1.1. Infinite-horizon deterministic consumption model without endogenous labor supply

- At each stage from here on out, use the same or similar utility and production functions unless otherwise indicated.
- Assume the individual receives a fixed wage and can save in a safe asset.

From Chapter 6, the infinite horizon deterministic consumption model without endogenous labor supply is illustrated as below.

We have the functional equation with stochastic wage (w) and a labor supply decision (n)

$$v(A,w) = \max_{A',n} U\left(A + wn - \frac{A'}{R}, n\right) + \beta E_{w'|w} v(A',w') \qquad \text{for all } (A,w).$$

The dynamic programming problem yields a simpler functional equation is:

$$v(A,w) = \max_{A'} Z(A,A',w) + \beta E_{w'|w} v(A',w'),$$

where

$$Z(A,A',w) \equiv U(A + w\varphi(A,w,A') - (A'/R), \varphi(A,w,A'))$$

With $c = A + wn - (A'/R)$ and $n = \varphi(A,w,A')$

Working hours equation:

$$\ln(n_t) = \frac{\ln w_t}{\omega_2 - 1} + \frac{1}{\omega_2 - 1}(\ln U_c(c_t,n_t) - \ln \gamma_{2t} - \ln \omega_2)$$

The Bellman equation for households:

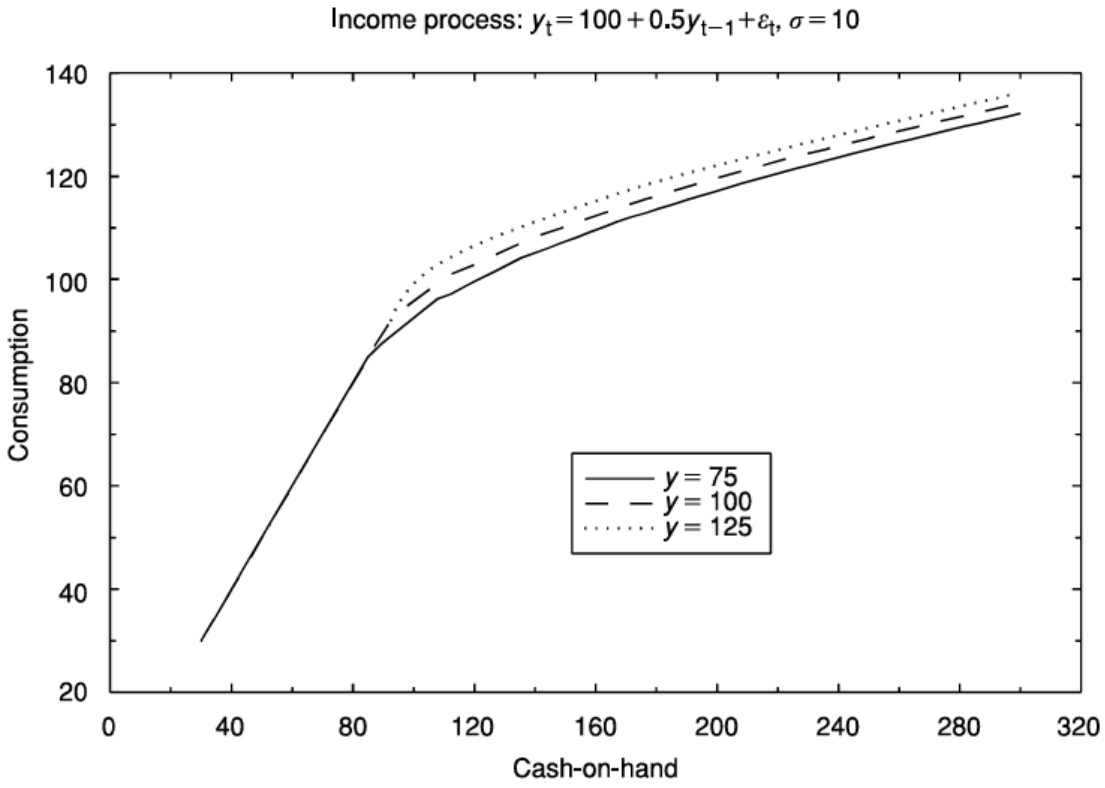
$$v(x) = \max_{0 \le c \le x} u(c) + \beta E_{y'} v(R(x-c) + y')$$

Wealth constraint in period 0:

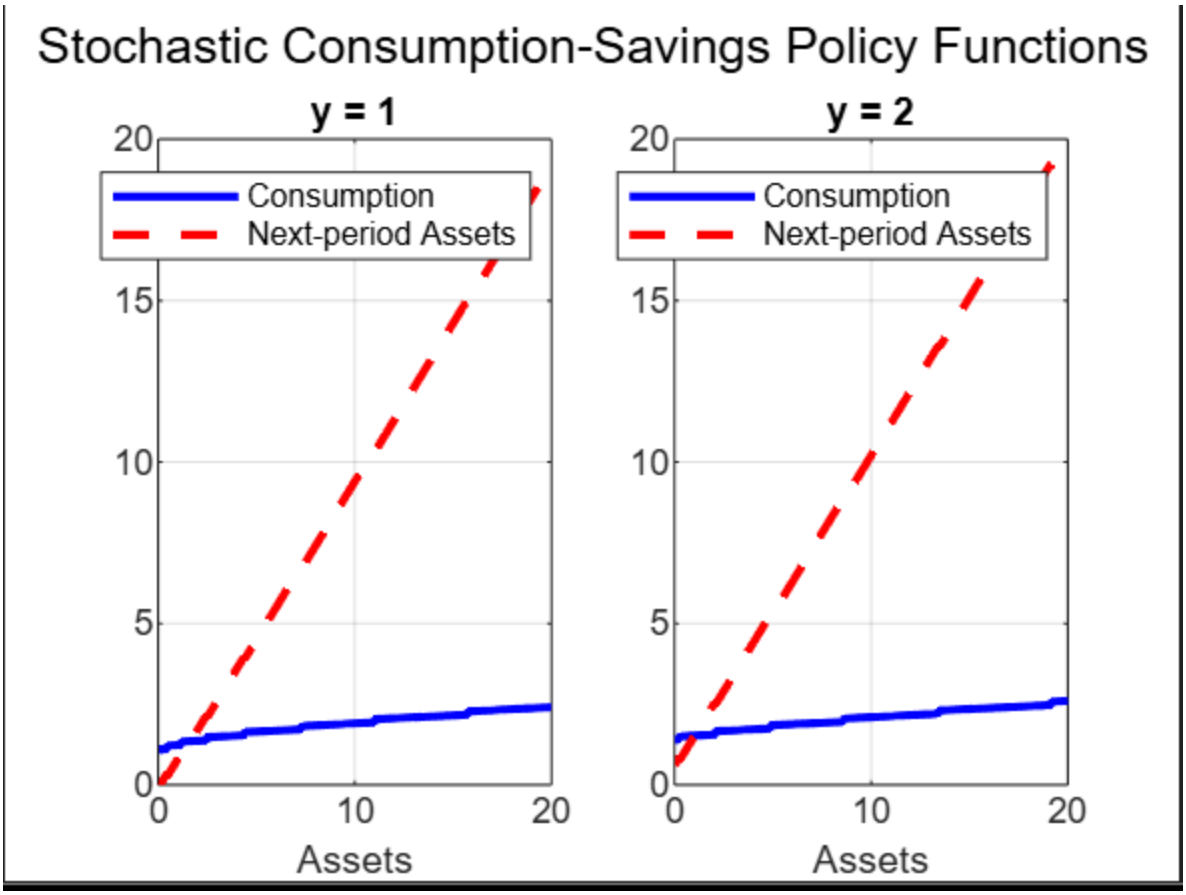
$$U_c(c_t,n_t) = \frac{\lambda_0}{\beta^t (1+r_1)\dots(1+r_t)}$$

In this stage all individuals have the same decision to become a worker and receive a fixed wage which will be allocated into two consumption and savings. Other than augmenting the resources available to finance present consumption and future riches, the amount of employment and labor wages has no dynamic component.

