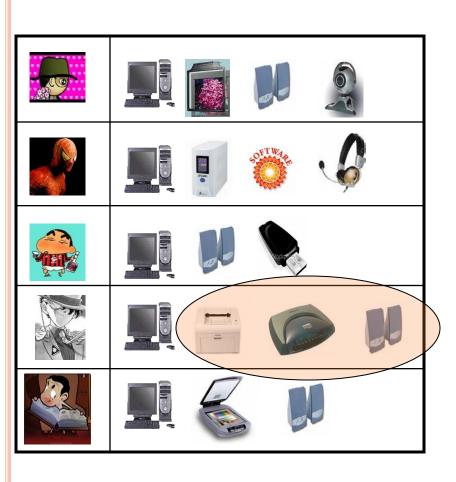
Rule form:

 $A \rightarrow B$ [support, confidence]

Where,

- A, B are frequent itemsets
- Support($A \rightarrow B$) = Support($A \cup B$) $\geq min_sup$
- Confidence(A \rightarrow B) = P(B|A) = Support(A U B)/Support(A) \geq min_conf

3. ASSOCIATION RULE REPRESENTATION





Support



Sup = 1





Sup = 1

Confidence

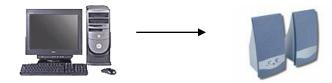
$$C(\boxed{)} \rightarrow \boxed{)} = \frac{4}{5}$$

4. MINING FREQUENT ITEMSETS/PATTERNS

- 2 steps in association rule mining
 - Finding frequent itemsets/patterns



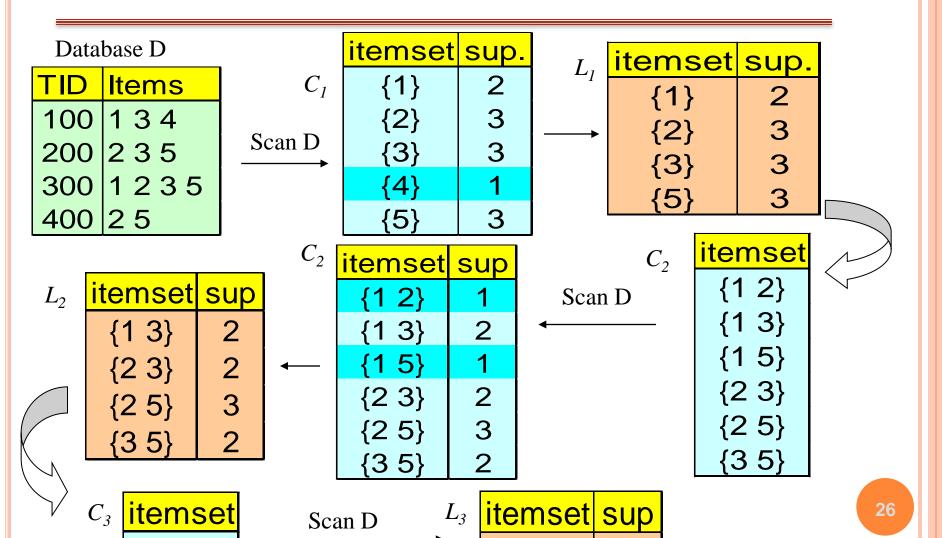
Mining association rules from frequent itemsets/patterns



4. MINING FREQUENT ITEMSETS/PATTERNS

- Apriori method: mining frequent itemsets with candidate itemsets using prior knowledge
 - R. Agrawal, R. Srikant. Fast algorithms for mining association rules. In VLDB 1994, pp. 487-499
- FP-Growth: using FP-tree
 - J. Han, J. Pei, Y. Yin. Mining frequent patterns without candidate generation. In SIGMOD 2000, pp. 1-12

- Use the prior knowledge about the characteristics of frequent itemsets
- Iterate the searching process to find frequent itemsets at each level (level-wise search)
 - k+1-itemsets: generated from k-itemsets
 - At each level, check all the data set to identify frequent itemsets
- Apriori property (to reduce the search space): All subsets of a frequent itemset are frequent itemsets
- Anti-monotone: If X is NOT a frequent itemset then neither does {X U Y}



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2 3 5

[2 3 5]

• AllElectronics dataset (after preprocessing)

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T800	11, 12, 13, 15
T900	11, 12, 13

