(As per New Syllabus of NCERT/CBSE)

Revised Edition

S. Chand's

# Principles of Physics

## FOR CLASS XII

With Value Based Questions

Also useful for Engineering & Medical Ent. Exams

S. CHAND School


V.K. MEHTA
ROHIT MEHTA

*As per the New CBSE Course Structure and New NCERT Guidelines.*

*S. Chand's*

# Principles of

# Physics

**For**

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**[with Value Based Questions]**

**[Senior Secondary Certificate Examinations of CBSE, Other State Boards of School Education and Various Engineering/Medical Entrance Examinations]**

**V.K. MEHTA**
**ROHIT MEHTA**

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*First Edition 1998*
*Subsequent Editions and Reprints 2000, 2001, 2002, 2003, 2004, 2006, 2007, 2008, 2009, 2011*
***Seventh Edition 2013***

**ISBN :** 81-219-1769-7 **Code :** 16C 235

PRINTED IN INDIA

*By Rajendra Ravindra Printers Pvt. Ltd., 7361, Ram Nagar, New Delhi -110 055*
*and published by S. Chand & Company Pvt. Ltd., 7361, Ram Nagar, New Delhi -110 055.*

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## UNIT-1
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negative charge on $C$ repels the electrons to $B$, leaving an equal positive charge at $A$ as shown in Fig. 1.23 (*i*).

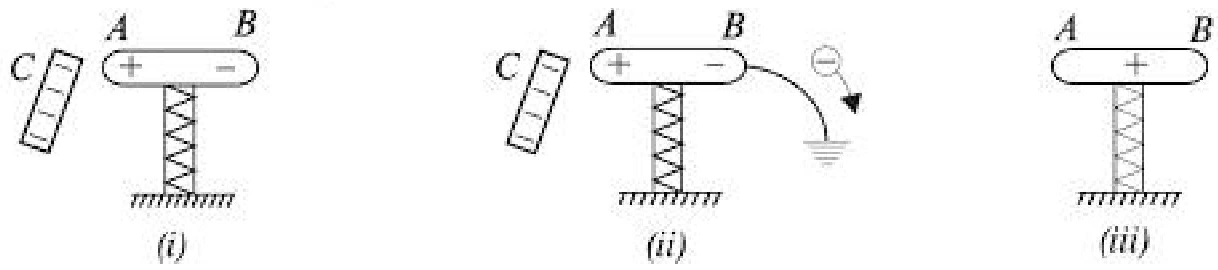



**Fig. 1.23**

(*ii*) Now touch the conductor at $B$. The electrons go to earth as shown in Fig. 1.23 (*ii*).

(*iii*) If we now remove the conductor $C$, a permanent positive charge is left on the conductor $AB$ as shown in Fig. 1.23 (*iii*).

**Q.7. The leaves of an electroscope always diverge when we bring a charged body near it, without touching it. Why?**

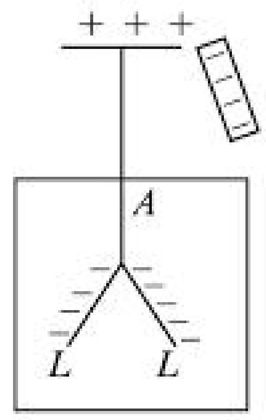



**Fig. 1.24**

**Ans.** A gold-leaf electroscope essentially consists of a metal rod $A$ to which gold leaves $L$ are attached as shown in Fig. 1.24. If, for example, we bring a negatively charged rod near the cap, it induces a positive charge on the cap and a negative charge on the leaves. Since the leaves are similarly charged, they repel each other.

**Q.8. A gold-leaf electroscope is charged negatively. If an unknown charge $X$ is brought near its cap, the divergence of the leaves increases. Is the unknown charge positive or negative?**

**Ans.** The unknown charge is negative. Fig. 1.25 (*i*) shows negatively charge gold-leaf electroscope. When the unknown charge is brought near the electroscope, the divergence of leaves increases. This is possible only if negative charge is induced on the leaves. This in turn means that unknown charge is negative.

