FSPL – Assignment 2

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1 Problem 1

1.1 Expression 1

1.1.1 normal order reduction sequence

$$
\begin{aligned}
(\lambda f.\lambda x f(f\,x))(\lambda b.\lambda x.\lambda y\,b\,y\,x)(\lambda z.\lambda w.z) &\to (\lambda b.\lambda x.\lambda y\,b\,y\,x)((\lambda b.\lambda x.\lambda y\,b\,y\,x)(\lambda z.\lambda w.z)) \\
&\to \lambda x.\lambda y((\lambda b.\lambda x.\lambda y\,b\,y\,x)(\lambda z.\lambda w.z))y\,x \quad (*) \Leftarrow canonical\,form \\
&\to \lambda x.\lambda y(\lambda s.\lambda t\,(\lambda z.\lambda w.z)\,t\,s)y\,x \\
&\to \lambda x.\lambda y(\lambda z.\lambda w.z)\,x\,y \\
&\to \lambda x.\lambda y.\,x
\end{aligned}
$$

1.1.2 canonical form

I have marked with asterisk above.

1.1.3 eager evaluation

$$
\begin{aligned}
(\lambda f.\lambda x f(f\,x))(\lambda b.\lambda x.\lambda y\,b\,y\,x)(\lambda z.\lambda w.z) &\to (\lambda b.\lambda x.\lambda y\,b\,y\,x)((\lambda b.\lambda x.\lambda y\,b\,y\,x)(\lambda z.\lambda w.z)) \\
&\to (\lambda b.\lambda x.\lambda y\,b\,y\,x)(\lambda x.\lambda y\,(\lambda z.\lambda w.z)\,y\,x) \\
&\to (\lambda b.\lambda x.\lambda y\,b\,y\,x)(\lambda x.\lambda y\,y) \\
&\to \lambda x.\lambda y\,(\lambda x.\lambda y\,y)\,y\,x \\
&\to \lambda x.\lambda y.\,x
\end{aligned}
$$

1.2 Expression 2

1.2.1 normal order reduction sequence

$$
\begin{aligned}
(\lambda d.\, d\, d)(\lambda f.\lambda x.f(f\, x)) &\to (\lambda f.\lambda x.f(f\, x))(\lambda f.\lambda y.f(f\, y)) \\
&\to \lambda x.(\lambda f.\lambda y.f(f\, y))((\lambda f.\lambda y.f(f\, y))\, x) \quad (*) \Leftarrow canonical\, form \\
&\to \lambda x.\lambda y.((\lambda f.\lambda y.f(f\, y))\, x)((\lambda f.\lambda y.f(f\, y))\, x\, y) \\
&\to \lambda x.\lambda y.(\lambda y.\, x(x\, y))((\lambda f.\lambda y.f(f\, y))\, x\, y) \\
&\to \lambda x.\lambda y.\, x(x\,((\lambda f.\lambda y.f(f\, y))\, x\, y)) \\
&\to \lambda x.\lambda y.\, x(x(x(x\, y))
\end{aligned}
$$

1.2.2 canonical form

I have marked with asterisk above.

1.2.3 eager evaluation

$$
\begin{aligned}
(\lambda d.\, d\, d)(\lambda f.\lambda x.f(f\, x)) &\to (\lambda f.\lambda x.f(f\, x))(\lambda f.\lambda y.f(f\, y)) \\
&\to \lambda x.(\lambda f.\lambda y.f(f\, y))((\lambda f.\lambda y.f(f\, y))\, x) \\
&\to \lambda x.(\lambda f.\lambda y.f(f\, y))(\lambda y.x(x\, y)) \\
&\to \lambda x.\lambda y.(\lambda y.x(x\, y))((\lambda y.x(x\, y))\, y) \\
&\to \lambda x.\lambda y.(\lambda y.x(x\, y))\, x(x\, y) \\
&\to \lambda x.\lambda y.\, x(x(x(x\, y))
\end{aligned}
$$