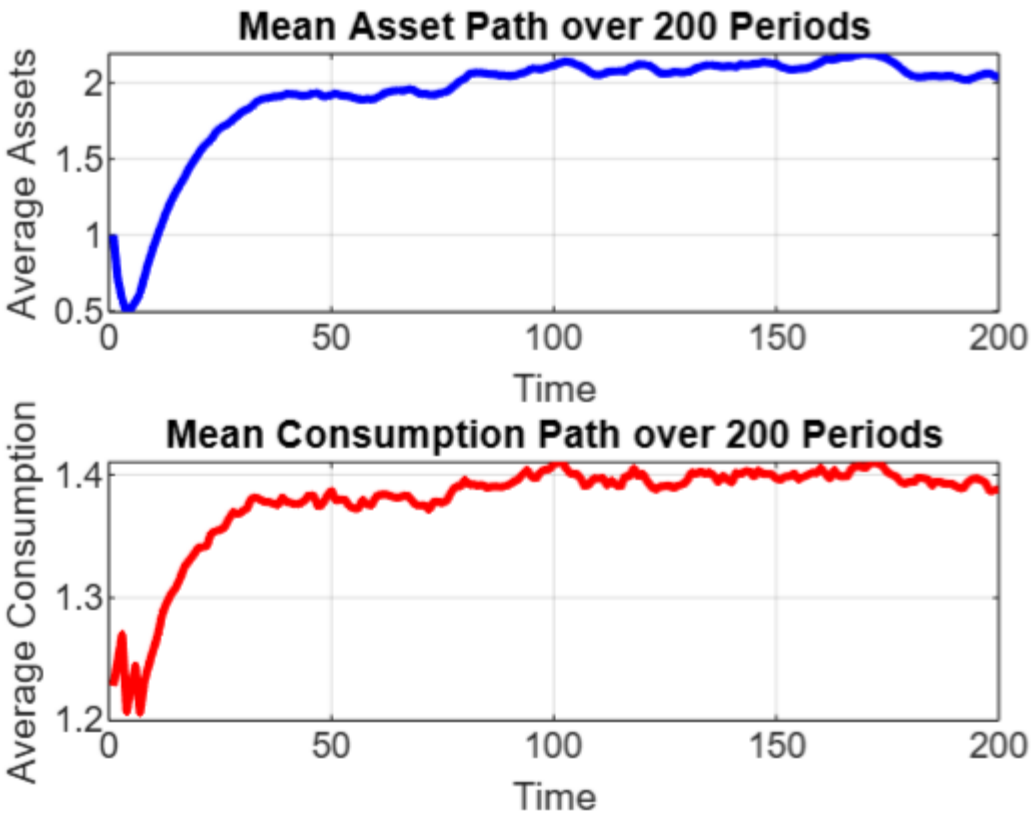
The figure below shows the optimal consumption rule in the case the problem has two state variables which are the current cash and the current income that provides the information for the future income of this agent. The policy rule shows three points for current income and 60 points for current cash, and for higher value on cash, the agent saves partly for the consumption which will happen in the future.



In the graph below, the blue line is the optimal consumption as a function of current assets, and the red one is the optimal next period asset which is hold as a'. According to this graph, as expected, the consumption level increases with assets in both states, and the higher income level leads to higher consumption at any level. In the next period, the curve becomes linear and steep which indicates that agent save more, especially with high assets level. As in the hypothesis, for the low income, the consumption will increase slowly and for the high income, the consumption will increase faster.



In order to have a deeper understanding of the current income of the agent and safe asset, we consider the graph below. The blue panel is average asset holding over time, and the red panel is the average consumption overtime. It is obvious that in the early dynamics, there is a sharp change in period 2 maybe since the agent starts the first period with no or low income and consumes in the next period. After this noticeable change, the asset holding rises steadily since the agent moves to a stable period. On the consumption path, there is a smooth upward trend that reflects the asset accumulation overtime. These graphs point out that both assets and consumption levels are stable, which illustrates the well-defined stationary distribution under the optimal policy.



After the age of 40, people tend to save more for their retirement.

1.2. The agent chooses to be an entrepreneur and invests in a project with uncertainty

Assume instead that the individual is an entrepreneur who decides to invest in a project with an unknown return, which makes the problem stochastic, as in the paper.

- The individual enters the period with wealth, invests capital in a project, consumes, and saves in a risk-free bond.

- The capital investment is used for a project with either high or low returns, but the individual won't know or receive this return until the next period.

Entrepreneurial activity is risky and poorly diversified, but the premium to entrepreneurial activity is surprisingly low, but many people still want to become entrepreneurs. The preference paper incorporates a simple dynamic occupational choice model that a borrowing

constrained agent chooses whether to be a worker or an entrepreneur. And in this stage, the agent chooses to become a worker or an entrepreneur and decides to invest in a project with uncertainty. This uncertainty comes from the income of an entrepreneur, which is decided by the project that he decides to invest in. Entrepreneurs receive the payoff from their chosen project in the next period workers earn wage income and returns on their savings.

