① Unique subset of 5 letters = 1 = {u, n, s, a, l}
No duplicates in set.

\# dif strings using 5 of 7 letters, (must have 5 of seven)

case 1 for 1 u its $5!$
case 2 for 2 u's its $5!/2!$
case 3 for 3 u's its $5!/3!$

$\underbrace{\qquad}_{\text{arranging linear orders of 5 with/without duplicates}}$ = ~~120~~

each case must select non-0 terms

so case 1 = $5! = 120$
case 2 = $\frac{5!}{2!} \cdot \binom{4}{3} = \frac{5!}{2} \cdot 4 \cdot 2 = 240$
case 3 = $\frac{5!}{3!} \cdot \binom{4}{2} = \frac{5!}{6} \cdot 6 = 120$

$120 + 240 + 120 = \boxed{480}$ unique strings

② Select pair values $\binom{13}{2}$ Select suits $\binom{4}{2}\binom{4}{2}$ Pair 1 Pair 2 Select last card $\binom{44}{1}$

$= \boxed{123552 \text{ ways}}$

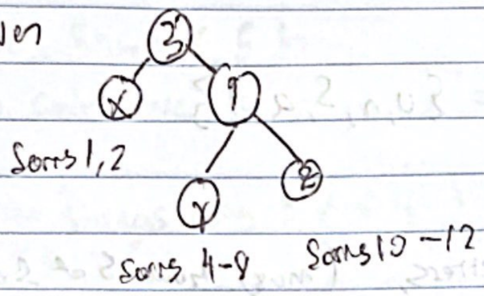
③ Stars and bars w/ 1 less candle and 1 less song. (so 15 songs among 6 candles)

$\binom{15+6-1}{15} = \binom{20}{5}$

Then select last song amongst all candles $\binom{7}{1}$

$\binom{20}{5}\binom{7}{1} = \boxed{108528 \text{ ways}}$

(4) given



• x = cases or $= \boxed{2}$

• y sorts 5 nodes (catalan #5) (math 432)

$$\frac{(2(\#Nodes))!}{((\#Nodes)+1)!\,(\#Nodes)!} = \frac{10!}{5!\,6!} = \frac{7\cdot 8\cdot 9\cdot 10}{2\cdot 3\cdot 4\cdot 5} = \boxed{42}$$

• z sorts 3 nodes $= \frac{6!}{4!\,3!} = \frac{5\cdot 6}{6} = \boxed{5}$

we get $42\cdot 2\cdot 5 = \boxed{420 \text{ trees}}$

(5) $\underset{9}{S(10,4)} + \underset{8}{S(10,3)} = \boxed{17}$

where $S(x,y)$ = set partitions of x into y into non empty subsets.
(math 432)