Scan D for count of each candidate

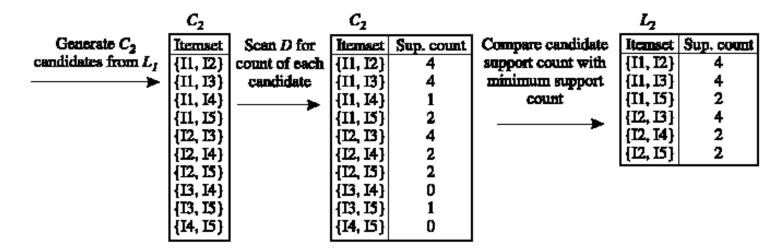
 $\begin{array}{c|cccc} C_I \\ \hline \textbf{Itemset} & \textbf{Sup. count} \\ \{I1\} & 6 \\ \{I2\} & 7 \\ \{I3\} & 6 \\ \{I4\} & 2 \\ \{I5\} & 2 \\ \end{array}$

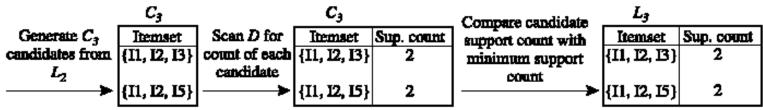
Compare candidate support count with minimum support count

<i>L</i> 1	
Sup. count	
6	
7	
6	
2	
2	

 $min_sup = 2/9$

minimum support count = 2





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• Pseudo-code:

```
C_b: Candidate itemset of size k
L_b: frequent itemset of size k
L_1 = \{ \text{frequent items} \};
While (L_k != \emptyset; k++) do begin
   C_{k+1} = candidates generated from L_k;
  for each transaction t in database do
        increment the count of all candidates in
   C_{k+1}
                           that are contained in t
  L_{k+1} = candidates in C_{k+1} with min_support
```

return $\cup_k L_k$;

end

- How to generate candidates?
 - Step 1: self-joining L_k
 - Step 2: pruning (Antimonotone)
- How to count supports of candidates?
- Example of Candidate-generation
 - L_3 ={abc, abd, acd, ace, bcd}
 - Self-joining: L_3*L_3
 - abcd from abc and abd
 - acde from acd and ace
 - Pruning:
 - ullet acde is removed because ade is not in L_3
 - ullet $C_4 = \{abcd\}$

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- Suppose the items in L_{k-1} are listed in an order
- o Step 1: self-joining L_{k-1} insert into C_k select $p.item_1$, $p.item_2$, ..., $p.item_{k-1}$, $q.item_{k-1}$ from L_{k-1} p, L_{k-1} q where $p.item_1$ = $q.item_1$, ..., $p.item_{k-2}$ = $q.item_{k-2}$, $p.item_{k-1}$ < $q.item_{k-1}$
- Step 2: pruning for all *itemsets* c *in* C_k do for all (k-1)-subsets s of c do if (s is not in L_{k-1}) then delete c from C_k

- Apriori's main characteristics
 - Create many candidates for frequent itemsets
 - o 10⁴ frequent 1-itemsets → more than 10^7 (≈10⁴(10⁴-1)/2) 2-itemsets as candidates
 - \circ For each k-itemset, it examines 2^k -1 itemsets of candidates
 - Scan the original data (D) many times for calculating the supports
 - Huge cost when the size of candidate itemsets increase
 - If k-itemsets is mined D is scanned k+1 times

Apriori's improvement techniques

- Techniques based on hashing: An k-itemset whose hashing bucket count is smaller than minimum support threshold **is not** a frequent itemset.
- Reduce the scan time: A transaction that does not contain **any** frequent k-itemset are not needed to be examined at later level (i.e., for identify k+1-itemset).
- Partitioning: An itemset must be frequent in at least one partition to be considered as frequent in the whole dataset.
- Sampling: Mine the frequent itemsets from samples (via sampling) with a smaller support threshold. It is necessary to have a method for validating the completeness.
- Dynamic itemset counting: Add an itemset to the candidates only iff all of its subsets are **predicted** to be "frequent".

