MECH876 – Intelligent Vehicle Control Systems

Unscented Kalman Filter based Super Twisting Control for Half-car Suspension system

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CONTENTS

- The presentation aims to provide the key concept, procedure and obtained results without being mathematically intensive.
- Details of the project can be found in the report.

Introduction – Project Motivation

Active suspension control:

- Better maneuver
- Safer
- Higher driving comfort
- Lower maintenance cost

Introduction – Project Objectives

- (Primary) To stabilize both vertical and pitch motions of a half-car suspension system
- (Auxiliary) To guarantee road holding and driving comfort

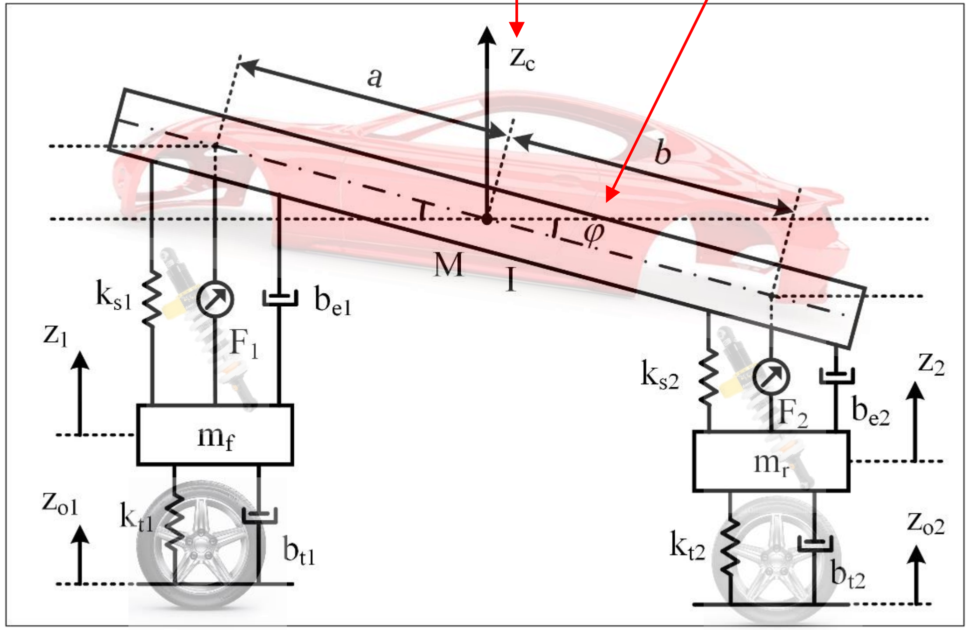


Figure 1: Modeling diagram of a half-car suspension system*

Why Half-car? - Fidelity

- Quarter-car: not sophisticated enough
- Full-car: too complicated

4 DOF system dynamics

- 4 Outputs: vertical z_c, pitch φ, front z_1, rear z_2
- 2 Inputs: force F_1, force F_2

Underactuated system
→ cannot control all 4 outputs!

(*) W. Sun, H. Pan, and H. Gao, "Filter-based adaptive vibration control for active vehicle suspensions with electrohydraulic actuators," IEEE Transactions on Vehicular Technology, vol. 65, no. 6, pp. 4619-4626, 2015.

Introduction- Obstacles and Solutions

(1) Robustness:
- Nonlinearity
- Parameter uncertainty
- External disturbance

→ Robust controller required! → Super twisting sliding mode control

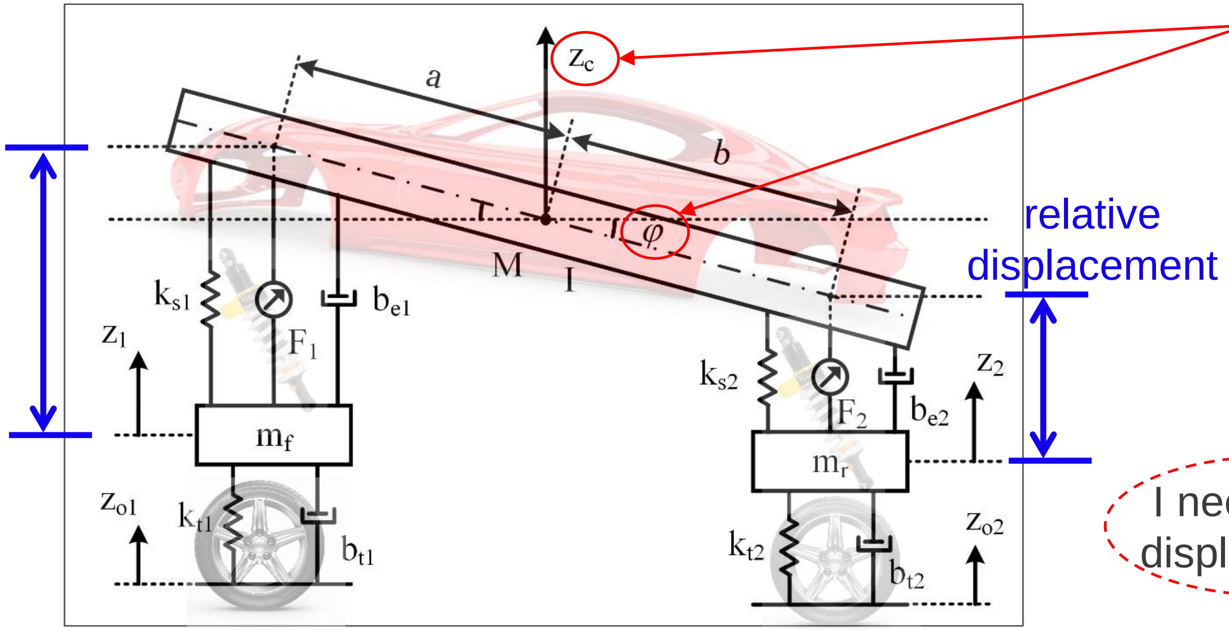


Figure 1: Modeling diagram of a half-car suspension system

(2) State availability:

Requirement of direct displacement & velocity feedback of vertical & pitch motion

- High cost of sensor installation
- Complexity in physical design

→ State estimation

- Higher-order sliding mode observer — I need direct displacement!
- Unscented Kalman filter — I need relative displacement!

