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# FREE vs FAST ADVERSARIAL TRAINING

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## ABSTRACT

In this work we propose a comparison between two methods for adversarial training: Free and Fast. The code is available at <https://github.com/not-a-genius/neuralNetworkExam>.

## 1 Introduction

Adversarial training is a method for training robust deep neural networks, training on adversarial examples as a defense against adversarial attacks. This is typically considered to be more expensive than standard training due to the necessity of constructing adversarial examples via a first-order method like projected gradient descent (PGD). The papers presented show that it is possible to train empirically robust models using methods no more costly than standard training in practice. These methods show results comparable to train a model against PGD but at lower cost in terms of computational time. The goal of adversarial training is to learn a model which is not only accurate on the data (accuracy on train and test data) but also accurate on adversarially perturbed versions of the data (i.e. accuracy on images perturbed using the PGD attack). This leads to particular attention at the results we obtained after adversarial training experiments, aware of the presence of an acceptable compromise in accuracy, given a large increase in robustness due to the tradeoff between robustness and generalization [1, 2, 3].

### 1.1 Related Works

Our work is based on “Adversarial training for free!” paper [4], that presented an algorithm able to eliminate the overhead cost of generating adversarial examples by recycling the gradient information computed when updating the model parameters. While researching related work in the field we discovered a more recent paper, called “Fast is better than free: Revisiting adversarial training” [5] presenting an already known method called FGSM, previously considered ineffective due to what the paper calls “catastrophic overfitting”. This failure condition is preventable with the use of random initialization points. Moreover the same paper reported a further acceleration in training even for free algorithm thanks to standard techniques for efficient training, including cyclic learning rate and mixed-precision arithmetic. This caught our interest and we decided to try to replicate the reported results and compare them with our implementation, in order to confirm or deny these thesis.

### 1.2 Dataset and architecture

We decided to choose CIFAR-10 as the dataset on which our experiments would have been conducted to adapt to the available hardware (our GPU is an Nvidia RTX 2070 super). The CIFAR-10 dataset consists of 60000 32x32 colour images in 10 classes, with 6000 images per class. There are 50000 training images and 10000 test images. The dataset is divided into five training batches and one test batch, each with 10000 images. The test batch contains exactly 1000 randomly-selected images from each class. The training batches contain the remaining images in random order, but some training batches may contain more images from one class than another. Between them, the training batches contain exactly 5000 images from each class.

We conducted adversarial training examples using as models ResNet50 and PreActResNet18. Both ResNet50 and PreActResnet18 results have been reported and compared between them, under the same conditions.

## 2 Adversarial Machine Learning

Adversarial machine learning is a machine learning technique that attempts to fool models by supplying deceptive input (called also adversarial example). Adversarial examples exploit the way artificial intelligence algorithms work to disrupt the behavior of artificial intelligence algorithms. In the past few years, adversarial machine learning has become an active area of research as the role of AI continues to grow in many of the applications we use. There's growing concern that vulnerabilities in machine learning systems can be exploited for malicious purposes. The main idea to deal with adversarial examples is represented by adversarial training; it can be traced back to [6], in which models were hardened by producing adversarial examples and injecting them into training data [4].

This technique can be summarized as follows: given a network  $f_\theta$  parametrized by  $\theta$ , a dataset  $(x_i, y_i)$ , a loss function  $l$  and a threat model  $\Delta$ , the learning problem consists in the following optimization problem:

$$\min_{\theta} \sum_i l(f_\theta(x_i + \delta), y_i)$$

A typical choice for the adversarial model is to take  $\Delta = \{\delta : \|\delta\|_\infty \leq \epsilon\}$  for some  $\epsilon > 0$ .

