Deterministic SEIR model for COVID in Greece

Introduction

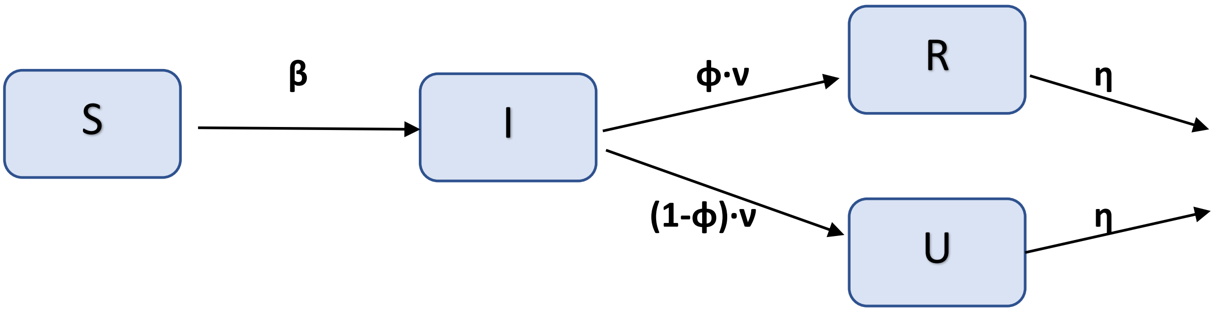
In 25/2/2020 national health organization of Greece (eody) has diagnosed people positive with corona virus 19 and an epidemic outbreak was to come. In Greece the pandemic does not seem to growth exponentially, but rather logistically. However a lot of cases with mild symptoms who have the virus can transmit it to other people and play a vital role to the spread of the pandemic. Due to the lack of tests to corona virus we do not know the exact number of those with mild symptoms and the purpose of this attempt is to study the number of those people.

The first cases have been reported at 25/2/2020 and more than 2000 cases have been reported in Greece [1]. In order to estimate the number of cases with mild symptoms we use the available data for reported cases using data from eody which have been published in 10/4/2020.

SEIR Model

In order to study the pandemic we created a deterministc SEIR model which has been used in an other study for this virus in China Wuhan [2]. As it is shown in the flow diagram below we split the population of Greece into 4 subgroups: "S": Susceptible population which we assume is the whole population in Greece and they can get infected by the virus, "I": the asymptomatically infected people who are infectious, "R": reported symptomatic people, "U": unreported symptomatic people with mild symptoms. We assume that the susceptibles can be infected with transmission rate β, which we tried to estimate, from either asymptomatically infected people "I" who spread the virus without knowing it and from infected people with mild symptoms "U". We assume that reported cases although they have severe symptoms they do not play an important role for spreading the virus as they become isolated. Asymptomatically infected people "I" become symptomatic with a rate ν and a percentage of those people φ have severe symptoms "R" and the rest percentage 1-φ have mild symptoms. People are removed from the model only after they cure with average cure rate η. We do not study people who are removed because they die.

