Data Science & Statistical Learning | II Level Master

“Swarming of Aerial Robots with Markov Random Field Optimization”

Paper Analysis

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Drone trajectories planning

A Quadrotors Aerial Robots – **Drone**

Image source: pngplay.com/it/image/8944

Five drones team trajectories in an obstacle field – Render

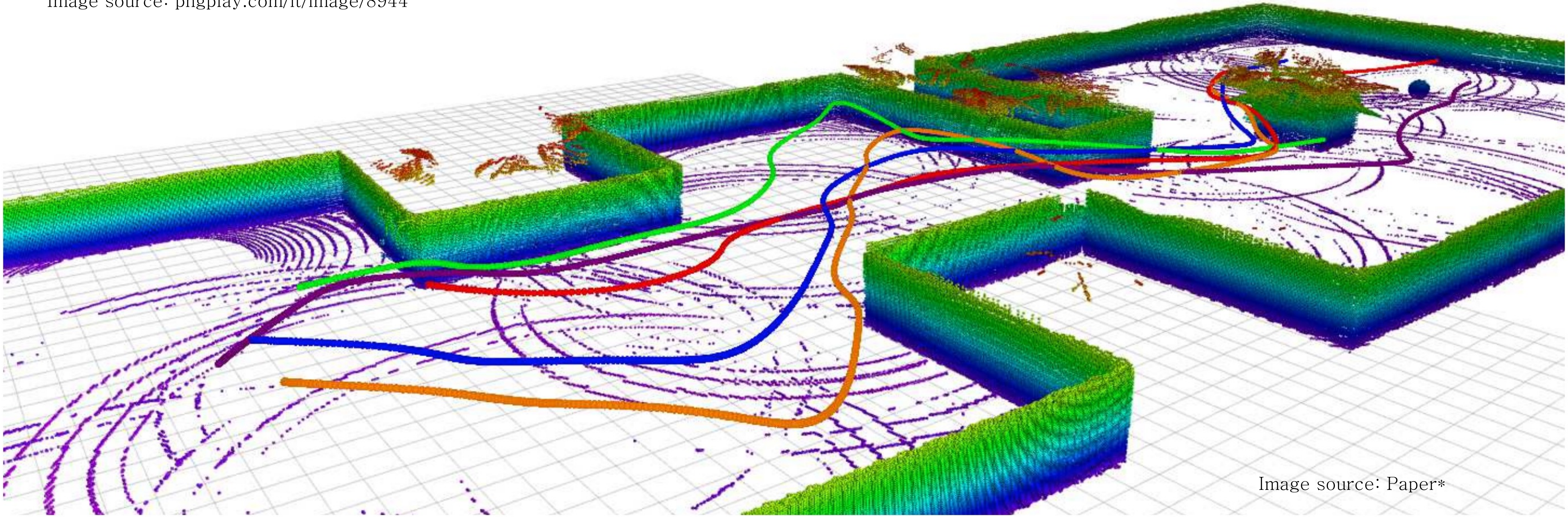


Image source: Paper*

Animal swarms **individual member behaviour**:

- Depends on the **topological neighborhood** rather than the distance.

Inspire to model natural-like interactions between **drones**:

- Where neighborhood are $\boldsymbol{k}$ **nearest** over N in swarm.

Images source: quora.com/How-do-ducks-fly-in-a-V-shaped-formation-coordinately

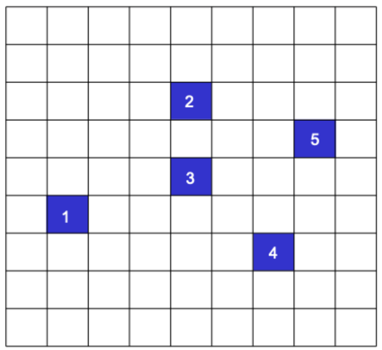
Drone location in the environment can be represented by an occupancy **grid map**:

- ➢ **E.g. 5 drones distribution** on a 2-D environment discretized into a 10×10 grid map

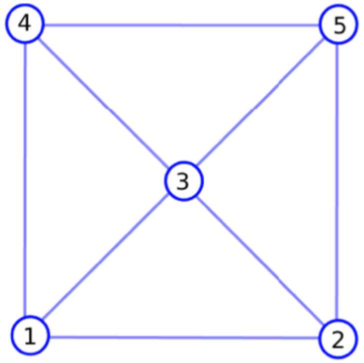
Interactions represented by **Undirected Graphical Model**:

- ➢ Bidirectional edge for two interacting drones.
  - Interactions graph G given the distribution on the grid map.