## 3 PGD attack

The PGD attack is a white-box attack, which means that the attacker has access to the model's weights, giving to the attacker much more power than a black-box attack (in which the model's weights are not known) as he can specifically craft the attack to fool the chosen model without having to rely on transfer attacks, which often result in human-visible perturbations. The key to understanding the PGD attack is to frame finding an adversarial example as a constrained optimisation problem. PGD attempts to find the perturbation that maximises the loss of a model on a particular input while keeping the size of the perturbation smaller than a specified amount referred to as epsilon. This constraint is usually expressed as the  $l^2$  or  $l^\infty$  norm of the perturbation and it is added so the content of the adversarial example is the same as the unperturbed sample. The PGD algorithm can be summarised with the 4 steps below although the attacker is free to apply any optimisation improvements such as momentum, Adam, multiple restarts etc...

1. Start from a random perturbation in the  $l_p$  ball around a sample;
2. Take a gradient step in the direction of the greatest loss;
3. Project perturbation back into the  $l_p$  ball if necessary;
4. Repeat steps 2-3 until convergence.

In our case, adversarial training consists simply of putting the PGD attack inside the training loop, applying a kind of "data augmentation"; in this case, instead of performing random transformations as a preprocessing step, we create specific perturbations that best fool the model and indeed adversarially trained models do exhibit less overfitting when trained on small datasets.

The PGD attack is based on the simplest version of what is called Fast Gradient Sign Method, used to approximate the inner maximization of Delta. This could be seen as a relatively inaccurate approximation of the inner maximization for  $l_\infty$  perturbations, and has the following closed form (as shown in [7]):

$$\delta^* = \epsilon \cdot \text{sign}(\nabla_x l(f(x), y)).$$

A better approximation of the inner maximization is to take multiple, smaller FGSM steps of size alpha. When the iteration leaves the threat model, it is projected back to the set  $\Delta$ . Multiple restarts within the threat model  $\Delta$  typically improve the approximation of the inner maximization even further. The combination of all these techniques is the PGD attack. The problem of such technique is that the number of gradient computations here is proportional to  $O(MK)$  in a single epoch, where  $M$  is the size of the dataset and  $K$  is the number of steps taken by the PGD adversary. This is  $K$  times greater than standard training so adversarial training is typically  $K$  times slower than standard training. The pseudo-code of the PGD attack is reported in Algorithm 1.

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**Algorithm 1** PGD adversarial training for  $T$  epochs, given some radius  $\epsilon$ , adversarial step size  $\alpha$  and  $K$  PGD steps and a dataset of size  $M$  for a network  $f_\theta$

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```

1: for  $t = 1 \dots T$  do
2:   for  $i = 1 \dots M$  do
3:     // Perform PGD adversarial attack
4:      $\delta = 0$  // or randomly initialized
5:     for  $j = 1 \dots K$  do
6:        $\delta = \delta + \alpha \cdot \text{sign}(\nabla_\delta l(f_\delta(x_i + \delta), y_i))$ 
7:        $\delta = \max(\min(\delta, \epsilon), -\epsilon)$ 
8:     end for
9:      $\theta = \theta - \nabla_\theta l(f_\theta(x_i + \delta), y_i)$  // Update model weights with some optimizer, e.g. SGD
10:  end for
11: end for

```

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The execution of K-PGD in [8] took four days on a Titan X (with model WideResNet and dataset CIFAR-10). This led us to consider the reported results [4],[5], obtained using K-PGD adversarial training, without running them directly.

## 4 Free Adversarial training for free!

Free adversarial training [4] claims to reduce the overhead compared to natural training. The idea behind the improved algorithm is the computation of ascent step by reusing the backward pass needed for the descent step. So the gradient with respect to the network parameters is computed on the backward pass and later on this same backward pass the gradient of the loss with respect to the input image is also computed.

In order to allow multiple adversarial updates on the same image the training is made on the same minibatch  $m$  times in a row. Every minibatch is repeated  $m$  times before switching to the next minibatch (“minibatch replays”), so the number of epochs can be considered as the number of epochs divided by  $m$ , such that the overall number of training iterations remains constant. Note that perturbations are not reset between mini-batches. The value of this parameter is shown to be crucial, since increasing  $m$  comes with the risk of increasing generalization error revealing the presence of a trade-off: robustness is increased at the cost of validation accuracy on natural images.

The pseudocode of Free is shown in Algorithm 2.

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**Algorithm 2** “Free” Adversarial Training (Free- $m$ )

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**Require:** Training samples  $X$ , perturbation bound  $\epsilon$ , learning rate  $\tau$ , hop steps  $m$

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1: Initialize  $\theta$ 
2:  $\delta \leftarrow 0$ 
3: for  $epoch = 1 \dots N_{ep}/m$  do
4:   for minibatch  $B \in X$  do
5:     for  $i = 1 \dots m$  do
6:       // Update  $\theta$  with stochastic gradient descent
7:        $g_\theta \leftarrow \mathbb{E}_{(x,y) \in B} \nabla_\theta [l(x + \delta, y, \theta)]$ 
8:        $g_{adv} \leftarrow \nabla_x l(x + \delta, y, \theta)$ 
9:        $\theta \leftarrow \theta - \tau g_\theta$ 
10:      // Use gradients calculated for the minimization step to update  $\delta$ 
11:       $\delta \leftarrow +\epsilon \cdot \text{sign}(g_{adv})$ 
12:       $\delta = \text{clip}(\delta, -\epsilon, \epsilon)$ 
13:    end for
14:  end for
15: end for