1.3 Expression 3

1.3.1 normal order reduction sequence

$$
\begin{aligned}
&(\lambda x.x(\lambda t.(\lambda s.s)t))(\lambda y.(\lambda z.z\,z\,z)(y\,(\lambda x.\lambda y.x)))(\lambda t.t)\\
&\to (\lambda y.(\lambda z.z\,z\,z)(y\,(\lambda x.\lambda y.x)))(\lambda t.(\lambda s.s)t)(\lambda t.t)\\
&\to (\lambda z.z\,z\,z)((\lambda t.(\lambda s.s)t)\,(\lambda x.\lambda y.x))(\lambda t.t)\\
&\to ((\lambda t.(\lambda s.s)t)\,(\lambda x.\lambda y.x))\,((\lambda t.(\lambda s.s)t)\,(\lambda x.\lambda y.x))\,((\lambda t.(\lambda s.s)t)\,(\lambda x.\lambda y.x))(\lambda t.t)\\
&\to (((\lambda s.s)(\lambda x.\lambda y.x)))\,((\lambda t.(\lambda s.s)t)\,(\lambda x.\lambda y.x))\,((\lambda t.(\lambda s.s)t)\,(\lambda x.\lambda y.x))(\lambda t.t)\\
&\to (\lambda x.\lambda y.x)\,((\lambda t.(\lambda s.s)t)\,(\lambda x.\lambda y.x))\,((\lambda t.(\lambda s.s)t)\,(\lambda x.\lambda y.x))(\lambda t.t)\\
&\to ((\lambda t.(\lambda s.s)t)\,(\lambda x.\lambda y.x))\,(\lambda t.t)\\
&\to (\lambda s.s)(\lambda x.\lambda y.x)(\lambda t.t)\\
&\to (\lambda x.\lambda y.x)(\lambda t.t)\\
&\to \lambda y.\lambda t.t \quad (*) \Leftarrow canonical\ form
\end{aligned}
$$

1.3.2 canonical form

I have marked with asterisk above.

1.3.3 eager evaluation

$$
\begin{aligned}
&(\lambda x.x(\lambda t.(\lambda s.s)t))(\lambda y.(\lambda z.z\,z\,z)(y\,(\lambda x.\lambda y.x)))(\lambda t.t)\\
&\to (\lambda x.x(\lambda t.t))(\lambda y.(\lambda z.z\,z\,z)(y\,(\lambda x.\lambda y.x)))(\lambda t.t)\\
&\to (\lambda y.(\lambda z.z\,z\,z)(y\,(\lambda x.\lambda y.x)))(\lambda t.t)(\lambda t.t)\\
&\to (\lambda y.((y\,(\lambda x.\lambda y.x))(y\,(\lambda x.\lambda y.x))(y\,(\lambda x.\lambda y.x))))(\lambda t.t)(\lambda t.t)\\
&\to (\lambda y.((y\,(\lambda x.\lambda y.x))(y\,(\lambda x.\lambda y.x))(y\,(\lambda x.\lambda y.x))))(\lambda t.t)(\lambda t.t)\\
&\to (((\lambda t.t)\,(\lambda x.\lambda y.x))((\lambda t.t)\,(\lambda x.\lambda y.x))((\lambda t.t)\,(\lambda x.\lambda y.x)))(\lambda t.t)\\
&\to (((\lambda x.\lambda y.x))((\lambda x.\lambda y.x))((\lambda x.\lambda y.x)))(\lambda t.t)\\
&\to (\lambda x.\lambda y.x)(\lambda x.\lambda y.x)(\lambda x.\lambda y.x)(\lambda t.t)\\
&\to (\lambda x.\lambda y.x)(\lambda t.t)\\
&\to \lambda y.\lambda t.t
\end{aligned}
$$