4.2. MINING FREQUENT ITEMSETS - FPGROWTH

- Compact the data into a Frequent Pattern tree (FP-tree)
 - Reduce the size of the examined dataset -> reduce the mining cost
 - Infrequent items are discarded early in the process
- Divide-and-conquer approach
 - The mining process is divided into smaller tasks
 - 1. Build the FP-tree
 - 2. Mine frequent itemsets using the FP-tree
- Avoid creating large candidate itemsets
 - Each mining task examines a small portion of the dataset

4.2. MINING FREQUENT ITEMSETS - FPGROWTH

- o 1. Building the FP-tree
 - 1.1. Scan D to find frequent 1-itemsets
 - 1.2. Order frequent 1-itemsets in the descending of support count (frequency)
 - 1.3. Scan D again (2nd scan), build the FP-tree
 - Initiate with root node, label it with "null" or {}
 - Each transaction/tuples will correspond to a branch in the FP-tree
 - Each node contains an item of the transaction
 - Items in a transaction are shorted in descending order
 - Each node associates with support count of the corresponding item
 - Transactions contain the same items will create shared prefix branches quangtran@hcmut.edu.vn

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4.2. MINING FREQUENT ITEMSETS - FPGROWTH

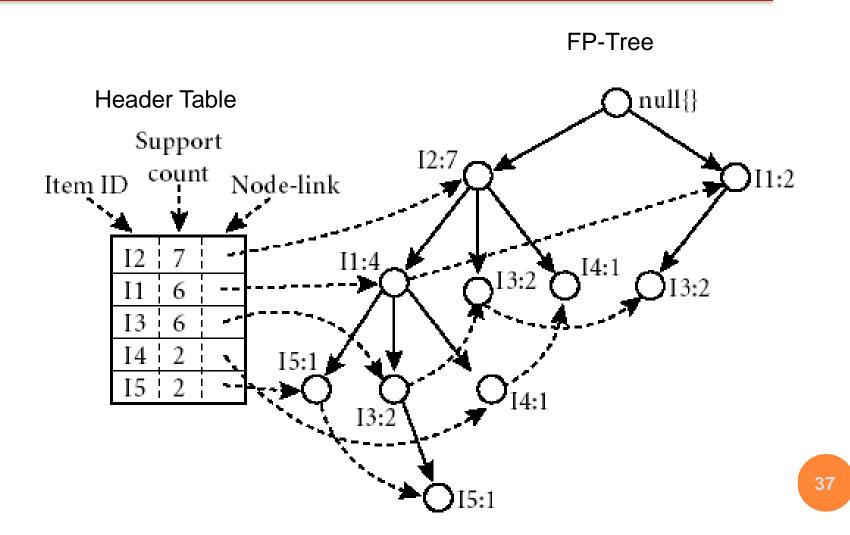
Input:

- D, a transaction database;
- min_sup, the minimum support count threshold.

Output: The complete set of frequent patterns.

Method:

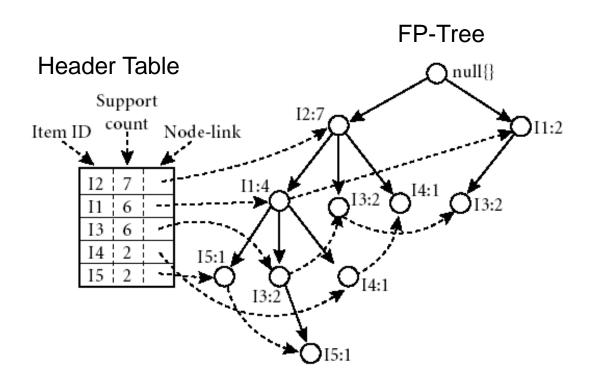
- The FP-tree is constructed in the following steps:
 - (a) Scan the transaction database D once. Collect F, the set of frequent items, and their support counts. Sort F in support count descending order as L, the list of frequent items.
 - (b) Create the root of an FP-tree, and label it as "null." For each transaction *Trans* in *D* do the following. Select and sort the frequent items in *Trans* according to the order of *L*. Let the sorted frequent item list in *Trans* be [p|P], where *p* is the first element and *P* is the remaining list. Call insert_tree([p|P], *T*), which is performed as follows. If *T* has a child *N* such that *N.item-name* = *p.item-name*, then increment *N*'s count by 1; else create a new node *N*, and let its count be 1, its parent link be linked to *T*, and its node-link to the nodes with the same *item-name* via the node-link structure. If *P* is nonempty, call insert_tree(*P*, *N*) recursively.
- 2. The FP-tree is mined by calling FP_growth(FP_tree, null), which is implemented as follows.