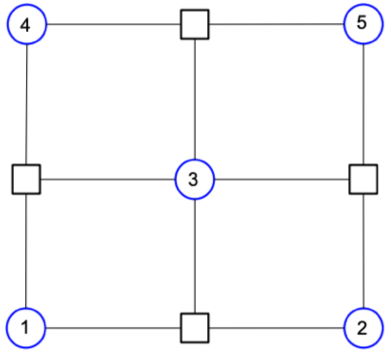
Factor graph for G where $\{c_i\} \in C$ is a maximal clique in the factorization.



GRID MAP



INTERACTIONS GRAPH



FACTOR GRAPH

Images source: Paper*

Markov Random Field - Clique

Undirected Graph – Markov Random Field

$G = (V, E)$:

- Vertices $V = \{1, 2, \ldots, N\}$, representing **navigable drone positions** x_i, with $i \in V$.
- Edges $E \subset \{V \times V\}$.
- Markovian blanket Ni of a node based on the local conditional independence of G :

$$P(\mathbf{x}_i | \{\mathbf{x}_j\}_{j \in \mathcal{V} \setminus i}) = P(\mathbf{x}_i | \{\mathbf{x}_j\}_{j \in \mathcal{N}_i})$$

- A **clique** c of an undirected graph G is a subset of vertices, which are all adjacent to each other.
- A **maximal clique** is a clique which cannot be extended by including one more adjacent vertex.

Clique factorization

- MRF joint probability distribution can be decomposed into a **product of clique potential functions** $\psi_c(x_c)$:

$$p(x) = \frac{1}{Z}\prod_{c\in C}(\psi_c(\mathbf{x}_c))$$

- c: maximal clique, with $c \in C$
- x_c: labelling of clique c
- C : set of all maximal cliques
- Z: partition function

Energy Minimization on Markov Random Fields

- Restricting $\psi_c(x_c)$ to be **strictly positive**:
  - Potential functions as **exponentials**:

$$\psi_c(\mathbf{x}_c) = \exp\left(-E(\mathbf{x}_c)\right)$$

$$p(x) = \frac{1}{Z}\exp\left(\sum_c -E(\mathbf{x}_c)\right)$$

- $E(x_c)$: energy function

- Clique energy computed using a set of super-imposed **artificial potential functions.**
- Partition function Z to ensure the distribution sum to 1 : $Z = \sum_{\mathbf{x}}\prod_{c\in C}\exp(\psi_c(\mathbf{x}_c))$

Swarm energy ε

Sum of **all maximal clique energies** of the interaction graph:

$$\varepsilon(\mathbf{x}) = \sum_{i=1}^{4} E_i(\mathbf{x}_i)$$

- According to **local Markov property**: Markovian blanket of drone *i*, N_i is the neighborhood of a drone *i*.
- Necessary and sufficient condition to **navigate drones cohesively**: $\forall i \in V, N_i \neq \emptyset$.
  - There aren't **independent vertices** in the interaction graph:
    - **Distances between drones** not exceed the limits of their communication ranges.
    - Drones **establish neighborhoods** at the time of initialization.

Artificial Potential Fields to:

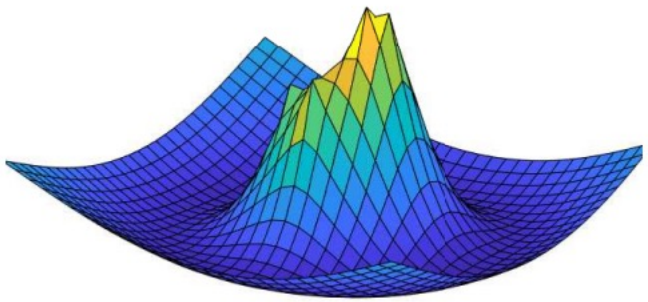
- **Model obstacles** in the environment.
- **Compute the paths** for drone groups:
  - **Multiple potential functions** to represent distinctive interactions:
    - Obstacles.
    - Neighbouring drones.
    - Goal position.

Drones:

- **Attracted** to a predefined **goal position.**
- **Repelled** by the **obstacles** in the environment.
  - Consisting in **two external potential fields**, affecting the swarm.
  - A potential field to model the swarm internal **interactions between** drones.