The value of being an entrepreneur at t = 1 can be expressed as

$$V_1^E(w_1) = \max_{w_2 \geq 0} \{u(w_1 - \frac{w_2}{A}) + \beta u(w_2)\}.$$

Looking at the graph below

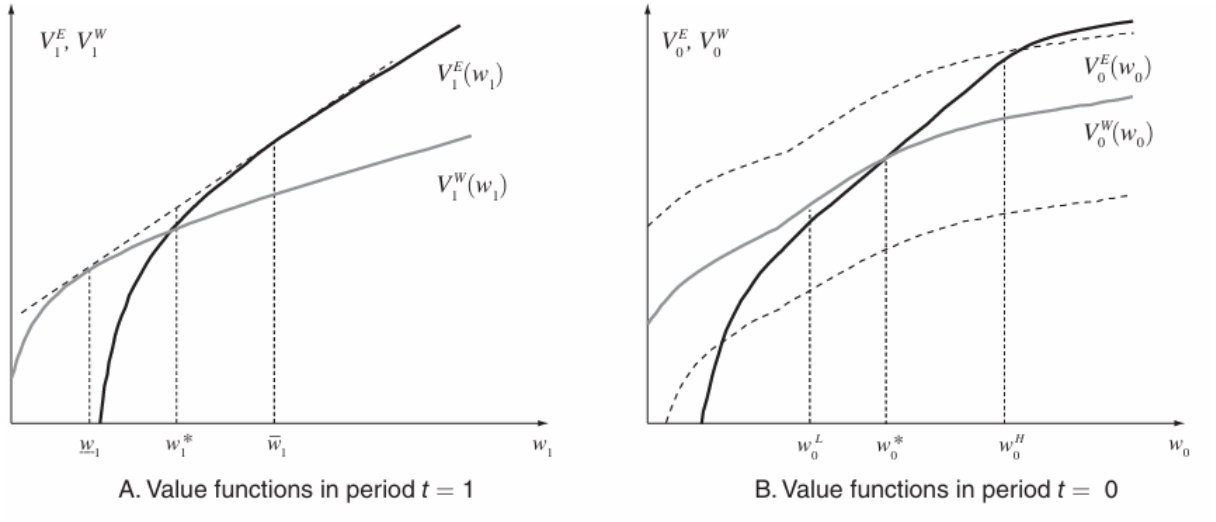


FIGURE 1. THE VALUES OF A WORKER AND AN ENTREPRENEUR IN PERIODS $t = 1$ AND $t = 0$

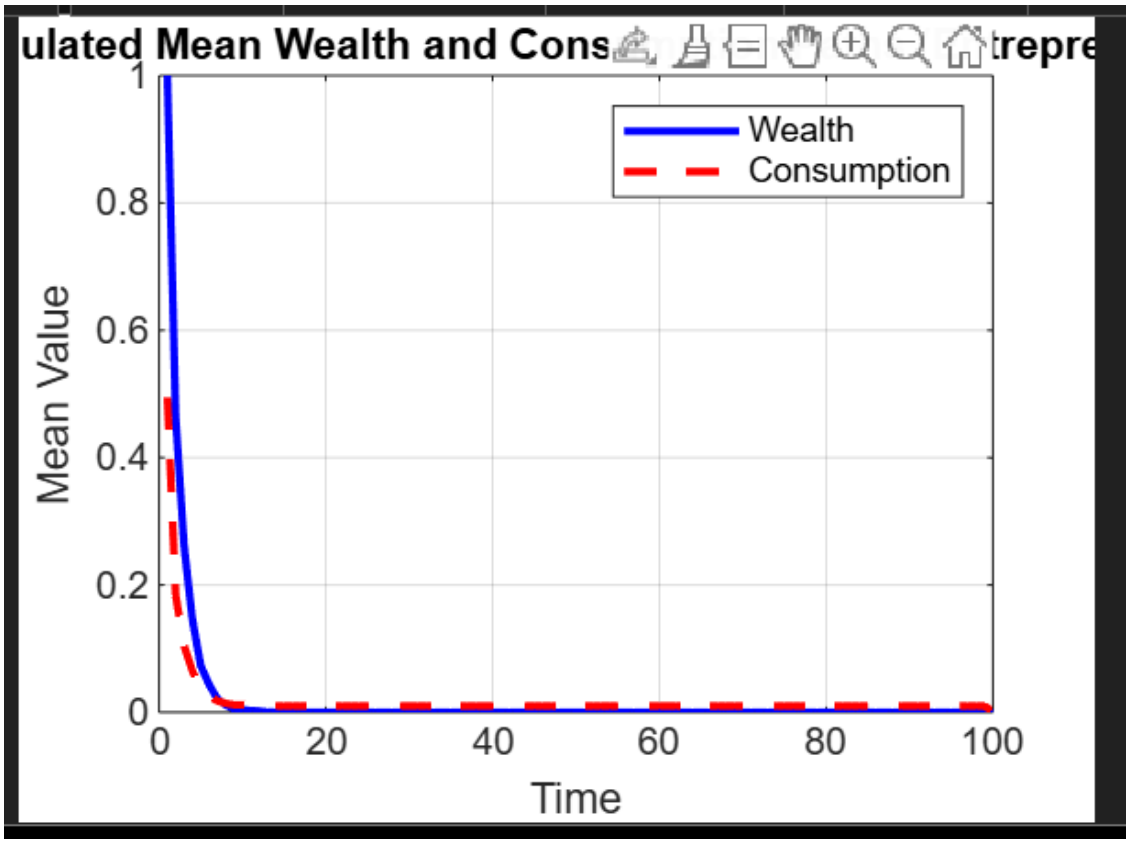
in which $V_1^E(w_1)$ is the value of being an entrepreneur

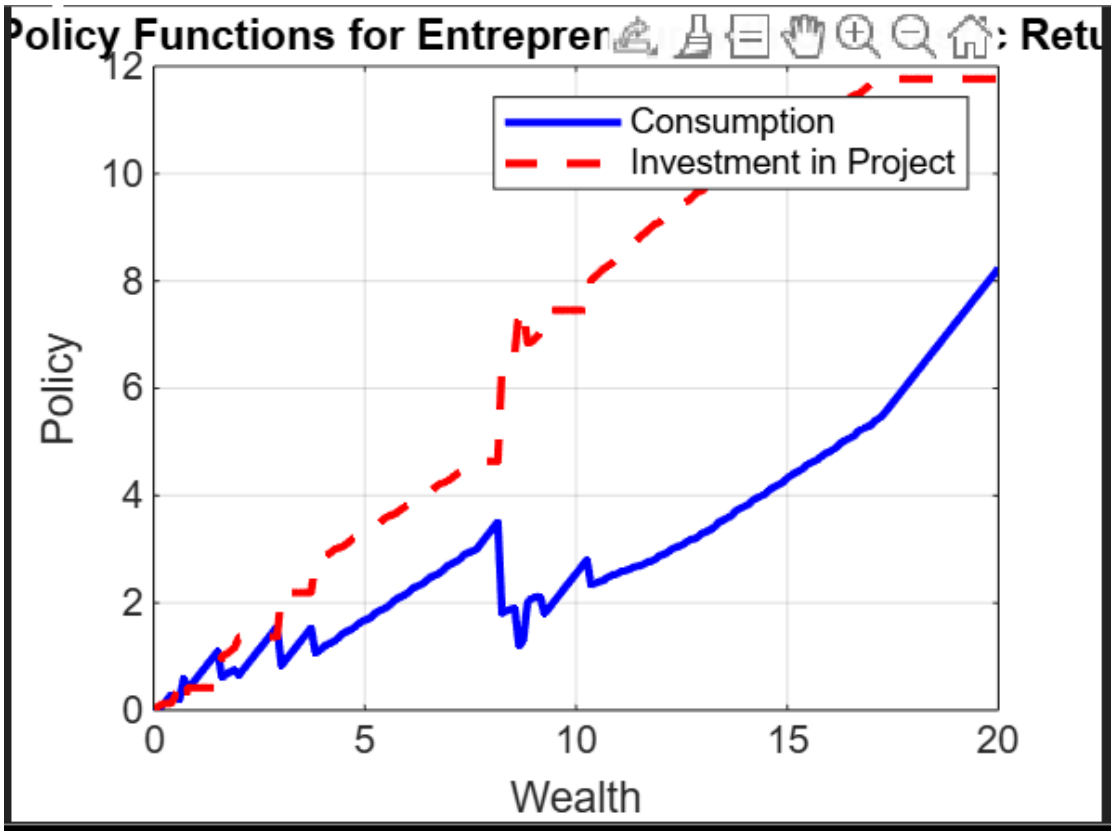
The value function of being an entrepreneur increases strictly concave and has an intersection with the value function of being a worker, but then the entrepreneur's value increases higher than the worker's one. From graph observations, the entrepreneur can increase the expected continuation value by investing in a risky instead of a safe project, which generates higher final wealth.