```

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In the choice of  $m$  one problem may arise: it’s possible that catastrophic forgetting happens, e.g. when all the “informative” images of one class are in the first few mini-batches. In this extreme case, we do not see useful examples for most of the epoch, and forgetting may occur. How much does mini-batch replay hurt generalization? To answer the question we followed [4] approach trying different values of  $m$ , reaching the same results: dropoff in accuracy for small  $m$ , while we see an inversion in PGD and test accuracy for high values of it. On CIFAR-10 effective robustness can be reached with values of  $m \leq 10$ , our best results have been obtained with  $m = 8$ .

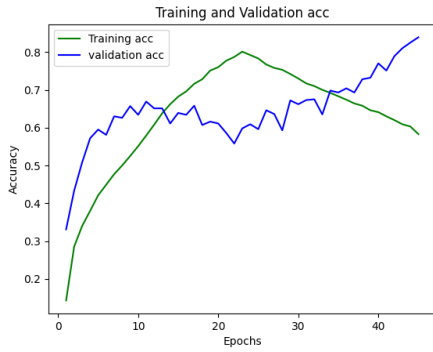
It is known that adversarially trained classifiers, e.g. PGD, have generative behaviours. When an image is perturbed

from one class to another, it adopts features that make it “look” like its adopted class, to a human eye. Free trained models exhibit this benefit and also smooth and flattened loss surface. Indeed, standard adversarially trained classifiers show the property to have a flatten and smooth loss landscape while some defenses work by “masking” the gradients. This techniques makes it difficult to identify adversarial examples using gradient methods, even though adversarial examples remain present. Reference [9] argues that gradient masking adds little security, free training does not operate by masking gradients and uses a rough loss surface. In the following table we present the results of our experiments conducted using Free method and varying the number of minibatch replays. We can notice that the train time increase as  $m$  increases as expected but in the case of  $m = 10$  we see a small inversion in the value of both natural image accuracy and robustness.

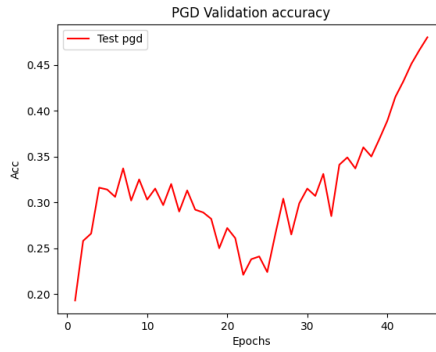
#### 4.1 Experiments results

All experiments using Free adversarial training in this section are carried out with the same number of epochs, epsilon value  $\epsilon = 8/255$ . All PGD adversaries used at evaluation are run with 1 random restarts for 10,20,50 iterations (with the same hyperparameters as those used by Shafahi et al. (2019) [4] but further strengthened with random restarts). Speedup with mixed-precision was incorporated with the Apex amp package at the O2 without loss scaling. The following table shows the results of our experiments:

