Problem Set 1

Question 1 (10%): Consider the time series model

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 c_{t-1} + \epsilon_t \ ,$$
$$c_t = \beta_0 + \beta_1 c_{t-1} + \beta_2 c_{t-2} + \beta_3 y_{t-1} + \nu_t \ ,$$

where $\xi_t \sim$ i.i.d $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$, with $\xi_t = \begin{bmatrix} \epsilon_t \\ \nu_t \end{bmatrix}, \mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, and $\mathbf{\Sigma} = \begin{bmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\nu^2 \end{bmatrix}$.

(a) Write the time series model in matrix form, with all matrices and vectors explicitly written out.

The time series model in matrix form is:

$$\begin{bmatrix} Y_t \\ C_t \\ C_{t-1} \end{bmatrix} = \begin{bmatrix} \alpha_0 \\ \beta_0 \\ 0 \end{bmatrix} + \begin{bmatrix} Y_{t-1} \\ C_{t-1} \\ C_{t-2} \end{bmatrix} \begin{bmatrix} \alpha_1 & \alpha_2 & 0 \\ \beta_3 & \beta_1 & \beta_2 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ v_t \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

(b) Convert the model to an AR(1) in matrix form:

The AR(1) model has format as:

$$\text{AR(1):} \qquad z_t = Az_{t-1} + \mathrm{C}\xi_t$$

From the matrix in part (a), we define A and C, and z_t, z_{t-1}, ξ_t as:

$$\mathrm{A} = \begin{bmatrix} \alpha_1 & \alpha_2 & 0 \\ \beta_3 & \beta_1 & \beta_2 \\ 0 & 1 & 0 \end{bmatrix} \quad ; \quad \mathrm{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad ; \quad \xi_t = \begin{bmatrix} \epsilon_t \\ v_t \end{bmatrix} \quad ;$$

$$z_t = \begin{bmatrix} \alpha_0 \\ \beta_0 \\ 0 \end{bmatrix} \quad ; \quad z_{t-1} = \begin{bmatrix} Y_{t-1} \\ C_{t-1} \\ C_{t-2} \end{bmatrix}$$

Therefore, the AR(1) model in matrix form is:

$$\text{AR(1):} \quad \begin{bmatrix} \alpha_0 \\ \beta_0 \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & 0 \\ \beta_3 & \beta_1 & \beta_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ C_{t-1} \\ C_{t-2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_t \\ v_t \end{bmatrix}$$

Question 2 (10%): Consider the Markov Diagram summarizing the Markov Chain for borrower classification

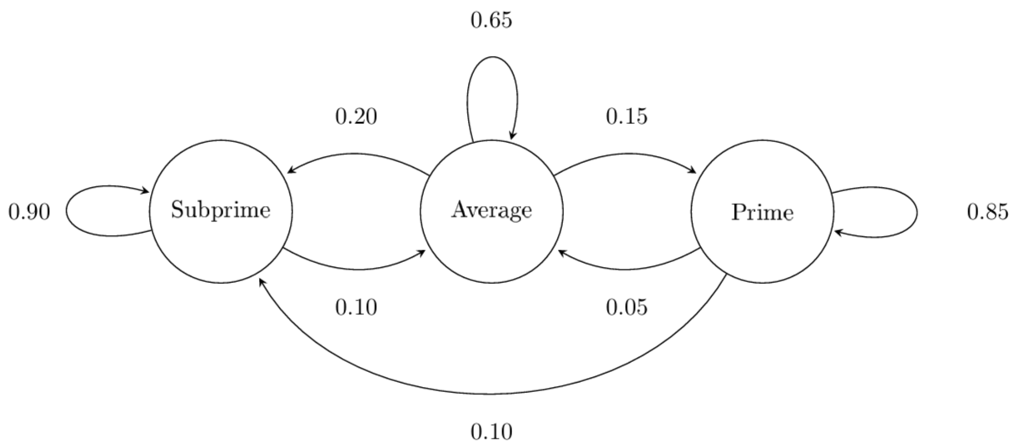


Figure 1: Markov Chain for Borrower Classification

(a) Construct and report the transition matrix and explain whether the transition matrix is valid or not.

Construct and report the transition matrix above:

1. The first state is "Subprime", the probability that it remains its position is 0.90, the probability that it can change to the next state "Average" is 0.10, and the probability it changes to the last state "Prime" is 0.
2. The second state "Average", the probability that it stays is 0.65, the probability that it changes to "Prime" is 0.15 and changes to "Subprime" is 0.20.
3. The third state is "Subprime", the probability that it stays is 0.85, the probability that it changes to "Average" is 0.05 and changes to "Subprime" is 0.10.

The transition matrix is:

$$\begin{bmatrix} 0.90 & 0.10 & 0 \\ 0.65 & 0.15 & 0.20 \\ 0.10 & 0.05 & 0.85 \end{bmatrix}$$

- No probability in this matrix is less than 0 and sum of each row (corresponding to each stage) is 1; therefore, this matrix is valid.

(b) Explain the Markov Diagram as if you were talking to a relative who is neither an economist nor financially literate.

- The Markov Diagram represents how borrowers' credit classification changes over time. There are three levels of credit classification which are Subprime (or poor credit classification), Average, and Prime (or excellent credit classification). The diagram shows the possibility that the credit levels can change to other states, the number of each arrow shows the transition probabilities.

1. In Subprime state (poor credit classification), the borrowers have:
    - 90% chance to stay in the current state Subprime
    - 10% chance to change to the next state Average
    - 0% chance to jump directly to Prime state
2. In Average state, the borrowers have:
    - 65% chance to stay in the current state Average
    - 15% chance to change to the next state Prime
    - 20% chance to drop to the previous state Subprime
3. In Prime state (excellent credit classification), the borrowers have:
    - 85% chance to stay in the current state Prime
    - 5% chance to drop to the previous state Average
    - 10% chance to drop directly to the first state Subprime

- The probability of each arrow represents the likelihood of changing in state of each credit classification to next periods or previous periods, depending on the current state of credit.

Question 3 (20%): Consider the AR(1) process

$$y_t = 0.5 + \gamma_1 y_{t-1} + \varepsilon_t \ ,$$

where $\varepsilon_t \sim$ i.i.d N(0,1) is white noise

(a) Write the pseudo-code for Rouwenhorst's Method to discretize the AR(1) process.

Inputs:

- γ_1 : AR(1) coefficient (persistence parameter)
- μ : unconditional mean of AR(1) process
- σ_ε : standard deviation of the white noise term (value 0.5 in the AR(1) process)
- N : number of state

Outputs:

- y : vector of length N which contains the states of Markov Chain
- Pn : N×N transition probability matrix, in which P[i,j] is the probability of moving from state i to state j

We have:

- p = q = $\frac{1-\gamma_1}{2}$; $\sigma_y = \frac{\sigma_\varepsilon}{\sqrt{1-\gamma_1^2}}$

Therefore:

- N = 2, we have the 2×2 matrix:

$$P = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

- N > 2, we have the N×N matrix:

$$P_N = p\begin{bmatrix} P_{n-1} & 0 \\ 0^T & 0 \end{bmatrix} + (1-p)\begin{bmatrix} 0 & P_{n-1} \\ 0 & 0^T \end{bmatrix} + q\begin{bmatrix} 0 & 0^T \\ 0 & P_{n-1} \end{bmatrix} + (1-q)\begin{bmatrix} 0^T & 0 \\ P_{n-1} & 0 \end{bmatrix}$$