Below is the value function for an entrepreneur

$$V_E(w) = \max_{k \geq 0, P \in \Omega(A)} \{u(w-k) + \beta EV(Zk)\},$$

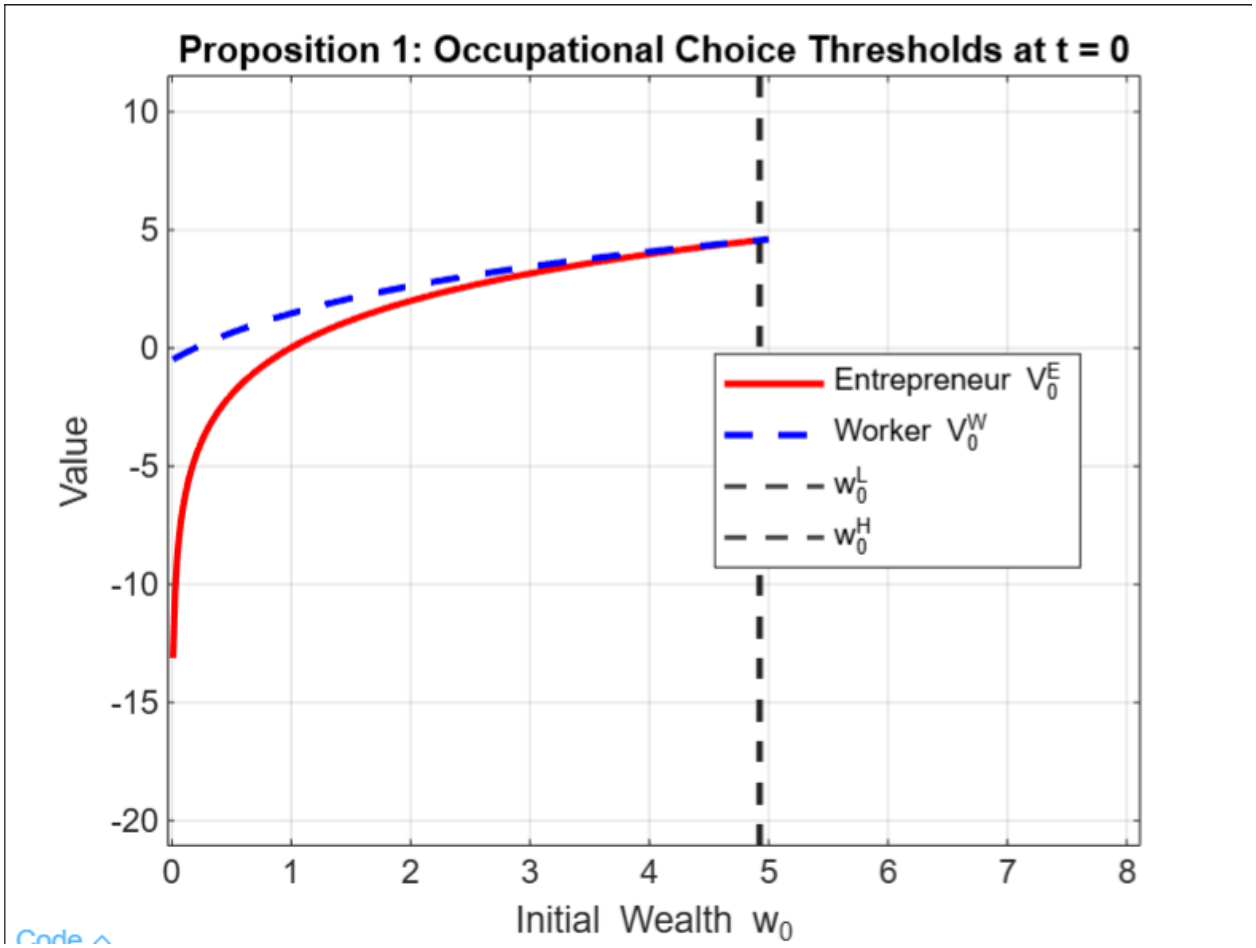
The entrepreneur chooses how much to invest an amount of k in their business and selects a project with random returns. The utility from current consumption and the discounted expected future value are maximized. Entrepreneurs balance current consumption with future returns from their investment. The choice of project risk allows them to manage their wealth trajectory and occupational options





Two graphs above reflect the economic activity of a person who chooses to be an entrepreneur and invests in a project with uncertain returns. The graph of the policy function reflects a highly nonlinear relationship between wealth and optimal actions: consumption and investment each exhibit sharp kinks and discontinuities, particularly at low to moderate wealth. This depicts the non-convexity of the value function for the entrepreneur because risk aversion in the event of investment failure gives way to risk taking at higher wealth levels when the agent is better able to withstand losses. Simulation of mean wealth and consumption over time provides evidence of the sensitivity of entrepreneurial trajectories under risk. Both lines slope down steeply, with wealth and consumption being driven to zero after only a finite number of periods. This failure implies that without insurance or external income, adverse shocks early in the entrepreneurial journey have the ability to lower economic wealth permanently. These forces are consistent with the underlying value function, where wealth in the future is selected by the entrepreneur at the cost of investment today, equating intertemporal utility with random project returns. In general, the results highlight the vulnerability of self-financed entrepreneurship in dangerous environments and underscore the influence of risk reduction tools such as business insurance or funding availability from outside.

The capital investment is used for a project with either high or low returns, but the individual won't know or receive this return until the next period. According to the preference paper, the uncertainty of entrepreneur's investment is shown as following. There exist $0 \leq w_0^L < w_0^H$ (L is low returns and H is high returns). Operates a safe project at t = 0 and becomes a worker at t = 1 if $w_0 \leq w_0^L$ (initial wealth is less than the high return level). Operates a risky project at t = 0 and, depending on the project's return, becomes either a worker or a safe entrepreneur in period t = 1 if $w_0 \in (w_0^L, w_0^H)$. Operates a safe project at t = 0 and continues as a safe entrepreneur at t = 1 if $w_0 \geq w_0^H$ (the initial wealth is more than or equal to high return level).

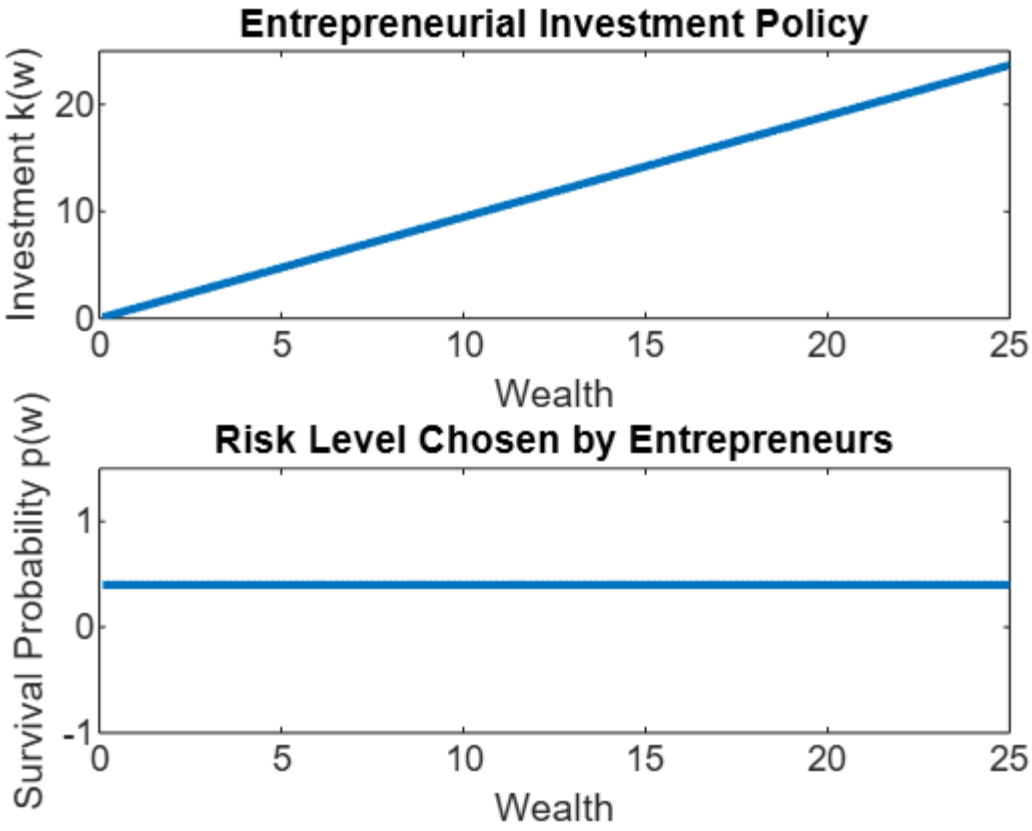


However, in the extension version of this model, it proves that patience leads to no risk taking, then the wealth value of becoming an entrepreneur will be higher than w_H.