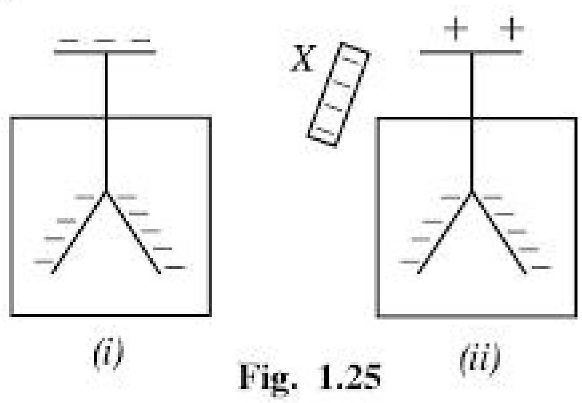



**Fig. 1.25**

**Q.9. Can we charge a body to have a charge of $15e/2$ ?**

**Ans.** No. Any charged body can have a charge equal to the integral multiple of $\pm e$, *i.e.*, $\pm e$, $\pm 2e$, $\pm 3e$.........

**Q.10. Any conducting object connected to earth is said to be grounded. Explain.**

**Ans.** Because earth is a very big source of electrons or sink, its potential remains constant whether electrons are given to it or removed from it. For this reason, electric potential of earth is assumed zero.

**Q.11. You cannot disturb the electrical neutrality of ordinary matter very much. Explain.**

**Ans.** Under ordinary conditions a body is electrically neutral, *i.e.*, it has the same amounts of positive charge and negative charge. When a body is charged (positively or negatively), this electrical neutrality is disturbed. Suppose you are charging a body positively by removing electrons from it. The positive charge on the body tends to pull the negative charge (*i.e.*, electrons) back. As the body gets more and more positive charge, the electrostatic force tending to pull the negative charge back also increases. Therefore, we cannot place a large charge (positive or negative) on a body.

## SHORT ANSWER QUESTIONS

**Q.1. When you run comb through dry hair, it attracts bits of paper. Why? What would you expect if the hair is wet or if it is a rainy day?**

**Ans.** When we run comb through hair, the comb gets charged due to friction and as a result it attracts bits of paper. If the hair is wet, friction between comb and hair is greatly reduced. Consequently, the comb is charged to lesser extent and it may not attract the bits of the paper. The same is true if it is a rainy day. It is because due to higher humidity, the hair do not remain perfectly dry.

**Q.2. Vehicles carrying inflammable materials usually have long chains that hang down and drag on the ground. Why?**

**Ans.** When a vehicle is in motion, its tyres rub against the road and get charged due to friction. Further, due to friction of air, the body of the vehicle also gets charged. If the accumulated charge becomes excessive, sparking may occur and the inflammable material may catch fire. Since the chain ropes are touching the ground, the charge leaks to the earth. Hence, the danger of fire is avoided.

**Q.3. Two point charges of + 2μC and + 6 μC repel each other with a force of 12N. If each is given an additional charge of –4μC, what will be the new force?**

**Ans.** $F = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q_1 q_2}{r^2}$. For same $r$ and $K$, $F \propto q_1 q_2$.

$$\therefore \quad \frac{F'}{F} = \frac{q'_1 q'_2}{q_1 q_2} = \frac{(+2-4)(+6-4)}{(2)(6)} = \frac{-4}{12}$$

$$\therefore \quad F' = \frac{-4}{12}F = \frac{-4}{12}\times 12 = -4\text{N} \quad \text{(attractive)}$$

**Q.4. Three charges each of + 1μC are placed at the corners of an equilateral triangle. If the force between any two charges be *F*, what will be the net force on each charge?**

**Ans.** On any charge, two equal forces each of $F$ act ; the angle between them being 60°.

$$\therefore \quad R = \sqrt{F^2 + F^2 + 2FF\cos 60^\circ} = \sqrt{F^2+F^2+F^2} = \sqrt{3}\,F$$

**Q.5. Is coulomb a big or small unit of charge?**

**Ans.** Coulomb is a very large unit of charge available for ordinary engineering computations.

$$F = 9\times 10^9 \frac{q_1 q_2}{r^2}$$

If $q_1 = q_2 = 1\text{C}$ and $r = 1\text{m}$, then $F = 9 \times 10^9$ N.

The reader may note the huge magnitude of force between two charges of 1C each.

**Q.6. Dielectric constant of water is 80. What is its permittivity?**

**Ans.** Since relative permittivity of a medium is also called dielectric constant, the relative permittivity of water is 80.

