Exercise - Rigid Body Simulation

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0.1 Rigid body attributes

- Mass: M = density * width * height * depth;
- Inertia tensor and its inverse: For a die, we can apply the formula of **Solid cuboid**'s inertia tensor

$$I = \begin{bmatrix} \frac{1}{12}m(h^2+d^2) & 0 & 0 \\ 0 & \frac{1}{12}m(w^2+d^2) & \\ 0 & 0 & \frac{1}{12}m(w^2+h^2) \end{bmatrix}$$

- Inertia tensor inverse in world space: $I^{-1}(t) = R(t)(I^0)^{-1}R(t)^T$

0.2 Force

The die always has gravity force $F = mg$ and at time step t=1, an instance Force f=(0.15,0.25,0.03).

0.3 Linear Momentum

For time integration:
$X(t+\delta t) = X(t) + \delta t * P(t)/M$
$P(t+\delta t) = P(t) + \delta t * F(t)$
The result shows a smooth parabolic motion in Figure 1

0.4 Torque

$\tau(t) = (r_0 - x(t)) \times F_i$ We have to change the vertex position from body space to world space first before calculate the torque: $r_(t) = R(t)r_i^0 + x(t)$.

0.5 Angular velocity

Calculate the Rotation matrix: $R(t+\delta t) = R(t) + \delta t(I^{-1}(t)L) \times R(t)$ and Angular Momemtum $L(t+\delta t) = L(t) + \delta t * \tau(t)$. The result is show in the Figure 2, where the die rotate in a smooth parabolic trajectory.

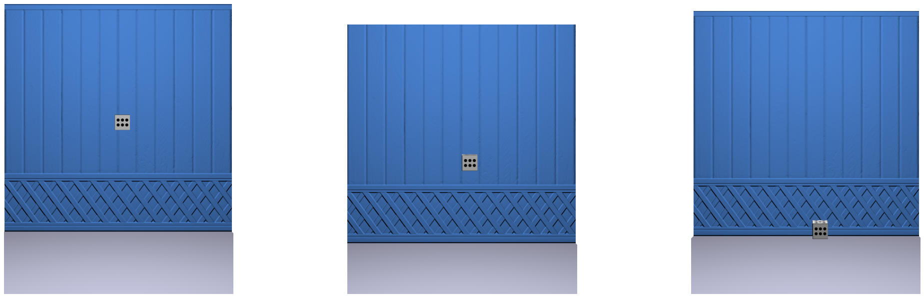

Figure 1: Linear momentum

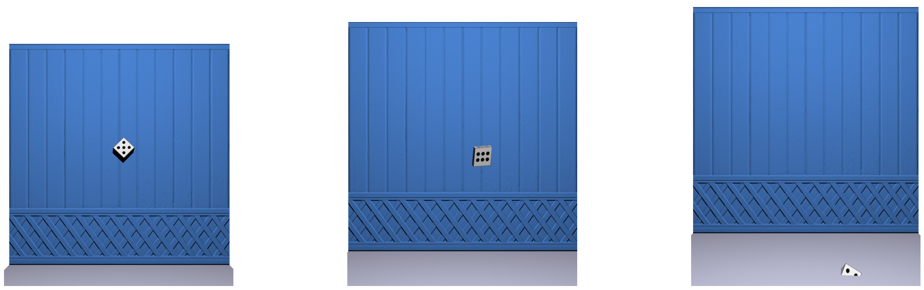

Figure 2: Angular momentum

0.6 Quaternion

$q(t) = q(t) + \delta t * \frac{1}{2} * (0, \omega(t))q(t)$
The rotation matrix R will be computed according to quaternion q, and quaternion will be normalized to prevent the rotation being distorted. The result is shown in Figure 3

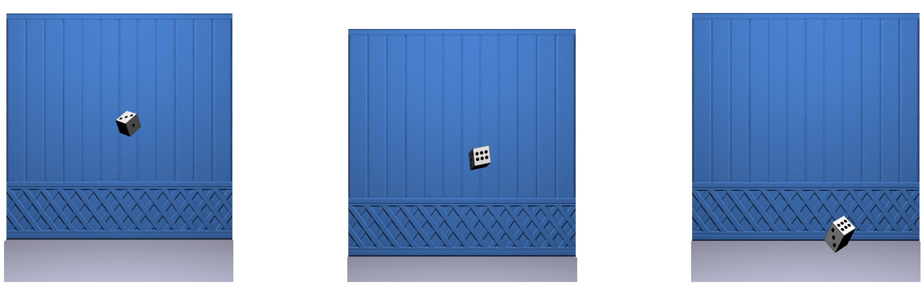

Figure 3: Quaternion