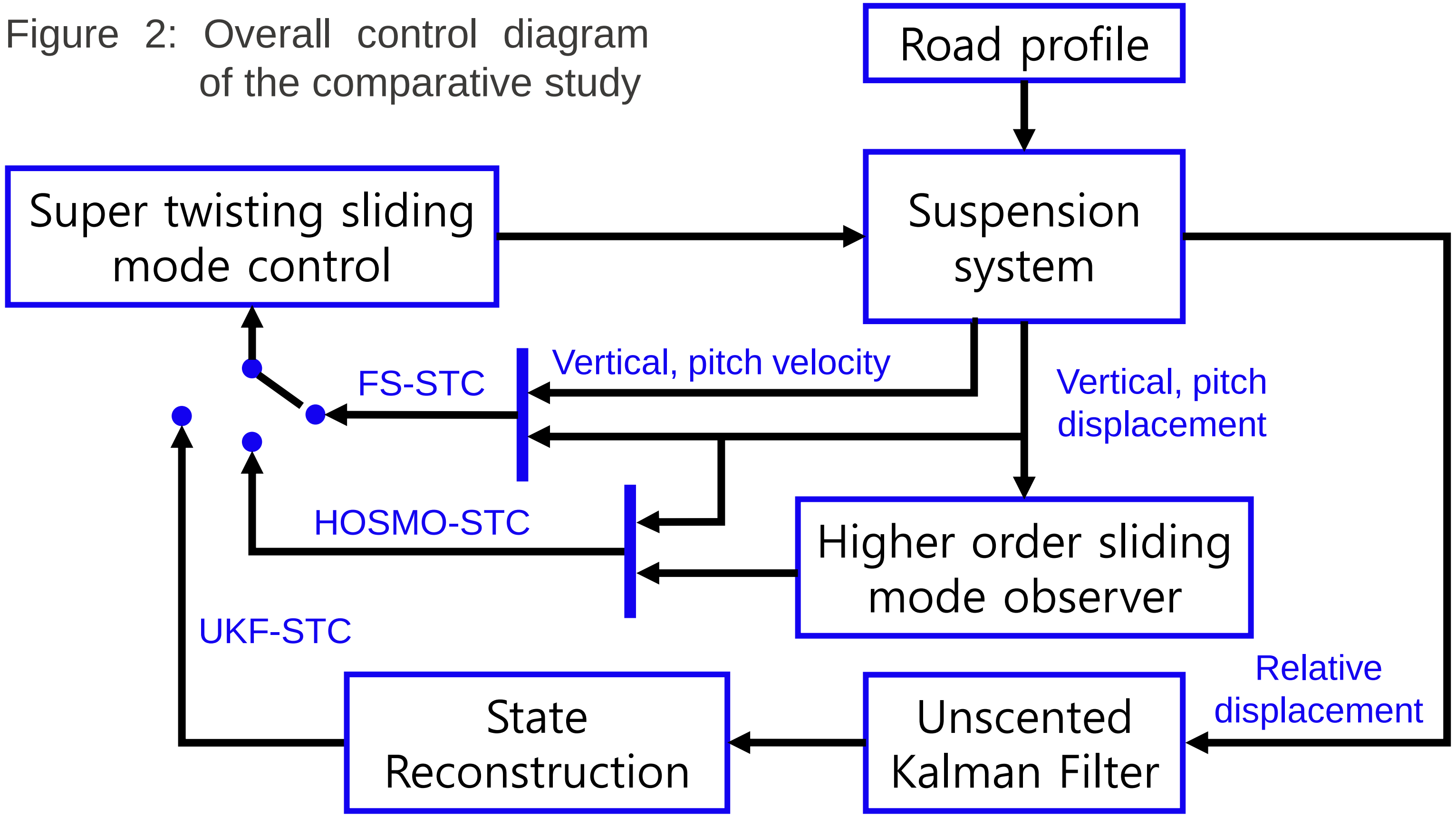
Methodology – Design Procedure

Control-Oriented Modeling → (Observer/Filter) + Controller → Comparative Simulation

Three controller candidates:

- FS-STC (Full state feedback): requires both direct displacement & velocity measurements of vertical and pitch motions
- HOSMO-STC (Semi state feedback): requires only direct displacement measurement of vertical and pitch motions
- UKF-STC (No state feedback): no direct measurement required, needs only relative displacement measurements

Figure 2: Overall control diagram of the comparative study



Design Procedure – Control-oriented Modeling

Step 1

First-principle modeling

Sprung mass subsystem: z_c, φ

including springs, shock absorbers, and force inputs

Unsprung mass subsystem: z_1, z_2

modeled as spring-mass-damper system

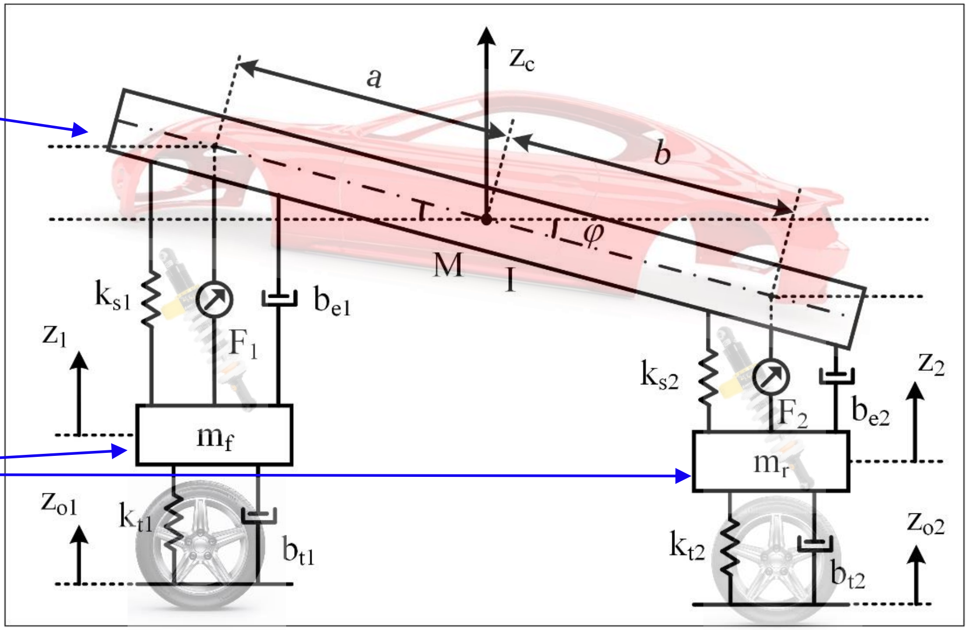


Figure 1: Modeling diagram of a half-car suspension system

Step 2

Control-oriented modeling

State defining:

[vertical, pitch]

$$\begin{cases} X_1 = [z_c,\ \varphi]^{\mathrm{T}} \\ X_2 = [\dot{z}_c,\ \dot{\varphi}]^{\mathrm{T}} \end{cases}$$

canonical format

Readily suitable for control design

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = H\left(X_1, X_2\right) + GU + D \end{cases}$$

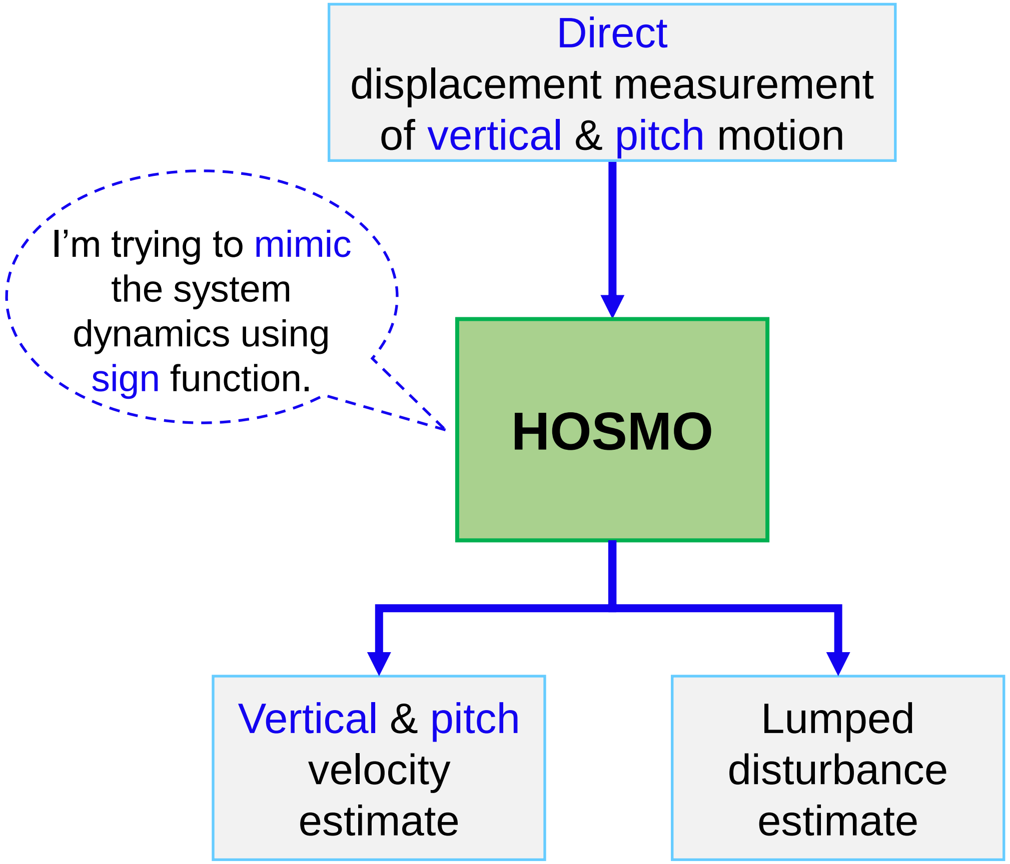
internal dynamics

control matrix

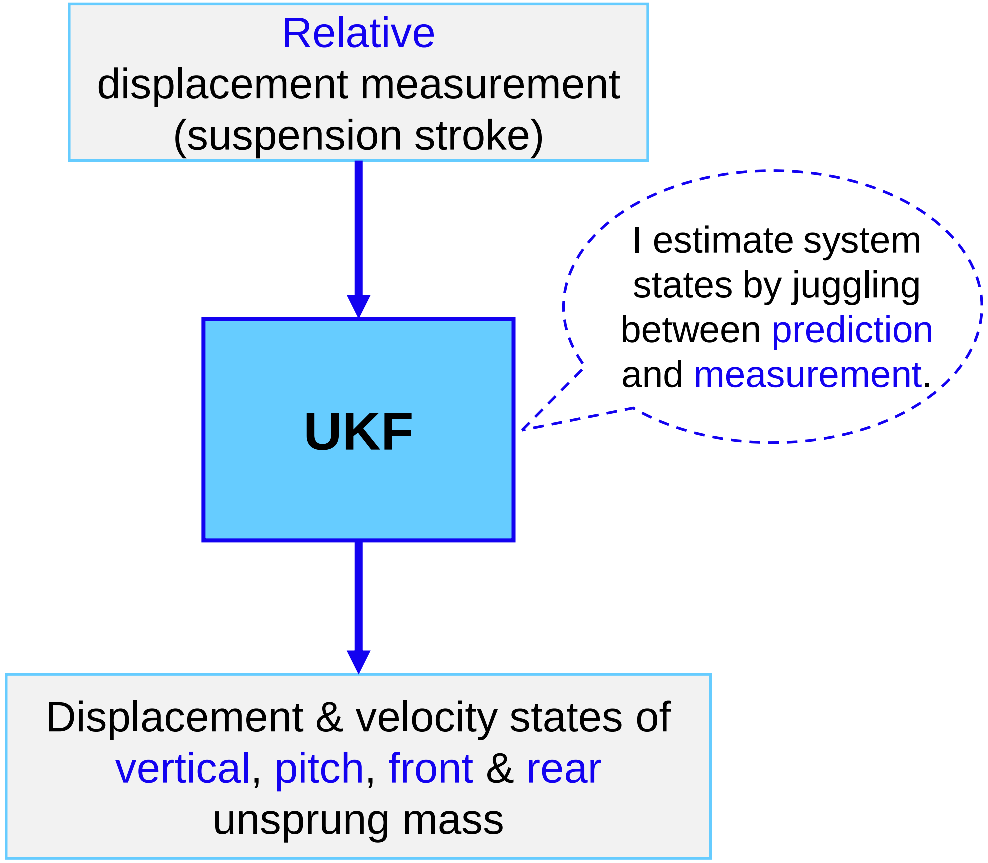
lumped disturbance

Design Procedure – Observer/Filter Design

Higher-order Sliding Mode Observer



Unscented Kalman Filter



Design Procedure – Controller Design

Step 1 | Standard Sliding Mode Control design | Step 2

Sliding surface design → **Control law derivation**

To design how the system states will behave after the sliding mode occurs

To ensure the sliding mode happens despite disturbance and dynamics

$$S = C_1 X_1 + X_2$$

FS-STC

- Straight forward; requires both direct displacement & velocity state measurement
- Control gains must be sufficiently large to compensate for unknown disturbance

$$\hat{S} = C_1 X_1 + \boxed{\hat{X}_2}$$

HOSMO-STC

- Requires only direct displacement state
- Uses estimates of velocity state and lumped disturbance
- Observer gains must be large enough for disturbance estimation

$$\hat{S} = C_1 \boxed{\hat{X}_1} + \boxed{\hat{X}_2}$$

UKF-STC

- No direct measurement required
- Reconstructs & utilizes system states and disturbance
- Control gains are small