Flow Diagram



Parameters

S: Susceptibles

I: Asymptomatic Infected

R: Reported Infected with severe symptoms

U: Unreported Infected with mild symptoms

β: transmission rate

ϕ: percentage of the asymptomatic which become reported infected

ν: rate from asymptomatic phase to symptomatic

η: removal rate

Equations

1) $\frac{dS(t)}{dt} = -\beta \cdot \frac{S \cdot [I+U]}{N}$

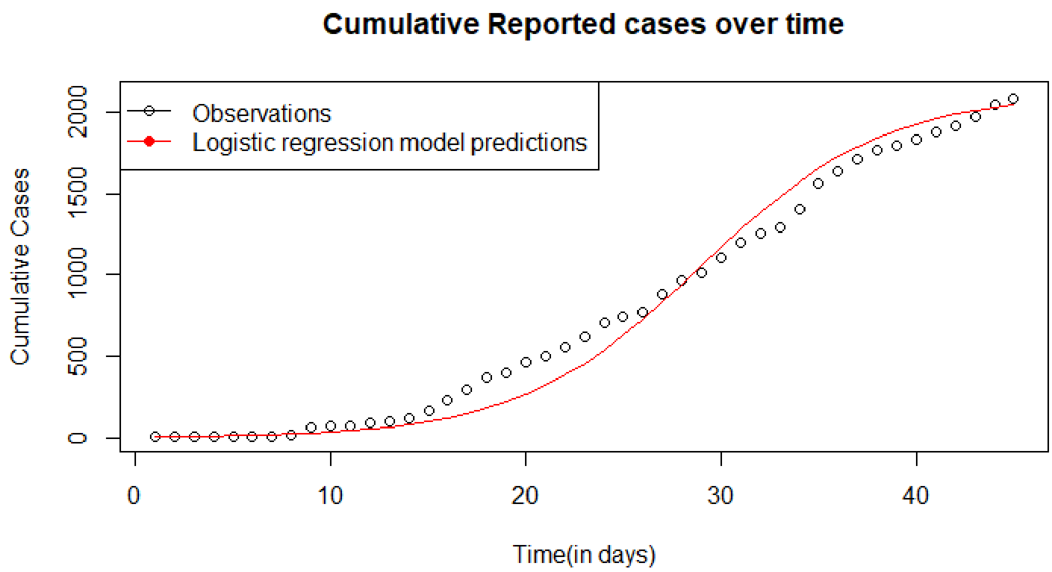
2) $\frac{dI(t)}{dt} = \beta \cdot \frac{S \cdot [I+U]}{N} - \nu \cdot I(t)$

3) $\frac{dR(t)}{dt} = \varphi \cdot \nu \cdot I(t) - \eta \cdot R(t)$

4) $\frac{dU(t)}{dt} = (1 - \varphi) \cdot \nu \cdot I(t) - \eta \cdot U(t)$

Data Analysis

We used data for reported cases we took from daily report from eody in 10/4/2020 [1]. From the scatterplot below we can assume that the observational cumulative cases: CR(t) have a logistic growth, which we predict by fitting a logistic model to the data.



Thus, $\mathrm{CR(t)} = \frac{c}{1+\mathrm{e}^{(\mathrm{b}-\mathrm{a}\cdot \mathrm{t})}} - \mathrm{x}_3$ (5) and by logistic regression we estimate c=30, b=29, a=4.65, x_3= 12.45. In the beginning of the outbreak we assume that $CR(t_0)=0 \Rightarrow t_0= b+ a\cdot\log(\frac{x_3}{c-x_3})$ = 5 and the 1st reported case was at 25/2/2020 so we assume that approximately these cases have the virus from 13/2/2020 , so t_0 means that the beginning of the outbreak is at 18/2/2020 and this is our initial date of studying the outbreak.

The cumulative number of reported cases is $CR(t)= \phi\cdot v\cdot\int_{t_0}^{t} \mathrm{I(u)du} \Rightarrow$ $\frac{dCR(t)}{dt} = \varphi \cdot \nu \cdot I(t)$ (6). But from the data we assume that CR(t) follows logistic growth as shown in equation (5) , thus from (6) we have:

$I(t)= \frac{c\cdot\alpha}{\nu\cdot\varphi} \cdot \frac{e^{(b-a\cdot t)}}{(1+e^{(b-a\cdot t)})^2} \Rightarrow I_0 = I(t_0) = \frac{c\cdot\alpha}{\nu\cdot\varphi} \cdot \frac{e^{(b-a\cdot t_0)}}{(1+e^{(b-a\cdot t_0)})^2}$ (7) from which we estimate the initial number of asymptomatic cases. Because S_0: the whole population of Greece is too large and due to the logistic growth we can assume that S(t) remains almost the same and equal to S_0, thus from equation (2) we find that U(t) = $\frac{1}{\beta \cdot S_0} \cdot [\mathrm{I}'(\mathrm{t}) + \nu \cdot \mathrm{I}(\mathrm{t})] - \mathrm{I}(\mathrm{t})$ (8) $\Rightarrow \mathrm{U}_0 = \mathrm{U}(t_0) = \frac{1}{\beta \cdot S_0} \cdot [\mathrm{I}'(t_0) + \nu \cdot \mathrm{I}(t_0)] - \mathrm{I}(t_0)$ (9) is the initial value of the unreported cases. In order for the model to have meaning it has to be $U_0 \geq 0 \Rightarrow \beta \leq \frac{I'(t)+\nu \cdot \mathrm{I}(\mathrm{t})}{\mathrm{I}(\mathrm{t}) \cdot S_0}$ (10) and we estimate β by using (8) and so the prediction of the model is close to the data.

