Deep Imbalanced Regression

Yuzhe Yang[1] Kaiwen Zha[1] Ying-Cong Chen[1] Hao Wang[2]
Dina Katabi[1]

[1]MIT Computer Science & Artificial Intelligence Laboratory
[2]Department of Computer Science, Rutgers University

ICML 2021

Presenter: Gianmarco Midena

26 November 2024

Overview

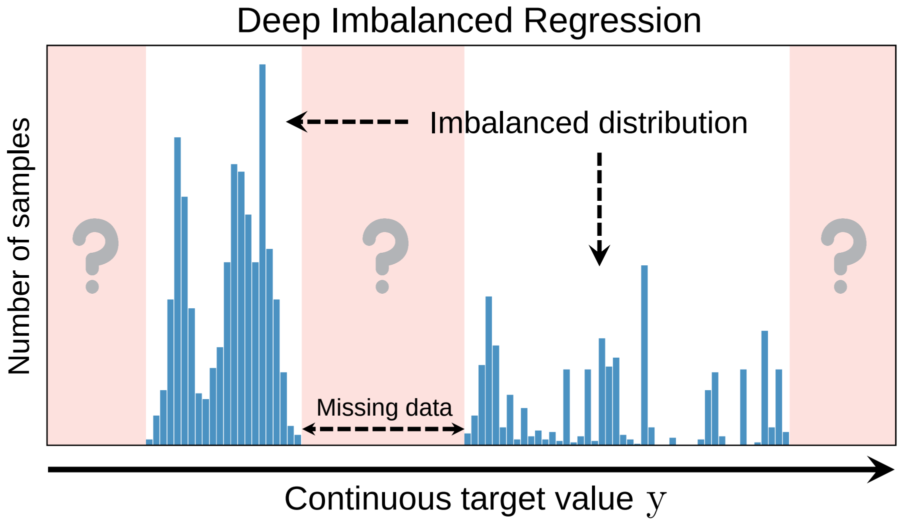


Image credit: *Yang et al. (2021)*

Problem Settings

- $\{(\mathbf{x}_i, y_i)\}_{i=1}^N$: training set
- $\mathbf{x}_i \in \mathbb{R}^d$: input
- $y_i \in \mathcal{Y}$: continuous label or target
- $b_i \in \mathcal{B}$: discrete label or target
- $\mathcal{Y} \subset \mathbb{R}$: continuous label space
- $\mathcal{B} = \{1, \ldots, M\} \subset \mathbb{Z}^+$: index space
  - divides $\mathcal{Y}$ into M groups (bins) with equal intervals $[t_j, t_{j+1})$
  - $\{[t_0, t_1), \ldots, [t_{M-1}, t_M)\}$: discrete label space
  - $t_k \in \mathcal{Y}$
  - minimum resolution
    - e.g., $\delta y \triangleq t_{j+1} - t_j = 1$ in age estimation
- $\hat{y}_i = g(\mathbf{z}_i) \in \mathbb{R}$: predicted continuous label
- $\mathbf{z}_i = f(\mathbf{x}_i; \theta) \in \mathbb{R}^{d'}$: learned representation
- θ: trainable model parameters

Evaluation

- Divide target space into disjoint regions (bins)
  - *Many-shot*: > 100 training examples
  - *Medium-shot*: 20-100 training examples
  - *Few-shot*: < 20 training examples
  - *Zero-shot*: 0 training examples
  - Inspired by Liu et al. 2019

- Metrics
  - Mean Absolute Error (MAE)
  - Mean Squared Error (MSE)
  - Pearson Correlation (PCC)
  - Geometric Mean Error (GM)

$$GM = \sqrt[n]{\prod_{i=1}^{n} |y_i - \hat{y}_i|}$$

★ Pros: $+$ fairness (uniformity) in prediction

Datasets - Overview

Dataset	Target type	Target range	Bin size	Max bin density	Min bin density	# Training set	# Val. set	# Test set
IMDB-WIKI-DIR	Age	0 - 186	1	7,149	1	191,509	11,022	11,022
AgeDB-DIR	Age	0 - 101	1	353	1	12,208	2,140	2,140
STS-B-DIR	Text similarity score	0 - 5	0.1	428	1	5,249	1,000	1,000
NYUD2-DIR	Depth	0.7 - 10	0.1	1.46×10^8	1.13×10^6	50,688 (3.51×10^9)	–	654 (8.70×10^5)
SHHS-DIR	Health condition score	0 - 100	1	275	0	1,892	369	369

