Karlsruher Institut für Technologie

Problem Set 5
Statistical Risk Factors and Principal Component Analysis

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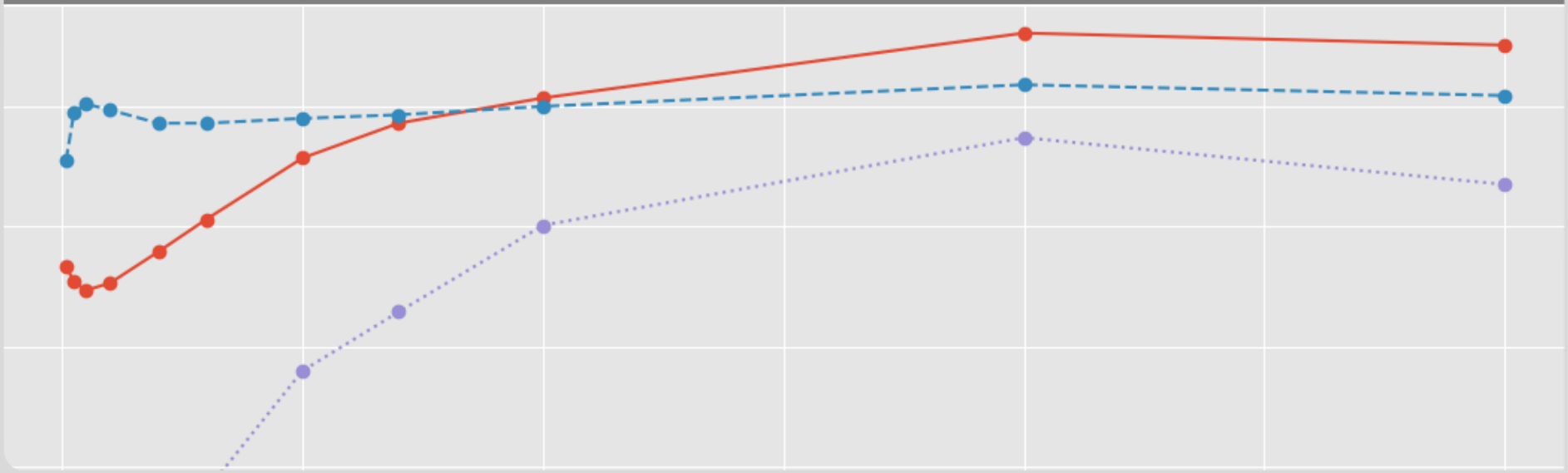
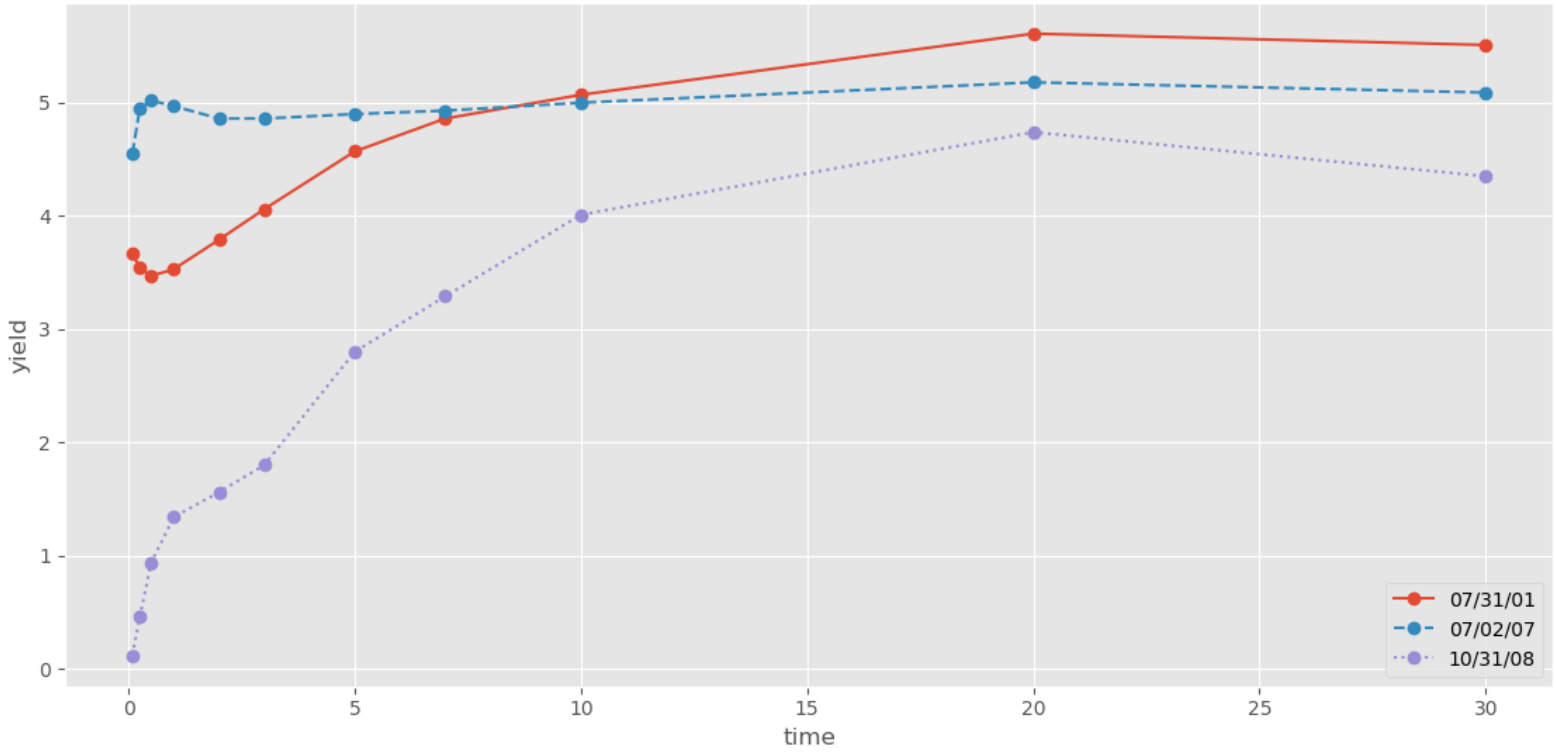
CRAM-Programming Lab WS 2017/18