→ State-feedback dependency reduction direction

Comparative Simulation Results

Simulation setups:

- Environment: MATLAB – Simulink
- Sampling time: 0.1 (ms)
- Solver: ODE 4 Runge-Kutta
- Disturbance inputs: road with 2 (cm)-bump at 1 (s)
- Controller gains: same level for all controllers
- Observer/Filter gains: trial-and-error

UKF-STC outperforms FS-STC and HOSMO-STC in convergence time, accuracy, and chattering attenuation.

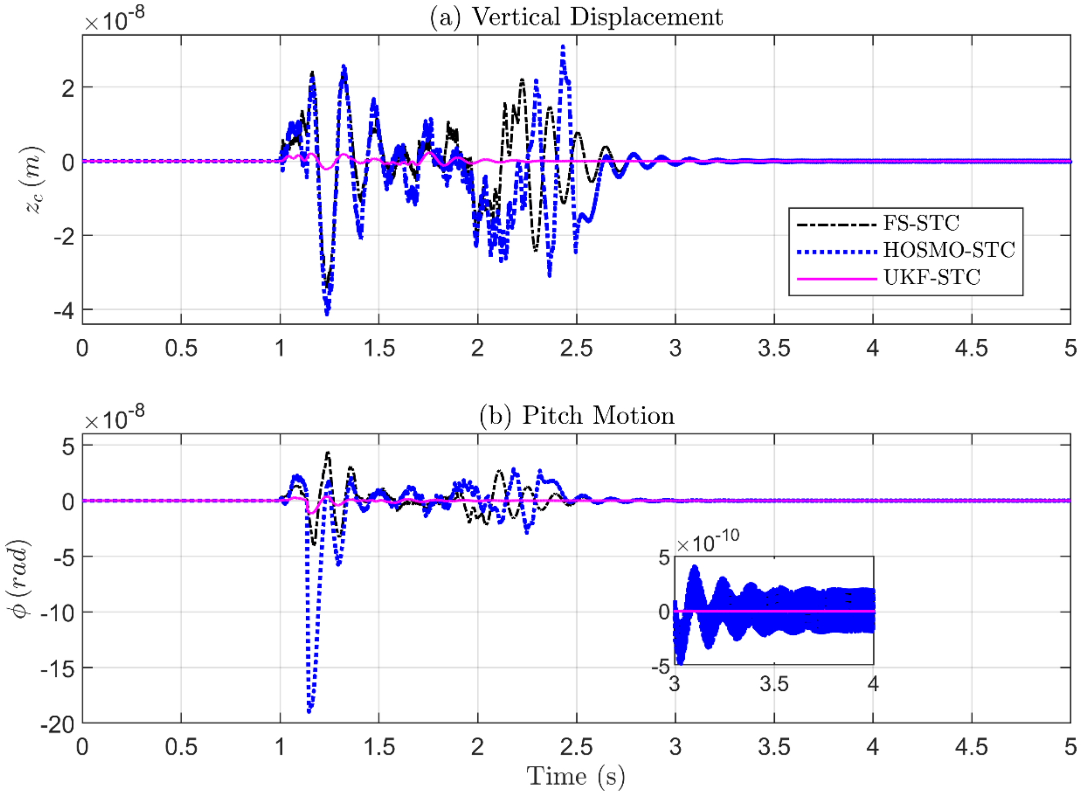


Figure 3: Stabilization performance of all control candidates under bumped road input at 1 (s).

Comparative Simulation Results

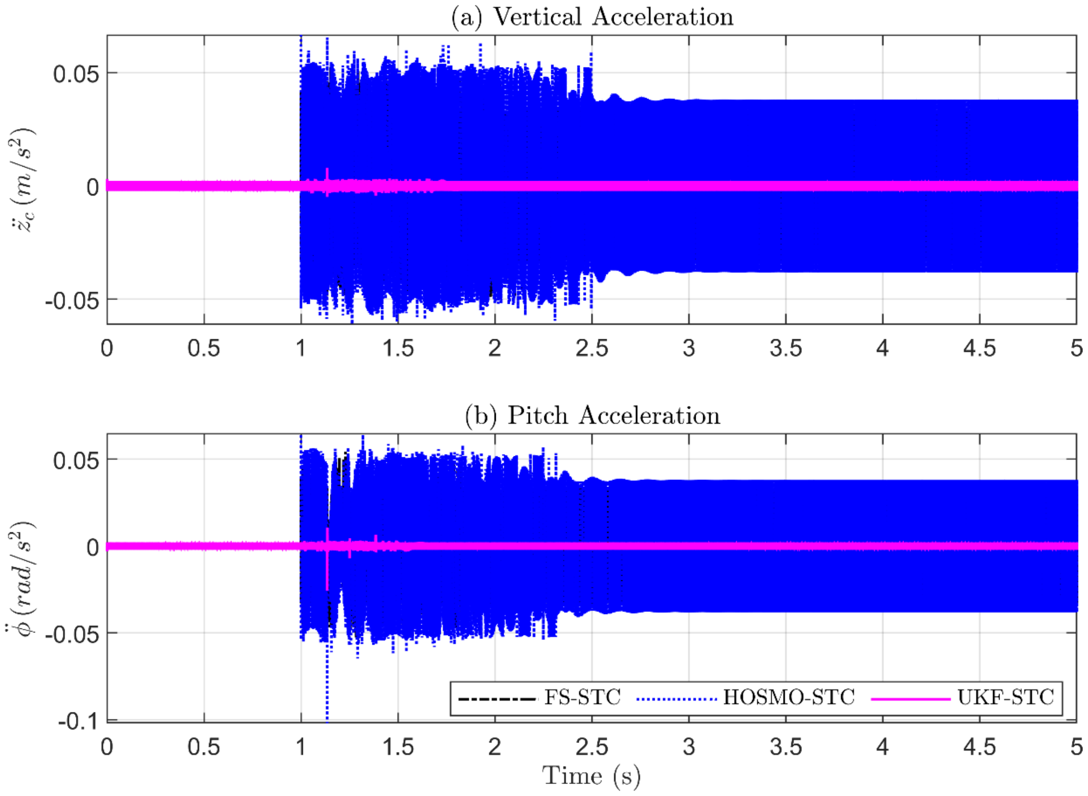


Figure 4: Driving comfort evaluation in term of acceleration minimization of all control candidates under bumped road input at.

