*CNRS - Concours chercheurs 2025*
DR Section 53 - Concours n° 53/01

# *Épistémologie des modèles distributionnels de langage par apprentissage machine*

## Explicabilité formelle et interprétabilité théorique

Juan Luis Gastaldi

`http://www.jlgastaldi.com/assets/gastaldi_cnrs_dr.pdf`

ETH zürich

22 Avril 2025

# Parcours et travaux

1997-2007 Recherche pré-doctorale
Argentine, France
(UNR, ENS, Paris 1, UPMC)

2008-2014 Recherche doctorale
France
(Bordeaux Montaigne)

2015-2022 Recherche post-doctorale
France, Suisse, Tchéquie, USA
(ETH, MSCA, CUNY, CMU)

2023-Présent Nouvelle recherche doctorale
Suisse
(ETH Zurich)

Sciences Politiques
Philosophie du language et pensée formelle

**Comment une science de l'homme est-elle possible?**

- Héritage **critique** kantien contre le positivisme
- Dépassement du partage **anthropologie/ontologie**
- Anthropologie ou théorie de l'**expression**
  (Foucault, 1954; Deleuze, 1954)
- Alliance entre **savoirs formels** et philosophie critique contemporaine

◇ Gastaldi (2009, 2010, 2011, 2015, 2016b, 2019)

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## Philosophie et histoire des sciences formelles

**Quelle critique pour les sciences formelles?**

- Épistémologie **historique**
- La notion de ***formel*** a elle-même une histoire (récente)
- En devenant formelle, la mathématisation n'entraîne pas la **naturalisation**

- ◇ Gastaldi (2014, 2016a, 2022, 2024a, 2024b, 2024d)
- ◇ Gastaldi (ed.) (2024a, 2024b)

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## Epistémologie des modèles de language par apprentissage machine

**Quelles pratiques formelles pour une théorie critique du langage?**

- Analyse **computationnelle** de **corpus** textuels
- Modèles **distributionnels** de langage

◇ Gastaldi (2021, 2024c), Gastaldi, Moot, and Rétoré (2024), and Gastaldi and Pellissier (2021)

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Fondements formels des modèles de language par apprentissage machine

**Quels fondements pour une science formelle du langage?**

- Fondements **formels** des pratiques computationnelles (catégories, types)
- Fondements **structuraux** des principes distributionnels (segmentation, structure)

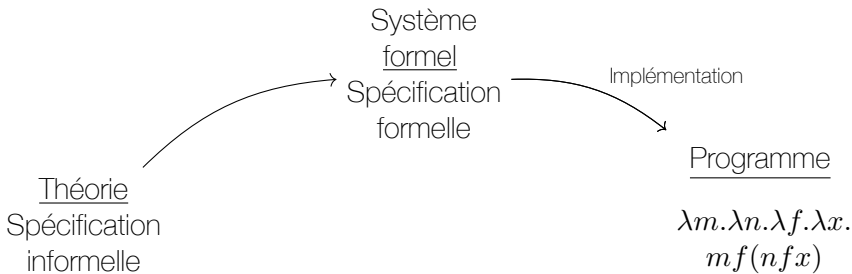
◇ Bradley et al. (2024), Gastaldi, Terilla, et al. (2024), Giulianelli et al. (2024), Vieira et al. (2024), and Zouhar et al. (2023a, 2023b)

# La structure implicite des données

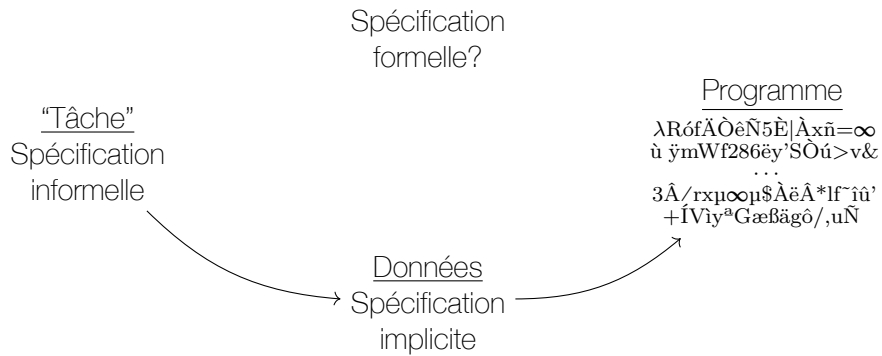
Programme

$$\lambda m.\lambda n.\lambda f.\lambda x.\ mf(nfx)$$

# La structure implicite des données



# La structure implicite des données



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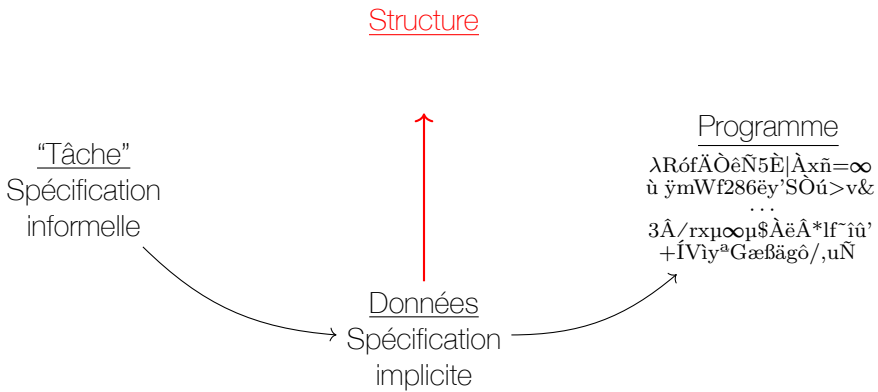

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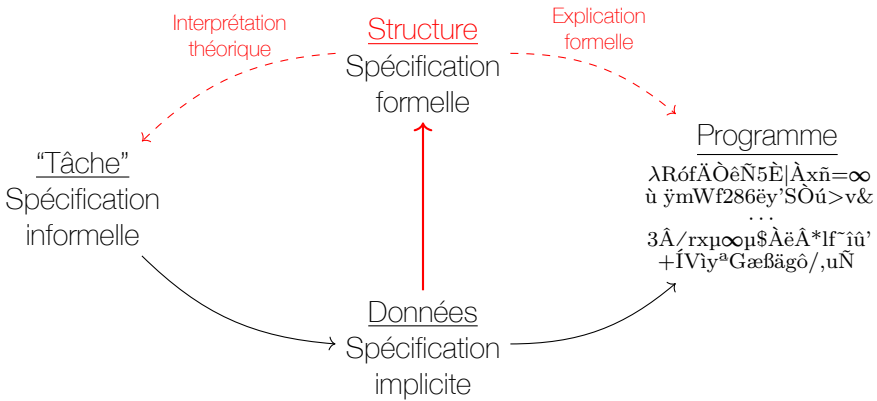