Minibatch-replay (m)	Accuracy using PreActResNet18				Epoch	Train Time (min)	Avg Epoch Time (sec)
	Natural Images	PGD-10	PGD-20	PGD-50			
$m = 2$	0.895	0.324	0.297	0.290	45	22	29.4
$m = 4$	0.867	0.471	0.457	0.451	45	44	58.2
$m = 8$	0.839	0.498	0.486	0.483	45	88	117
$m = 10$	0.821	0.490	0.482	0.479	45	108	143.4



(a) Train and validation accuracies results.



(b) PGD validation accuracy

Figure 1: Free Adversarial training using PreActResnet18 (epsilon = 8/255,  $m = 8$ , batch size = 128, epochs = 45)

The following results are related to the training and evaluation based on ResNet50 model instead of PreActResnet18. The number of epochs has been increased to 90 to see the impact on the accuracy in the long run.

Minibatch-replay (m)	Accuracy using ResNet50		Epoch	Train Time (min)	Avg Epoch Time (sec)	Batch size
	Natural Images	PGD-50				
$m = 8$	0.735	0.404	90	118	78.6	128

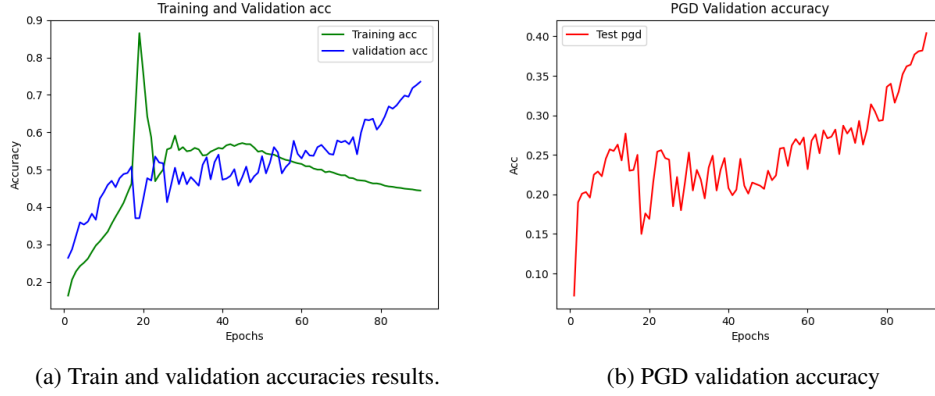


Figure 2: Free Adversarial training using ResNet50 (epsilon = 8/255, m = 8, batch size = 128, epochs = 90)

## 5 Fast is better than free

Wong et al. (2020) [5] claim to spend shorter computational time than Shafai et al. (2019) [4] “Free” method using FGSM, with time very distant from 10 hours (200 epochs) to learn a robust CIFAR-10 [5], while standard training requires minutes. Similarly for Imagenet. A key difference between FGSM and free adversarial training is that in the latter the perturbation from the previous minibatch is a reasonable starting point for the next minibatch, while FGSM can be used as it is just starting from a non-zero initial perturbation. So “Fast” adversarial training is just standard FGSM with random initialization for the perturbation and this is claimed to be as effective as PGD adversarial training [5]. For this adversarial training one crucial parameter is the step size  $\alpha$  that shows results on par with “Free” ones only choosing values in a particular interval. Indeed, two failure modes arise:

- a step size too small ( $\alpha = \epsilon$ ) would mean a zero-initialized perturbation and lead to a too weak defense
- a step size too large ( $\alpha = 2\epsilon$ ) will lead to a model overfitted, restricted to specific threats (“catastrophic overfitting”).

A good compromise is shown to be:  $\alpha = 1.25$ ,  $\epsilon = 10/255$ . Another key difference is that FGSM does not need to repeat mini-batches but needs two backward passes to compute gradients separately from the perturbation and the model weights. From a computational complexity point of view this means that an epoch of FGSM is equivalent to two epochs of standard training.