- 2. Mining frequent itemsets from the FP-tree
 - 2.1. Generate conditional pattern base for each node of the FP-tree
 - Accumulate the prefix paths with frequency of that node
 - 2.2. Create conditional FP-tree from each conditional pattern base
 - Accumulate frequency for each item in each base
 - Build conditional FP-tree for frequent items of that base
 - 2.3. Mine the conditional FP-tree for frequent itemsets recursively
 - If conditional FP-tree has a single path then list all itemsets

• FP-Growth algorithm

```
procedure FP_growth(Tree, \alpha)
(1)
       if Tree contains a single path P then
           for each combination (denoted as \beta) of the nodes in the path P
(2)
               generate pattern \beta \cup \alpha with support\_count = minimum support count of nodes in <math>\beta;
(3)
       else for each a_i in the header of Tree {
(4)
           generate pattern \beta = a_i \cup \alpha with support\_count = a_i.support\_count;
(5)
           construct \beta's conditional pattern base and then \beta's conditional FP_tree Tree_{\beta};
(6)
           if Tree_{\beta} \neq \emptyset then
(7)
               call FP_growth(Tree_{\beta}, \beta); }
(8)
```

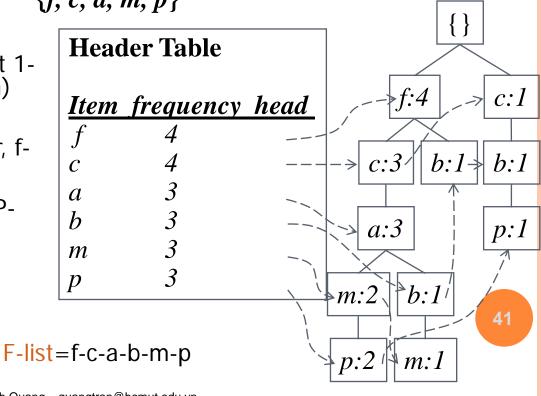


Item	Conditional Pattern Base	Conditional FP-tree	Frequent Patterns Generated
15	{{I2, I1: 1}, {I2, I1, I3: 1}}	⟨I2: 2, I1: 2⟩	{I2, I5: 2}, {I1, I5: 2}, {I2, I1, I5: 2}
I4	$\{\{I2, I1: 1\}, \{I2: 1\}\}$	⟨I2: 2⟩	{I2, I4: 2}
13	{{I2, I1: 2}, {I2: 2}, {I1: 2}}	\langle I2: 4, I1: 2 \rangle , \langle I1: 2 \rangle	{I2, I3: 4}, {I1, I3: 4}, {I2, I1, I3: 2}
I1	{{I2: 4}}	\langle I2: 4 \rangle	{I2, I1: 4}

4.2. MINING FREQUENT ITEMSETS – FPGROWTH (ANOTHER EXAMPLE)

TID	Items bought ((ordered) frequent items	
100	$\{f, a, c, d, g, i, m, p\}$	$\{f, c, a, m, p\}$	
200	$\{a, b, c, f, l, m, o\}$	$\{f, c, a, b, m\}$	
300	$\{b, f, h, j, o, w\}$	$\{f, b\}$	min_support = 3
400	$\{b, c, k, s, p\}$	$\{c, b, p\}$	- 11
500	$\{a, f, c, e, l, p, m, n\}$	$\{f, c, a, m, p\}$	[]

- 1. Scan DB once, find frequent 1-itemset (single item pattern)
- Sort frequent items in frequency descending order, flist
- 3. Scan DB again, construct FP-tree



4.2. FP-GROWTH EXAMPLE: PARTITION PATTERNS AND DATABASES

• Frequent patterns can be partitioned into subsets according to f-list

f-list=f-c-a-b-m-p

Patterns containing p

Patterns having m but no p

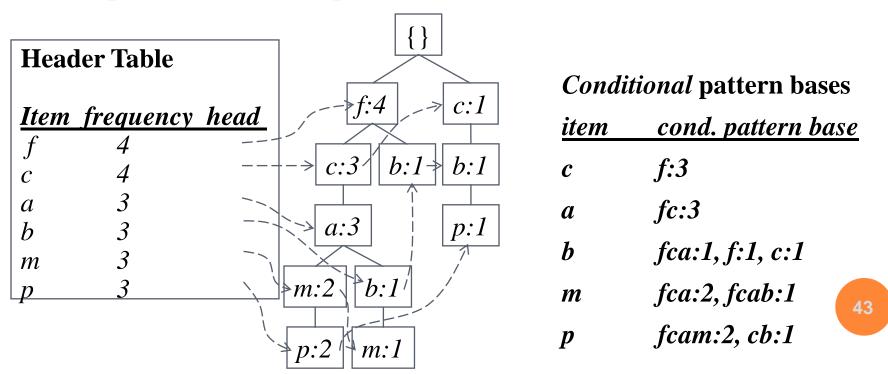
. . .

