FREE VS FAST ADVERSARIAL TRAINING

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ABSTRACT

In this work we propose a comparison between two methods for adversarial training: Free and Fast. We started from [Shafahi et al.] [4] paper, trying to reach their conclusions, in particular the choice of m as a crucial parameter in its algorithm. Then we did the same for [Wong et al.] [5] paper, with regard to the crucial parameter α. We then chose from our experiments the best among these two parameters to enforce a comparison between the two methods presented in the above cited papers, comparing validation and robustness accuracies along with required train times. Finally we analyzed and compared Resnet50 and PreActResNet18 results both in Free and Fast adversarial training. The code is available at `https://github.com/not-a-genius/neuralNetworkExam`.

1 Introduction

Adversarial training is a method for training robust deep neural networks, training on adversarial examples as a defense against adversarial attacks. This is typically considered to be more expensive than standard training due to the necessity of constructing adversarial examples via a first-order method like projected gradient descent (PGD). The papers presented show that it is possible to train empirically robust models using methods no more costly than standard training in practice. These methods show results comparable to train a model against PGD but at lower cost in terms of computational time. The goal of adversarial training is to learn a model which is not only accurate on the data (accuracy on train and test data) but also accurate on adversarially perturbed versions of the data (i.e. accuracy on images perturbed using the PGD attack). This leads to particular attention at the results we obtained after adversarial training experiments, aware of the presence of an acceptable compromise in accuracy, given a large increase in robustness due to the tradeoff between robustness and generalization [1, 2, 3].

1.1 Related Works

Our work is based on "Adversarial training for free!" paper by [Shafahi et al.] [4], that presented an algorithm able to eliminate the overhead cost of generating adversarial examples by recycling the gradient information computed when updating the model parameters. While researching related work in the field we discovered a more recent paper, called "Fast is better than free: Revisiting adversarial training" by [Wong et al.] [5] presenting an improvement of an already known method called FGSM, previously considered ineffective due to what the paper calls "catastrophic overfitting". This failure condition is preventable with the use of random initialization points. Moreover the same paper reported a further acceleration in training even for the Free algorithm, thanks to standard techniques for efficient training, including cyclic learning rate and mixed-precision arithmetic. This caught our interest and we decided to try to replicate the reported results and compare them with our implementation, in order to confirm or deny these thesis.

1.2 Dataset and architecture

We decided to choose CIFAR-10 as the dataset on which our experiments have been conducted. All of them are run on a single NVIDIA GeForce RTX 2070-Super GPU.
The CIFAR-10 dataset consists of 60000 32x32 colour images in 10 classes, with 6000 images per class. There are 50000 training images and 10000 test images. The dataset is divided into five training batches and one test batch, each

with 10000 images. The test batch contains exactly 1000 randomly-selected images from each class. The training batches contain the remaining images in random order, but some training batches may contain more images from one class than another. Between them, the training batches contain exactly 5000 images from each class.

We conducted adversarial training examples using as models ResNet50 and PreActResNet18. Both ResNet50 and PreActResnet18 results have been reported and compared between them, under the same conditions.

2 Adversarial Machine Learning

Adversarial machine learning is a machine learning technique that attempts to fool models by supplying deceptive input (called also adversarial example). Adversarial examples exploit the way artificial intelligence algorithms work to disrupt the behavior of such algorithms. In the past few years, adversarial machine learning has become an active area of research as the role of AI continues to grow in many of the applications we use. There's growing concern that vulnerabilities in machine learning systems can be exploited for malicious purposes. The main idea to deal with adversarial examples is represented by adversarial training; it can be traced back to [6], in which models were hardened by producing adversarial examples and injecting them into training data [4].

This technique can be summarized as follows: given a network f_θ parametrized by θ, a dataset (x_i, y_i), a loss function l and a threat model Δ (where $\delta \in \Delta$ is the single perturbation), the learning problem consists in the following optimization problem:

$$min_\theta \sum_i l(f_\theta(x_i + \delta), y_i)$$

A typical choice for the adversarial model is to take $\Delta = \{\delta : ||\delta||_\infty \leq \epsilon\}$ for some $\epsilon > 0$.