1.3. The agent chooses between being a worker or an entrepreneur and comparison of two results

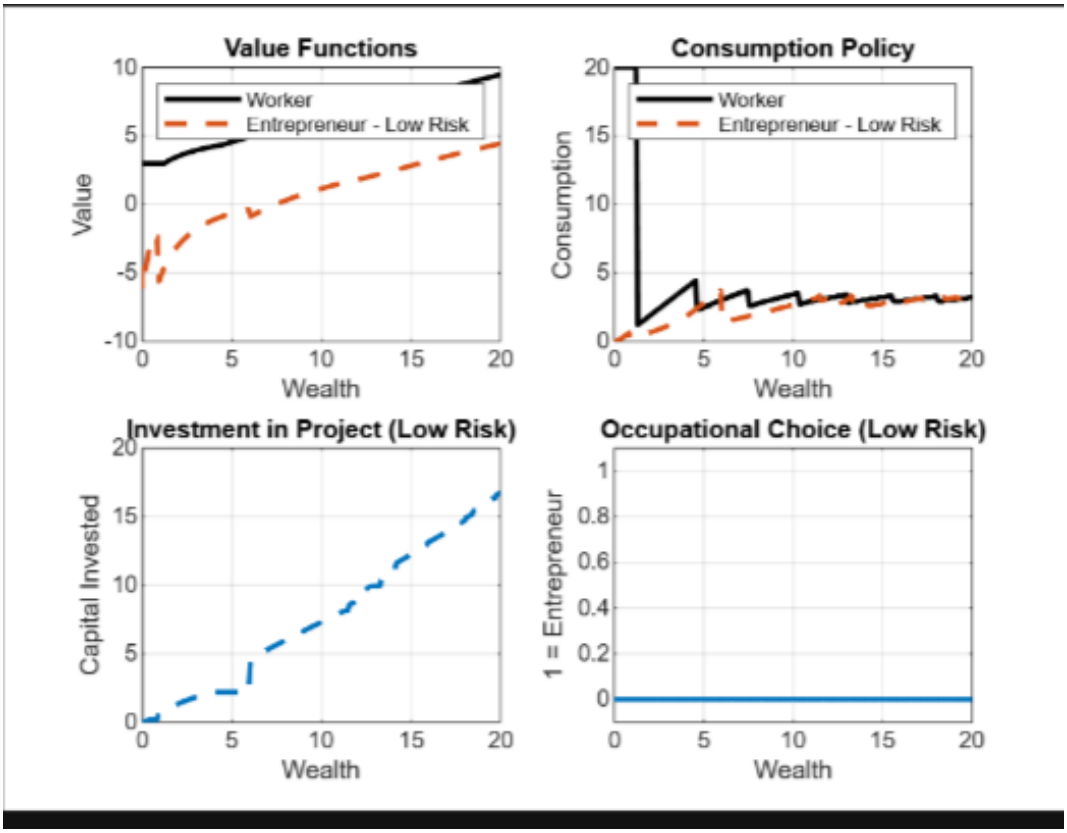
In this stage, the agent can choose between becoming a worker and receiving a fixed wage or becoming an entrepreneur and receiving uncertainties. In general, becoming a worker yields no risk in the first period, and becoming an entrepreneur has to receive the risk on investment. In the first stage, the agent chooses to become a worker and receive the fixed wage, and they cannot consume or save more than their current wage. And the second stage, the agent chooses to become an entrepreneur and invest in a safe project.

The graphs below show the entrepreneurial investment and risk-taking decision rules as functions of wealth. In the top panel, the investment policy k(w) is linearly increasing in wealth, indicating that entrepreneurs invest a proportionally larger amount of capital in their projects as their wealth rises. This is consistent with models in which investment is unbounded and rises with available funds, exhibiting greater capacity for taking risks at higher wealth levels. In contrast, the lower panel shows that the survival probability or risk level p(w) chosen by entrepreneurs is constant across all levels of wealth. That is, risk is either exogenous or not optimal in the current model. The lack of variation in p(w) means that entrepreneurs are not altering project risk in response to their financial position, a simplifying assumption that can provide a benchmark before endogenous risk choice is introduced. Combined, the policy functions suggest a model where investment decisions are sensitive to wealth, and yet risk exposure is invariant regardless of financial resources.



The figure presents four panels that jointly characterize the economic behavior of agents under a model where individuals choose between being workers or low-risk entrepreneurs. The top-left panel compares value functions for workers and entrepreneurs across varying levels of wealth. It shows that, at all wealth levels, the value function for workers (solid black line) dominates that of entrepreneurs (dashed red line), indicating that individuals prefer to remain workers rather than engage in low-risk entrepreneurship. This is reinforced by the top-right panel, which plots the consumption policy functions. While both worker and entrepreneur consumption levels converge at higher wealth, workers exhibit higher consumption at lower levels of wealth, further contributing to their superior value function.

The bottom-left panel depicts the investment policy of low-risk entrepreneurs, showing that capital investment increases monotonically with wealth, suggesting that investment decisions are positively correlated with entrepreneurial capacity. However, despite this increasing investment pattern, the bottom-right panel shows that the occupational choice remains constant at zero across all wealth levels - implying that no agents opt into entrepreneurship in the low-risk setting. This finding suggests that the return from low-risk entrepreneurial activity is insufficient to outweigh the benefits of employment, even at higher wealth levels. Overall, the graph highlights that under this particular model calibration, entrepreneurship is consistently dominated by wage work in terms of both utility and consumption, rendering it an unattractive occupational choice.



In the preference paper, Vereshchagina and Hopenhayn (2009) presume that occupational choice between being an entrepreneur or a worker is driven by wealth dynamics and borrowing constraints, and entrepreneurial risk is determined endogenously. The individual is either a worker or an entrepreneur as a binary and exclusive decision, where workers get a fixed wage and entrepreneurs have access to production technology whose returns are random. Both the amount of capital to invest and the riskiness of the project are selected by the entrepreneur - defined by a mean-preserving spread of returns with fixed expected value but volatile variance.

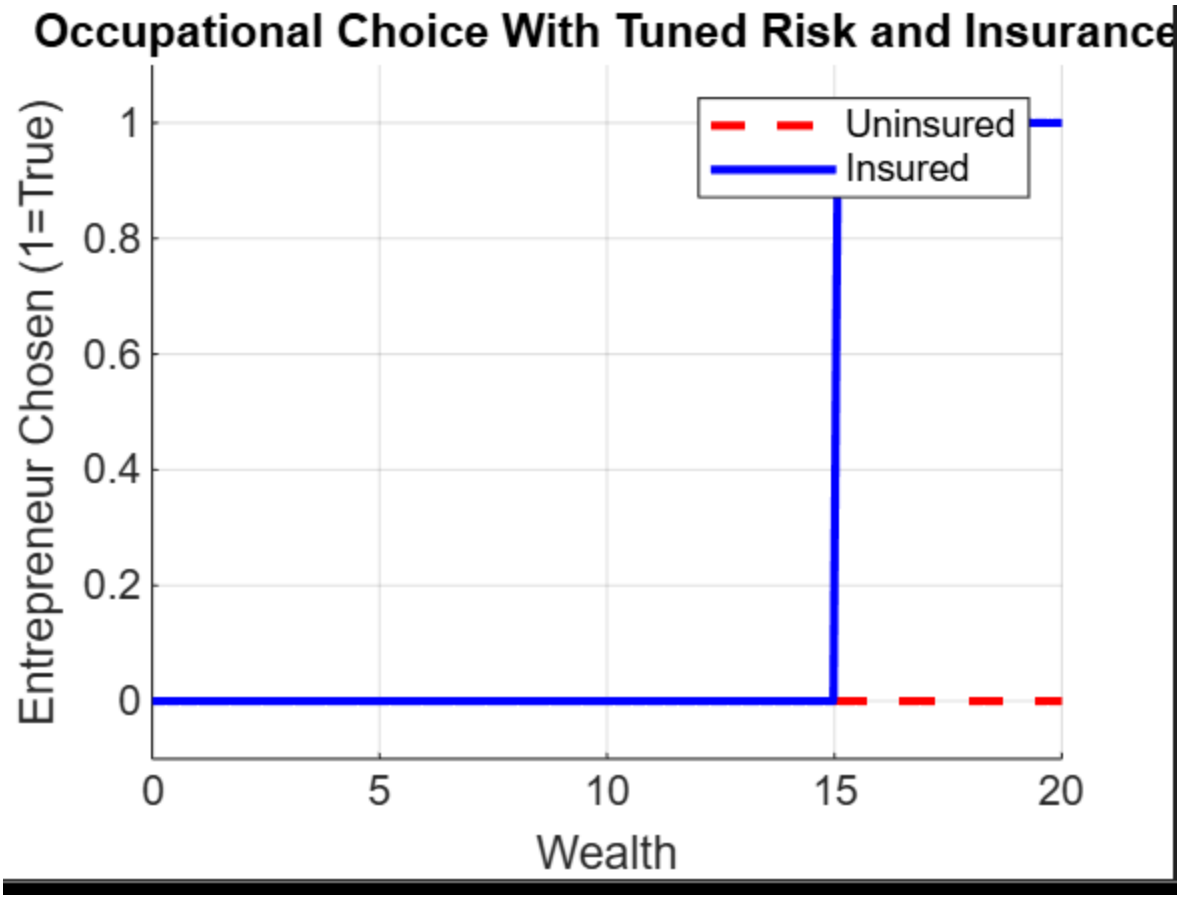
Workers have income stability and can riskless save to accumulate wealth. Entrepreneurs, on the other hand, carry the burden of having to finance their businesses through self-finance with no access to external borrowing. Hence, poorer agents, not being capable of investing sufficiently, would rather have the stability of wage employment. Richer agents, with more amounts of capital, however, prefer entrepreneurship as they can achieve higher expected utility through investment. But because of occupational choice discreteness and non-concavities in the value function, some agents close to the wealth threshold between occupations opt to become entrepreneurs even when the return does not carry a premium. A key implication of the model is that entrepreneurial risk-taking is not only due to preference heterogeneity but endogenously may arise through occupational choice and financing frictions. Poor entrepreneurs are more inclined to choose riskier projects - not for higher payoffs, but because risk provides a "lottery" to wealth in the future that smooths non-concavities in the value function. In fact, by choosing a risky venture, they bet on the chance of remaining entrepreneurs; failing, they resort to wage labor. With increased wealth, entrepreneurs shift to less risky ventures because they can get more utility without risking exit.