Model – Potential Functions

Interaction Potential



- **Morse** potential function to model the interaction between drones.
- Output energy given the **distance between the agents**:
  - **Repulsion** dominant if distance to low.
  - **Attraction** dominant if distance is high.
    - **Graph**: **valley** around the given arbitrary drones formation is a **globally minimum energy zone** for drones configuration when the they are not subjected to any other potential energy.
- For any two neighboring drones, i and j, at p_i and p_j :
  - Interaction energy ρ_I :

$$\rho_I(p_i, p_j) = \rho'_I(p_i, p_j) + \overset{\text{To restrict potential to be positive}}{\zeta},$$

$$\rho'_I(p_i, p_j) = \underbrace{-a_I \exp\left(-\left\|\frac{d_{ij}}{k_a}\right\|\right)}_{\text{Attractive term}} + \underbrace{b_I \exp\left(-\left\|\frac{d_{ij}}{k_r}\right\|\right)}_{\text{Repulsive term}}$$

- d_{ij}: distance between drones i & j
- All other terms are positive constants

Image source: Paper*

Goal Potential

- Static and positive potential function, that attracts the drones toward a **global minimum energy at goal region.** Goal potential:

$$\rho_G(p) = a_G \exp(\frac{\|p - p_g\|}{k_G})$$

- p_g: target region
- p : current position
- All other terms are positive constants

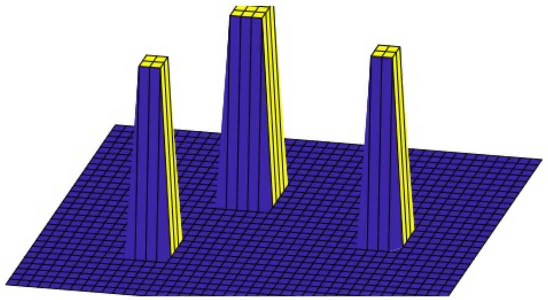
Obstacle Potential

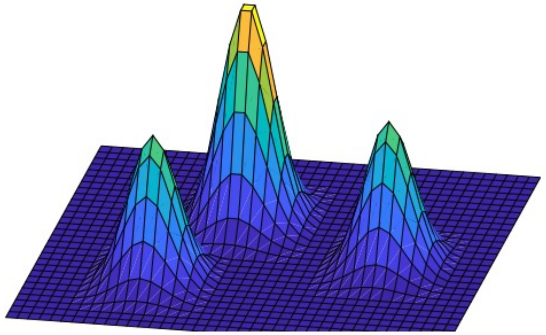
- To navigate the drones away from the obstacles:
  - **Highpass filter** and **Gaussian kernel** (G) function on the occupancy grid map, with x, y coordinate:
    - Filter separates **obstacle and free space** by thresholding.
    - Gaussian kernel on the map to have **smooth** and positive obstacle potential field.

$$\hat{m}_{xy} = \begin{cases} \Gamma & P(m_{xy}) \geq 0.5, \quad \textbf{Where } \Gamma > 0 \\ 0 & \textbf{Otherwise.} \end{cases}$$

$$\rho_O(p) = \hat{m}_{xy} \circledast \mathbf{G}$$

Images source: Paper*





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Combined Potential Field

- Goal, obstacle and interaction, $\rho_{com} = \rho_I + \rho_O + \rho_G$.
- **Potential field dynamically changes** as drones move in the environment.
  - This property is used to search for **collision free paths** using an iterative and local search.
- Graph:
  - **Valley** in the map represents the **goal position** at (65, 15).
  - The higher energy area represents the drones starting position and their interaction energies.