3 PGD attack

The PGD attack is a white-box attack, which means that the attacker has access to the model's weights, giving to the attacker much more power than a black-box attack (in which the model's weights are not known), as he can specifically craft the attack to fool the chosen model without having to rely on transfer attacks, which often results in human-visible perturbations. The key for understanding the PGD attack is to frame the research of an adversarial example as a constrained optimization problem. PGD attempts to find the perturbation that maximizes the loss of a model on a particular input while keeping the size of the perturbation smaller than a specified amount referred to as ϵ. This constraint is usually expressed as the l_2 or l_∞ norm of the perturbation and it is added so that the content of the adversarial example is as similar as possible to the unperturbed sample. The PGD algorithm can be summarised with the 4 steps below (although the attacker is free to apply any optimisation improvements such as momentum, Adam, multiple restarts, etc...):

1. Start from a random perturbation in the l_p ball around a sample;
2. Take a gradient step in the direction of the greatest loss;
3. Project perturbation back into the l_p ball if necessary;
4. Repeat steps 2-3 until convergence.

In our case, adversarial training consists simply of putting the PGD attack inside the training loop and applying a kind of "data augmentation"; in this case, instead of performing random transformations as a preprocessing step, we create specific perturbations that best fool the model and indeed adversarially trained models do exhibit less overfitting when trained on small datasets.

The PGD attack is based on the simplest version of what is called Fast Gradient Sign Method, used to approximate the inner maximization over Δ. This could be seen as a relatively inaccurate approximation of the inner maximization for l_∞ perturbations, and has the following closed form (as shown in [7]):

$$\delta^\star = \epsilon \cdot sign(\nabla_x l(f(x), y)).$$

A better approximation of the inner maximization is to take multiple, smaller FGSM steps of size alpha. When the iteration leaves the threat model, it is projected back to the set Δ. Multiple restarts within the threat model Δ typically improve the approximation of the inner maximization even further. The combination of all these techniques is the PGD attack. The problem of such technique is that the number of gradient computations here is proportional to O(MK) in a single epoch, where M is the size of the dataset and K is the number of steps taken by the PGD adversary. This is K times greater than standard training so adversarial training is typically K times slower than standard training. The pseudo-code of the PGD attack is reported in Algorithm 1.

Algorithm 1 PGD adversarial training for T epochs, given some radius ϵ, adversarial step size α and K PGD steps and a a dataset of size M for a network f_θ

1: **for** $t = 1...T$ **do**
2: **for** $i = 1...M$ **do**
3:   // Perform PGD adversarial attack
4:   $\delta = 0$ // or randomly initialized
5:   **for** $j = 1...K$ **do**
6:    $\delta = \delta + \alpha \cdot sign(\nabla_\delta l(f_\delta(x_i + \delta), y_i))$
7:    $\delta = max(min(\delta, \epsilon), -\epsilon)$
8:   **end for**
9:   $\theta = \theta - \nabla_\theta l(f_\theta(x_i + \delta), y_i)$ // Update model weights with some optimizer, e.g. SGD
10: **end for**
11: **end for**

The execution of K-PGD in [8] took four days on a Titan X (with model WideResNet and dataset CIFAR-10). This led us to consider the reported results [4],[5], obtained using K-PGD adversarial training, without running them directly.

4   DAWNBench improvements

The top submissions to the DAWNBench competition have shown that CIFAR10 (and also ImageNet) classifiers can be trained spending significantly less computational time and at much lower cost than traditional learning methods. There are two general approaches that have an effective impact on the convergence rate and computational speed of standard training. They are:

- **Cyclic Learning Rate**: used for improving convergence and reducing the amount of tuning required when training networks. When a cyclic schedule is used, the number of epochs required for training deep networks is drastically reduced, in particular in the case of a CIFAR10 classifier the convergence is achieved in tens of epochs instead of hundreds.

- **Mixed-precision arithmetic**: this improvement can be used only with newer GPU architectures (which come with tensor cores specifically built for rapid half-precision calculations). The key idea is that using mixed-precision arithmetic when training deep networks can also provide significant speedups for standard training. Moreover, this drastically reduces memory utilization.

Both these techniques are adopted for use in our experiments, allowing us to drastically reduce the number of training epochs as well as the runtime on GPU infrastructure with tensor cores, while using modest amounts of computational resources.