- *UKF-STC outperforms FS-STC and HOSMO-STC in acceleration minimization, chattering attenuation, trajectory consistency.*
- *FS-STC & HOSMO-STC exhibits chattering phenomenon even after the bump, indicating a sacrifice of smoothness for robustness.*

Comparative Simulation Results

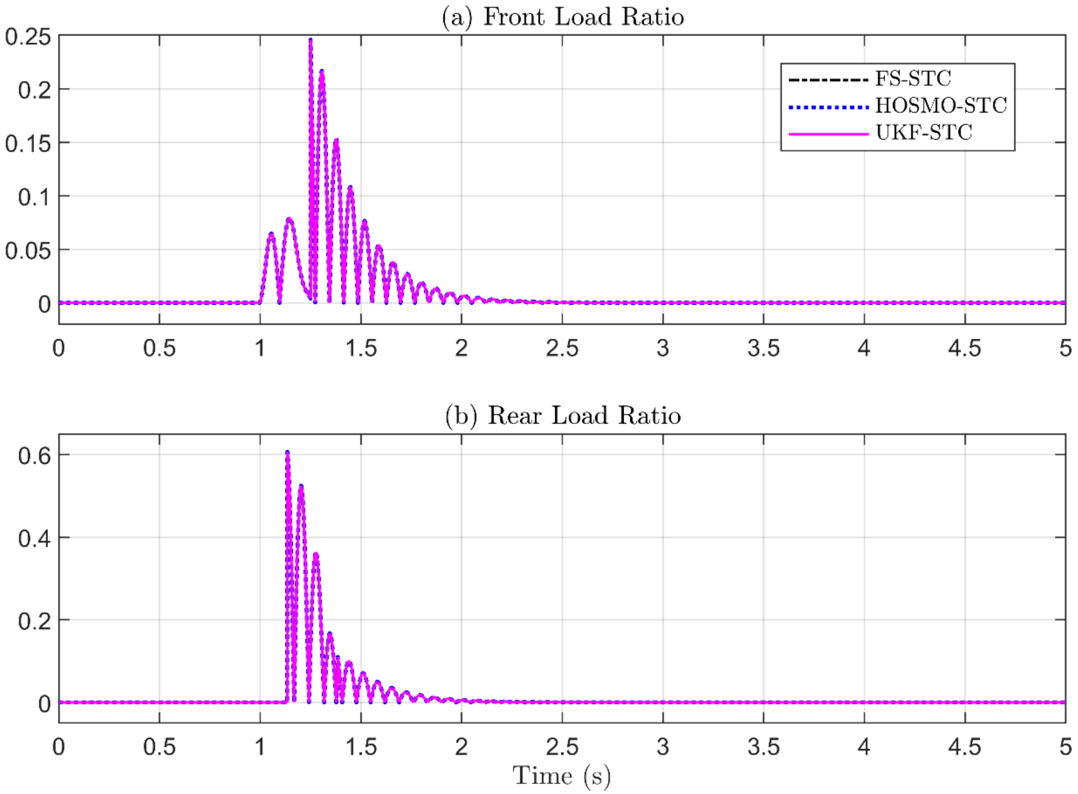


Figure 5: Road holding assessment with dynamic-static load ratio of all control candidates under bumped road input.

$$\text{Road} - \text{holding ratio} = \left|\frac{\text{Dynamic tire load}}{\text{Static tire load}}\right| < 1$$

Keeps the tires on the ground!

All controllers satisfy the road-holding requirement.

Comparative Simulation Results

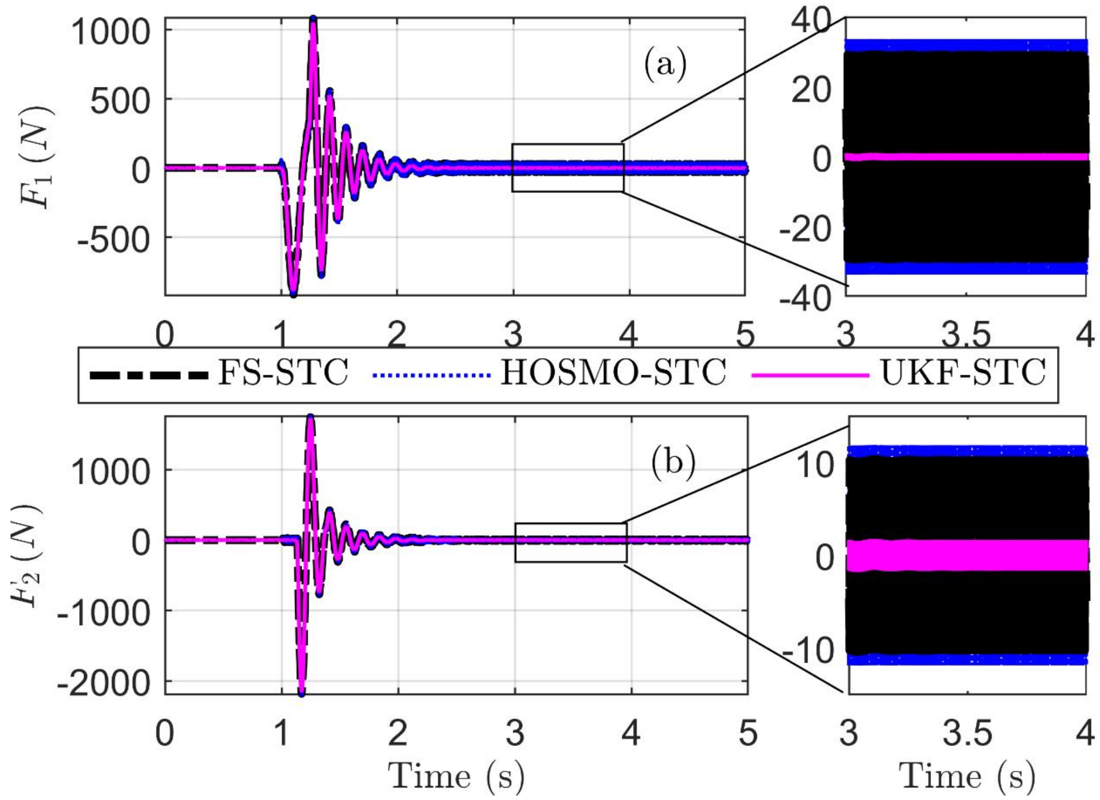


Figure 6: Control inputs in the form of ideal force generator of all control candidates under bumped road input.