Table credit: *Yang et al. (2021)*

(Training) Datasets - Label Distributions

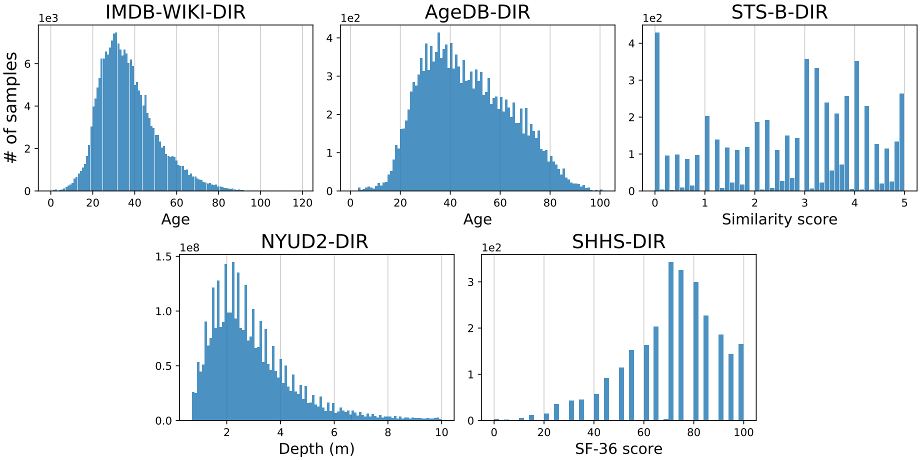
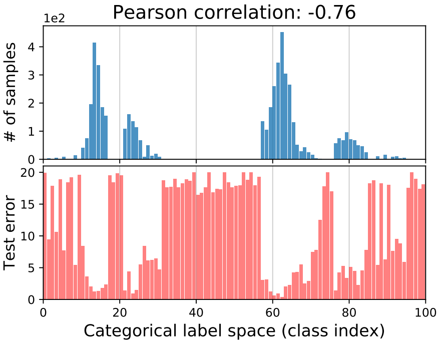


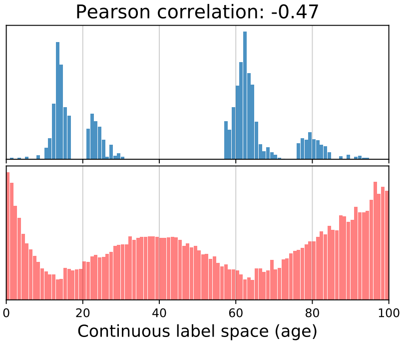
Image credit: *Yang et al. (2021)*

Label Distribution Smoothing (LDS)

Imbalanced Categorical vs. Continuous Label Space (1/3)



(a) Classification



(b) Regression

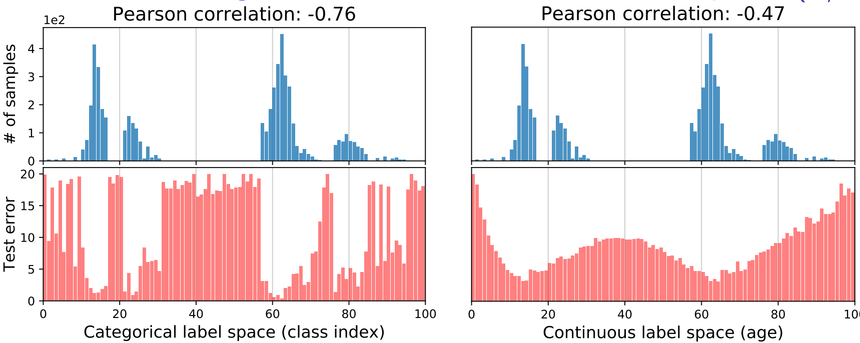
- task: picture ⟶ class
- data souce: CIFAR-100

- task:
  person's picture ⟶ person's age
- age subrange: 0-99
- data souce: IMDB-WIKI

- Simulated label imbalance
- Label density distributions forced to be equal
- Balanced test sets

Image credit: *Yang et al. (2021)*

Imbalanced Categorical vs. Continuous Label Space (2/3)