5   Free Adversarial training for free!

Free adversarial training [4] claims to reduce the overhead compared to natural training. The idea behind the improved algorithm is the computation of ascent step by reusing the backward pass needed for the descent step. So the gradient with respect to the network parameters is computed on the backward pass and later on this same backward pass the gradient of the loss with respect to the input image is also computed.
In order to allow multiple adversarial updates on the same image, the training is made on the same minibatch *m* times in a row. Every minibatch is repeated *m* times before switching to the next minibatch ("minibatch replays"), so the number of epochs can be considered as the number of epochs divided by *m*, such that the overall number of training iterations remains constant. Note that perturbations are not reset between minibatches. The value of this parameter is shown to be crucial, since increasing *m* comes with the risk of increasing generalization error revealing the presence of a trade-off: robustness is increased at the cost of validation accuracy on natural images.
The pseudocode of Free is shown in Algorithm 2.

Algorithm 2 "Free" Adversarial Training (Free-m)

Require: Training samples X, perturbation bound ϵ, learning rate τ, hop steps m
1: Initialize θ
2: $\delta \leftarrow 0$
3: **for** $epoch = 1...N_{ep}/m$ **do**
4:      **for** $minibatch\ B \in X$ **do**
5:          **for** $i = 1...m$ **do**
6:              // Update θ with stochastic gradient descent
7:              $g_\theta \leftarrow \mathbb{E}_{(x,y)\in B} \nabla_\theta[l(x+\delta, y, \theta)]$
8:              $g_{adv} \leftarrow \nabla_x l(x+\delta, y, \theta)$
9:              $\theta \leftarrow \theta - \tau g_\theta$
10:             // Use gradients calculated for the minimization step to update δ
11:             $\delta \leftarrow +\epsilon \cdot sign(g_{adv})$
12:             $clip(\delta, -\epsilon, \epsilon)$
13:          **end for**
14:      **end for**
15: **end for**

In the choice of *m* one problem may arise: it's possible that catastrophic forgetting happens, e.g. when all the "informative" images of one class are in the first few minibatches. In this extreme case, we do not see useful examples for most of the epoch, and forgetting may occur.

How much does minibatch replay hurt generalization? To answer the question we followed [4] approach trying different values of *m*, reaching the same results: dropoff in accuracy for small *m*, while we see an inversion in PGD and test accuracy for high values of it. On CIFAR-10 effective robustness can be reached with values of $m \leq 10$, our best results have been obtained with $m = 8$.

It is known that adversarially trained classifiers, e.g. classifiers trained against PGD, have generative behaviours. When an image is perturbed from one class to another, it adopts features that make it "look" like its adopted class, to a human eye. Free trained models exhibit this benefit and also smooth and flattened loss surface. Indeed, standard adversarially traininned classifiers show the property to have a flatten and smooth loss landscape while some defenses work by "masking" the gradients. This techniques makes it difficult to identify adversarial examples using gradient methods, even though adversarial examples remain present. Reference [9] argues that gradient masking adds little security, free training does not operate by masking gradients and uses a rough loss surface.

5.1 Experiments results

In this section, we present the results of our experiments conducted using Free method and varying the number of minibatch replays. We can notice that the train time increase as *m* increases as expected but in the case of $m = 10$ we got values of accuracies lower than $m = 8$.

All experiments using Free adversarial training in this section are carried out with the same number of epochs and epsilon value $\epsilon = 8/255$. We are interested in evaluating the accuracy of the model against the natural images and the accuracy against images perturbed using the PGD attack. In particular, we ran PGD using three different values for the number *K* of iterations, which are $\{10, 20, 50\}$. The number of random restarts used for PGD is 1; all the other hyper-parameters are the same used by [Shafahi et al.] in [4]. The implementation of Free provided by [Wong et al.] in [5] includes also the optimization of cyclic learning rate and mixed-precision.

The following table shows the results when the used model is PreactResNet18:

Minibatch-replay (m)	Accuracy using PreActResNet18				Epoch	Train Time (min)	Avg Epoch Time (sec)
	Natural Images	PGD-10	PGD-20	PGD-50			
$m = 2$	0.895	0.324	0.297	0.290	45	22	29.4
$m = 4$	0.867	0.471	0.457	0.451	45	44	58.2
$m = 8$	0.839	0.498	0.486	0.483	45	88	117
$m = 10$	0.821	0.490	0.482	0.479	45	108	143.4