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### Algorithm 3 FGSM adversarial training for T epochs

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**Require:** Dataset of size M, radius  $\epsilon$ , K PGS steps, step size  $\alpha$

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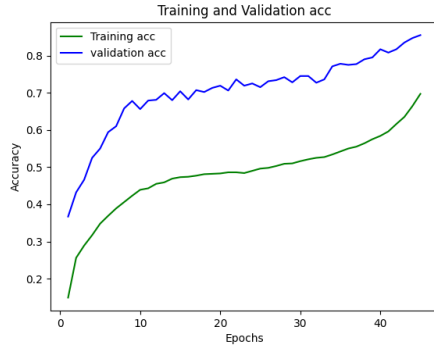
1: for  $t = 1 \dots T$  do
2:   for  $i = 1 \dots M$  do
3:     // Perform FGSM adversarial attack
4:      $\delta = \text{Uniform}(-\epsilon, \epsilon)$ 
5:      $\delta = \delta + \alpha \cdot \text{sign}(\nabla_{\delta} l(f_{\theta}(x_i + \delta), y_i))$ 
6:      $\delta = \max(\min(\delta, \epsilon), -\epsilon)$ 
7:     // Update model weights with some optimizer, e.g. SGD
8:      $\theta = \theta - \nabla_{\theta} l(f_{\theta}(x_i + \delta), y_i)$ 
9:   end for
10: end for
```

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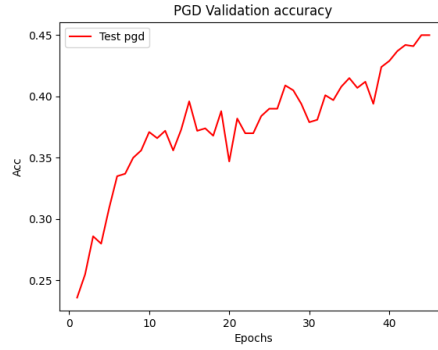
### 5.1 Experiments results

All experiments using FGSM adversarial training in this section are carried out with the same number of epochs, epsilon value  $\epsilon = 8/255$ . All PGD adversaries used at evaluation are run with 1 random restarts for 10,20,50 iterations (with some of the same hyperparameters as those used by Wong et al. (2019) but further strengthened with random restarts). Speedup with mixed-precision was incorporated with the Apex amp package at the O2 without loss scaling.

Step-size	Accuracy using PreActResNet18				Epoch	Train Time (min)	Avg Epoch Time (sec)	Batch size
	Natural Images	PGD-10	PGD-20	PGD-50				
$\alpha = 10$	0.847	0.480	0.463	0.450	45	21	28.8	128
$\alpha = 10$	0.850	0.454	0.438	0.434	45	20	25.8	256
$\alpha = 16$	0.856	0	0	0	45	21	28.2	128
$\alpha = 16$	0.87	0	0	0	45	20	26.4	256



(a) Train and validation accuracies results.

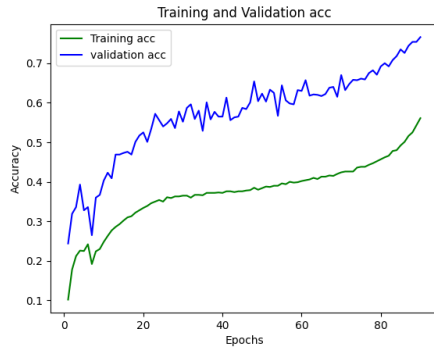


(b) PGD validation accuracy

