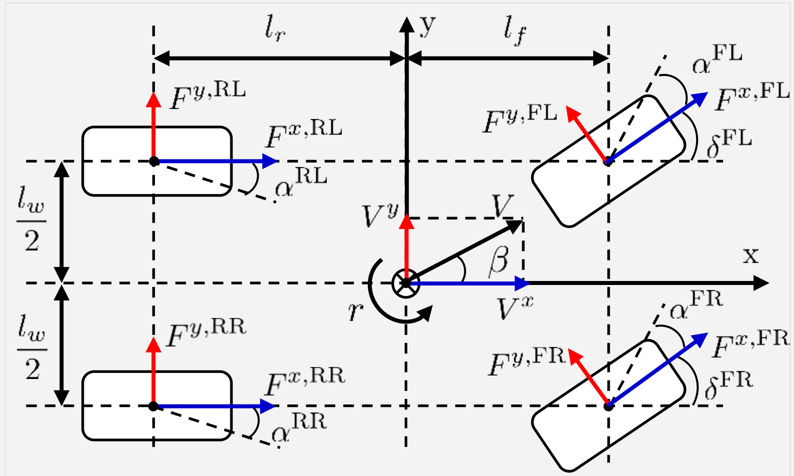
Vehicle Planar Dynamics

Force System of 4-wheel-drive electric vehicle



Fully expressed continuous-time vehicle planar dynamics:

$$
\begin{aligned}
\dot{V}_t^x &= \frac{1}{m}\left(\left(F_t^{x,\mathrm{FL}} + F_t^{x,\mathrm{FR}}\right)\cos(\delta_t) - \left(F_t^{y,\mathrm{FL}} + F_t^{y,\mathrm{FR}}\right)\sin(\delta_t) + F_t^{x,\mathrm{RL}} + F_t^{x,\mathrm{RR}}\right) + V_t^y r_t \\
\dot{V}_t^y &= \frac{1}{m}\left(\left(F_t^{x,\mathrm{FL}} + F_t^{x,\mathrm{FR}}\right)\sin(\delta_t) + \left(F_t^{y,\mathrm{FL}} + F_t^{y,\mathrm{FR}}\right)\cos(\delta_t) + F_t^{y,\mathrm{RL}} + F_t^{y,\mathrm{RR}}\right) - V_t^x r_t \\
\dot{r}_t &= \frac{1}{I_z}\left(\left(\left(F_t^{y,\mathrm{FL}} + F_t^{y,\mathrm{FR}}\right)\cos(\delta_t) + \left(F_t^{x,\mathrm{FL}} + F_t^{x,\mathrm{FR}}\right)\sin(\delta_t)\right) l_f - \left(F_t^{y,\mathrm{RL}} + F_t^{y,\mathrm{RR}}\right) l_r \right. \\
&\quad \left. + \left(\left(F_t^{x,\mathrm{FR}} - F_t^{x,\mathrm{FL}}\right)\cos(\delta_t) + \left(F_t^{y,\mathrm{FL}} - F_t^{y,\mathrm{FR}}\right)\sin(\delta_t) + \left(F_t^{x,\mathrm{RR}} - F_t^{x,\mathrm{RL}}\right)\right)\frac{l_w}{2}\right)
\end{aligned}
$$

Reference: R. N. Jazar, Vehicle dynamics. Springer, 2008, vol. 1.

Vehicle Planar Dynamics

Control-oriented vehicle dynamics:

$$\dot{V}_t^y = \frac{1}{m}\left((T_t^{\mathrm{F}}/r_w)\sin(\delta_t) + (F_t^{y,\mathrm{FL}} + F_t^{y,\mathrm{FR}})\cos(\delta_t) + F_t^{y,\mathrm{RL}} + F_t^{y,\mathrm{RR}}\right) - V_t^x r_t$$

$$\dot{r}_t = \frac{1}{I_z}\Bigg(\left((F_t^{y,\mathrm{FL}} + F_t^{y,\mathrm{FR}})\cos(\delta_t) + (T_t^{\mathrm{F}}/r_w)\sin(\delta_t)\right) l_f - \left(F_t^{y,\mathrm{RL}} + F_t^{y,\mathrm{RR}}\right) l_r$$

$$+ \left((u_1/r_w)\cos(\delta_t) + (F_t^{y,\mathrm{FL}} - F_t^{y,\mathrm{FR}})\sin(\delta_t) + (u_2/r_w)\right)\frac{l_w}{2}\Bigg)$$

$$\dot{x}_t = \left[\dot{V}_t^y, \dot{r}_t\right]^\top$$

Front torque sum
$T_t^{\mathrm{F}} = T_t^{\mathrm{FL}} + T_t^{\mathrm{FR}}$

Front torque difference
$u_1 = T_t^{\mathrm{FR}} - T_t^{\mathrm{FL}}$

Rear torque difference
$u_2 = T_t^{\mathrm{RR}} - T_t^{\mathrm{RL}}$

Induced constraints: $-T_{\mathrm{vec}}^{\max} \leq u_1, u_2 \leq T_{\mathrm{vec}}^{\max}$ $\quad T_{\mathrm{vec}}^{\max}$: Maximum vectoring torque