(a) Train and validation accuracies results.
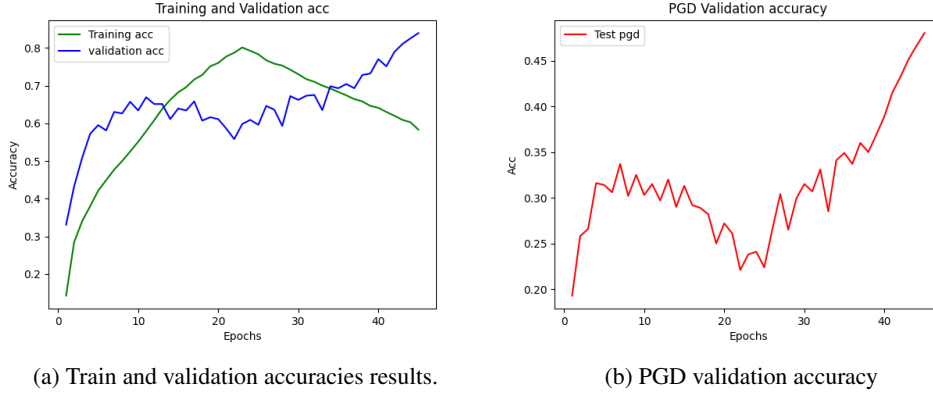
(b) PGD validation accuracy

Figure 1: Free Adversarial training using PreActResnet18 (epsilon = 8/255, m = 8, batch size = 128, epochs = 45)

The following results are related to the training and evaluation based on ResNet50 model instead of PreActResnet18. The number of epochs has been increased to 90 to see the impact on the accuracy in the long run.

Minibatch-replay (m)	Accuracy using ResNet50		Epoch	Train Time (min)	Avg Epoch Time (sec)	Batch size
	Natural Images	PGD-50				
$m = 8$	0.735	0.404	90	118	78.6	128

(a) Train and validation accuracies results.
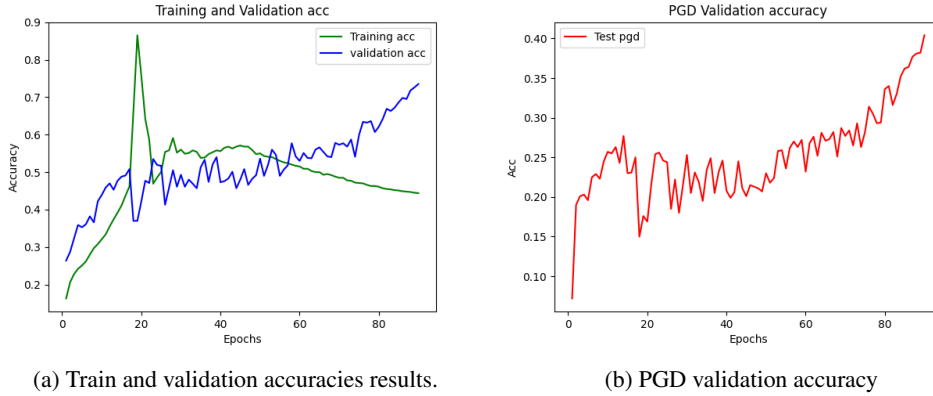
(b) PGD validation accuracy

Figure 2: Free Adversarial training using ResNet50 (epsilon = 8/255, m = 8, batch size = 128, epochs = 90)

6   Fast is better than free

[Wong et al.] in [5] consider it possible to have shorter adversarial training times than the Free method proposed by [Shafahi et al.] in [4], using a modified version of the simple FGSM method, called Fast; if we compare the data from the two original papers [Shafahi et al.] in [4] and [Wong et al.] in [5], we can see that the Free method requires 785 minutes for achieving a good robustness on CIFAR-10 against PGD while the Fast method, in order to achieve the same robustness, requires 12.17 minutes. Similar results about the speed-up in adversarial training time can been observed if the chosen dataset is Imagenet.

A key difference between Fast/FGSM and Free adversarial training, is that in the latter the perturbation from the previous minibatch is a reasonable starting point for the next minibatch, while FGSM can be used as it is, just starting from a non-zero initial perturbation.