(a) Classification

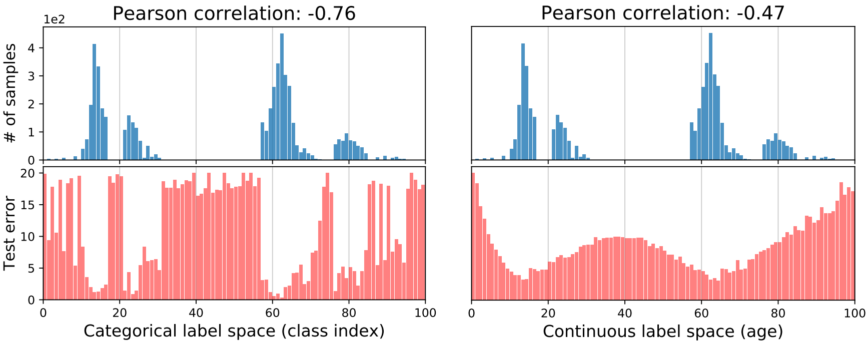
(b) Regression

- the error distribution *correlates* with the label density distribution
- majority classes with more examples are better learned than minority classes

- the error distribution DOES NOT *correlate* well with the label density distribution
- smoother error distribution

Image credit: *Yang et al. (2021)*

Imbalanced Categorical vs. Continuous Label Space (3/3)



(a) Classification

(b) Regression

- Compensating for the imbalance in the empirical label density distribution WORKS WELL.

- Compensating for the imbalance in the empirical label density distribution is INACCURATE.
- The empirical density does not accurately reflect the imbalance as seen by the model.
- Intuition: dependence between features at nearby labels.
- Proposed solution: **Label Distribution Smoothing (LDS)**

Image credit: *Yang et al. (2021)*

Label Distribution Smoothing (LDS) - Overview

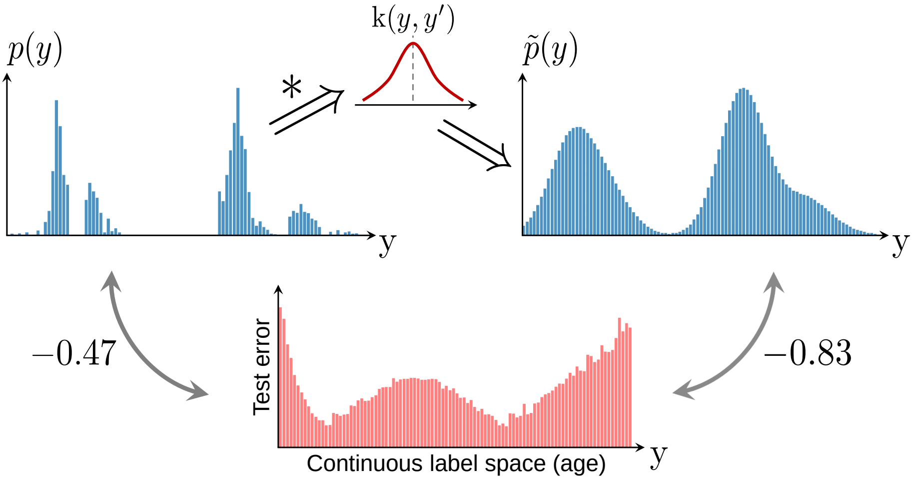


Image credit: *Yang et al. (2021)*

Label Distribution Smoothing (LDS)

- Starting points
  - Dependence between features at nearby continuous labels
  - Expected density estimation
    - Significant literature in statistics (Parzen 1962)
    - Kernel density estimation
- Functioning
  - Convolves a symmetric kernel with the empirical label density distribution.
  - Extracts a kernel-smoothed label density accounting for the feature overlap of neighbouring labels.
- Symmetric kernel
  - E.g., Gaussian or Laplacian kernel
  - Similarity between target values w.r.t. their distance in the target space.
- *Effective label density distribution*

$$\tilde{p}(y') \triangleq \int_{\mathcal{Y}} k(y, y')p(y)dy$$

  where
  - $p(y)$: nr. occurrences of label y in training data
- How to use it in practice?
  - Possible direct adaptation of class imbalance techniques.
  - E.g., loss weighted by inverse effective label density

Feature Distribution Smoothing (FDS) - Preliminaries

Feature Distribution Smoothing (FDS) - Preliminaries

- Starting points
  1. Continuity in the **target** space $\longleftrightarrow$ Continuity in the **feature** space

Feature Distribution Smoothing (FDS) - Preliminaries

- Starting points
  1. Continuity in the **target** space $\longleftrightarrow$ Continuity in the **feature** space
  2. Data balance $\Longrightarrow$ close feature statistics of nearby targets