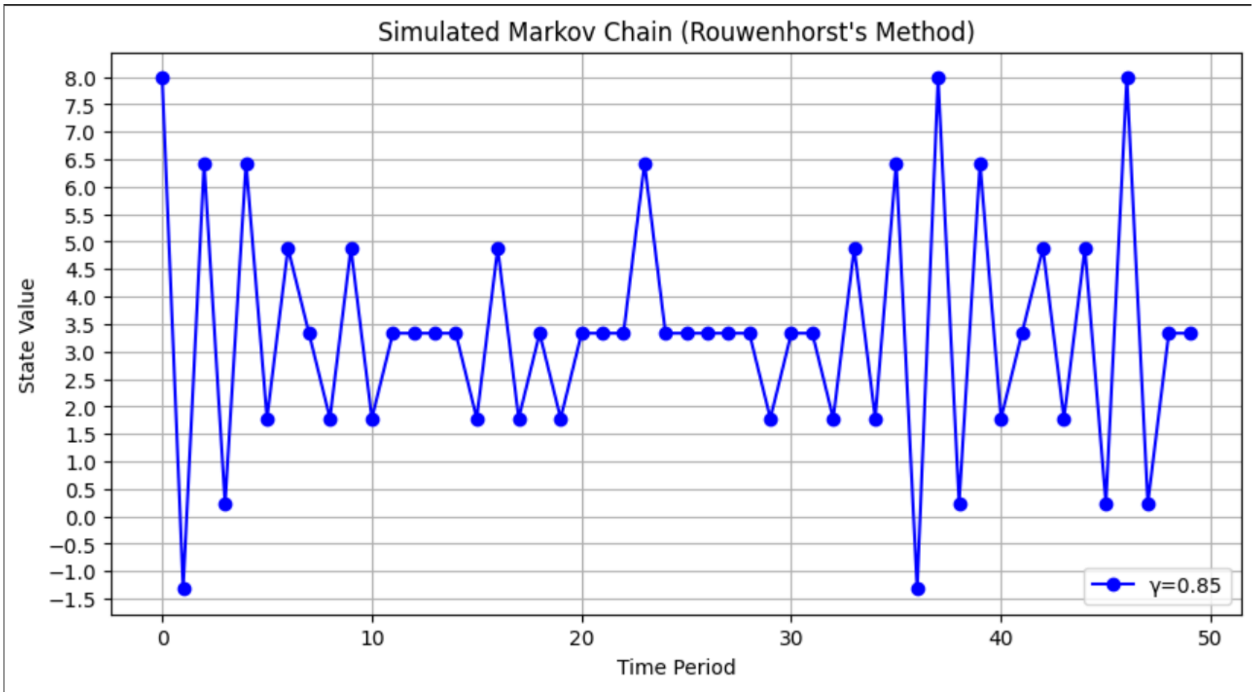
(b) Set $\gamma_1 = 0.85$. Discretize the AR(1) process into a 7-state Markov Chain using Rouwenhorst's Method. Report the transition matrix and state vector.

Here is the result of this question (code is submitted with the PDF file)

```
Min value:  -1.3165722164194378
Max value:  7.983238883086104
State vector:
[-1.31657222  0.2333963   1.78336482  3.33333333  4.88330185  6.43327037
  7.98323888]
Trans Matrix:
[[1.77978516e-07 1.31704102e-05 4.06087646e-04 6.67788574e-03
  6.17704431e-02 3.04734186e-01 6.26398049e-01]
 [2.19506836e-06 1.35540527e-04 3.34991821e-03 4.14510205e-02
  2.57284098e-01 6.46988197e-01 5.07890310e-02]
 [2.70725098e-05 1.33996729e-03 2.49249353e-02 2.07171636e-01
  6.59504720e-01 1.02913639e-01 4.11802954e-03]
 [3.33894287e-04 1.24353062e-02 1.55378727e-01 6.63704146e-01
  1.55378727e-01 1.24353062e-02 3.33894287e-04]
 [4.11802954e-03 1.02913639e-01 6.59504720e-01 2.07171636e-01
  2.49249353e-02 1.33996729e-03 2.70725098e-05]
 [5.07890310e-02 6.46988197e-01 2.57284098e-01 4.14510205e-02
  3.34991821e-03 1.35540527e-04 2.19506836e-06]
 [6.26398049e-01 3.04734186e-01 6.17704431e-02 6.67788574e-03
  4.06087646e-04 1.31704102e-05 1.77978516e-07]]
Row Sums: [1. 1. 1. 1. 1. 1. 1.]
All Rows Sum to 1: True
Contains Negative Values: False
```

(c) Set the seed for simulation to 2025. Draw the initial states from the state vector, assuming the initial unconditional distribution is uniform. Simulate and plot the Markov Chain in (b) for 50 periods.

Here is the result of this question (code is submitted with the PDF file)



- The Markov chain has fluctuation between different state values, showing a mean-reverting behavior, which is expected for an AR(1) process.
- The AR(1) process has been effectively discretized into a finite set of states by the use of Rouwenhorst's approach.

(d) Repeat (c) for $\gamma_1 = 0.75, 0.85, 0.95, 0.99$ and comment on your findings. Ensure the seed is the same as in (c) and plot the simulations in one graph.

```
Rho:  0.75
Min value:  -1.7032803990902057
Max value:  5.703280399090206
Row Sums: [1. 1. 1. 1. 1. 1. 1.]
All Rows Sum to 1: True
Contains Negative Values: False
------------------------------
Rho:  0.85
Min value:  -1.3165722164194378
Max value:  7.983238883086104
Row Sums: [1. 1. 1. 1. 1. 1. 1.]
All Rows Sum to 1: True
Contains Negative Values: False
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```

```
Rho:  0.95
Min value:  2.155354594472631
Max value:  17.844645405527352
Row Sums: [1. 1. 1. 1. 1. 1. 1.]
All Rows Sum to 1: True
Contains Negative Values: False
------------------------------
Rho:  0.99
Min value:  32.63602759480299
Max value:  67.36397240519693
Row Sums: [1. 1. 1. 1. 1. 1. 1.]
All Rows Sum to 1: True
Contains Negative Values: False
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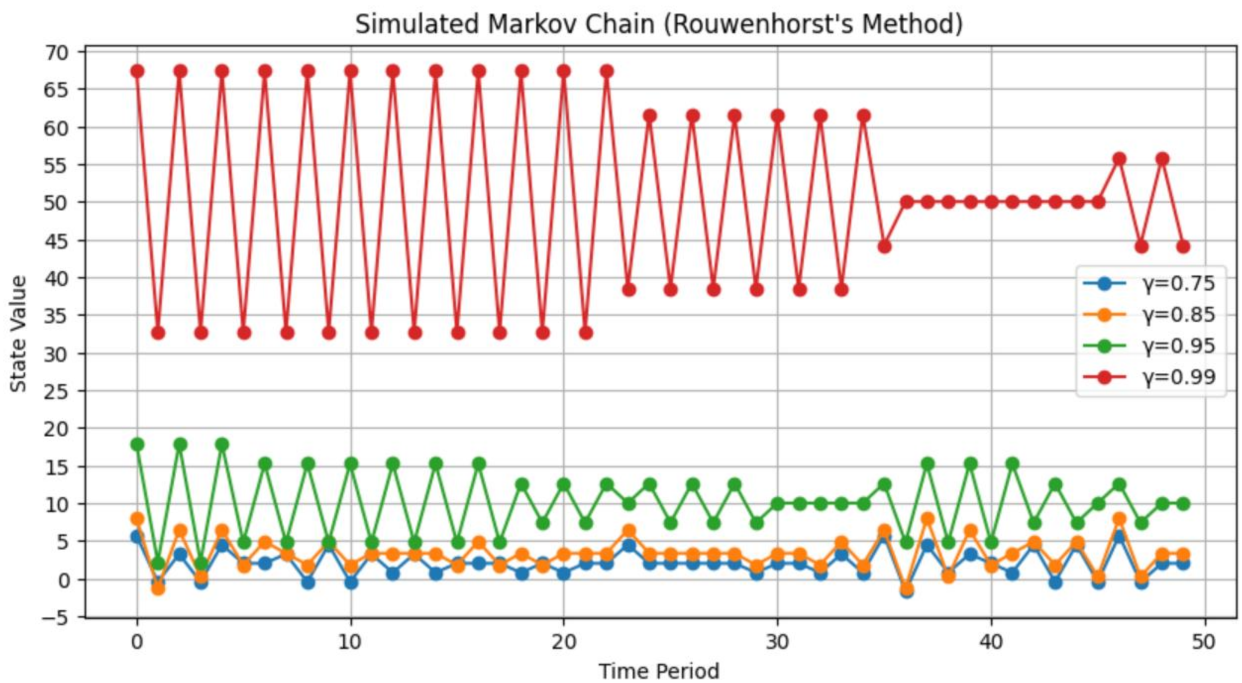
Here is the result of this question (code is submitted with the PDF file)



The dispersion and durability of the state values clearly show the impact of varying values of γ.