So, "Fast" adversarial training is just standard FGSM with random non-zero initialization for the perturbation and this is claimed to be as effective as PGD adversarial training. This idea of non-zero random initial perturbation was already

introduced by [Tramèr et al.] in [10] but, the key difference between this implementation and Fast is that the first one uses a more restricted random initialization and step size, which doesn't give a robust model against PGD attack. So, for this adversarial training, one crucial parameter is the step size α that shows results on par with "Free" ones only choosing values in a particular interval. Indeed, two failure modes arise:

- a step size too small ($\alpha = \epsilon$) would mean a zero-initialized perturbation and lead to a too weak defense
- a step size too large ($\alpha = 2\epsilon$) will lead to a model overfitted, restricted to specific threats ("catastrophic overfitting").

A good compromise for these parameters is shown to be: $\alpha = 1.25$, $\epsilon = 10/255$.
Another key difference is that FGSM does not need to repeat minibatches, but needs two backward passes to compute gradients separately from the perturbation and the model weights. From a computational complexity point of view, this means that an epoch of FGSM is equivalent to two epochs of standard training.

Algorithm 3 FGSM adversarial training for T epochs

Require: Dataset of size M, radius ϵ, K PGS steps, step size α
1: **for** $t = 1...T$ **do**
2:      **for** $i = 1...M$ **do**
3:          // Perform FGSM adversarial attack
4:          $\delta = Uniform(-\epsilon, \epsilon)$
5:          $\delta = \delta + \alpha \cdot sign(\nabla_\delta l(f_\theta(x_i + \delta), y_i))$
6:          $\delta = max(min(\delta, \epsilon), -\epsilon)$
7:          // Update model weights with some optimizer, e.g. SGD
8:          $\theta = \theta - \nabla_\theta l(f_\theta(x_i + \delta), y_i)$
9:      **end for**
10: **end for**

6.1 Experiments results

All experiments using FGSM adversarial training in this section are carried out with the same number of epochs and epsilon value $\epsilon = 8/255$. We ran PGD using three different values for the number K of iterations, which are $\{10, 20, 50\}$. The number of random restarts used for PGD is one while the remaining hyperparameters are those used by [Wong et al.] in [5]. Speedup is determined also by the optimization of cyclic learning rate and mixed-precision.
The following results are related to the training and evaluation based on PreActResnet18 model.

	Accuracy using PreActResNet18							
Step-size	**Natural Images**	**PGD-10**	**PGD-20**	**PGD-50**	**Epoch**	**Train Time (min)**	**Avg Epoch Time (sec)**	**Batch size**
$\alpha = 10$	0.847	0.480	0.463	0.450	45	21	28.8	128
$\alpha = 10$	0.850	0.454	0.438	0.434	45	20	25.8	256
$\alpha = 16$	0.856	0	0	0	45	21	28.2	128
$\alpha = 16$	0.87	0	0	0	45	20	26.4	256

(a) Train and validation accuracies results.
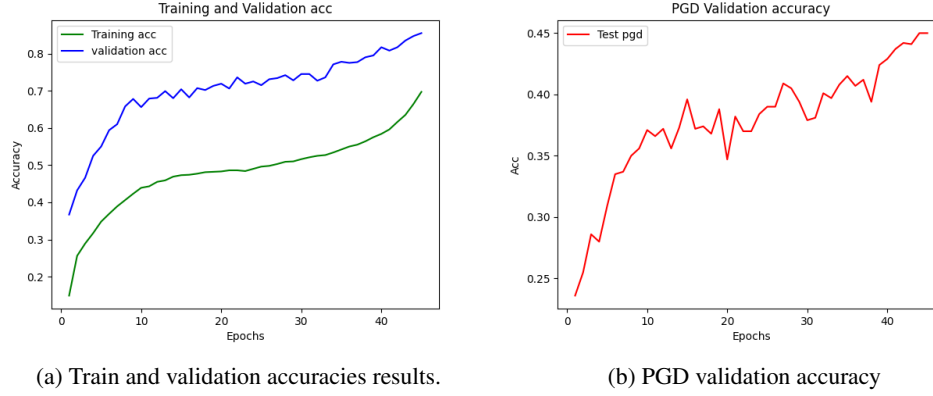
(b) PGD validation accuracy

Figure 3: FGSM Adversarial training using PreActResNet18 (epsilon = 8/255, m = 8, batch size = 128, epochs = 45)

The following results are related to the training and evaluation based on ResNet50 model instead of PreActResnet18. The number of epochs has been increased to 90 to see the impact on the accuracy in the long run.

