Chat Token Vector
Università Ca' Foscari
Venice, Italy

Toward a Critical Formalism

Philosophical and Theoretical Effects of a Mathematical Critique of LLMs

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Outline

Intro: Critique and Formalism

Epistemological Critique: LLMs as Formal Objects

Theoretical Critique: Formal Explainability
- The Algebra Behind the Embeddings
- The Structure Behind the Algebra
- The Categories Behind the Structure

Conclusion

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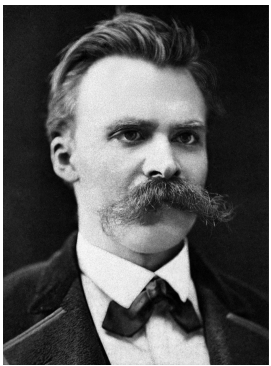
  - Underwood (2023):
    “The beautiful irony of this situation [...] is that a generation of humanists trained on Foucault have now rallied around “On the Dangers of Stochastic Parrots” to oppose a theory of language that their own disciplines invented, just at the moment when computer scientists are reluctantly beginning to accept it.”

The Birth of Contemporary Critique



"In some remote corner of the universe, flickering in the light of the countless solar systems into which it had been poured, there was once a planet on which clever animals invented cognition. It was the most arrogant and most mendacious minute in the 'history of the world'…"

"On Truth and Lying in a Non-Moral Sense"
(Nietzsche, 1873)

The Critical Argumentative Matrix

Knowledge depends on language

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Critique and Formalism

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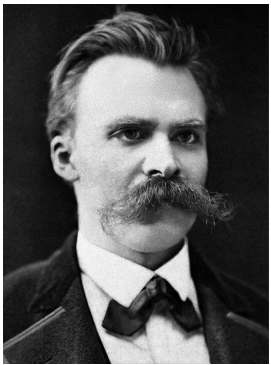
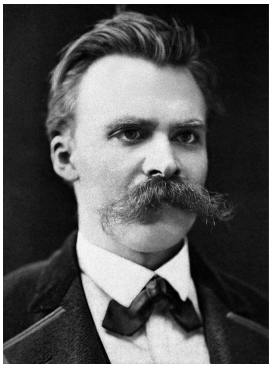
Critique vs Formalism



- “their eyes merely glide across the surface of things and see ‘forms’; nowhere does their perception lead into truth; instead it is content to receive stimuli and, as it were, to play with its fingers on the back of things.”

Critique vs Formalism



- "their eyes merely glide across the surface of things and see 'forms'; nowhere does their perception lead into truth; instead it is content to receive stimuli and, as it were, to play with its fingers on the back of things."
- "...the great edifice of concepts exhibits the rigid regularity of a *Roman columbarium*, while logic breathes out that air of severity and coolness which is peculiar to mathematics."
- "...if we are forced to comprehend all things under these forms alone, then it is no longer wonderful that what we comprehend in all these things is actually nothing other than these very forms; for all of them must exhibit the laws of number, and number is precisely that which is most astonishing about things."
- "...the only things we really know about them [laws of nature] are things which we bring to bear on them: time and space, in other words, relations of succession and number."

For a Critical Formalism

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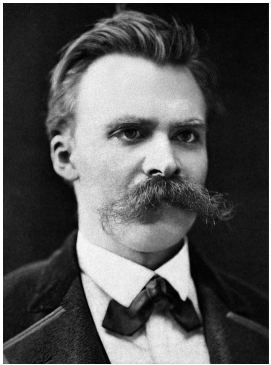
- We need a new strategy: Elaborate a critical formalism

For a Critical Formalism



"...all peoples have just such a mathematically divided firmament of concepts above them [...] Here, one can certainly admire humanity as a mighty architectural genius who succeeds in erecting the infinitely complicated cathedral of concepts on moving foundations, or even, one might say, on flowing water; admittedly, in order to rest on such foundations, it has to be like a thing constructed from cobwebs, so delicate that it can be carried off on the waves and yet so firm as not to be blown apart by the wind."

(Nietzsche, 1873)

For a Critical Formalism

- In the case of AI, a critical formalism can provide:
  - New epistemological tools countering dogmatic perspectives stemming from within the field
  - New theoretical tools contributing to the non-dogmatic positive production of knowledge

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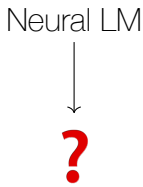
Neural LMs as Computable Functions



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Neural LM

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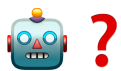

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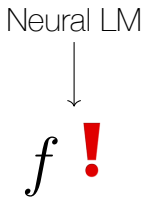

Neural LMs as Computable Functions

Neural LM

Function

$$f : \mathbb{R}^n \xrightarrow{f_1} \mathbb{R}^{n_1} \xrightarrow{f_2} \cdots \xrightarrow{f_K} \mathbb{R}^{n_K} \xrightarrow{g} \mathbb{R}^m$$

Neural LMs as Computable Functions

Neural LM

↓

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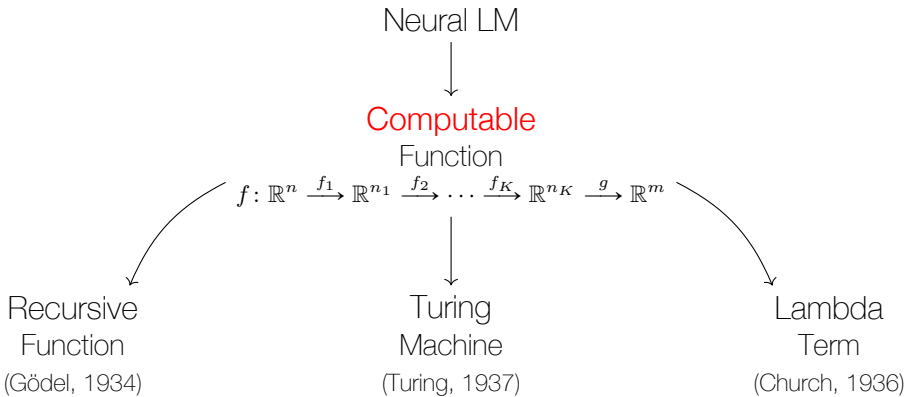
↓

Turing

Machine

(Turing, 1937)

Neural LMs as Computable Functions



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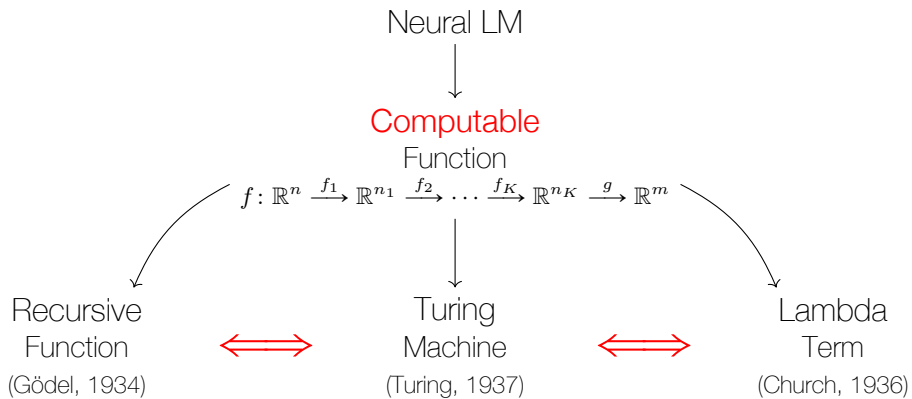

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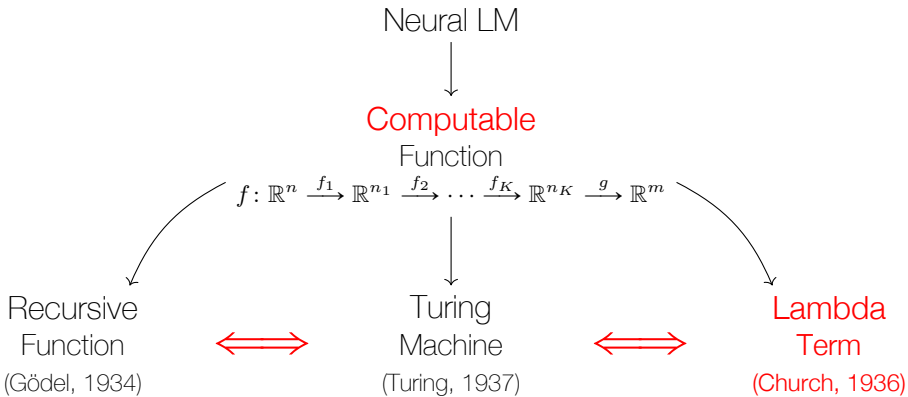

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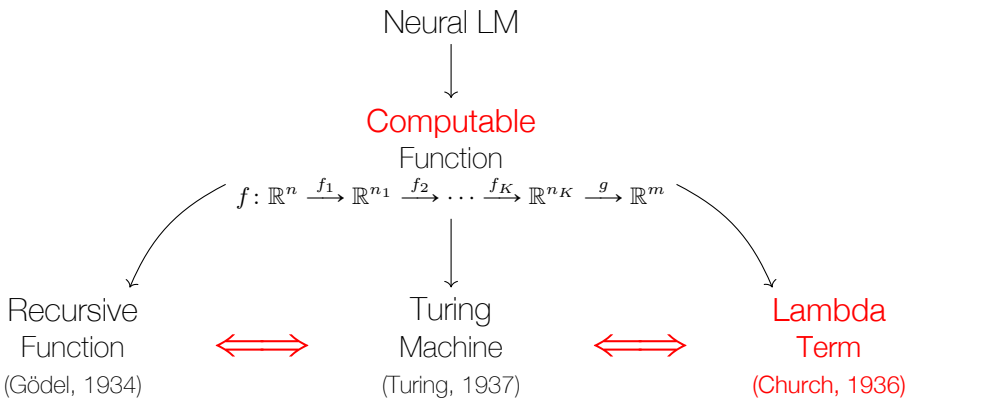

λ-abstraction and β-reduction in λ-calculus

$$yxz$$

λ-abstraction and β-reduction in λ-calculus

$$\lambda x.yxz$$

λ-abstraction and β-reduction in λ-calculus

$$(\lambda x.yxz)t$$

λ-abstraction and β-reduction in λ-calculus

$$(\lambda x.yxz)t$$

$$\downarrow$$

$$ytz$$

$P := \lambda m.\lambda n.\lambda f.\lambda x.mf(nfx)$

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0: $\lambda f.\lambda x.x$

1: $\lambda f.\lambda x.fx$

2: $\lambda f.\lambda x.f(fx)$

3: $\lambda f.\lambda x.f(f(fx))$

4: $\lambda f.\lambda x.f(f(f(fx)))$

5: $\lambda f.\lambda x.f(f(f(f(fx))))$

$\ldots$

n: $\lambda f.\lambda x.\underbrace{f(\ldots(f}_{n \text{ times}}x)\ldots)$

Empirical Evaluation

$P := \lambda m.\lambda n.\lambda f.\lambda x.mf(nfx)$

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$P' := \lambda r.\lambda s.\lambda f.\lambda x.f(f(f(f(fx))))$

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$\lambda r.\lambda s.\lambda f.\lambda x.f(f(f(f(fx))))(\lambda f.\lambda x.f(fx))(\lambda f.\lambda x.f(f(fx)))$

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$\ldots$

n: $\lambda f.\lambda x.\underbrace{f(\ldots(f}_{n \text{ times}}\, x)\ldots)$

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$\vdots$

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$$\lambda n.\lambda f.\lambda x.(\lambda g.\lambda y.g(gy))f(nfx)(\lambda h.\lambda z.h(h(hz)))$$
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$$\lambda f.\lambda x.(\lambda g.\lambda y.g(gy))f((\lambda h.\lambda z.h(h(hz)))fx)$$
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$$\lambda f.\lambda x.(\lambda y.f(fy))((\lambda z.f(f(fz)))x)$$
$$\lambda f.\lambda x.(\lambda y.f(fy))(f(f(fx)))$$
$$\lambda f.\lambda x.f(f(f(f(fx))))$$

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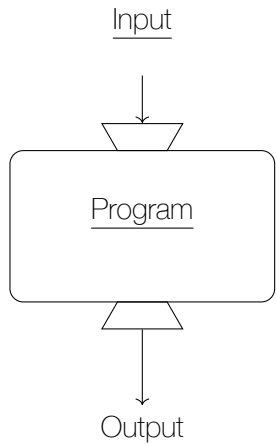

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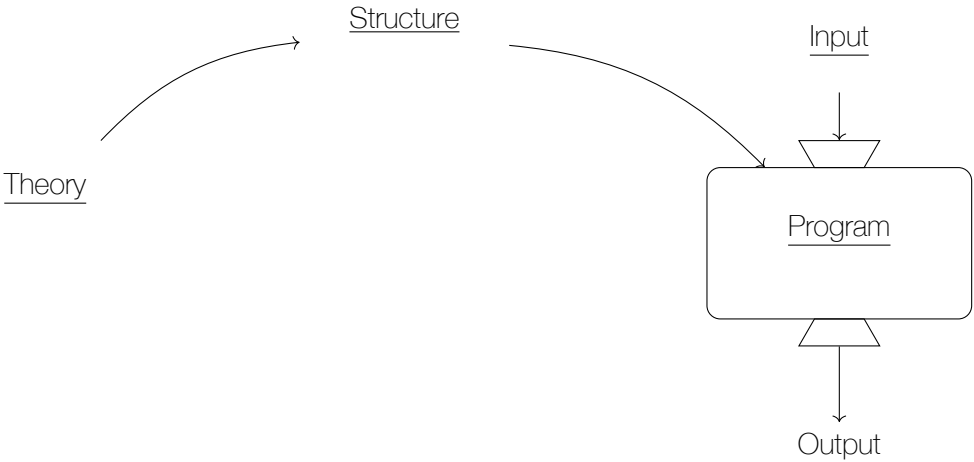