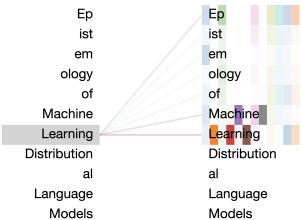
# Axe 1: Explicabilité formelle

Tokenisation (Sennrich et al., 2016) → Embedding (Mikolov et al., 2013) → Attention (Vaswani et al., 2017)

```
Epistemology of Machine Learning
Distributional Language Models
```

(https://tiktokenizer.vercel.app)


(https://projector.tensorflow.org)




(https://github.com/jessevig/bertviz)

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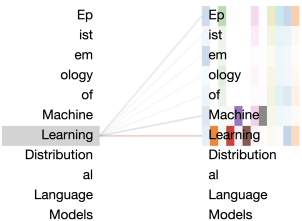


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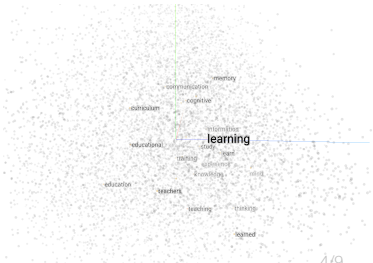
# Embedding et SVD



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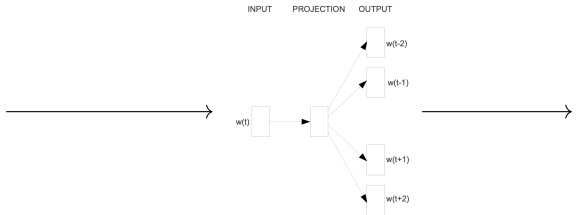
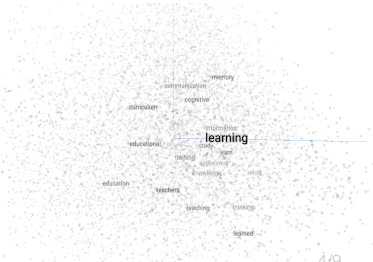
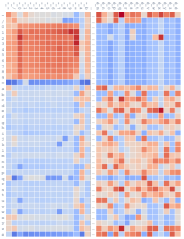
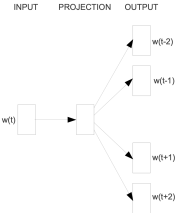


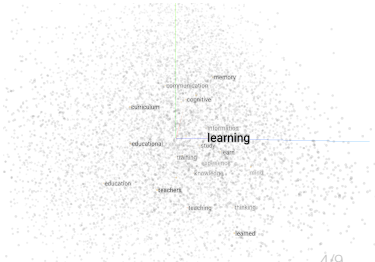



# Embedding et SVD

Matrice PMI



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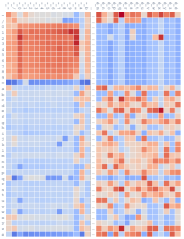
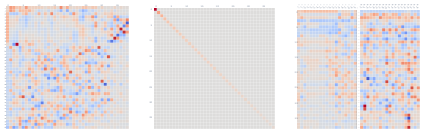
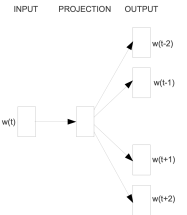
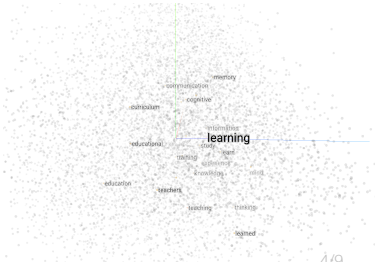




SVD

WIKIPEDIA
The Free Encyclopedia



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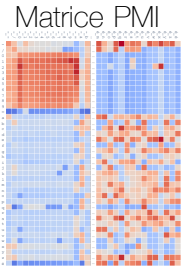


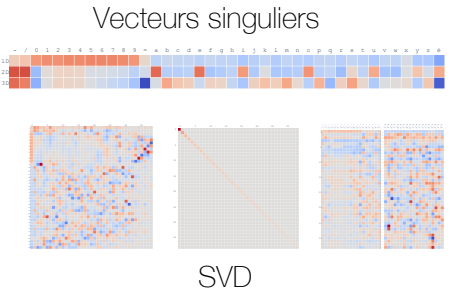



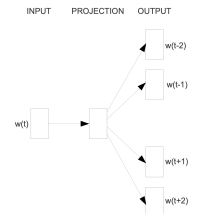


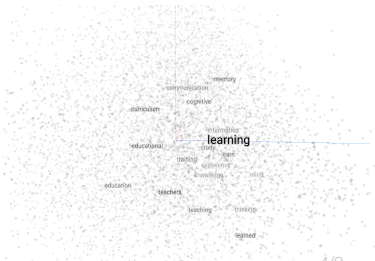

# Embedding et SVD

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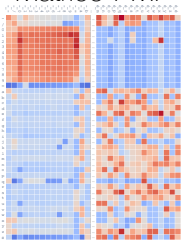
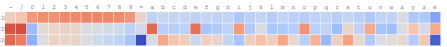
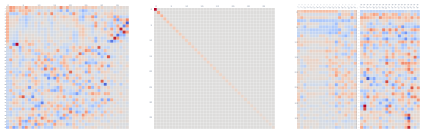


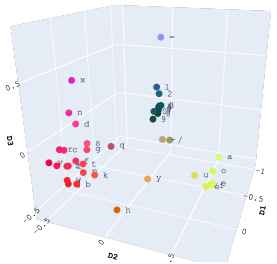
Vecteurs singuliers





SVD

Embedding SVD

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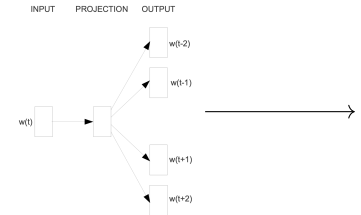
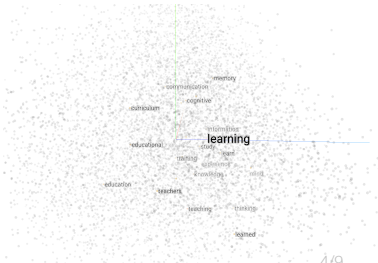
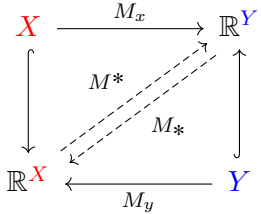
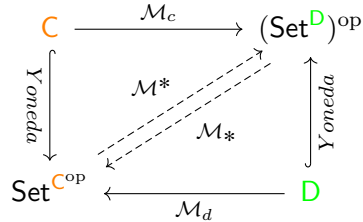
The Free Encyclopedia



