# FSPL – Assignment 4

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## 3.1

1. $y = x^2$
2. $y = z \wedge x = w$
3. $a \leq b \wedge y = ax$
4. $\boldsymbol{true}$
5. $\boldsymbol{false}$

## 3.2

The program $P$ is $c := a\,; d := b;$ $\boldsymbol{while}$ $(d > 0)$ $\boldsymbol{do}$ $(r := c\ \boldsymbol{rem}\ d; c := d; d := r)$.
The loop invariant is $d \geq 0 \wedge gcd(c, d) = gcd(a, b)$.
Proof:

1. $[a \geq 0 \wedge b \geq 0 \wedge gcd(a, b) = gcd(a, b)] \quad c := a \quad [a \geq 0 \wedge b \geq 0 \wedge gcd(c, b) = gcd(a, b)]$ AS
2. $[a \geq 0 \wedge b \geq 0 \wedge gcd(c, b) = gcd(a, b)] \quad d := b \quad [d \geq 0 \wedge gcd(c, d) = gcd(a, b)]$ AS
3. $[a \geq 0 \wedge b \geq 0 \wedge gcd(a, b) = gcd(a, b)] c := a; d := b; [d \geq 0 \wedge gcd(c, d) = gcd(a, b)]$ SC,1,2
4. $[d \geq 0 \wedge gcd(c, d) = gcd(a, b) \wedge d > 0 \wedge d = z] r := c\ \boldsymbol{rem}\ d [d \geq 0 \wedge gcd(d, r) = gcd(a, b) \wedge d > 0 \wedge d = z]$ AS
5. $[d \geq 0 \wedge gcd(d, r) = gcd(a, b) \wedge d > 0 \wedge d = z] \quad c := d \quad [d \geq 0 \wedge gcd(c, r) = gcd(a, b) \wedge d > 0 \wedge d = z]$ AS
6. $[d \geq 0 \wedge gcd(c, r) = gcd(a, b) \wedge d > 0 \wedge d = z] \quad d := r \quad [d \geq 0 \wedge gcd(c, d) = gcd(a, b) \wedge d > 0 \wedge d < z]$ AS
7. $[d \geq 0 \wedge gcd(c, d) = gcd(a, b) \wedge d > 0 \wedge d = z]$
   $r := c\ \boldsymbol{rem}\ d; c := d$
   $[d \geq 0 \wedge gcd(c, r) = gcd(a, b) \wedge d > 0 \wedge d = z]$ SC,4,5
8. $[d \geq 0 \wedge gcd(c, d) = gcd(a, b) \wedge d > 0 \wedge d = z]$
   $r := c\ \boldsymbol{rem}\ d; c := d; d := r$
   $[d \geq 0 \wedge gcd(c, d) = gcd(a, b) \wedge d > 0 \wedge d < z]$ SC,7,6
9. $d \geq 0 \wedge gcd(c, d) = gcd(a, b) \wedge d > 0 \Rightarrow d \geq 0$
10. $[d \geq 0 \wedge gcd(c, d) = gcd(a, b)]$
    $\boldsymbol{while}$ $(d > 0)$ $\boldsymbol{do}$ $(r := c\ \boldsymbol{rem}\ d; c := d; d := r)$
    $[d \leq 0 \wedge d \geq 0 \wedge gcd(c, d) = gcd(a, b)]$ WHT,8,9

11. $[a \geq 0 \land b \geq 0 \land gcd(a,b) = gcd(a,b)]$
$c := a; d := b;$ ***while*** $(d > 0)$ ***do*** $(r := c$ ***rem*** $d; c := d; d := r)$
$[d \leq 0 \land d \geq 0 \land gcd(c,d) = gcd(a,b)]$ SC 3,10

12. $[d \leq 0 \land d \geq 0 \land gcd(c,d) = gcd(a,b)] \Rightarrow [gcd(c,0) = gcd(a,b)]$

13. $[a \geq 0 \land b \geq 0 \land gcd(a,b) = gcd(a,b)]$
$c := a; d := b;$ ***while*** $(d > 0)$ ***do*** $(r := c$ ***rem*** $d; c := d; d := r)$
$[gcd(c,0) = gcd(a,b)]$ WC,11,12

14. $gcd(c,0) = gcd(a,b) \Rightarrow c = gcd(a,b)$

15. $[a \geq 0 \land b \geq 0 \land gcd(a,b) = gcd(a,b)]$
$c := a; d := b;$ ***while*** $(d > 0)$ ***do*** $(r := c$ ***rem*** $d; c := d; d := r)$
$[c = gcd(a,b)]$ WC,13,14

16. $a \geq 0 \land b \geq 0 \Rightarrow a \geq 0 \land b \geq 0 \land gcd(a,b) = gcd(a,b)$

17. $[a \geq 0 \land b \geq 0]$
$c := a; d := b;$ ***while*** $(d > 0)$ ***do*** $(r := c$ ***rem*** $d; c := d; d := r)$
$[c = gcd(a,b)]$ SP,15,16

## 3.4

The loop invariant is $x \geq 0 \land y = y_0 + \frac{(x_0-x)(x_0+x-1)}{2}$.

1. $[x \geq 0 \land y = y_0 + \frac{(x_0-x)(x_0+x-1)}{2} \land x \neq 0 \land x = z] x := x - 1 [x \geq 0 \land y = y_0 + \frac{(x_0-x-1)(x_0+x)}{2} \land x < z]$ AS

2. $[x \geq 0 \land y = y_0 + \frac{(x_0-x-1)(x_0+x)}{2} \land x < z] \quad y := y + x \quad [x \geq 0 \land y = y_0 + \frac{(x_0+x-1)(x_0-x)}{2} \land x < z]$ AS

3. $[x \geq 0 \land y = y_0 + \frac{(x_0-x)(x_0+x-1)}{2} \land x \neq 0 \land x = z]$
$x := x - 1; y := y - x;$
$[x \geq 0 \land y = y_0 + \frac{(x_0-x)(x_0+x-1)}{2} \land x < z]$ SC,1,2

4. $x \geq 0 \land y = y_0 + \frac{(x_0-x)(x_0+x-1)}{2} \land x \neq 0 \Rightarrow x \geq 0$

5. $[x \geq 0 \land y = y_0 + \frac{(x_0-x)(x_0+x-1)}{2}]$
***while*** $x \neq 0$ ***do*** $(x := x - 1; y := y - x)$
$[x \geq 0 \land y = y_0 + \frac{(x_0-x)(x_0+x-1)}{2} \land x = 0]$ WHT,3,4

6. $x \geq 0 \land x = x_0 \land y = y_0 \Rightarrow x \geq 0 \land y = y_0 + \frac{(x_0-x)(x_0+x-1)}{2}$

7. $[x \geq 0 \land x = x_0 \land y = y_0]$
***while*** $x \neq 0$ ***do*** $(x := x - 1; y := y - x)$
$[x \geq 0 \land y = y_0 + \frac{(x_0-x)(x_0+x-1)}{2} \land x = 0]$ SP,5,6

8. $x \geq 0 \land y = y_0 + \frac{(x_0-x)(x_0+x-1)}{2} \land x = 0 \Rightarrow y = y_0 + \frac{x_0(x_0-1)}{2}$

9. $[x \geq 0 \land x = x_0 \land y = y_0]$ ***while*** $x \neq 0$ ***do*** $(x := x - 1; y := y - x)$ $[y = y_0 + \frac{x_0(x_0-1)}{2}]$ WC,7,8