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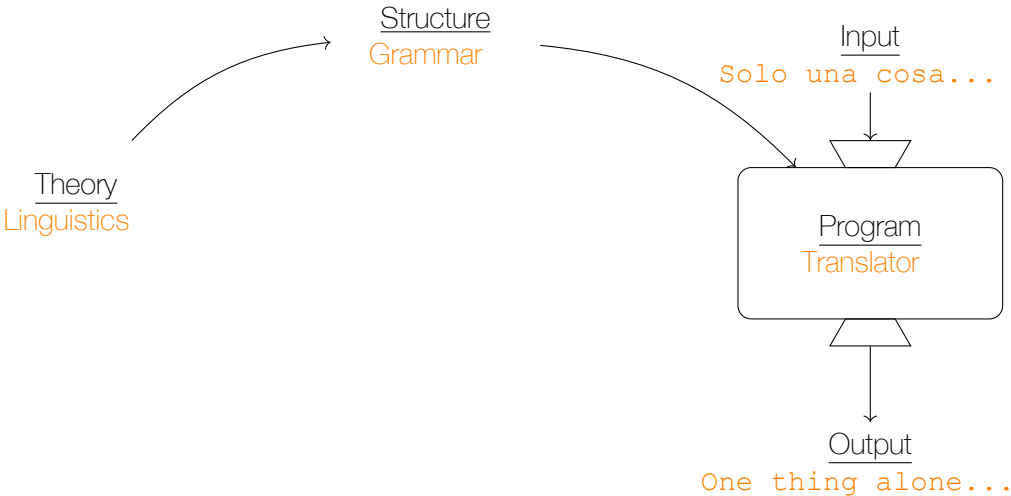

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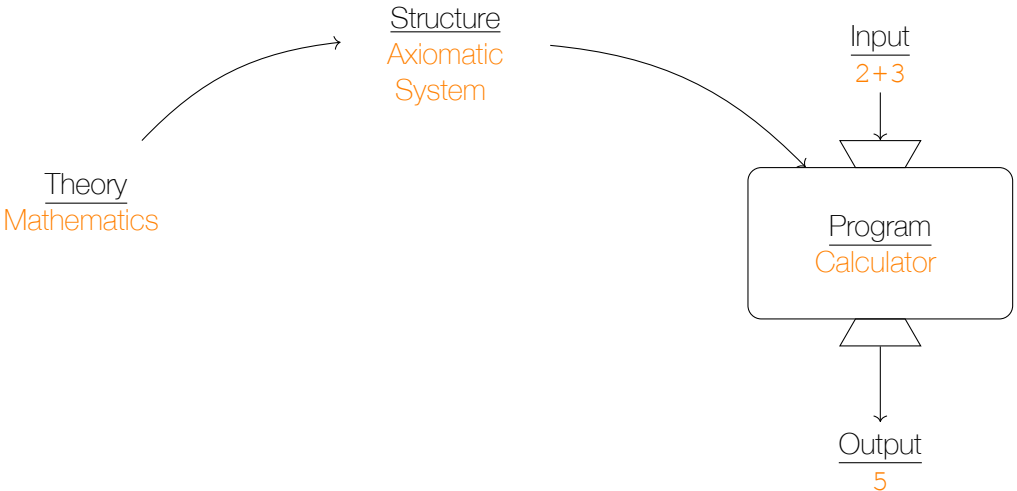

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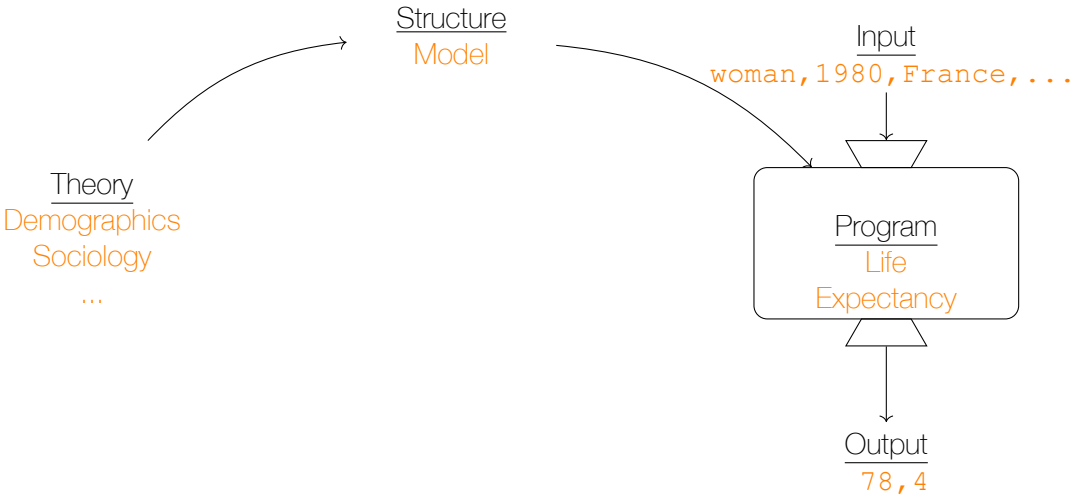

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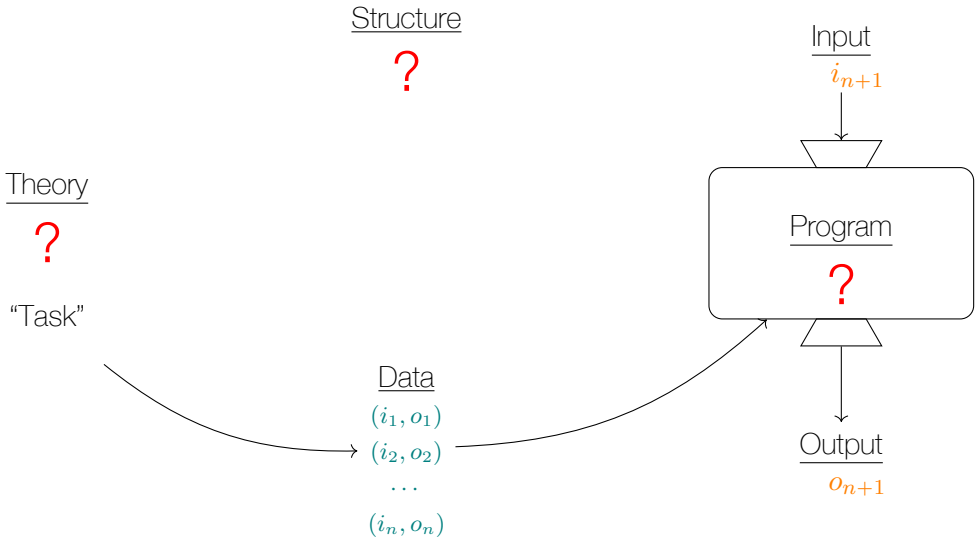

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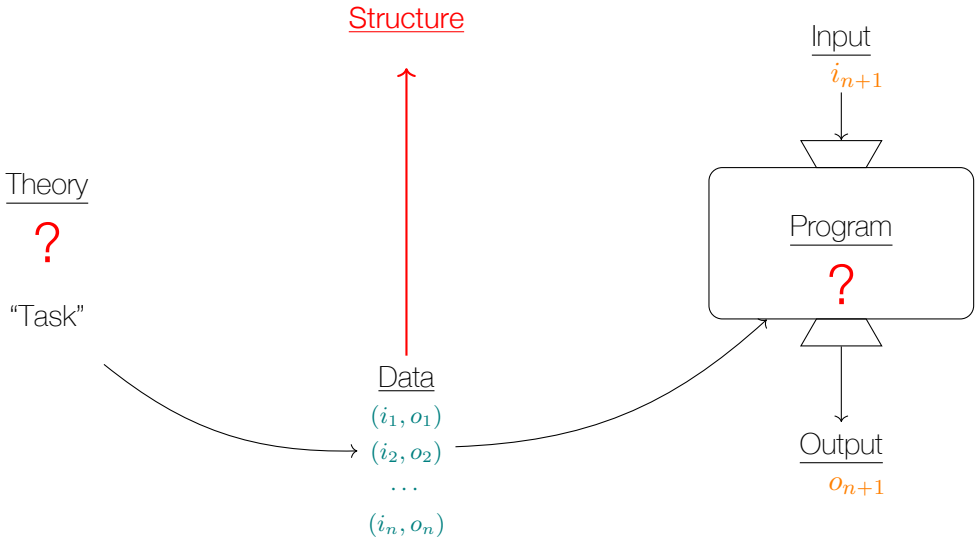

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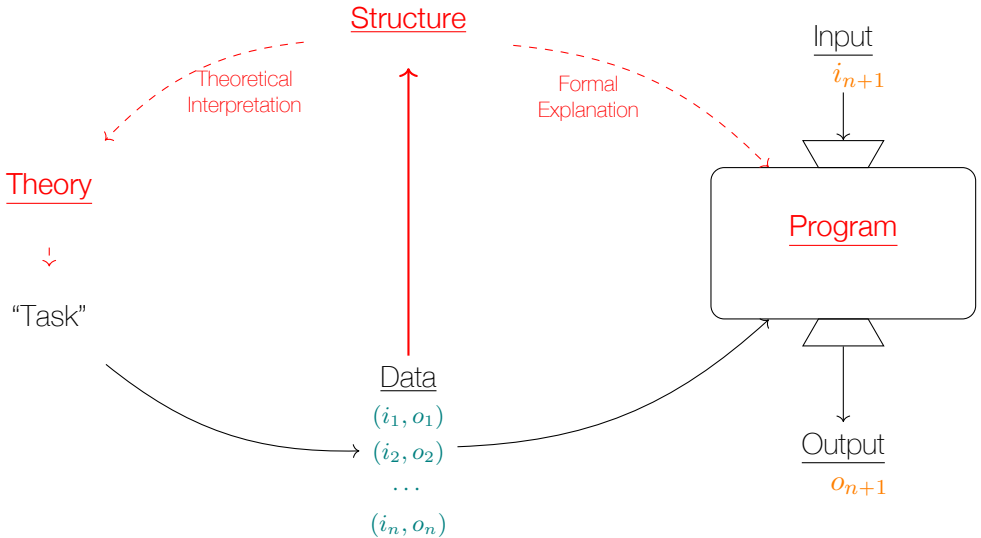

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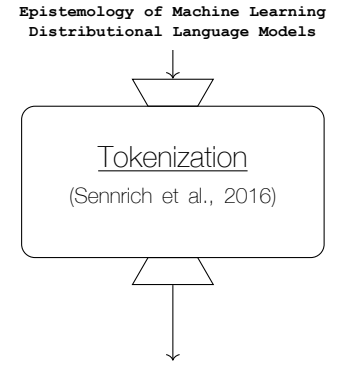
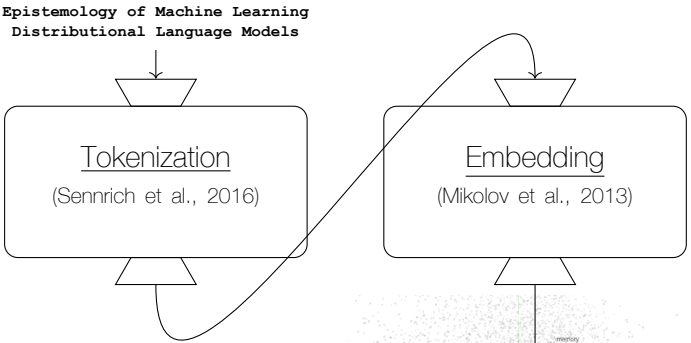


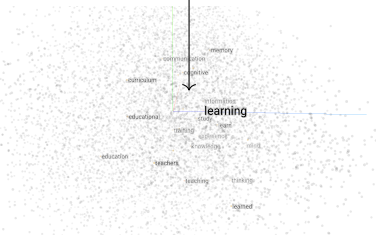
(https://tiktokenizer.vercel.app)

Formal Explainability



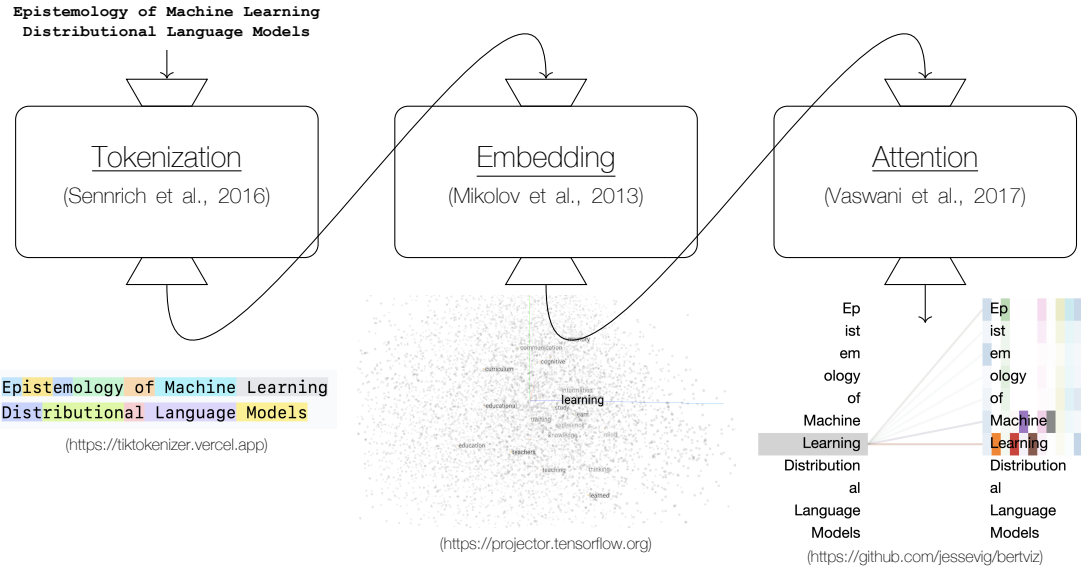
Epistemology of Machine Learning
Distributional Language Models

(https://tiktokenizer.vercel.app)

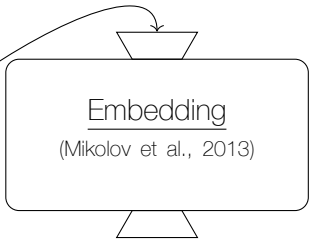


(https://projector.tensorflow.org)

Formal Explainability



Formal Explainability



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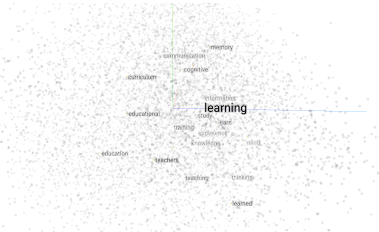
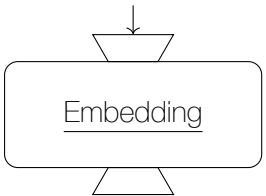
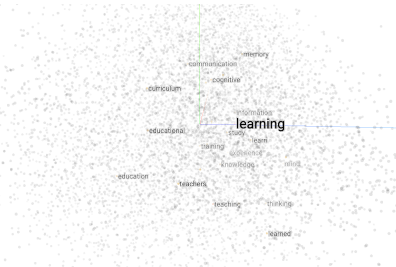


(https://projector.tensorflow.org)

The Structure of Embeddings

Epistemology of Machine Learning
Distributional Language Models





The Structure of Embeddings

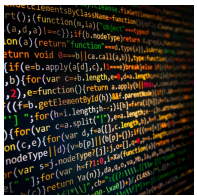
Structure

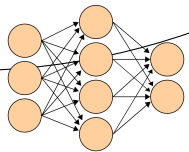
?