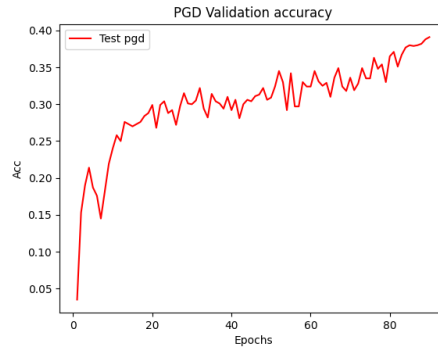
Figure 3: FGSM Adversarial training using Resnet50 (epsilon = 8/255, m = 8, batch size = 128, epochs = 45)

The following results are related to the training and evaluation based on ResNet50 model instead of PreActResnet18. The number of epochs has been increased to 90 to see the impact on the accuracy in the long run.

Step-size	Accuracy using Resnet50			Epoch	Train Time (min)	Avg Epoch Time (sec)	Batch size
	Natural Images	PGD-50	PGD-10				
$\alpha = 10$	0.766	0.391	0.381	90	28.18	18.78	128



(a) Train and validation accuracies results.



(b) PGD validation accuracy

Figure 4: FGSM Adversarial training using Resnet50 (epsilon = 8/255, m = 8, batch size = 128, epochs = 90)

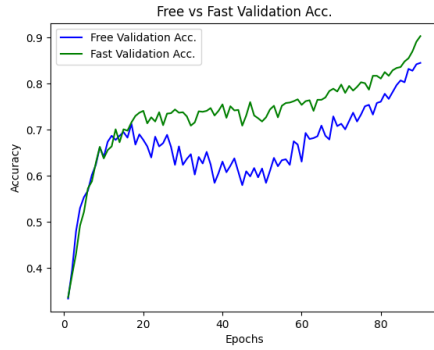
## 6 Final Experiments

### 6.1 Free vs Fast using PreActResnet18

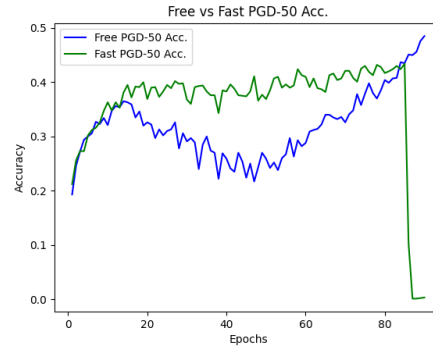
In this section we compare the best results we obtained from the precedent experiments, then showing the evolution of validation accuracy in these two methods.

The following table contains the best  $m$  and batch size values for free method, obtained from the different free experiments compared with the best step size and batch size for FGSM.

	Minibatch-replays ( $m$ ) / step size ( $\alpha$ )	Accuracy using PreactResnet18		Epoch	Train Time (min)	Avg Epoch Time (sec)	Batch size
		Natural Images	PGD-50				
<b>FREE</b>	$m = 8$	0.845	0.485	90	176	117	128
<b>FAST</b>	$\alpha = 10/255$	0.836	0.434	90	43	28	128



(a) Free vs Fast, Validation accuracies.

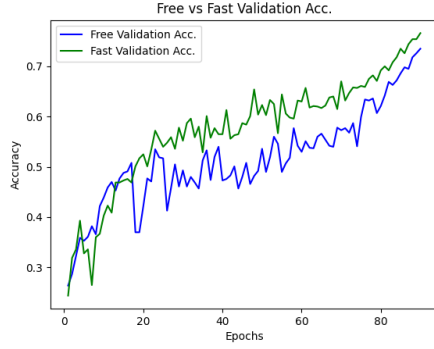


(b) Free vs Fast, PGD validation accuracies.

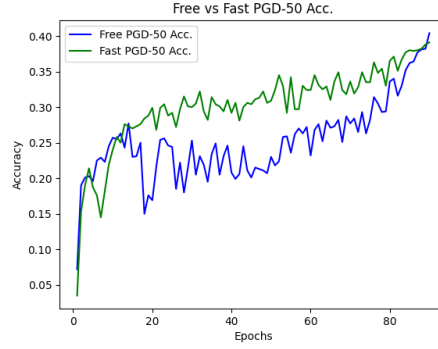
Figure 5: Comparing results for Adversarial training using PreActResNet18 in Free vs Fast.

### 6.2 Free vs Fast using ResNet50

	Minibatch-replays ( $m$ ) / step size ( $\alpha$ )	Accuracy using ResNet50		Epoch	Train Time (min)	Avg Epoch Time (sec)	Batch size