$\therefore$ Permittivity of water, $\varepsilon = \varepsilon_0 K = 8.854 \times 10^{-12} \times 80 = 708.32 \times 10^{-12}\ \text{C}^2\,\text{N}^{-1}\,\text{m}^{-2}$

**Q.7. Force of attraction between two point charges at a distance *d* is *F*. What distance apart should they be kept in the same medium so that force between them is *F*/3?**

**Ans.** $F = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q_1 q_2}{d^2}$. As per conditions of the problem, $F \propto \dfrac{1}{d^2}$

Now $\quad \dfrac{F'}{F} = \dfrac{d^2}{d'^2}$ or $\dfrac{F/3}{F} = \dfrac{d^2}{d'^2}$ or $d'^2 = 3\,d^2 \quad \therefore \quad d' = \sqrt{3}\,d$

**Q.8. Two point charges placed at a distance *r* in air exert a force *F* on each other. Find the distance *r*′ at which these charges will exert the same force in a medium of dielectric constant *K*.**

**Ans.** Force in air, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2}$ ; Force in medium, $F' = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q_1 q_2}{(r')^2}$

Now, $\quad \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2} = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q_1 q_2}{(r')^2}$ or $\dfrac{1}{r^2} = \dfrac{1}{K(r')^2} \quad \therefore \quad r' = \dfrac{r}{\sqrt{K}}$

## MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. The dielectric constant of metals is **[PMT MP 1998]**
   (*a*) 1 (*b*) greater than 1 but finite
   (*c*) zero (*d*) infinite

2. Three charges each equal to $q$ are placed at the corners of an equilateral triangle. If force between any two charges is $F$, then net force on the triangle will be **[AFMC 2000]**
   (*a*) $3F$ (*b*) $2F$
   (*c*) zero (*d*) $\sqrt{2}\,F$

3. Two charged spheres separated by a distance $d$ exert force $F$ on each other. If they are immersed in a liquid of dielectric constant 2, then what is the force exerted if all other conditions are same ? **[AIIMS 1997]**
   (*a*) $F/2$ (*b*) $F$
   (*c*) $2F$ (*d*) $4F$

4. Four charges are arranged at the corners of a square $ABCD$ as shown in Fig. 1.26. The force on a charge kept at the centre $O$ is **[BHU PMT 1999]**

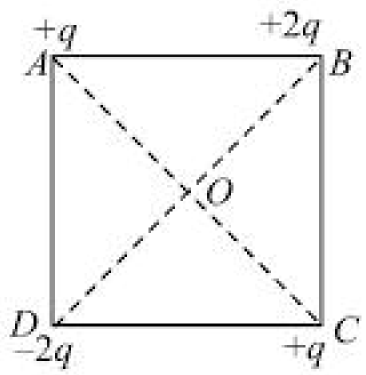



**Fig. 1.26**

   (*a*) zero (*b*) along diagonal $BD$
   (*c*) along diagonal $AC$
   (*d*) perpendicular to side $AB$

5. When air is replaced by a medium of dielectric constant $K$, the force of attraction between two charges separated by a distance $r$ **[BHU PMT 1998]**
   (*a*) decreases $K$ times (*b*) remains unchanged
   (*c*) increases $K$ times (*d*) increases $K^{-2}$ times

6. If distance between two charged particles is doubled, then the force will **[BHU PMT 1997]**
   (*a*) be doubled (*b*) be halved
   (*c*) be one-fourth (*d*) remain the same

7. A charge $q$ is placed at the centre of the line joining two exactly equal positive charges $Q$. The system of three charges will be in equilibrium if $q$ is equal to **[CBSE PMT 1995]**
   (*a*) $-Q/4$ (*b*) $+Q$
   (*c*) $-Q$ (*d*) $Q/2$

8. Point charges $+4q$, $-q$ and $+4q$ are kept on the $x$-axis at $x = 0$, $x = a$ and $x = 2a$ respectively. **[CBSE PMT 1992]**
   (*a*) all the charges are in stable equilibrium.
   (*b*) only $-q$ is in stable equilibrium.
   (*c*) none of the charges is in equilibrium.
   (*d*) all the charges are unstable.

