

# Math Bits

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## Costs For Determining Preemption

Our algorithm involves two different costs which are used to evaluate whether or not a job  $j_i$  is killed on preemption—cost of killing ( $C_k$ ) and cost of using a checkpoint with a potential migration to a new processor ( $C_p$ ).

Even if we kill a process, it will eventually need to be re-run. Therefore the cost of killing involves a factor of how much time a job has already been running as whatever work a processor has already done on that job will have to be done again. Because the assumption is that there is no overhead associated with killing, we can say that  $C_k$  is simply

$$C_k = t_i$$

where  $t_i$  is the amount of time job  $j_i$  has been worked on.

We now consider the cost of using a previous checkpoint. Assume we checkpoint at interval  $l$ . If we use a previous checkpoint, then we only have to re-run the job from a previous checkpoint as opposed to the whole job. However we also have to consider a migration cost. In the event a job switches to a new processor, an amount of overhead is associated with writing the memory of the checkpoint to the new processor. If we assume that a processor has an equal likelihood of being scheduled onto any processor, then the probability of incurring a migration cost is  $(m - 1)/m$  where  $m$  is the number of processors. Therefore our cost  $C_p$  is

$$C_p = (t_i - t_p) + \frac{o_m(m - 1)}{m}$$

where  $t_p$  is the amount of time job  $j_i$  has been worked on at the most recent checkpoint and  $o_m$  is the overhead associated with migration. This value is constant. We know that  $t_i - t_p$  must be bound by  $l$  and therefore we can say that

$$C_p < l + \frac{o_m(m - 1)}{m}.$$

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For later analysis it is important to note that the mean and standard deviations for amount of migrations with  $n_p$  preemptions using checkpoints are

$$\mu_p = n_p \frac{m-1}{m}$$
$$\sigma_p = \frac{\sqrt{n_p(m-1)}}{m}.$$