		Natural Images	PGD-50				
<b>FREE</b>	$m = 8$	0.735	0.404	90	176	117	128
<b>FAST</b>	$\alpha = 10/255$	0.766	0.391	90	29	19	128



(a) Free vs Fast, Validation accuracies.

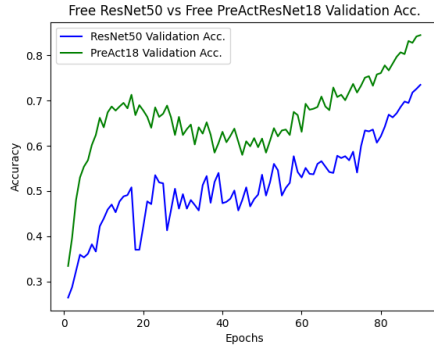


(b) Free vs Fast, PGD validation accuracies.

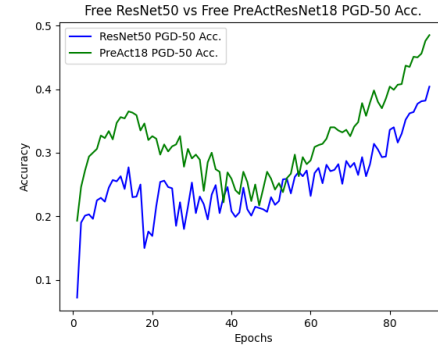
Figure 6: Comparing results for Adversarial training using Resnet50 in Free vs Fast.

### 6.3 PreAct18 vs ResNet50

In the following two charts we compare the evolution of Free adv.learning using the two different models ResNet50 and PreActResnet18.

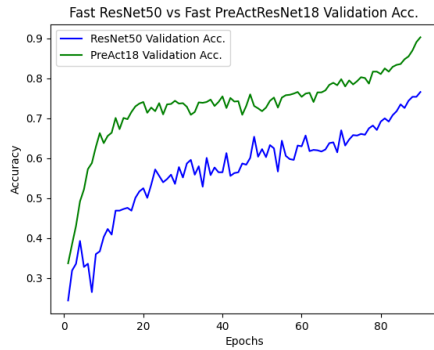


(a) Preact18 vs Resnet50 Validation accuracies.

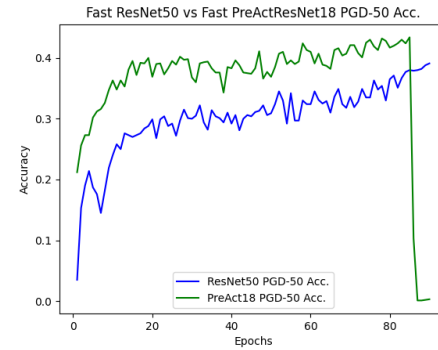


(b) Preact18 vs Resnet50 PGD validation accuracies.

Figure 7: Free Adversarial training (epsilon = 8/255, m = 8, batch size = 128, epochs = 90)



(a) Preact18 vs Resnet50 Validation accuracies.



(b) Preact18 vs Resnet50 PGD validation accuracies.

Figure 8: Fast Adversarial training (epsilon = 8/255,  $\alpha = 10/255$ , batch size = 128, epochs = 90)



## 7 Conclusions

Wong et al.'s paper [5] was useful to reconsider FGSM as a feasible method for adversarial training once step size parameter is tuned correctly, and how simple optimization operations on learning rate adaptation (Cyclic Learning Rate) or based on hardware compatibility with mixed precision computation (Mixed-precision arithmetic) can decrease computation time, with small effort. FGSM shows results similar to the ones obtained from the "Free" method, considering the same number of epochs, but the total training time suggests FGSM as a faster and better method to speed up adversarial training compared to free, based on our experiments on CIFAR10.

## References

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