INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

23 April 2018 (am)

Subject CT5 – Contingencies
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 13 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

> *In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.*

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1   Calculate $_{2.75}q_{84.5}$ using the method of Constant Force of Mortality.

Basis: AM92   [3]

2   Describe three types of reversionary bonus that may be awarded to a with-profits contract.   [4]

3   A life aged 50 exact purchases a single-premium temporary annuity. The annuity pays 7,500 annually in arrears for a term of 10 years, ceasing on death, if earlier.

Calculate the reserve for the annuity at the end of the first policy year, using the net retrospective method and the following basis:

Mortality   AM92 Select
Interest   4% per annum
Expenses   Ignore

                                                                                                [4]

4   Describe the impact of occupation on mortality and morbidity.   [5]

5   On 1 January 2010, a life insurance company issued single life annuities to policyholders then aged 65 exact.

Each annuity is for 30,000 payable annually in arrears.

At the beginning of 2017, there were 5650 policyholders alive and during 2017, 80 policyholders died.

The company calculates its reserves using the following basis:

Mortality   PMA92C20
Interest   4% per annum
Expenses   Ignore

(i)   Calculate the mortality profit or loss for these annuities for the year 2017.   [4]

(ii)   Comment on your answer in part (i).   [2]
                                                                                   [Total 6]

6 (i) Give expressions, using random variables, for the expected present value of:

   (a) an annual temporary annuity-due.

   (b) a joint whole-life insurance payable immediately on first death.

   Define any symbols that you use.

   [4]

   A life insurance company issues continuous reversionary annuities based on two lives, x and y. Payments begin on the death of the first life, x, provided the second life, y, is alive, and cease on the death of the second life, y.

   (ii) (a) State, using random variables, the present value of this annuity.

   (b) Give an expression for the expected present value of this annuity in terms of life table functions.

   [4]
   [Total 8]

7 An employer wishes to introduce a lump sum retirement benefit scheme.

   Employees will receive a lump sum benefit of 1,500 for each year of service, payable immediately on retirement at age 65 or earlier in normal health. In the event of ill-health retirement the benefit is 1,000 for each year of service payable immediately. Service includes proportionate parts of a year.

   (i) Give a formula in terms of commutation functions to value the total benefits (past and future service) for an employee currently aged x with n years of past service. Definition of terms is not required. [3]

   (ii) Calculate the expected present value of the benefits for an employee currently aged 45 exact with exactly 15 years' past service. [3]

   (iii) Calculate the level annual contribution payable continuously throughout this employee's future service to fund the benefits. [2]

   Basis: Pension Scheme from the Formulae and Tables for Actuarial Examinations

   [Total 8]

8 A term assurance policy is issued on a life aged x for a term of 20 years.

Under this policy a sum assured, payable immediately on death, is 10,000 for the first 10 years increasing to 20,000 for the subsequent 10 years.

(i) Calculate the expected present value of the benefits. [3]

(ii) Determine the variance of the present value of the benefits. [5]

Basis:

Mortality $\mu_x = 0.03$ for all x
Force of Interest $\partial = 5\%$ throughout

[Total 8]

9 A life insurance company issues 20-year critical illness term assurance policies. The benefits, payable during the policy term, are a lump sum of 50,000 payable immediately on diagnosis of a critical illness, or a benefit of 100,000 immediately on death if earlier.

Premiums are payable monthly in advance during the term of the policy, ceasing on any claim.

(i) Draw a transition state model for this policy labelling your diagram. [3]

A policy is issued to a life aged 45 exact, with an annual premium rate of 750.

(ii) Calculate the value of the policy to the company. [6]

Basis:

Mortality $m_x = 0.004$ for all x
Critical Illness $s_x = 0.002$ for all x
Force of Interest $\partial = 4\%$ throughout

[Total 9]

10 A life insurance company sells a special deferred annuity policy to a life aged 60.75 years exact (i.e. 60 years and 9 months). The policy is funded by quarterly premiums in advance ceasing at age 65 (no premium is payable at age 65 exact).

