

Workshop on AI & DH
CRIHN, Université de Montréal
Montréal, Canada

Sous les distributions, la structure!

Unearthing structural(ist) features from linguistic distributions

Juan Luis Gastaldi



November 28, 2024

Outline

Introduction

Word Embeddings

The Structure behind Word Embeddings

Example: Wikipedia

Which Structure?

What Computation?

Why Language?

Conclusion

Outline

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Example: Wikipedia

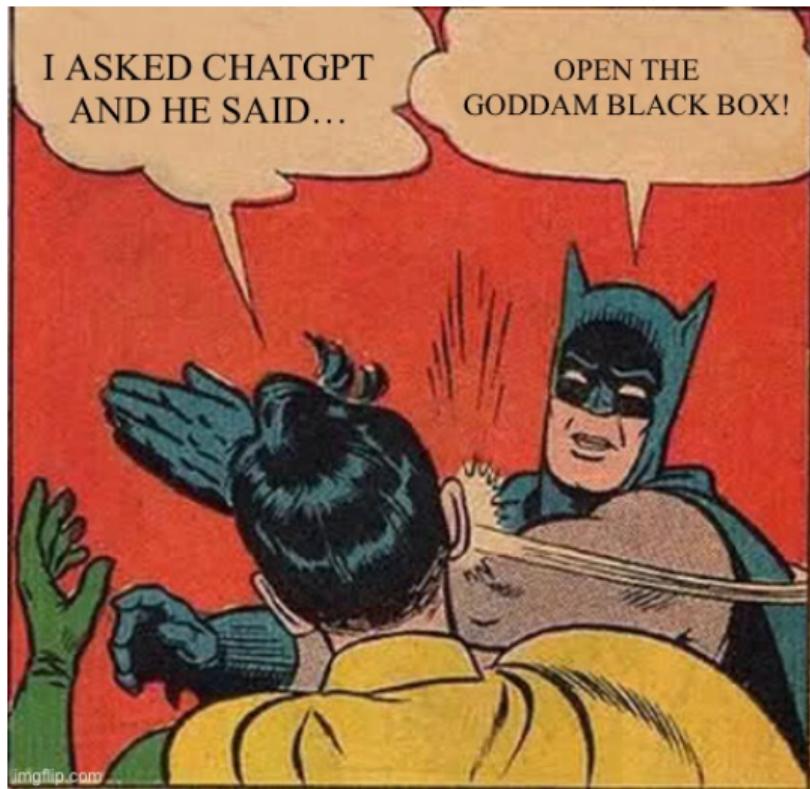
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- ◊ When it is not ideologically motivated, the black box narrative relies on a **categorical mistake**.
 - ML models are not *natural objects* but *formal objects*.

Stochastic Parrots vs. AI Consciousness



Language models are not like us,
therefore they do not and can not have any relation to meaning.



Language models have a relation to meaning,
therefore they are like us.

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- ◊ ~~What do LLMs understand?~~
 - What do we understand about LLMs ?
 - What can we understand *about language* through them?

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- ◊ ~~What do LLMs understand?~~
 - What do we understand about LLMs ?
 - What can we understand *about language* through them?
- ◊ LLMs tell us something about the **Computational Structure of Language**

Opening the Black-Box

Subword Tokenization
(Sennrich et al., 2016)

Word Embeddings
(Mikolov, Sutskever, Chen, Corrado, and Dean, 2013)

Self-Attention
(Vaswani et al., 2017)

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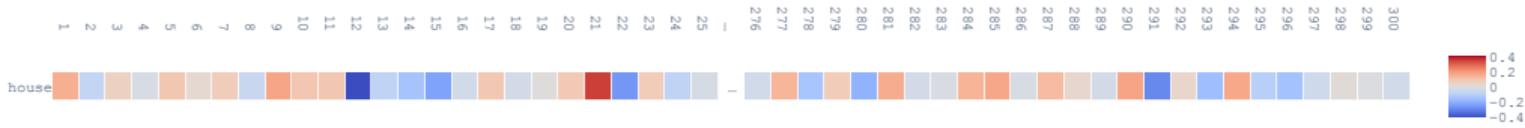
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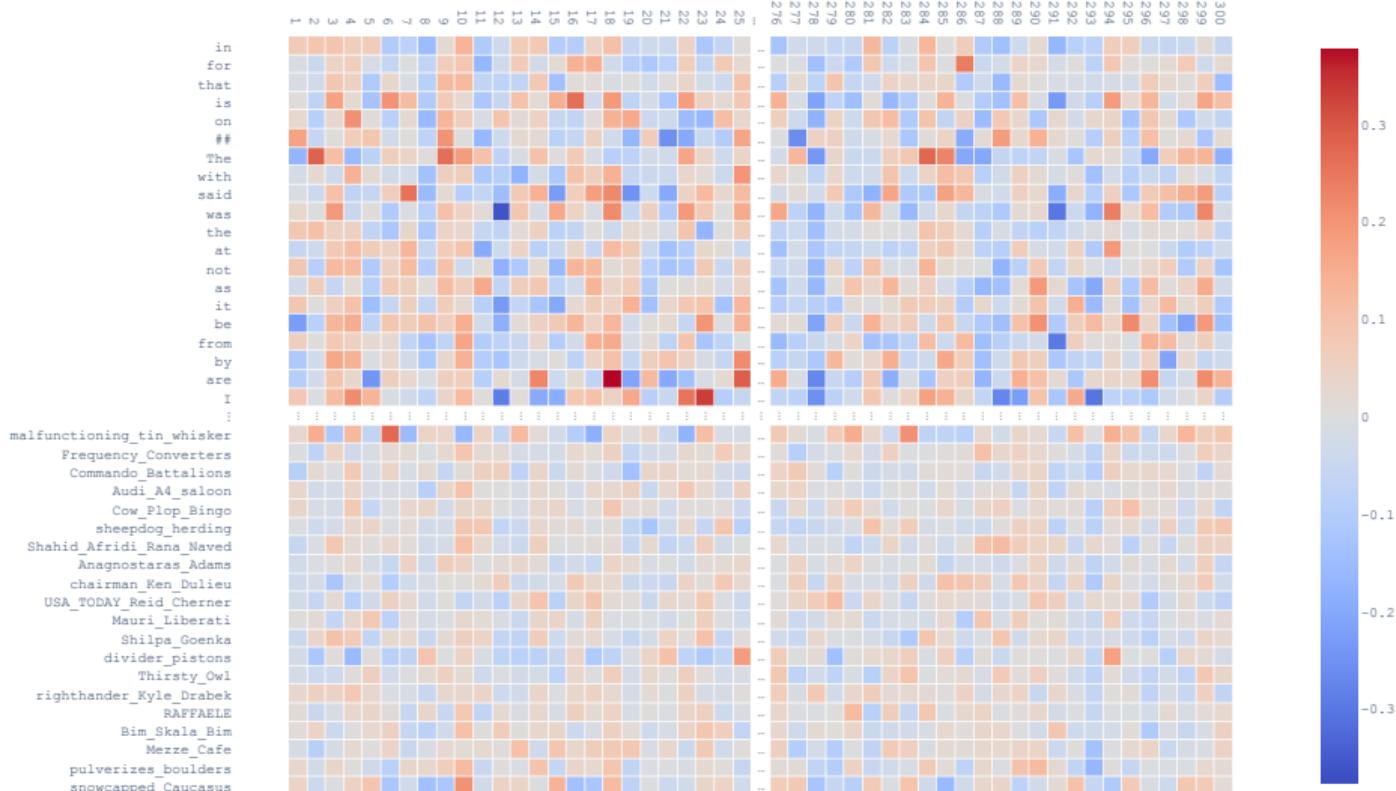
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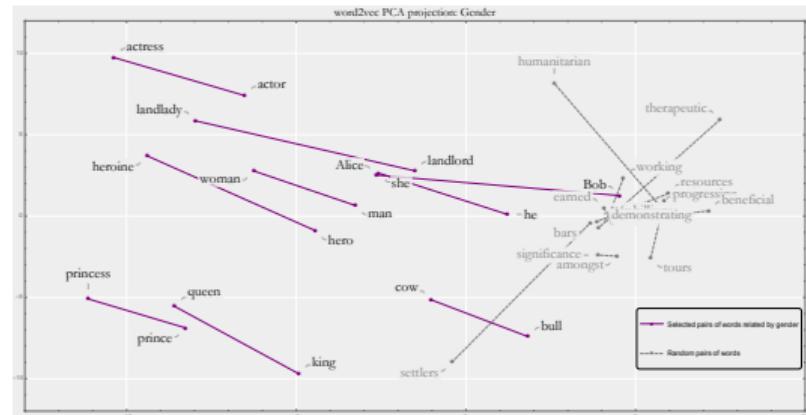
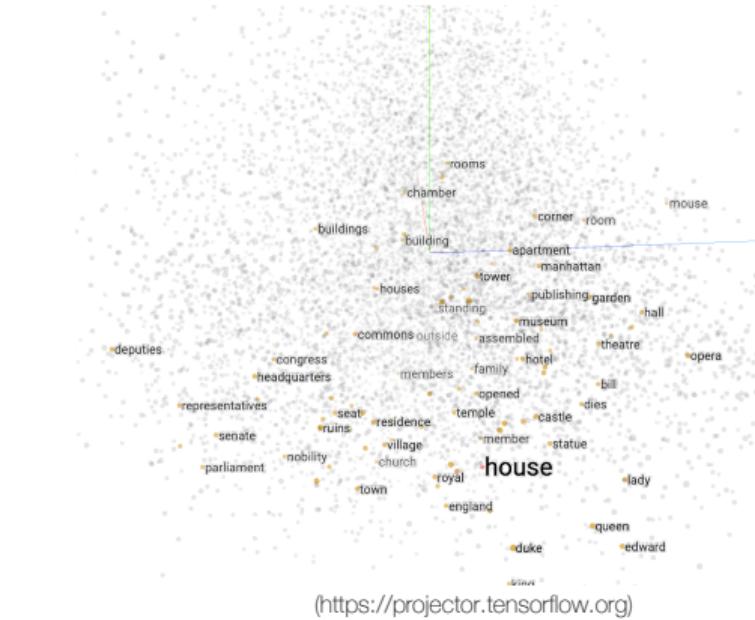
Word Embeddings: Vector



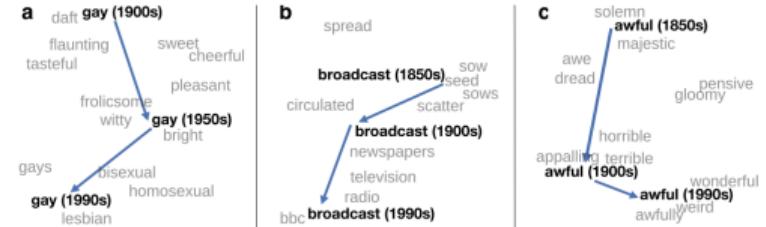
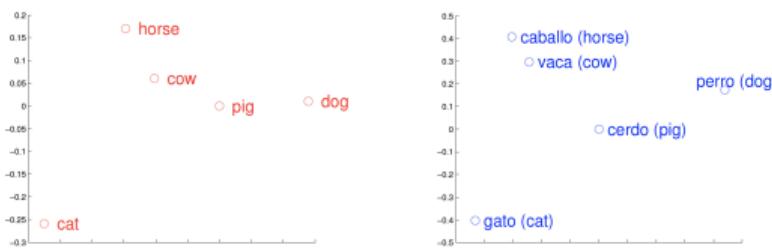
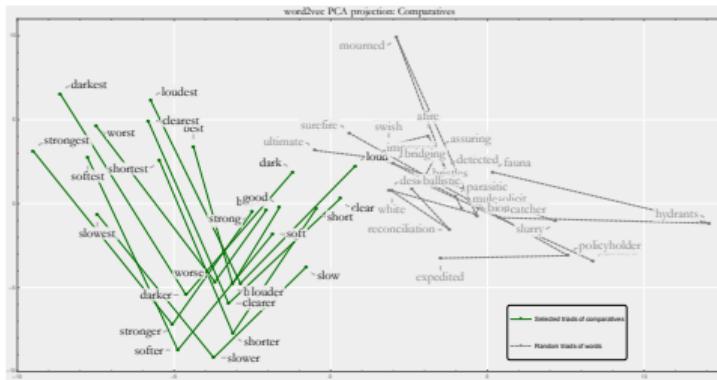
Word Embeddings: Matrix