	Accuracy using Resnet50					
Step-size	Natural Images	PGD-50	Epoch	Train Time (min)	Avg Epoch Time (sec)	Batch size
α = 10/255	0.766	0.391	90	28.18	18.78	128

(a) Train and validation accuracies results.
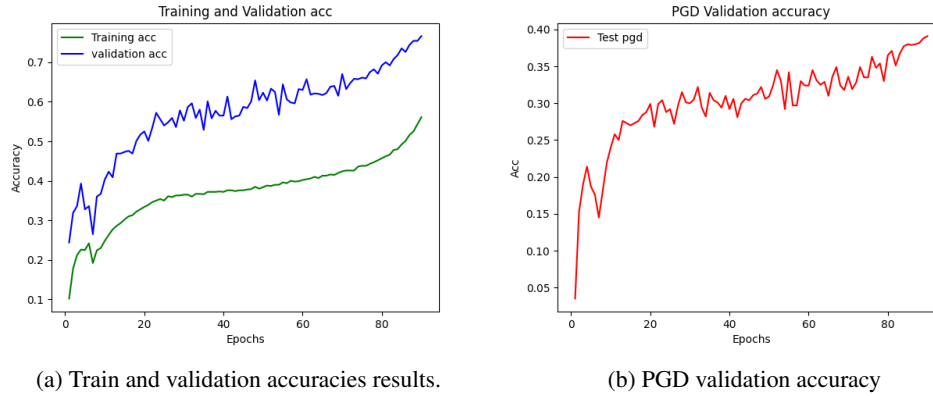
(b) PGD validation accuracy

Figure 4: FGSM Adversarial training using Resnet50 (epsilon = 8/255, m = 8, batch size = 128, epochs = 90)

7   Final Experiments

In order to ease the training and the experiments using different parameters we also developed a single python script that load user preferences (hyperparameters and general parameters) from an editable config file. In "configs.ini" file the user can choose among three different modalities (sequential running of Free adv. training and then fgsm, only Free adv. training or only fast) and change hyperparameters (batch size, epsilon, etc...) along with common parameters (name of the model, output filename, etc...).
Here there is an extract from the script:

```python
def main():

    print("Starting experiments")
    config = parse_config()

    os.chdir('CIFAR10')

```

```
 8   mode = int(config["SCRIPT"]["mode"])
 9   keysFree= [c for c in config["FREE"] ]
10   keysFast= [c for c in config["FAST"] ]
11
12   freeCommand = ["python3 free.py"] +
13     ["--"+k+" "+config["FREE"][k] for k in keysFree]
14   freeCommand = " ".join(freeCommand)
15
16   fastCommand = ["python3 fast.py"] +
17     ["--"+k+" "+config["FAST"][k] for k in keysFast]
18   fastCommand = " ".join(fastCommand)
19
20   if(mode==0): # BOTH FREE AND FAST
21     print("-"*40,"FREE","-"*40)
22     print("Launching",freeCommand)
23     subprocess.call(freeCommand, shell=True)
24
25     print("-"*40,"FAST","-"*40)
26     print("Launching",fastCommand)
27     subprocess.call(fastCommand, shell=True)
28
29   elif(mode==1): # ONLY FREE
30     print("-"*40,"FREE","-"*40)
31     print("Launching",freeCommand)
32     subprocess.call(freeCommand, shell=True)
33
34   elif(mode==2): # ONLY FAST
35     print("-"*40,"FAST","-"*40)
36     print("Launching",fastCommand)
37     subprocess.call(fastCommand, shell=True)
```

Listing 1: main_script.py

7.1 Free vs Fast using PreActResnet18

In this section we compare the best results we obtained from the precedent experiments, then showing the evolution of validation accuracy in these two methods using PreactResnet18 as model.

The following table contains the best *m* and batch size values for Free method, obtained from the different Free experiments compared with the best step size and batch size for FGSM.