# De l'algèbre linéaire aux catégories




$M_*M^* \colon \mathbb{R}^X \to \mathbb{R}^X$

$M^*M_* \colon \mathbb{R}^Y \to \mathbb{R}^Y$

$M_*M^*u_i = \lambda_i u_i$

$M^*M_*v_i = \lambda_i v_i$

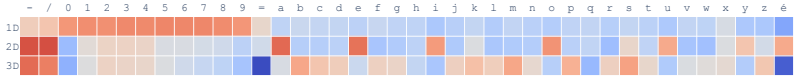
$\mathcal{M}_*\mathcal{M}^* \colon \mathsf{Set}^{\mathsf{C}^{\mathrm{op}}} \to \mathsf{Set}^{\mathsf{C}^{\mathrm{op}}}$

$\mathcal{M}^*\mathcal{M}_* \colon (\mathsf{Set}^{\mathsf{D}})^{\mathrm{op}} \to (\mathsf{Set}^{\mathsf{D}})^{\mathrm{op}}$

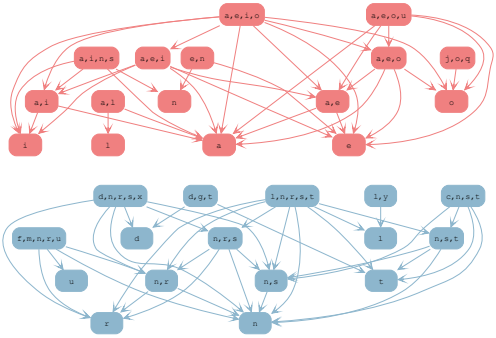
$\mathrm{Fix}(\mathcal{M}_*\mathcal{M}^*) := \{f \in \mathsf{Set}^{\mathsf{C}^{\mathrm{op}}} | \mathcal{M}_*\mathcal{M}^*(f) \cong f\}$

$\mathrm{Fix}(\mathcal{M}^*\mathcal{M}_*) := \{g \in (\mathsf{Set}^{\mathsf{D}})^{\mathrm{op}} | \mathcal{M}^*\mathcal{M}_*(g) \cong g\}$

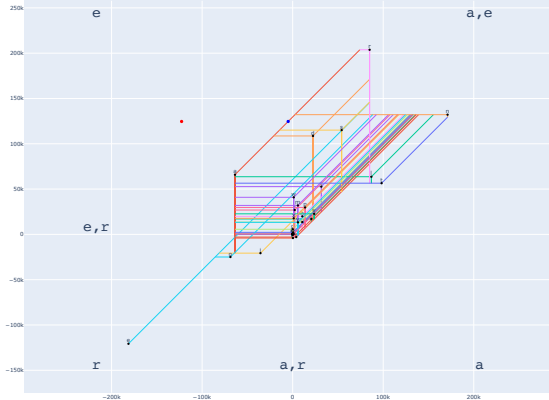
## Algèbre linéaire




$\mathcal{M}^* : 2^{C^{op}} \leftrightarrows (2^D)^{op} : \mathcal{M}_*$




$\mathcal{M}^* : \bar{\mathbb{R}}^{C^{op}} \leftrightarrows (\bar{\mathbb{R}}^D)^{op} : \mathcal{M}_*$



# Axe 2: Interprétabilité théorique

**Hypothèse distributionnelle**

Le contenu des unités linguistiques est déterminé par leur **distribution** dans un corpus.

→

**Hypothèse structurale**

Le contenu linguistique est l'effet d'une **structure** virtuelle dérivée des pratiques linguistiques dans une communauté.

# Axe 2: Interprétabilité théorique

## Hypothèse distributionnelle

Le contenu des unités linguistiques est déterminé par leur **distribution** dans un corpus.

## Hypothèse structurale

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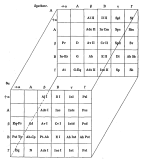
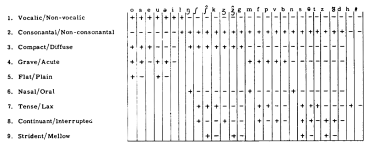
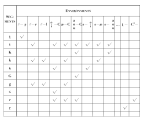


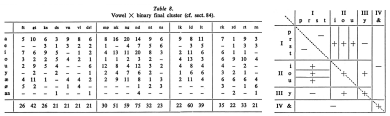
(Hjelmslev, 1935)




(Jakobson et al., 1952)



(Harris, 1960)




(Spang-Hanssen, 1959)

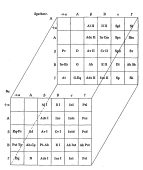
# Axe 2: Interprétabilité théorique

## Hypothèse distributionnelle

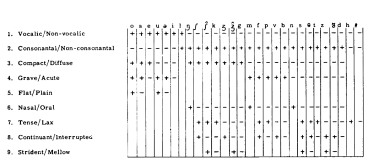
Le contenu des unités linguistiques est déterminé par leur **distribution** dans un corpus.

## Hypothèse structurale

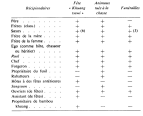
Le contenu linguistique est l'effet d'une **structure** virtuelle dérivée des pratiques linguistiques dans une communauté.



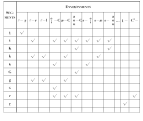
(Hjelmslev, 1935)



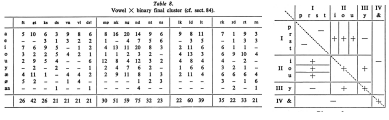

(Jakobson et al., 1952)



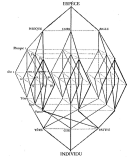
(Lévi-Strauss, 1949)



(Harris, 1960)



(Spang-Hanssen, 1959)



(Lévi-Strauss, 1962)

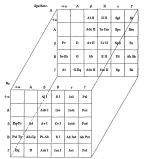
# Axe 2: Interprétabilité théorique

## Hypothèse distributionnelle

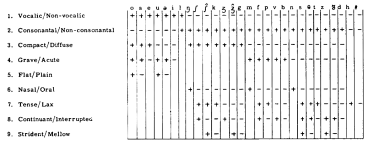
Le contenu des unités linguistiques est déterminé par leur **distribution** dans un corpus.