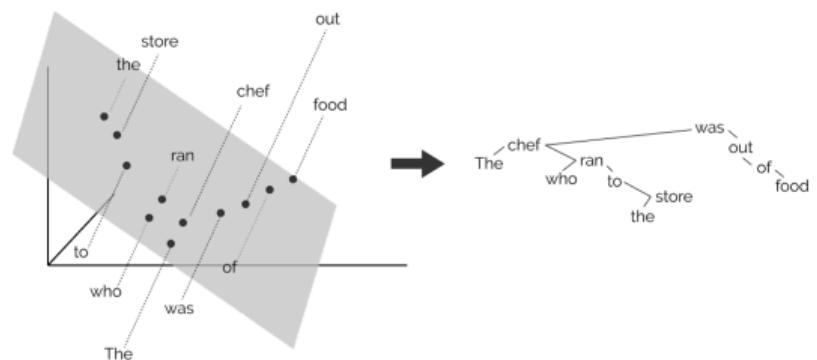
Embedding Space: Similarity and Analogy



Embedding Space: Other Applications

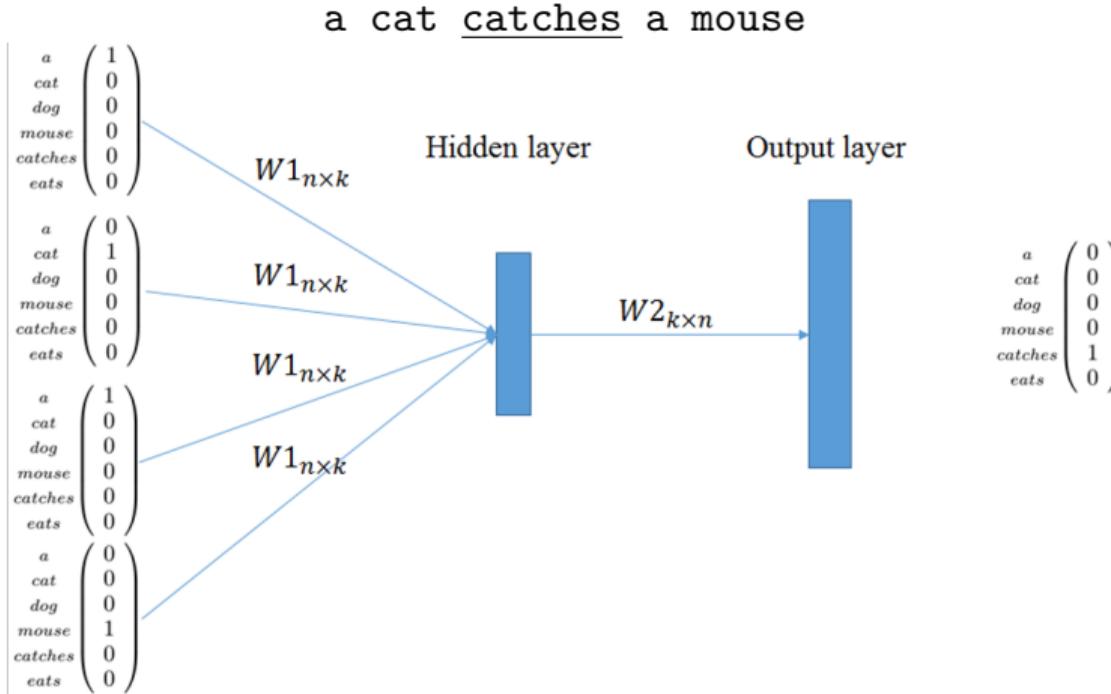


(Hamilton et al., 2016)



(<https://nlp.stanford.edu/~johnhew/structural-probe.html>)

word2vec Models



Credit: Ferrone et al., 2017

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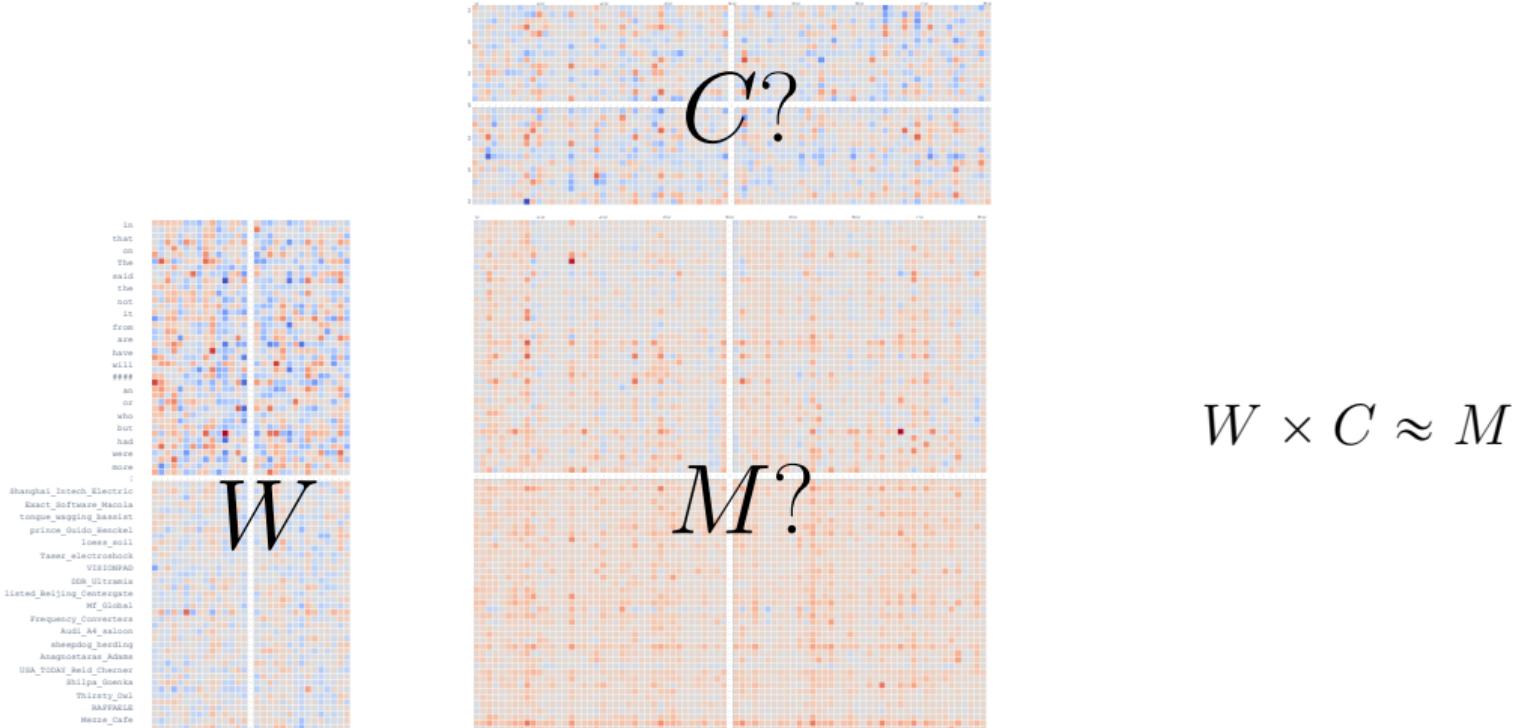
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word2vec as Implicit Matrix Factorization (Levy and Goldberg, 2014)



word2vec as Implicit Matrix Factorization

(Levy and Goldberg, 2014)



word2vec Explained (Levy and Goldberg, 2014)

$$\ell = \sum_{w \in V_w} \sum_{c \in V_c} \#(w, c) (\log \sigma(\vec{w} \cdot \vec{c}) + k \cdot \mathbb{E}_{c_N \sim P_D} [\log \sigma(-\vec{w} \cdot \vec{c}_N)])$$

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Three results:

- ◊ $M = PMI(w, c) - \log k$ (Pointwise Mutual Information)

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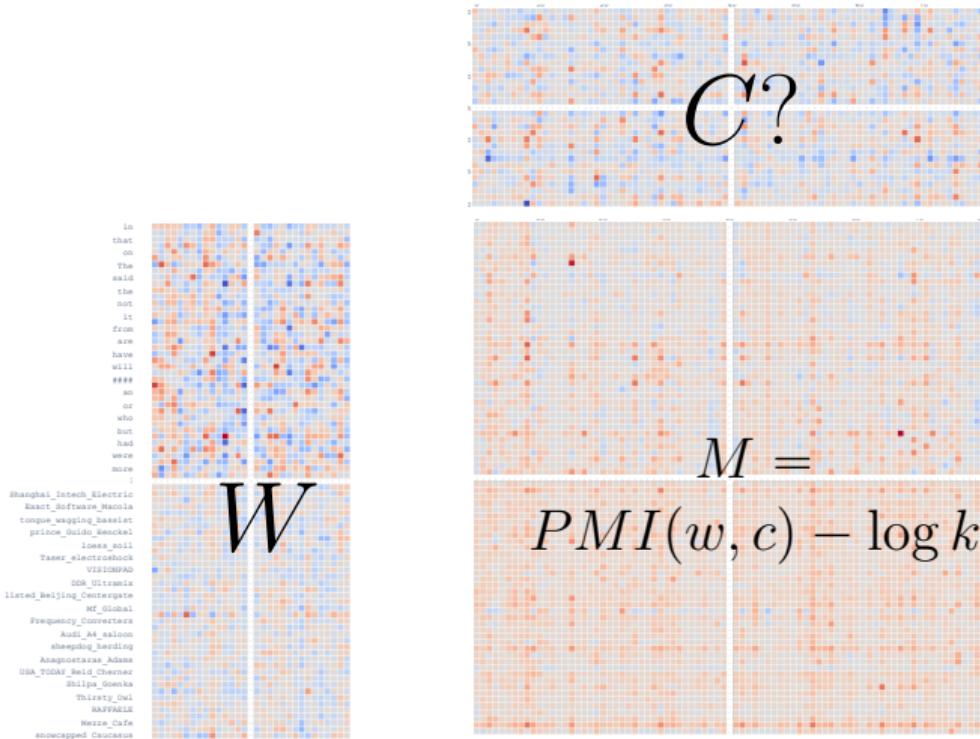
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Three results:

- ◊ $M = PMI(w, c) - \log k$ (Pointwise Mutual Information)
- ◊ W is low dimensional
- ◊ The Singular Value Decomposition (SVD) provides an exact solution to find W

Pointwise Mutual Information (PMI)



$$PMI(w, c) = \log \frac{p(w, c)}{p(w)p(c)}$$

Singular Value Decomposition (SVD)

$$M = U\Sigma V^*$$

Where:

M = $m \times n$ (real or complex) matrix

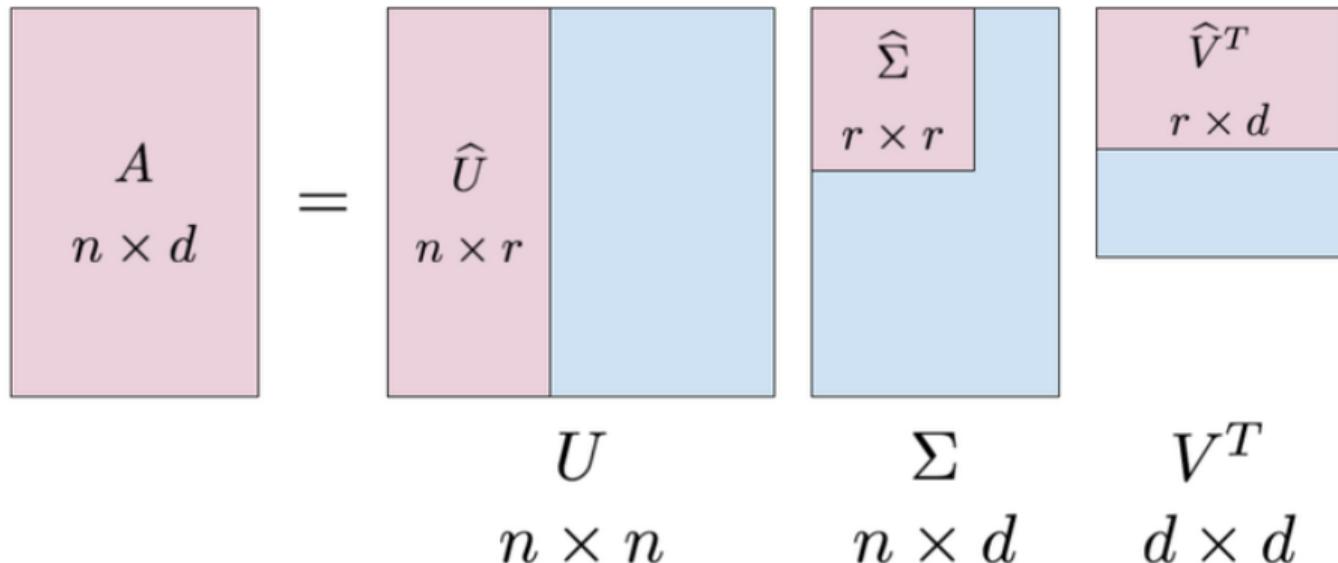
U = $m \times m$ unitary matrix

Σ = $m \times n$ non-negative real rectangular diagonal matrix

V^* = conjugate transpose of V , a $n \times n$ unitary matrix

Truncated SVD

$$M = U\Sigma V^*$$

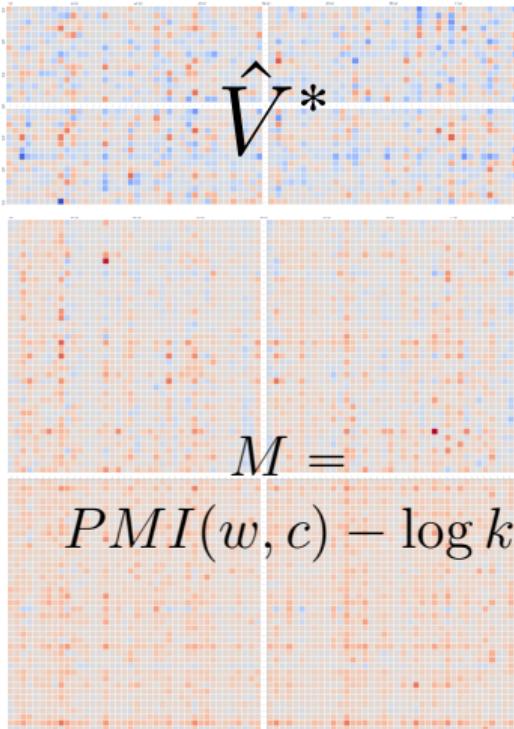


Credit: Angela Ju

Word Embeddings as Truncated SVD

in
on
The
said
the
not
it
from
are
have
will

an
or
who
But
had
were
more
Shanghai_Shanghai
Kunming_Kunming
Computer_Science
Physics_Physics
Electron_Electron
VISIONPAD
DE_Ultramix
listed_Beijing_Centergate
MC_Global
Frequency_Converters
Audi_Audi
Sheets_Material
Anastasiia_Adana
USA_TGDFR_Wild_Cheese
Philip_Greens
Thirsty_Owl
RAFVALE
Mezze_Cafe
snowcapped_Caucasus



