Deep Imbalanced Regression

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Presenter: Gianmarco Midena

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Overview

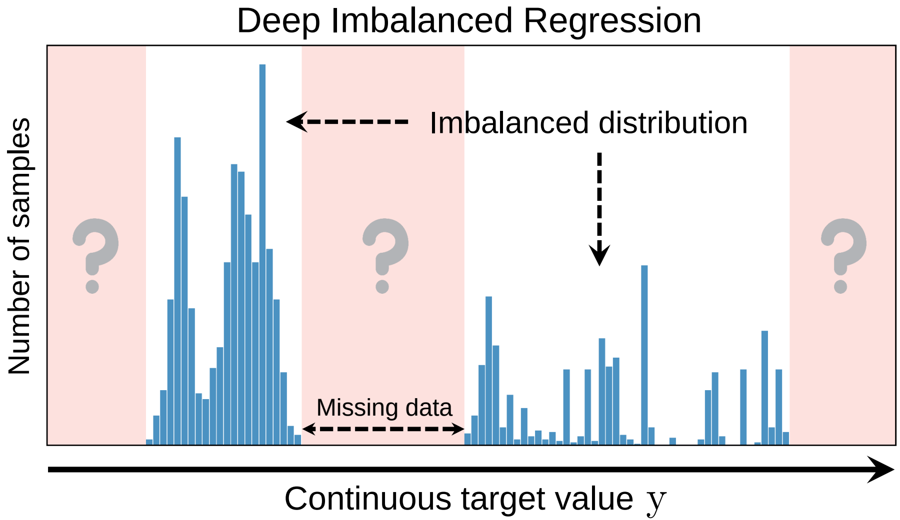


Image credit: *Yang et al. (2021)*

Problem Settings

- $\{(\mathbf{x}_i, y_i)\}_{i=1}^N$: training set
- $\mathbf{x}_i \in \mathbb{R}^d$: input
- $y_i \in \mathcal{Y}$: continuous label or target
- $b_i \in \mathcal{B}$: discrete label or target
- $\mathcal{Y} \subset \mathbb{R}$: continuous label space
- $\mathcal{B} = \{1, \dots, M\} \subset \mathbb{Z}^+$: index space
  - divides $\mathcal{Y}$ into M groups (bins) with equal intervals $[t_j, t_{j+1})$
  - $\{[t_0, t_1), \dots, [t_{M-1}, t_M)\}$: discrete label space
  - $t_k \in \mathcal{Y}$
  - minimum resolution
    - e.g., $\delta y \triangleq t_{j+1} - t_j = 1$ in age estimation
- $\hat{y}_i = g(\mathbf{z}_i) \in \mathbb{R}$: predicted continuous label
- $\mathbf{z}_i = f(\mathbf{x}_i; \theta) \in \mathbb{R}^{d'}$: learned representation
- θ: trainable model parameters

Evaluation

- Divide target space into disjoint regions (bins)
  - *Many-shot*: > 100 training examples
  - *Medium-shot*: 20-100 training examples
  - *Few-shot*: < 20 training examples
  - *Zero-shot*: 0 training examples
  - Inspired by Liu et al. 2019

- Metrics
  - Mean Absolute Error (MAE)
  - Mean Squared Error (MSE)
  - Pearson Correlation (PCC)
  - Geometric Mean Error (GM)

$$GM = \sqrt[n]{\prod_{i=1}^{n} |y_i - \hat{y}_i|}$$

  - ★ Pros: $+$ fairness (uniformity) in prediction

Datasets - Overview

Dataset	Target type	Target range	Bin size	Max bin density	Min bin density	# Training set	# Val. set	# Test set
IMDB-WIKI-DIR	Age	0 - 186	1	7,149	1	191,509	11,022	11,022
AgeDB-DIR	Age	0 - 101	1	353	1	12,208	2,140	2,140
STS-B-DIR	Text similarity score	0 - 5	0.1	428	1	5,249	1,000	1,000
NYUD2-DIR	Depth	0.7 - 10	0.1	1.46×10^8	1.13×10^6	50,688 (3.51×10^9)	–	654 (8.70×10^5)
SHHS-DIR	Health condition score	0 - 100	1	275	0	1,892	369	369

Table credit: *Yang et al. (2021)*

(Training) Datasets - Label Distributions

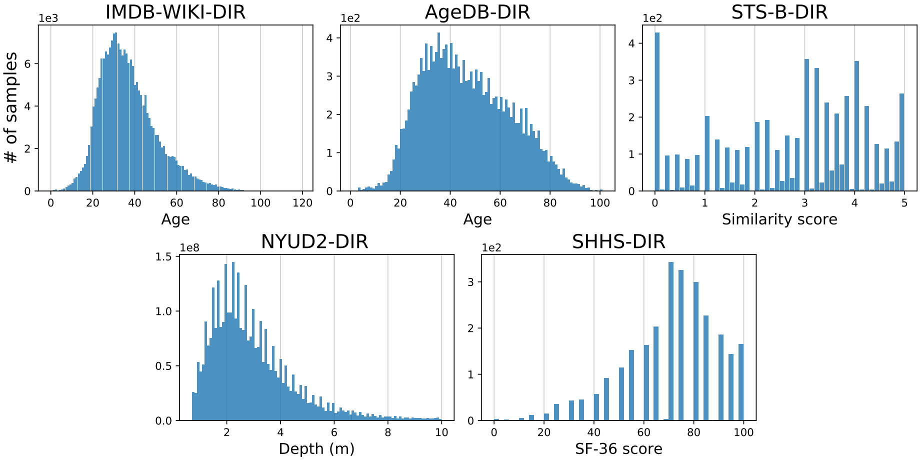
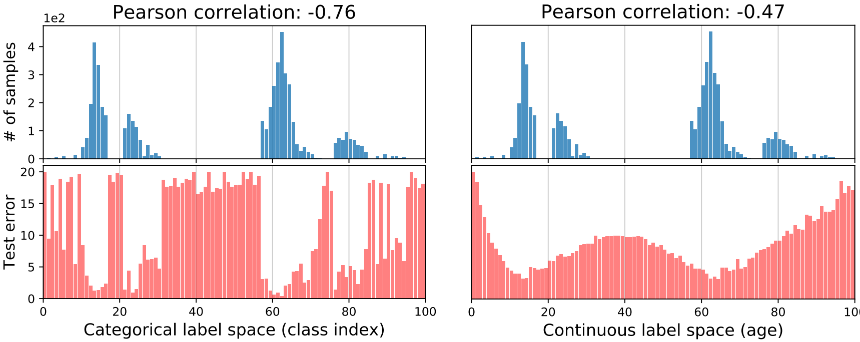