Patterns having c but no a nor b, m, p
Pattern f

Completeness and non-redundance

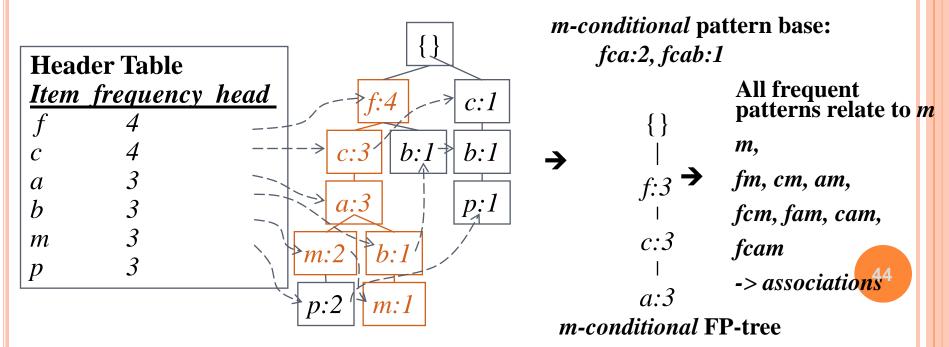
4.2. FP-GROWTH EXAMPLE: FIND PATTERNS HAVING P FROM P-CONDITIONAL DATABASE

- Starting at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item *p*
- Accumulate all of *transformed prefix paths* of item *p* to form *p*'s conditional pattern base



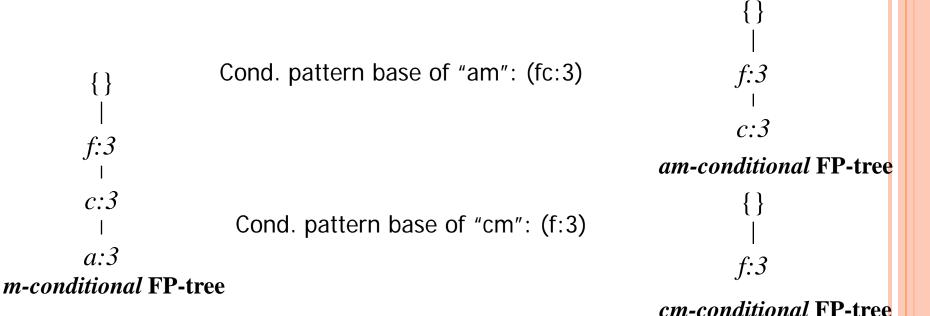
4.2. FP-GROWTH EXAMPLE: FROM CONDITIONAL PATTERN-BASES TO CONDITIONAL FP-TREES

- For each pattern-base
 - Accumulate the count for each item in the base
 - Construct the FP-tree for the frequent items of the pattern base



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4.2. FP-GROWTH EXAMPLE: RECURSION: MINING EACH CONDITIONAL FP-TREE



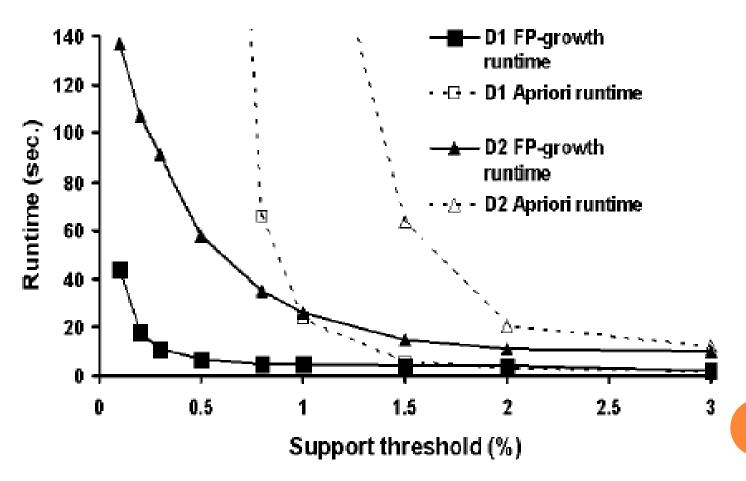
Cond. pattern base of "cam": (f:3)