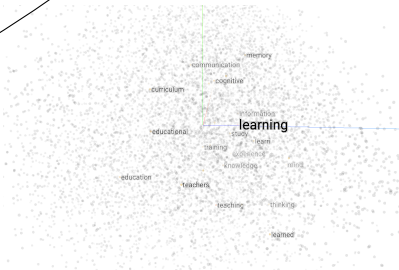
Epistemology of Machine Learning
Distributional Language Models

Embedding

Data







The Structure of Embeddings

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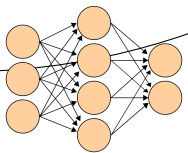
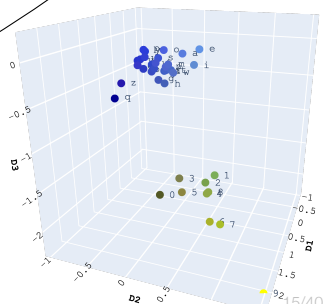
?

{-, /, 0, 1, 2, . . . , 8, 9, =,
a, b, c, . . . , w, x, y, z, é}

Embedding

Data





Outline

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Theoretical Critique: Formal Explainability
- The Algebra Behind the Embeddings
- The Structure Behind the Algebra
- The Categories Behind the Structure

Conclusion

word2vec Explained

(Levy and Goldberg, 2014)

$$\ell = \sum_{w \in V_w} \sum_{c \in V_c} \#(w,c)(\log \sigma(\vec{w} \cdot \vec{c}) + k \cdot \mathbb{E}_{c_N \sim P_D}[\log \sigma(-\vec{w} \cdot \vec{c}_N)])$$

$$\frac{\partial \ell}{\partial(\vec{w} \cdot \vec{c})} = 0 \quad \text{when} \quad \vec{w} \cdot \vec{c} \quad = \log \left(\frac{\#(w,c) \cdot |D|}{\#(w) \cdot \#(c)} \right) - \log k$$

- Word2vec performs an implicit, low-dimensional factorization of a pointwise mutual information (pmi), word-context matrix.

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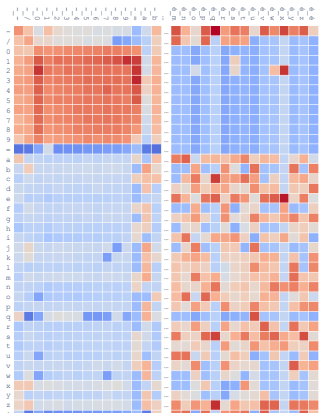
- Word2vec performs an implicit, low-dimensional factorization of a pointwise mutual information (pmi), word-context matrix.
- The Singular Value Decomposition (SVD) provides an exact solution to this problem.

Example: Characters in Wikipedia

$$W = \{\texttt{-}, \texttt{/}, \texttt{0}, \texttt{1}, \texttt{2}, \texttt{3}, \texttt{4}, \texttt{5}, \texttt{6}, \texttt{7}, \texttt{8}, \texttt{9}, \texttt{=}, \texttt{a}, \texttt{b}, \texttt{c}, \ldots, \texttt{w}, \texttt{x}, \texttt{y}, \texttt{z}, \texttt{é}\}$$

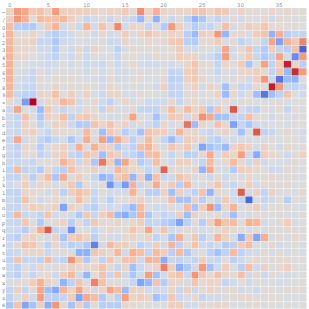
$$C = X \times X = \{\texttt{(-,-)}, \texttt{(-,/)}, \texttt{(-,0)}, \ldots, \texttt{(é,z)}, \texttt{(é,é)}\}$$

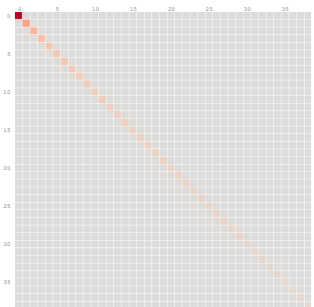
$$\begin{aligned} M_{wc} &= \operatorname{pmi}(w, c) \\ &= \log \frac{p(w, c)}{p(w)p(c)} \end{aligned}$$

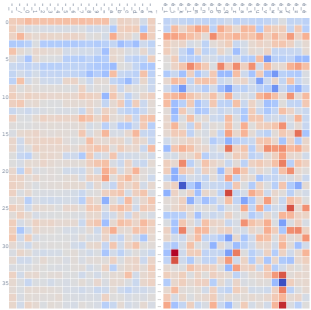


SVD of Wikipedia Character PMI Matrix

$$U \qquad \Sigma \qquad V^T$$

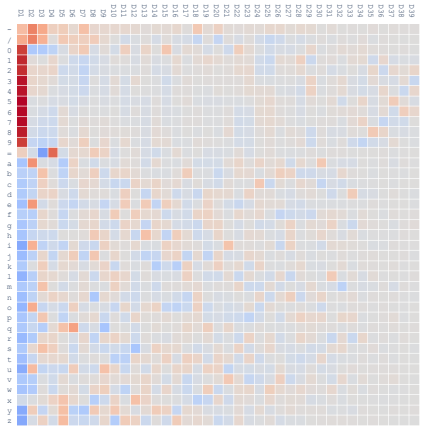






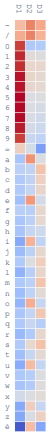
Truncate

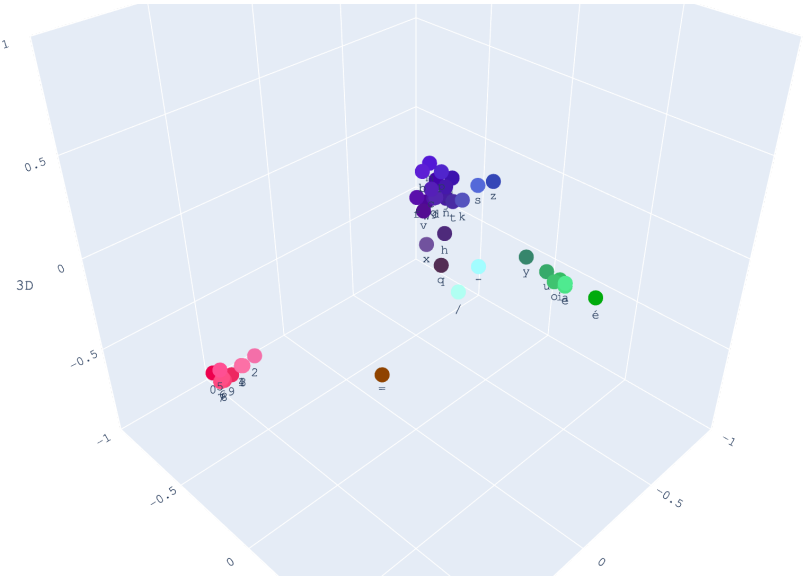
$$U \times \Sigma$$

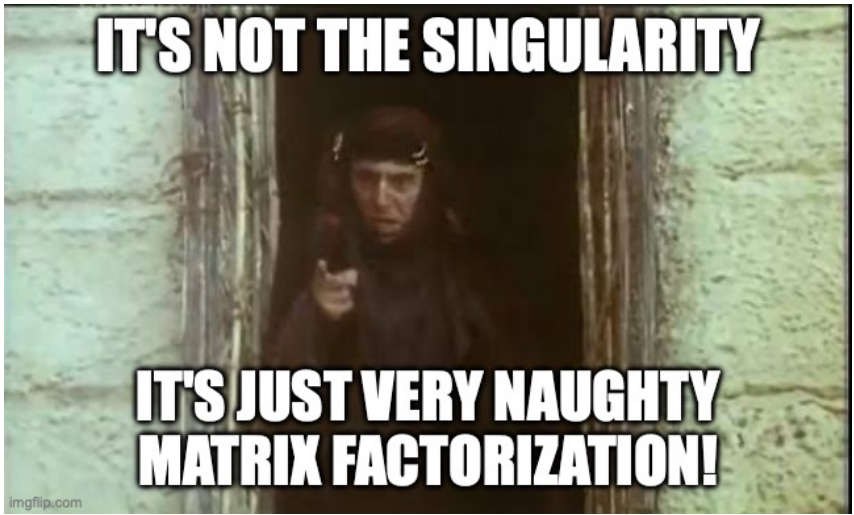


$$\hat{U} \times \hat{\Sigma}$$

$$\hat{U} \times \hat{\Sigma}$$






IT'S NOT THE SINGULARITY
IT'S JUST VERY NAUGHTY
MATRIX FACTORIZATION!
imgflip.com

4 Why does this produce good word representations?

Good question. We don't really know.

The distributional hypothesis states that words in similar contexts have similar meanings. The objective above clearly tries to increase the quantity $v_w \cdot v_c$ for good word-context pairs, and decrease it for bad ones. Intuitively, this means that words that share many contexts will be similar to each other (note also that contexts sharing many words will also be similar to each other). This is, however, very hand-wavy.

Can we make this intuition more precise? We'd really like to see something more formal.

(Goldberg and Levy, 2014)

The Structure of Embeddings

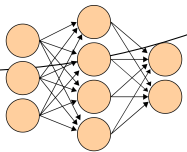
Structure

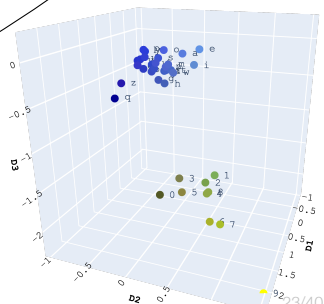
?

{-, /, 0, 1, 2, . . . , 8, 9, =,
a, b, c, . . . , w, x, y, z, é}

Embedding

Data





The Structure of Embeddings

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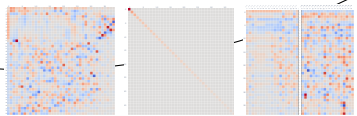
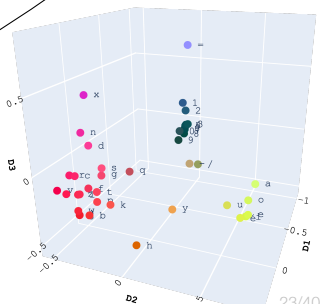
Embedding

Data



SVD



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Embeddings as Functions Over Sets

$$X = \{\texttt{-}, \texttt{/}, \texttt{0}, \texttt{1}, \texttt{2}, \texttt{3}, \texttt{4}, \texttt{5}, \texttt{6}, \texttt{7}, \texttt{8}, \texttt{9}, \texttt{=}, \texttt{a}, \texttt{b}, \texttt{c}, \ldots, \texttt{w}, \texttt{x}, \texttt{y}, \texttt{z}, \texttt{é}\}$$

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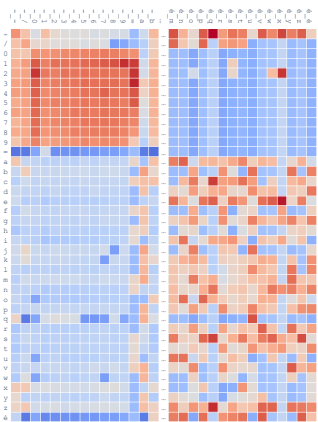
$$M \colon X \times Y \to \mathbb{R}$$

$$(x, y) \mapsto \mathrm{pmi}(x, y)$$



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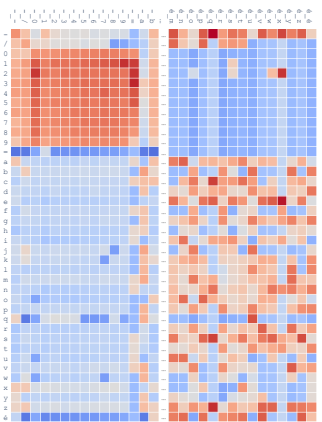
$$M_x \colon X \to \mathbb{R}^Y$$

$$x \mapsto M(x, -)$$



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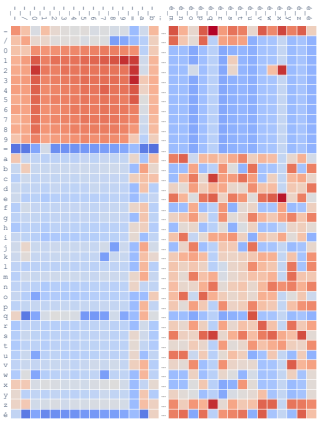