- γ = 0.75 (blue): The process returns to the mean rapidly, showing lesser persistence, and the variations are somewhat modest
- γ = 0.85 (orange) exhibits considerable persistence while still mean-reverting within a tolerable range
- γ = 0.95 (green): There is a greater degree of persistence, since the state values stay high or low for longer before changing
- γ = 0.99 (red): The state values take a lot longer to return to the mean, indicating extreme persistence. Long-lasting deviations result from the values' apparent strong autocorrelation

Question 4 (15%): Vietnam's military regularly ensures that its tanks are combat-ready and may choose to modernize its forces when needed. Tanks have an effective age, z_t, which depends on the number of years since the manufacturing date, the number of years in active service, and measures of its utilization. Every year, the military must decide whether to maintain a tank or buy a new one.

(a) The per-period utility function of the military.

We have:

- Operation cost of a tank is $\boldsymbol{c(z_t)}$
- Maintenance cost of a tank is $\boldsymbol{a(z_t)}$
- Cost in event a tank breakdown is $\boldsymbol{b(z_t)}$
- Cost of a new tank is $\boldsymbol{D_t + c(z_t)}$

Assume age of a new tank is $\boldsymbol{z_0}$, and probability of the event a tank breakdown is $\boldsymbol{p(z_t)}$

Define d_t, in which $d_t \in \{0; 1\}$, is whether the military buys the new tank ($d_t = 1$) or maintains the current tanks ($d_t = 0$)

The military will minimize the cost for tanks; therefore, the utility function will be the negative value of costs. The military has two options: (1) they will maintain the current tanks, which leads to the increase in maintenance cost and cost in the event a tank breakdown ($d_t = 0$); or (2) they will buy the new tank, which leads to the increase in cost of new tank ($d_t = 1$)

The utility function if the military chooses to maintain the current tanks is:

$$U_{maintain} = -c(z_t) = -a(z_t) - p(z_t).b(z_t)$$

The utility function if the military chooses to buy the new tank is:

$$U_{new} = -D_t - c(z_0)$$

Therefore, the per-period utility function of the military is:

$$U = \begin{cases} -a(z_t) - p(z_t).b(z_t)\ , & if\ d_t = 0 \\ -D_t - c(z_0) & if\ d_t = 1 \end{cases}$$

(b) The Bellman Equation that describes the military's optimization problem, modeling ϵ_t as i.i.d. taste shocks

- The military has two choices which are (1) maintain the current tanks $d_t = 0$ and (2) buy the new tanks $d_t = 1$
- The states are the effective age of the tank z_t and the taste shocks ϵ_t
- The transition of the effective age from z_t to z_{t+1} and taste shocks from ϵ_t to ϵ_{t+1}

The Bellman Equation is:

$$V(z_t, \epsilon_t) = max_{d_t \in \{0;1\}}\{u(z_t, d_t) + \epsilon_t(d_t) + \beta E(V(z_{t+1}, \epsilon_{t+1})|z_t, d_t)\}$$

In which:

β is the discount factor

$E(V(z_{t+1}, \epsilon_{t+1})$ is the expected value that the military expects to take over

1. If the military decides to maintain the current tanks ($d_t = 0$)
   - The cost of this decision is $-c(z_t) = -a(z_t) - p(z_t).b(z_t)$
   - $d_t = 0 \ \rightarrow \epsilon_t(0)$
   - The military's expected value is $\beta E[V(z_{t+1}, \epsilon_{t+1})]$, the effective age of the current tank is z_{t+1} and the taste shock is ϵ_{t+1} in maintenance decision
2. If the military decides to buy the new tanks ($d_t = 1$)
   - The cost of this decision is $-D_t \ - c(z_0)$
   - $d_t = 1 \ \rightarrow \epsilon_t(1)$

- The military's expected value is $\beta E[V(z_0, \epsilon_{t+1})]$, the effective age of a new tank is z_0 and the taste shock now is ϵ_{t+1}

Therefore, the Bellman Equation that describes the military's optimization problem is:

$$V(z_t, \epsilon_t) = max\{-c(z_t) + \epsilon_t(0) + \beta E\big(V(z_{t+1}, \epsilon_{t+1})\big) - D_t - c(z_0) + \epsilon_t(1) + \beta E\big(V(z_0, \epsilon_{t+1})\big)\}$$

(c) In words, describe the transition probabilities for z_t

From part (b), the state is the current effective age of the tank, and the control is the decision to maintain or buy the new tank

The z_t will transit to z_{t+1} (the change in state as known as the change in effective age of tank)

1. If the military decides to maintain the current tank $d_t = 0$
   - In this case, the effective age of the current tank will increase an amount which is equal to Δz; therefore, the effective age of the current tank increases to $z_{t+1} = z_t + \Delta z$
   - The probability that the effective age increases from z_t to z_{t+1} given that the $d_t = 0$ is equal to 1 => if the military decides to maintain the current tank, the effective age z_t(the state) will increase in a period and depends on both state z_t and control d_t (decision to maintain)

$$P(z_{t+1} = z_t + \Delta z \mid z_t, d_t = 0) = 1$$

2. If the military decides to buy the new tank $d_t = 1$
   - In this case, the effective age of the new tank will be z_0; therefore, the effective age of the new tank is $z_{t+1} = z_0$
   - The probability that the effective age will be equal to z_0 given that $d_t = 1$ is equal to 1 because the military chose to buy the new tank $d_t = 1$ and the transition probabilities for z_t depends on both state z_t and control d_t (decision to buy new one)

$$P(z_{t+1} = z_0 \mid z_t, d_t = 1) = 1$$

Now we have two states $z_{t+1} = z_t + \Delta z$ and $z_{t+1} = z_0$, and control are two choices $d_t = 0$ and $d_t = 1$. As proved above, the transition probabilities for z_t depends on both states and control factors. When the z_t reaches its maximum value, the tank is completely worn down => the tank cannot be maintained and be forced to replace by a new one => the probability of maintenance will be zero; therefore, the probabilities of transition of z_t to any states $z_{t+1} = z_t + \Delta z$ and $z_{t+1} = z_0$ in any control $d_t = 0$ and $d_t = 1$ is zero.

Conclusion: the transition probabilities for z_t depends on both states and controls, and the probabilities are equal to zero.

Question 5 (45%): Study about the context of economic growth and development of Vietnam.

(a) Background of the economy

Vietnam's economic development since 1986 has been marked by its transition from a centrally planned economy to a socialist-oriented market economy with the beginning of Doi Moi reforms. This transition has transformed Vietnam from one of the world's poorest countries to a lower middle-income country, this leads to the rapid industrialization, trade liberalization, and integration into global markets. And Vietnam is now one of the most dynamic countries in East Asia and the Pacific.

The history background of economic development in Vietnam can be divided into three stages. To begin with it was from 1986 to 1995 - the Doi Moi Reforms. The innovative policies of this period focused primarily on Agricultural reforms, Price and Trade liberalization, Foreign Direct Investment (FDI) and development of Private Sector besides the growth of State-owned/ Public Sector, and Monetary & Fiscal Reforms. The results of Reforms were significant with accelerated economic growth, remarkable growth in agricultural sector that made Vietnam become the world's second-largest rice exporter by the 1990s, and population's living standards improved overtime. The second stage of Vietnam economic development is from 1996 to 2010. In this period, Vietnam economy underwent two financial crises which were Asian Financial Crisis in 1997 – 1998 and Global Financial Crisis in 2008 – 2009. During this time, Vietnam actively participated in many trade agreements: Vietnam joined ASEAN in 1995, Vietnam and the US signed trade agreement in 2000, the Vietnam – US Bilateral Trade Agreement (BTA) signed in 2000, Vietnam joined the World Trade Organization (WTO) in 2007, etc. These agreements played an important role in boosting export of Vietnam, promote GDP growth, FDI inflows surged, helped Vietnam emerged technology and manufacturing, and especially, the stock market was established in 2000 in Ho Chi Minh City. However, due to the fluctuations and difficulties of the global economy during this period, especially when Vietnam underwent two serious financial crises, the national economy still had to cope with several challenge. The last period is from 2011 to present. At the beginning of this period, the Vietnam economy had a slowdown, especially in 2011, Vietnam recorded the highest inflation ever at nearly 19%. GDP at that time was slowed but stable, inflation gradually under control. From 2016, the Comprehensive and Progressive Agreement for Trans-Pacific Partnership (CPTPP) and EU-Vietnam Free Trade Agreement (EVFTA) opened more markets, which helped Vietnam expand the trade markets around the world. From 2018, the trade war happened but Vietnam still had stable growth and promoted digital transformation in many fields, developed high-tech industries, and focused on environment problems & climate change. From 2020 to 2022, despite the COVID-19 pandemic, the Vietnam economy still recorded the stable growth.

