STATS100B – Introduction Mathmatical Statistics
Homework 1

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Exercise 1

Solution:

$X \sim \Gamma(\alpha, \beta)$, so

$$\begin{aligned} f(x|(\alpha,\beta)) &= \frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^\alpha \Gamma(\alpha)} \\ &= \frac{1}{\beta^\alpha \Gamma(\alpha)} e^{(\alpha-1)lnx} e^{-\frac{x}{\beta}} \\ &= \frac{1}{\beta^\alpha \Gamma(\alpha)} exp\{(\alpha-1)lnx - \frac{1}{\beta}x\} \end{aligned}$$

where $h(x) = 1,\ c(\boldsymbol{\theta}) = \dfrac{1}{\beta^\alpha \Gamma(\alpha)},\ \sum_{i=1}^k (w_i(\boldsymbol{\theta}) t_i(x)) = (\alpha-1)lnx - \frac{1}{\beta}x.$

Exercise 2

Solution:

Let $F_Y(y)$ be the cdf of Y and suppose that g is a monotonic reversible function.

$$\begin{aligned} F_Y(y) &= P(Y \le y) \\ &= P(g(X) \le y) \\ &= P(X \le g^{-1}(y)) \\ &= \int_0^{g^{-1}(y)} \frac{2}{\sqrt{2\pi}} e^{-\frac{1}{2}[g^{-1}(y)]^2}\, dx \end{aligned}$$

$$p_Y(y) = F_Y(y)' = \frac{2}{\sqrt{2\pi}} \frac{\mathrm{d}g^{-1}(y)}{\mathrm{d}y} e^{-\frac{1}{2}[g^{-1}(y)]^2}$$

Because the pdf of gamma distribution is

$$p_Y(y) = \frac{y^{\alpha-1}e^{-\frac{y}{\beta}}}{\beta^\alpha \Gamma(\alpha)},$$

let $g^{-1}(y) = c\sqrt{y}$, where c is constant.

Therefore,

$$-\frac{y}{\beta} = -\frac{1}{2}[g^{-1}(y)]^2 = -\frac{1}{2}cy^2$$