- Similar shape of control trajectory for all controllers.
- UKF-STC exhibits much less chattering! → advantage if the actuator dynamics is included in control design.

HOSMO State Estimation Results

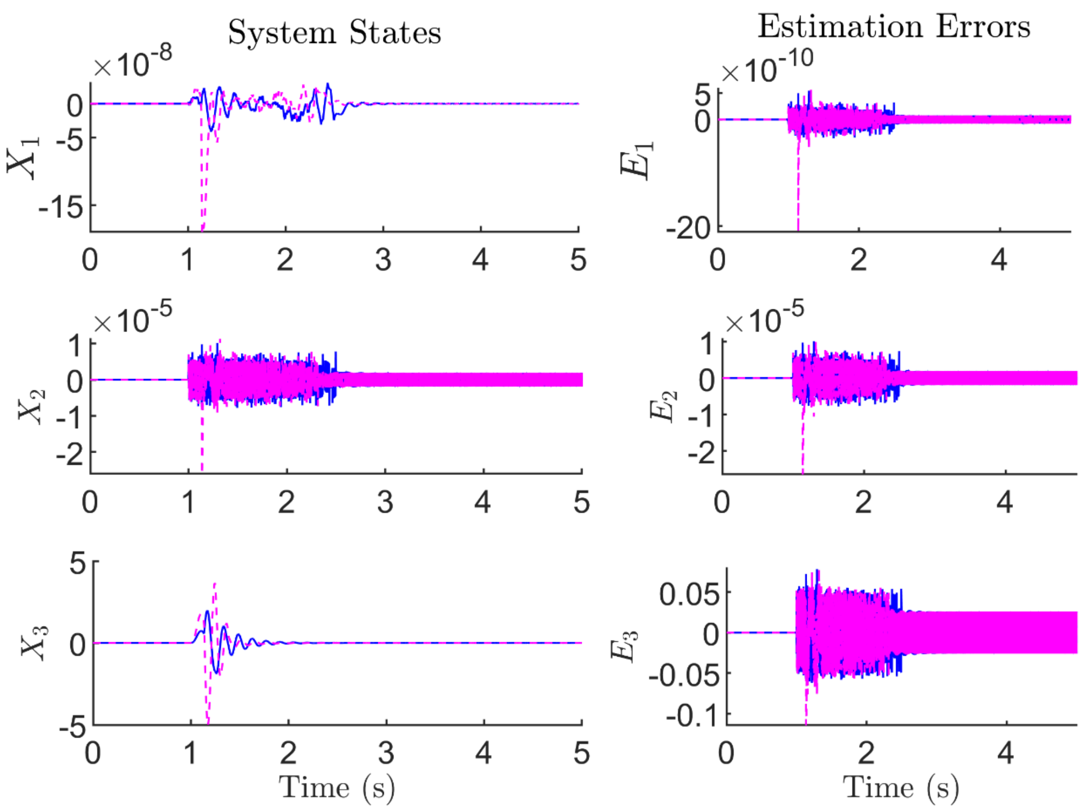


Figure 7: State & lumped disturbance estimation performance of HOSMO under bumped road input.

RMS Error

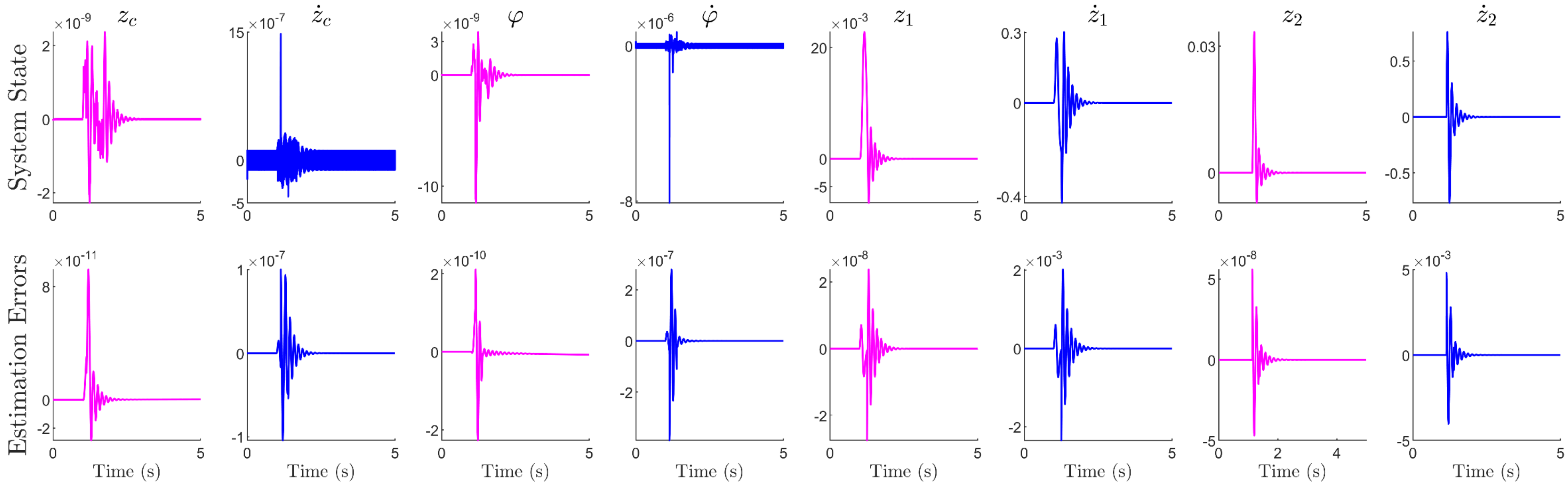
$[7.9749, 10.593] \times 10^{-11}$

$[2.2114, 2.3431] \times 10^{-6}$

$[1.87, 1.90] \times 10^{-2}$

UKF State Estimation Results

Figure 8: State estimation precision of UKF under bumped road input.