Some factors can be considered to have a deep insight into Vietnam economy development according to Kaldor's Facts.

1. Steady Growth in Output: exhibit long-term growth in total output, or Gross Domestic Product (GDP)

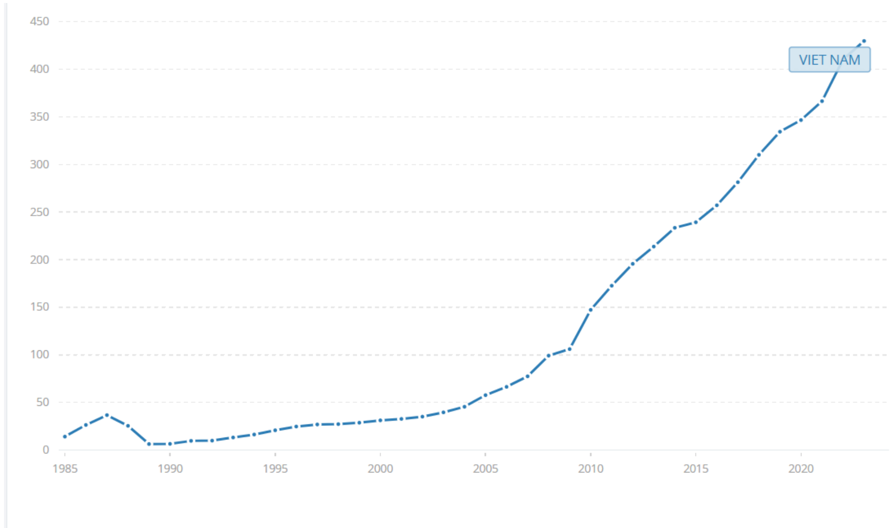


Figure 1 - GDP (current US$) - Viet Nam from 1985 to 2023 (World Bank)

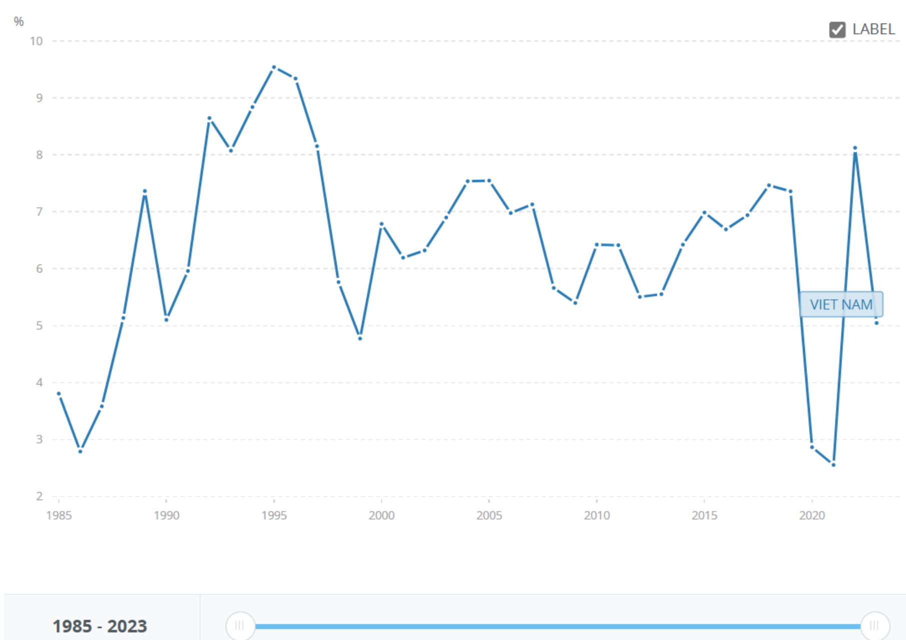


Figure 2 - GDP growth (annual %) - Viet Nam from 1985 to 2023 (World Bank)

- The GDP of Vietnam has stable development overtime, but the GDP growth rate is fluctuating over the long term.

2. Growth in Capital Stock: suggesting that ongoing investment and accumulation of capital as a driver of economic growth

According to Vietstock Report (2024), Vietnam's capital markets have seen significant growth over the past decade. By encouraging effective capital allocation, robust capital market expansion would also produce favorable externalities for the whole economy. Vietnam has the potential to increase its economic growth by leveraging money from foreign markets in addition to making better use of its own resources.

3. Stable Capital/Output Ratio: this ratio tends to be stable over long periods

Vietnam's Incremental Capital Output Ratio (ICOR) decreased from 6.42 in 2016 to 6.08 in 2019, according to statistics from the General Statistics Office. Compared to the ICOR of 6.25 for the 2011–2015 period, the average ICOR during the 2016–2019 period was 6.13. However, the Covid-19 pandemic's detrimental effects caused the ICOR to rise to 14.28 in 2020, and then to 15.54 in 2021. 2020 and 2021 had economic growth rates of 2.91% and 2.58%, respectively. The ICOR dropped to 5.92 in 2022 as a result of the economic recovery and an 8.02% GDP growth rate. Compared to 2020 and 2021, this is a comparatively low level, suggesting that total investment efficacy is increasing.

4. Growth in Labor Productivity: consistent increase in output per worker and growth in productivity

Overall, the labor productivity in Vietnam has a consistent growth. These figures indicate a positive trend in labor productivity.