Image credit: *Yang et al. (2021)*

Label Distribution Smoothing (LDS)

Imbalanced Categorical vs. Continuous Label Space (1/3)



(a) Classification

- task: picture ⟶ class
- data souce: CIFAR-100

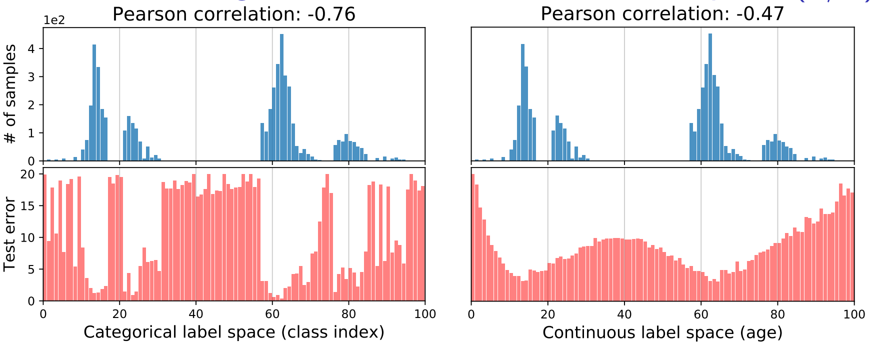
(b) Regression

- task:
  person's picture ⟶ person's age
- age subrange: 0-99
- data souce: IMDB-WIKI

- Simulated label imbalance
- Label density distributions forced to be equal
- Balanced test sets

Image credit: *Yang et al. (2021)*

Imbalanced Categorical vs. Continuous Label Space (2/3)



(a) Classification

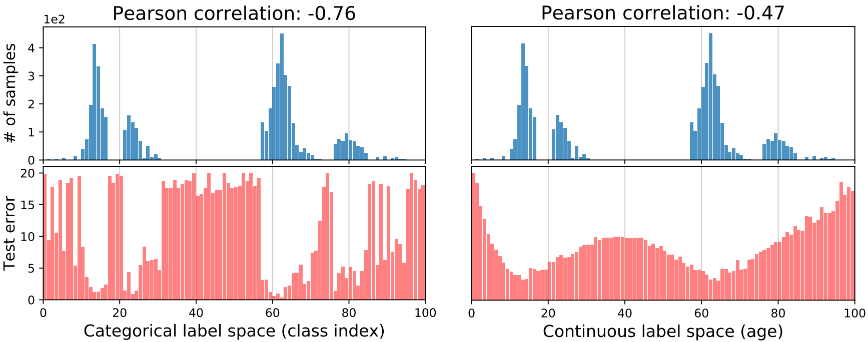
(b) Regression

- the error distribution *correlates* with the label density distribution
- majority classes with more examples are better learned than minority classes

- the error distribution DOES NOT *correlate* well with the label density distribution
- smoother error distribution

Image credit: *Yang et al. (2021)*

Imbalanced Categorical vs. Continuous Label Space (3/3)



(a) Classification

(b) Regression

- Compensating for the imbalance in the empirical label density distribution WORKS WELL.

- Compensating for the imbalance in the empirical label density distribution is INACCURATE.
- The empirical density does not accurately reflect the imbalance as seen by the model.
- Intuition: dependence between features at nearby labels.
- Proposed solution: **Label Distribution Smoothing (LDS)**

Image credit: *Yang et al. (2021)*

Label Distribution Smoothing (LDS) - Overview

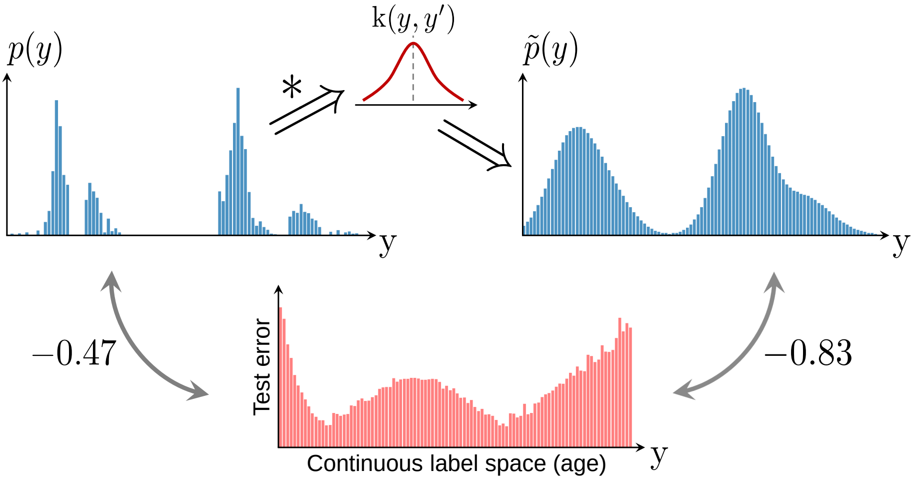


Image credit: *Yang et al. (2021)*

Label Distribution Smoothing (LDS)