## Hypothèse structurale

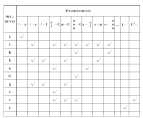
Le contenu linguistique est l'effet d'une **structure** virtuelle dérivée des pratiques linguistiques dans une communauté.



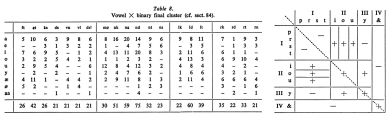
(Hjelmslev, 1935)



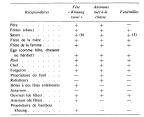

(Jakobson et al., 1952)



(Harris, 1960)

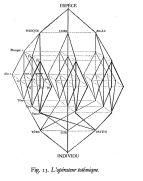


(Spang-Hanssen, 1959)



(Lévi-Strauss, 1949)

(Bourdieu, 1979)



(Lévi-Strauss, 1962)

(Bourdieu, 1994)

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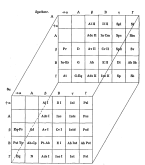
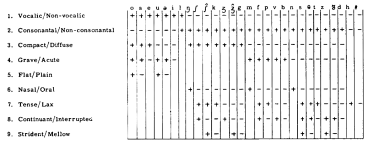
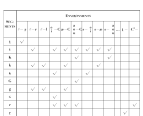


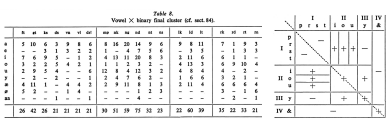
(Hjelmslev, 1935)




(Jakobson et al., 1952)



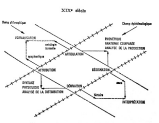
(Harris, 1960)



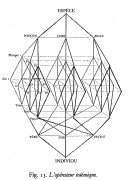
(Spang-Hanssen, 1959)

(Lévi-Strauss, 1949)

(Bourdieu, 1979)

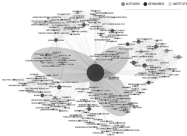


(Foucault, 1966)



(Lévi-Strauss, 1962)

(Bourdieu, 1994)



(Latour et al., 2012)

# Intégration

## LIPN, UMR 7030 (Paris)

- Équipe **LoCal** (Logique et Calcul)
- Accent sur les **fondements**
  (théorie des types, théorie de catégories, TAL)
- **Rapprochement** de différentes équipes
  (eg. axe "Sc. des données")
- Forte **interdisciplinarité**
  (santé, linguistique, physique, philosophie)

## LIRMM, UMR 5506 (Montpellier)

- Équipe **TEXTE** (Exploration et exploitation de données textuelles)
- Accent sur les **applications**
  (grammaires catégorielles, TAL, th. des types)
- Activités **transversales**
  (eg. axe "IA et Sc. des données")
- Forte **interdisciplinarité**
  (projet Muse: "Nourrir, Soigner, Protéger")

## Dans les deux cas

- **Collaboration** et **contact** avec des membres et la direction
- Intégration des aspects **épistémologiques** et **sociétaux** dans la recherche
- Présentation de mon travail aux équipes

# Travaux en cours

## Publications en cours

- “Content from expressions” (accepté à *Synthèse*)
- “LMs over Canonical BPEs” (soumis à ICML)
- “Distributional typing” (en cours)
- Chapitre sur LLMs (en cours, Oxford Handbook)

## Collaborations et projets

- Cluster “Foundations of AI” (CUNY, Simons Foundation)
- Projet “Human Forms” (soumis à E. Schmidt Foundation)

## Conférences à venir

- ICLR 2025 (Singapour, 24-28/04)
- Séminaire “IA et créativité” (Strasbourg, 13/05)
- Symposium “Chat Token Vector” (Venise, 11-13/06 - Keynote)
- Séminaire “Linguistics and Language Models” (Dagstuhl, 21-25/07)

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*CNRS - Concours chercheurs 2025*
DR Section 53 - Concours n° 53/01

# *Épistémologie des modèles distributionnels de langage par apprentissage machine*

## Explicabilité formelle et interprétabilité théorique

Juan Luis Gastaldi

`http://www.jlgastaldi.com/assets/gastaldi_cnrs_dr.pdf`

ETH zürich

22 Avril 2025

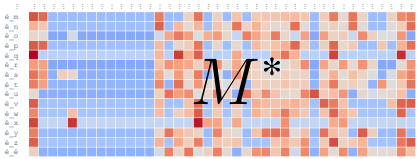
# Axe 1: Objectifs

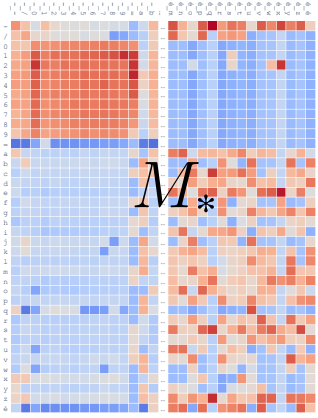
- Short Term
  - Systematizing, Formalizing, and Testing
  - Enriching Over $\bar{\mathbb{R}}$
  - Defining a Product over the Nucleus
- Medium Term
  - Characterizing Tokenization from a Structural Standpoint
  - Scaling for Real World Data
- Long Term
  - Assessment of state-of-the-art DNN language models in the light of the proposed formal framework (performance and efficiency comparison, limit properties, verification guarantees).
  - Exploration of other base categories and measures.
  - Further optimization.
  - Development of software packages and a usable integrated software framework for the scientific community.

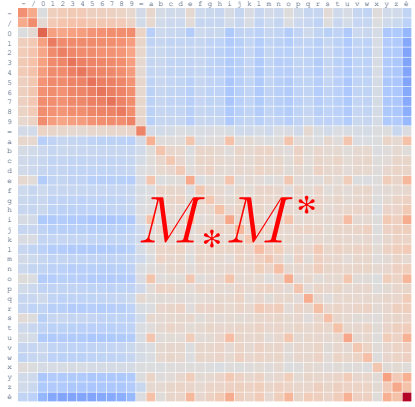
# Axe 2: Objectifs

- Short Term
  - Characterizing Epistemological Stakes through a Historical Lens
  - Interpreting Paradigms as Types
  - Adressing Semantic Aspects
- Medium Term
  - Revisiting the Theory of Distinctive Features and Exploring Possible Extensions
  - Studying the Syntacting properties of the Nucleus' compositional Structure
- Long Term
  - The study of pragmatic limits to the formal content captured by the set of types definable through a structural analysis of linguistic data.
  - The evaluation of the effects on ML models of the bias towards written language.
  - The assessment of other classically structuralist principles such as segmentation and double patterning.
  - A critique of the Symbolist-Connectionist debate in light of the results established in this program.
  - The exploration of possible applications of the framework proposed to non-linguistic data (sounds, images, formal languages, etc.)
  - The study of the conditions for the generalization of the use of the model proposed across scientific disciplines, specially in the Social Sciences and the Humanities.