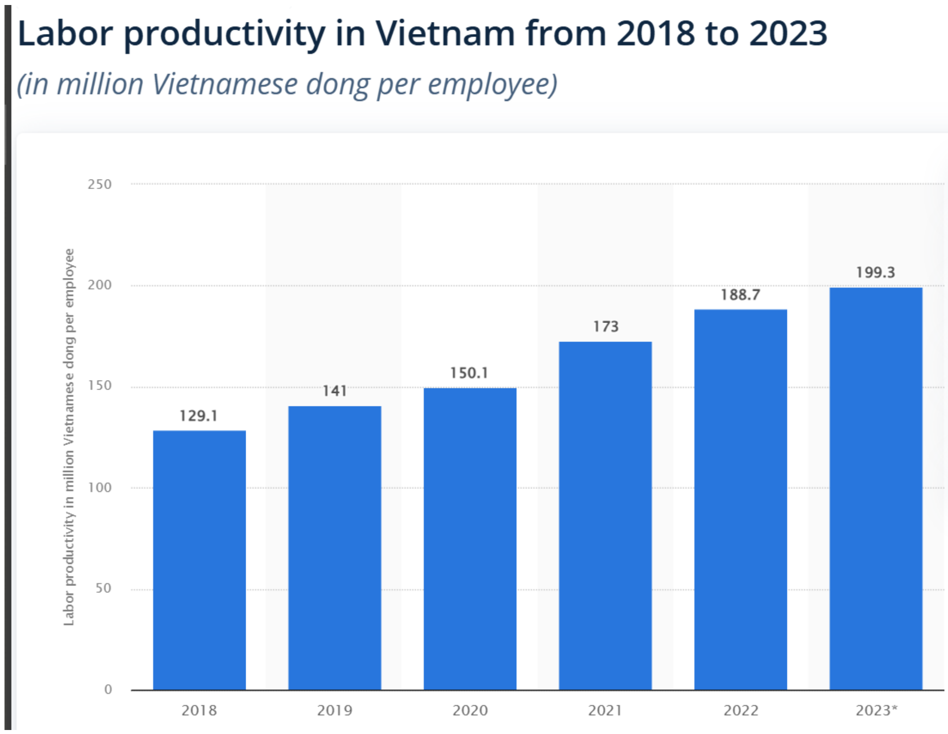


Figure 3 – Labor productivity of Vietnam from 2018 to 2023

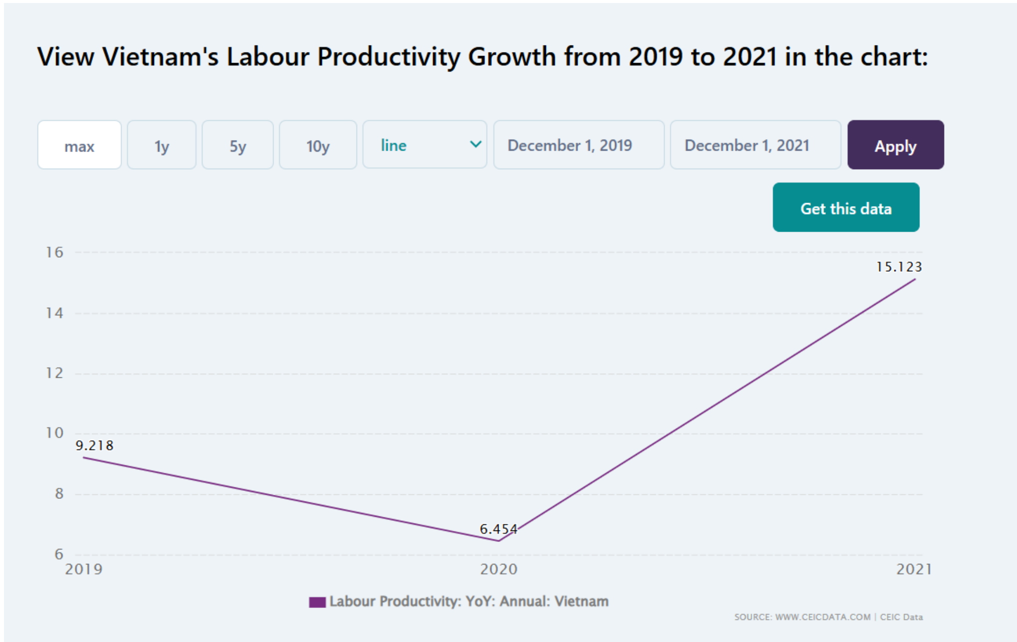


Figure 4 – Labor productivity growth of Vietnam from 2019 to 2021 (CECI Data)

5. Real Wage Growth: tends to increase in the long run

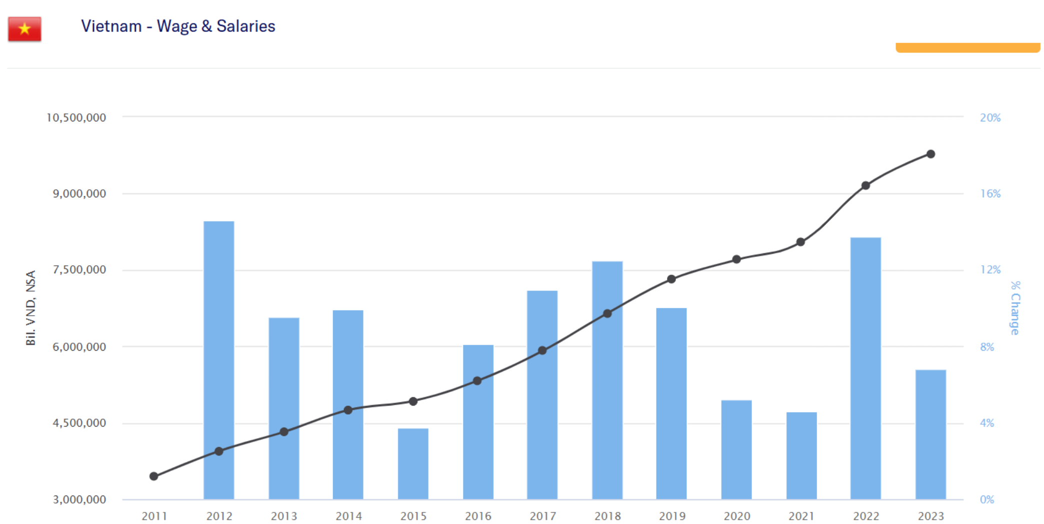


Figure 5 – Wage and Salaries of Vietnam from 2011 to 2023

The real labor wage tends to have a stable increase during the period from 2011 to 2023.

(b) Analysis of growth factors and potential mechanisms that policymakers may target to stimulate or maintain economic growth

Vietnam economy is driven by several growth factors

1. Foreign Direct Investment (FDI)

Vietnam economy benefits a lot from the low labor costs and abundant labor resources, and Vietnam also actively participates in several strong trade agreements to attract Foreign Direct Investment into national economy as a key driven in economic growth and technological growth. FDI of Vietnam is diverse with investment into many industries and professional fields such as high-technology, manufacturing and processing industry, apparel manufacturing, etc. According to the Vietnam General Statistics Office (2025), Vietnam is in the top 15 developing countries for FDI inflows.

However, Vietnam is still facing to the situation that we are over-reliance on FDI to have capital and new technology to develop country. Vietnam needs to address directly these challenges to have solutions, need stronger local supply chains but still maintains FDI attractions.

2. Business Environment and Entrepreneurship (policies, corporate tax, business environment)

   Through economic forums, experts identified the favorable factors for Vietnam's economy need to focus on to have strong growth in 2025. In other words, in anticipation of the change in global supply chains, the government is actively refining and coordinating regulations and the business climate. This could also be an opportunity to broaden the market, participate more deeply in the global supply chain and attract investment when Vietnam actively engaged in free trade agreements (FTAs).

3. Infrastructure, Technology, and Manufacturing Development (including public investment, digital transformation, establishment of special/ concentrated economic zones)

Public investment is considerably increasing in infrastructure such as airports, ports, lands, digital factors, etc. to boost economic development, especially in technology fields. However, the rapid development of technology and digital transformations make a large gap in labor skills for high-technology sectors which requires labor to have more skill sets to adapt with new technologies. Besides, the asynchronous infrastructure development also makes traffic in many areas inconvenient.

The Solow Model analyzes the changes in population growth rate, capital accumulation, and technological progress changes that lead to change in output level of an economy over time. Today, Vietnam economy has a positive change in technological progress which contributed a lot from Foreign Direct Investment (World Bank), and the national gross capital growth has increased over time (as illustrated by figure below).

