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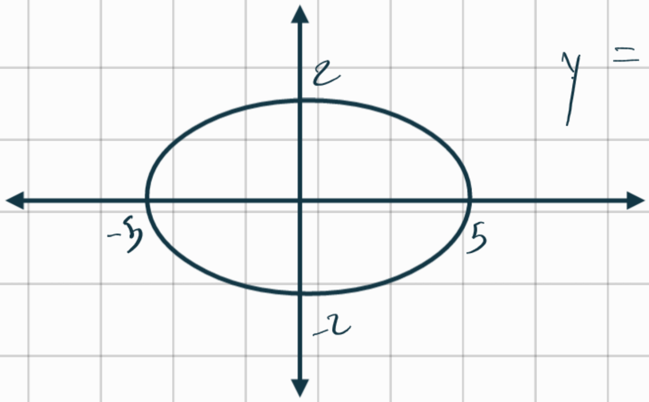
#121260

Assigment 2.1

MATH 1360 Secc. 80
Prof. Milena L. Gomez

1. Determine el área encerrada por la elipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$



$$y = \tfrac{2}{5}\sqrt{25 - x^2}$$

$$\frac{x^2}{5^2} + \frac{y^2}{2^2} = 1$$

$$\frac{y^2}{4} = 1 - \frac{x^2}{25}$$

$$\frac{y}{4} = \frac{\sqrt{25-x^2}}{5}$$

$$y = \tfrac{4}{5}\sqrt{25 - x^2}$$

$$f(x) = \tfrac{4}{5}\sqrt{25 - x^2}$$

$$A = \int_{x=0}^{5} f(x)\,dx$$

$$A = \int_0^5 \tfrac{4}{5}\sqrt{25 - x^2}\,dx$$

$$= \tfrac{4}{5}\int_0^5 \sqrt{5^2 - x^2}\,dx$$

$$\left[\because \int \sqrt{a^2 - x^2}\,dx = \frac{a^2}{2}\sin^{-1}\left(\tfrac{x}{a}\right) + \tfrac{1}{2}\sin\left(2\sin^{-1}\left(\tfrac{x}{a}\right)\right) + C\right]$$

$$A = \frac{4}{5}\left[\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\Big|_0^5 + \frac{1}{2}\sin\left(2\sin^{-1}\left(\frac{x}{5}\right)\right)\Big|_0^5\right]$$

$$= \frac{4}{5}\left[\frac{25}{2}\left[\sin^{-1}\left(\frac{5}{5}\right) - \sin^{-1}(0)\right] + \frac{1}{2}\sin\left(2\sin^{-1}\left(\frac{5}{5}\right) - \sin^{-1}(0)\right)\right]$$

$$\sin^{-1}(1) - 0 \qquad\qquad 2\sin^{-1}(1) - 0$$

$$= \qquad \downarrow \qquad\qquad\qquad \downarrow$$

$$\left(\frac{\pi}{2}\right) \qquad\qquad 2\left(\frac{\pi}{2}\right)$$

$$= \frac{4}{5}\left[\frac{25}{2}\left(\frac{\pi}{2}\right) + \frac{1}{2}\sin\left(2\left(\frac{\pi}{2}\right)\right)\right]$$

$$\sin(\pi)$$

$$= \frac{4}{5}\left[\frac{25}{2}\left(\frac{\pi}{2}\right) + 0\right] = \frac{4}{5}\left(\frac{25}{4}\right)\pi = 5\pi$$

$$A = 5\pi$$

2. Determine el área de la región bajo la curva

$$y = \frac{1}{x^3+x}, \quad 1 \leq x \leq 2$$

$$\text{Area} = \int_1^2 y \, dx$$

$$= \int_1^2 \frac{1}{x^2+x} \, dx$$

$$= \int_1^2 \frac{1}{x(x+1)} \, dx$$

$$= \int_1^2 \left[\frac{1}{x} - \frac{1}{x+1} \right] dx$$

$u = x+1$

$du = dx$

x	1	2
u	2	3

$$= \int_1^2 \frac{dx}{x} - \int_1^2 \frac{dx}{x+1}$$

$$= \Big[\ln x \Big]_1^2 - \int_2^3 \frac{du}{u}$$

$$= (\ln 2 - \ln 1) - \Big[\ln u \Big]_2^3$$

$$= \ln 2 - (\ln 3 - \ln 2)$$

$$= 2 \ln 2 - \ln 3$$

$$= \ln 4 - \ln 3 = \ln\left(\frac{4}{3}\right)$$