STATS100B – Introduction to Mathematical Statistics
Homework 4

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Exercise 1

(a) Solution:

$$M_{\overline{X}}(t) = \left(1 - \frac{\beta}{n} t\right)^{-n\alpha}$$

$$\begin{aligned} M_{\overline{X}}\left(\frac{2n}{\beta} t\right) &= \left(1 - \frac{\beta}{n} \times \frac{2n}{\beta} t\right)^{-n\alpha} \\ &= (1-2t)^{-\frac{2n\alpha}{2}} \\ &= M_{\frac{2n}{\beta}\overline{X}}(t) \end{aligned}$$

(b) Solution:

$$\begin{aligned} F_Y(y) &= P(aX + b \leq y) \\ &= P(X \leq \frac{y-b}{a}) \\ &= \int_{-\infty}^{\frac{y-b}{a}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\dfrac{(x-\mu)^2}{2\sigma^2}} dx \end{aligned}$$

$$\begin{aligned} p_Y(y) &= F_Y'(y) \\ &= \frac{1}{\sqrt{2\pi}\sigma} e^{-\dfrac{\left[\frac{y-b}{a} - \mu\right]^2}{2\sigma^2}} \frac{1}{a} \\ &= \frac{1}{\sqrt{2\pi}(a\sigma)} e^{-\frac{1}{2(a\sigma)^2}[y-(a\mu+b)]^2} \\ &\therefore aX + b \sim N(a\mu + b, a\sigma) \end{aligned}$$

Exercise 2

Solution:

$$M_{lnx}(t) = E(e^{tlnx}) = E(X^t)$$

Exercise 3

$$M_X(t) = (1 - \beta t)^{-\alpha}$$

$$M_T(t) = \prod_{i=1}^{n} M_{X_i}(t) = (1 - \beta t)^{-n\alpha}$$

$$\therefore T \sim \Gamma(n\alpha, \beta)$$

$$M_{\bar{X}}(t) = \prod_{i=1}^{n} M_{X_i}\left(\frac{t}{n}\right) = \left(1 - \frac{\beta}{n} t\right)^{-n\alpha}$$

$$\therefore T \sim \Gamma\left(n\alpha, \frac{\beta}{n}\right)$$