$$M \approx \hat{U} \times \hat{\Sigma} \times \hat{V}^*$$

$$PMI(w, c) - \log k$$

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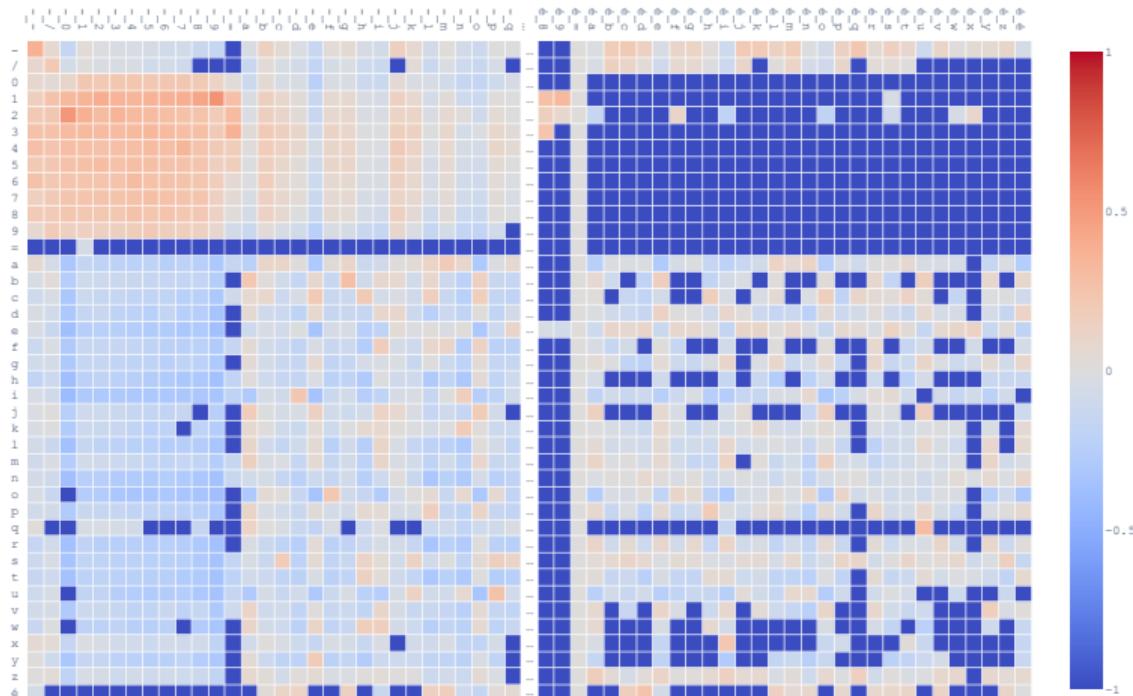
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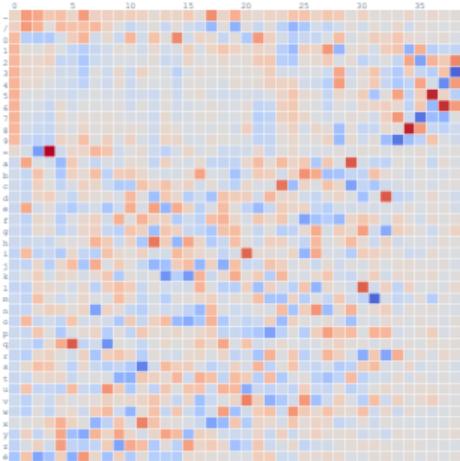
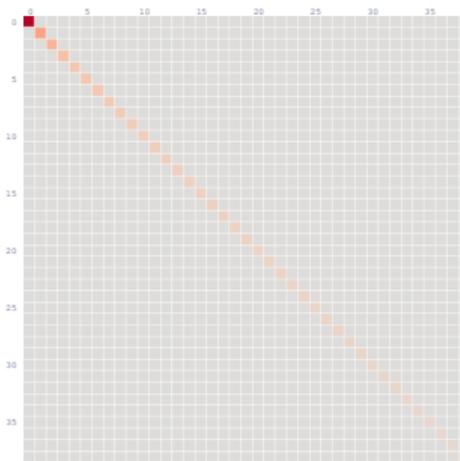
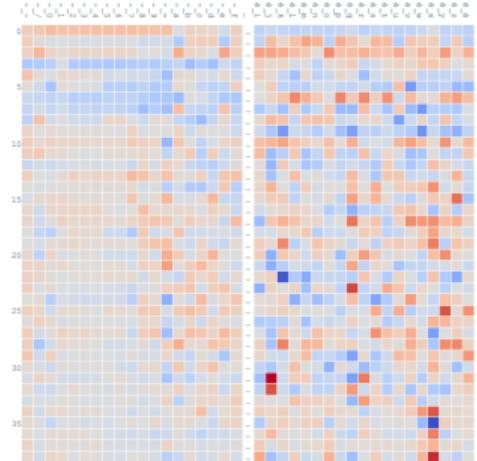
Example: Characters in Wikipedia

$$PMI(w, c) =$$

$$\log \frac{p(w,c)}{p(w)p(c)}$$

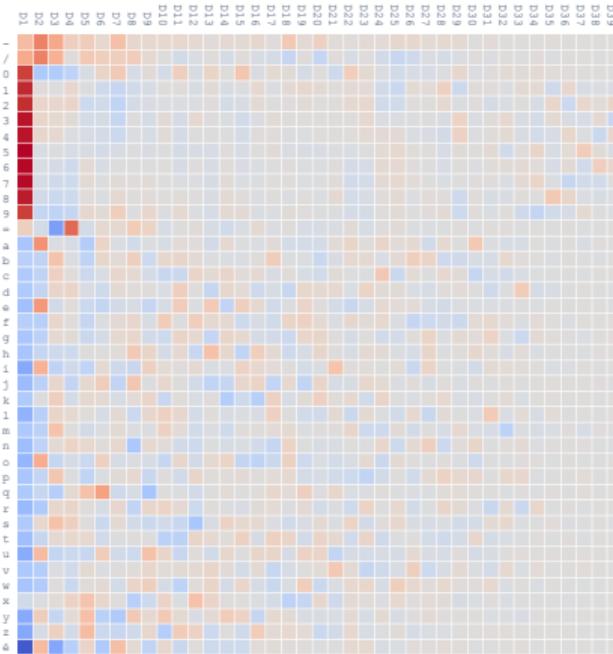


SVD of Wikipedia Character PMI Matrix

 U  Σ  V^* 

Truncate and Embed

$$U \times \Sigma$$



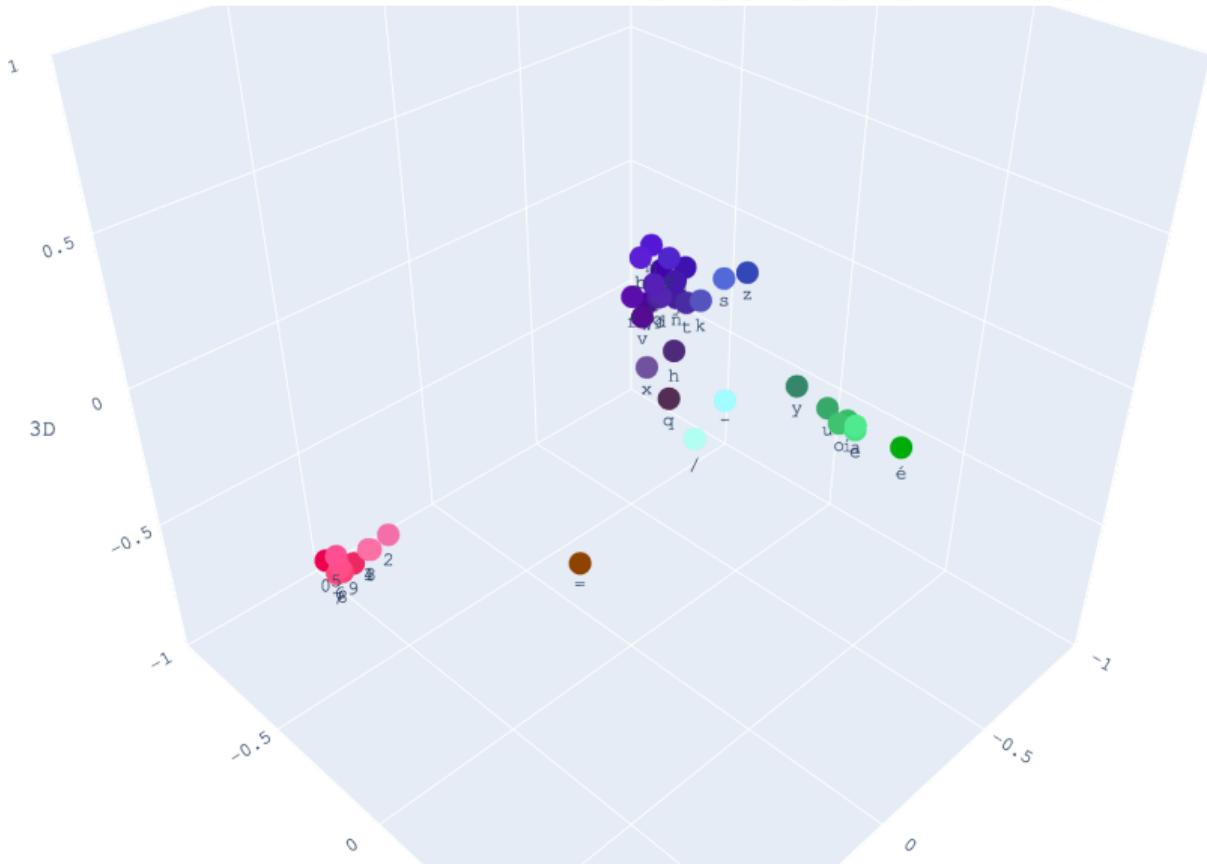
Truncate and Embed

$$\hat{U} \times \hat{\Sigma}$$



Truncate and Embed

$$\hat{U} \times \hat{\Sigma}$$



But Why?

4 Why does this produce good word representations?

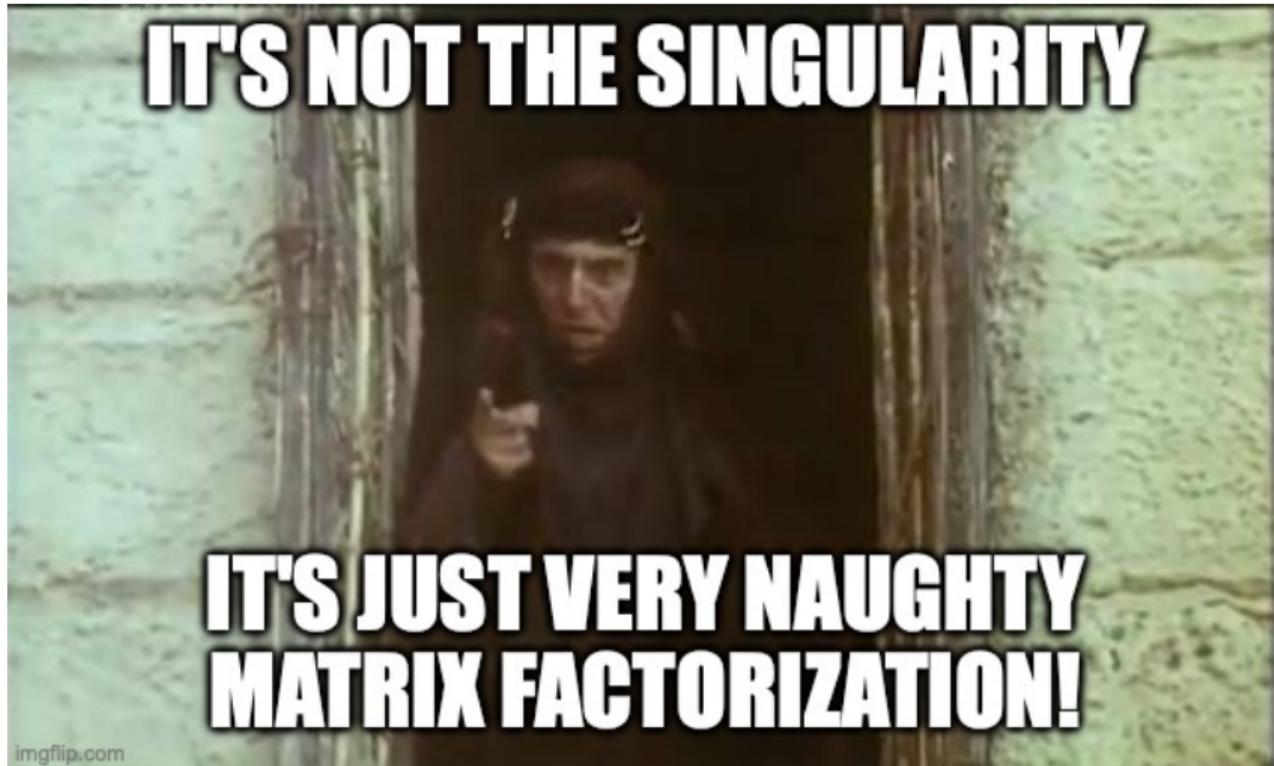
Good question. We don't really know.