9. The study of the effects associated with electric charges at rest is called **[AIIMS 1997]**
   (*a*) electromagnetism (*b*) electrostatics
   (*c*) magnetostatics (*d*) none of these

10. The number of electrons in one coulomb of charge will be **[MP PMT 1998]**
   (*a*) $5.46 \times 10^{29}$ (*b*) $6.25 \times 10^{18}$
   (*c*) $1.6 \times 10^{19}$ (*d*) $9 \times 10^{11}$

11. Two spheres $A$ and $B$ of exactly same mass are given equal positive and negative charges respectively. Their masses after charging **[Rajasthan PMT 1996]**
   (*a*) remain unaffected
   (*b*) mass of $A >$ mass of $B$
   (*c*) mass of $A <$ mass of $B$
   (*d*) nothing can be said

12. Two point charges of 10μC and –5μC are separated in air by 1 m. The ratio of force exerted by one on the other is **[MP PMT 1995]**
   (*a*) 1 : 2 (*b*) 2 : 1
   (*c*) 1 : 1 (*d*) none of the above

13. Two charges are placed a certain distance apart. A metal sheet is placed between them. What will happen to the force between the charges ? **[CBSE PMT 1992]**
   (*a*) will decrease (*b*) will increase
   (*c*) will remain unchanged
   (*d*) either (*a*) or (*b*)

14. Two point charges of + 2μC and + 6μC repel each other with a force of 12 N. If each is given an additional charge of –4μC, then new force will be **[Rajasthan PMT 1998]**
   (*a*) + 4 N (*b*) 3 N
   (*c*) 48 N (*d*) – 4 N

15. Three charges each of +1μC are placed at the corners of an equilateral triangle. If the force between any two charges be $F$, then the net force on each charge is **[MP PMT 1993]**
   (*a*) $\sqrt{2}\,F$ (*b*) $3\,F$
   (*c*) $\sqrt{3}\,F$ (*d*) $2\,F$

**Ans.** $Vm^{-1} = \dfrac{J}{C}m^{-1} = \dfrac{Nm}{C}m^{-1} = NC^{-1}$

**Q.25. What is the dimensional formula of potential gradient?**

**Ans.** Potential gradient $= \dfrac{V}{r} = \dfrac{ML^2T^{-3}A^{-1}}{L} = [MLT^{-3}A^{-1}]$

**Q.26. Give two examples of conservative forces.**

**Ans.** (*i*) Gravitational force (*ii*) Electrostatic force.

**Q.27. The electric potential is constant in a region. What can you say about electric field there?**

**Ans.** $E = -\dfrac{dV}{dr}$. Since electric potential $V$ is constant, electric field $E$ is zero.

**Q.28. What is an equipotential surface?**

**Ans.** Any surface over which electric potential is the same at every point is called an equipotential surface.

**Q.29. What is the shape of equipotential surfaces for a point charge?**

**Ans.** For a point charge, the equipotential surfaces are concentric spheres whose centres are located at the point charge.

**Q.30. What is the shape of equipotential surfaces for a uniform electric field?**

**Ans.** For a uniform electric field, the equipotential surfaces are parallel planes at right angles to the direction of electric field.

**Q.31. How much work is done in moving a charge of 500 μC between two points on an equipotential surface?**

**Ans.** Work done = Charge × Potential difference. Since all points on an equipotential surface are at the same potential, the potential difference between any two points on such a surface is zero. Therefore, work done in moving a charge on an equipotential surface is zero.

**Q.32. No two equipotential surfaces intersect each other. Why?**

**Ans.** If two equipotential surfaces intersect, it would mean that there will be two values of electric potential at the point of intersection which is not possible.

**Q.33. What is the direction of electric field w.r.t. equipotential surface?**

**Ans.** Electric field is normal to the equipotential surface.

**Q.34. Two parallel surfaces are at the same potential. Distance between these surfaces is $r$. A charge $q$ is taken from one surface to the other. What is the work done in this case?**

**Ans.** Work done = Charge × Potential difference. Since the potential difference between the two surfaces is zero, work done is zero.

**Q.35. What is the SI unit of line integral of electric field?**

**Ans.** $\int \vec{E}.\vec{dl} = NC^{-1}m = JC^{-1}$ (or V)

Thus the line integral of electric field represents the physical quantity electric potential.