- Starting points
  - Dependence between features at nearby continuous labels
  - Expected density estimation
    - Significant literature in statistics (Parzen 1962)
    - Kernel density estimation
- Functioning
  - Convolves a symmetric kernel with the empirical label density distribution.
  - Extracts a kernel-smoothed label density accounting for the feature overlap of neighbouring labels.
- Symmetric kernel
  - E.g., Gaussian or Laplacian kernel
  - Similarity between target values w.r.t. their distance in the target space.
- *Effective label density distribution*

$$\tilde{p}(y') \triangleq \int_{\mathcal{Y}} k(y, y') p(y) dy$$

  where
  - $p(y)$: nr. occurrences of label y in training data
- How to use it in practice?
  - Possible direct adaptation of class imbalance techniques.
  - E.g., loss weighted by inverse effective label density

Feature Distribution Smoothing (FDS) - Preliminaries

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  1. Continuity in the **target** space $\longleftrightarrow$ Continuity in the **feature** space

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  1. Continuity in the **target** space $\longleftrightarrow$ Continuity in the **feature** space
  2. Data balance $\Longrightarrow$ close feature statistics of nearby targets

- Feature statistics: mean and variance (or covariance) w.r.t. each bin

$$\{\boldsymbol{\mu}_b, \boldsymbol{\sigma}_b\}_{b=1}^{B}$$

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  1. Continuity in the **target** space $\longleftrightarrow$ Continuity in the **feature** space
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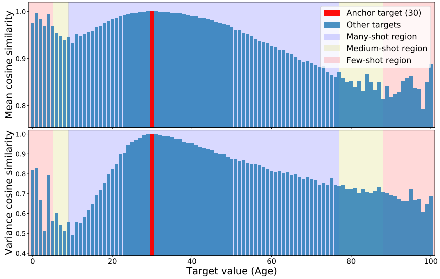
- Feature statistics: mean and variance (or covariance) w.r.t. each bin

$$\{\boldsymbol{\mu}_b, \boldsymbol{\sigma}_b\}_{b=1}^{B}$$

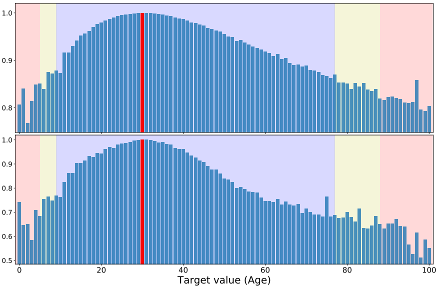
- (next slides) Feature statistics similarity: cosine similarity of feature statistics between one anchor bin b_0 and all other bins
  - $b_0 = 0, 30, 60, 90$ (age): chosen anchor bins
  - different target densities:
    many ($>$100), medium (20-100), few ($<$20) examples
  - task: person's picture $\longrightarrow$ person's age
  - data source: IMDB-WIKI

Feature statistics similarity (1/4)

Anchor age 30



(a) Baseline



(b) FDS

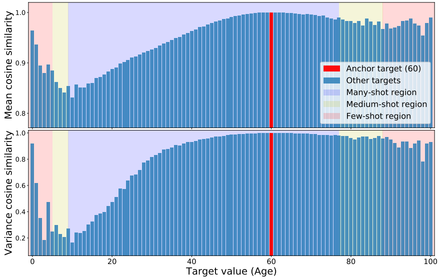
- High similarity in neighbourhood
- High similarities with further regions
- Lower similarities with some closer regions

- Improved feature statistics calibration:
  - High similarity only in neighbourhood
  - "The further the region the lower the similarity"
  - More gradual similarity change

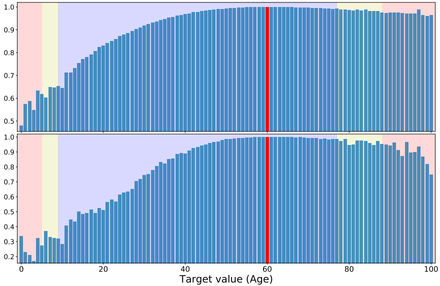
Image credit: *Yang et al. (2021)*

Feature statistics similarity (2/4)

Anchor age 60



(a) Baseline



(b) FDS

- High similarity in neighbourhood
- High similarities with further regions
- Lower similarities with some closer regions

- Improved feature statistics calibration:
  - High similarity only in neighbourhood
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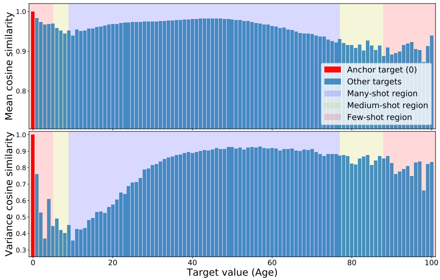
  - More gradual similarity change