The distributional hypothesis states that words in similar contexts have similar meanings. The objective above clearly tries to increase the quantity $v_w \cdot v_c$ for good word-context pairs, and decrease it for bad ones. Intuitively, this means that words that share many contexts will be similar to each other (note also that contexts sharing many words will also be similar to each other). This is, however, very hand-wavy.

Can we make this intuition more precise? We'd really like to see something more formal.

(Goldberg and Levy, 2014)

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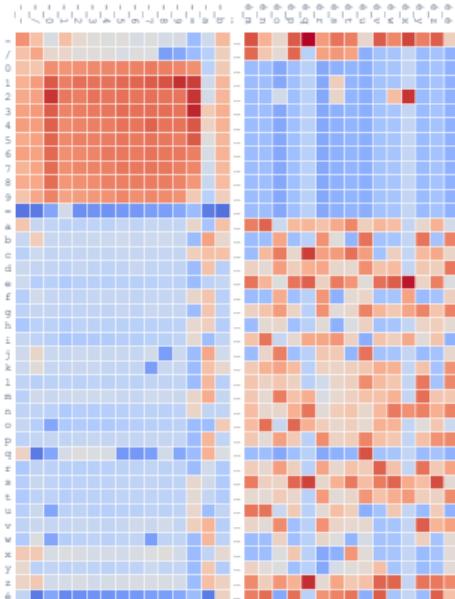
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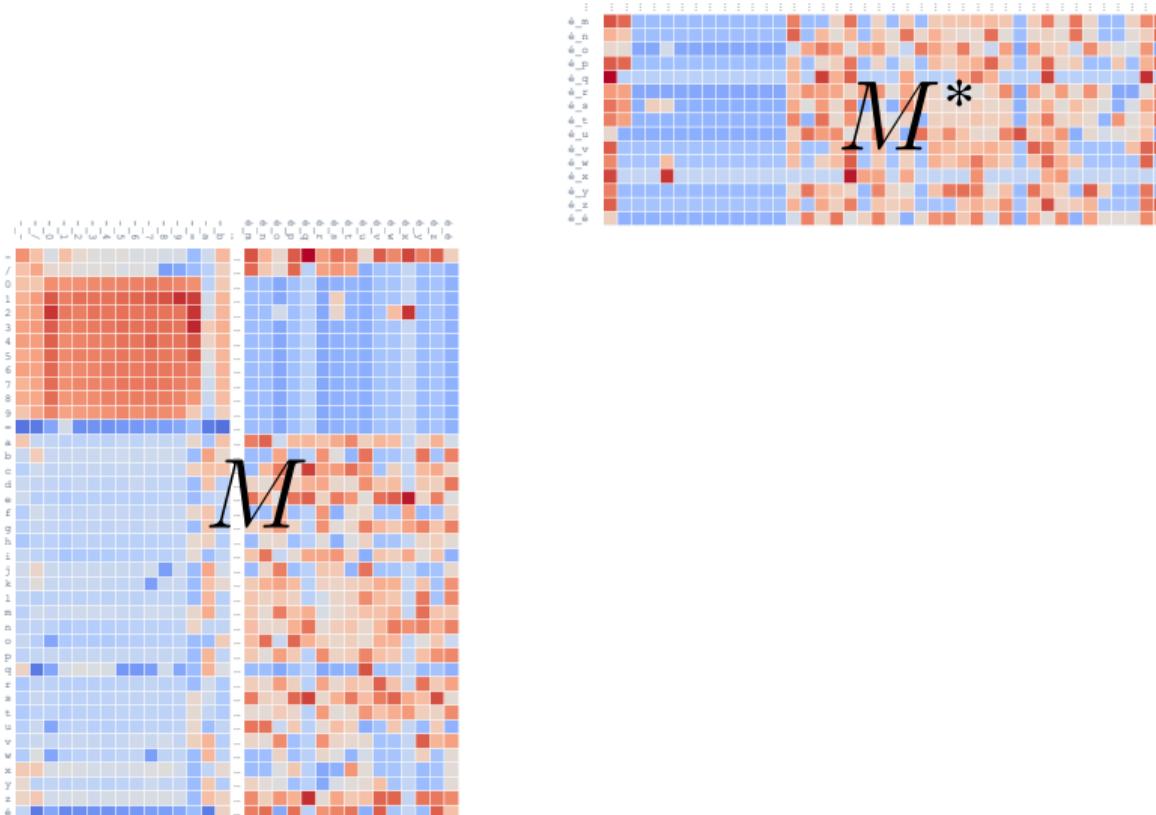
In particular:

- ◊ The columns of U (left singular vectors) are **eigenvectors of MM^***
- ◊ The rows of V^* (right singular values) are **eigenvectors of M^*M**
- ◊ The non-zero elements of Σ (non-zero singular values) are the square roots of the non-zero **eigenvalues of MM^* or M^*M**

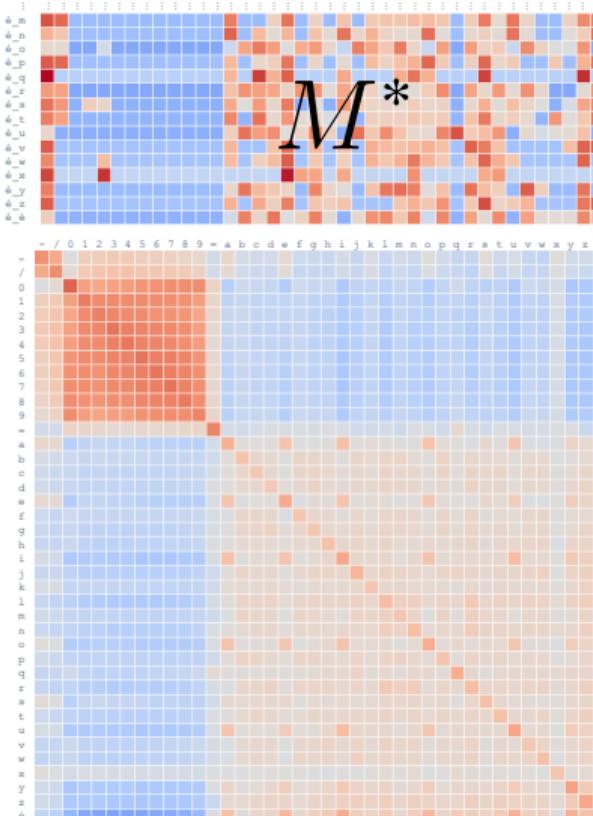
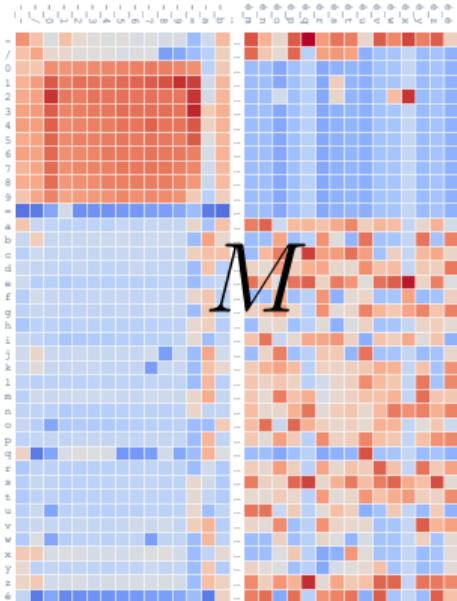
$M \times M^*$ as a Covariance Matrix



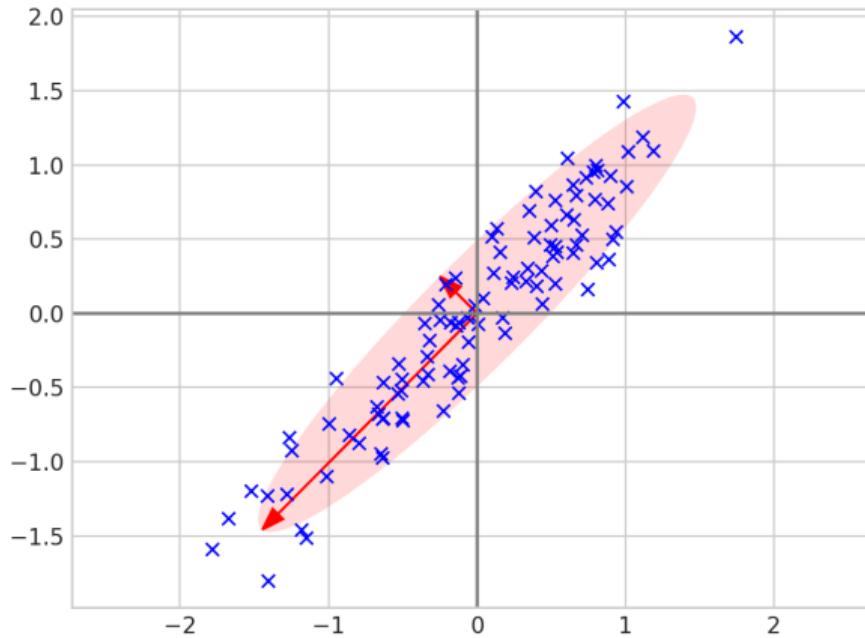
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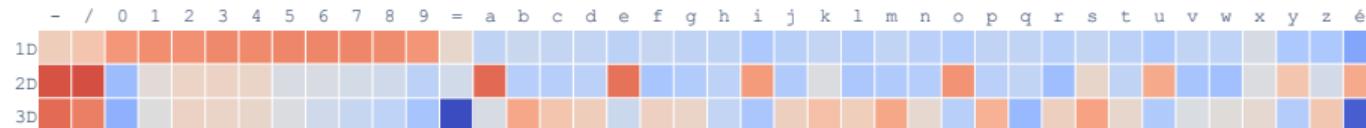
Eigenvectors and Eigenvalues



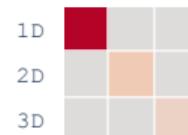
Credit: Joel Laity

Structural Features

Eigenvectors of $M \times M^*$:



Eigenvalues of $M \times M^*$:



Eigenvectors of $M^* \times M$:

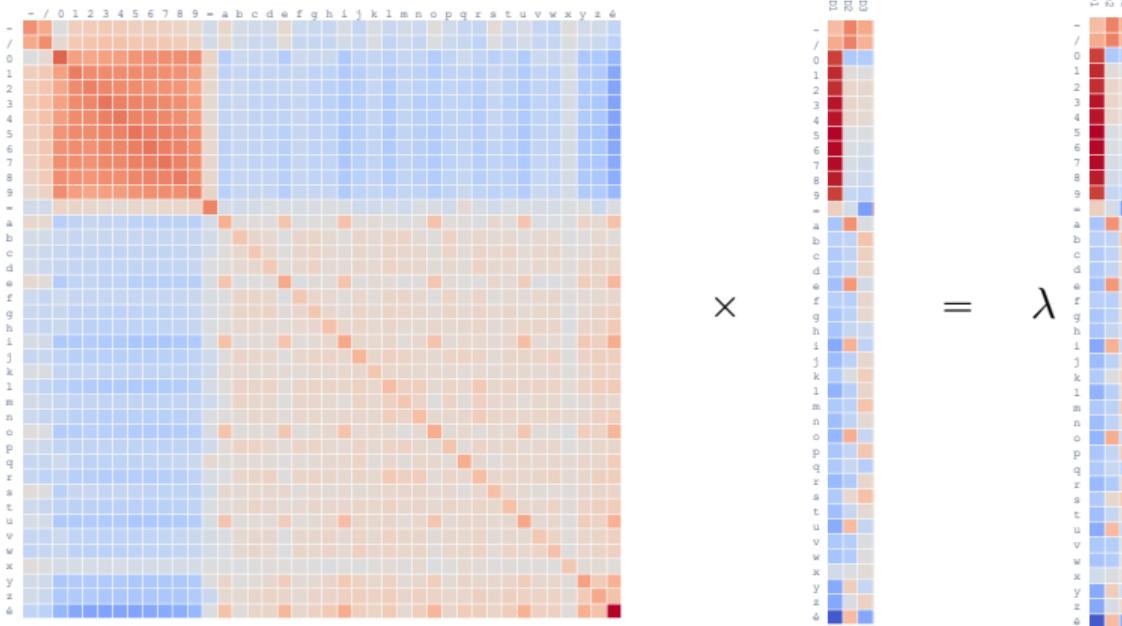


Eigenvectors as Fixed Points

$$(M \times M^*)\mathbf{v} = \lambda \mathbf{v}$$

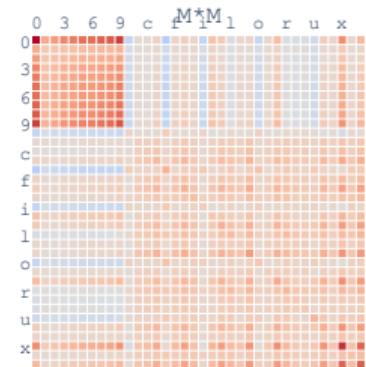
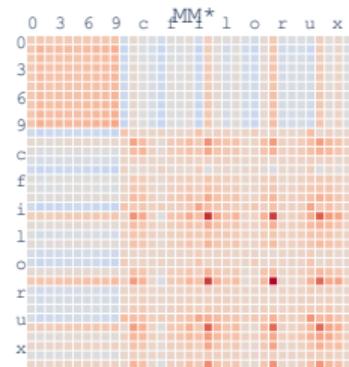
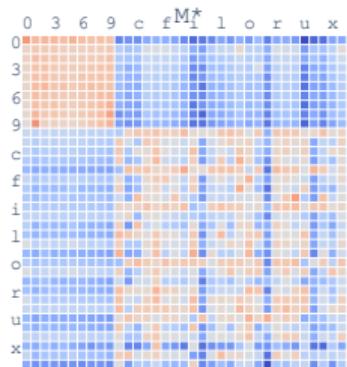
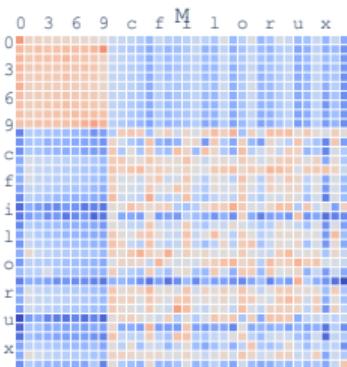
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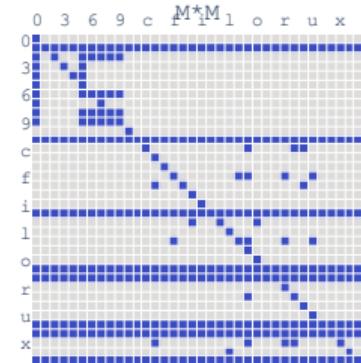
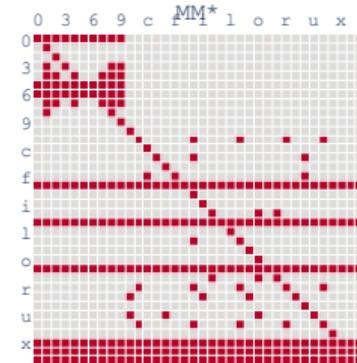
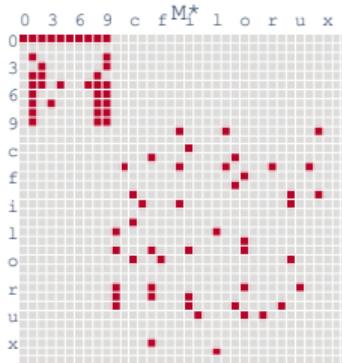
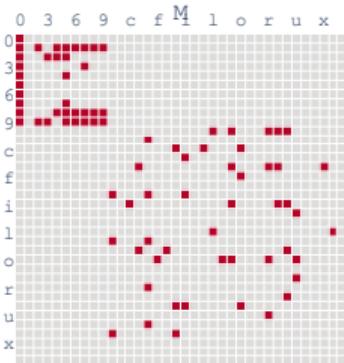
Binary Matrices

$$(M_t \star M_t^*) \star v = v$$



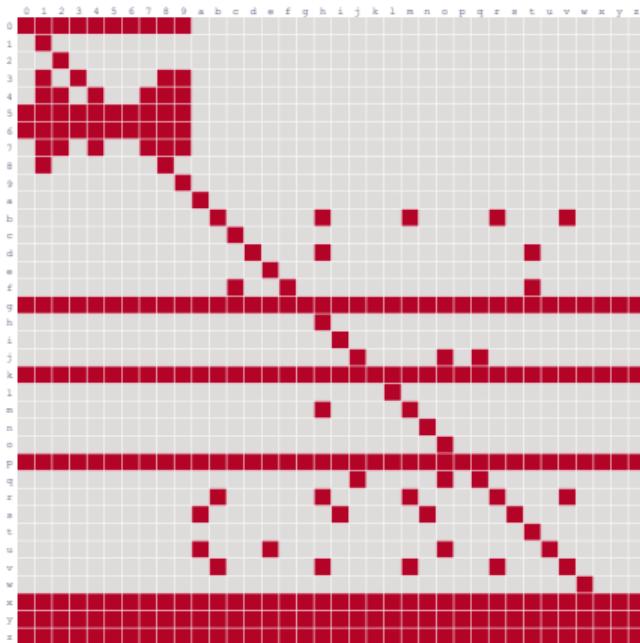
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Binary Fixed Points

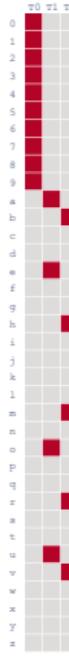
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\star

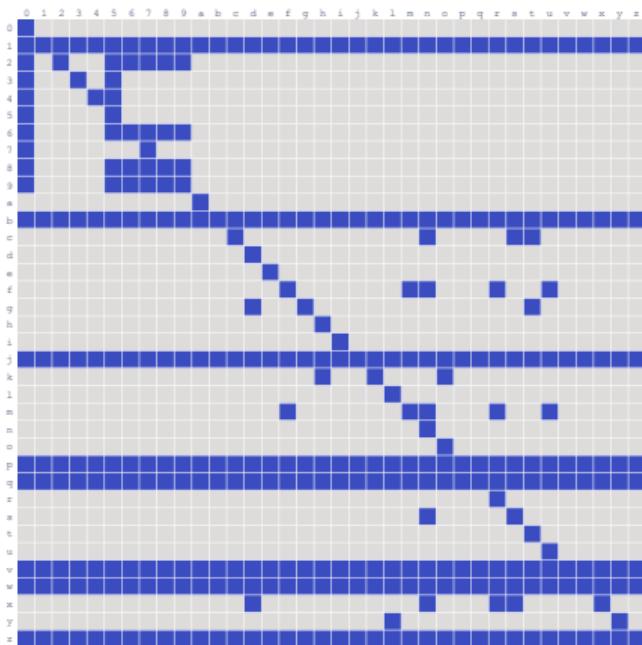


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Binary Fixed Points

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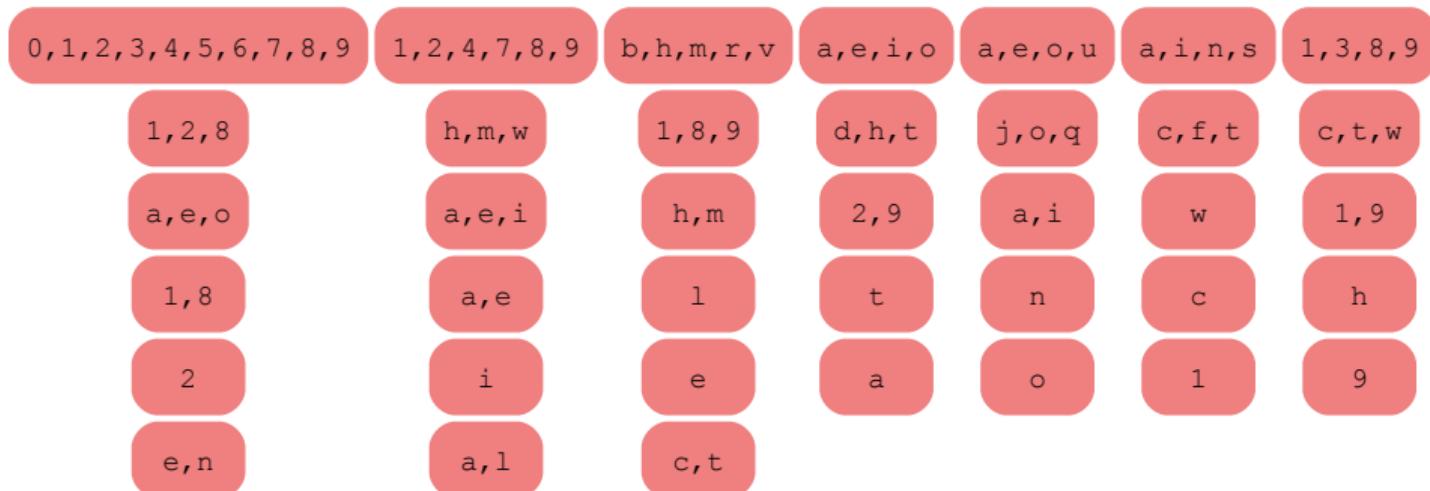
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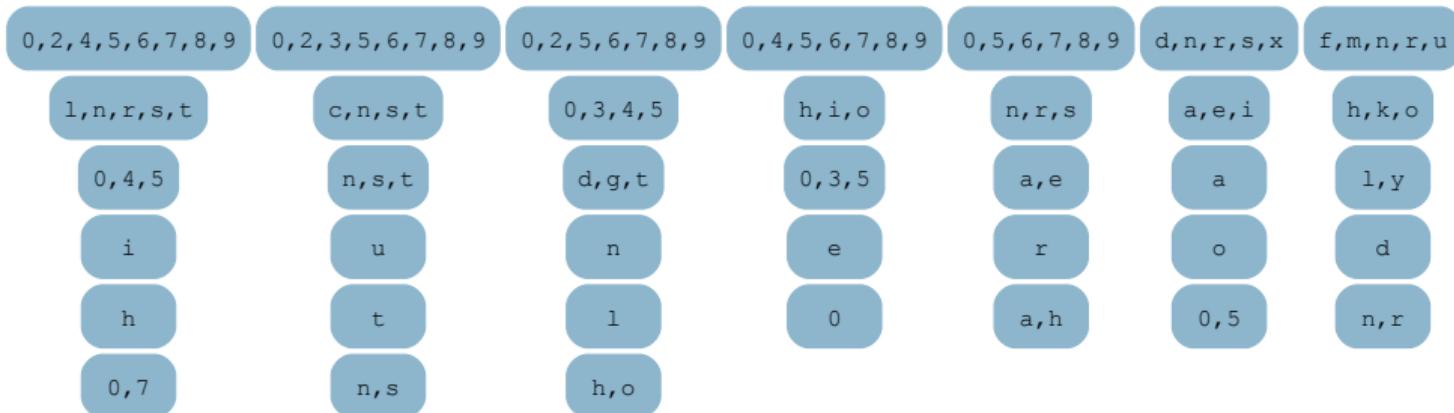
“Eigensets”

$$(M_t \star M_t^*) \star \textcolor{red}{v} = \textcolor{red}{v}$$

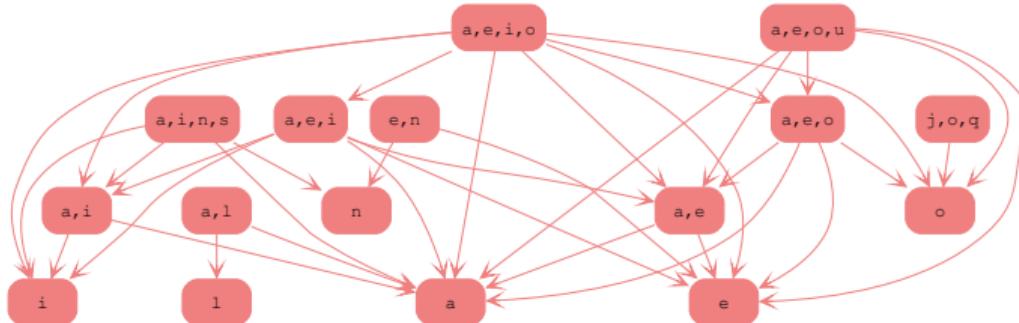
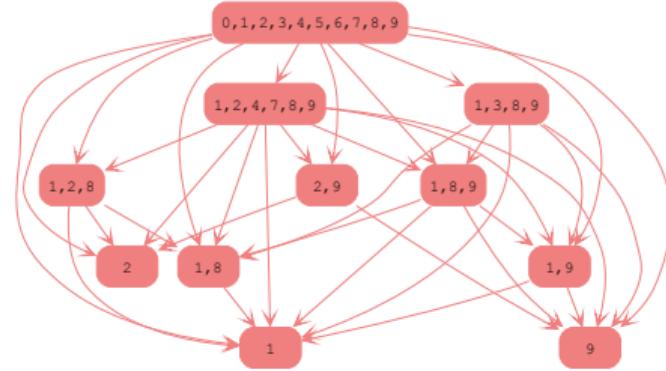
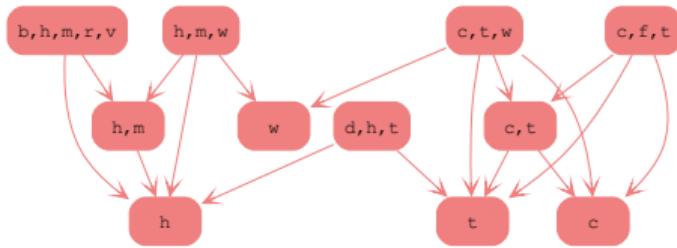


