Time Stepper

$u(x,t) \longrightarrow$ | Euler with Step Size Δt
 $\cdot + \Delta t\, f(\cdot)$ | $\longrightarrow u(x, t + \Delta t)$

Neural Operator

$u(\cdot, t)$

$u(x, t + \Delta t)$

x

$\Downarrow$

$\swarrow$

① Black Box Time Stepper Operator

$u(x,t)$ | Black Box with
 Step Size Δt | $\mathcal{S}_{\Delta t}[u](x)$

$\Rightarrow$

$u \longrightarrow$ B. B. $\longrightarrow \mathcal{S}_{\Delta t}[u]$

$u + \delta v \rightarrow$ B. B. $\rightarrow \mathcal{S}_{\Delta t}[u + \delta v]$

②

$\left. \vphantom{\begin{array}{c} a \\ b \end{array}} \right\} \approx D\mathcal{S}_{\Delta t}[u] \cdot v$

$\Downarrow$

Finite Differences Approximation
to the Jacobian $D\mathcal{S}_{\Delta t}$

$\Downarrow$

③ $\Leftarrow$

Matrix-Free Linear Algebra:
Newton-Krylov, Arnoldi, . . .

④

$v_{n+1} = D\mathcal{S}_{\Delta t}[u] \cdot v_n$

$D\,\mathcal{S}_{\Delta t}[u_n] \cdot (u_{n+1} - u_n) = -\mathcal{S}_{\Delta t}[u_n]$

u_n u_{n+1} u^* u