The simulations suggest that when the potential for risky projects exists, agents with middle wealth - those at the occupational cutoff - choose high-variance projects. On the other hand, when only low-risk projects are available, less entrepreneurship is taking place, as illustrated by the graphical results you presented. The investment function remains growing in wealth, but occupational choice is at "worker" because low-risk entrepreneurship is less appealing. In addition, the model proposes that entrepreneurship risk-taking vanishes if agents are extremely patient (i.e., high discount factors), waiting and accumulating capital before entering safely. Impatient agents, as opposed to this, enter earlier and are more likely to engage in riskier entrepreneurship, especially when subjected to credit constraints.

1.4. Extension

As mentioned above, entrepreneurial activity is risky and poorly diversified, but the premium to entrepreneurial activity is surprisingly low, but many people still want to become entrepreneurs. The risk premium efficiency is not too popular in entrepreneurial activity.

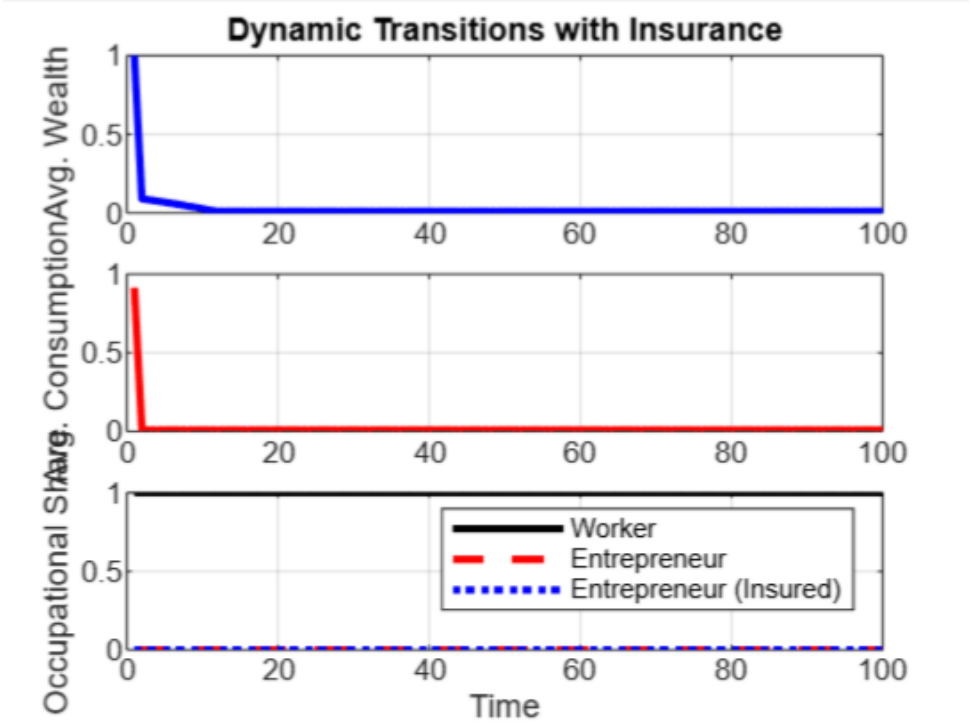
The graph below illustrates how access to insurance affects the decision to become an entrepreneur as a function of individual wealth. The vertical axis indicates the occupational choice, where 1 represents choosing entrepreneurship and 0 represents choosing to remain a worker. The horizontal axis shows increasing levels of wealth.



This graph illustrates two cases: the uninsured case (red dashed line) and the insured case (solid blue line). In the uninsured scenario, agents do not choose entrepreneurship at any wealth level up to 20; the curve remains flat at zero. This suggests that, without insurance, the risk associated with entrepreneurship is prohibitively high - even for wealthier individuals - discouraging occupational switching. In contrast, in the insured scenario, we observe agents reacting with a steep threshold response: agents with wealth below approximately 15 have a greater chance of being workers, but beyond this cutoff wealth, there is a steep increase in the probability of deciding to be an entrepreneur to 1. This is in line with the theory we observe by Vereshchagina and Hopenhayn (2009), where risk aversion, credit constraints, and the prevalence of an alternative opportunity (wage labor) intersect to decide occupation. Insurance practically eliminates the

downside risk of business failure, but the entrepreneurial path only becomes attractive after sufficient wealth is accumulated to justify entry.

The graph below illustrates the dynamic dynamics of average wealth, consumption, and occupational choice over time in a covered economy. The top panel illustrates that average wealth declines steeply and converges to a low figure, illustrating that there is minimal capital accumulation in the presence of risk insurance. Similarly, the middle panel shows mean consumption has declining sharply in early years and remaining low forever, which suggests that agents are spending current income heavily with little savings or investment returns. The lower panel graphs occupation shares and shows that almost all agents always choose to remain workers with no entry into entrepreneurship—insured or uninsured. These findings highlight that, even with the contribution of insurance towards lowering entrepreneurial risk, occupational mobility is limited and wage employment dominates. These outcomes corroborate the theoretical argument of Vereshchagina and Hopenhayn (2009), wherein the presence of borrowing constraints and the need for substantial initial wealth prevents most agents from pursuing entrepreneurship. Thus, insurance in and of itself is not sufficient to encourage entrepreneurship on a mass scale unless complemented by greater returns or re-allocation of wealth.