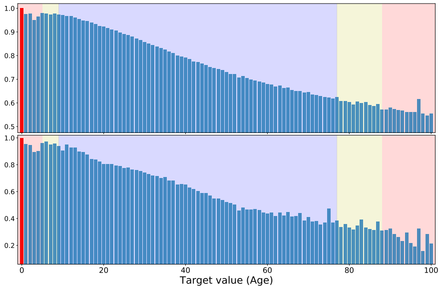
Image credit: *Yang et al. (2021)*

Feature statistics similarity (3/4)

Anchor age 0



(a) Baseline



(b) FDS

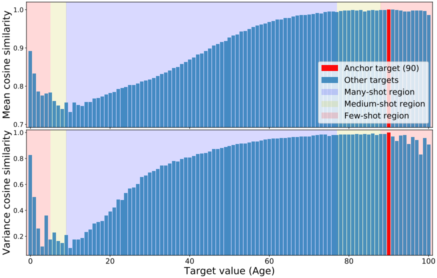
- High similarity in neighbourhood for mean
- High similarities with further regions
- Lower similarities with some closer regions, e.g., variance neighbourhood

- Improved feature statistics calibration:
  - High similarity only in neighbourhood
  - "The further the region the lower the similarity"
  - More gradual similarity change

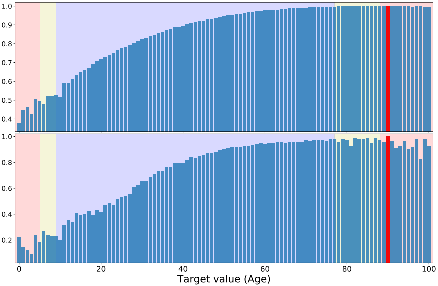
Image credit: *Yang et al. (2021)*

Feature statistics similarity (4/4)

Anchor age 90



(a) Baseline



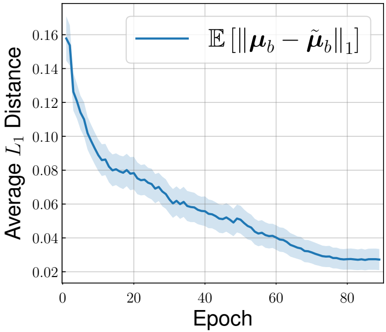
(b) FDS

- High similarity in neighbourhood, esp. for mean
- High similarities with further regions
- Lower similarities with some closer regions

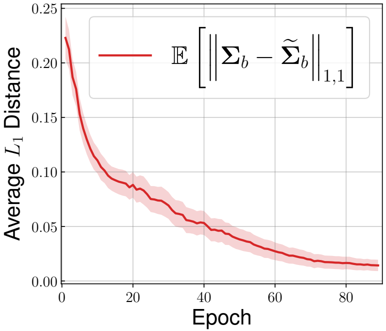
- Improved feature statistics calibration:
  - High similarity only in neighbourhood
  - "The further the region the lower the similarity"
  - More gradual similarity change

Image credit: *Yang et al. (2021)*

Change of feature statistics w.r.t. epoch



(a) Mean



(b) Covariance

- $\boldsymbol{\mu}, \boldsymbol{\Sigma}$: Running mean and covariance
- $\tilde{\boldsymbol{\mu}}, \tilde{\boldsymbol{\Sigma}}$: Smoothed mean and covariance

Image credit: *Yang et al. (2021)*

Feature Distribution Smoothing (FDS) - Overview

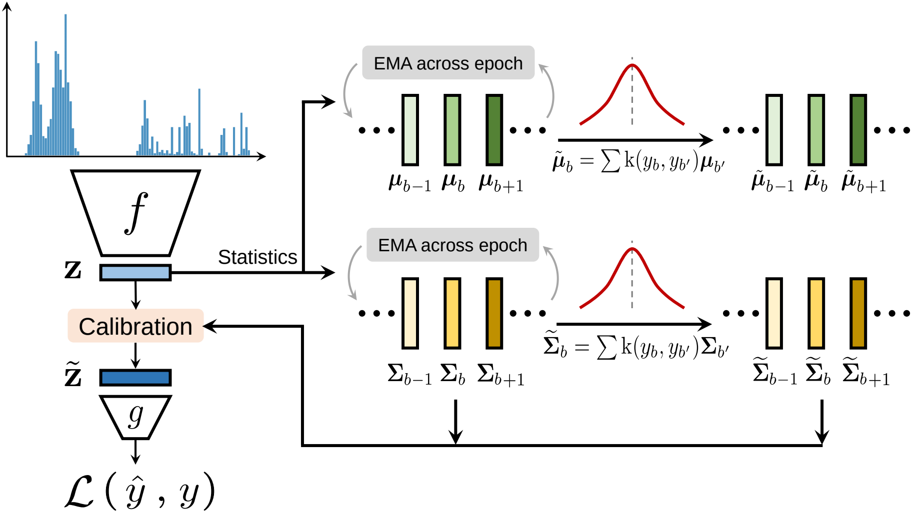


Image credit: *Yang et al. (2021)*

Feature Distribution Smoothing (FDS)

- Transfers the feature statistics between nearby bins.
- Aim: calibrate the potentially biased estimates of feature distribution, esp. for underrepresented target values in training data.
- General functioning
  - Estimates mean $\boldsymbol{\mu}_b$ and covariance $\boldsymbol{\Sigma}_b$ feature statistics by each target bin.
  - Smooths the feature statistics over the target bins $\mathcal{B}$ by a symmetric kernel $k(y_b, y'_b)$. Obtains the smoothed mean $\tilde{\boldsymbol{\mu}}_b$ and covariance $\tilde{\boldsymbol{\Sigma}}_b$ feature statistics.
  - Whitens features (Sun et al. 2016):

$$\boldsymbol{z}^w = \boldsymbol{\Sigma}_b^{-\frac{1}{2}}(\boldsymbol{z} - \boldsymbol{\mu}_b)$$