FBV, Chair of Financial Economics and Risk Management



Question 1a: Treasury Yield Curves



Question 1a: Treasury Yield Curves

- 31/7/2001 and 31/10/2000
  - Normal yield curve, where longer maturity bonds have a higher yield compared to short-term bonds due to the risks associated with time
  - The yield curve for 31/10/2008 changes dramatically because global financial crisis in 10/2008 caused the the yield for the treasury in 2008 to be very low
- 2/7/2007
  - Flat Yield Curve, where the short- and long-term yields are very close to each other

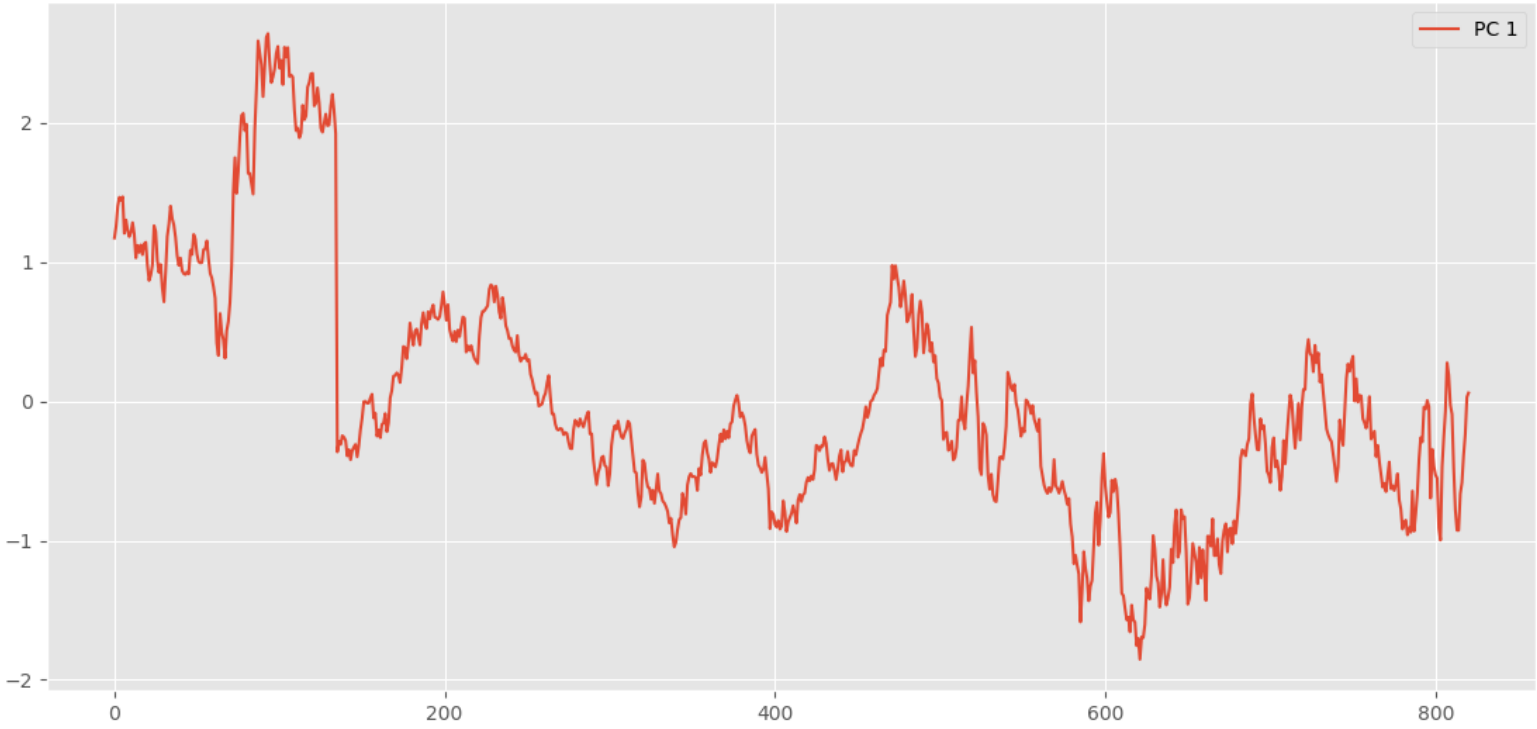
Question 1b: PCA Intuition

- Methodology: Reduce the dimensionality of correlated data without giving up a significant amount of information about data‘s second moment
- Main goals of PCA:
  - The transformed TxK data has a diagonal KxK covariance matrix
  - Each orthogonal factor explains the maximal variance possible
  - The transformed data contains the same information as X
- Specific applications of the PCA in risk and asset management:
  - Extract statistical risk factors
  - Improve diversification of porfolio