# $M_* M^*$ comme matrice de covariance

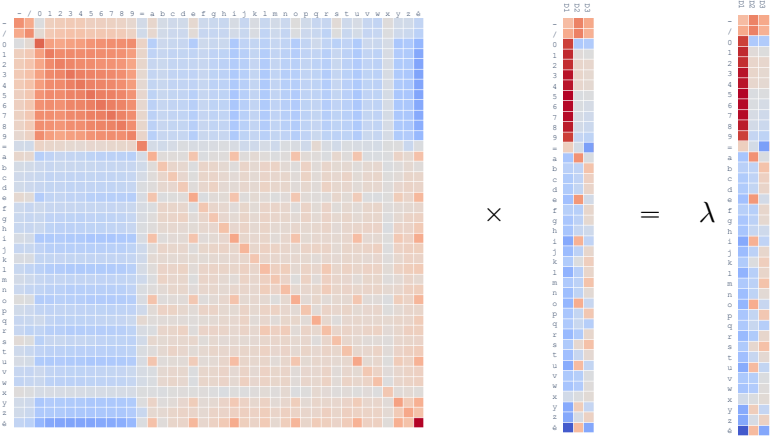


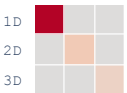
# Vecteurs propres comme points fixes

$$M_* M^* u = \lambda u$$

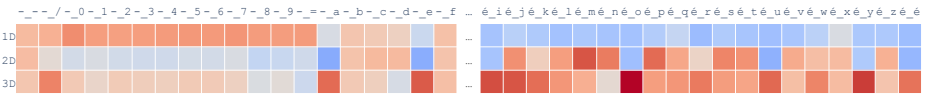


Eigenvectors of $M_* M^*$:

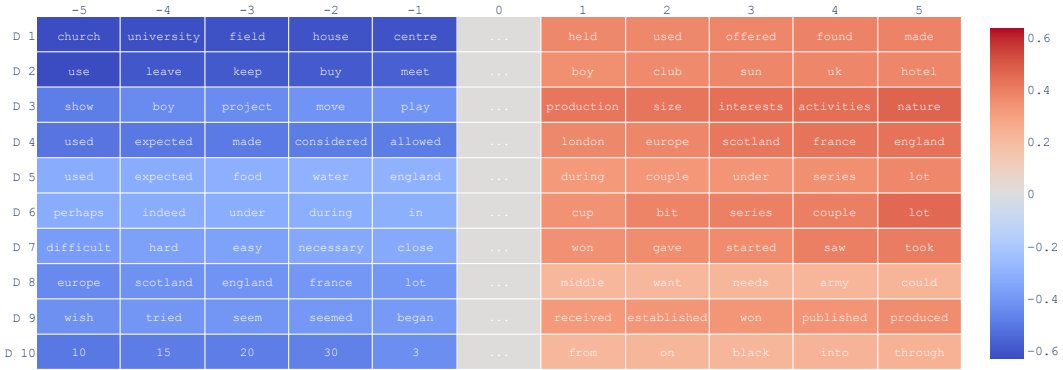

Eigenvalues of $M_* M^*$ and $M^* M_*$:




Eigenvectors of $M^* M_*$:

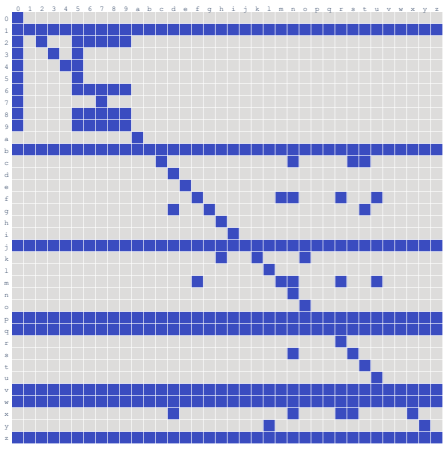


# Mots


-5 -4 -3 -2 -1 0 1 2 3 4 5
D 1 church university field house centre ... held used offered found made
D 2 use leave keep buy meet ... boy club sun uk hotel
D 3 show boy project move play ... production size interests activities nature
D 4 used expected made considered allowed ... london europe scotland france england
D 5 used expected food water england ... during couple under series lot
D 6 perhaps indeed under during in ... cup bit series couple lot
D 7 difficult hard easy necessary close ... won gave started saw took
D 8 europe scotland england france lot ... middle want needs army could
D 9 wish tried seem seemed began ... received established won published produced
D 10 10 15 20 30 3 ... from on black into through
0.6
0.4
0.2
0
-0.2
-0.4
-0.6

# Points fixes booléens

$$M_i^* M_*^i d = d$$

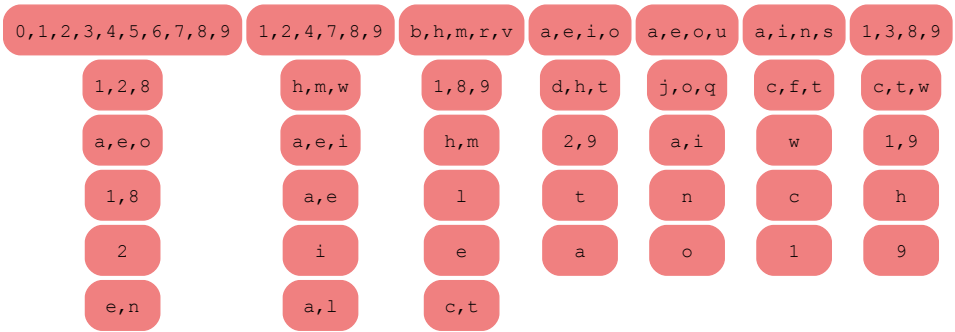


$\star$

$\stackrel{?}{=}$

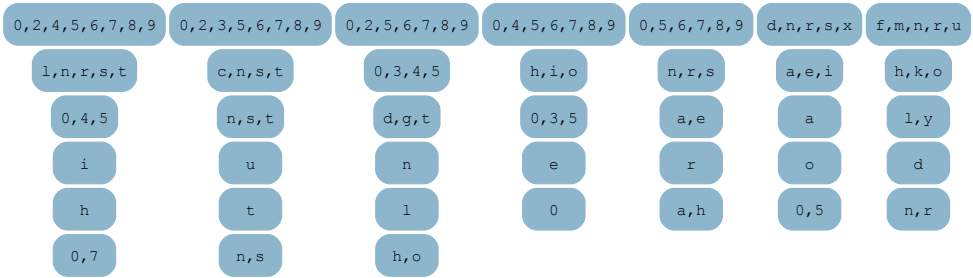
# “Eigensets”