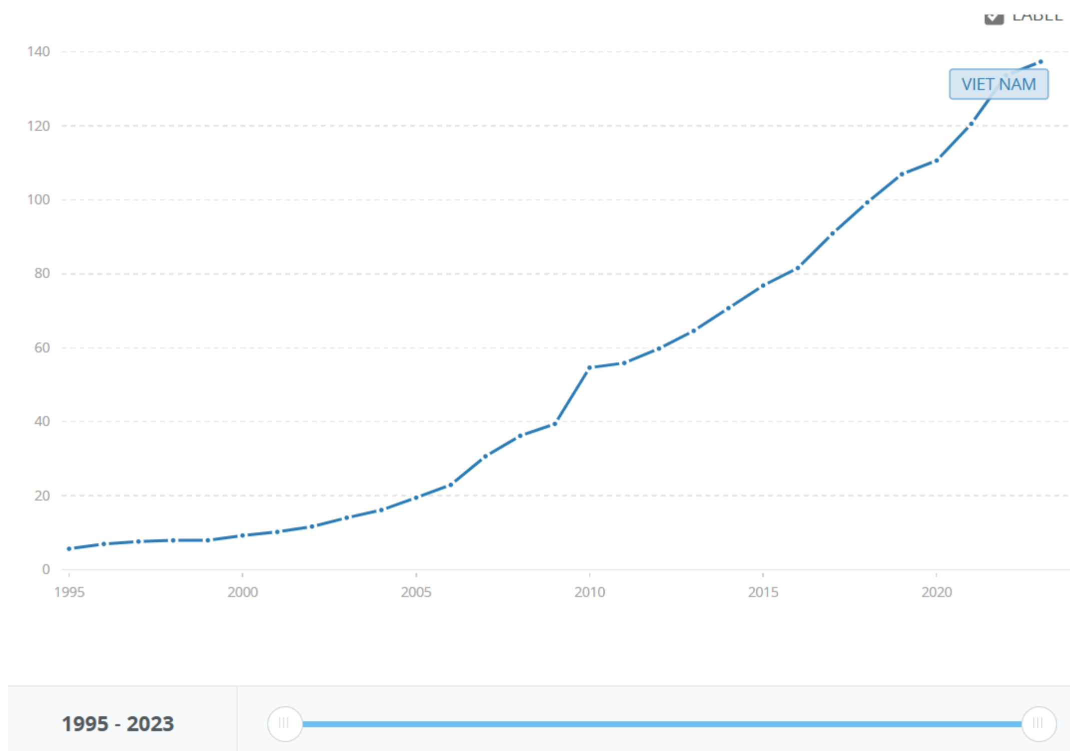


Figure 6 – National Gross Capital Growth of Vietnam (World Bank)

As mentioned in the previous part, all factors in Solow Model that need to boost the increase in output of Vietnam: labor force/ labor productivity, capital accumulation (primarily from high investment rate, FDI inflows, and public investment), and technological change also have positive changes over time (World Bank). Therefore, these changes lead to the increase in output of Vietnam economy. In conclusion, the Solow Model suggests that Vietnam economy has stable development which is primarily driven by capital, labor, and technological changes; and by continuing this orientation, Vietnam can keep leveraging the principles of this model and tend to attain sustained economic expansion and advancement.

4. Environmental Issues and Adaptation Plan with Climate Change

Vietnam is one of the countries most affected by climate change in the world. Today, Vietnam must cope with several serious consequences caused by climate change in both economic and social aspects such as the rising of sea levels, extreme weather, salinity intrusion in agricultural areas, etc. Climate change will have a direct negative impact on Vietnam's economy, especially in the agricultural sector. According to General Statistics Office of Vietnam, as a country with agriculture accounting for a considerable proportion of the economy and agricultural products still account for a significant proportion of export turnover, with traditional consumption markets, climate change is a huge challenge for Vietnam in maintaining stable economic development and ensuring national food security.

Through green programs, international agreements, and government regulations, Vietnam is taking a proactive approach and plan a collective action approach to fight climate change. "The Vietnamese public sector has taken multiple policy commitments to address these threats, though not enough has been done to translate this into practice" (Phan Quang Anh Bui, 2020).

(c) Extend the stochastic growth model to incorporate the mechanisms

In this exercise, I choose Vietnam economy to study about; therefore, in this question, I will use the tax rates and other factors which is related to tax as factor to expand the stochastic growth model to incorporate the mechanisms. In an economy, there are primarily three agents which are the government, the firms, and the household; and each agent has different preferences and constraints.

Let tax rate is $\boldsymbol{\xi_t}$

1. The households:

Firstly, we assume the household preference is U(c, g), in which *c* and *g* are respectively private and public consumption. Let labor input of household is *n*. The tax rate which imposed by the government is exogenous and stochastic

We have the household's budget constraint is:

$$c_t + i_t = (1 - \xi_t^k) r_t k_t^* + (1 - \xi_t^n) w_t^* n_t + \delta \xi_t^k k_t + T_t$$

In which:

- c_t is the household's consumption in a period
- i_t is the investment
- $r_t k_t^*$ is the capital income in time *t*, price of renting capital is *r*, capital stock is k^*
- $w_t^* n_t$ is the labor income in time *t*, w^* is wage rate, n_t is labor input in time *t*
- ξ_t^n is tax rate of labor income in time *t*
- ξ_t^k is tax rate of capital in time *t*
- T_t is the lump-sum transfer payments of government in time *t*
- $\delta \xi_t^k k_t$ is the depreciation allowances of final income, in which the rate of capital depreciation $\delta \in (0, 1]$ is constant
- 1 is the price of consumption and investment

The CRRA form for the household's constraint is:

$$U(c, g) = \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{g_t^{1-\sigma}}{1-\sigma}$$

2. The government:

Let's consider the government's constraints. The government has the right to decide tax and tax rate on other factors in the economy, they may impose taxes on the rental income of other factors to finance their expenditures and make sure it follows a stochastic stream. The tax revenue accounts for a considerable proportion of the government's total revenue. As mentioned in the previous part, T_t is the lump-sum transfer payments of government in time *t*.

Therefore, the transfer to households is:

$$T_t = \xi_t^k r_t k_t + \xi_t^n w_t n_t - \delta \xi_t^k k_t - g_t$$

The government's budget constraint is:

$$T_t + g_t = \xi_t^k r_t k_t + \xi_t^n w_t n_t - \delta \xi_t^k k_t$$

The government's budget is balanced in each period

3. The firms:

We have Cobb-Douglass production function of the firms in intensive form is:

$$y_t = A_t k_t^\alpha$$

The function of conversion of investment and stocks from period capital k_t to the next period k_{t+1} is:

$$k_{t+1} = (1-\delta)k_t + i_t$$

in which the rate of capital depreciation $\delta \in (0, 1]$ is constant

Let the variation of labor productivity is A_t, the AR(1) Model is:

$$\log(A_{t+1}) = \mu + \rho \log(A_t) + \varepsilon_{t+1}$$

From all equations of three factors above, we have the optimization problem equation:

$$\max_{(c_t,k_{t+1})} E_0 \sum_{t=1}^{\infty} \beta_t \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{g_t^{1-\sigma}}{1-\sigma}$$

such that $k_t \geq 0, c_t > 0, n_t = 1$ and $t = 1, 2, \ldots$

Therefore, the recursive problem is (also based on the Bellman Equation):

$$V_t(k_t, A_t) = max_{(c_t,k_{t+1})} \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{(1-g_t)^{1-\sigma}}{1-\sigma} + \beta E_t[V_{t+1}(k_{t+1}, A_{t+1})]$$

(each element of this equation is from all equations which are stated clearly in the previous part)

(d) & (e)

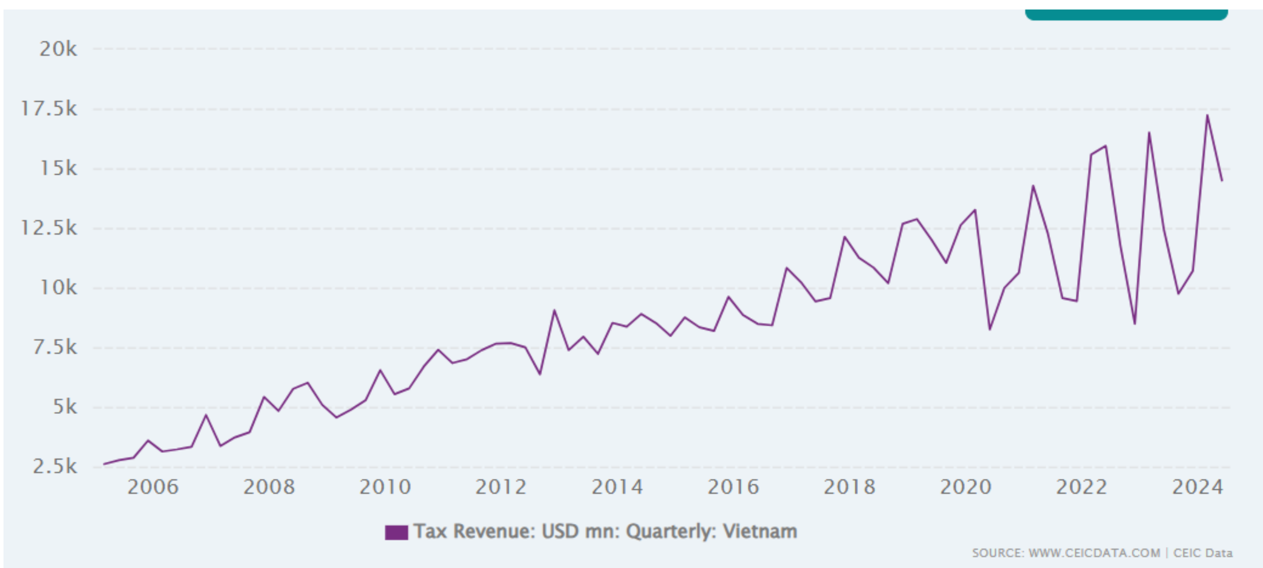


Figure 7 – Tax Revenue of Vietnam from 2005 to 2024 (CEIC Data)