  - Re-colors whitened features (Sun et al. 2016):

$$\boldsymbol{z}^r = \tilde{\boldsymbol{\Sigma}}_b \boldsymbol{z}^w + \tilde{\boldsymbol{\mu}}_b$$

- Integration into deep learning
  - Feature calibration layer after the final feature map.
  - Momentum update running statistics $\{\boldsymbol{\mu}_b, \boldsymbol{\Sigma}_b\}$ across each epoch.
    - ★ Exponential Moving Average (EMA)
  - Smoothed statistics $\{\tilde{\boldsymbol{\Sigma}}_b, \tilde{\boldsymbol{\mu}}_b\}$ updated across different but fixed within each training epoch.

Baselines (1/2)

- Vanilla: neglects data imbalance

- Synthetic samples
  - SMOTER (Torgo et al. 2013)
    1. Defines frequent and rare regions using label density.
    2. Creates synthetic samples for pre-defined rare regions by linearly interpolating both inputs and labels.
  - SMOGN (Branco et al. 2017): augments SMOTER with Gaussian noise

- Focal-R

$$\frac{1}{n}\sum_{i=1}^{n}\sigma(|\beta e_i|)^{\gamma}e_i$$

  - Error-aware loss
  - Maps the absolute error into $[0, 1]$.
  - e_i: L_1 error for the i-th sample
  - β, γ: hyper-parameters
  - Inspired by Focal Loss (Lin 2017) for classification

Baselines (2/2)

- Regressor re-training (RRT)
  - Two-stage training
    1. Train encoder
    2. Re-train regressor with inverse re-weighting and frozen encoder.
  - Inspired by Kang et al. 2019

- Cost-sensitive re-weighting: re-weighting schemes based on label distribution
  - Inverse-frequency weighting (INV)
  - Square-root weighting variant (SQINV)

Results

Inferring Age from Images

IMDB-WIKI

Metrics	MAE ↓				GM ↓			
Shot	All	Many	Med.	Few	All	Many	Med.	Few
VANILLA	8.06	7.23	15.12	26.33	4.57	4.17	10.59	20.46
SMOTER (Torgo et al. 2013)	8.14	7.42	14.15	25.28	4.64	**4.30**	9.05	19.46
SMOGN (Branco et al. 2017)	8.03	**7.30**	14.02	25.93	4.63	**4.30**	8.74	20.12
SMOGN + **LDS**	8.02	7.39	13.71	23.22	4.63	4.39	8.71	15.80
SMOGN + **FDS**	8.03	7.35	14.06	23.44	4.65	4.33	8.87	16.00
SMOGN + **LDS** + **FDS**	**7.97**	7.38	**13.22**	**22.95**	**4.59**	4.39	**7.84**	**14.94**
FOCAL-R	7.97	7.12	15.14	26.96	4.49	4.10	10.37	21.20
FOCAL-R + **LDS**	7.90	**7.10**	14.72	25.84	**4.47**	**4.09**	10.11	19.14
FOCAL-R + **FDS**	7.96	7.14	14.71	26.06	4.51	4.12	10.16	19.56
FOCAL-R + **LDS** + **FDS**	**7.88**	**7.10**	**14.08**	**25.75**	**4.47**	4.11	**9.32**	**18.67**
RRT	7.81	7.07	14.06	25.13	4.35	4.03	8.91	16.96
RRT + **LDS**	7.79	7.08	13.76	24.64	4.34	**4.02**	8.72	16.92
RRT + **FDS**	**7.65**	**7.02**	12.68	23.85	**4.31**	4.03	7.58	16.28
RRT + **LDS** + **FDS**	**7.65**	7.06	**12.41**	**23.51**	**4.31**	4.07	**7.17**	**15.44**
SQINV	7.87	7.24	12.44	22.76	4.47	4.22	7.25	15.10
SQINV + **LDS**	7.83	7.31	**12.43**	22.51	4.42	4.19	7.00	13.94
SQINV + **FDS**	7.83	7.23	12.60	22.37	4.42	4.20	**6.93**	13.48
SQINV + **LDS** + **FDS**	**7.78**	**7.20**	12.61	**22.19**	**4.37**	**4.12**	7.39	**12.61**
Ours (best) VS. VANILLA	**+0.41**	**+0.21**	**+2.71**	**+4.14**	**+0.26**	**+0.15**	**+3.66**	**+7.85**

- Either LDS, FDS, or both leads to performance gains.
- LDS + FDS often achieves the best results:
  - maintains or improves performance overall and on many-shot regions,
  - boosts performance for medium-shot and few-shot regions.