		Accuracy using PreactResnet18					
	Minibatch-replays (m) / step size (α)	Natural Images	PGD-50	Epoch	Train Time (min)	Avg Epoch Time (sec)	Batch size
FREE	$m = 8$	0.845	0.485	90	176	117	128
FAST	$\alpha = 10/255$	0.836	0.434	84	43	28	128

(a) Free vs Fast, Validation accuracies.
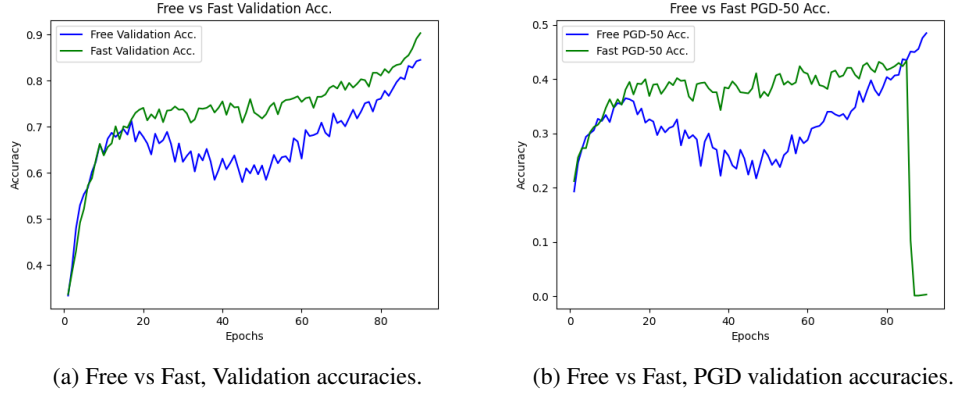
(b) Free vs Fast, PGD validation accuracies.

Figure 5: Comparing results for Adversarial training using PreActResNet18 in Free vs Fast.

As we can see from the previous data, Free can be considered the best solution since, even if there is a substantial difference between the training times associated to the two methods, the great difference between the two values associated to the accuracy against PGD-50 suggests to choose Free as algorithm for adversarial training.

7.2 Free vs Fast using ResNet50

In this section we compare the best results we obtained from the precedent experiments, then showing the evolution of validation accuracy in these two methods when using Resnet50 as model.

The following table contains the best m and batch size values for Free method, obtained from the different Free experiments compared with the best step size and batch size for FGSM.

		Accuracy using ResNet50					
	Minibatch-replays (m) / step size (α)	Natural Images	PGD-50	Epoch	Train Time (min)	Avg Epoch Time (sec)	Batch size
FREE	$m = 8$	0.735	0.404	90	118	79	128
FAST	$\alpha = 10/255$	0.766	0.391	90	29	19	128

(a) Free vs Fast, Validation accuracies.
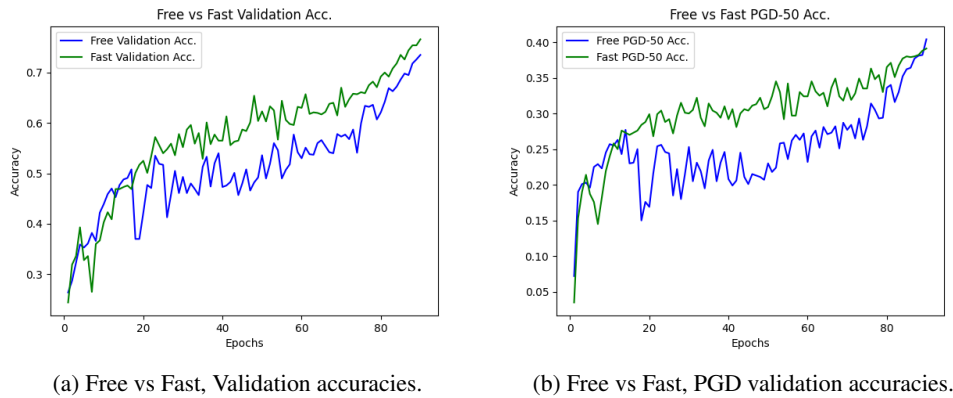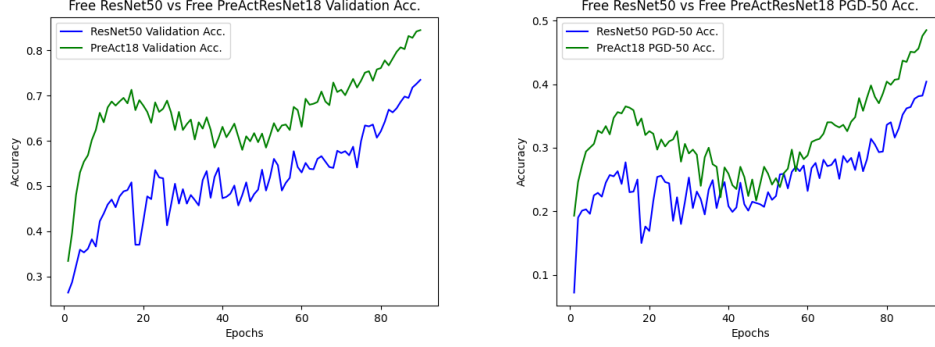
(b) Free vs Fast, PGD validation accuracies.

