KIMATHI UNIVERSITY COLLEGE OF TECHNOLOGY

UNIVERSITY EXAMINATIONS 2012/2013 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SMA 2100: DISCRETE MATHEMATICS

15th AUGUST, 2012 DURATION: 2 HOURS

Instructions To Candidates

1. Answer Question One and any other Two Questions.
2. Show clearly the number of questions attempted.

Question One (30 Marks)

a) Define the following terms:
   i) universal set
   ii) function
   iii) real number (3 Marks)

b) Let P and Q be the sets:

$$P=\{x: x=4k, k\in\mathbb{N}, 0<k<5\} \text{ and } Q=\{1,4,9,16,\ldots\}$$

Determine
   i) $P\cap Q$
   ii) $P-Q$
   iii) Q in set–builder form (6 Marks)

c) (i) List the element of each of the following sets:

$A=\{x: x^2=16 \text{ and } 2x=6\}$

$B=\{x: x\neq x\}$

$B=\{x: x+5=5\}$ (3 Marks)

(ii) Determine which of the sets $\varnothing$, $\{0\}$, $\{\varnothing\}$ are equal. (1 Mark)

d) Given that A and B are sets such that $n(U)=22$, $n(A^c)=12$, $n(B)=14$, $n(A^c\cap B^c)=5$. Find $n(A\cap B^c)$ (3 Marks)

e) (i) Define a rational number

(ii) Prove that $\sqrt{2}+a$ is an irrational number where $a \in \mathbb{Q}$. (4 Marks)

f) (i) Let $f:\mathbb{Z}\to\mathbb{Q}$ and $g:\mathbb{Z}\to\mathbb{Q}$

where $f(x)=\frac{1}{2x+1}$ and $g(x)=\frac{3}{x-2}$

Find which of the relations are functions. (2 Marks)

(ii) The functions $f(x)=x^2+2$ and $g(x)=x+2$ are defined from $\mathbb{R}$ to $\mathbb{R}$.

Find $g\circ f$, $(g\circ f)(-2)$, and $f^{-1}(x)$ (3 Marks)

g) (i) Define the following terms:

Proposition

Predicate (2 Marks)

(ii) Let p = " misers like money" and

q = "my cousins are not greedy".

Write the following in ordinary language as simply as possible:

$p \wedge q$, $p \to \sim q$, $p \wedge \sim q$

(3 Marks)

Question Two (20 Marks)

a) Use set algebra to prove that

$A \Delta (A \cap B) = A - B$ (6 Marks)

b) Use the set notation to prove that

$(A \cup B)^c = A^c \cap B^c$ (7 Marks)

c) A college soccer team is expected to: have clean boots , practice, and arrive on time. During a certain week 7 players had clean boots, 9 practiced and 9 arrived on time. Six had clean boots and practiced, while 6 had clean boots and arrived on time. Seven practiced and arrived on time. Two players did not obey any of the rules. Use a Venn diagram to determine how many obeyed all the three rules. (7 Marks)

Question Three (20 Marks)

a) Let A = B = { 1 ,2, 3, 4 }and $f_1, f_2, \ldots f_4$ be relations from A to B

If f_1 = { (1, 2), (2, 3), (3, 4) }

f_2 = { (1, 4), (2, 1), (3, 2), (4, 4) }

f_3 = {(1, 2), (2, 3), (3, 4), (4, 1)}

f_4 = { (1 ,4), (2, 3), (3, 2), (4, 1) }

Find which of the relations are:

(i) functions (ii) injections

(iii) surjections and (iv) bijections (8 Marks)

b) Let the functions $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be defined by

$f(x) = \sqrt{(x-2)}$ and $g(x) = x^2 + 4$.

Find (i) fog (ii) gof (4 Marks)

c) Given $f:\mathbb{R}\to\mathbb{R}$ and f(x) = $\frac{x}{2}+3$, find g(x) and h(x) such that (hog)(x) = f(x). Use these results to determine the inverse of f(x). (4 Marks)

d) Define $f:\mathbb{N}\to\mathbb{Z}$, $g:\mathbb{Z}\to\mathbb{Z}$ by f(x) = 2x – 4 and g (x) = x^2

(i) Show that f is injective but not surjective.

(ii) Show that g is not injective.

(4 Marks)

Question Four (20 Marks)

a) State the Axiom of mathematical induction (3 Marks)

b) Proof by mathematical induction that:

(i) 3n + 2n -1 is divided by 4

(ii) (1 x 2) + (2 x 3) + (3 x 4) + **…**+ n(n + 1) = $\frac{n}{3}$ (n+1)(n + 2) (14 Marks)

c) Prove that;

If a, b, m and n are integers and if c | a and c | b then c | (ma + n b). (3 Marks)

Question Five (20 Marks)

a) Consider the statement " If this car is made in Kenya then it is a good car"

Write down a linguistic statement which is

(i) the negation

(ii) the converse

(iii) the contra positive of the statement (3 Marks)

b) Show that the propositions

~ (p Λ q) and (~ p Λ ~ q) are logically equivalent.

Write down the equivalent statement in the laws of sets

(6 Marks)

c) Construct the truth table for the proposition

(p→ q) OR (p→ ~q)

Hence determine if the proposition is a contradiction, a tautology or logically indeterminate

(6Marks)

d) Prove that $A\subset B \Rightarrow A\cap B = A$ (5 Marks)