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$$X \xrightarrow{M_x} \mathbb{R}^Y$$

$$\mathbb{R}^X \xleftarrow[M_y]{} Y$$

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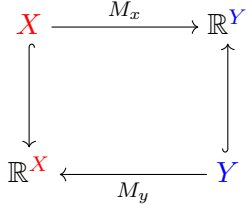


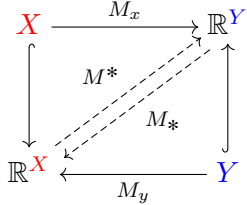
Embeddings as Functions Over Sets

$$X = \{\texttt{-}, \texttt{/}, \texttt{0}, \texttt{1}, \texttt{2}, \texttt{3}, \texttt{4}, \texttt{5}, \texttt{6}, \texttt{7}, \texttt{8}, \texttt{9}, \texttt{=}, \texttt{a}, \texttt{b}, \texttt{c}, \ldots, \texttt{w}, \texttt{x}, \texttt{y}, \texttt{z}, \texttt{é}\}$$
$$Y = X \times X = \{\texttt{(-,-)}, \texttt{(-,/)}, \texttt{(-,0)}, \ldots, \texttt{(é,z)}, \texttt{(é,é)}\}$$

$$M\colon X \times Y \to \mathbb{R}$$
$$(x, y) \mapsto \mathrm{pmi}(x, y)$$

$$M_x\colon X \to \mathbb{R}^Y$$
$$x \mapsto M(x, -)$$

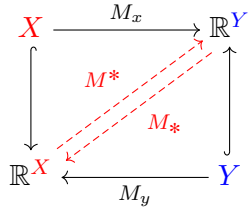
$$M_y\colon Y \to \mathbb{R}^X$$
$$y \mapsto M(-, y)$$



$$M^*\colon \mathbb{R}^X \to \mathbb{R}^Y$$
$$M_*\colon \mathbb{R}^Y \to \mathbb{R}^X$$

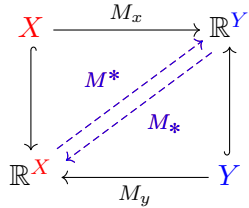
Embeddings as Functions Over Sets

$M_* M^* : \mathbb{R}^X \to \mathbb{R}^X$



Embeddings as Functions Over Sets

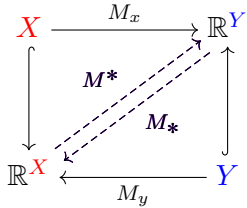
$$M_* M^* : \mathbb{R}^X \to \mathbb{R}^X$$
$$M^* M_* : \mathbb{R}^Y \to \mathbb{R}^Y$$



Embeddings as Functions Over Sets

$$M_* M^* \colon \mathbb{R}^X \to \mathbb{R}^X$$
$$M^* M_* \colon \mathbb{R}^Y \to \mathbb{R}^Y$$

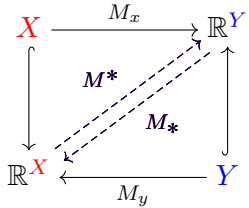
$$\{u_1, \ldots, u_m\} \subset \mathbb{R}^X$$
$$\{v_1, \ldots, v_n\} \subset \mathbb{R}^Y$$
$$\{\lambda_1, \ldots, \lambda_{\min(m,n)}, 0, \ldots, 0\}$$



Embeddings as Functions Over Sets

$$M_* M^* \colon \mathbb{R}^X \to \mathbb{R}^X$$
$$M^* M_* \colon \mathbb{R}^Y \to \mathbb{R}^Y$$

$$\{u_1, \ldots, u_m\} \subset \mathbb{R}^X$$
$$\{v_1, \ldots, v_n\} \subset \mathbb{R}^Y$$
$$\{\lambda_1, \ldots, \lambda_{\min(m,n)}, 0, \ldots, 0\}$$



$$M = U \Sigma V^T$$

$$U := [u_1, \ldots, u_m]$$
$$V := [v_1, \ldots, v_n]$$
$$\Sigma := \begin{bmatrix} \sqrt{\lambda_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{\lambda_r} \end{bmatrix}$$

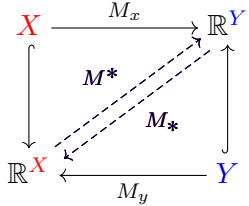
Embeddings as Functions Over Sets

$$M_* M^* \colon \mathbb{R}^X \to \mathbb{R}^X$$
$$M^* M_* \colon \mathbb{R}^Y \to \mathbb{R}^Y$$

$$\{u_1, \ldots, u_m\} \subset \mathbb{R}^X$$
$$\{v_1, \ldots, v_n\} \subset \mathbb{R}^Y$$
$$\{\lambda_1, \ldots, \lambda_{\min(m,n)}, 0, \ldots, 0\}$$

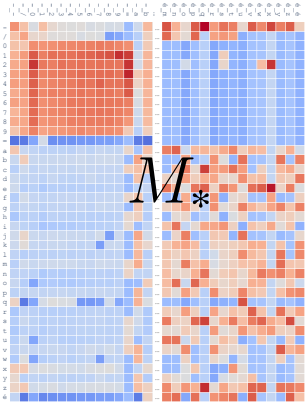
$$M_* M^* u_i = \lambda_i u_i$$
$$M^* M_* v_i = \lambda_i v_i$$

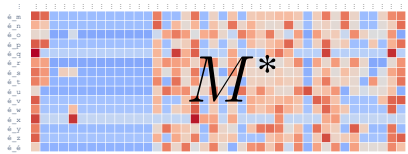
The u_i and v_i are (linear) fixed points!

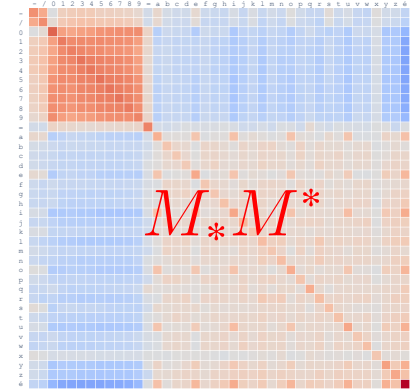


$$M = U \Sigma V^T \qquad U := [u_1, \ldots, u_m] \qquad V := [v_1, \ldots, v_n] \qquad \Sigma := \begin{bmatrix} \sqrt{\lambda_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{\lambda_r} \end{bmatrix}$$

M_*M^* as a Covariance Matrix

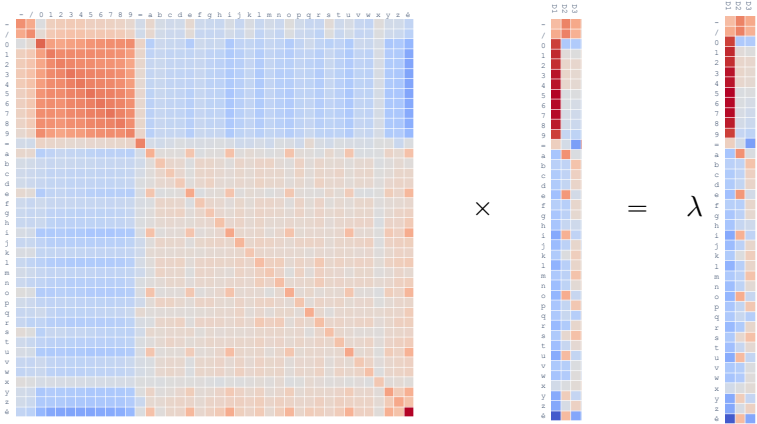




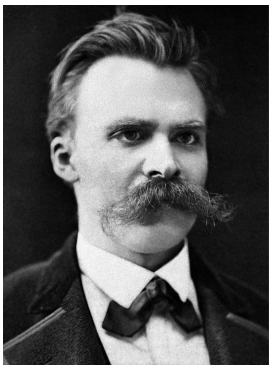


Eigenvectors as Fixed Points

$$M_* M^* u = \lambda u$$



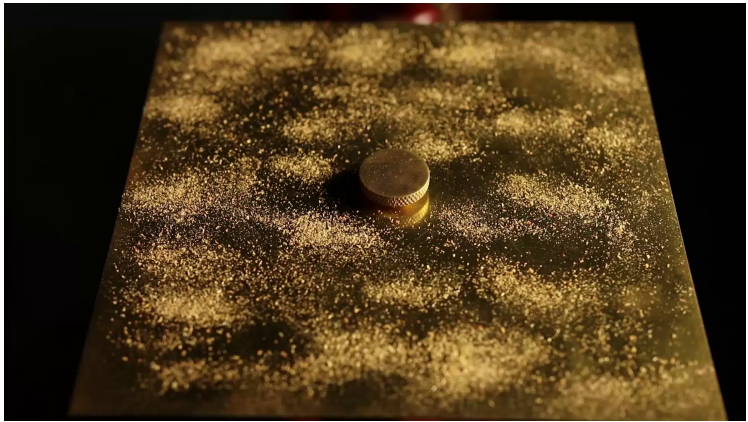
Chladni Figures



"One can conceive of a profoundly deaf human being who has never experienced sound or music; just as such a person will gaze in astonishment at the Chladnian sound-figures in sand, find their cause in the vibration of a string, and swear that he must now know what men call sound — this is precisely what happens to all of us with language."

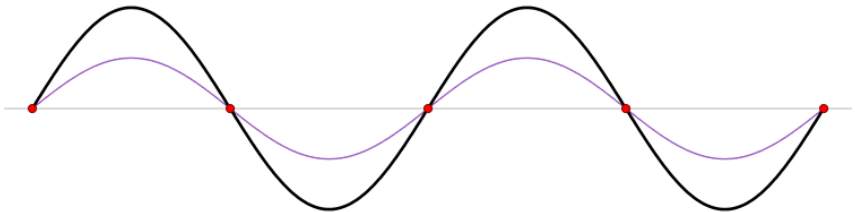
(Nietzsche, 1873)

Chladni Figures

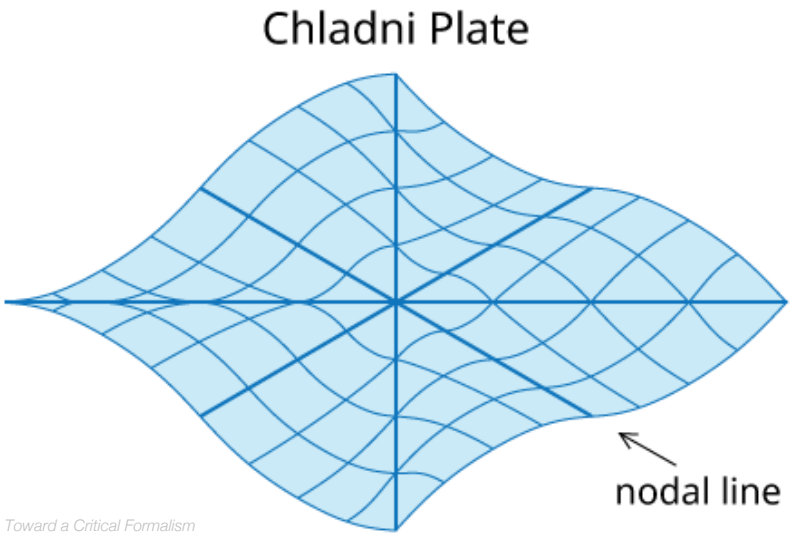


Chladni Figures

Chladni Figures

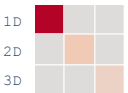


Chladni Figures

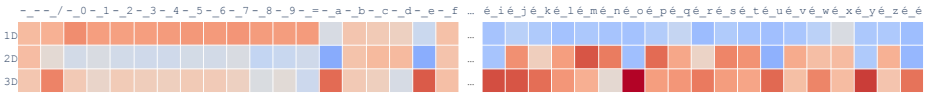


Eigenvectors of $M_* M^*$:

Eigenvalues of $M_* M^*$ and $M^* M_*$:

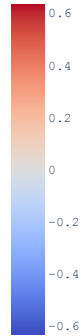


Eigenvectors of $M^* M_*$:



Words

	-5	-4	-3	-2	-1	0	1	2	3	4	5
D 1	church	university	field	house	centre	...	held	used	offered	found	made
D 2	use	leave	keep	buy	meet	...	boy	club	sun	uk	hotel
D 3	show	boy	project	move	play	...	production	size	interests	activities	nature
D 4	used	expected	made	considered	allowed	...	london	europe	scotland	france	england
D 5	used	expected	food	water	england	...	during	couple	under	series	lot
D 6	perhaps	indeed	under	during	in	...	cup	bit	series	couple	lot
D 7	difficult	hard	easy	necessary	close	...	won	gave	started	saw	took
D 8	europe	scotland	england	france	lot	...	middle	want	needs	army	could
D 9	wish	tried	seem	seemed	began	...	received	established	won	published	produced
D 10	10	15	20	30	3	...	from	on	black	into	through



Outline

Intro: Critique and Formalism

Epistemological Critique: LLMs as Formal Objects

Theoretical Critique: Formal Explainability

- The Algebra Behind the Embeddings
- The Structure Behind the Algebra
- The Categories Behind the Structure

Conclusion

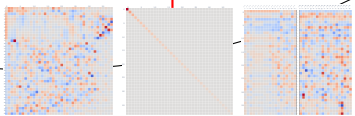
The Structure of Embeddings

Structure

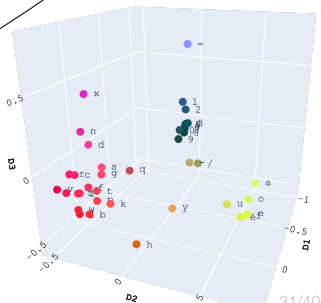
{-, /, 0, 1, 2, . . . , 8, 9, =,
a, b, c, . . . , w, x, y, z, é}

Embedding

Data

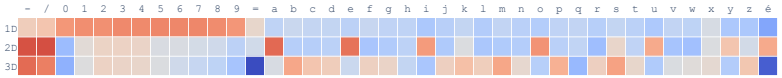


SVD



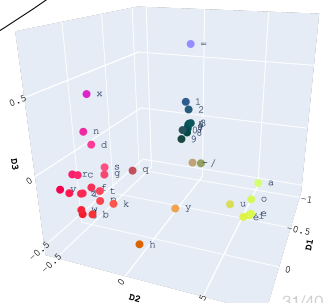
The Structure of Embeddings

Structure

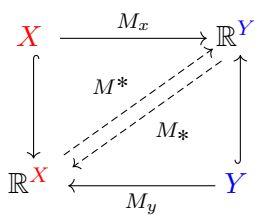


$\{-, /, 0, 1, 2, \ldots, 8, 9, =,$
$a, b, c, \ldots, w, x, y, z, é\}$

Embedding



Data



The Structure of Embeddings

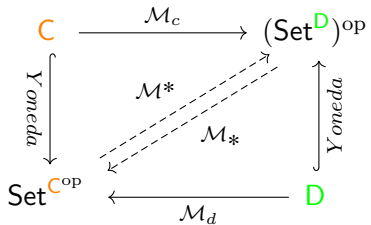
Structure

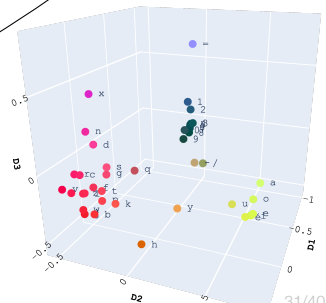
?