$$\mathcal{M}_*\mathcal{M}^* f = f$$

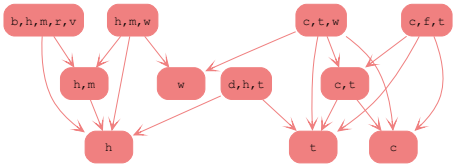


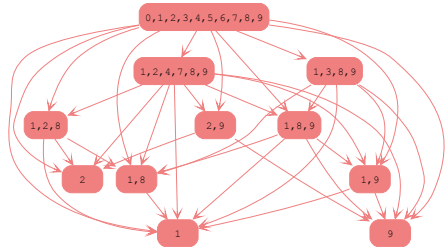
# “Eigensets”

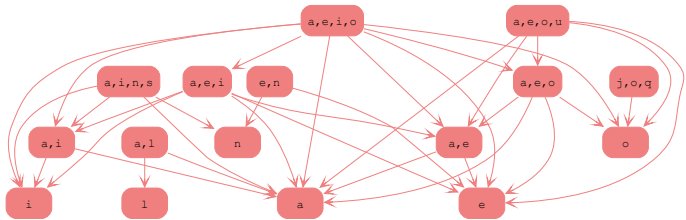
$$M_i^* M_*^i d = d$$



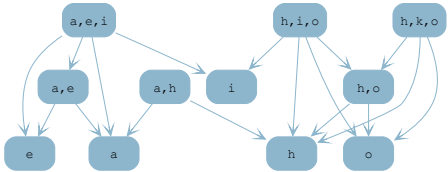
# Quelle Structure?

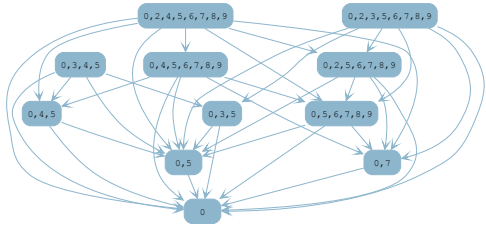


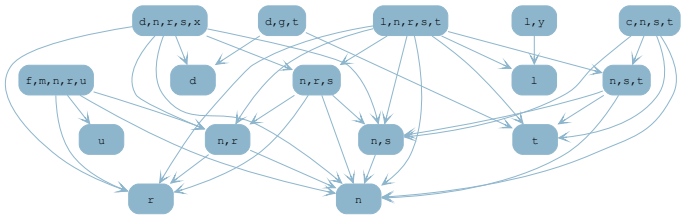



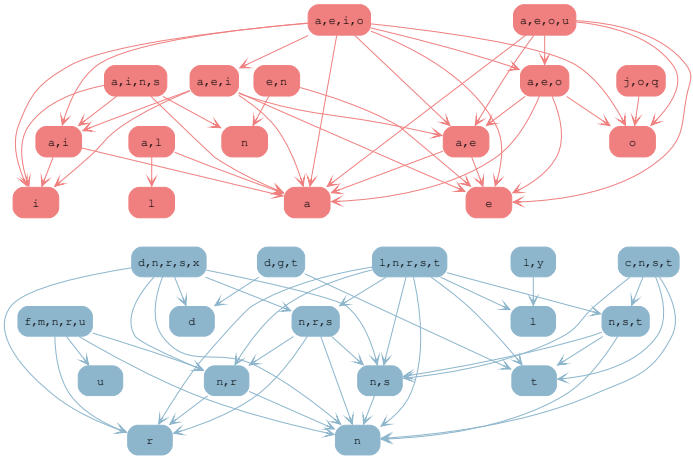
# Quelle Structure?



