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Homework 2

Binary Classification and Statistical Learning Theory

1. Problem of Binary Classification:

Binary classification is a core problem in supervised learning where the goal is to classify input data into one of two possible classes. Formally, let the input space be X (space of instances) and the label space Y = {-1, +1} (binary labels). The task is to find a function f: X -> Y, called the classifier, which can correctly predict the label for new instances based on training data. Given a set of training examples (X1, Y1), ..., (Xn, Yn) where each Xi belongs to X and Yi belongs to Y, the objective is to learn a function that minimizes the number of misclassified examples. This is typically achieved by minimizing some loss function.

2. Statistical Learning Theory (SLT) Framework:

SLT provides a rigorous mathematical framework to solve the binary classification problem. It focuses on two main goals: - Minimizing the empirical risk (training error) based on the available data. - Ensuring good generalization, meaning the classifier should perform well on unseen data. To formalize this, SLT defines the risk or expected loss of a classifier f as: R(f) = E[l(f(X), Y)] where l(f(X), Y) is the loss function (e.g., 0-1 loss) and the expectation is over the unknown probability distribution P(X, Y). Since P is unknown, SLT uses the principle of Empirical Risk Minimization (ERM), which minimizes the empirical risk: Remp(f) = (1/n) * sum of l(f(Xi), Yi) over all i from 1 to n SLT also introduces the concept of the Bayes classifier, which is the optimal classifier that minimizes the risk: fBayes(x) = 1 if P(Y=1 | X=x) >= 0.5, else -1 Though the Bayes classifier cannot be directly computed due to the unknown distribution, SLT helps approximate it by controlling the complexity of the function space (capacity) using tools like VC dimension. This ensures good generalization and prevents overfitting.

SLT also addresses generalization—how well a classifier performs on new, unseen data—by controlling the complexity of the function space through measures like the VC dimension . This ensures that the classifier not only fits the training data but also works well in practice, preventing overfitting. By offering a structured approach to minimizing both training error and ensuring good generalization, SLT forms the mathematical foundation for many machine learning algorithms.

Conclusion SLT offers a robust mathematical foundation for binary classification by guiding the creation of classifiers that not only perform well on training data but also generalize to unseen data. Its focus on risk minimization, generalization, and capacity control makes it a cornerstone in the theory of machine learning.