{-, /, 0, 1, 2, . . . , 8, 9, =,
a, b, c, . . . , w, x, y, z, é}

Embedding

Data





The Structure of Embeddings

Structure

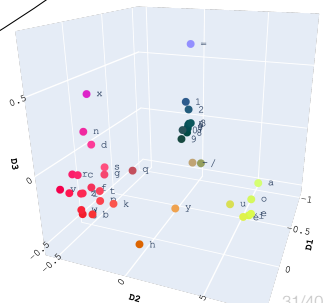
?

Data

$C^{op} \times D \rightarrow Set$

{-, /, 0, 1, 2, . . . , 8, 9, =,
a, b, c, . . . , w, x, y, z, é}

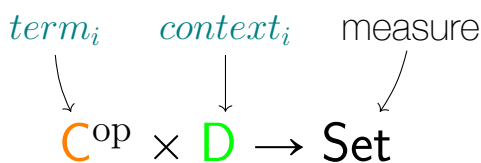
Embedding



Nucleus and Types

Structure

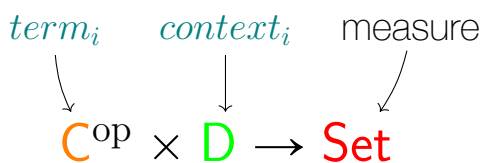
?



Nucleus and Types

Structure

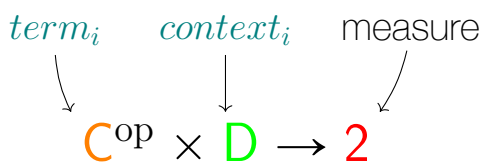
?



Nucleus and Types

Structure

?

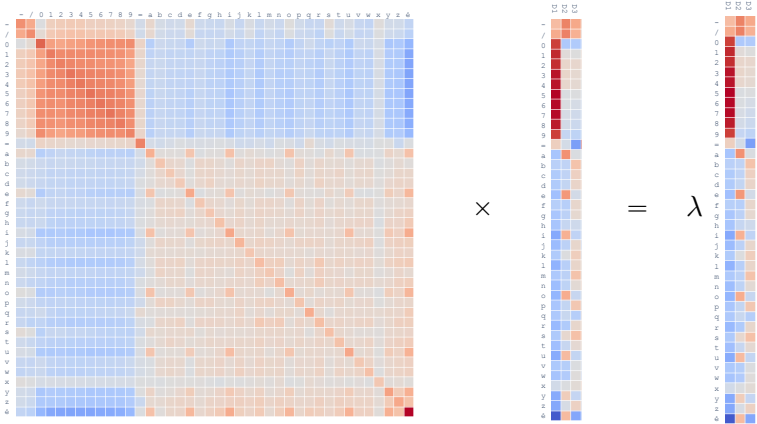


Structure

$$\begin{array}{c} \mathsf{C}^{\mathrm{op}} \times \mathsf{D} \longrightarrow 2 \\ \Downarrow \\ \mathcal{M}^* : 2^{\mathsf{C}^{\mathrm{op}}} \leftrightarrows (2^{\mathsf{D}})^{\mathrm{op}} : \mathcal{M}_* \end{array}$$

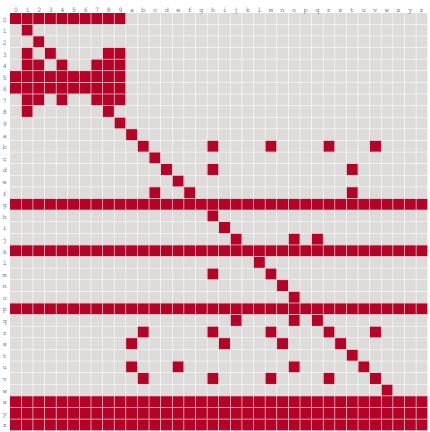
Binary Fixed Points

$$M_* M^* u = \lambda u$$

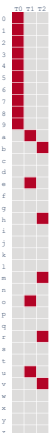


Binary Fixed Points

$$\mathcal{M}_*\mathcal{M}^* f = f$$



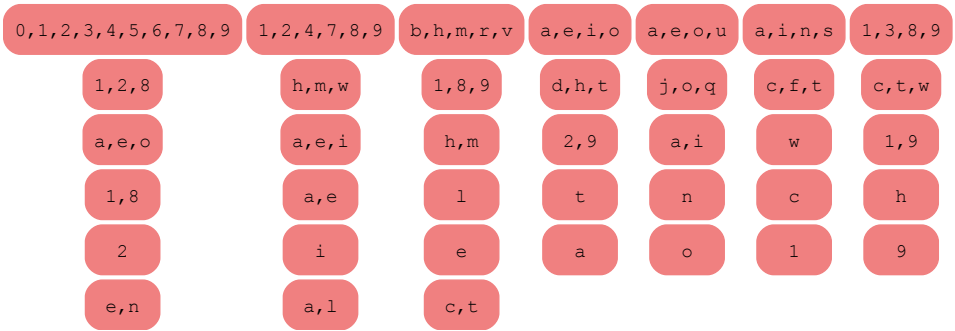
$\star$



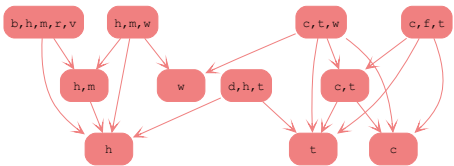
$\stackrel{?}{=}$

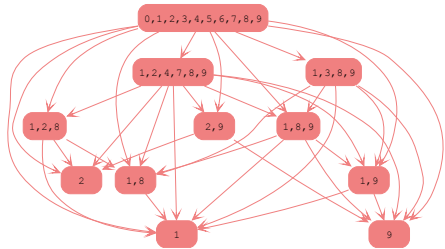
“Eigensets”

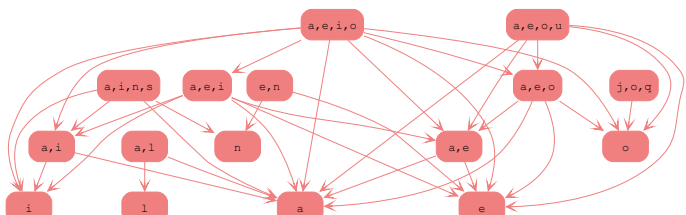
$$\mathcal{M}_* \mathcal{M}^* f = f$$



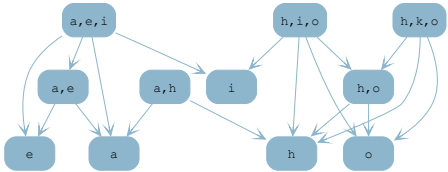
Partial Order Structure

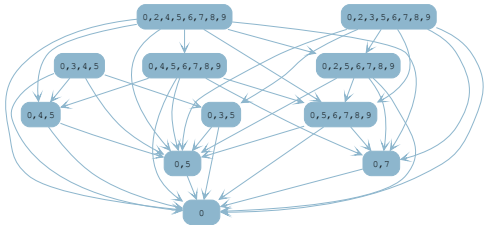


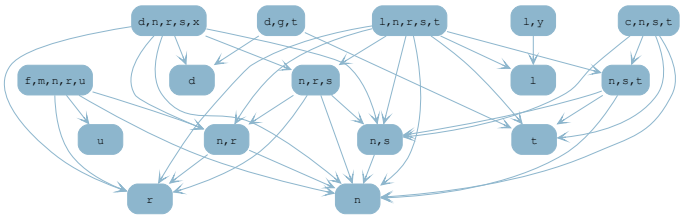




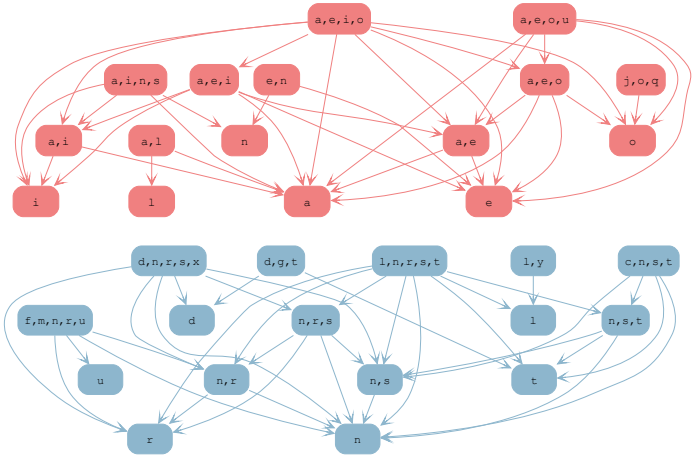
Dual Partial Order



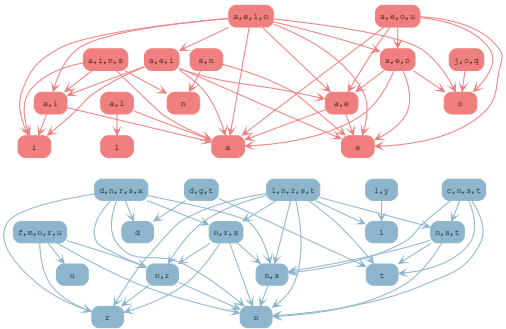




Paring of Partial Ordered Fixed Points



Structure



$$\mathsf{C}^{\mathrm{op}} \times \mathsf{D} \longrightarrow 2$$

$$\Downarrow$$

$$\mathcal{M}^* \colon 2^{\mathsf{C}^{\mathrm{op}}} \leftrightarrows \left(2^{\mathsf{D}}\right)^{\mathrm{op}} \colon \mathcal{M}_*$$

Structure

?

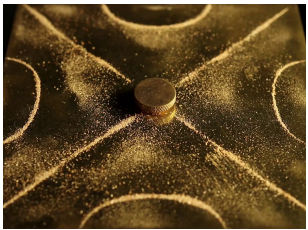
$$\mathsf{C}^{\mathrm{op}} \times \mathsf{D} \longrightarrow \bar{\mathbb{R}}$$

$$\Downarrow$$

$$\mathcal{M}^* : \bar{\mathbb{R}}^{\mathsf{C}^{\mathrm{op}}} \leftrightarrows (\bar{\mathbb{R}}^{\mathsf{D}})^{\mathrm{op}} : \mathcal{M}_*$$

Structure

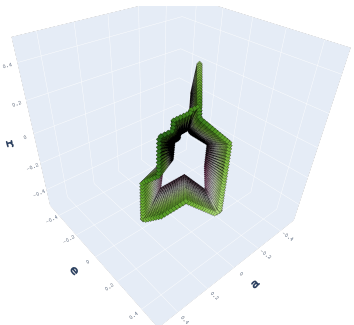
?