{} | f:3

cam-conditional FP-tree

- FP-Growth's main characteristics
 - Does not create candidate itemsets
 - FP-tree is a compact data structure
 - Reduce the cost for scanning the original dataset
 - Main costs come from building FP-tree (at the initial) and mining it
 - → Effective and scalable for mining long and sort frequent itemsets

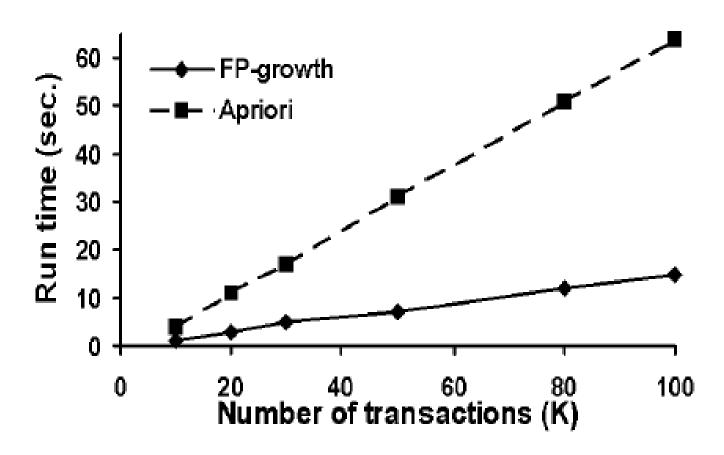
Comparison between Apriori and FP-Growth



FP-growth scales well with support threshold

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Comparison between Apriori and FP-Growth



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4.3 VARIANCE OF FREQUENT ITEMSET MINING

- Some variances of frequent itemsets
 - Frequent itemsets/subsequences/substructures
 - Closed frequent itemsets
 - Maximal frequent itemsets
 - Constrained frequent itemsets
 - Approximate frequent itemsets
 - Top-k frequent itemsets

5. MINING ASSOCIATION RULES FROM FREQUENT ITEMSETS

- o Strong association rules A→B
 - Support($A \rightarrow B$) = Support($A \cup B$) >= min_sup
 - Confidence(A→B) = Support(A U B)/Support(A) =
 P(B | A) >= min_conf
 - \rightarrow Support(A \rightarrow B) = Support_count(A U B) >= min_sup
 - → Confidence(A→B) = P(B | A) = Support_count(AUB)/Support_count(A) >= min_conf

5. MINING ASSOCIATION RULES FROM FREQUENT ITEMSETS

- Process to generate strong association rules from frequent itemsets
 - For each frequent itemset *l*, create non-empty subsets of l.

$$Support_count(l) \ge min_sup$$

• For each non-empty subset s of l, create rule " $s \rightarrow (l$ s)" if:

$$Support_count(l)/Support_count(s) >= min_conf_{51}$$

5. MINING ASSOCIATION RULES FROM FREQUENT ITEMSETS

Frequent Patterns Generated

```
{I2, I5: 2}, {I1, I5: 2}, {I2, I1, I5: 2}

{I2, I4: 2}

{I2, I3: 4}, {I1, I3: 4}, {I2, I1, I3: 2}

{I2, I1: 4}
```

```
l = \{I1, I2, I5\}
```

nonempty subsets of l are {I1, I2}, {I1, I5}, {I2, I5}, {I1}, {I2}, and {I5}

$$I1 \land I2 \Rightarrow I5$$
, $confidence = 2/4 = 50\%$
 $I1 \land I5 \Rightarrow I2$, $confidence = 2/2 = 100\%$
 $I2 \land I5 \Rightarrow I1$, $confidence = 2/2 = 100\%$
 $I1 \Rightarrow I2 \land I5$, $confidence = 2/6 = 33\%$
 $I2 \Rightarrow I1 \land I5$, $confidence = 2/7 = 29\%$
 $I3 \Rightarrow I1 \land I2$, $confidence = 2/2 = 100\%$