Table credit: *Yang et al. (2021)*

Inferring Age from Images

AgeDB

Metrics	MAE ↓				GM ↓			
Shot	All	Many	Med.	Few	All	Many	Med.	Few
VANILLA	7.77	6.62	9.55	13.67	5.05	4.23	7.01	10.75
SMOTER (Torgo et al. 2013)	8.16	7.39	8.65	12.28	5.21	4.65	5.69	8.49
SMOGN (Branco et al. 2017)	8.26	7.64	9.01	12.09	5.36	4.90	6.19	8.44
SMOGN + **LDS**	7.96	7.44	8.64	11.77	5.03	4.68	5.69	7.98
SMOGN + **FDS**	8.06	7.52	8.75	11.89	5.02	4.66	5.63	8.02
SMOGN + **LDS** + **FDS**	**7.90**	**7.32**	**8.51**	**11.19**	**4.98**	**4.64**	**5.41**	**7.35**
FOCAL-R	7.64	6.68	9.22	13.00	4.90	4.26	6.39	9.52
FOCAL-R + **LDS**	7.56	**6.67**	8.82	12.40	4.82	4.27	5.87	8.83
FOCAL-R + **FDS**	7.65	6.89	8.70	**11.92**	4.83	4.32	5.89	**8.04**
FOCAL-R + **LDS** + **FDS**	**7.47**	6.69	**8.30**	12.55	**4.71**	**4.25**	**5.36**	8.59
RRT	7.74	6.98	8.79	11.99	5.00	4.50	5.88	8.63
RRT + **LDS**	7.72	7.00	8.75	11.62	4.98	4.54	5.71	8.27
RRT + **FDS**	7.70	**6.95**	8.76	11.86	4.82	**4.32**	5.83	8.08
RRT + **LDS** + **FDS**	**7.66**	6.99	**8.60**	**11.32**	**4.80**	4.42	**5.53**	**6.99**
SQINV	7.81	7.16	8.80	11.20	4.99	4.57	5.73	7.77
SQINV + **LDS**	7.67	**6.98**	8.86	10.89	4.85	4.39	5.80	7.45
SQINV + **FDS**	7.69	7.10	8.86	**9.98**	4.83	4.41	5.97	**6.29**
SQINV + **LDS** + **FDS**	**7.55**	7.01	**8.24**	10.79	**4.72**	**4.36**	**5.45**	6.79
Ours (best) VS. VANILLA	**+0.30**	**-0.05**	**+1.31**	**+3.69**	**+0.34**	**-0.02**	**+1.65**	**+4.46**

- Either LDS, FDS, or both leads to performance gains.
- LDS + FDS often achieves the best results:
  - maintains or improves performance overall and on many-shot regions,
  - boosts performance for medium-shot and few-shot regions.

Table credit: *Yang et al. (2021)*

Inferring Text Similarity Score

STS-B

Metrics	MSE ↓				Pearson correlation (%) ↑			
Shot	All	Many	Med.	Few	All	Many	Med.	Few
Vanilla	0.974	0.851	1.520	0.984	74.2	72.0	62.7	75.2
SmoteR (Torgo et al. 2013)	1.046	0.924	1.542	1.154	72.6	69.3	65.3	70.6
SMOGN (Branco et al. 2017)	0.990	0.896	1.327	1.175	73.2	70.4	65.5	69.2
SMOGN + **LDS**	0.962	0.880	1.242	1.155	74.0	71.5	65.2	69.8
SMOGN + **FDS**	0.987	0.945	**1.101**	1.153	73.0	69.6	**68.5**	69.9
SMOGN + **LDS** + **FDS**	**0.950**	**0.851**	1.327	**1.095**	**74.6**	**72.1**	65.9	**71.7**
Focal-R	0.951	0.843	1.425	0.957	74.6	72.3	61.8	76.4
Focal-R + **LDS**	0.930	**0.807**	1.449	0.993	**75.7**	**73.9**	62.4	75.4
Focal-R + **FDS**	**0.920**	0.855	**1.169**	1.008	75.1	72.6	**66.4**	74.7
Focal-R + **LDS** + **FDS**	0.940	0.849	1.358	**0.916**	74.9	72.2	66.3	**77.3**
RRT	0.964	0.842	1.503	0.978	74.5	72.4	62.3	75.4
RRT + **LDS**	0.916	0.817	1.344	0.945	75.7	73.5	64.1	76.6
RRT + **FDS**	0.929	0.857	**1.209**	1.025	74.9	72.1	**67.2**	74.0
RRT + **LDS** + **FDS**	**0.903**	**0.806**	1.323	**0.936**	**76.0**	**73.8**	65.2	**76.7**
Inv	1.005	0.894	1.482	1.046	72.8	70.3	62.5	73.2
Inv + **LDS**	0.914	0.819	1.319	0.955	75.6	73.4	63.8	76.2
Inv + **FDS**	0.927	0.851	**1.225**	1.012	75.0	72.4	**66.6**	74.2
Inv + **LDS** + **FDS**	**0.907**	**0.802**	1.363	**0.942**	**76.0**	**74.0**	65.2	**76.6**
Ours (best) vs. Vanilla	+.071	+.049	+.419	+.068	+1.8	+2.0	+5.8	+2.1

- Both LDS and FDS improve results for various methods, esp. medium- and few-shot regions.

Table credit: *Yang et al. (2021)*

Inferring Depth

NYUD2

Metrics	RMSE ↓				δ_1 ↑			
Shot	All	Many	Med.	Few	All	Many	Med.	Few
VANILLA	1.477	0.591	0.952	2.123	0.677	0.777	0.693	0.570
VANILLA + **LDS**	1.387	0.671	0.913	1.954	0.672	0.701	0.706	0.630
VANILLA + **FDS**	1.442	**0.615**	0.940	2.059	0.681	**0.760**	0.695	0.596
VANILLA + **LDS** + **FDS**	**1.338**	0.670	**0.851**	**1.880**	**0.705**	0.730	**0.764**	**0.655**
Ours (best) VS. VANILLA	**+.139**	**-.024**	**+.101**	**+.243**	**+.028**	**-.017**	**+.071**	**+.085**

FDS and LDS

- alleviates overfitting on many-shot regions,
- generalizes better to all regions,
- slightly degrades many-shot region,
- boosts other regions.