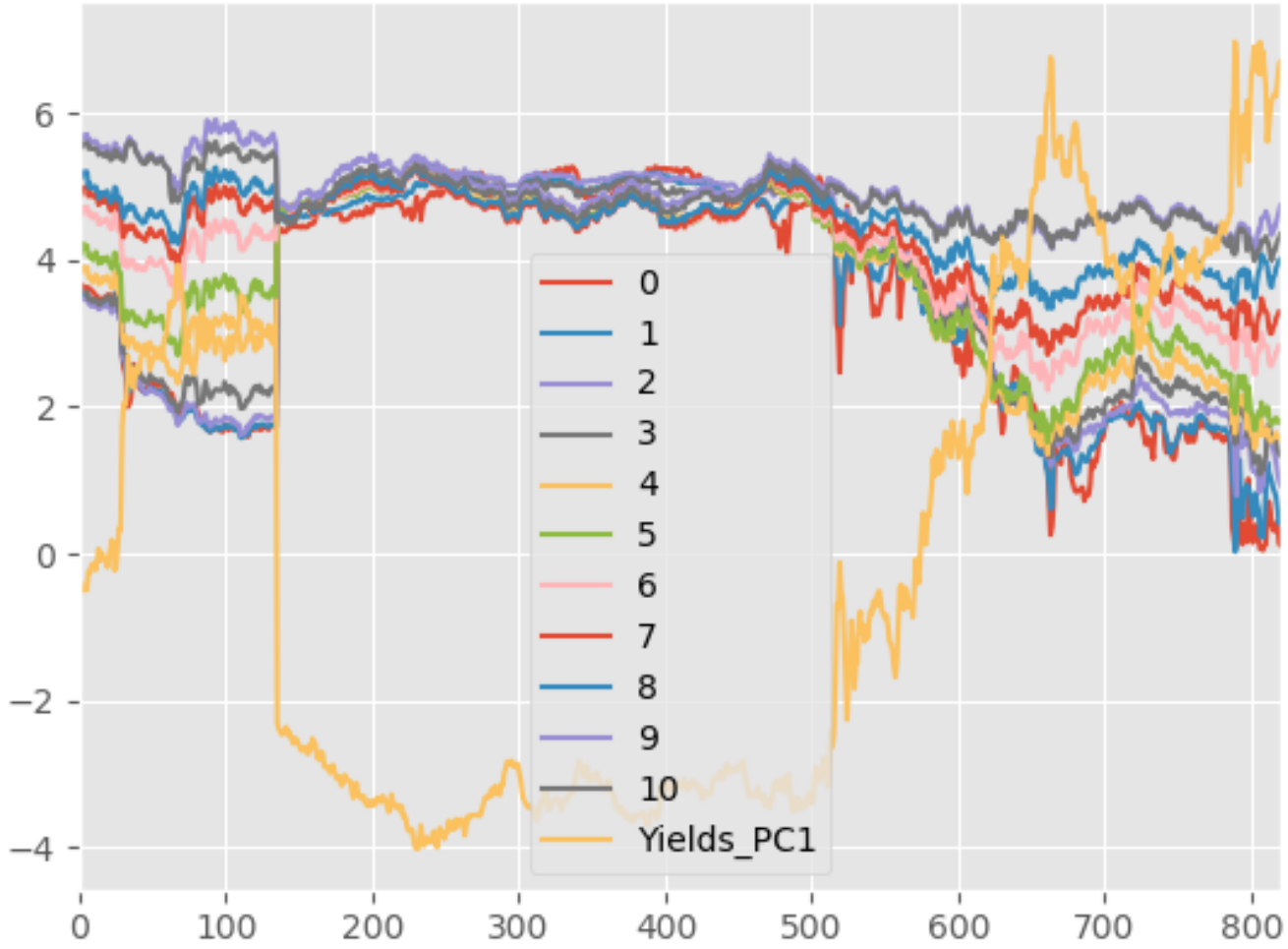
Question 1c: Class ‘PCA‘

- *See code.*

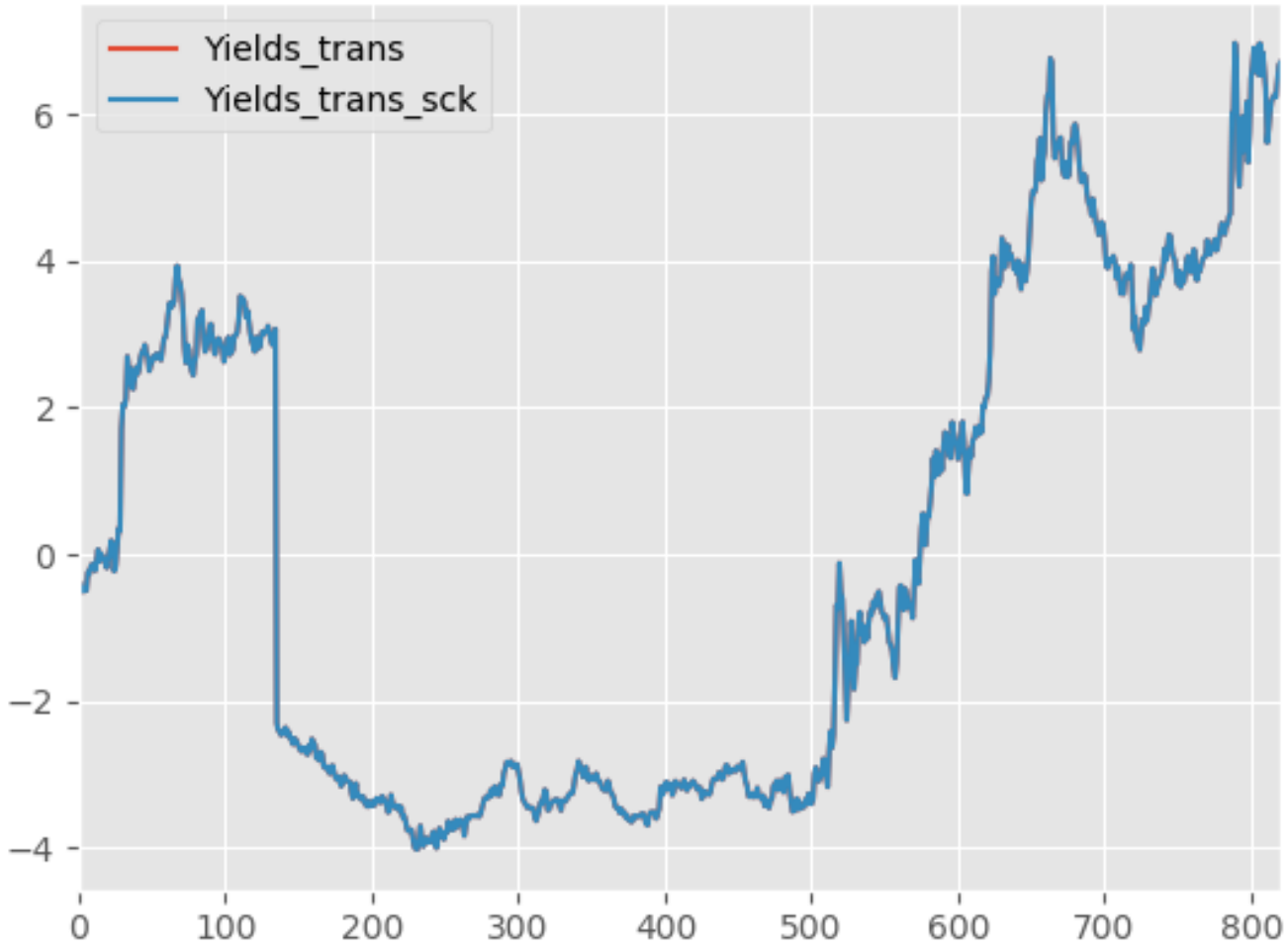
Question 1d: Principal Components of Yields



Question 1d: Principal Components of Yields



Question 1d: Principal Components of Yields



Question 1d: Principal Components of Yields

	0	1	2	3	4	5 \
0	1.000000	0.992274	0.980586	0.974837	0.943761	0.910039
1	0.992274	1.000000	0.995143	0.990757	0.959826	0.924269
2	0.980586	0.995143	1.000000	0.997001	0.963748	0.924868
3	0.974837	0.990757	0.997001	1.000000	0.979884	0.948334
4	0.943761	0.959826	0.963748	0.979884	1.000000	0.991700
5	0.910039	0.924269	0.924868	0.948334	0.991700	1.000000
6	0.827853	0.838397	0.832623	0.864946	0.942311	0.975436
7	0.734527	0.741194	0.729865	0.768078	0.869992	0.922169
8	0.641692	0.648847	0.637259	0.679618	0.797190	0.861675
9	0.256500	0.249254	0.221731	0.270859	0.430274	0.533190
10	0.276857	0.265329	0.232635	0.277096	0.426721	0.525575
Yields_PC1	-0.981521	-0.990371	-0.988389	-0.993831	-0.988143	-0.967185

	6	7	8	9	10	Yields_PC1
0	0.827853	0.734527	0.641692	0.256500	0.276857	-0.981521
1	0.838397	0.741194	0.648847	0.249254	0.265329	-0.990371
2	0.832623	0.729865	0.637259	0.221731	0.232635	-0.988389
3	0.864946	0.768078	0.679618	0.270859	0.277096	-0.993831
4	0.942311	0.869992	0.797190	0.430274	0.426721	-0.988143
5	0.975436	0.922169	0.861675	0.533190	0.525575	-0.967185
6	1.000000	0.984161	0.949609	0.700157	0.684056	-0.902241
7	0.984161	1.000000	0.987884	0.811746	0.790748	-0.819309
8	0.949609	0.987884	1.000000	0.879523	0.853961	-0.737426
9	0.700157	0.811746	0.879523	1.000000	0.983053	-0.355724
10	0.684056	0.790748	0.853961	0.983053	1.000000	-0.363069
Yields_PC1	-0.902241	-0.819309	-0.737426	-0.355724	-0.363069	1.000000

