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#121260

MATH 1360 - 80

Prof. Milena Gomez

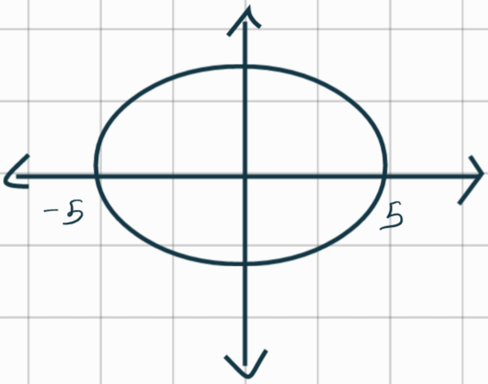
1. Determine el área encerrada por la elipse $\frac{x^2}{25}+\frac{y^2}{4}=1$

2. Determine el área de la región bajo la curva dada $y=\frac{1}{x^2+x}$, $1 \le x \le 2$

1) $\underline{A = 10\pi}$

2) $\underline{A = 0.288}$

1)



$$4x^2 + 25y^2 = 100$$

$$y = \sqrt{\frac{100-4x^2}{25}}$$

$$= \sqrt{\frac{4(25-x^2)}{25}}$$

$$= \frac{2}{5}\sqrt{25-x^2}$$

$x_1 = 0 \quad x_2 = 5$

$$\frac{A}{4} = \int_{x_1}^{x_2} y\,dx$$ ← Formula

$$A/4 = \int_0^5 \frac{2}{5}\sqrt{25-x^2}\,dx$$

$$A/4 = \frac{2}{5}\int_0^5 \sqrt{5^2-x^2}\,dx$$

$x = 5\operatorname{sen}(z) \quad dx = 5\cos(z)$

$$A/4 = \frac{2}{5}\int_{z_1}^{z_2} \sqrt{5^2-5^2\operatorname{sen}^2(z)}(5)\cos(z)\,dz$$

$$= \frac{2}{5}\int_{z_1}^{z_2} 25\sqrt{1-\operatorname{sen}^2(z)}\cos(z)\,dz$$

$$= \frac{2}{5}(25)\int_{z_1}^{z_2} \cos^2 z\,dz$$

$z_1 = 0 \qquad z_2 = \frac{\pi}{2}$

$$\frac{A}{4} = 10 \int_0^{\frac{\pi}{2}} \frac{\cos(2z)+1}{2} \, dz$$

$$= 5\left[-\frac{\operatorname{sen}(2z)}{2} + z\right]_{z_1}^{z_2}$$

$$= 5\left(\frac{-\operatorname{sen}}{2} + \frac{\pi}{2} - \frac{-\operatorname{sen} 0}{2} - 0\right)$$

$$= 5\left(0 + \frac{\pi}{2} - 0 - 0\right) = \frac{5\pi}{2}$$

$$\cos(2z) = \cos^2(z) - \operatorname{sen}^2(z)$$
$$= \cos^2(z) - (1 - \cos^2(z))$$
$$= 2\cos^2(z) - 1$$
$$\cos^2(z) = \frac{\cos(2z)+1}{2}$$

Segunda Curva

$$A = \int_1^2 \frac{dx}{x^2+x} = \int_1^2 \frac{dx}{x(x+1)}$$

$$\frac{1}{x(x+1)} = \frac{a}{x} + \frac{b}{x+1}$$

$$\frac{1}{x(x+1)} = \frac{a(x+1)+bx}{x(x+1)}$$

$$1 = ax + a + bx$$
$$= x(a+b) + a$$

$a + b = 0 \qquad a = 1$

$b = -1$

$\Rightarrow$

$$A = \int_1^2 \left(\frac{1}{x} - \frac{1}{(x+1)}\right) dx$$
$$= \left[\ln(x) - \ln(x+1)\right]_1^2$$
$$= \left[\ln\left(\frac{x}{x+1}\right)\right]_1^2$$
$$= \ln\left(\frac{2}{3}\right) - \ln\left(\frac{1}{2}\right)$$
$$= 0.288$$

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2) $\underline{\underline{A = 0.288}}$