**Q.36. What is electric flux?**

**Ans.** Electric flux through a surface in an electric field represents the total number of electric field lines that pass through the surface normally.

**Q.37. Define electric flux quantitatively.**

**Ans.** The electric flux ($\phi_E$) is defined as the product of the magnitude of electric field ($E$) and the surface area ($S$) perpendicular to the electric field.

**Q.38. How will you express electric flux mathematically?**

**Ans.** Electric flux, $\phi_E = \vec{E}.\vec{S} = ES\cos\theta$

Here $\vec{E}$ is the electric field vector and $\vec{S}$ is the area vector. Also $\theta$ is the angle between $\vec{E}$ and $\vec{S}$.

**Q.39. What is the SI unit of electric flux?**

**Ans.** Electric flux = Electric field × Area $= \dfrac{\text{Force}}{\text{Charge}} \times \text{Area}$. Therefore, the SI unit of electric flux is $Nm^2C^{-1}$ or $JmC^{-1}$.

**14. A cubical box sits in a uniform electric field as shown in Fig. 3.77. Find the net flux coming out of the box.**

**Hint.** Since the field lines simply skim the four sides of the box, the flux through them is zero. The flux through the top is

$$\phi_{top} = \vec{E}.\vec{A_t} = E A_t$$

Note that in this case $\theta = 0°$ so that $\cos\theta = \cos 0° = 1$. The flux through the bottom is

$$\phi_{bottom} = \vec{E}.\vec{A_b} = E A_b \cos 180° = -E A_b$$

The negative sign arises because the flux is into the bottom area, not out of it.

Since $A_t = A_b$, the net flux from the box is

$$\phi_{total} = E A_t - E A_b = 0$$

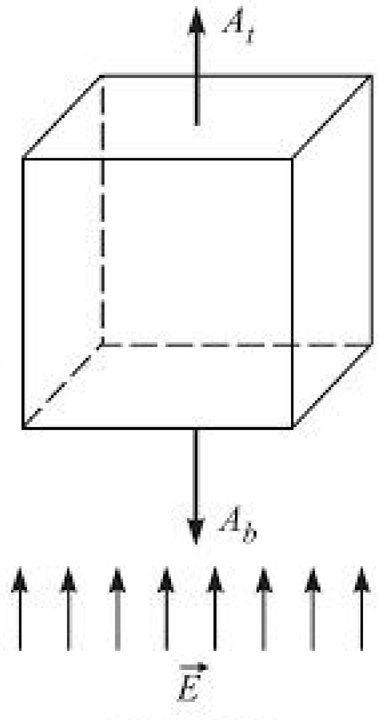


**Fig. 3.77**

**15. Fi.g 3.78 shows a Gaussian surface in the form of a cylinder of radius *R* immersed in a uniform electric field $\vec{E}$ with the cylinder axis parallel to the field. What is the net flux through this closed surface?**

**Hint.** We can write the flux as the sum of three terms : integrals over the left cap *a*, the cylindrical surface *b* and the right cap *c*.

$$\phi = \oint \vec{E}.\vec{dS} = \int_a \vec{E}.\vec{dS} + \int_b \vec{E}.\vec{dS} + \int_c \vec{E}.\vec{dS}$$

For left cap, the angle $\theta$ between $\vec{E}$ and $\vec{dS}$ is 180° for all points and magnitude $E$ of the field is constant.

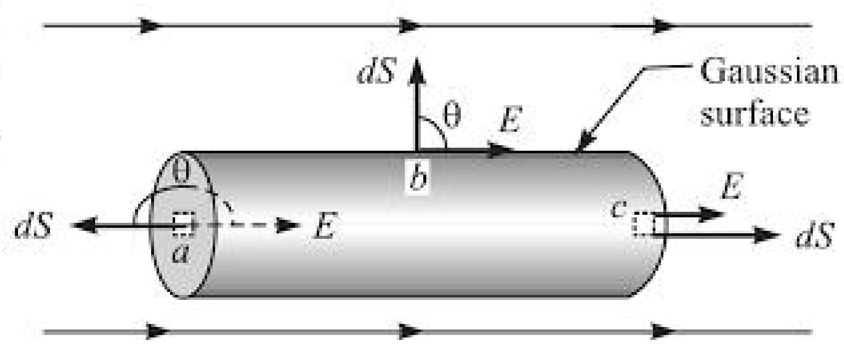


**Fig. 3.78**

$$\therefore \quad \oint_a \vec{E}.\vec{dS} = \int E\, dS \cos 180°$$

$$= -ES$$

where $\int dS = S =$ area of cap $(= \pi r^2)$

Also $\int_c \vec{E}.\vec{dS} = \int E\, dS \cos 0° = ES$

Note that for right cap, angle $\theta$ between $\vec{E}$ and $\vec{dS}$ is 0° for all points.