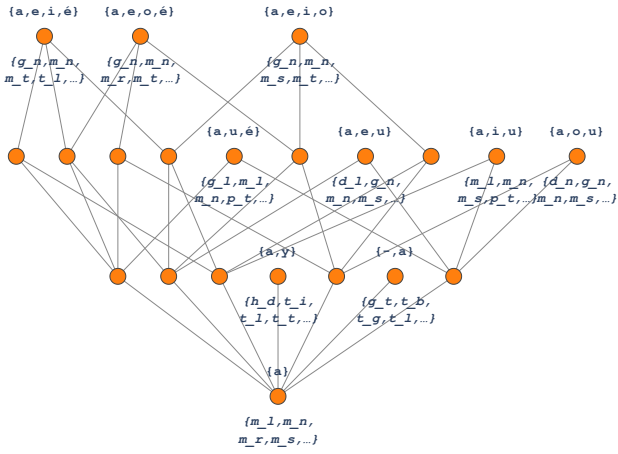



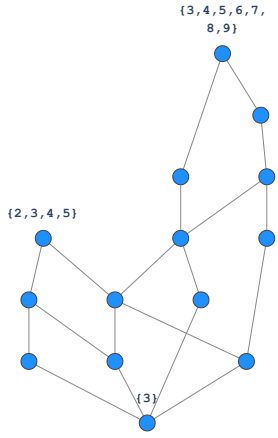



# Quelle Structure?

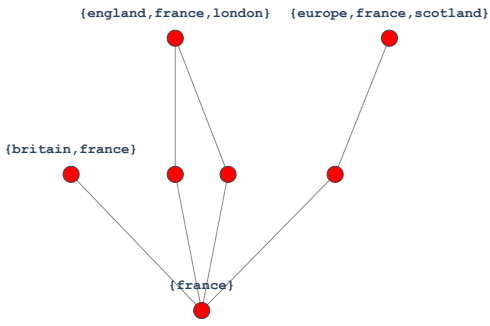


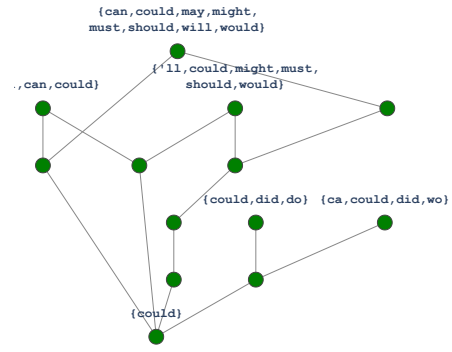
# Concepts formels



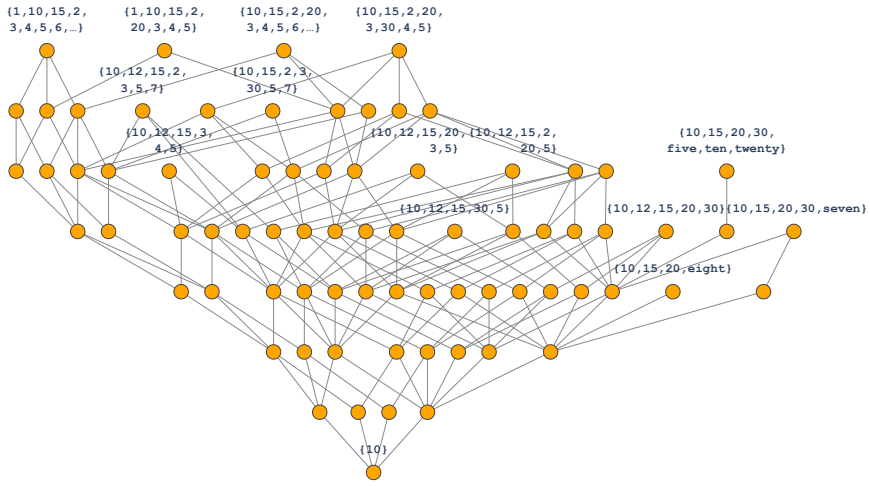



# Concepts formels (mots)



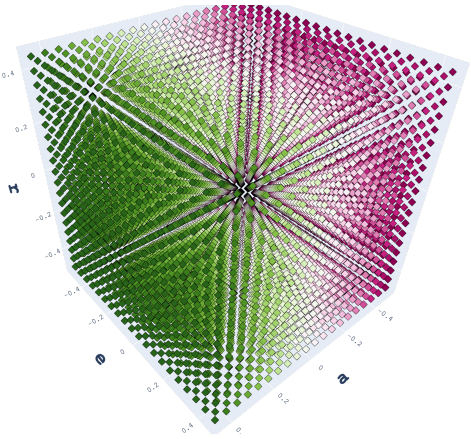



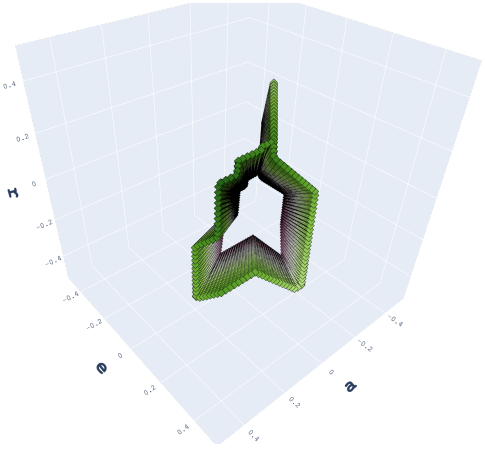
# Concepts formels (mots)



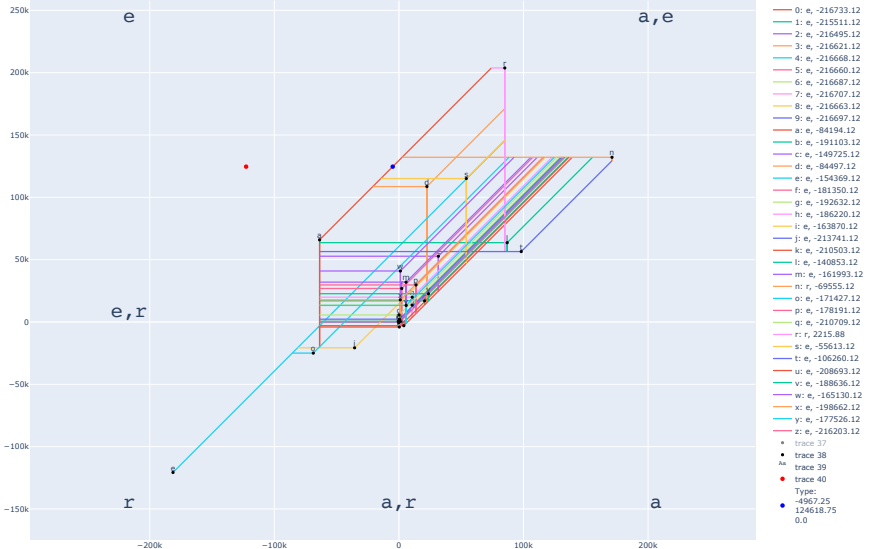
Noyeau (nucleus)

$$\bar{\mathbb{R}}\{a,e,r\} \xrightarrow{\mathcal{M}_*\mathcal{M}^*} \bar{\mathbb{R}}\{a,e,r\}$$



# Structure interne du noyeau



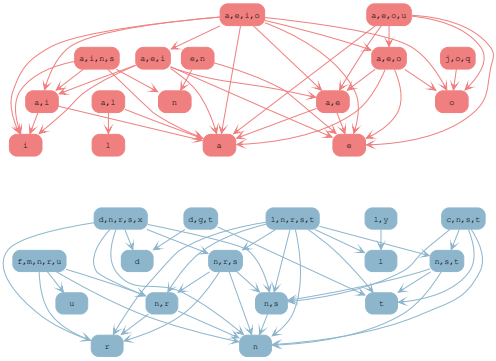
# Théorie des types computationnels

**Definition (Polaire/Orthogonal - Girard, 2006)**

[É]tant donnée une fonction binaire $a, b \rightsquigarrow \langle a|b\rangle \colon A \times B \to C$ et un sous-ensemble $P \subset C$ (le « pôle »), on peut définir le *polaire* $X^\perp \subset B$ d'un sous-ensemble $X \subset A$ (resp. $Y^\perp \subset A$ d'un sous-ensemble $Y \subset B$) par :

$$X^\perp := \{y \in B \colon \forall x \in X, \langle a|b\rangle \in P\}$$
$$Y^\perp := \{x \in A \colon \forall y \in Y, \langle a|b\rangle \in P\}$$

- L'application « polaire » est décroissante: $X \subset X' \Rightarrow X'^\perp \subset X^\perp$.
- L'ensemble $\mathrm{Pol}(A) \subset \mathcal{P}(A)$ des ensembles *polaires*, i.e., de la forme $Y^\perp$, est stable par intersections arbitraires. En particulier, $A$ est polaire et $X^{\perp\perp}$ est le plus petit ensemble polaire contenant $X$.
- En conséquence, $X^{\perp\perp\perp} = X^\perp$.