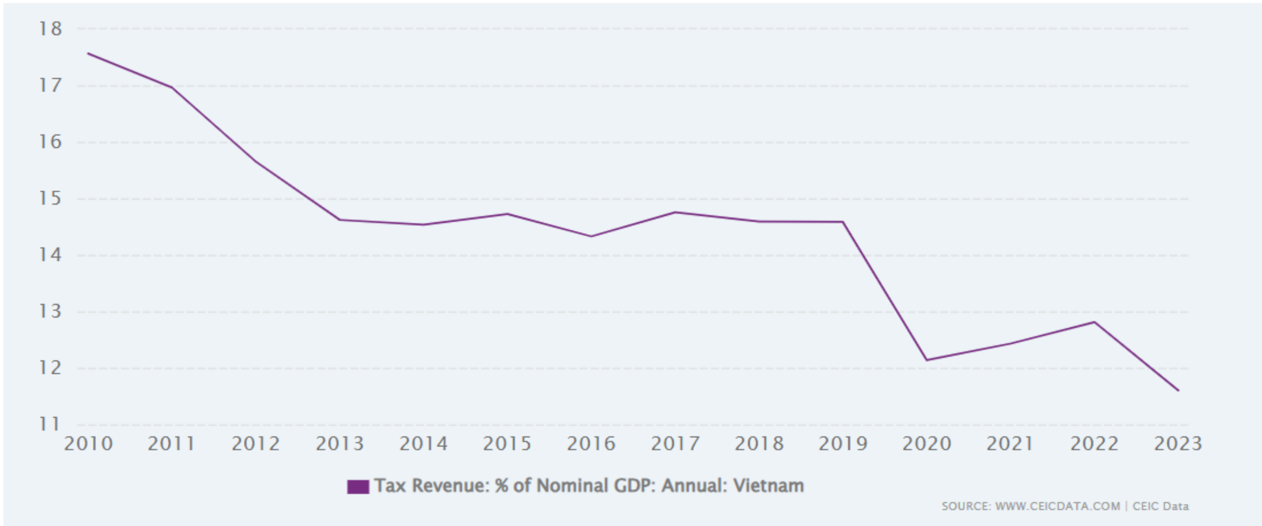


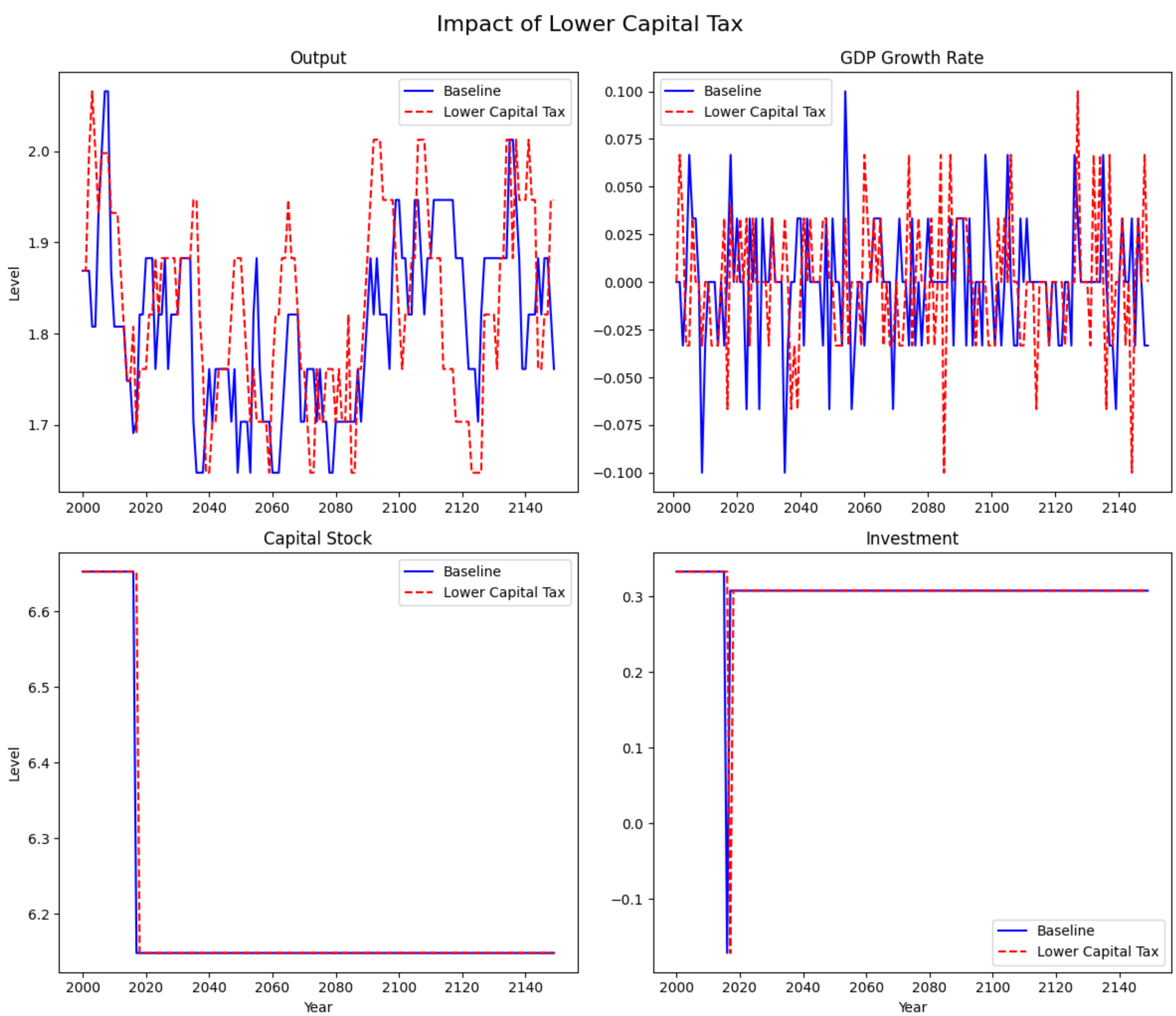
Figure 8 – Proportion of Tax Revenue of Vietnam in GDP (CEIC Data)