6. MINING ASSOCIATION RULES BASED ON CONSTRAINTS

- Constraints
 - Instruct the processes of mining patterns and rules
 - Limit the search space of mining process
 - Some common constraints
 - Knowledge type constraints
 - Data constraints
 - Level/dimension constraints
 - Interestingness constraints
 - Rule constraints

6. MINING ASSOCIATION RULES BASED ON CONSTRAINTS

- Knowledge type constraints
 - What kind of knowledge we want to consider when mining rules Ex. Mining association rules or correlation rules
- Data constraints
 - The characteristics of the data to be mined
- Level/dimension constraints
 - What kind of dimensions/features or level of abstractions that we want to know when mining association rules
- Interestingness constraints: Definition of the measure, what is the threshold,...
- Rule constraints: The types of rules to be mined

6. MINING ASSOCIATION RULES BASED ON CONSTRAINTS

- The mining process becomes more effective and efficient
 - Rules are mined based on business needs/user requirements -> More effective
 - Optimizers can be utilized to improve the efficiency

- Strong association rules $A \Rightarrow B$ are mined based on:
 - Occurrence frequency of A and B (min_sup)
 - The conditional probability of B based on A (min_conf)
 - → minsupport and minconfidence are set in accordance with the subjective views of users
 - →A large amount of rules will be returned.
 - → Within 10,000 transactions, 6,000 ones for *computer games*, 7,500 for *videos*, and 4,000 for both *computer games* and *videos*
 - →Buys(X, "computer games") \Rightarrow Buys (X, "videos") [support = 40%, confidence = 66%]

- Correlation analysis for association rule $A \Rightarrow B$
 - Examine the correlation and dependency between A and B
 - Based on data statistics
 - Measures are objective, do not subjectively depend on users' views
 - → Withing 10,000 transactions, 6,000 for *computer games*, 7,500 for *videos*, and 4,000 for both *computer games* and *videos*
 - →Buys(X, "computer games") \Rightarrow Buys (X, "videos") [support = 40%, confidence = 66%]
 - →P("videos") = 75% > 66%: "computer games" and "videos" are negatively correlated with each other.

- Correlation rules: $A \Rightarrow B$ [support, confidence, correlation]
 - correlation: measuring the correlation between A and B.
 - Common correlation measures: lift, χ^2 (Chi-square), $all_confidence$, cosine
 - *lift*: Validate the independent occurrence between A and B based on probability
 - χ^2 (Chi-square): Validate the independence between A and B based on expected value and the observed value.
 - *all_confidence*: Validate rules based on maximum support
 - *cosine*: similar to *lift* but it help to mitigate the dependency to the total number of transactions (i.e., the size of the dataset)
 - *→ all_confidence* and *cosine* are good for large datasets, do not depented on transactions that do not contain any itemsets being validated (null-transactions)

• lift

- lift(A, B) < 1: A and B are negatively correlated
- lift(A, B) > 1: A and B are positively correlated
- lift(A, B) = 1: A and B are independent

$$lift(A, B) = \frac{P(A \cup B)}{P(A)P(B)} = P(B \mid A) / P(B) = confidence(A \Rightarrow B) / support(B)$$

	game	game	Σ_{row}	$P(\{game\}) = 0.60,$
video	4,000	3,500	7,500	$P(\{video\}) = 0.75$
video	2,000	500	2,500	$I\left(\{viaeo_f\}\right) = 0.75$
Σ_{col}	6,000	4,000	10,000	$P(\{game, video\}) = 0.40$

 $P(\{game, video\})/(P(\{game\}) \times P(\{video\})) = 0.40/(0.60 \times 0.75) = 0.89.$

 $lift(\{game\}=>\{video\}) = 0.89 < 1 \rightarrow \{game\} \text{ and } \{video\} \text{ are negatively correlated.}$

8. SUMMARY

- Association rule mining
 - Considered as one of the most important contributions from database communities in KDD
- Rule types: logical/quantitative rules, single dimension/multiple dimensions, single level/multiple levels of abstraction, association/correlation rules
- Forms of item/pattern: Frequent itemsets/subsequences/substructures, Closed frequent itemsets, Maximal frequent itemsets, Constrained frequent itemsets, Approximate frequent itemsets, Top-k frequent itemsets
- Mine frequent itemsets: Apriori and FP-Growth algorithms

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