Feature Distribution Smoothing (FDS) - Preliminaries

- Starting points
  1. Continuity in the **target** space $\longleftrightarrow$ Continuity in the **feature** space
  2. Data balance $\Longrightarrow$ close feature statistics of nearby targets

- Feature statistics: mean and variance (or covariance) w.r.t. each bin

$$\{\boldsymbol{\mu}_b, \boldsymbol{\sigma}_b\}_{b=1}^{B}$$

Feature Distribution Smoothing (FDS) - Preliminaries

- Starting points
  1. Continuity in the **target** space $\longleftrightarrow$ Continuity in the **feature** space
  2. Data balance $\Longrightarrow$ close feature statistics of nearby targets

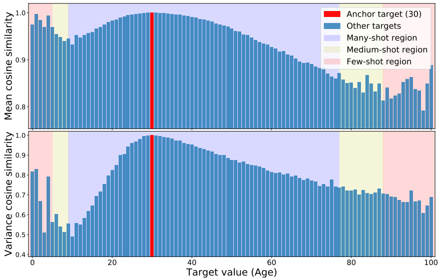
- Feature statistics: mean and variance (or covariance) w.r.t. each bin

$$\{\boldsymbol{\mu}_b, \boldsymbol{\sigma}_b\}_{b=1}^{B}$$

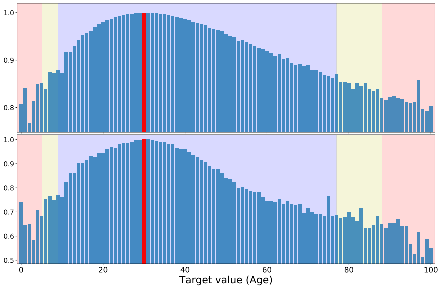
- (next slides) Feature statistics similarity: cosine similarity of feature statistics between one anchor bin b_0 and all other bins
  - $b_0 = 0, 30, 60, 90$ (age): chosen anchor bins
  - different target densities:
    many (>100), medium (20-100), few (<20) examples
  - task: person's picture $\longrightarrow$ person's age
  - data source: IMDB-WIKI

Feature statistics similarity (1/4)

Anchor age 30



(a) Baseline



(b) FDS

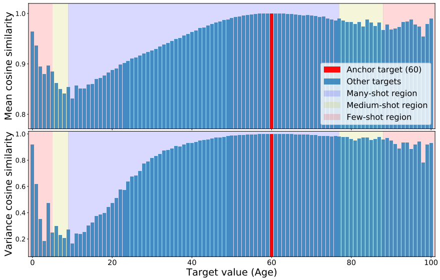
- High similarity in neighbourhood
- High similarities with further regions
- Lower similarities with some closer regions

- Improved feature statistics calibration:
  - High similarity only in neighbourhood
  - "The further the region the lower the similarity"
  - More gradual similarity change

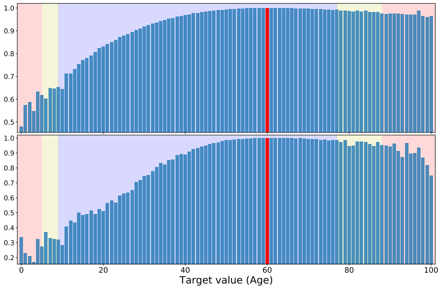
Image credit: *Yang et al. (2021)*

Feature statistics similarity (2/4)

Anchor age 60



(a) Baseline



(b) FDS

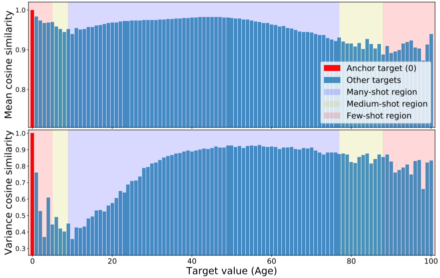
- High similarity in neighbourhood
- High similarities with further regions
- Lower similarities with some closer regions

- Improved feature statistics calibration:
  - High similarity only in neighbourhood
  - "The further the region the lower the similarity"
  - More gradual similarity change

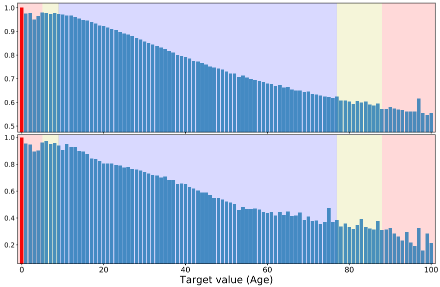
Image credit: *Yang et al. (2021)*

Feature statistics similarity (3/4)