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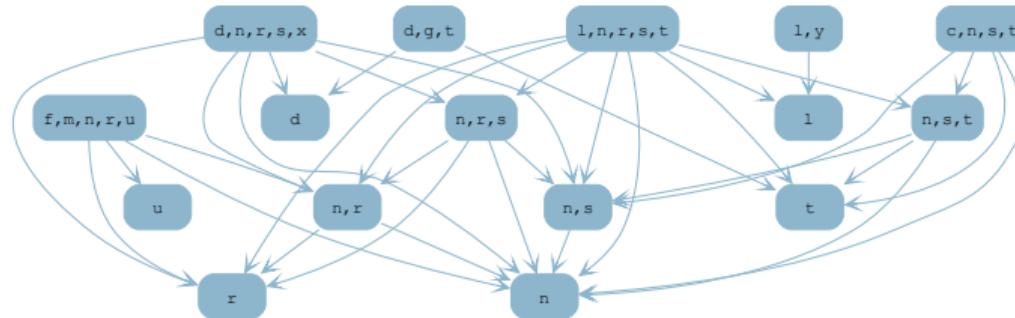
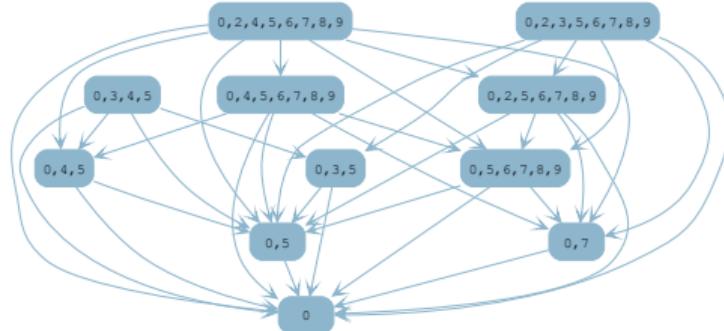
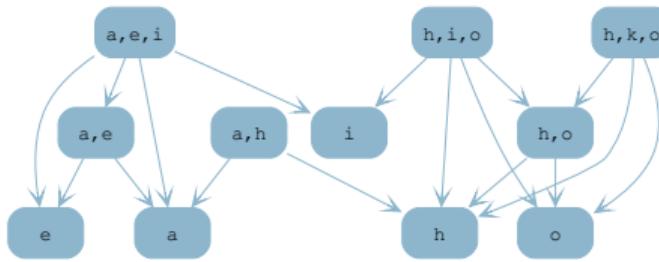
$$(M_t^* \star M_t) \star \textcolor{red}{v} = \textcolor{red}{v}$$



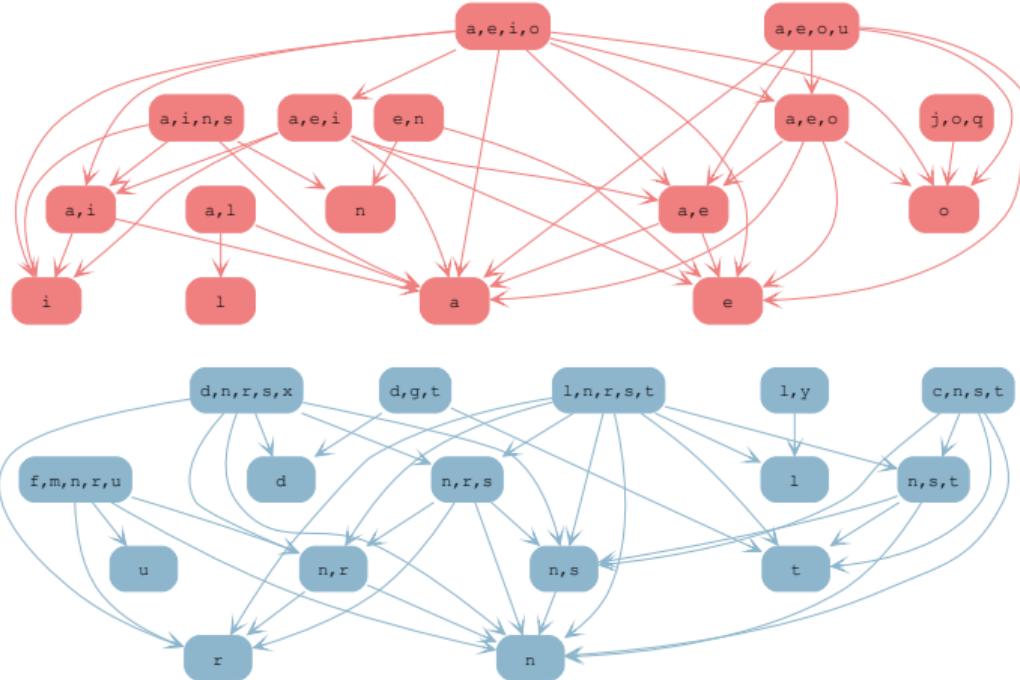
Which Structure?



Which Structure?



Which Structure?



Which Structure?

Profunctor

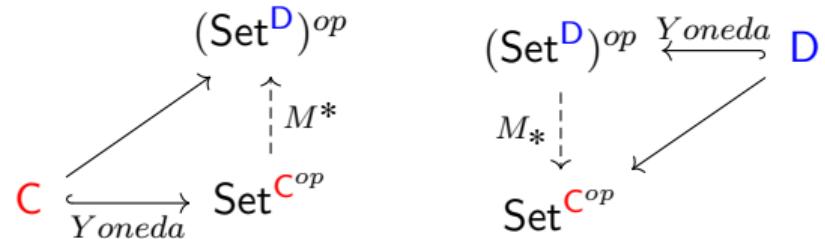
$$f: \mathbf{C}^{\text{op}} \times \mathbf{D} \rightarrow \mathbf{Set}$$

$$\begin{aligned} \mathbf{C} &\rightarrow (\mathbf{Set}^{\mathbf{D}})^{\text{op}} \\ \mathbf{D} &\rightarrow \mathbf{Set}^{\mathbf{C}^{\text{op}}} \end{aligned}$$

$$\begin{aligned} M^*: \mathbf{Set}^{\mathbf{C}^{\text{op}}} &\rightarrow (\mathbf{Set}^{\mathbf{D}})^{\text{op}} \\ M_*: (\mathbf{Set}^{\mathbf{D}})^{\text{op}} &\rightarrow \mathbf{Set}^{\mathbf{C}^{\text{op}}} \end{aligned}$$

Isbell adjunction

$$M^*: \mathbf{Set}^{\mathbf{C}^{\text{op}}} \leftrightarrows (\mathbf{Set}^{\mathbf{D}})^{\text{op}}: M_*$$



$$M^* \mathbf{c}_i \cong \mathbf{d}_i \text{ and } M_* \mathbf{d}_i \cong \mathbf{c}_i.$$

(Bradley et al., 2024)

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Theory of Computational Types

Definition (Polar/Orthogonal - Girard, 2011)

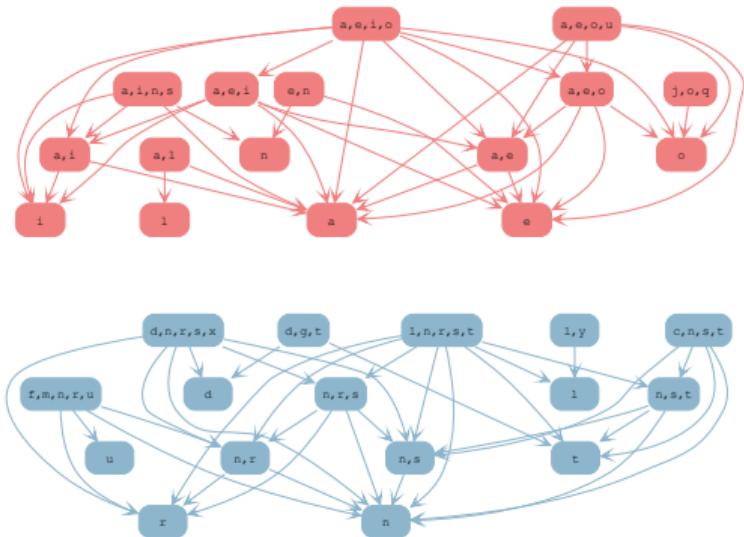
[G]iven a binary operation, noted

$a, b \rightsquigarrow \langle a|b \rangle : A \times B \rightarrow C$ and a subset $P \subset C$ (the 'pole')
 one can define the *polar* $X^\perp \subset B$ of a subset $X \subset A$
 (resp. $Y^\perp \subset A$ of a subset $Y \subset B$) by :

$$X^\perp := \{y \in B : \forall x \in X, \langle a|b \rangle \in P\}$$

$$Y^\perp := \{x \in A : \forall y \in Y, \langle a|b \rangle \in P\}$$

- ◊ The map 'polar' is decreasing:
 $X \subset X' \Rightarrow X'^\perp \subset X^\perp$.
- ◊ The set $\text{Pol}(A) \subset \mathcal{P}(A)$ of *polar* sets, i.e., of the form Y^\perp , is closed under arbitrary intersections. In particular, A is polar and $X^{\perp\perp}$ is the smallest polar set containing X .
- ◊ As a consequence, $X^{\perp\perp\perp} = X^\perp$.



Theory of Computational Types

Definition (Polar/Orthogonal - Girard, 2011)

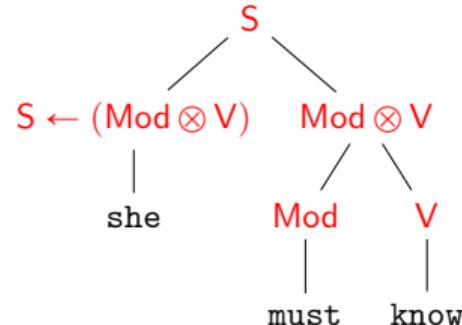
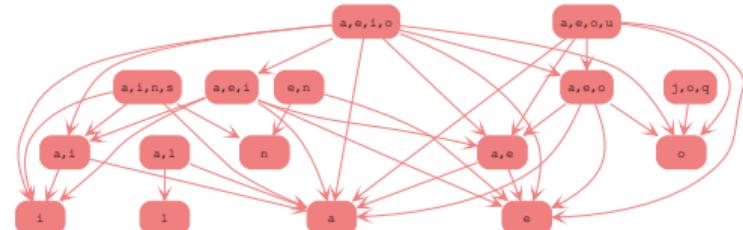
[G]iven a binary operation, noted

$a, b \rightsquigarrow \langle a|b \rangle : A \times B \rightarrow C$ and a subset $P \subset C$ (the 'pole')
 one can define the *polar* $X^\perp \subset B$ of a subset $X \subset A$
 (resp. $Y^\perp \subset A$ of a subset $Y \subset B$) by :

$$X^\perp := \{y \in B : \forall x \in X, \langle a|b \rangle \in P\}$$

$$Y^\perp := \{x \in A : \forall y \in Y, \langle a|b \rangle \in P\}$$

- ◊ The map 'polar' is decreasing:
 $X \subset X' \Rightarrow X'^\perp \subset X^\perp$.
- ◊ The set $\text{Pol}(A) \subset \mathcal{P}(A)$ of *polar* sets, i.e., of the form Y^\perp , is closed under arbitrary intersections. In particular, A is polar and $X^{\perp\perp}$ is the smallest polar set containing X .
- ◊ As a consequence, $X^{\perp\perp\perp} = X^\perp$.



(Gastaldi and Pellissier, 2021)

Outline

Introduction

Word Embeddings

The Structure behind Word Embeddings

Example: Wikipedia

Which Structure?

What Computation?

Why Language?

Conclusion

Structuralist Semiology: Language (*Langue*) and Text

A **Language** [...] is the **Paradigmatic** of a Denotative Semiotic whose Paradigms are Manifested by all Purports.

(Hjelmslev, 1975, Df. 38)

A **Text** [...] is the **Syntagmatic** of a Denotative Semiotic whose Chains are Manifested by all Purports.

(Hjelmslev, 1975, Df. 39)

Structuralist Semiology: Language (*Langue*) and Text

A **Language** [...] is the **Paradigmatic** of a Denotative Semiotic whose Paradigms are Manifested by all Purports.

(Hjelmslev, 1975, Df. 38)

A **Paradigmatic** or **Sign-System** [...] is a Semiotic System.

(Hjelmslev, 1975, Df. 35)

A **Text** [...] is the **Syntagmatic** of a Denotative Semiotic whose Chains are Manifested by all Purports.

(Hjelmslev, 1975, Df. 39)

A **Syntagmatic** or **Sign-Process** [...] is a Semiotic Process.

(Hjelmslev, 1975, Df. 33)

Structuralist Semiology: Semiotic

A **Semiotic** [...] is a Hierarchy, any of whose Components admits of a further Analysis into Classes defined by mutual Relation, so that any of these classes admits of an analysis into Derivates defined by mutual **Mutation**.

(Hjelmslev, 1975, Df. 24)

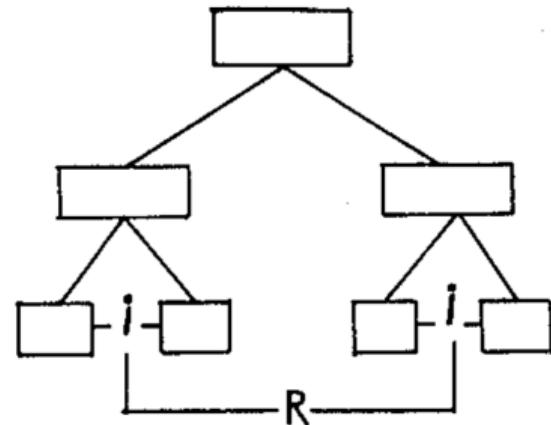
Structuralist Semiology: Semiotic

A **Semiotic** [...] is a Hierarchy, any of whose Components admits of a further Analysis into Classes defined by mutual Relation, so that any of these classes admits of an analysis into Derivates defined by mutual **Mutation**.

(Hjelmslev, 1975, Df. 24)

Mutation [...] is a Function existing between first-Degree Derivates of one and the same Class, a *function that has Relation to a function* between other first-degree derivates of one and the same class and belonging to the same Rank.