Finally, $\int_b \vec{E}.\vec{dS} = \int E\, dS \cos 90° = 0$

$$\therefore \quad \phi = -ES + 0 + ES = 0$$

This result is exactly what we expect from Gauss's law because no charges are located within the cylindrical surface. Notice that there is electric field within the cylinder but the net flux is zero because the inward flux and outward flux are equal (as many field lines pass into the surface as pass out of the surface).

**16. The magnitude of the average electric field normally present in the earth's atmosphere just above the surface of earth is about 150 N/C, directed downward.**

charges establish an electric field $\vec{E_i}$ within the conductor which opposes the external field $\vec{E}_{ex}$. When $\vec{E_i}$ becomes equal to $\vec{E}_{ex}$ [See Fig. 4.1 (*ii*)], further movement of free electrons stops. Since applied electric field is equal and opposite to the induced electric field, the net electric field inside the metallic conductor is zero. Under such conditions, the conductor is said to be in electrostatic equilibrium.

**2. The net charge inside a conductor is zero.** In other words, charge resides on the outer surface of the conductor. We shall prove this fact in two ways :

(*i*) Consider a metallic body hanging from an insulating thread and having charge $q$ as shown in Fig. 4.2. Imagine a closed surface (Gaussian surface) just inside the actual surface of the conductor. According to Gauss's law,

$$\oint \vec{E} \cdot \vec{dS} = q / \varepsilon_0$$

Since electric field inside a conductor is zero (*i.e.*, $\vec{E} = 0$),

$$\therefore \quad 0 = q/\varepsilon_0$$

$$\text{or} \quad q = 0$$

Thus, net charge inside a charged conductor is zero; it resides only on the outer surface of the conductor.

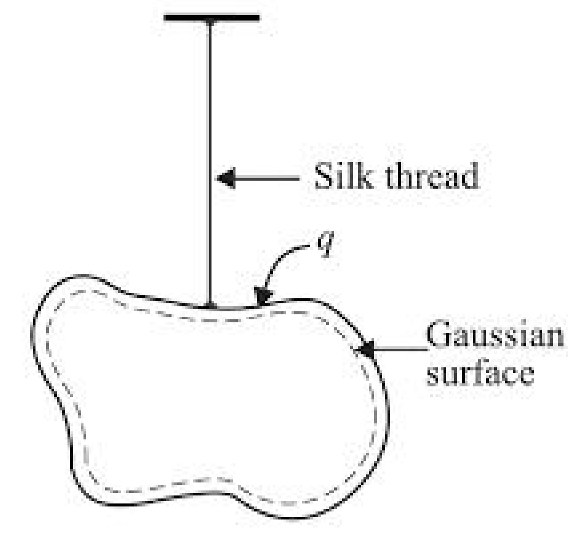



Fig. 4.2

(*ii*) When a conductor is charged (say positively), like charges repel each other. Therefore, charges try to get as far away from each other as they can. As a result, charges move to the surface of the conductor. Hence, charge always resides on the outer surface of a conductor. None of the charge will be found to be within the body of the conductor.

**3. The electric field on the surface or just outside the charged conductor is perpendicular to the surface of the conductor at every point.** When a metallic conductor is charged, it quickly attains electrostatic equilibrium. In other words, there is no movement of charges (free electrons) on the surface of the conductor. This means that component of electric field along the tangent to the surface is *zero. This in turn means that electric field is perpendicular to the surface of the conductor as shown in Fig. 4.3. This is an important point to keep in mind and is true regardless of the shape of the conductor.