Anchor age 0



(a) Baseline



(b) FDS

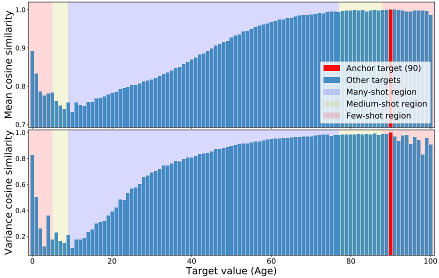
- High similarity in neighbourhood for mean
- High similarities with further regions
- Lower similarities with some closer regions, e.g., variance neighbourhood

- Improved feature statistics calibration:
  - High similarity only in neighbourhood
  - "The further the region the lower the similarity"
  - More gradual similarity change

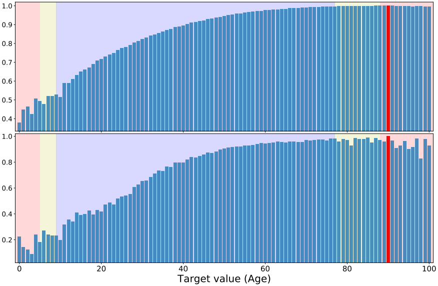
Image credit: *Yang et al. (2021)*

Feature statistics similarity (4/4)

Anchor age 90



(a) Baseline



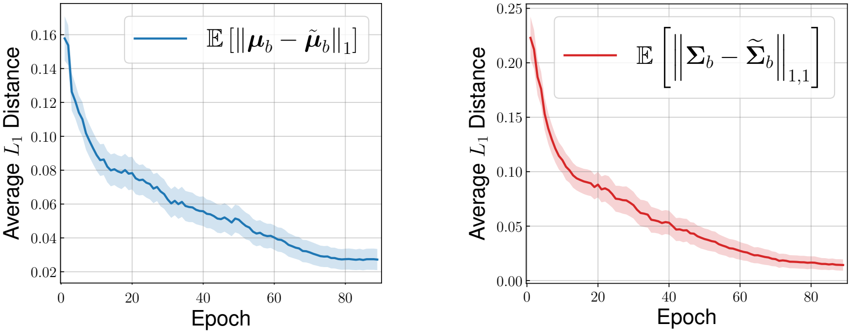
(b) FDS

- High similarity in neighbourhood, esp. for mean
- High similarities with further regions
- Lower similarities with some closer regions

- Improved feature statistics calibration:
  - High similarity only in neighbourhood
  - "The further the region the lower the similarity"
  - More gradual similarity change

Image credit: *Yang et al. (2021)*

Change of feature statistics w.r.t. epoch



(a) Mean

(b) Covariance

- $\boldsymbol{\mu}, \boldsymbol{\Sigma}$: Running mean and covariance
- $\tilde{\boldsymbol{\mu}}, \tilde{\boldsymbol{\Sigma}}$: Smoothed mean and covariance

Image credit: *Yang et al. (2021)*

Feature Distribution Smoothing (FDS) - Overview

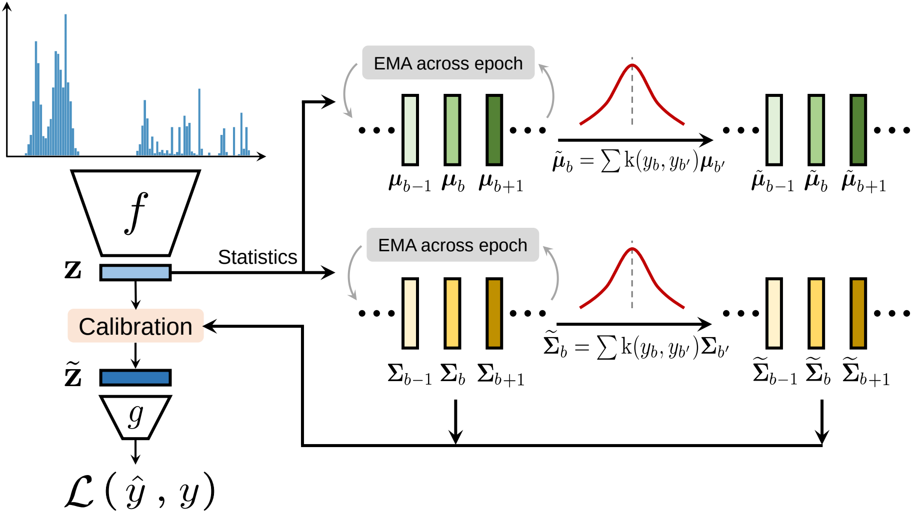


Image credit: *Yang et al. (2021)*

Feature Distribution Smoothing (FDS)