Torque extraction:

$$T_t^{\mathrm{FL}} = 0.5\left(T_t^{\mathrm{F}} - u_1\right) \qquad T_t^{\mathrm{RL}} = 0.5\left(T_t^{\mathrm{cmd}} - T_t^{\mathrm{F}} - u_2\right)$$

$$T_t^{\mathrm{FR}} = 0.5\left(T_t^{\mathrm{F}} + u_1\right) \qquad T_t^{\mathrm{RR}} = 0.5\left(T_t^{\mathrm{cmd}} - T_t^{\mathrm{F}} + u_2\right)$$

Vehicle Planar Dynamics

Pacejka tire model:

Reference: Pacejka, Hans. Tire and vehicle dynamics. Elsevier, 2005.

$$F_t^{y,i} = D^i \sin\left(C \tan^{-1}\left(B^i \alpha_t^i - E\left(B^i \alpha_t^i - \tan^{-1}\left(B^i \alpha_t^i\right)\right)\right)\right) \qquad i = \{\mathrm{FL}, \mathrm{FR}, \mathrm{RL}, \mathrm{RR}\}$$

$$D^i = \mu D F_t^{z,i} \qquad B^i = \frac{C_\alpha}{C D^i}$$

Tire side slip angles:

$$\alpha_t^{\mathrm{FL}} = -\tan^{-1}\left(\frac{V_t^y + r_t l_f}{V_t^x - r_t \frac{l_w}{2}}\right) + \delta_t^{\mathrm{FL}} \qquad \alpha_t^{\mathrm{RL}} = -\tan^{-1}\left(\frac{V_t^y - r_t l_r}{V_t^x - r_t \frac{l_w}{2}}\right)$$

$$\alpha_t^{\mathrm{FR}} = -\tan^{-1}\left(\frac{V_t^y + r_t l_f}{V_t^x + r_t \frac{l_w}{2}}\right) + \delta_t^{\mathrm{FR}} \qquad \alpha_t^{\mathrm{RR}} = -\tan^{-1}\left(\frac{V_t^y - r_t l_r}{V_t^x + r_t \frac{l_w}{2}}\right)$$

Vertical loads:

$$F_t^{z,\mathrm{FL}} = 0.5\left(mgl_r - ma_t^x h - ma_t^y hl_r\right)/L,$$

$$F_t^{z,\mathrm{FR}} = 0.5\left(mgl_r - ma_t^x h + ma_t^y hl_r\right)/L,$$

$$F_t^{z,\mathrm{RL}} = 0.5\left(mgl_f + ma_t^x h - ma_t^y hl_f\right)/L,$$

$$F_t^{z,\mathrm{RR}} = 0.5\left(mgl_f + ma_t^x h + ma_t^y hl_f\right)/L.$$

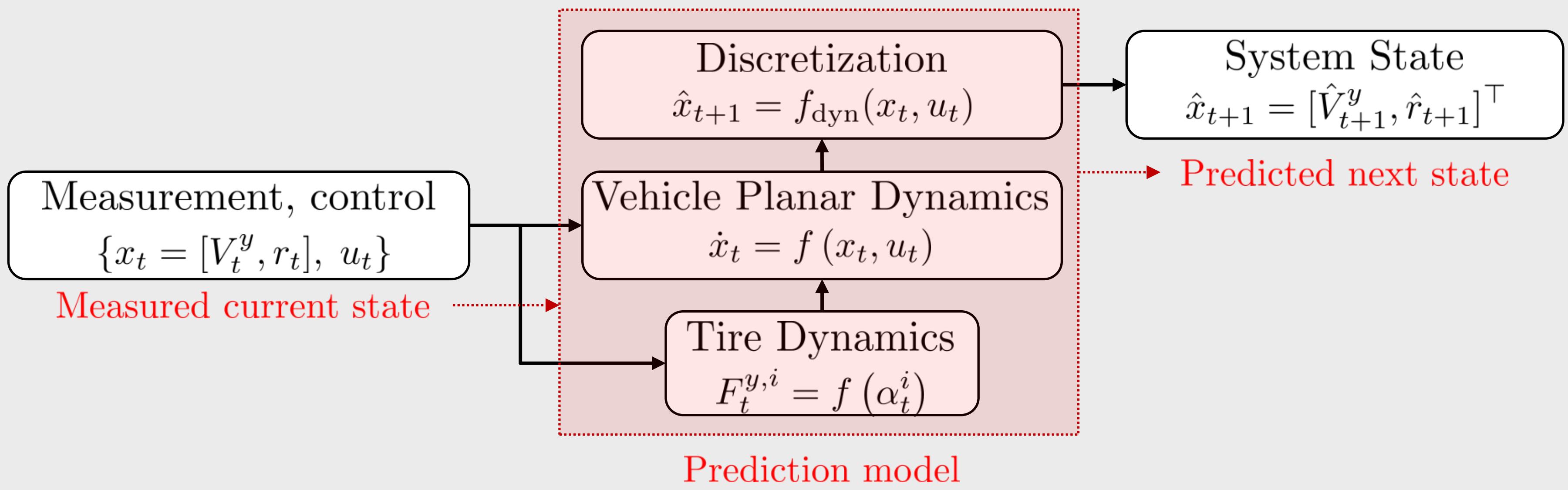
Static load

Load transfer due to longitudinal acceleration

Load transfer due to lateral acceleration

Vehicle Planar Dynamics

State Prediction with Nominal (Physics) Dynamics:



Constraints and Objectives

Pure Torque related Constraints:

Torque bounds: $|T_t^i| \leq T_t^{i,\max},\ i = \{\mathrm{FL}, \mathrm{FR}, \mathrm{RL}, \mathrm{RR}\}$

Torque rate of change: $|\dot{T}_t^i| \leq \dot{T}_t^{i,\max},\ i = \{\mathrm{FL},\ \mathrm{FR},\ \mathrm{RL},\ \mathrm{RR}\}$

Torque increment: $|\Delta T_t^i| \leq \Delta T_t^{i,\max},\ i = \{\mathrm{FL},\ \mathrm{FR},\ \mathrm{RL},\ \mathrm{RR}\}$

Vectoring torque: $|T_t^{\mathrm{FL}} - T_t^{\mathrm{FR}}| \leq T_{\mathrm{vec}}^{\max}, \qquad |T_t^{\mathrm{RL}} - T_t^{\mathrm{RR}}| \leq T_{\mathrm{vec}}^{\max}$

Torque command: $T_t^{\mathrm{FL}} + T_t^{\mathrm{FR}} + T_t^{\mathrm{RL}} + T_t^{\mathrm{RR}} = T_t^{\mathrm{cmd}}$

(1) Yaw rate reference tracking:

- Yaw rate reference:

$$|r_t^{\mathrm{ref}}| = |V_t^x \delta_t / L| \leq 0.85 \mu_x g / |V_t^x|$$

- Objective function:

$$\text{Minimize } \left\| r_t - r_t^{\mathrm{ref}} \right\|^2$$

Reference: Rajamani, Rajesh. Vehicle dynamics and control. Springer Science & Business Media, 2011.

(2) Torque increment minimization:

- Torque increment:

$$\Delta T_t^i = T_t^i - T_{t-1}^i$$

$$i = \{\mathrm{FL}, \mathrm{FR}, \mathrm{RL}, \mathrm{RR}\}$$

- Objective function:

$$\text{Minimize } \left\| \Delta T_t^i \right\|^2$$

$$i = \{\mathrm{FL}, \mathrm{FR}, \mathrm{RL}, \mathrm{RR}\}$$

Optimal Control Problem

Nominal dynamics-based OCP:

Objective function →

$$\underset{u_{k|t}, x_{k|t}}{\text{Minimize}} \sum_{k=0}^{N_p} Q_r (r_{k|t} - r_t^{\text{ref}})^2 + \sum_{k=0}^{N_c-1} \|\Delta T_{k|t}\|_R^2$$

Vehicle dynamics →

$$\text{s.t.} \begin{cases} x_{k+1|t} = f_{\text{dyn}}(x_{k|t}, u_{k|t}), \text{ for } 0 \leq k \leq N_p \\ x_{k|t} = \left[V_{k|t}^y ; r_{k|t}\right], \ x_{0|t} = x_t, \\ u_{k|t} = \left[T_{k|t}^{\text{FR}} - T_{k|t}^{\text{FL}} ; T_{k|t}^{\text{RR}} - T_{k|t}^{\text{RL}}\right], \end{cases}$$

$$u_{k|t} = \begin{cases} u_{k|t} & \text{for } 0 \leq k \leq N_c - 1 \\ u_{N_c-1|t} & \text{for } N_c \leq k \leq N_p \end{cases},$$

Torque bound constraints →

$$|T_{k|t}^i| \leq T_t^{i,\max}, \ i = \{\text{FL}, \text{FR}, \text{RL}, \text{RR}\}, \text{for } 0 \leq k \leq N_c - 1$$

Torque increment constraints →

$$\left|\Delta T_{k|t}^i\right| \leq \Delta T_t^{i,\max}, \ i = \{\text{FL}, \text{FR}, \text{RL}, \text{RR}\}, \ \text{for } 0 \leq k \leq N_c - 1$$

Vectoring torque constraints →

$$\begin{cases} \left|T_{k|t}^{\text{FL}} - T_{k|t}^{\text{FR}}\right| \leq T_{\text{vec}}^{\max}, \ \text{for } 0 \leq k \leq N_c - 1 \\ \left|T_{k|t}^{\text{RL}} - T_{k|t}^{\text{RR}}\right| \leq T_{\text{vec}}^{\max}, \ \text{for } 0 \leq k \leq N_c - 1 \end{cases}$$

Torque command constraints →

$$T_{k|t}^{\text{FL}} + T_{k|t}^{\text{FR}} + T_{k|t}^{\text{RL}} + T_{k|t}^{\text{RR}} = T_t^{\text{cmd}}$$