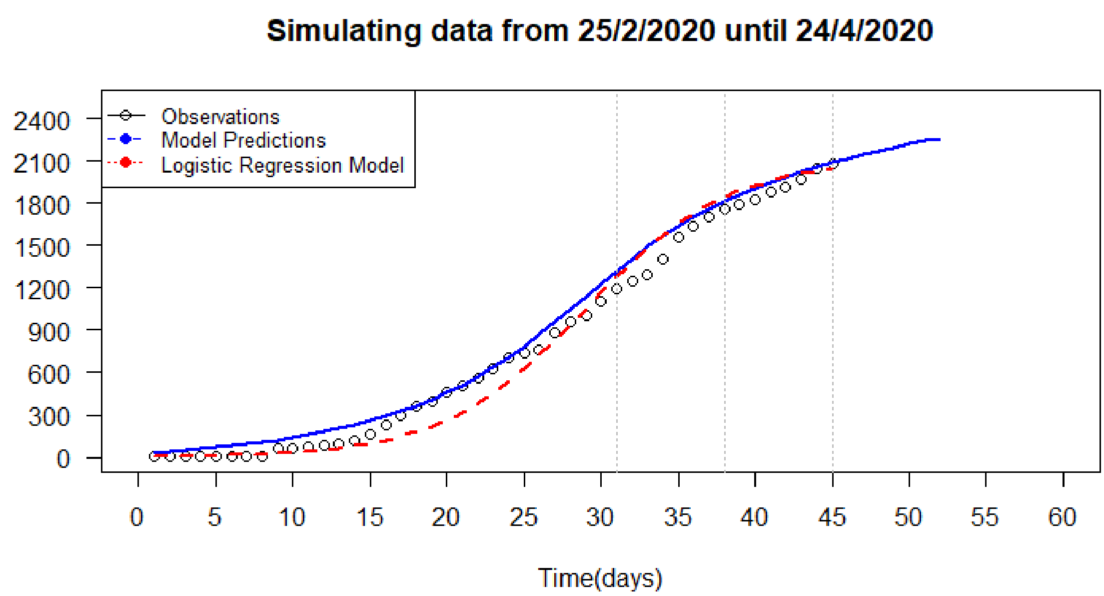
Parameters

In order to run the model and start the simulations we take some fixed values for some parameters and we estimate some other parameters as shown from the equations above.

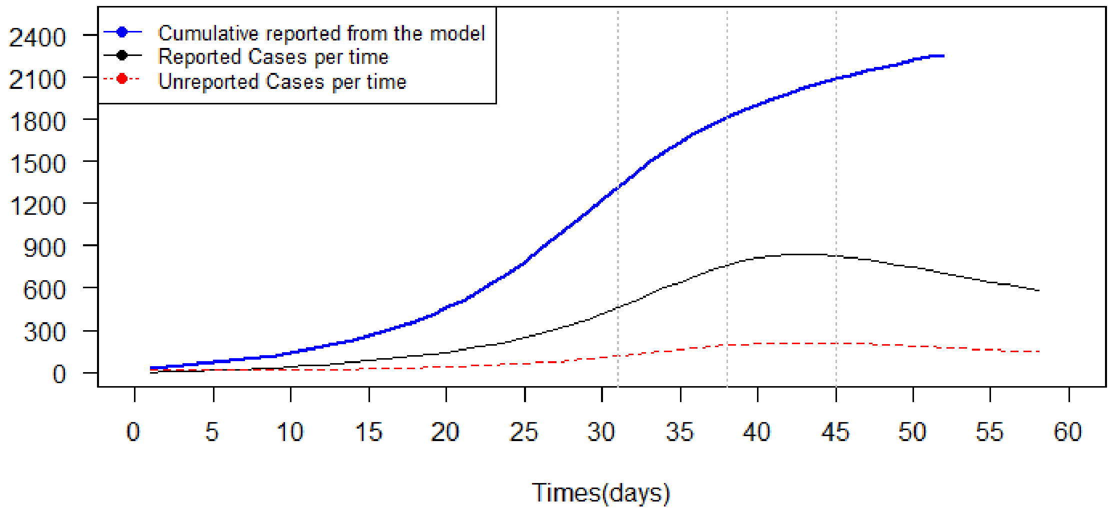
Parameters	Symbol	Values	Source
Transmission rate	β	–	Fitted to data
Percentage from asymptomatic becomes symptomatic	φ	0.8	Fixed [2]
Rate from asymptomatic to symptomatic phase(days)	ν	1/5.2	Fixed [1]
Removal rate(days)	η	1/14	Fixed [1]
Initial number of susceptibles	S_0	1e07	Fixed as the whole population of Greece
Initial number of asymptomatically infected	I_0	10.17309	Fitted : equation (7)