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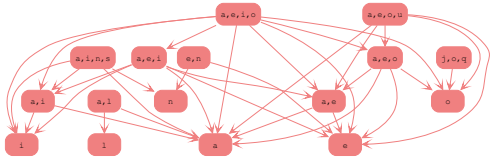
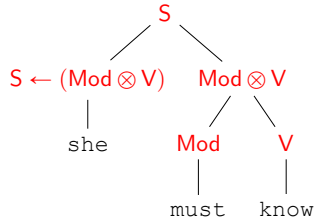



(Gastaldi and Pellissier, 2021)

# Matrice et analogie

| | |
|---|---|
| a = your<br>c = my | w = apartment<br>x = house<br>y = chair<br>z = stool |

| | … | w | x | y | z | … |
|---|---|---|---|---|---|---|
| … | … | 0 | 0 | 0 | 0 | … |
| a | … | 0 | 1 | 1 | 0 | … |
| b | … | 0 | 0 | 1 | 1 | … |
| c | … | 1 | 0 | 0 | 1 | … |
| … | … | 0 | 0 | 0 | 0 | … |

your : house
my : apartment

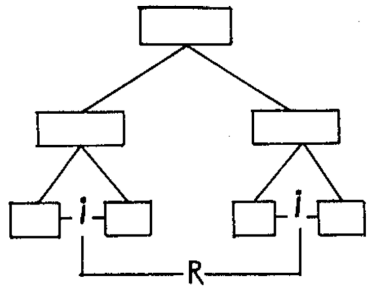
# Semiotique Structurale

Une sémiotique [...] est une hiérarchie dont chacune des composantes admet une analyse ultérieure en classes définies par relation mutuelle, de telle sorte que chacune de ces classes admette une analyse en dérivés définis par mutation mutuelle.

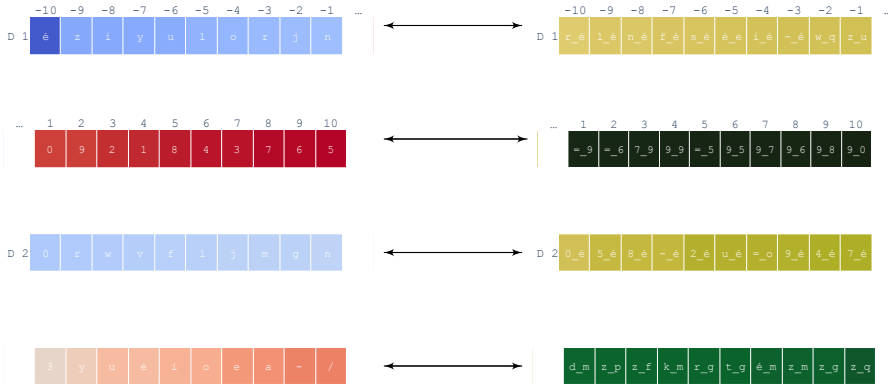
(Hjelmslev, 1975, Df. 24)

Une mutation [...] est une fonction existant entre des dérivés du premier degré d'une seule et même classe, une fonction qui a une relation à une fonction entre d'autres dérivés de premier degré d'une seule et même classe et appartenant au même rang.

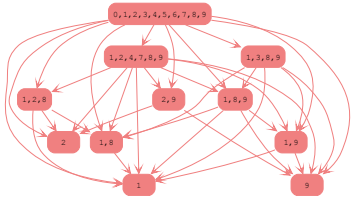
(Hjelmslev, 1975, Df. 23)

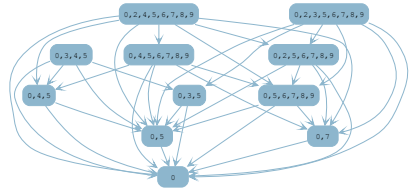


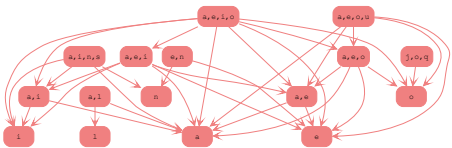
# Syntagmatique et Texte (Vecteurs)

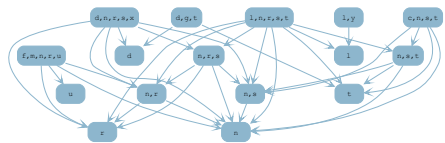


# Syntagmatique et Texte (Noyeau/Types)

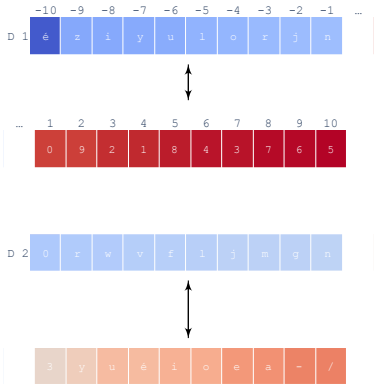


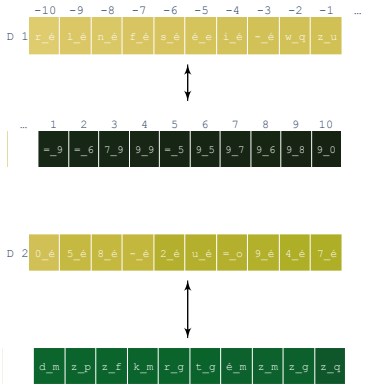



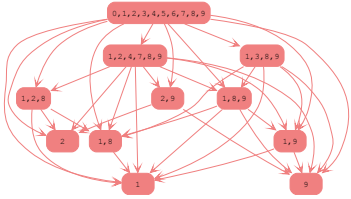



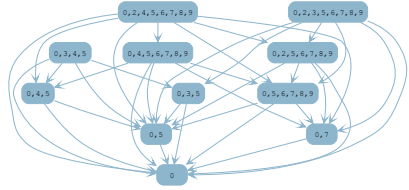
# Paradigmatique et Langue (Vecteurs)

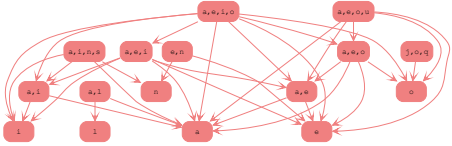


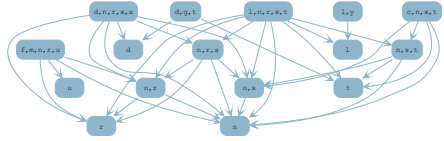



# Paradigmatique et Langue (Noyau/Types)



# Illustration du contenu formel

(Gastaldi and Pellissier, 2021)