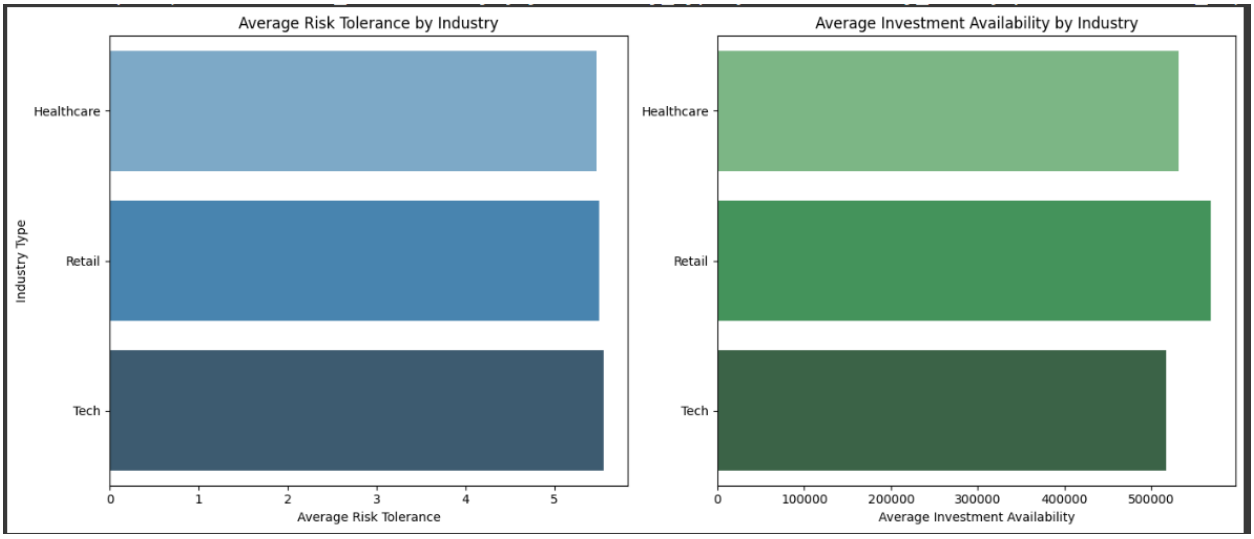


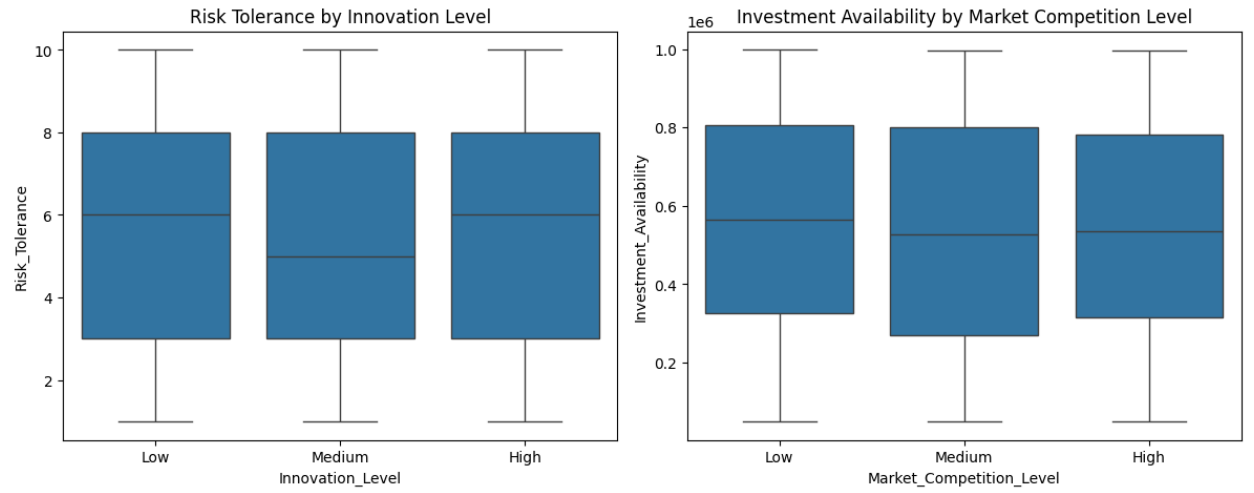
Example of Entrepreneurship Risk Tolerance

Using data from Kaggle website about Entrepreneurship Decision, we can see some correlations between the willingness to take risk of entrepreneurs and their growth rate.

The comparative analysis of average risk tolerance and investment availability across industry sectors reveals notable patterns. Although there is minimal variation in average risk tolerance among the Healthcare, Retail, and Tech industries, Tech firms exhibit a marginally higher propensity for risk-taking, followed closely by Retail, with Healthcare slightly trailing. This suggests that risk tolerance may be relatively stable across industries, potentially reflecting broader managerial or cultural norms that transcend sector-specific characteristics. In contrast, investment availability shows more pronounced differences. Retail firms have the highest average investment availability, surpassing both Healthcare and Tech sectors. Despite the capital-intensive nature of

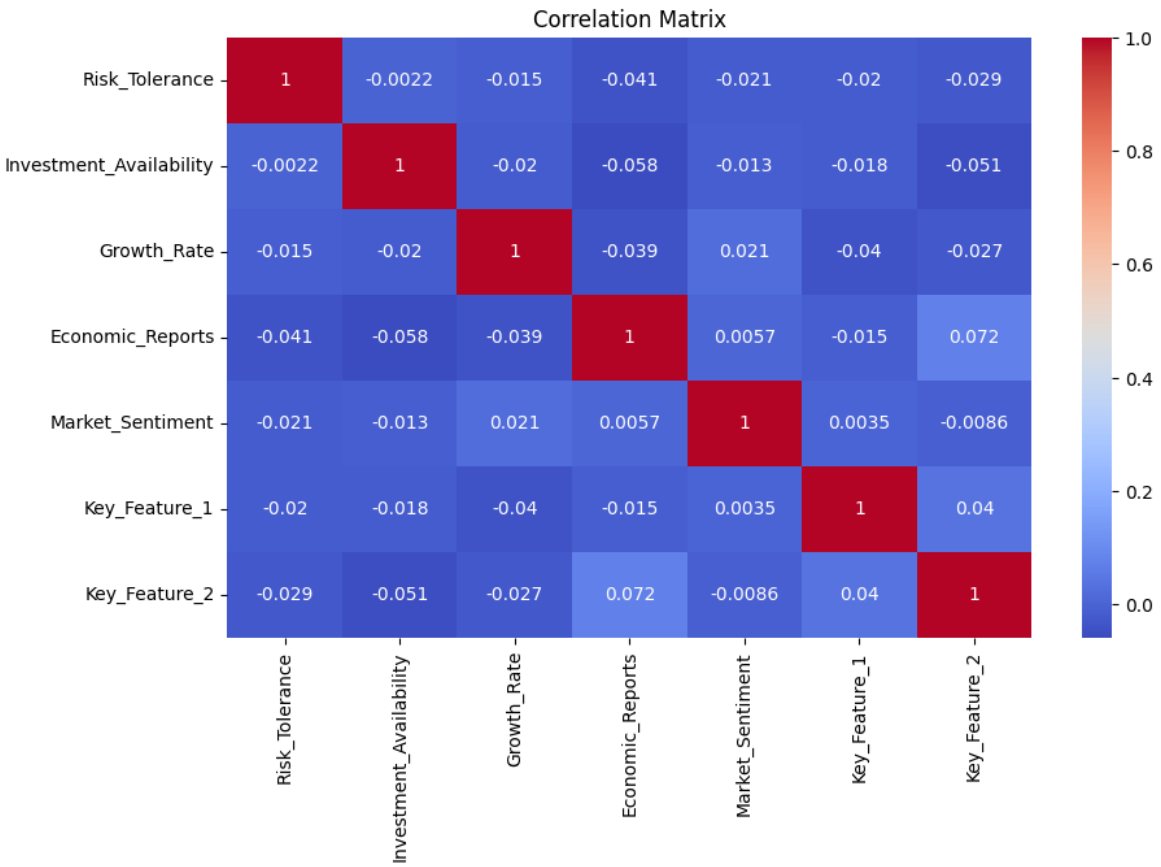
technology ventures, Tech firms report the lowest average investment availability, which may indicate investor caution or the early-stage nature of many firms in this sector. Healthcare firms occupy a middle ground, likely benefiting from stable demand and institutional support. These findings suggest that while risk attitudes may not differ significantly across industries, access to investment capital is more closely tied to sector-specific financial dynamics and perceived investor confidence.