Table credit: *Yang et al. (2021)*

Inferring Health Score

SHHS-DIR

Metrics	MAE ↓				GM ↓			
Shot	All	Many	Med.	Few	All	Many	Med.	Few
VANILLA	15.36	12.47	13.98	16.94	10.63	8.04	9.59	12.20
FOCAL-R	14.67	11.70	13.69	17.06	9.98	7.93	8.85	11.95
FOCAL-R + **LDS**	14.49	12.01	12.43	16.57	9.98	7.89	8.59	11.40
FOCAL-R + **FDS**	14.18	**11.06**	13.56	15.99	9.45	**6.95**	8.81	11.13
FOCAL-R + **LDS** + **FDS**	**14.02**	11.08	**12.24**	**15.49**	**9.32**	7.18	**8.10**	**10.39**
RRT	14.78	12.43	14.01	16.48	10.12	8.05	9.71	11.96
RRT + **LDS**	14.56	12.08	13.44	16.45	9.89	7.85	9.18	11.82
RRT + **FDS**	14.36	11.97	13.33	16.08	9.74	7.54	9.20	11.31
RRT + **LDS** + **FDS**	**14.33**	**11.96**	**12.47**	**15.92**	**9.63**	**7.35**	**8.74**	**11.17**
INV	14.39	11.84	13.12	16.02	9.34	7.73	8.49	11.20
INV + **LDS**	14.14	11.66	12.77	16.05	9.26	7.64	8.18	11.32
INV + **FDS**	13.91	**11.12**	12.29	15.53	8.94	**6.91**	7.79	10.65
INV + **LDS** + **FDS**	**13.76**	**11.12**	**12.18**	**15.07**	**8.70**	6.94	**7.60**	**10.18**
Ours (best) VS. VANILLA	+1.60	+1.41	+1.80	+1.87	+1.93	+1.13	+1.99	+2.02

- Both FDS and LDS are effective.
- FDS + LDS often get highest gains over all tested regions.
- Note: SMOTER and SMOGN not directly applicable.

Table credit: *Yang et al. (2021)*

Could LDS + FDS help when the label distribution is skewed with one or more Gaussian peaks?

- Experimental setup
  - Curated skewed label distributions with 1-4 Gaussian peaks on IMDB-WIKI-DIR
  - Compared with the vanilla model
- Findings
  - Robustness to distribution change
  - Brings improvement

Skewed label distribution with one Gaussian peak

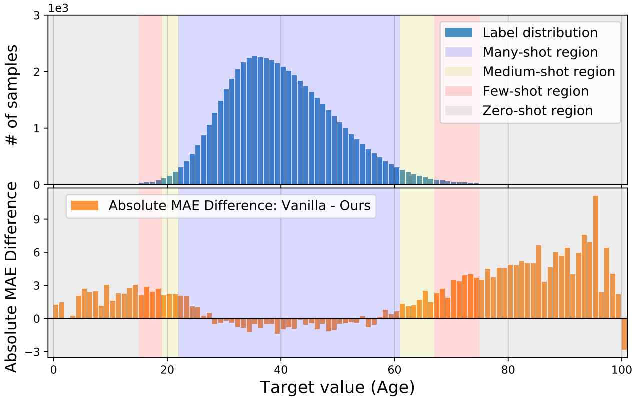


Figure: MAE gains of LDS + FDS over the vanilla model.

- Performance gains, esp. for extrapolation & interpolation

Image credit: *Yang et al. (2021)*

Skewed label distribution with two Gaussian peaks

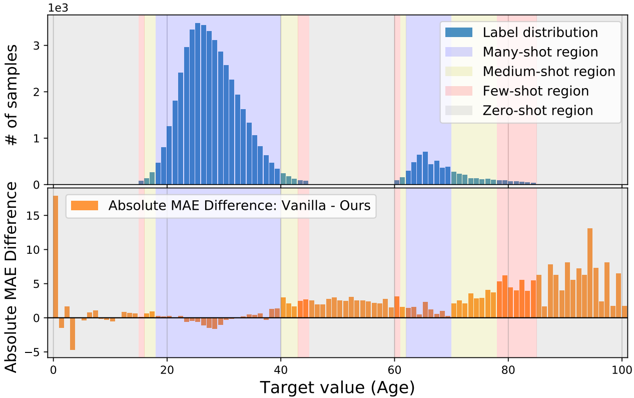


Figure: MAE gains of LDS + FDS over the vanilla model.

- Performance gains, esp. for extrapolation & interpolation

Image credit: *Yang et al. (2021)*

Skewed label distribution with three Gaussian peaks

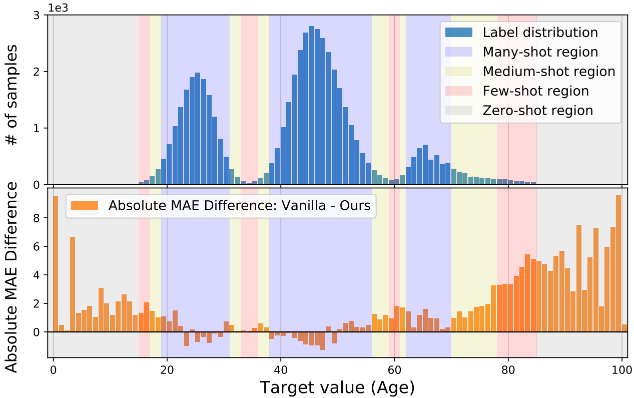


Figure: MAE gains of LDS + FDS over the vanilla model.