Figure 6: Comparing results for Adversarial training using Resnet50 in Free vs Fast.

The results of both Free and Fast using ResNet50 are not on par with the ones using PreActResnet18 (both for natural images and PGD-50) but, in this particular study case, Fast outperforms Free for what concerns the training time; on the other side, the difference between Free and Fast about the accuracies is not so relevant, so we can choose Fast as best method when using ResNet50 as training model.
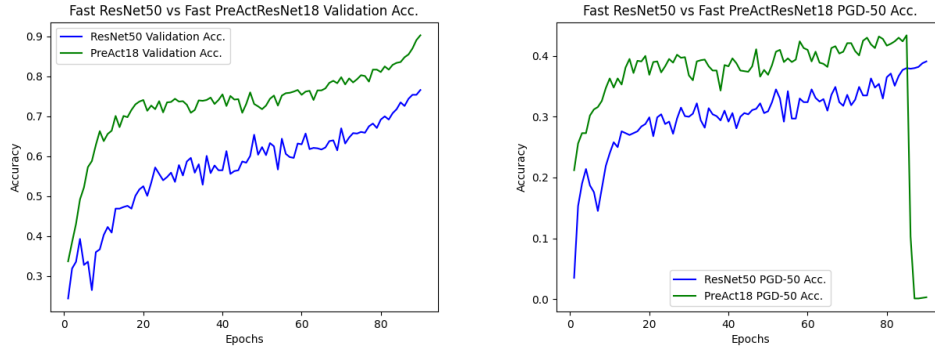
7.3 PreAct18 vs ResNet50

In the following two charts we compare the evolution of Free adv. learning using the two different models ResNet50 and PreActResnet18.

(a) Preact18 vs Resnet50 Validation accuracies. (b) Preact18 vs Resnet50 PGD validation accuracies.

Figure 7: Free Adversarial training (epsilon = 8/255, m = 8, batch size = 128, epochs = 90)

(a) Preact18 vs Resnet50 Validation accuracies. (b) Preact18 vs Resnet50 PGD validation accuracies.

Figure 8: Fast Adversarial training (epsilon = 8/255, α = 10/255, batch size = 128, epochs = 90)

As we can see from the previous charts, for each of the study cases, PreActResnet18 seems to be the best model to use according to the results both on test validation accuracy and PGD validation accuracy. Even if the time required to train the model based on ResNet50 is less than the one required to train the model based on PreActResnet18 (indeed this difference is evident from the two tables shown before), we decided to focus mainly on the latter, since this difference in time is not so high while, on the other side, there is a significant difference between the two models in terms of accuracies (both on natural images and perturbed ones). Obviously, these results are based on the usage of CIFAR-10 as dataset for the experiments but, as we can see from the original papers, the previous claim could not be valid when the used dataset is bigger than the one considered in this case (for example when using ImageNet).

8   Conclusions

Our experiments confirm minibatch-replays (m) as a crucial parameter regarding Free method, that needs to be tuned trying to reach a trade-off between test and robustness accuracies. Instead in Fast, step size (α) value is a crucial parameter, highly impactful on robust accuracy. We reached a "catastrophic overfitting" obtaining a null PGD accuracy value passing from a step size of value 10/255 to a step size of value 16/255. [Wong et al.]'s paper [5] was useful to reconsider FGSM as a feasible method for adversarial training once step size parameter is tuned correctly, and how simple optimization operations on learning rate adaptation (Cyclic Learning Rate) or based on hardware compatibility with mixed precision computation (Mixed-precision arithmetic) can decrease computation time, with small efforts. FGSM shows results similar to the ones obtained from the Free method, considering the same number of epochs, but the total training time suggests FGSM as a faster and better method to speed up adversarial training compared to Free, moreover PreActResnet18 showed better results than Resnet50 for all the considered epochs, based on our experiments on CIFAR-10.

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