2 Problem 2

$$
\begin{aligned}
&(\textbf{while}\, x < 4\, \textbf{do}\, x := x + 2,\ \sigma\{x \rightsquigarrow 1\})\\
&\rightarrow (\textbf{if}\, x < 4\, \textbf{then}\, (x := x + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2)\, \textbf{else}\, \textbf{skip}\,, \sigma\{x \rightsquigarrow 1\})\\
&\rightarrow (\textbf{if}\, 1 < 4\, \textbf{then}\, (x := x + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2)\, \textbf{else}\, \textbf{skip}\,, \sigma\{x \rightsquigarrow 1\})\\
&\rightarrow (\textbf{if}\ \textbf{true}\ \textbf{then}\, (x := x + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2)\, \textbf{else}\, \textbf{skip}\,, \sigma\{x \rightsquigarrow 1\})\\
&\rightarrow (x := x + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2, \sigma\{x \rightsquigarrow 1\})\\
&\rightarrow (x := 1 + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2, \sigma\{x \rightsquigarrow 1\})\\
&\rightarrow (x := 3; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2, \sigma\{x \rightsquigarrow 1\})\\
&\rightarrow (\textbf{skip}\,; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2, \sigma\{x \rightsquigarrow 3\})\\
&\rightarrow (\textbf{if}\, x < 4\, \textbf{then}\, (x := x + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2)\, \textbf{else}\, \textbf{skip}\,, \sigma\{x \rightsquigarrow 3\})\\
&\rightarrow (\textbf{if}\, 3 < 4\, \textbf{then}\, (x := x + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2)\, \textbf{else}\, \textbf{skip}\,, \sigma\{x \rightsquigarrow 3\})\\
&\rightarrow (\textbf{if}\ \textbf{true}\ \textbf{then}\, (x := x + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2)\, \textbf{else}\, \textbf{skip}\,, \sigma\{x \rightsquigarrow 3\})\\
&\rightarrow (x := x + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2, \sigma\{x \rightsquigarrow 3\})\\
&\rightarrow (x := 3 + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2, \sigma\{x \rightsquigarrow 3\})\\
&\rightarrow (x := 5; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2, \sigma\{x \rightsquigarrow 3\})\\
&\rightarrow (\textbf{skip}\,; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2, \sigma\{x \rightsquigarrow 5\})\\
&\rightarrow (\textbf{if}\, x < 4\, \textbf{then}\, (x := x + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2)\, \textbf{else}\, \textbf{skip}\,, \sigma\{x \rightsquigarrow 5\})\\
&\rightarrow (\textbf{if}\, 5 < 4\, \textbf{then}\, (x := x + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2)\, \textbf{else}\, \textbf{skip}\,, \sigma\{x \rightsquigarrow 5\})\\
&\rightarrow (\textbf{if}\ \textbf{false}\ \textbf{then}\, (x := x + 2; \textbf{while}\, x < 4\, \textbf{do}\, x := x + 2)\, \textbf{else}\, \textbf{skip}\,, \sigma\{x \rightsquigarrow 5\})\\
&\rightarrow (\textbf{skip}\,, \sigma\{x \rightsquigarrow 5\})
\end{aligned}
$$

3 Problem 3

3.1 (a)

$$
\begin{aligned}
(x := (x++) + (x++), \sigma\{x \rightsquigarrow 2\}) &\rightarrow (x := 2 + (x++), \sigma\{x \rightsquigarrow 2 + 1\})\\
&\rightarrow (x := 2 + (x++), \sigma\{x \rightsquigarrow 3\})\\
&\rightarrow (x := 2 + 3, \sigma\{x \rightsquigarrow 3 + 1\})\\
&\rightarrow (x := 5, \sigma\{x \rightsquigarrow 4\})\\
&\rightarrow (\textbf{skip}\,, \sigma\{x \rightsquigarrow 5\})
\end{aligned}
$$

3.2 (b)

$$\frac{\sigma\, x = \lfloor \mathbf{n} \rfloor}{(++x, \sigma) \to (n+1, \sigma\{x \to \lfloor \mathbf{n} \rfloor + 1\})}$$

4 Problem 4

It is not hard to find the c_1 and σ' so that

$$(c_1, \sigma) \to^* (\mathbf{skip}\,, \sigma_1),\ \sigma' \neq \sigma_1$$

Construct $c_2 = \mathbf{while}\, b\, \mathbf{do}\, c,$ where $\sigma\, b =\ \mathbf{true}\,,\ (c, \sigma_1) \to (c, \sigma')$.
So we have

$$\begin{aligned}
(c_1; c_2, \sigma) &\to (\mathbf{skip}\,; c_2, \sigma_1) \\
&\to (\mathbf{skip}\,; \mathbf{while}\, b\, \mathbf{do}\, c, \sigma_1) \\
&\to (\mathbf{if}\, b\, \mathbf{then}\, (c; \mathbf{while}\, b\, \mathbf{do}\, c)\ \mathbf{else}\, \mathbf{skip}\,, \sigma_1) \\
&\to (c; \mathbf{while}\, b\, \mathbf{do}\, c, \sigma_1) \\
&\to (\mathbf{while}\, b\, \mathbf{do}\, c, \sigma') = (c_2, \sigma')
\end{aligned}$$

Therefore, we have $(c_1; c_2, \sigma) \to (c_2, \sigma')$ but $(c_1, \sigma) \to^* (\mathbf{skip}\,, \sigma_1) \neq (\mathbf{skip}\,, \sigma')$.