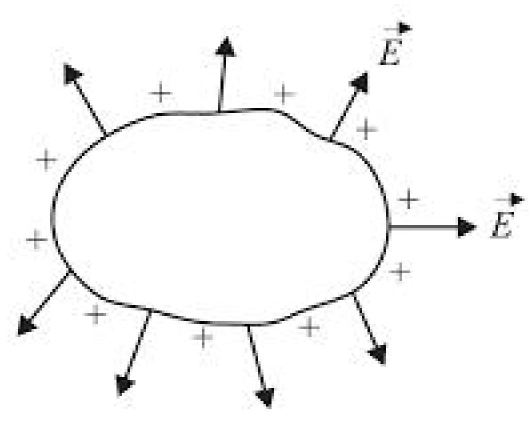



Fig. 4.3

**4. The magnitude of electric field just outside a charged conductor is $\sigma/\varepsilon_0$,** where $\sigma$ is surface charge density. This result applies to any conductor shape and suggests that we find large electric fields ($\vec{E}$) where the charge density on a conductor is high (*e.g.*, pointed portions on a conductor).

**5. The electric potential is the same (*i.e.* constant) at the surface and inside a charged conductor.** We know that inside a charged conductor, electric field is zero, *i.e.*, $\vec{E} = 0$. This in turn means that :

* If the component of electric field along the tangent to the surface were not zero, then this tangential component would exert forces on the surface charges, causing them to move. But this is not possible because the conductor is in electrostatic equilibrium.

**Solution.** Capacitance of earth, $C = 4\pi\,\varepsilon_0\, r$

Here, $4\pi\varepsilon_0 = \dfrac{1}{9\times10^9}$; $r = 6400\text{ km} = 6.4\times10^6\text{ m}$

$$\therefore \qquad C = \frac{6.4\times10^6}{9\times10^9} = 0.711\times10^{-3}\text{ F} = 711\ \mu\text{F}$$

This shows that farad is a very large unit of capacitance.

**Note.** Since capacitance of earth is quite large, we choose earth as a level of zero potential for practical purposes. Think about this !

**Example 4.2. An isolated sphere has a capacitance of 50pF. (*i*) Calculate its radius. (*ii*) How much charge should be placed on it to raise its potential to $10^4$V?**

**Solution.** (*i*) Capacitance of sphere, $C = 4\,\pi\,\varepsilon_0\, r$

$\therefore$ Radius of sphere, $r = \dfrac{1}{4\pi\varepsilon_0}\times C = (9\times10^9)\times(50\times10^{-12}) = 45\times10^{-2}\text{ m} = 45\text{ cm}$

(*ii*) Charge to be placed, $q = CV = (50\times10^{-12})\times10^4 = 5\times10^{-7}\text{ C} = 0.5\ \mu\text{C}$

**Example 4.3. Twenty seven spherical drops, each of radius 3mm and carrying $10^{-12}$C of charge are combined to form a single drop. Find the capacitance and potential of the bigger drop.**

**Solution.** Let $r$ and $R$ be the radii of smaller and bigger drops respectively.

Volume of bigger drop = 27 × Volume of smaller drop

or $$\frac{4}{3}\pi R^3 = 27\times\frac{4}{3}\pi r^3$$

or $$R = 3r = 3\times3 = 9\text{ mm} = 9\times10^{-3}\text{ m}$$

Capacitance of bigger drop, $C = 4\pi\varepsilon_0\, R = \dfrac{1}{9\times10^9}\times9\times10^{-3} = 10^{-12}\text{ F} = 1\text{pF}$

Since charge is conserved, the charge on the bigger drop is $27\times10^{-12}$ C.

$$\therefore \qquad \text{Potential of bigger drop, } V = \frac{q}{C} = \frac{27\times10^{-12}}{10^{-12}} = 27\text{V}$$

## PROBLEMS FOR PRACTICE

1. Calculate the radius of a spherical conductor of capacitance 1 farad. **[$9\times10^6$ km]**
2. Can a metal sphere of radius 1 cm hold a charge of 1C ? **[No]**
3. Calculate the capacitance of a conducting sphere of radius 10 cm situated in air. How much charge is required to raise it to a potential of 1000 V ? **[11 pF; $1.1\times10^{-8}$ C]**
4. When $1.0\times10^{12}$ electrons are transferred from one conductor to another of a capacitor, a potential difference of 10V develops between the two conductors. Calculate the capacitance of the capacitor. **[$1.6\times10^{-8}$ F]**
5. A potential difference of 250V is applied across the plates of a 25μF capacitor. Calculate the charge on the plates of capacitor. **[$6.25\times10^{-3}$ C]**