RMS Error	1.0823×10^{-11}	1.4967×10^{-8}	2.8875×10^{-11}	3.8779×10^{-8}	3.2234×10^{-9}	2.7451×10^{-4}	5.9705×10^{-9}	5.1278×10^{-4}
HOSMO	7.9749×10^{-11}	2.2114×10^{-6}	10.593×10^{-11}	2.3431×10^{-6}				

Comparative Simulation Results – Noisy Measurement

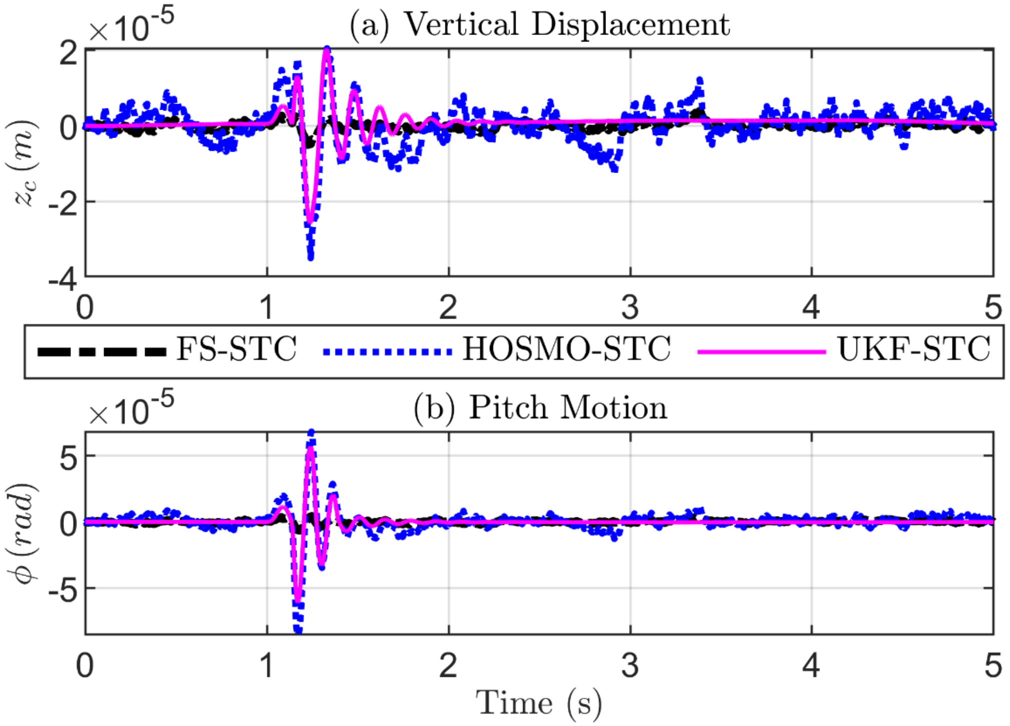


Figure 9: Stabilization performance of all control candidates under bumped road input with noisy measurement.

- Control precision degrades due to noise
- UKF-STC is still comparable to FS-STC and HOSMO-STC in terms of convergence time, accuracy.
- UKF-STC trajectory is more stable.

Controller Comparison

	Advantage	Disadvantage
FS-STC	• Straight-forward control design	• Requires direct displacement & velocity me asurements of both vertical & pitch motion • Large control gains associated with *sign* function → prone to chattering
HOSMO-STC	• Velocity state of vertical & pitch motion is estimated → less dependent on direct state-feedback information • Lumped disturbance is estimated and used in control design	• Still requires direct displacement measurement of both vertical & pitch motion • Large observer gains associated with *sign* function → prone to chattering*
UKF-STC	• Uses only easily accessible relative displacement (suspension stroke) • All system states can be estimated → internal dynamics & disturbance compensation → small control gains → less chattering	• Control design is indirect by state estimates → performance strongly depends on UKF accuracy → careful parameter tuning

Conclusion & Future Work

Summary:

- Objective: Vertical & pitch stabilization of a half-car suspension system & ensuring road holding and driving comfort
- Methodology: Super twisting sliding mode control (main controller), implementation of HOSMO (observer) or UKF (filter) to be less dependent on state feedback
- Result: UKF-STC demonstrates superior performance over FS-STC and HOSMO-STC

Limitations suggesting future work:

- Lack of intensive assessment on influence of noise and parameter uncertainty on control performance
- Missing tuning guideline for process & measurement noise covariance of UKF
- Exclusion of actuator dynamics in control design

경청해주셔서 감사합니다.

Appendix – First-principle Modeling

Unsprung mass subsystem:

(modeled as spring-mass damper system)

$$\begin{cases} m_f \ddot{z}_1 - k_{s1}\Delta y_1 - b_{e1}\Delta \dot{y}_1 + k_{t1}(z_1 - z_{o1}) + b_{t1}(\dot{z}_1 - \dot{z}_{o1}) = -F_1 \\ m_r \ddot{z}_2 - k_{s2}\Delta y_2 - b_{e2}\Delta \dot{y}_2 + k_{t2}(z_2 - z_{o2}) + b_{t2}(\dot{z}_2 - \dot{z}_{o2}) = -F_2 \end{cases}$$

Relative displacement

$$\Delta y_1 = z_c + a\sin\varphi - z_1, \quad \Delta \dot{y}_1 = \dot{z}_c + a\dot{\varphi}\cos\varphi - \dot{z}_1$$

$$\Delta y_2 = z_c - b\sin\varphi - z_2, \quad \Delta \dot{y}_2 = \dot{z}_c - b\dot{\varphi}\cos\varphi - \dot{z}_2$$

Relative velocity

Sprung mass subsystem:

(including springs, shock absorbers, and force inputs)

$$\begin{cases} M\ddot{z}_c = -\phi_1(t) + F_1 + F_2 \\ I\ddot{\varphi} = -\phi_2(t) + aF_1 - bF_2 \end{cases}$$

$$\phi_1(t) = k_{s1}\Delta y_1 + k_{s2}\Delta y_2 + b_{e1}\Delta \dot{y}_1 + b_{e2}\Delta \dot{y}_2$$

$$\phi_2(t) = a(k_{s1}\Delta y_1 + b_{e1}\Delta \dot{y}_1) - b(k_{s2}\Delta y_2 + b_{e2}\Delta \dot{y}_2)$$

