Jorge A. Serrano　　　Assessment 2.1　　　MATH-1360 Secc. 80

#121260　　　11/30/2023

Profa. Milena L. Gomez

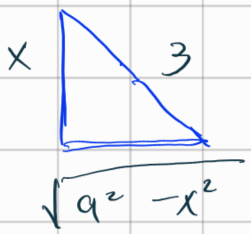
1. $\int \frac{dx}{x^2\sqrt{9-x^2}}$

$u = x \qquad A = 3$

$x = 3\,\text{sen}\,\theta$

$= x/3 = \frac{\text{Opuesto}}{\text{hipo.}}$

$dx = 3\cos\theta\, d\theta$



$= \int \frac{3\cos\theta\, d\theta}{9\,\text{sen}^2\theta\sqrt{9-9\,\text{sen}^2\theta}}$

$= \int \frac{3\cos\theta\, d\theta}{9\,\text{sen}^2\theta\; 3\cos\theta}$

$= 1/9 \int \frac{d\theta}{\text{sen}^2\theta} \qquad = 1/9 \int \csc^2\theta\, d\theta$

$= -1/9 \cot\theta + C$

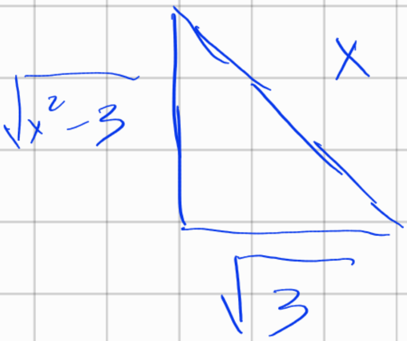
$= \int \frac{dx}{x^2\sqrt{9-x^2}} = -1/9 \cot\theta + C$

$= 1/9\, \frac{\sqrt{9^2-x^2}}{x} + C$

2. $\int \frac{\sqrt{x^2-3}}{x}\,dx$

$U = x \qquad A = \sqrt{3}$

$$x = \sqrt{3}\sec\theta$$
$$= \sec\theta = \frac{U}{a}$$
$$= \frac{x}{\sqrt{3}} = \frac{\text{hipo.}}{\text{ady.}}$$



$$\int \frac{\sqrt{x^2-3}}{x}\,dx$$
$$= \int \sqrt{3\sec^2\theta - 3}\,\frac{\sqrt{3}\sec\theta\tan\theta}{\sqrt{3}\sec\theta}\,d\theta$$
$$= \sqrt{3}\int \tan^2\theta\,d\theta$$
$$= \sqrt{3}\int (\sec^2\theta - 1)\,d\theta$$
$$= \sqrt{3}(\tan\theta - \theta) + C$$
$$= \sqrt{3}\left(\sqrt{\sec^2\theta - 1} - \sec^{-1}\left(\frac{x}{\sqrt{3}}\right)\right)$$
$$= \sqrt{x^2-3} - \sqrt{3}\sec^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

3. $\int_0^3 \frac{x^3}{\sqrt{x^2+9}}\,dx$

$U = X^2$

$V = U + 9$

$dV = dU$

$X = \sqrt{U} = x^2$

$$= \int \frac{U}{2\sqrt{+9}}\,dV$$

$$= \frac{1}{2} \int \frac{V-9}{\sqrt{V}}\,dV$$

$$= \frac{1}{2}\left(\int \sqrt{V}\,dV - \frac{9}{\sqrt{V}}\,dV\right)$$

$$= \frac{1}{2}\left(\frac{2}{3} V^{\frac{2}{3}} - 9 \times 2\sqrt{V} + C\right)$$

$$= \frac{1}{3} V^{\frac{2}{3}} - 9\sqrt{V} + C$$

$$= \frac{1}{3}(V - 27)\sqrt{V} + C$$

$$= \frac{1}{3}(x^2 - 18) - \sqrt{x^2+9}$$

$$= -9\sqrt{2} + 18 = 9(9 - \sqrt{2})$$

$$= 18 - 9\sqrt{2}$$

4. $\int \frac{dx}{x^2+2x}$

$u = x+1$

$du = dx$

$= \int \frac{du}{u^2-1}$

$= -\int \frac{du}{1-u^2}$

$= -\tanh^{-1}(u)$

$= -\tanh^{-1}(x+1)$

5. $\int \frac{2x^2+13}{(1-x)(x^2+4)}\,dx$

$$\frac{A}{1-x} + \frac{B}{x^2+4}$$

$$2x^2+13 = A(x^2+4) + Bx(1-x) + C(1-x)$$

$$2+13 = A(1+4) + B\,1(1-1) + C(1-1)$$

$$15 = A\,5$$

$$A = \frac{15}{5} = 3$$

$x=0$

$$2(0)+13 = 3(0^2+4) + B(0) + C\,1$$

$$13 = 12 + C$$

$$C = 13 - 12$$

$$C = 1$$

$$2x^2+13 = 3(x^2+4) + Bx(1-x) + (1-x)$$

$$2x^2+13 = 3x^2+12+1-x + Bx(1-x)$$

$$2x^2+13-3x^2-13+x = Bx(1-x)$$

$$x - x^2 = Bx(1-x)$$

$$x(1-x) = Bx(1-x)$$

$$B = 1$$

$A=3$ $\quad B=\underline{1}$ $\quad C=\underline{1}$

$$= \int\left(\frac{3}{1-x} + \frac{x+1}{x^2+9}\right)dx$$

$$= 3\int\frac{dx}{1-x} + \frac{1}{2}\int\frac{2x}{x^2+4}\,dx + \int\frac{dx}{x^2+4}$$

$$= -3\ln|1-x| + \frac{1}{2}\ln|x^2+4| + \frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

6. $\int \frac{x^4}{x^2-1} dx$

$$= \int \frac{(x^4 - 1 + 1)}{x^2 - 1} dx$$

$$= \int \frac{\cancel{(x^2-1)}(x^2+1) + 1}{\cancel{x^2-1}} dx$$

$$= \int \left((x^2+1) + \frac{1}{x^2-1} \right) d$$

$$= \frac{x^3}{3} + x + \frac{1}{2} \ln\left(\frac{x-1}{x+1}\right) + C$$