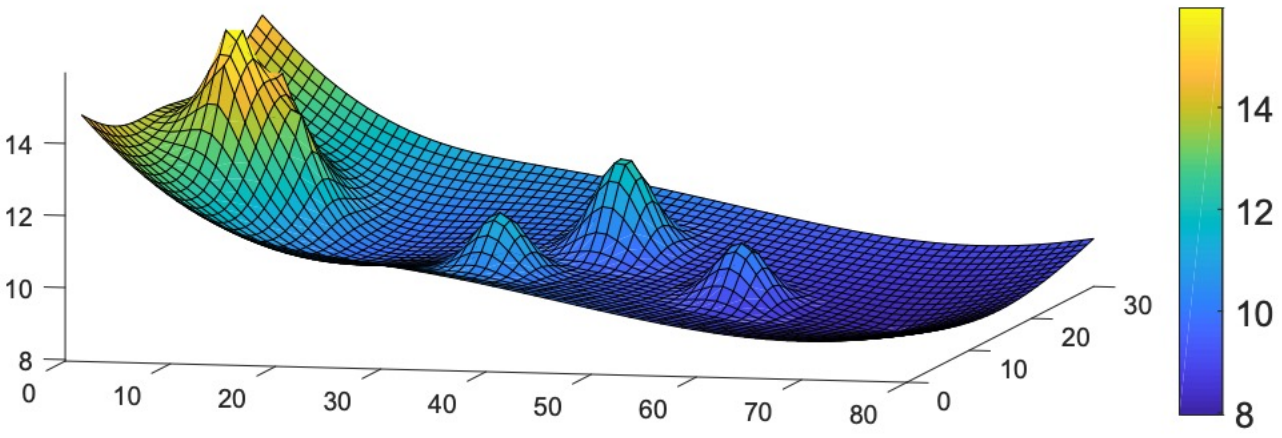

Image source: Paper*

Markov Random Field Optimization

- **Dynamic Markov Random Field** given interaction graph $G = \{V, E\}$ and $|V| = N$.
- Where each vertex $i \in V$ is associated with a latent random variable $\boldsymbol{x}_i^t \in \boldsymbol{X}_i^t$ local search space.

- Iteratively updated **at each optimization step** t:
  - **Interaction graph**, thus the random field.
  - $\boldsymbol{X}_i^t$ local search space for a given drone.
  - N **locally optimum position nodes** of a set of collision free **discrete paths.**

For any cell of the grid map

$$X_i^t = \{(x_j, y_j) \in m : (x_i - x_j)^2 + (y_i - y_j)^2 \leq n\}$$

So, search for the *global minimum of the MRF energy via local search.*

- As t grows, **latent values that minimizes the clique potentials** are obtained.
- For a group of N drones, the **combined local search space** at time t can be defined as:

$$X^t = \bigcup_{i=1}^{N} X_i^t$$

- It is used to perform MRF optimization to generate collision free paths for *multi drones teams.*

Iteratively searches for the locally optimum label $\boldsymbol{x_i^{*t}}$ that minimizes the **clique energy Ec**.

Computational complexity grows with the order of the **local search space**, **n**.

Limit search space size by heuristics at different stages of the trajectory:

- **Avoid** any further **collisions.**
- Drone directions permit to navigate only toward the goal, by trimming the preceding half.

Inter-drone collision avoidance properties:

  - **No two drones share the same position** in occupancy grid at the same time-step.
  - Drone **paths may not intersect in between two adjacent time-steps**.

Performing MRF optimization over combined local search space Xt iteratively until the swarm energy $\boldsymbol{\varepsilon}$ is converged, **returns a series of nodes that can be used as waypoints for individual drones.**

Computed **points are connected**, unless the path intersects with an obstacle or another drone's trajectory.

Finally, **path optimization is performed to smooth** the trajectory of drones considering constant traversal velocity and altitude of the drones till they converge to the goal position.

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Matlab – ROS/Gazebo experiment

- The **maximal clique factorization** of the interaction graph is performed using **Bron–Kerbosch algorithm** with pivoting, an enumeration algorithm for finding all maximal cliques in an undirected graph.
- The initial drone positions are sampled from a **multivariate Gaussian distribution** with mean set to a fixed starting position for the drone team.

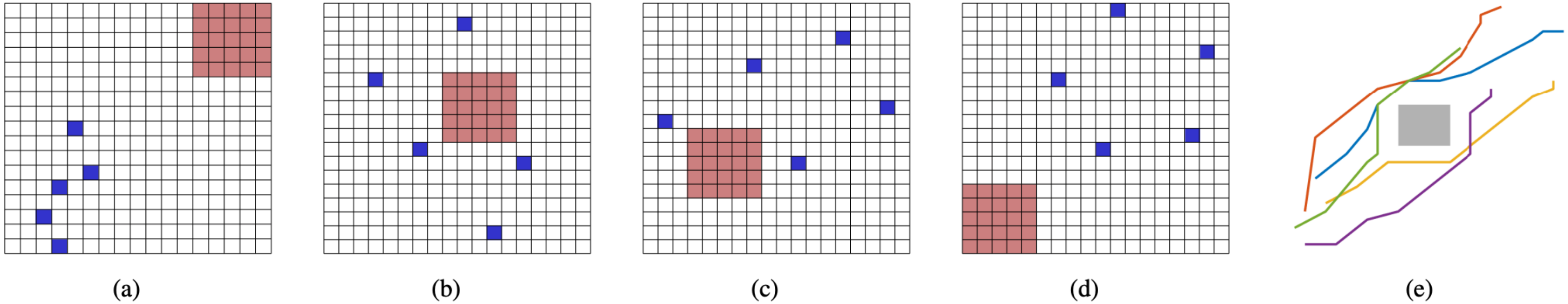

Fig. 5: MRF optimization steps for 5 robots with a single obstacle. (a) Initial robot distribution. (b) (c) (d) After 4, 7 and 14 optimization steps. (e) Resulting discrete trajectories after waypoint pruning.