- Performance gains, esp. for extrapolation & interpolation

Image credit: *Yang et al. (2021)*

Skewed label distribution with four Gaussian peaks

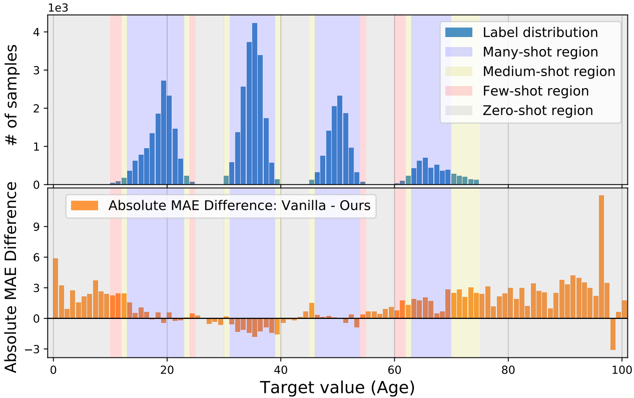


Figure: MAE gains of LDS + FDS over the vanilla model.

- Performance gains, esp. for extrapolation & interpolation

Image credit: *Yang et al. (2021)*

Skewed label distribution with two Gaussian peaks on IMDB-WIKI-DIR

Metrics	MAE ↓				GM ↓			
Shot	All	w/ data	Interp.	Extrap.	All	w/ data	Interp.	Extrap.
VANILLA	11.72	9.32	16.13	18.19	7.44	5.33	14.41	16.74
VANILLA + **LDS**	10.54	8.31	14.14	17.38	6.50	4.67	12.13	15.36
VANILLA + **FDS**	11.40	8.97	15.83	18.01	7.18	5.12	14.02	16.48
VANILLA + **LDS** + **FDS**	**10.27**	**8.11**	**13.71**	**17.02**	**6.33**	**4.55**	**11.71**	**15.13**
Ours (best) VS. VANILLA	**+1.45**	**+1.21**	**+2.42**	**+1.17**	**+1.11**	**+0.78**	**+2.70**	**+1.61**

Table: Interpolation & extrapolation results

- Best results by smoothing both label & feature distributions

Table credit: *Yang et al. (2021)*

Different skewed label distributions on IMDB-WIKI-DIR

Metrics	MAE ↓							GM ↓						
Shot	All	Many	Med.	Few	Zero	Interp.	Extrap.	All	Many	Med.	Few	Zero	Interp.	Extrap.
1 peak:														
Vanilla	11.20	6.05	11.43	14.76	22.67	–	22.67	7.02	**3.84**	8.67	12.26	21.07	–	21.07
Vanilla + **LDS**	10.09	6.26	9.91	12.12	19.37	–	19.37	6.14	3.92	6.50	8.30	16.35	–	16.35
Vanilla + **FDS**	11.04	**5.97**	11.19	14.54	22.35	–	22.35	6.96	**3.84**	8.54	12.08	20.71	–	20.71
Vanilla + **LDS** + **FDS**	**10.00**	6.28	**9.66**	**11.83**	**19.21**	–	**19.21**	**6.09**	3.96	**6.26**	**8.14**	**15.89**	–	**15.89**
2 peaks:														
Vanilla	11.72	6.83	11.78	15.35	16.86	16.13	18.19	7.44	3.61	8.06	12.94	15.21	14.41	16.74
Vanilla + **LDS**	10.54	6.72	9.65	12.60	15.30	14.14	17.38	6.50	3.65	**5.65**	9.30	13.20	12.13	15.36
Vanilla + **FDS**	11.40	6.69	11.02	14.85	16.61	15.83	18.01	7.18	**3.50**	7.49	12.73	14.86	14.02	16.48
Vanilla + **LDS** + **FDS**	**10.27**	**6.61**	**9.46**	**11.96**	**14.89**	**13.71**	**17.02**	**6.33**	3.54	5.68	**8.80**	**12.83**	**11.71**	**15.13**
3 peaks:														
Vanilla	9.83	7.01	9.81	11.93	20.11	–	20.11	6.04	3.93	6.94	9.84	17.77	–	17.77
Vanilla + **LDS**	9.08	**6.77**	8.82	10.48	18.43	–	18.43	**5.35**	**3.78**	5.63	7.49	15.46	–	15.46
Vanilla + **FDS**	9.65	6.88	9.58	11.75	19.80	–	19.80	5.86	3.83	6.68	9.48	17.43	–	17.43
Vanilla + **LDS** + **FDS**	**8.96**	6.88	**8.62**	**10.08**	**17.76**	–	**17.76**	5.38	3.90	**5.61**	**7.36**	**14.65**	–	**14.65**
4 peaks:														
Vanilla	9.49	7.23	9.73	10.85	12.16	8.23	18.78	5.68	3.45	6.95	8.20	9.43	6.89	16.02
Vanilla + **LDS**	8.80	**6.98**	8.26	10.07	11.26	8.31	**16.22**	5.10	**3.33**	**5.07**	7.08	8.47	6.66	**12.74**
Vanilla + **FDS**	9.28	7.11	9.16	10.88	11.95	8.30	18.11	5.49	3.36	6.35	8.15	9.21	6.82	15.30
Vanilla + **LDS** + **FDS**	**8.76**	7.07	**8.23**	**9.54**	**11.13**	**8.05**	16.32	**5.05**	3.36	**5.07**	**6.56**	**8.30**	**6.34**	13.10

Table credit: *Yang et al. (2021)*

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