- Transfers the feature statistics between nearby bins.
- Aim: calibrate the potentially biased estimates of feature distribution, esp. for underrepresented target values in training data.
- General functioning
  - Estimates mean $\boldsymbol{\mu}_b$ and covariance $\boldsymbol{\Sigma}_b$ feature statistics by each target bin.
  - Smooths the feature statistics over the target bins $\mathcal{B}$ by a symmetric kernel $k(y_b, y'_b)$. Obtains the smoothed mean $\tilde{\boldsymbol{\mu}}_b$ and covariance $\tilde{\boldsymbol{\Sigma}}_b$ feature statistics.
  - Whitens features (Sun et al. 2016):

$$\boldsymbol{z}^w = \boldsymbol{\Sigma}_b^{-\frac{1}{2}}(\boldsymbol{z} - \boldsymbol{\mu}_b)$$

  - Re-colors whitened features (Sun et al. 2016):

$$\boldsymbol{z}^r = \tilde{\boldsymbol{\Sigma}}_b \boldsymbol{z}^w + \tilde{\boldsymbol{\mu}}_b$$

- Integration into deep learning
  - Feature calibration layer after the final feature map.
  - Momentum update running statistics $\{\boldsymbol{\mu}_b, \boldsymbol{\Sigma}_b\}$ across each epoch.
    - ⋆ Exponential Moving Average (EMA)
  - Smoothed statistics $\{\tilde{\boldsymbol{\Sigma}}_b, \tilde{\boldsymbol{\mu}}_b\}$ updated across different but fixed within each training epoch.

Baselines (1/2)

- Vanilla: neglects data imbalance

- Synthetic samples
  - SMOTER (Torgo et al. 2013)
    1. Defines frequent and rare regions using label density.
    2. Creates synthetic samples for pre-defined rare regions by linearly interpolating both inputs and labels.
  - SMOGN (Branco et al. 2017): augments SMOTER with Gaussian noise

- Focal-R

$$\frac{1}{n}\sum_{i=1}^{n}\sigma(|\beta e_i|)^{\gamma}e_i$$

  - Error-aware loss
  - Maps the absolute error into $[0, 1]$.
  - e_i: L_1 error for the i-th sample
  - β, γ: hyper-parameters
  - Inspired by Focal Loss (Lin 2017) for classification

Baselines (2/2)

- Regressor re-training (RRT)
  - Two-stage training
    1. Train encoder
    2. Re-train regressor with inverse re-weighting and frozen encoder.
  - Inspired by Kang et al. 2019
- Cost-sensitive re-weighting: re-weighting schemes based on label distribution
  - Inverse-frequency weighting (INV)
  - Square-root weighting variant (SQINV)

Results

Could LDS + FDS help when the label distribution is skewed with one or more Gaussian peaks?

- Experimental setup
  - Curated skewed label distributions with 1-4 Gaussian peaks on IMDB-WIKI-DIR
  - Compared with the vanilla model
- Findings
  - Robustness to distribution change
  - Brings improvement

Skewed label distribution with one Gaussian peak

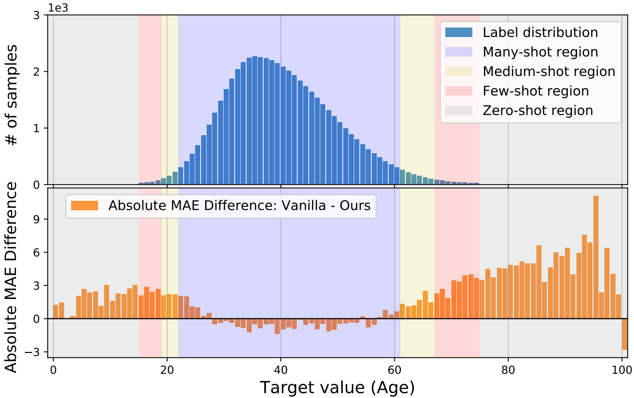


Figure: MAE gains of LDS + FDS over the vanilla model.