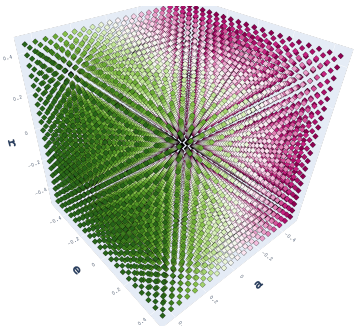
$$\mathsf{C}^{\mathrm{op}} \times \mathsf{D} \longrightarrow \bar{\mathbb{R}}$$

$$\mathcal{M}^* \colon \bar{\mathbb{R}}^{\mathsf{C}^{\mathrm{op}}} \leftrightarrows \left(\bar{\mathbb{R}}^{\mathsf{D}}\right)^{\mathrm{op}} \colon \mathcal{M}_*$$

Structure



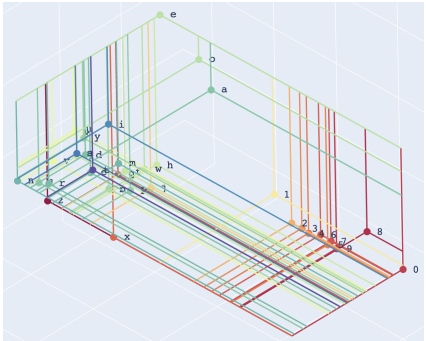
$\mathcal{M}_* \mathcal{M}^*$



$$\mathsf{C}^{\mathrm{op}} \times \mathsf{D} \to \bar{\mathbb{R}}$$

$$\mathcal{M}^* \colon \bar{\mathbb{R}}^{\mathsf{C}^{\mathrm{op}}} \leftrightarrows (\bar{\mathbb{R}}^{\mathsf{D}})^{\mathrm{op}} \colon \mathcal{M}_*$$

Structure

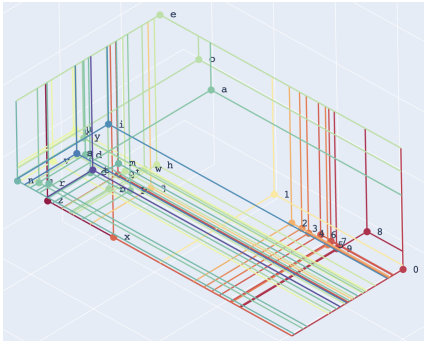


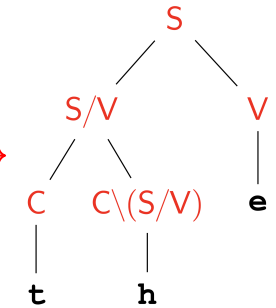
$$\mathsf{C}^{\mathrm{op}} \times \mathsf{D} \longrightarrow \bar{\mathbb{R}}$$

$$\Downarrow$$

$$\mathcal{M}^* \colon \bar{\mathbb{R}}^{\mathsf{C}^{\mathrm{op}}} \leftrightarrows \left(\bar{\mathbb{R}}^{\mathsf{D}}\right)^{\mathrm{op}} \colon \mathcal{M}_*$$

Structure





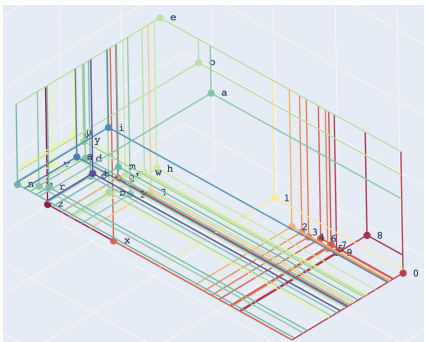
$$\mathsf{C}^{\mathrm{op}} \times \mathsf{D} \longrightarrow \bar{\mathbb{R}}$$

$$\Downarrow$$

$$\mathcal{M}^* : \bar{\mathbb{R}}^{\mathsf{C}^{\mathrm{op}}} \leftrightarrows \left(\bar{\mathbb{R}}^{\mathsf{D}}\right)^{\mathrm{op}} : \mathcal{M}_*$$

Formal Explainability

Structure

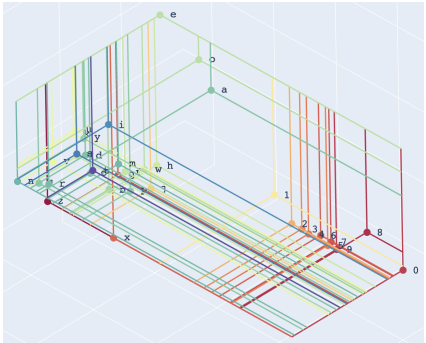


Embedding

Nucleus and Types

Formal Explainability

Structure



Tokenization
Renormalization

Embedding
Nucleus and Types

Attention
Tensor Product

Outline

Intro: Critique and Formalism

Epistemological Critique: LLMs as Formal Objects

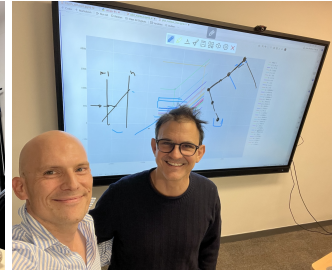
Theoretical Critique: Formal Explainability
- The Algebra Behind the Embeddings
- The Structure Behind the Algebra
- The Categories Behind the Structure

Conclusion

Conclusion: For a Critical Formalism

- It is urgent to address the epistemological dimension of the critical project in its own terms
- This requires to develop a critical approach within formal sciences where formalization is not assumed to lead to naturalization
  - The new role of data within formal sciences is crucial in this sense
- A critical formalism will be incomplete if it remains disconnected from the political, and even the artistic dimension of the critical program
  - We need a new alliance between the formal sciences, the human sciences, and the arts.

Collaborations



J. Terilla (CUNY), T.-D. Bradley (SandboxAQ), L. Pellissier (Paris-Est Créteil), Th. Seiller (CNRS), S. Jarvis (CUNY)

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Chat Token Vector
Università Ca' Foscari
Venice, Italy

Toward a Critical Formalism

Philosophical and Theoretical Effects of a Mathematical Critique of LLMs

Juan Luis Gastaldi

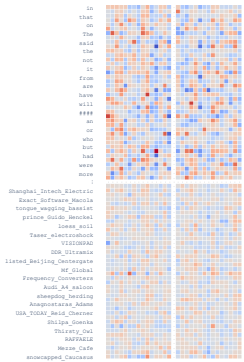
www.giannigastaldi.com

ETH zürich

June 12, 2025

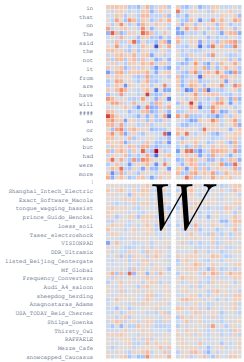
word2vec as Implicit Matrix Factorization

(Levy and Goldberg, 2014)



word2vec as Implicit Matrix Factorization

(Levy and Goldberg, 2014)



W

$C?$

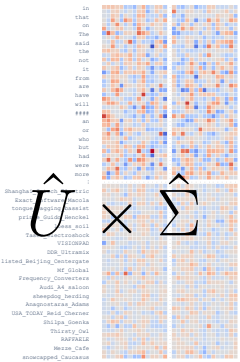
$M =$

$\text{pmi}(w, c) - \log k$

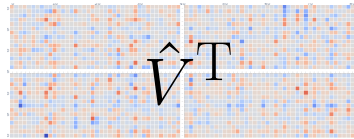
$W \times C \approx M$

Word Embeddings as Truncated SVD

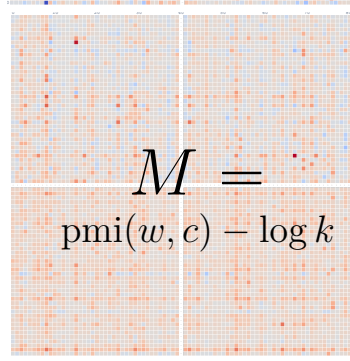
(Levy and Goldberg, 2014)



$$\hat{U} \times \hat{\Sigma}$$



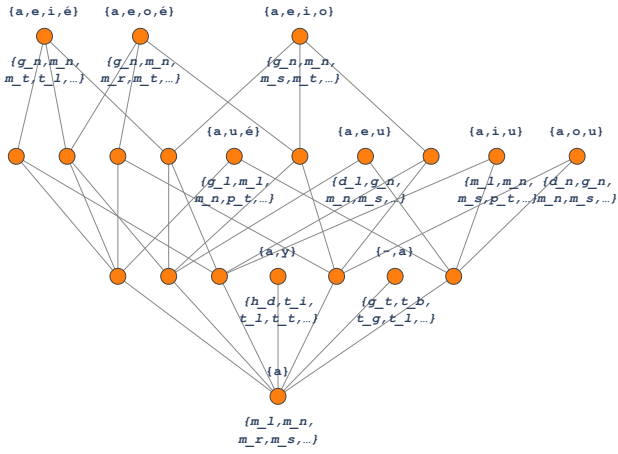
$$\hat{V}^{\mathrm{T}}$$

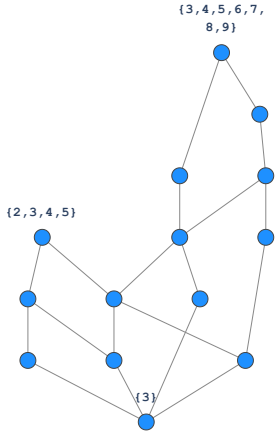


$$M = \mathrm{pmi}(w, c) - \log k$$

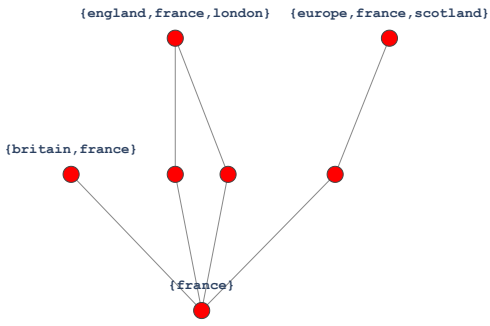
$$M \approx \hat{U} \times \hat{\Sigma} \times \hat{V}^{\mathrm{T}}$$

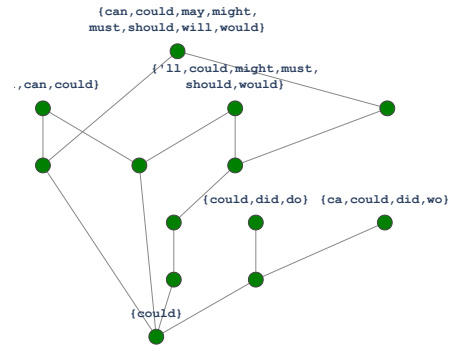
Formal Concepts



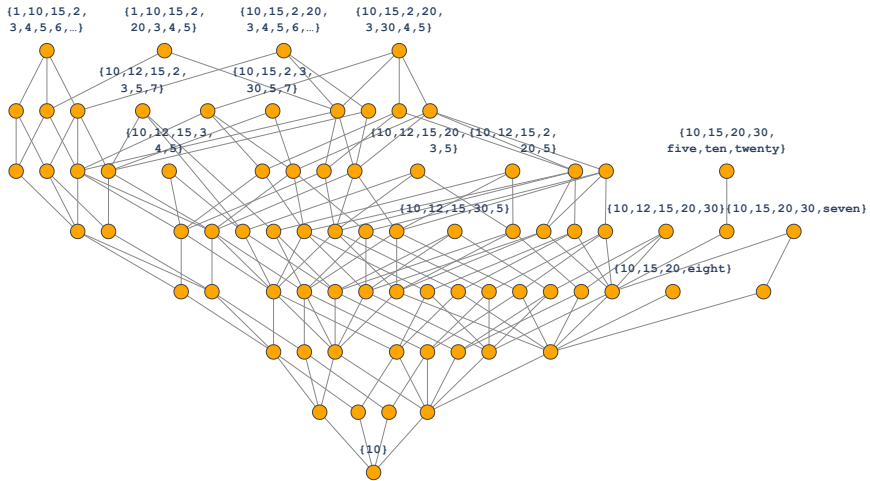


Formal Concepts (words)



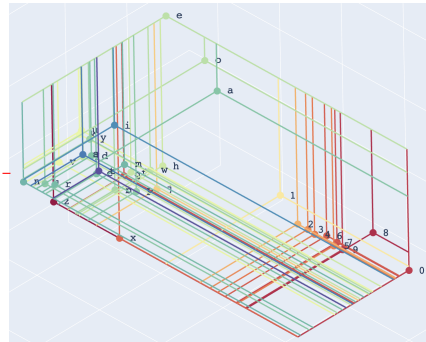


Formal Concepts (words)



Theoretical Interpretability

Structure



Theory

"Task"

?

Theoretical Interpretability

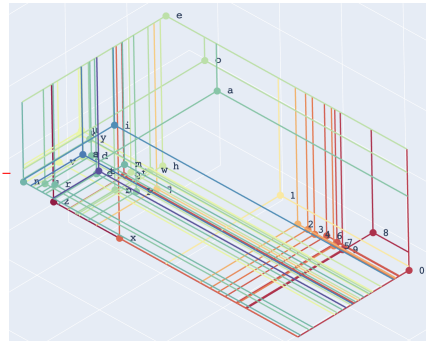
$$C^{op} \times D \rightarrow \bar{\mathbb{R}}$$

Distributional Hypothesis

The content of linguistic units is determined by their *distribution* in a corpus.

Theory
"Task"

Structure



Theoretical Interpretability

$$C^{op} \times D \rightarrow \bar{\mathbb{R}}$$

Theory
"Task"

Distributional Hypothesis

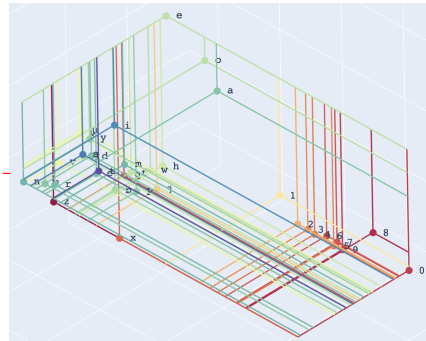
The content of linguistic units is determined by their *distribution* in a corpus.