Appendix – HOSMO Formulation

HOSMO

$$\begin{cases} \dot{\hat{X}}_1 = \hat{X}_2 + Z_1 \\ \dot{\hat{X}}_2 = \hat{X}_3 + \hat{H} + GU + Z_2 \\ \dot{\hat{X}}_3 = Z_3 \end{cases}$$

Lumped disturbance estimate

Correction terms

$$Z_1 = K_1 |E_1|^{2/3} \mathrm{sign}(E_1)$$
$$Z_2 = K_2 |E_1|^{1/3} \mathrm{sign}(E_1)$$
$$Z_3 = K_3 \mathrm{sign}(E_1)$$

$$E_1 = X_1 - \hat{X}_1$$

First state estimation error

Observer gains K_i *(i=1,2,3)* must be appropriately selected for the convergence of the observer.

$$K_1 = \rho_1 L^{1/3}$$
$$K_2 = \rho_2 L^{1/2}$$
$$K_3 = \rho_3 L,$$

$\rho_i (i = 1,2,3)$: tuning parameters

L: Lipschitz constant

must be sufficiently large for high precision!

Appendix – Standard UKF Formulation*

Initialization:

$$\hat{X}_0 = \mathrm{E}[X_0], \quad P_0 = \mathrm{E}[(X_0 - \hat{X}_0)(X_0 - \hat{X}_0)^{\mathrm{T}}]$$

Scaling factors:

$$\eta = \sqrt{N + \lambda}$$
$$\lambda = N(\alpha^2 - 1)$$

Sigma points calculation:

$$X_{k-1} = [\ \hat{X}_{k-1} \quad \hat{X}_{k-1} + \eta\sqrt{P_{k-1}} \quad \hat{X}_{k-1} - \eta\sqrt{P_{k-1}}\ ]$$

weights:

$$\omega_0^{(m)} = \lambda/(N+\lambda), \quad \omega_0^{(c)} = \lambda/(N+\lambda) + (1 - \alpha^2 + \beta)$$
$$\omega_i^{(m)} = \omega_i^{(c)} = 1/\{2(N+\lambda)\}, \quad i = 1, 2, ..., 2N$$

Time update:

$$X_{k|k-1} = F(X_{k-1}, U_{k-1})$$
$$\hat{X}_k^- = \sum_{i=0}^{2N} \omega_i^{(m)} X_{i,k|k-1}$$
$$P_k^- = \sum_{i=0}^{2N} \omega_i^{(c)} [X_{i,k|k-1} - \hat{X}_k^-][X_{i,k|k-1} - \hat{X}_k^-]^T + Q$$
$$Y_{k|k-1} = J(X_{k|k-1})$$
$$\hat{Y}_k^- = \sum_{i=0}^{2N} \omega_i^{(m)} Y_{i,k|k-1}$$

Measurement update:

$$P_{\tilde{Y}_k \tilde{Y}_k} = \sum_{i=0}^{2N} \omega_i^{(c)} [Y_{i,k|k-1} - \hat{Y}_k^-][Y_{i,k|k-1} - \hat{Y}_k^-]^{\mathrm{T}} + R$$
$$P_{X_k Y_k} = \sum_{i=0}^{2N} \omega_i^{(c)} [X_{i,k|k-1} - \hat{X}_k^-][Y_{i,k|k-1} - \hat{Y}_k^-]^{\mathrm{T}}$$
$$\mathrm{K}_k = P_{X_k Y_k} P_{\tilde{Y}_k \tilde{Y}_k}^{-1}$$
$$\hat{X}_k = \hat{X}_k^- + \mathrm{K}_k (Y_k - \hat{Y}_k^-)$$
$$P_k = P_k^- - \mathrm{K}_k P_{\tilde{Y}_k \tilde{Y}_k} \mathrm{K}_k^{\mathrm{T}}$$

(*) R. Van Der Merwe and E. A. Wan, “The square-root unscented Kalman filter for state and parameter-estimation,” in 2001 IEEE international conference on acoustics, speech, and signal processing. Proceedings (Cat. No. 01CH37221), vol. 6. IEEE, 2001, pp. 3461–3464.

Appendix – UKF Design Procedure in Simulink

Step 1	UKF	Standard first order discrete time
Step 2	State defining	8 states {displacement, velocity} of vertical, pitch, front & rear unsprung mass
Step 3	State transition function	Discretize state transition function from time k to (k+1)
Step 4	Measurement function	Front & rear relative displacement (suspension strokes) only!
Step 5	Process & measurement noise covariance tuning	Repeat until satisfying performance
Step 6	Initialization, sigma point calculation, time update, measurement update	

Appendix – Controller Formulation

	Sliding surface	Control law
FS-STC	$S = C_1\boxed{X_1}+\boxed{X_2}$	large, large $U = G^{-1}\left(-C_1\boxed{X_2}-\boxed{H}-\lambda_1\lvert S\rvert^{1/2}sign\left(S\right)-\int_0^t \lambda_2 sign\left(S\right)d\tau\right)$ Direct state feedback
HOSMO-STC	$\hat{S} = C_1\boxed{X_1}+\boxed{\hat{X}_2}$	Cause of chattering $Z_2 = K_2\lvert E_1\rvert^{1/3}\mathrm{sign}\left(E_1\right)$ $U = G^{-1}\left(-C_1\boxed{\hat{X}_2}-\boxed{\hat{X}_3}-\boxed{\hat{H}}+\boxed{Z_2}-\lambda_1\left\lvert\hat{S}\right\rvert^{1/2}sign(\hat{S})+\int_0^t \lambda_2 sign(\hat{S})d\tau\right)$ State estimate, Disturbance estimate, small, small
UKF-STC	$\hat{S} = C_1\boxed{\hat{X}_1}+\boxed{\hat{X}_2}$	small, small $U = G^{-1}(-C_1\boxed{\hat{X}_2}-\boxed{\hat{H}}-\boxed{\hat{D}}-\lambda_1\lvert S\rvert^{\frac{1}{2}}sign\left(S\right)-\int_0^t \lambda_2 sign\left(S\right)d\tau)$ State estimate, Disturbance estimate