(Hjelmslev, 1975, Df. 23)



Syntagmatic and Text (Vectors)

D 1	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	...
	é	z	i	y	u	l	o	r	j	n	

D 1	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	...
	r_é	l_é	n_é	f_é	s_é	é_e	i_é	-_é	w_q	z_u	

...	1	2	3	4	5	6	7	8	9	10	
	0	9	2	1	8	4	3	7	6	5	

...	1	2	3	4	5	6	7	8	9	10	
	=_9	=_6	7_9	9_9	=_5	9_5	9_7	9_6	9_8	9_0	

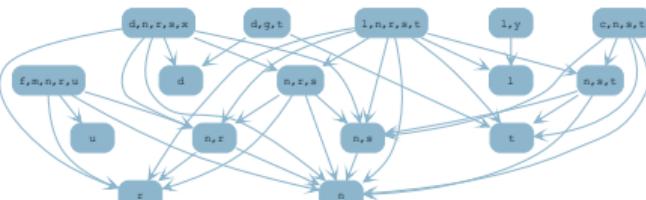
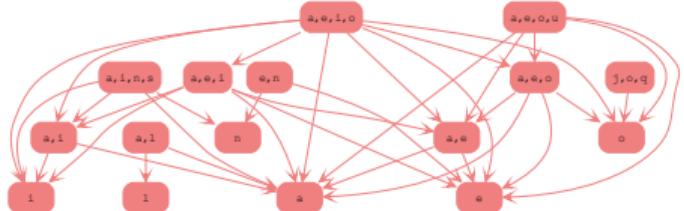
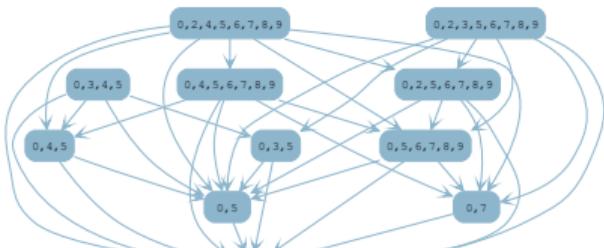
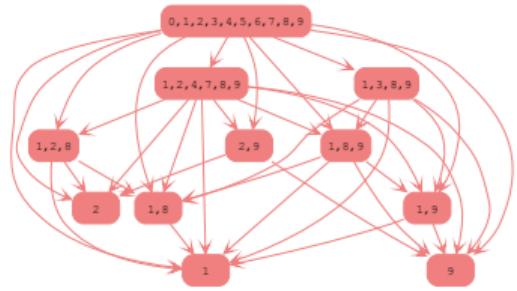
D 2	0	r	w	v	f	l	j	m	g	n	
	0_é	5_é	8_é	-_é	2_é	u_é	=_o	9_é	4_é	7_é	

D 2	0	r	w	v	f	l	j	m	g	n	
	0_é	5_é	8_é	-_é	2_é	u_é	=_o	9_é	4_é	7_é	

3	y	u	é	i	o	e	a	-	/	
	d_m	z_p	z_f	k_m	x_g	t_g	é_m	z_m	z_g	z_q

3	y	u	é	i	o	e	a	-	/	
	d_m	z_p	z_f	k_m	x_g	t_g	é_m	z_m	z_g	z_q

Syntagmatic and Text (Nucleai/Types)



Paradigmatic and *Langue* (Vectors)

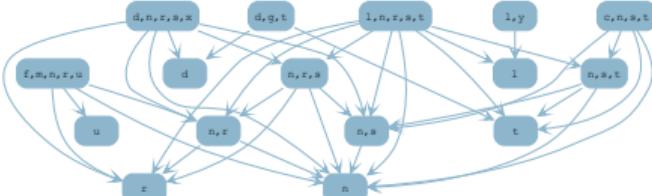
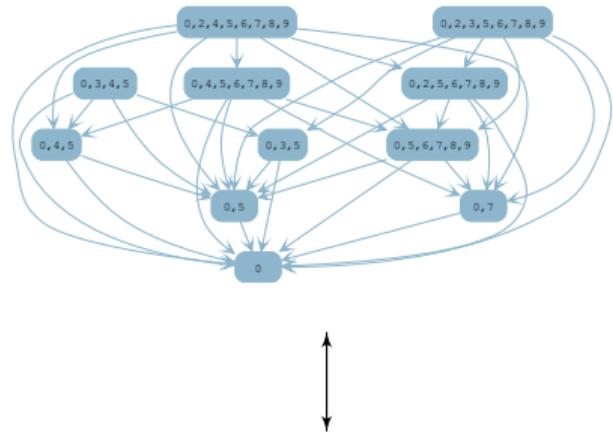
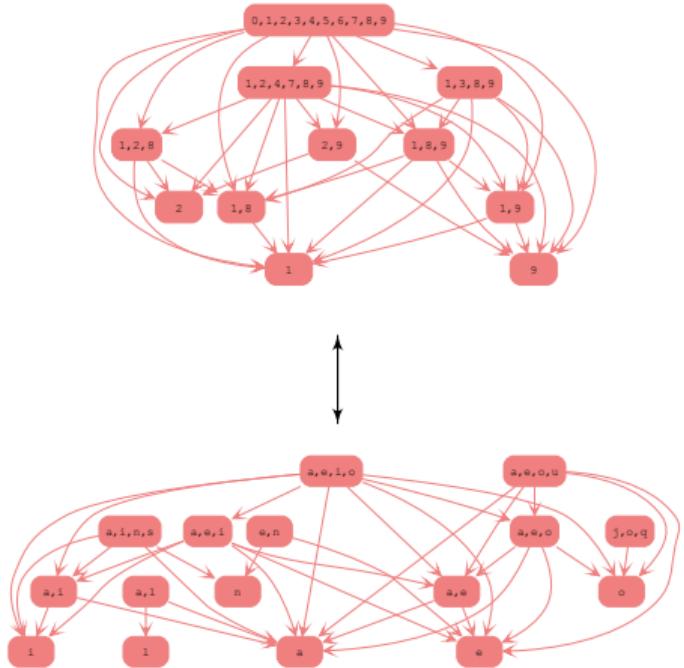
	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	...
D 1	é	z	i	y	u	l	o	r	j	n	
...	1	2	3	4	5	6	7	8	9	10	
	0	9	2	1	8	4	3	7	6	5	

	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	...
D 1	r_é	l_é	n_é	f_é	s_é	é_e	i_é	-_é	w_q	z_u	
...	1	2	3	4	5	6	7	8	9	10	
	=_9	=_6	7_9	9_9	=_5	9_5	9_7	9_6	9_8	9_0	

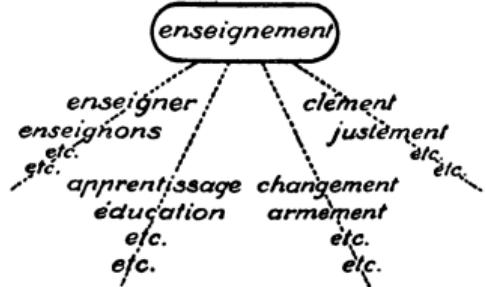
	0	r	w	v	f	l	j	m	g	n	
D 2	0	r	w	v	f	l	j	m	g	n	
...	3	y	u	é	i	o	e	a	-	/	

	0_é	5_é	8_é	-_é	2_é	u_é	=_o	9_é	4_é	7_é	
D 2	0_é	5_é	8_é	-_é	2_é	u_é	=_o	9_é	4_é	7_é	
...	d_m	z_p	z_f	k_m	r_g	t_g	é_m	z_m	z_g	z_q	

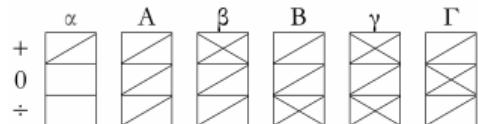
Paradigmatic and *Langue* (Nucleai/Types)



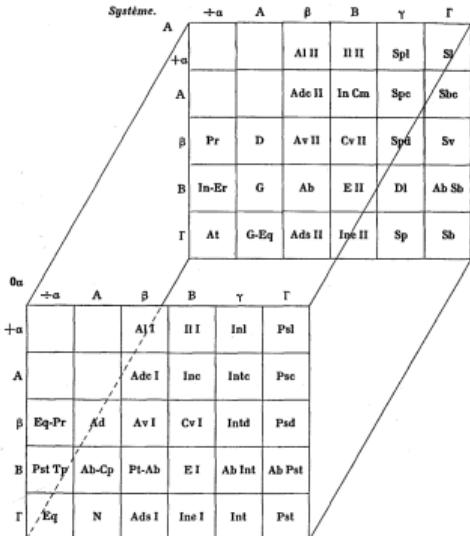
Structuralism and Formalism



(Saussure, 1980)



(Hjelmslev, 1975)



(Hjelmslev, 1935)

SEGMENTS	ENVIRONMENTS										
	# - r	# - r	# - l	e i	- C æ - C	a o u	- Cs -	e i	s - a	s - a o u	... t -
t	✓										
t		✓		✓	✓	✓	✓	✓	✓	✓	
K						✓					✓
k		✓	✓		✓						✓
K				✓				✓			
G							✓				
g		✓	✓		✓						
G						✓					
r					✓	✓	✓				✓
Γ											✓

(Harris, 1960)

Structuralism and Formalism

	a	b	d	e	f	g	h	i
a	aa	ab	ad		af	ag	ah	
b	ba							bi
d	da			de				di
e		eb	ed			eg		
f				fe				gi
g								hi
h	ha							hi
i			id			ih	i	

Diagram 1.

	b	d	f	g	h	a	e	i
f						fa	fe	
h						ha	hi	
g						ga	ge	gi
b						ba	be	bi
d						da	de	di
a	ab	ad	af	ag	ah	aa		
e	eb	ed	ef	eg				
i	ib	id	ig	ih				i

Diagram 3.

	I	II	III	IV
p	r	s	t	
r	-			
s		+	+	-
t	-			
i	+			
o	+			
u	+			
III	y	-	+	+
IV	&	-		+

Diagram 2.

(SpangHanssen1959)

Structuralism and Formalism

Table 8.
Vowel × binary final cluster (cf. sect. 84).

	ft	gt	ks	ds	vn	vl	dr!	mp	nk	ng	nd	nt	ns	lk	ld	lt	rk	rd	rt	rn	S	T	jC	
a	5	10	6	3	9	8	6	8	16	20	14	9	6	9	8	11	7	1	9	3	168	281	3	a
e	-	-	3	1	3	2	2	1	-	4	7	5	6	-	3	5	-	1	3	3	49	95	33	e
i	7	6	9	5	-	1	2	4	13	11	20	8	3	2	11	6	6	1	1	-	116	171	-	i
o	3	2	2	5	4	2	1	1	1	2	3	2	-	4	13	3	6	9	10	4	77	120	-	o
u	2	9	5	4	-	-	6	12	8	4	12	3	2	4	8	4	4	-	2	-	89	143	-	u
y	-	2	-	2	-	-	1	2	4	7	6	2	-	1	6	6	3	2	1	-	45	56	-	y
æ	4	11	1	-	4	4	2	2	9	11	8	1	3	2	11	4	6	6	6	4	99	145	-	æ
ø	5	2	-	-	1	4	-	-	-	-	1	2	3	-	-	-	3	-	1	6	28	47	10	ø
aa	-	-	-	1	-	-	1	-	-	-	4	-	-	-	-	-	-	2	-	1	9	11	-	aa
	26	42	26	21	21	21	21	30	51	59	75	32	23	22	60	39	35	22	33	21	680	1069	46	

(SpangHanssen1959)

Formal Content

(Gastaldi and Pellissier, 2021; Gastaldi, Forthcoming 2024c)

Form ~~vs.~~ and ~~Meaning~~ Content

Kant, Hegel, Frege, Saussure, Hjelmslev, etc.

Formal Content: The dimension of content which finds its source in the internal relations holding between the expressions of a language.

Formal Content

(Gastaldi and Pellissier, 2021; Gastaldi, Forthcoming 2024c)

Form ~~vs.~~ and ~~Meaning~~ Content

Kant, Hegel, Frege, Saussure, Hjelmslev, etc.

Formal Content: The dimension of content which finds its source in the internal relations holding between the expressions of a language.