Structuralist Hypothesis

Linguistic content is the effect of a virtual *structure* underlying linguistic practices within a community

$$\bar{\mathbb{R}}^{C^{op}} \leftrightarrows (\bar{\mathbb{R}}^{D})^{op}$$

Structure

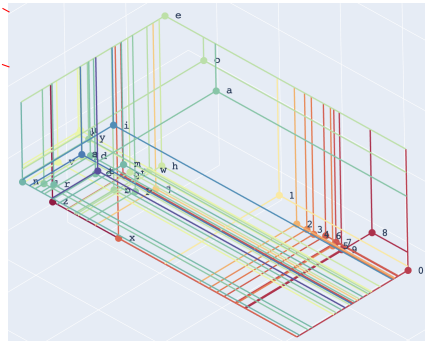


Theoretical Interpretability

Units
Classes
Relations

Semantics
Syntax
Morphology
Phonology

Structure



Theoretical Interpretability

Units
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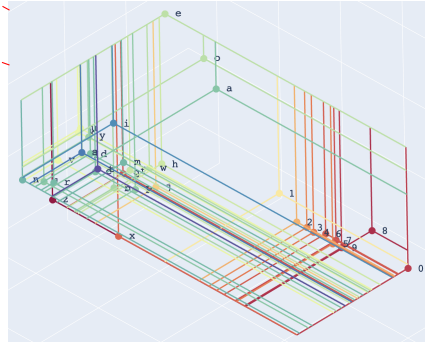
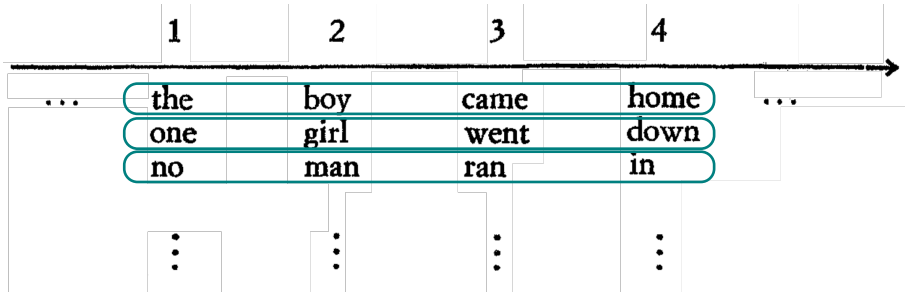


	o	a	e	u	ɑ	i	ŋ	ʃ	k	ʒ	ð	q	m	f	p	v	b	n	s	θ	t	z	d	h	ŋ
1. Vocalic/Non-vocalic	+	+	+	+	+	+	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−
2. Consonantal/Non-consonantal	−	−	−	−	−	−	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	−
3. Compact/Diffuse	+	+	+	−	−	−	+	+	+	+	+	+	−	−	−	−	−	−	−	−	−	−	−	−	
4. Grave/Acute	+	+	−	+	+	−							+	+	+	+	−	−	−	−	−	−	−	−	
5. Flat/Plain	+	−		+	−																				
6. Nasal/Oral							+	−	−	−	−	−	+	−	−	−	−	+	−	−	−	−	−	−	
7. Tense/Lax								+	+	+	−	−		+	+	−	+	−	−	+	+	+	−	−	−
8. Continuant/Interrupted								+	−	−	+	−		+	−	+	−			+	+	−	+	+	
9. Strident/Mellow									+	−		+	−							+	−	+	−		

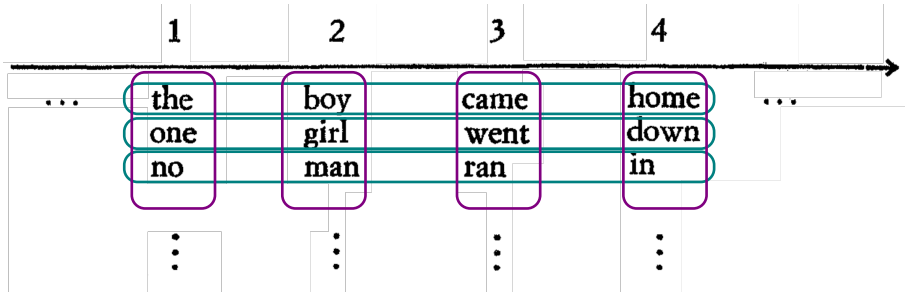
(Jakobson et al., 1952)

Syntagmes et Paradigmes



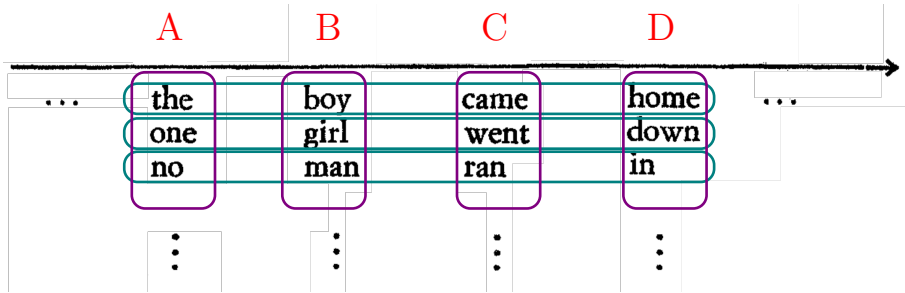
(Hjelmslev, 1971)

Syntagmes et Paradigmes



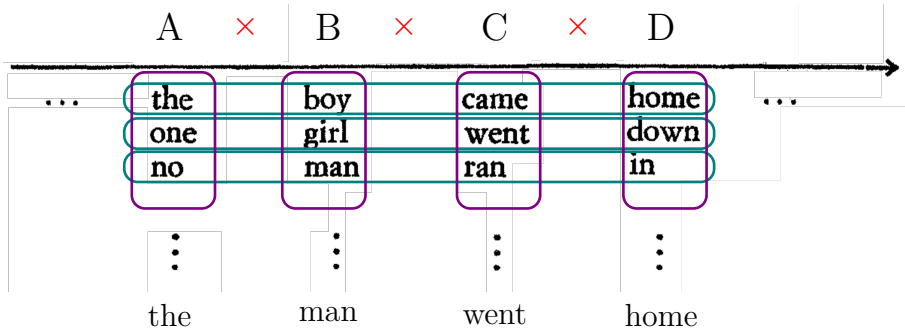
(Hjelmslev, 1971)

Syntagmes et Paradigmes

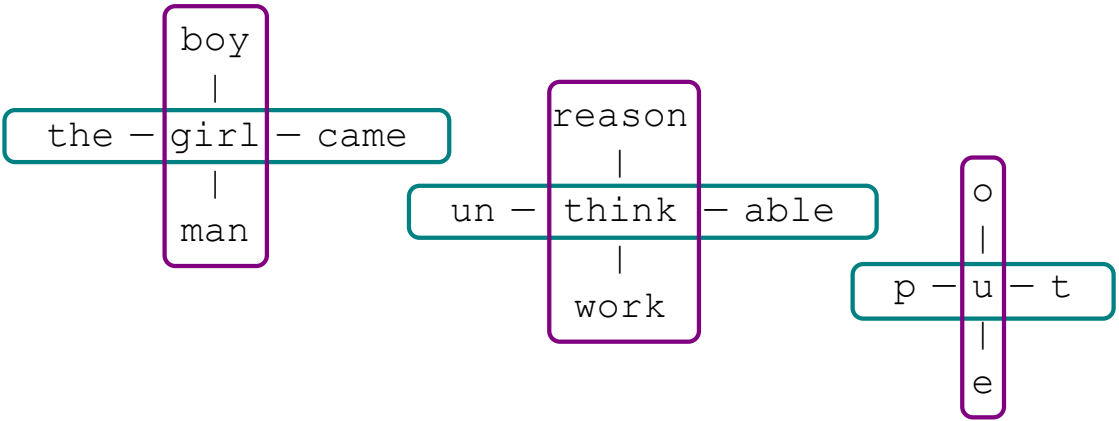


(Hjelmslev, 1971)

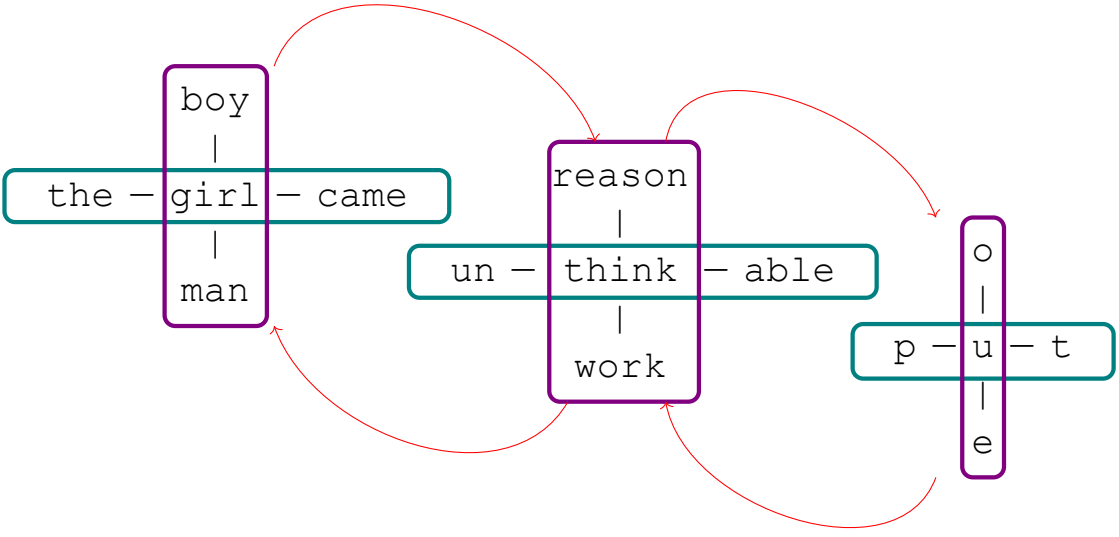
Syntagmes et Paradigmes



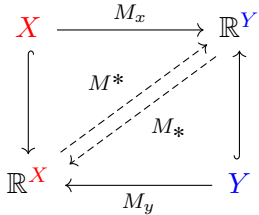
(Hjelmslev, 1971)

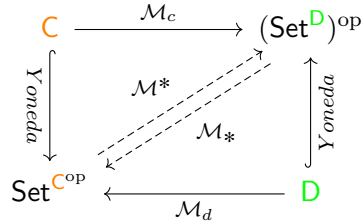

boy
the — girl — came
man
reason
un — think — able
work
o
p — u — t
e

Stratification



De l'algèbre linéaire aux catégories





$M_*M^* \colon \mathbb{R}^X \to \mathbb{R}^X$

$M^*M_* \colon \mathbb{R}^Y \to \mathbb{R}^Y$

$M_*M^*u_i = \lambda_i u_i$

$M^*M_*v_i = \lambda_i v_i$

$\mathcal{M}_*\mathcal{M}^* \colon \mathsf{Set}^{\mathsf{C}^{\mathrm{op}}} \to \mathsf{Set}^{\mathsf{C}^{\mathrm{op}}}$

$\mathcal{M}^*\mathcal{M}_* \colon (\mathsf{Set}^{\mathsf{D}})^{\mathrm{op}} \to (\mathsf{Set}^{\mathsf{D}})^{\mathrm{op}}$

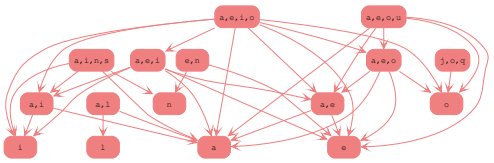
$\mathrm{Fix}(\mathcal{M}_*\mathcal{M}^*) := \{f \in \mathsf{Set}^{\mathsf{C}^{\mathrm{op}}} | \mathcal{M}_*\mathcal{M}^*(f) \cong f\}$

$\mathrm{Fix}(\mathcal{M}^*\mathcal{M}_*) := \{g \in (\mathsf{Set}^{\mathsf{D}})^{\mathrm{op}} | \mathcal{M}^*\mathcal{M}_*(g) \cong g\}$

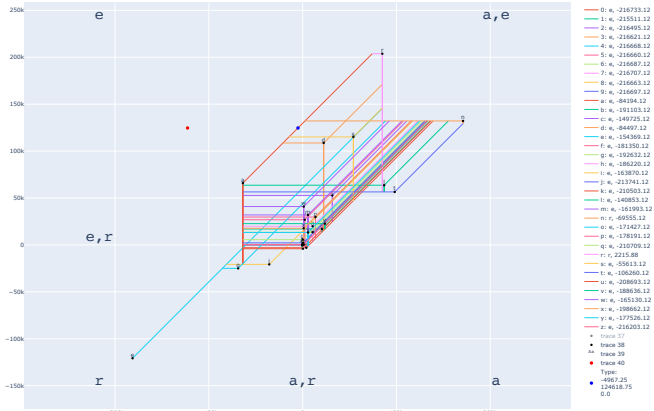
Structures catégoriques

Algèbre linéaire

$\mathcal{M}^*\colon 2^{C^{\mathrm{op}}} \leftrightarrows (2^D)^{\mathrm{op}}\colon \mathcal{M}_*$



$\mathcal{M}^*\colon \bar{\mathbb{R}}^{C^{\mathrm{op}}} \leftrightarrows (\bar{\mathbb{R}}^D)^{\mathrm{op}}\colon \mathcal{M}_*$



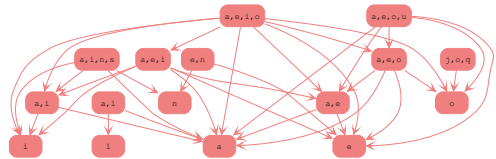
Théorie des types computationnels

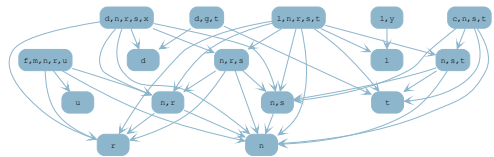
Definition (Polaire/Orthogonal - Girard, 2006)

[É]tant donnée une fonction binaire $a, b \rightsquigarrow \langle a|b\rangle \colon A \times B \to C$ et un sous-ensemble $P \subset C$ (le « pôle »), on peut définir le *polaire* $X^\perp \subset B$ d'un sous-ensemble $X \subset A$ (resp. $Y^\perp \subset A$ d'un sous-ensemble $Y \subset B$) par :

$$X^\perp := \{y \in B \colon \forall x \in X, \langle a|b\rangle \in P\}$$
$$Y^\perp := \{x \in A \colon \forall y \in Y, \langle a|b\rangle \in P\}$$

- L'application « polaire » est décroissante: $X \subset X' \Rightarrow X'^\perp \subset X^\perp$.
- L'ensemble $\mathrm{Pol}(A) \subset \mathcal{P}(A)$ des ensembles *polaires*, i.e., de la forme $Y^\perp$, est stable par intersections arbitraires. En particulier, A est polaire et $X^{\perp\perp}$ est le plus petit ensemble polaire contenant X.
- En conséquence, $X^{\perp\perp\perp} = X^\perp$.