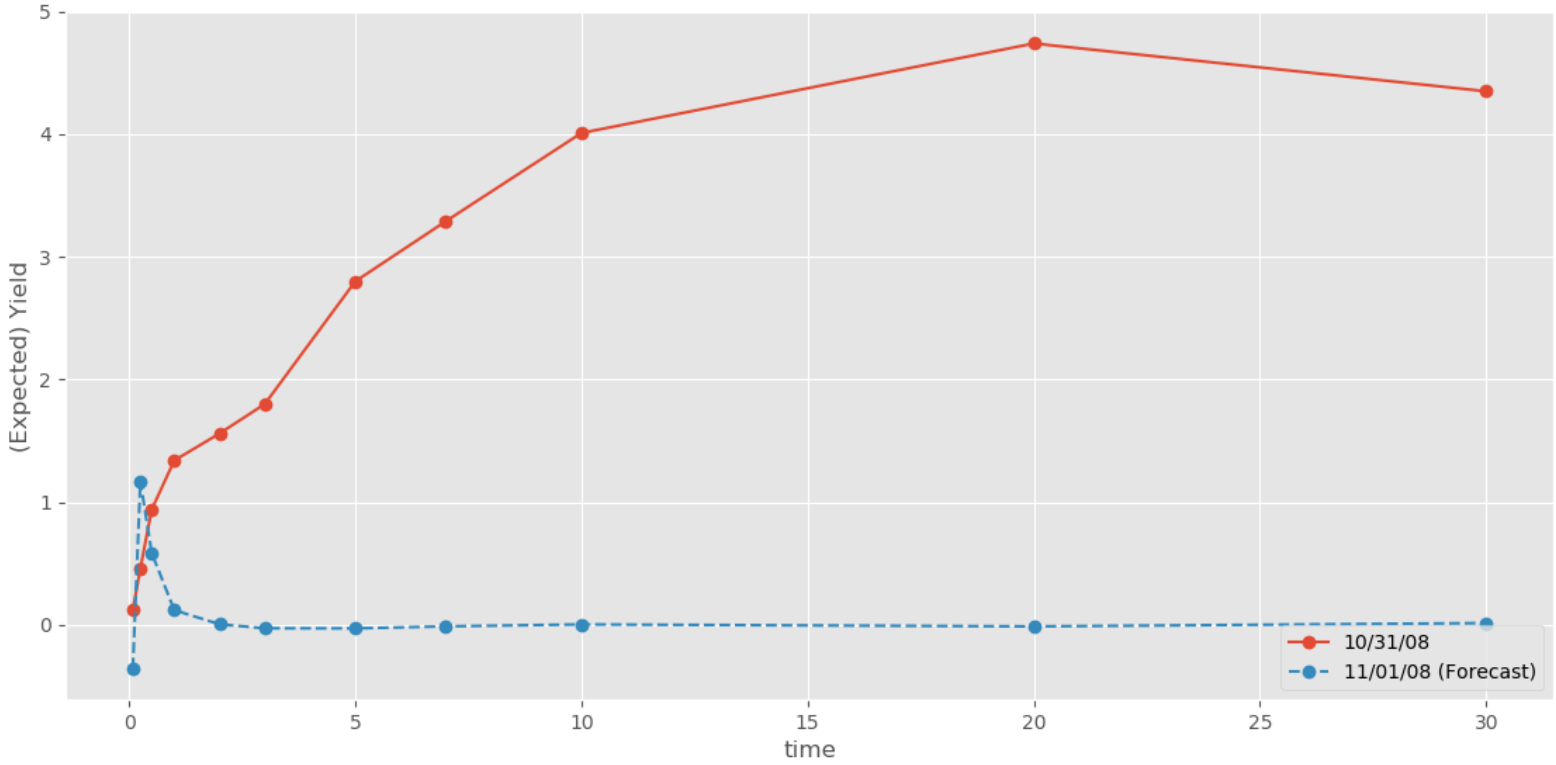
Question 1e: Importance of the PCs

- Explained variance ratio of PCs:

  [0.928 0.062 0.008 0.002 0. 0. 0. 0. 0. 0. 0.]

  -> we choose the first two PCs as 99% of the variance of the data is explained by both PCs

Question 1f: Forecasting the Yield Curve



- Invest into 3-month-bond as it shows highest expected yield