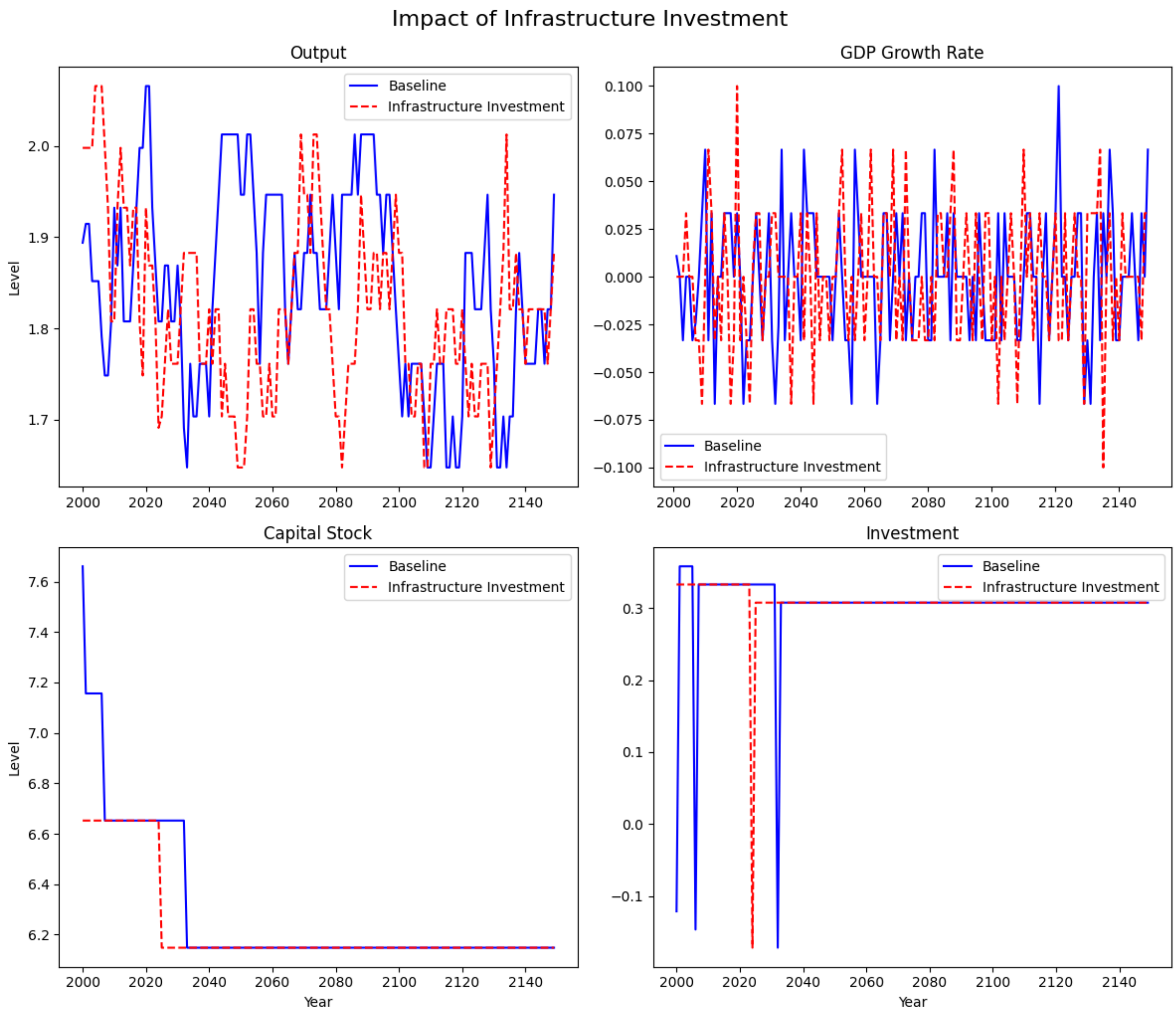
From these figures, it is obvious that the government tax revenue is increasing by the time; however, the proportion of tax revenue in GDP is decreasing. Therefore, the change in tax rate which is imposed by the government plays an important role in economic development and this change can lead to other significant changes in other sectors of the economy.

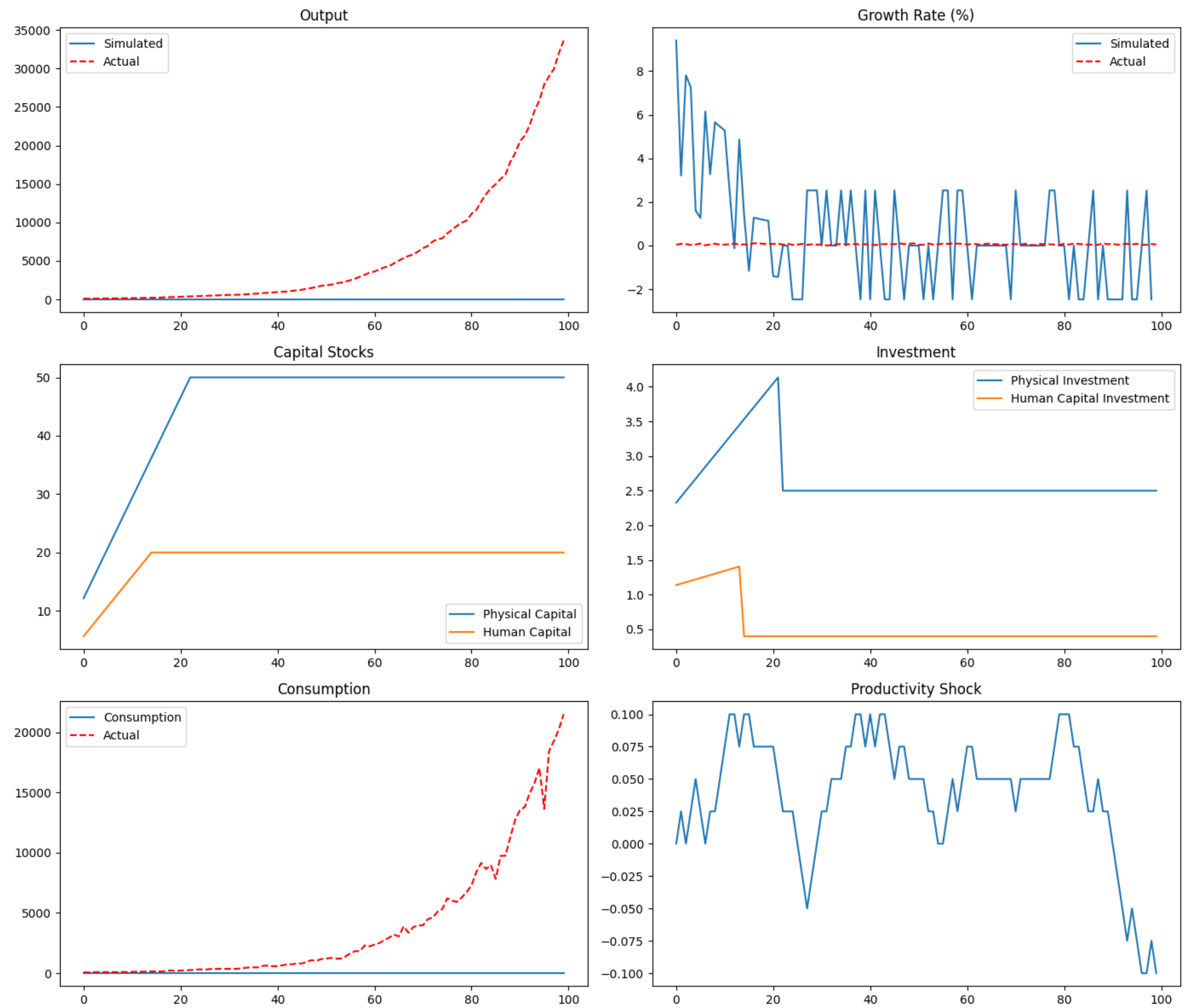
Here is the result of the questions (code is submitted with the PDF file)

- In both policy interventions, the GDP growth rate and output show significant fluctuations over time.
- The capital stock and investment graphs show a sharp decline at the beginning.
- The baseline and policy scenarios seem to differ very little; more analysis is required to determine whether the policy impact is what was anticipated.
- It seems that reducing capital tax has no effect on long-term patterns in capital stock or production under the reduced capital tax policy. The GDP growth rate changes, however, imply that while the policy may have short-term effects, it has no discernible effect on the trajectory of long-term growth.

- When compared to the baseline, the infrastructure investment strategy clearly demonstrates a change in production and capital stock. Compared to the capital tax policy, the early impact appears to be more noticeable, suggesting a greater short-term influence on growth. It would be helpful to examine why infrastructure investment does not result in long-term capital accumulation, nevertheless, if capital stock and investment decrease over time.




Impact of Infrastructure Investment
Output
Baseline
Infrastructure Investment
Level
GDP Growth Rate
Capital Stock
Investment
Year



Output
Simulated
Actual
Growth Rate (%)
Simulated
Actual
Capital Stocks
Physical Capital
Human Capital
Investment
Physical Investment
Human Capital Investment
Consumption
Consumption
Actual
Productivity Shock

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