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Assessment 1.1

1. $\int x^3 \ln x \, dx$
2. $\int \cos^{-1} x \, dx$
3. $\int (t^4 + 2t^3 + t^2 - t + 5) e^{3t} dt$
4. $\int sen^6 \theta \cos^3 \theta d\theta$
5. $\int sen^3 \theta \cos^{2/3} \theta \, d\theta$
6. $\int \sec^4 3x \tan^3 3x dx$
7. $\int \frac{\sec^2 t}{\tan^2 t} dt$

$$\int u \, dv = uv - \int v \, du$$

Buscar
derivada de u
integral de dv

ILATE

1) $\int \overset{E^{dv}}{x^3} \, \overset{L^{u}}{\ln x} \, dx$

$u = \ln x \qquad dv = x^3 dx$

$du = \frac{1}{x} \qquad v = \frac{x^4}{4}$

$$= \ln x \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$$

$$= \frac{x^4}{4} \ln x - \int \frac{1}{4} x^3 dx$$

$$= \frac{x^4}{4} \ln x - \frac{x^4}{16} + C$$

2) $\int \overset{I^{u}}{\cos^{-1} x} \, \overset{dv}{dx}$

$t = 1 - x^2$

$\frac{1}{t^{1/2}} = t^{-1/2}$

$u = \cos^{-1} x \qquad dv = dx$

$du = \frac{-1}{\sqrt{1-x^2}} dx \qquad v = x$

$\frac{x}{1} \cdot \frac{-1}{\sqrt{1-x^2}}$

$$= \cos^{-1} x \cdot x - \int x \cdot \frac{-1}{\sqrt{1-x^2}} dx$$

$$= x \cos^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$$

$$= x \cos^{-1} x - \frac{1}{2} \int \frac{1}{\sqrt{t}} dt$$

$$= x \cos^{-1} x - \frac{1}{2} \int t^{-1/2} dt$$

$\frac{1}{2} \cdot \frac{t^{1/2}}{1/2} \quad t^{1/2}$

$$= x \cos^{-1} x - \sqrt{1-x^2} + C$$

3) $\int \underbrace{(t^4 + 2t^3 + t^2 - t + 5)}_{v} \underbrace{e^{3t}\, dt}_{dv}$

$du = (4t^3 + 6t^2 + 2t - 1)$

$v = \frac{1}{3} e^{3t}$

$\int \frac{1}{3}(t^4 + 2t^3 + t^2 - t + 5)e^{3t} - \int \frac{1}{3}(4t^3 + 6t^2 + 2t - 1)e^{3t}\, dt$

Simplify:

$\frac{1}{9}(t^4 + 2t^3 + t^2 - t + 5)e^{3t} - \frac{1}{27}(t^3 + 2t^2 + t - 1)e^{3t} + C$

$e^{3t}\left(\frac{t^4}{9} + \frac{2t^3}{9} + \frac{t^2}{9} - \frac{t}{9} + \frac{5}{9}\right) - \left(\frac{t^3}{27} - \frac{2t^2}{27} - \frac{t}{27} + \frac{1}{27}\right) + C$

$\left(\frac{t^4 + 2t^3 + t^2}{9} + \frac{5}{9} - \frac{t}{9} - \frac{t^3 - 2t^2 - t}{27} + \frac{1}{27}\right)$

$e^{3t}\left(\frac{t^4 + 2t^3 + t^2}{9} - \frac{t}{9} - \frac{t^3 - 2t^2}{27} - \frac{t}{27}\right) + \frac{14}{27} + C$

$e^{3t}\left(\frac{t^4 + 2t^3 + t^2}{9} - \frac{t^3 - 2t^2}{27}\right) - \frac{4t}{27} + \frac{14}{27} + C$

$e^{3t}\left[\left(\frac{3t^4 + 6t^3 - 3t^2 - t^3 - 2t^2}{27}\right) - \frac{4t}{27} + \frac{14}{27}\right] + C$

$e^{3t}\left[\left(\frac{3t^4 + 6t^3 - t^3}{27}\right) - \frac{5t^2}{27} - \frac{4t}{27} + \frac{14}{27}\right] + C$

$e^{3t}\left(\frac{3t^4 + 5t^3 - 5t^2 - 4t + 14}{27}\right) + C$

4) $\int \text{sen}^6 \theta \cos^3 \theta \, d\theta$

$= \int \text{sen}^6 \theta \cdot \cos^2 \theta \cdot \cos \theta \, d\theta$

$= \int \text{sen}^6 \theta (1 - \text{sen}^2 \theta) \cos \theta \, d\theta$

$t = \text{sen}\, \theta \qquad dt = \cos \theta \, d\theta$

$= \int t^6 (1 - t^2)\, dt$

$= \int (t^6 - t^8)\, dt$

$= \frac{t^7}{7} - \frac{t^9}{9} + C$

$= \frac{1}{7} \text{sen}^7 \theta - \frac{1}{9} \text{sen}^9 \theta + C$

5) $\int \text{sen}^3 \theta \cos^{2/3}\theta \, d\theta$

$= \int \text{sen}^2\theta \cos^{2/3}\theta \ \text{sen}\,\theta \, d\theta$

$= \int (1-\cos^2\theta)\cos^{2/3}\theta \ \text{sen}\,\theta \, d\theta$

$t = \cos\theta$

$dt = -\text{sen}\,\theta \, d\theta$

$= \int (1-t^2)\, t^{2/3} (-dt)$

$= -\int (t^{2/3} - t^{8/3})\, dt$

$= -\left[\frac{t^{5/3}}{5/3} - \frac{t^{11/3}}{11/3}\right] + C$

$= 3/11 \, t^{11/3} - 3/5 \, t^{5/3} + C$

$= \frac{3}{11} \cos^{11/3} - \frac{3}{5} \cos^{5/3}\theta + C$

6) $\int \sec^4 3x \tan^3 3x \, dx$

$= \int \sec^2 3x \tan^3 3x \sec^2 3x \, dx$

$= \int (1+\tan^2 3x)\tan^3 3x \sec^2 3x \, dx$

$t = \tan 3x$

$dt = 3\sec^2 3x$

$= \int (1+t^2)\, t^3 \, dt/3$

$= 1/3 \int (t^3 + t^5)\, dt$

$= 1/3 \left[\frac{t^4}{4} + \frac{t^6}{6}\right] + C$

$= 1/12 \tan^4 3x + 1/18 \tan^6 3x + C$

7) $\int \frac{\sec^2 t}{\tan^2 t} \, dt$

$x = \tan t$

$dx = \sec^2 t \, dt$

$= \int \frac{dx}{x^2} = \int x^{-2} dx = \frac{x^{-1}}{-1} - C = -1/y + C = -1/\tan t + C$

$= \frac{-1}{\tan t} + C$