Images source: Paper*

Experiment & Result

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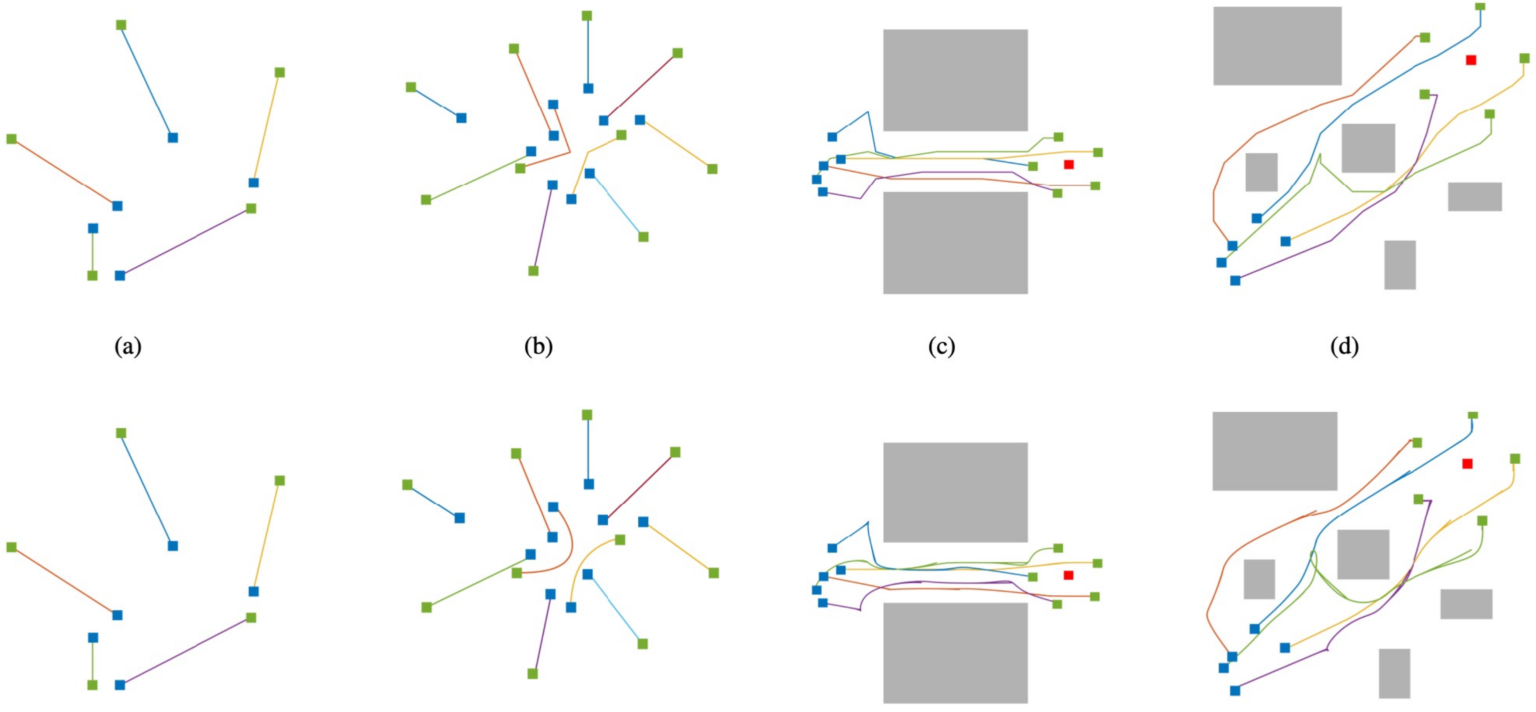
- Drones maintaining a tight cohesive **formations such as pentagons.**
- Due to the topological neighborhood, we observed that the **cohesive behavior in large swarms** lead the drones to **wait until the other drones get closer or mover over**, similar to natural aggregations.

- Green and blue markers show the start and end positions of the drones respectively.
- Grey areas denote the obstacles.
- (a)(b) Groups of 5 and 10 drones converging to a minimum energy formation with k = 3 and k = 7.
- **Sharp edges denote stop-start scenarios.**



(a) (b) (c) (d)

(e) (f) (g) (h)

Images source: Paper*

Probably the **cutest** drone you will ever see

Tamagotchi inspired

Image source: boredeye.design

Thank you

Gianmarco