Initial number of unreported cases	U_0	7.121166	Fitted: equation (9)
Initial value of reported cases	R_0	0	Fixed

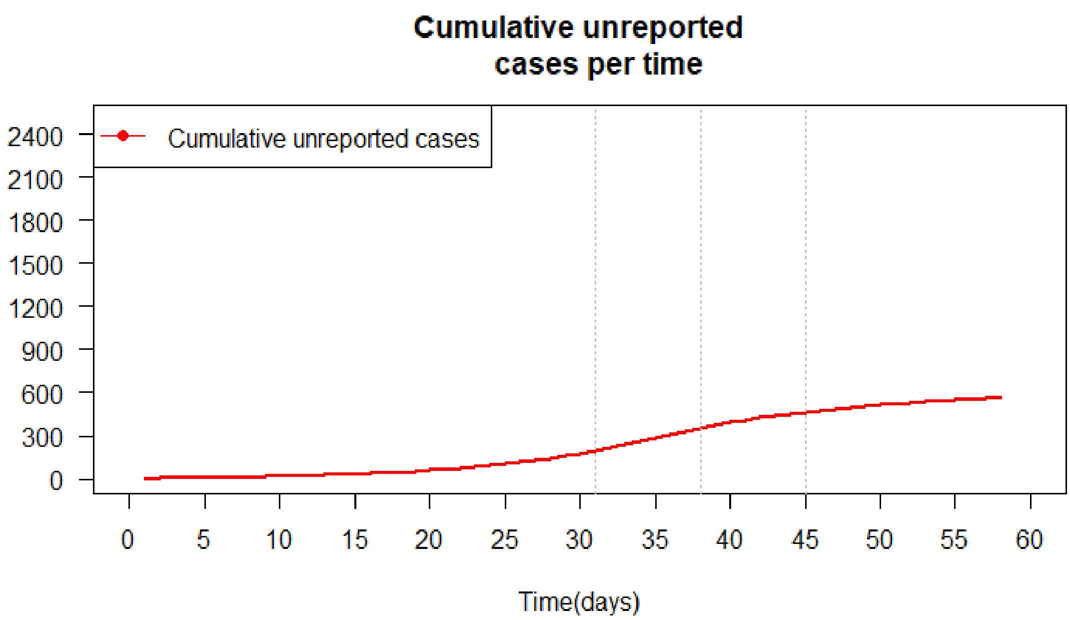
We estimate the transmission rate: β by simulating for the first 25 days from 25/2/2020-22/3/2020 until the carantinte measures have started and we find approximately β= 2.380347e-08. Then after the new measures we simulate again for the next 7 days by finding a new β= 1.586898e-08, which is smaller than before. We do this because we assume that carantine measures have not been taken into account until the 1st week and because we want to predict the cumulative number of reported cases to be as much closer to the data as we can, in order to find a better estimation for the unreported cases, which is the aim of this research. After the 1st week of the carantine we simulate again and we estimate β= 6.800991e-09. The simulated data are shown in the diagram below.



In the diagram below we see how the cumulative reported cases chahge over time, as well as the reported and the unreported cases.



Finally we estimate the cumulative number of unreported cases in the diagram below:



From which we predict that the unreported cases are approximately 565 people.

Discussion

In the current study we have made a lot of assumptions in order for the model to fit the data good and some of them are no reasonable, thus the results we found are no good to predict in the long term. However it is a good estimate for the unreported cases and it can be a start for further studies, by using data from eody as time passes and more and more tests will be made in the long term and so we will be able to try estimate better the transmission rate and the proportion ϕ, which make an important role to predict the future of this pandemic.

References

[1] National Health Center of Greece (eody). Data available online: https://eody.gov.gr/

[2] Zhihua Liu, Pierre Magal, Ousmane Seydi and Glenn Webb. Understanding Unreported Cases in the COVID-19 Epidemic Outbreak in Wuhan, China, and the Importance of Major Public Health Interventions. Biology. (2020), 9, 50; doi:10.3390/biology9030050.