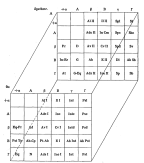
Axe 2: Interprétabilité théorique

Hypothèse distributionnelle

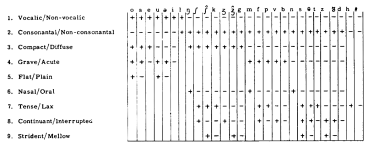
Le contenu des unités linguistiques est déterminé par leur **distribution** dans un corpus.

Hypothèse structurale

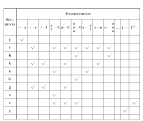
Le contenu linguistique est l'effet d'une **structure** virtuelle dérivée des pratiques linguistiques dans une communauté.



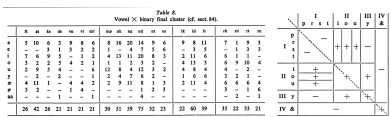
(Hjelmslev, 1935)



(Jakobson et al., 1952)



(Harris, 1960)



(Spang-Hanssen, 1959)

(Lévi-Strauss, 1949)

(Bourdieu, 1979)

(Foucault, 1966)

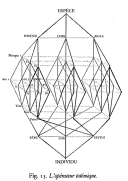
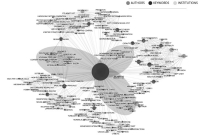


Fig. 11. *L'opération totémique.*

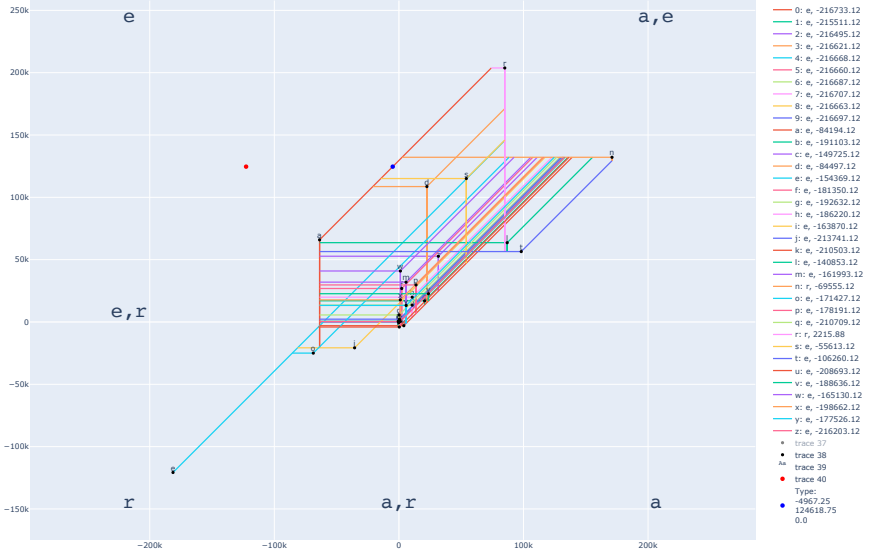
(Lévi-Strauss, 1962)

(Bourdieu, 1994)

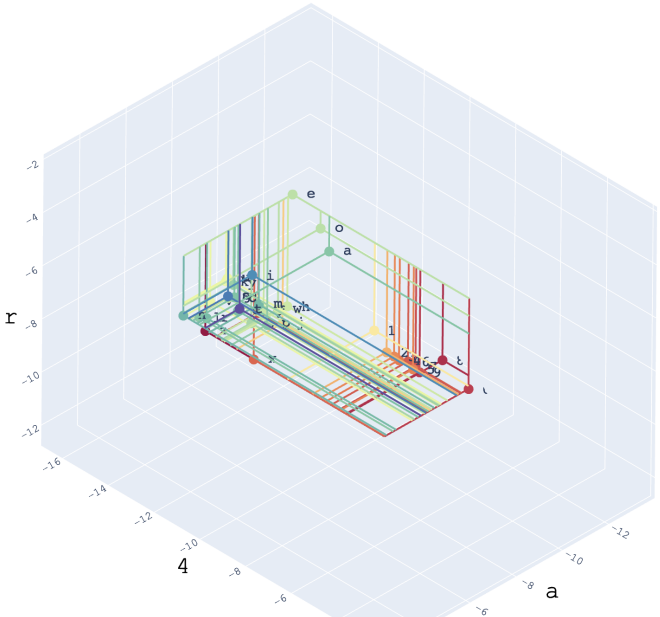


(Latour et al., 2012)

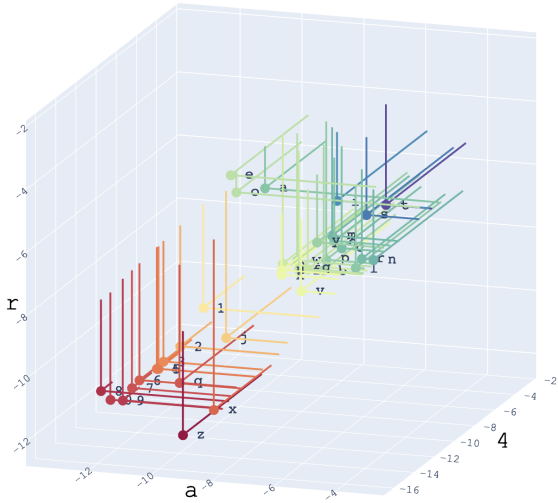
Structure interne du noyeau



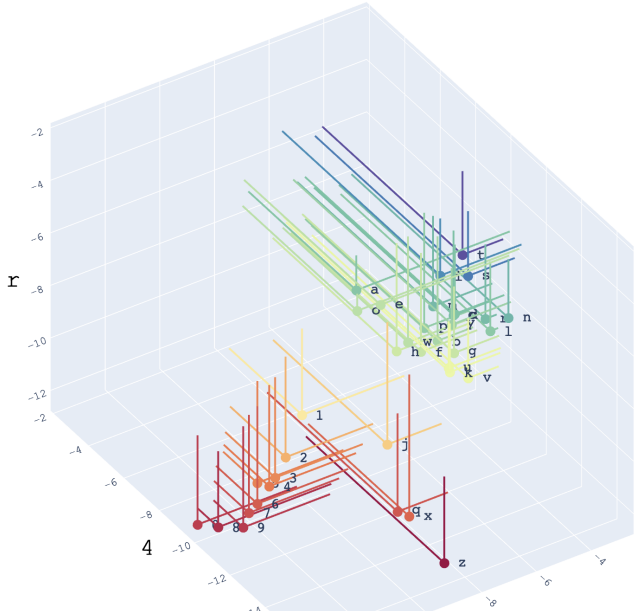
Structure interne du noyeau



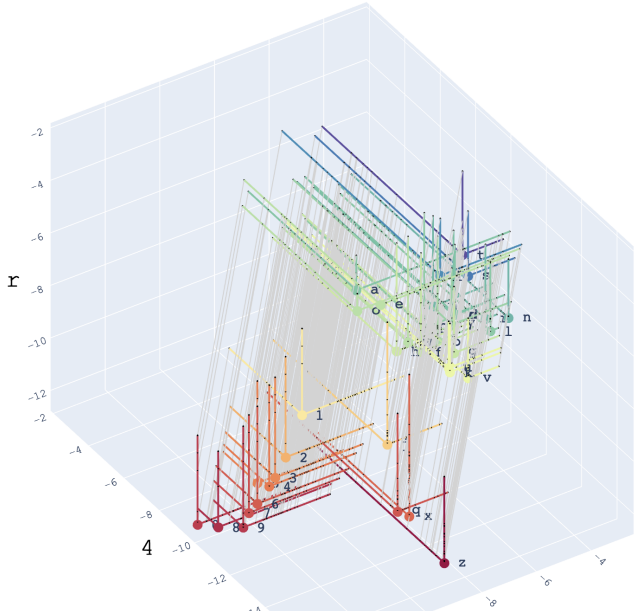
Structure interne du noyeau



Structure interne du noyeau



Structure interne du noyeau



Matrice et analogie

a = your c = my	w = apartment x = house y = chair z = stool

	…	w	x	y	z	…
…	…	0	0	0	0	…
a	…	0	1	1	0	…
b	…	0	0	1	1	…
c	…	1	0	0	1	…
…	…	0	0	0	0	…

your : house
my : apartment

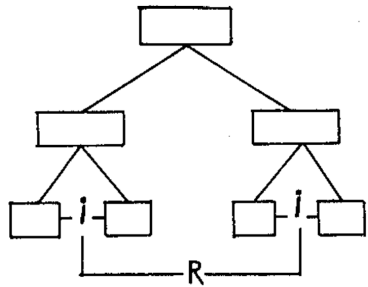
Semiotique Structurale

Une sémiotique [...] est une hiérarchie dont chacune des composantes admet une analyse ultérieure en classes définies par relation mutuelle, de telle sorte que chacune de ces classes admette une analyse en dérivés définis par mutation mutuelle.

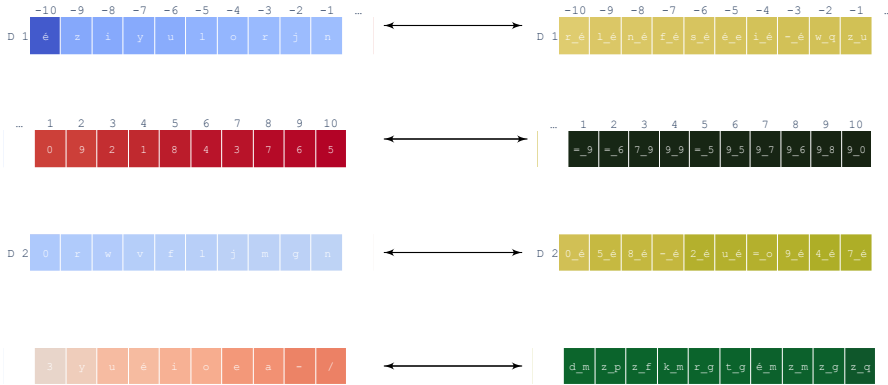
(Hjelmslev, 1975, Df. 24)

Une mutation [...] est une fonction existant entre des dérivés du premier degré d'une seule et même classe, une fonction qui a une relation à une fonction entre d'autres dérivés de premier degré d'une seule et même classe et appartenant au même rang.

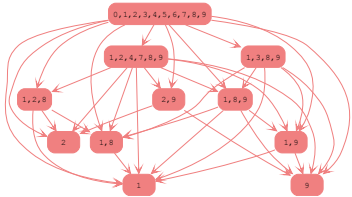
(Hjelmslev, 1975, Df. 23)

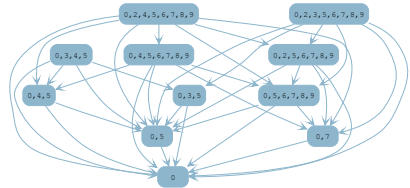


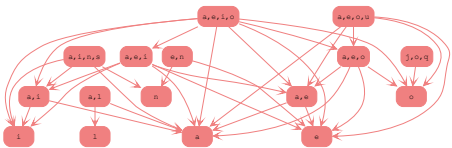
Syntagmatique et Texte (Vecteurs)

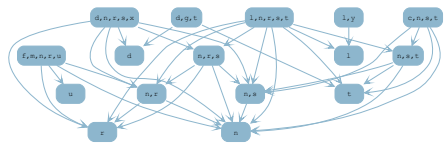


Syntagmatique et Texte (Noyeau/Types)

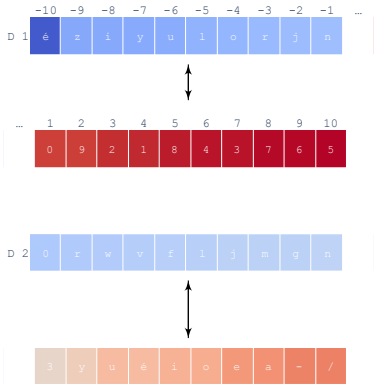


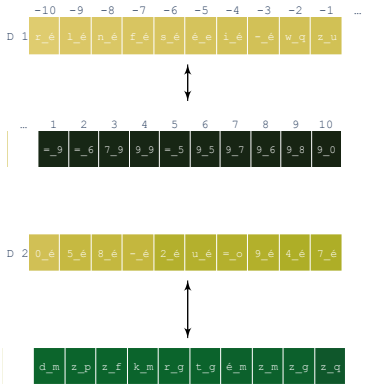




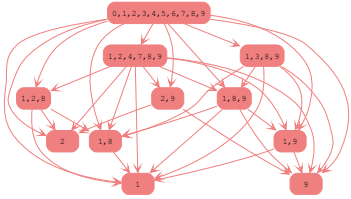


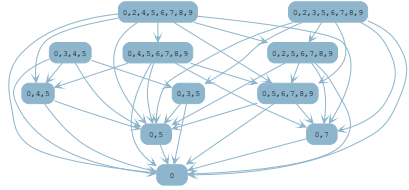
Paradigmatique et Langue (Vecteurs)

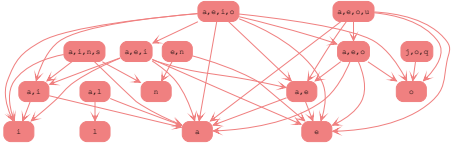




Paradigmatique et Langue (Noyau/Types)







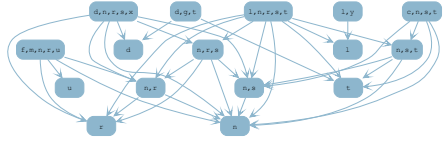


Illustration du contenu formel

(Gastaldi and Pellissier, 2021)