- Performance gains, esp. for extrapolation & interpolation

Image credit: *Yang et al. (2021)*

Skewed label distribution with two Gaussian peaks

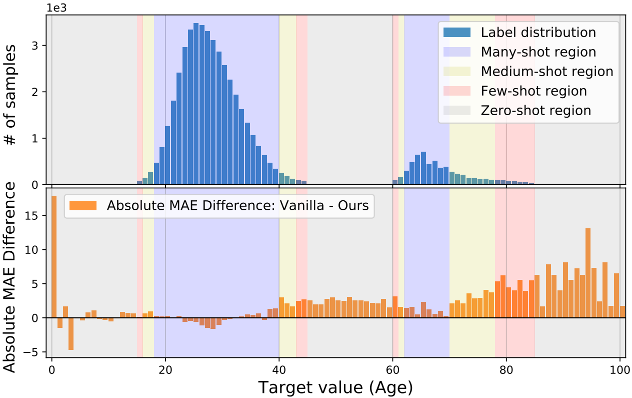


Figure: MAE gains of LDS + FDS over the vanilla model.

- Performance gains, esp. for extrapolation & interpolation

Image credit: *Yang et al. (2021)*

Skewed label distribution with three Gaussian peaks

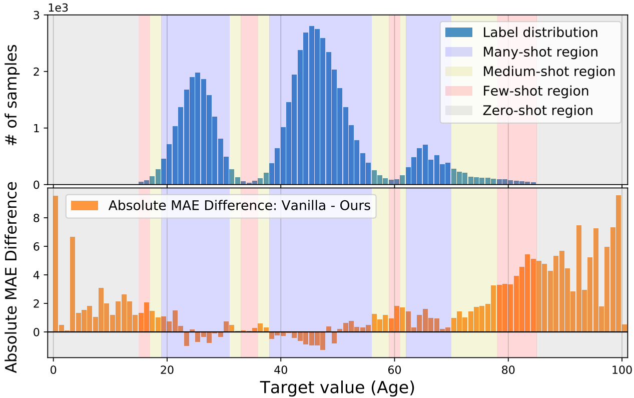


Figure: MAE gains of LDS + FDS over the vanilla model.

- Performance gains, esp. for extrapolation & interpolation

Image credit: *Yang et al. (2021)*

Skewed label distribution with four Gaussian peaks

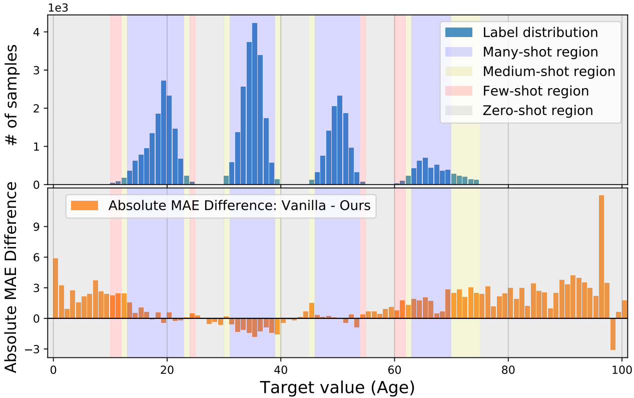


Figure: MAE gains of LDS + FDS over the vanilla model.

- Performance gains, esp. for extrapolation & interpolation

Image credit: *Yang et al. (2021)*

Skewed label distribution with two Gaussian peaks on IMDB-WIKI-DIR

Metrics	MAE ↓				GM ↓			
Shot	All	w/ data	Interp.	Extrap.	All	w/ data	Interp.	Extrap.
VANILLA	11.72	9.32	16.13	18.19	7.44	5.33	14.41	16.74
VANILLA + **LDS**	10.54	8.31	14.14	17.38	6.50	4.67	12.13	15.36
VANILLA + **FDS**	11.40	8.97	15.83	18.01	7.18	5.12	14.02	16.48
VANILLA + **LDS** + **FDS**	**10.27**	**8.11**	**13.71**	**17.02**	**6.33**	**4.55**	**11.71**	**15.13**
Ours (best) VS. VANILLA	**+1.45**	**+1.21**	**+2.42**	**+1.17**	**+1.11**	**+0.78**	**+2.70**	**+1.61**