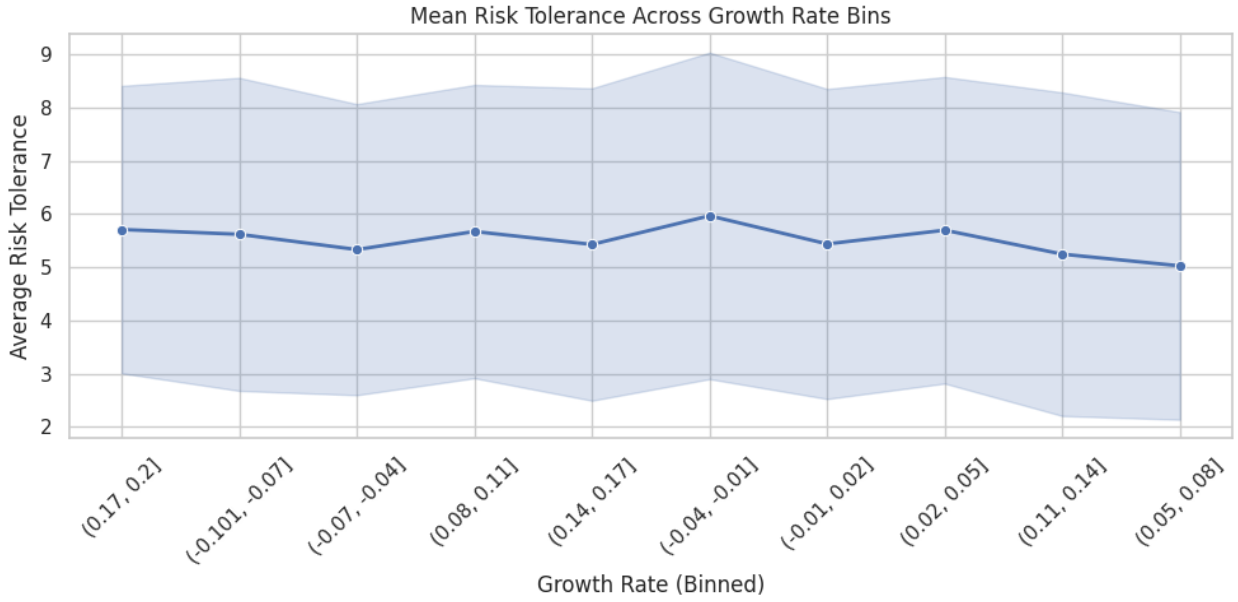


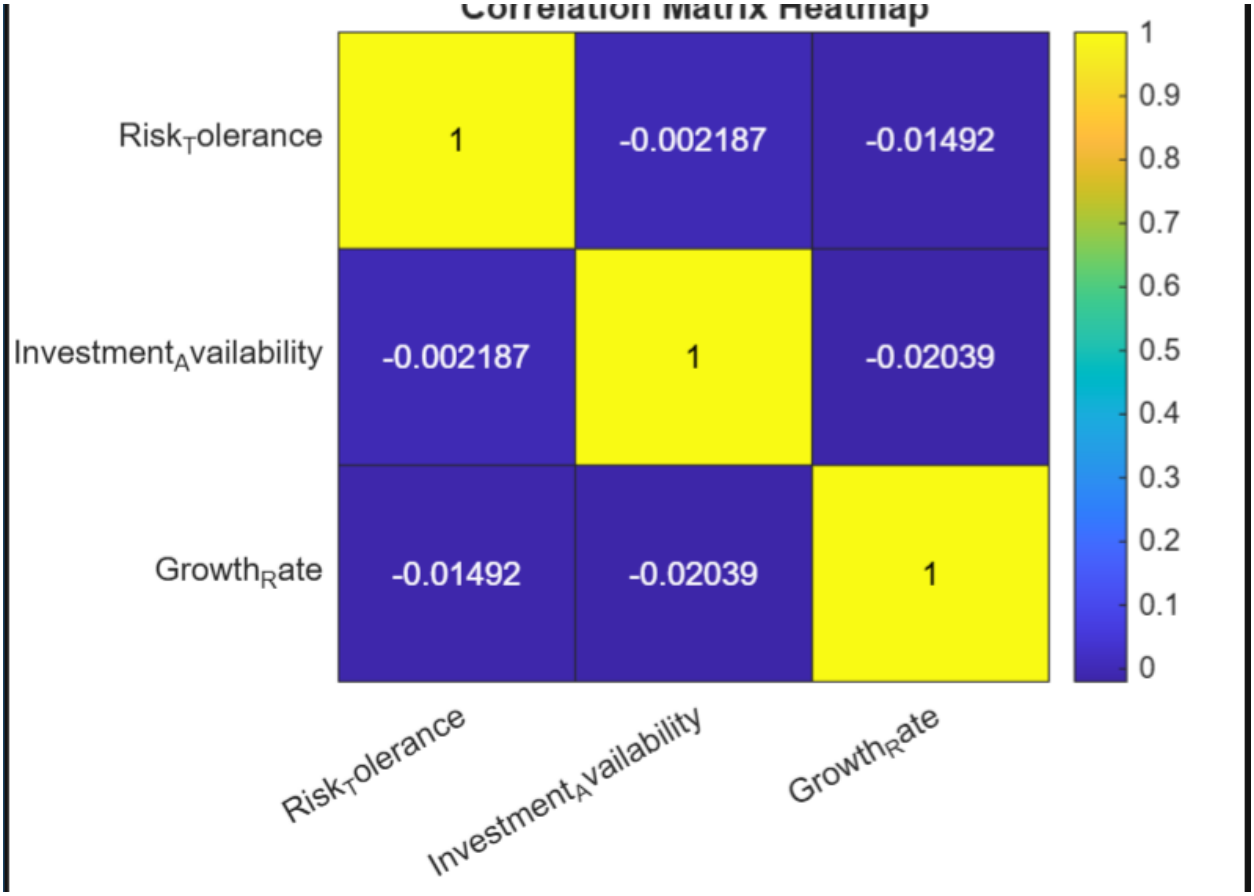


The correlation matrix heatmap below shows the strength and direction of linear relationships among several key variables, including risk tolerance, investment availability, growth rate, economic reports, market sentiment, and two additional key

features. The results reveal that none of the variables exhibit strong correlations with one another. Notably, risk tolerance shows negligible correlations with all other variables, including growth rate (r = -0.015), investment availability (r = -0.0022), and economic reports (r = -0.041), suggesting that a firm's willingness to take risks operates independently of these factors. Similarly, investment availability demonstrates weak negative correlations with economic reports (r = -0.058) and key feature 2 (r = -0.051), implying only a minor inverse relationship. The growth rate variable also lacks substantial association with the rest of the features, with all correlation coefficients hovering close to zero. These findings indicate an overall absence of strong linear interdependencies among the variables, suggesting that each factor may influence entrepreneurial decisions in distinct, potentially non-linear or context-specific ways. The lack of multicollinearity also supports the validity of these features being analyzed separately in further predictive or inferential modeling.







The analysis of the relationship between risk tolerance and growth rate among firms reveals no statistically significant linear association. A Pearson correlation coefficient of

approximately -0.015 indicates an extremely weak and effectively negligible negative correlation between the two variables. This finding suggests that firms with higher risk tolerance do not consistently experience higher growth rates, nor do firms with lower risk tolerance demonstrate slower growth. Visualizations using binned growth rate intervals and corresponding average risk tolerance levels further support this conclusion, showing only minor, inconsistent fluctuations without any discernible trend. These results imply that risk tolerance may not be a primary driver of firm growth within this dataset and may instead be influenced by other factors such as industry characteristics, innovation capacity, or market dynamics. Consequently, the relationship between risk behavior and growth appears to be more complex and potentially moderated by external variables not captured solely by a direct correlation.

Since higher risk tolerance is not associated with higher growth rate in this dataset, we cannot conclude that if becoming an entrepreneur or becoming a worker is better and help the agents receives higher return (worker receives return from saving and entrepreneur receives return from project investment) in the future.