The benefits under the policy are as follows:

*   At age 65, an annuity of 12,000 a year is payable half-yearly in advance. Payment is guaranteed for five years and then continues to the age of 75 when it reduces to 10,000 a year under the same payment method until the death of the policyholder.

*   At age 75, an additional lump sum benefit of 10,000 is payable. The annuity then reduces to 10,000 a year, ceasing on the death of the policyholder.

Determine that the quarterly premium is approximately 9,030.

Basis:

Mortality	PFA92C20 (assume the uniform distribution of deaths method)
Rate of Interest	4% per annum
Expenses	Ignore

[9]

11 A life insurance company issues 30-year term assurance policies to lives aged 35 exact. For each policy, the initial sum assured is 90,000 which increases by 45,000 at the start of the 16th policy year. The sum assured is payable immediately on death. Level monthly premiums are payable in advance throughout the term of these policies or until earlier death.

The company uses the following basis for calculating premiums and reserves:

Mortality	AM92 Select
Interest	4% per annum
Initial commission	50% of the total premium payable in the first policy year
Initial expenses	375 at policy commencement date
Renewal commission	2.5% of each premium from the start of the second policy year
Renewal expenses	72 per annum, inflating at 4% per annum compound, at the start of the second and subsequent policy years (the renewal expense quoted is at outset and the increases due to inflation start immediately)
Claim expense	295 on death

Calculate the monthly premium for the policy. [10]

12 On 1 January 2013, a life insurance company issued a with-profits whole-life policy to a life then aged 25 exact. Under the policy, the basic sum assured of 150,000 and attaching bonuses are payable immediately on death. The company declares simple reversionary bonuses at the start of each year (i.e. the death benefit includes the bonus relating to the policy year of death). Level premiums are payable annually in advance under the policy.

(i) Give an expression for the gross future loss random variable under the policy at the outset using the basis below. [3]

(ii) Calculate the gross annual premium, using the equivalence principle and the same basis.

Basis:

Mortality		AM92 Select
Interest		6% per annum
Bonus loading		4% per annum simple
Expenses	Initial	265
	Renewal	5% of each premium payable in the second and subsequent years
	Claim	315

[4]

On 31 December 2017 the policy is still in force. The actual bonuses added to the policy have been as follows:

Year	Simple bonus per annum
2013	4.0%
2014	4.0%
2015	3.75%
2016	3.5%
2017	3.0%

(iii) Determine the gross premium prospective reserve for the policy as at 31 December 2017 using the following basis:

Mortality	AM92 Ultimate
Interest	4% per annum
Bonus loading	3% per annum simple
Renewal expenses	5% of each premium
Claim expenses	325

[4]
[Total 11]

13 A life insurance company issues a three-year unit-linked endowment assurance contract to a male life aged 62 exact under which level annual premiums of 6,000 are payable in advance throughout the term of the policy or until earlier death. 90% of each year's premium is invested in units at the offer price.

There is a bid-offer spread in unit values, with the bid price being 95% of the offer price.

There is an annual management charge of 1% of the bid value of units. Management charges are deducted at the end of each year, before death or maturity benefits are paid.

On the death of the policyholder during the term of the policy, the benefit, payable at the end of the year of death, is 12,000, or the bid value of the units if greater. The policyholder may surrender the policy only at the end of each year immediately before a premium is payable. On surrender, the bid value of the units is payable at the end of the year of exit. On maturity, 110% of the bid value of the units is payable.

The company holds unit reserves equal to the full bid value of the units. It sets up non-unit reserves to zeroise any negative non-unit fund cash flows, other than those occurring in the first year.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	5% per annum
Rate of interest on non-unit fund cash flows	3% per annum
Mortality	AM92 Ultimate
Surrenders	10% at the end of the first policy year, 5% at the end of the second policy year based on policies in force at that time
Initial expenses	225 plus 5% of the first premium (all incurred on policy commencement)
Renewal expenses	65 at the start of each of the second and third policy years plus 2.5% of the second and third premiums
Risk discount rate	7% per annum

Calculate the profit margin on the contract. [15]

END OF PAPER