Table: Interpolation & extrapolation results

- Best results by smoothing both label & feature distributions

Table credit: *Yang et al. (2021)*

Different skewed label distributions on IMDB-WIKI-DIR

Metrics	MAE ↓							GM ↓						
Shot	All	Many	Med.	Few	Zero	Interp.	Extrap.	All	Many	Med.	Few	Zero	Interp.	Extrap.
1 peak:														
VANILLA	11.20	6.05	11.43	14.76	22.67	–	22.67	7.02	**3.84**	8.67	12.26	21.07	–	21.07
VANILLA + **LDS**	10.09	6.26	9.91	12.12	19.37	–	19.37	6.14	3.92	6.50	8.30	16.35	–	16.35
VANILLA + **FDS**	11.04	**5.97**	11.19	14.54	22.35	–	22.35	6.96	**3.84**	8.54	12.08	20.71	–	20.71
VANILLA + **LDS** + **FDS**	**10.00**	6.28	**9.66**	**11.83**	**19.21**	–	**19.21**	**6.09**	3.96	**6.26**	**8.14**	**15.89**	–	**15.89**
2 peaks:														
VANILLA	11.72	6.83	11.78	15.35	16.86	16.13	18.19	7.44	3.61	8.06	12.94	15.21	14.41	16.74
VANILLA + **LDS**	10.54	6.72	9.65	12.60	15.30	14.14	17.38	6.50	3.65	**5.65**	9.30	13.20	12.13	15.36
VANILLA + **FDS**	11.40	6.69	11.02	14.85	16.61	15.83	18.01	7.18	**3.50**	7.49	12.73	14.86	14.02	16.48
VANILLA + **LDS** + **FDS**	**10.27**	**6.61**	**9.46**	**11.96**	**14.89**	**13.71**	**17.02**	**6.33**	3.54	5.68	**8.80**	**12.83**	**11.71**	**15.13**
3 peaks:														
VANILLA	9.83	7.01	9.81	11.93	20.11	–	20.11	6.04	3.93	6.94	9.84	17.77	–	17.77
VANILLA + **LDS**	9.08	**6.77**	8.82	10.48	18.43	–	18.43	**5.35**	**3.78**	5.63	7.49	15.46	–	15.46
VANILLA + **FDS**	9.65	6.88	9.58	11.75	19.80	–	19.80	5.86	3.83	6.68	9.48	17.43	–	17.43
VANILLA + **LDS** + **FDS**	**8.96**	6.88	**8.62**	**10.08**	**17.76**	–	**17.76**	5.38	3.90	**5.61**	**7.36**	**14.65**	–	**14.65**
4 peaks:														
VANILLA	9.49	7.23	9.73	10.85	12.16	8.23	18.78	5.68	3.45	6.95	8.20	9.43	6.89	16.02
VANILLA + **LDS**	8.80	**6.98**	8.26	10.07	11.26	8.31	**16.22**	5.10	**3.33**	**5.07**	7.08	8.47	6.66	**12.74**
VANILLA + **FDS**	9.28	7.11	9.16	10.88	11.95	8.30	18.11	5.49	3.36	6.35	8.15	9.21	6.82	15.30
VANILLA + **LDS** + **FDS**	**8.76**	7.07	**8.23**	**9.54**	**11.13**	**8.05**	16.32	**5.05**	3.36	**5.07**	**6.56**	**8.30**	**6.34**	13.10

Table credit: *Yang et al. (2021)*

References

Branco, Paula, Luís Torgo, and Rita P Ribeiro (2017). "SMOGN: a pre-processing approach for imbalanced regression". In: *First international workshop on learning with imbalanced domains: Theory and applications*. PMLR, pp. 36–50.

Kang, Bingyi et al. (2019). "Decoupling representation and classifier for long-tailed recognition". In: *arXiv preprint arXiv:1910.09217*.

Lin, T (2017). "Focal Loss for Dense Object Detection". In: *arXiv preprint arXiv:1708.02002*.

Liu, Ziwei et al. (2019). "Large-scale long-tailed recognition in an open world". In: *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 2537–2546.

Parzen, Emanuel (1962). "On estimation of a probability density function and mode". In: *The annals of mathematical statistics* 33.3, pp. 1065–1076.

Sun, Baochen, Jiashi Feng, and Kate Saenko (2016). "Return of frustratingly easy domain adaptation". In: *Proceedings of the AAAI conference on artificial intelligence*. Vol. 30. 1.

Torgo, Luís et al. (2013). "Smote for regression". In: *Portuguese*