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#121260

MATH-1860-80

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Self-Assessment 9.1

Calcule el largo de arco en los intervalos que se indican:

1. $r = 5\cos\theta \quad \frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$
2. $r = 1 + sen\theta \quad 0 \le \theta \le 2\pi$

$$\frac{dr}{d\theta} = -5\sin\theta$$

1) $$L = \int_{-\pi/2}^{\pi/2} \sqrt{(5\cos\theta)^2 + (-5\sin\theta)^2}\, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \sqrt{25\cos^2\theta + 25\sin^2\theta}\, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \sqrt{25(\cos^2\theta + \sin^2\theta)}\, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \sqrt{25}\, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} 5\, d\theta$$

$$= 5[\theta]_{-\pi/2}^{\pi/2}$$

$$= 5\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)$$

$$\boxed{L = 5\pi}$$

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2) $$L_2 = \int_0^{2\pi} \sqrt{(1+\sin\theta)^2 + (\cos\theta)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{(1+\sin\theta)^2 + \cos^2\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{2+2\sin\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{2(1+\sin\theta)}\, d\theta$$

$$= \sqrt{2}\int_0^{2\pi} \sqrt{1+\sin\theta}\, d\theta$$

$1+\sin$
$= 2\cos^2\left(\frac{\theta}{2}\right)$

$$= 2\int_0^{2\pi} \left|\cos\left(\theta/2\right)\right| d\theta$$

$$= 2\left(\int_0^{\pi} \cos\left(\frac{\theta}{2}\right) d\theta + \int_{\pi}^{2\pi} -\cos\left(\frac{\theta}{2}\right) d\theta\right) = 2(2+2) = \boxed{L = 8}$$