CS 330, Spring 2023, Homework 1

Due: Wednesday 2/1 11:59 pm on Gradescope

Collaboration policy Collaboration on homework problems is permitted, you are allowed to discuss each problem with at most 3 other students currently enrolled in the class. Before working with others on a problem, you should think about it yourself for at least 45 minutes. Finding answers to problems on the Web or from other outside sources (these include anyone not enrolled in the class) is strictly forbidden.

You must write up each problem solution by yourself without assistance, even if you collaborate with others to solve the problem. You must also identify your collaborators. If you did not work with anyone, you should write "Collaborators: none." It is a violation of this policy to submit a problem solution that you cannot orally explain to an instructor or TA.

Typesetting Solutions should be typed and submitted as a PDF file on Gradescope. You may use any program you like to type your solutions. LaTeX, or "Latex", is commonly used for technical writing (`overleaf.com` is a free web-based platform for writing in Latex) since it handles math very well. Word, Google Docs, Markdown or other software are also fine.

Problem 1 *Order functions by asymptotic growth (10 points)*

Rank the functions given to you (in terms of Θ) in increasing order of complexity. Some of the functions are equivalent. In that case, write them on the same line. For each function (or group of equivalent functions), give as simple a function as you can that has the same (Θ) complexity.

For this problem we don't ask for any explanations or proofs other than listing the Θ value.

2^n	$\binom{n}{4} - 2n + 9$	$\log_5(5^{2n})$	$\sqrt{n}$	$(n-1)!$
$n^{3.01}$	$\log_2(n^n)$	$\log_3(n^3)$	$\sum_{i=1}^{n} i$	$3^{\frac{n}{100}}$
$n!$	$\log_2(n)$	$n! + 2^n$	$n^3 + n\log n + 1$	$\log_2(n^2!)$

Here is an example on the format. Suppose that the functions are

$$n^2 + 2, \quad 3n^3, \quad n + \log(n), \quad 5n^3 + n^2 + 2, \quad n^2 + \log(n).$$

Then you would write:

1. $n + \log(n) = \Theta(n)$
2. $n^2 + 2$, $n^2 + \log(n), \sum_{i=1}^{n} i = \Theta(n^2)$
3. $3n^3$, $5n^3 + n^2 + 2 = \Theta(n^3)$

Problem One Answers:

1. $\log_2(n), log_3(n^3)= \Theta(log(n))$
2. $\sqrt{n}= \Theta(n^{1/2})$
3. $log_5(5^{2n}) = (n)$
4. $log_2(n^n) = \Theta(nlog(n))$
5. $log_2(n^2!) = \Theta(n^2log(n))$
6. $\sum_{i=1}^{n} i = \Theta(n^2)$
7. $n^3 + n\log n + 1= \Theta(n^3)$
8. $n^{3.01}= \Theta(n^{3.01})$
9. $\binom{n}{4} - 2n + 9 = \Theta(n^4)$
10. $3^{\frac{n}{100}}= \Theta(3^{\frac{n}{100}})$
11. $2^n= \Theta(2^n)$
12. $(n-1)!= \Theta((n-1)!)$
13. $n!, n! + 2^n= \Theta(n!)$

Problem 2 *Compare functions (10 points)*

For each of the following pairs of functions state which one grows faster and why.

1. $(f(n/2))^3$ or $(f(n/3))^2$ where $f(n) = 2^n$
2. $4^{\log n}$ or $n^{1.5}$
3. $(1.2)^{n-1} + \sqrt{n} + \log n$ or $n^{1.5}$
4. $5^2 + 5^3 + \cdots + 5^{\log n}$ or n^3
5. $n(2 + \frac{\log n}{n^{0.5}})$ or $n \log\log n$

Problem Two Answers:

1. $(f(n/2))^3$ grows faster because the two functions have the same base but $(f(n/2))^3$ has a larger exponent (3n > 2n).
2. Since $4^{\log n}$ is raised to the power of $log(n)$ it will grow at a slower rate compared to $n^{1.5}$. For example, if n=16, $4^{\log 16} = 5.31$ whereas $16^{1.5} = 64$. This pattern continues for larger values of n as $n^{1.5}$ consistently grows faster than $4^{\log n}$.

3. $(1.2)^{n-1} + \sqrt{n} + \log n$ grows faster because the $(1.2)^{n-1}$ part of the function grows at exponential time. When comparing this function to $n^{1.5}$ which grows at quadratic time, the $(1.2)^{n-1} + \sqrt{n} + \log n$ increases faster.
4. n^3 grows at a faster rate. The function $5^2 + 5^3 + \cdots + 5^{\log n}$ always has a base 5 and sums up the values until it reaches $5^{\log n}$. Therefore the function could be represented through a partial summation. When comparing the summation growth to n^3, n^3 grows at a faster rate.
5. $n \log \log n$ grows at a faster rate. Since both $n(2 + \frac{\log n}{n^{0.5}})$ and $n \log \log n$ are multiplied by n, the main comparison between the two functions is between $\log \log n$ and $2 + \frac{\log n}{n^{0.5}}$. When looking at the growth of the two, the value of $2 + \frac{\log n}{n^{0.5}}$ decreases over time whereas the value of $\log \log n$ increases. Therefore $n \log \log n$ grows at a faster rate.

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