University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Homework 6

Answer the following questions:

a. Let $X_1, X_2, \ldots, X_n$ be i.i.d.Poisson(λ) and let $\bar{X}$ and S^2 be the sample mean and sample variance respectively. Each one of these two estimators has expected value equal to λ (why?). Which estimator is better?

b. Let $X_1, \ldots, X_n$ be i.i.d. $N(\theta, \theta), \theta > 0$. For this model both $\bar{X}$ and cS are unbiased estimators of θ, where $c = \frac{\sqrt{n-1}\Gamma(\frac{n-1}{2})}{\sqrt{2}\Gamma(\frac{n}{2})}$. Show that for any α the estimator $\alpha\bar{X}+(1-\alpha)cS$ is also unbiased estimator of θ. For what value of α this estimator has the minimum variance?

c. Let $X_1, \ldots, X_n$ be i.i.d. random variables with $X_i \sim \Gamma(\alpha, \beta)$ with α known. Find an unbiased estimator of $\frac{1}{\beta}$. (Find $E\frac{1}{\bar{X}}$ and then adjust it to be unbiased of $\frac{1}{\beta}$.)

d. Let X_1, X_2, X_3, X_4 be i.i.d. random variables with $X_i \sim N(0, 1)$. Show that

$$\sum_{i=1}^{4}(X_i - \bar{X})^2 = \frac{(X_1 - X_2)^2}{2} + \frac{[X_3 - \frac{(X_1+X_2)}{2}]^2}{\frac{3}{2}} + \frac{[X_4 - \frac{(X_1+X_2+X_3)}{3}]^2}{\frac{4}{3}}.$$

Show that the three terms in the RHS are independent each with a χ^2_1 distribution.