- ◊ Characteristic Content: The content resulting from the **inclusion** of a unit **in a class of other units** by which it accepts to be substituted in given contexts
- ◊ Syntactic Content: The content a unit receives as a result of the multiple **dependencies** it can maintain with respect **to other units** in its context
- ◊ Informational Content: The content related to the **non-uniform distribution** of units within those substitutability classes

Illustration of Formal Contents

(Gastaldi and Pellissier, 2021; Gastaldi, Forthcoming 2024c)

Characteristic Content

```
{cat, dog, spider,  
gavagai}
```

Atomic Type

Syntactic Content

"the gavagai is on the
mat"

Profunctor Nucleus

Informational Content

```
{cat:0.059%,  
dog:0.012%,  
spider:0.009%  
gavagai:0.000%}
```

Probability Distribution

Formal Interpretability

Subword Tokenization
(Sennrich et al., 2016)

Word Embeddings
(Mikolov, Sutskever, Chen, Corrado, and Dean, 2013)

Self-Attention
(Vaswani et al., 2017)

Formal Interpretability

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Syntactic Content

Outline

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The Structure behind Word Embeddings

Example: Wikipedia

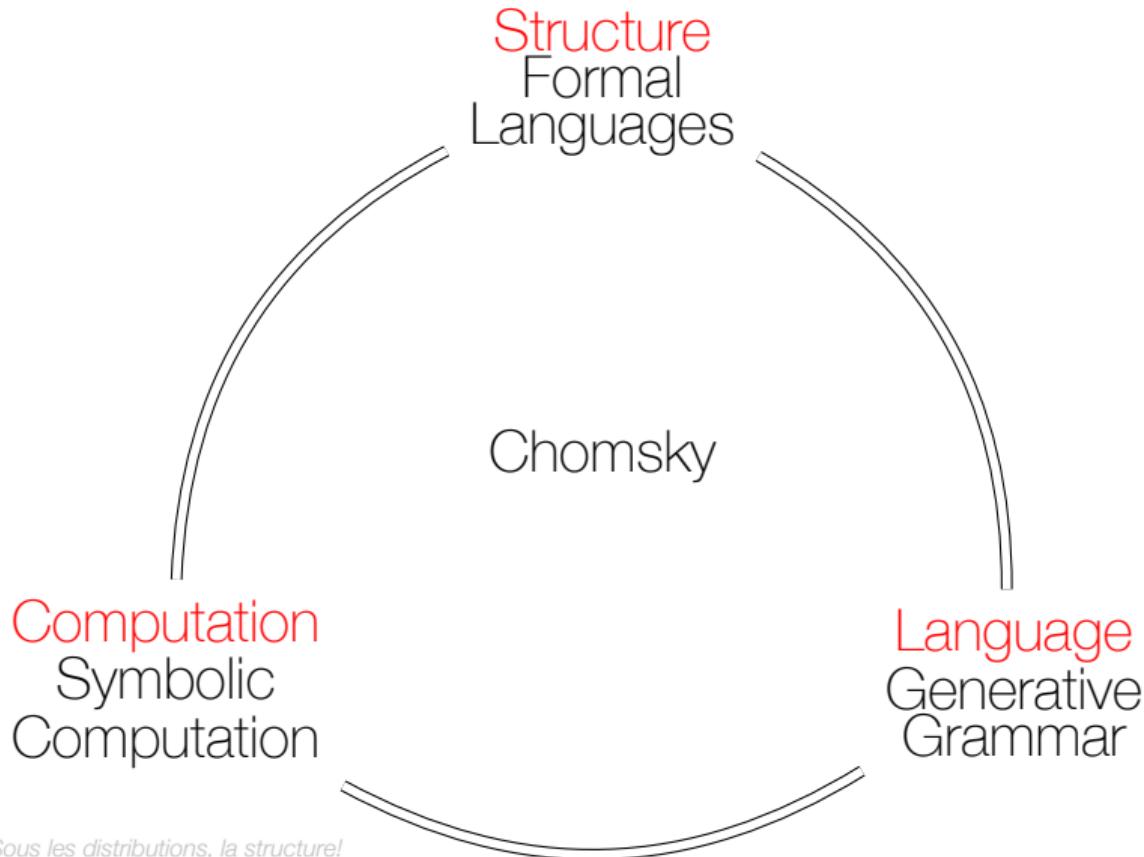
Which Structure?

What Computation?

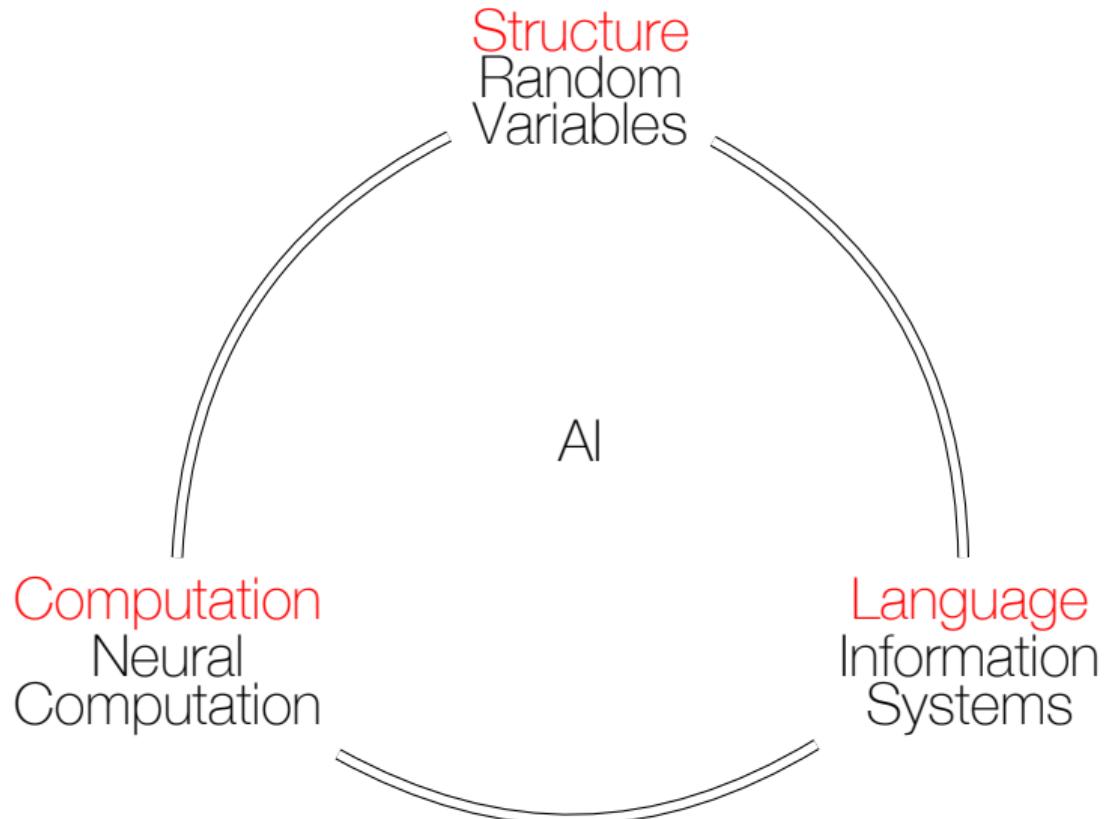
Why Language?

Conclusion

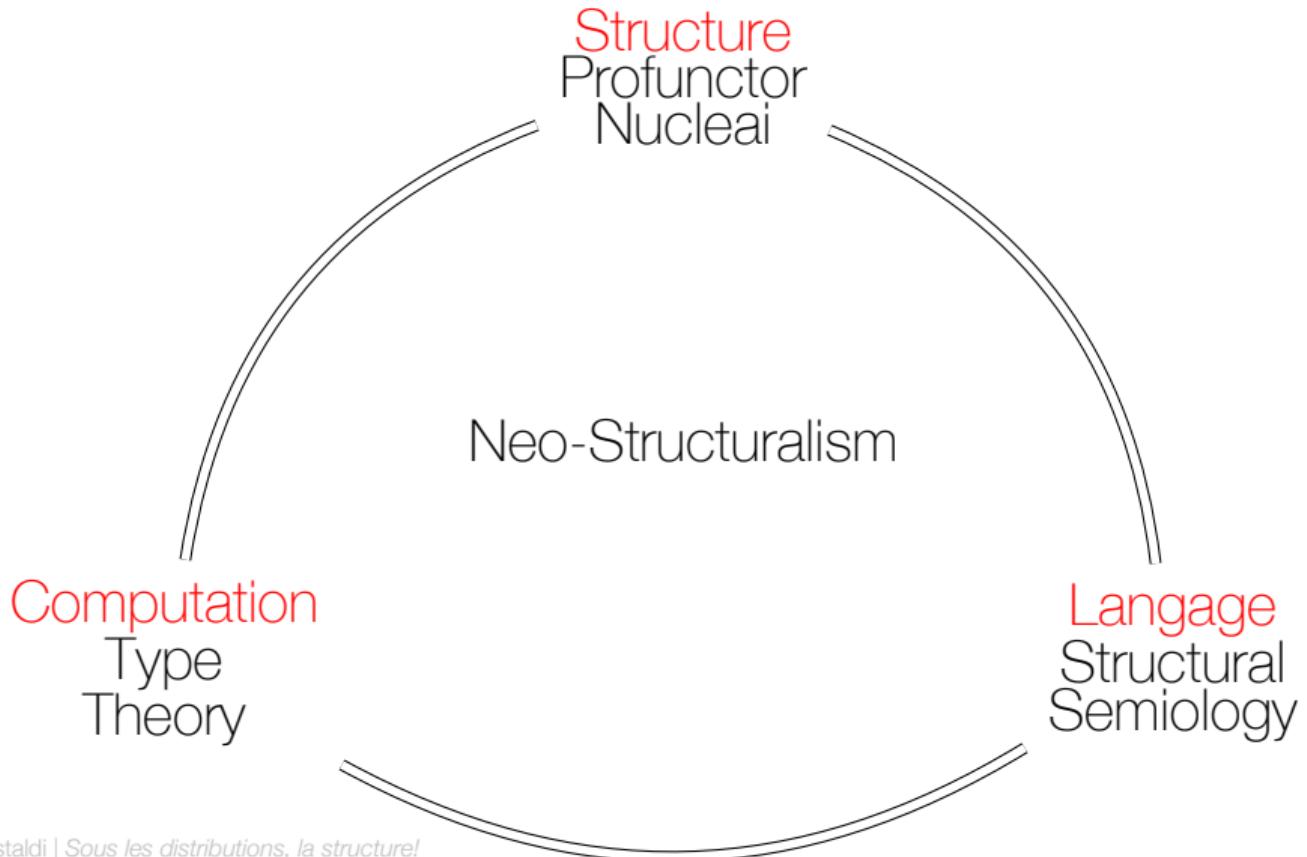
Conclusion



Conclusion



Conclusion



Collaborations



J. Terilla (CUNY), T.-D. Bradley (SandboxAQ), L. Pellissier (Paris-Est Créteil), Th. Seiller (CNRS), S. Jarvis (CUNY)

Reference Papers

- ◊ Gastaldi, J. L. (2020). Why can computers understand natural language?: The structuralist image of language behind word embeddings. *Philosophy & Technology*
- ◊ Gastaldi, J. L., & Pellissier, L. (2021). The calculus of language: Explicit representation of emergent linguistic structure through type-theoretical paradigms. *Interdisciplinary Science Reviews*.
<https://doi.org/10.1080/03080188.2021.1890484>
- ◊ Bradley, T.-D., Gastaldi, J. L., & Terilla, J. (2024). The structure of meaning in language: Parallel narratives in linear algebra and category theory. *Notices of the American Mathematical Society*.
<https://api.semanticscholar.org/CorpusID:263613625>

References |

- Bradley, T.-D., Gastaldi, J. L., & Terilla, J. (2024). The structure of meaning in language: Parallel narratives in linear algebra and category theory. *Notices of the American Mathematical Society*.
<https://api.semanticscholar.org/CorpusID:263613625>
- Gastaldi, J. L. (2020). Why can computers understand natural language?: The structuralist image of language behind word embeddings. *Philosophy & Technology*.
- Gastaldi, J. L. (Forthcoming 2024c). Content from Expressions. The Place of Textuality in Deep Learning Approaches to Mathematics. *Synthese (under review)*.
- Gastaldi, J. L., & Pellissier, L. (2021). The calculus of language: Explicit representation of emergent linguistic structure through type-theoretical paradigms. *Interdisciplinary Science Reviews*.
<https://doi.org/10.1080/03080188.2021.1890484>
- Girard, J.-Y. (2011, September). *The blind spot*. European Mathematical Society.
- Goldberg, Y., & Levy, O. (2014). Word2vec explained: Deriving mikolov et al.'s negative-sampling word-embedding method. *CoRR, abs/1402.3722*.
- Hamilton, W. L., Leskovec, J., & Jurafsky, D. (2016). Diachronic word embeddings reveal statistical laws of semantic change. *CoRR, abs/1605.09096*.
- Harris, Z. (1960). *Structural linguistics*. University of Chicago Press.
- Hjelmslev, L. (1935). *La catégories des cas*. Wilhelm Fink Verlag.
- Hjelmslev, L. (1975). *Résumé of a Theory of Language*. Nordisk Sprog-og Kulturforlag.
- Levy, O., & Goldberg, Y. (2014). Neural word embedding as implicit matrix factorization. *Proceedings of the 27th International Conference on Neural Information Processing Systems - Volume 2*, 2177–2185.

References II

- Mikolov, T., Sutskever, I., Chen, K., Corrado, G., Dean, J., Le, Q., & Strohmann, T. (2013). *Learning representations of text using neural networks. NIPS deep learning workshop 2013 slides.*
- Mikolov, T., Sutskever, I., Chen, K., Corrado, G., & Dean, J. (2013). Distributed representations of words and phrases and their compositionality. *CoRR, abs/1310.4546.*
- Saussure. (1980). *Cours de linguistique générale*. Payot.
- Sennrich, R., Haddow, B., & Birch, A. (2016). Neural machine translation of rare words with subword units. *Proceedings of the 54th Annual Meeting of the ACL*, 1715–1725.
- Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A. N., Kaiser, Ł., & Polosukhin, I. (2017). Attention is all you need. In I. Guyon, U. V. Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, & R. Garnett (Eds.), *Advances in neural information processing systems* (Vol. 30). Curran Associates, Inc. https://proceedings.neurips.cc/paper_files/paper/2017/file/3f5ee243547dee91fb053c1c4a845aa-Paper.pdf

Workshop on AI & DH
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Montréal, Canada

Sous les distributions, la structure!

Unearthing structural(ist) features from linguistic distributions

Juan Luis Gastaldi



November 28, 2024