## 4.8. CAPACITANCE OF A SPHERICAL CAPACITOR

A spherical capacitor consists of two concentric hollow metallic spheres $A$ and $B$ which do not

Suppose the capacitance of parallel plate air capacitor is measured to be 2μF. When air is replaced by mica, the measured value comes out to be 12μF. Therefore, relative permittivity $K$ of mica is

$$K = \frac{C_m}{C_{air}} = \frac{12\,\mu\text{F}}{2\,\mu\text{F}} = 6$$

**Discussion.** The following points are worth noting :

(*i*) The relative permittivity $K$ of a dielectric can be defined in the following three ways :

$$(a)\ K = \frac{\varepsilon}{\varepsilon_0} \qquad (b)\ K = \frac{F_{air}}{F_m} \qquad (c)\ K = \frac{C_m}{C_{air}}$$

(*ii*) For metals, $K$ approaches infinity.

(*iii*) The value of $K$ (or $\varepsilon_r$) for common dielectrics are :

Mica = 6 − 7; Water = 81; Paper = 2 − 3

Amber = 2.8 ; Transformer oil = 2.24

**Example 4.6. The plates of a parallel plate air capacitor are separated by a distance of 1 mm. What must be the plate area if the capacitance of the capacitor is to be 1F?**

**Solution.** The capacitance of a parallel plate air capacitor is given by ;

$$C = \frac{\varepsilon_0 A}{d}$$

Here $d = 1\text{ mm} = 10^{-3}\text{ m}; \quad A = ?; \quad C = 1\text{F}$

$$\therefore \quad A = \frac{Cd}{\varepsilon_0} = \frac{1 \times 10^{-3}}{8.854 \times 10^{-12}} = \mathbf{1.1 \times 10^8\ m^2}$$

Note the enormous magnitude of plate area required to have a capacitance of 1F. This shows that farad is a very large unit of capacitance.

**Example 4.7. What distance apart should the two plates each of area 0.2 m × 0.1 m of a parallel plate air capacitor be placed in order to have the same capacitance as a spherical conductor of radius 0.5m?**

**Solution.** Area of plate, $A = 0.2 \times 0.1 = 0.02\ \text{m}^2$

Radius of sphere, $r = 0.5$ m

For parallel plate capacitor, $C = \varepsilon_0 A/d$

For spherical conductor, $C = 4\pi\,\varepsilon_0\, r$

Since the capacitance of the two capacitors is the same,

$$\therefore \quad \frac{\varepsilon_0 A}{d} = 4\pi\varepsilon_0 r$$

$$\text{or} \quad d = \frac{A}{4\pi r} = \frac{0.02}{4\pi \times 0.5} = 3.18 \times 10^{-3}\text{ m} = \mathbf{3.18\ mm}$$

**Example 4.8. Calculate the capacitance of a parallel plate air capacitor of plate area 30 m$^2$ the plates being separated by a dielectric 2 mm thick and of relative permittivity 6. If the electric field strength between the plates is 500 V/mm, calculate the charge on each plate.**

**Solution.** Capacitance, $C = \dfrac{\varepsilon_0 K A}{d} = \dfrac{(8.854 \times 10^{-12})(6)(30)}{2 \times 10^{-3}} = 0.797 \times 10^{-6}\text{ F} = \mathbf{0.797\ \mu F}$

P.D. across plates, $V = E \times d = 500 \times 2 = 1000$ volts

$\therefore$ Charge on each plate, $q = CV = (0.797 \times 10^{-6})1000 = 0.797 \times 10^{-3}\text{ C} = \mathbf{0.797\ mC}$

$$\frac{1}{C_S} = \frac{1}{0.5} + \frac{1}{0.3} + \frac{1}{0.2} = \frac{31}{3}$$

or $$C_S = 3/31\ \mu F$$

$\therefore$ $$\frac{C_P}{C_S} = \mathbf{\frac{31}{3}}$$

**Example 4.12. Two capacitors of capacitance 15 μF and 20 μF are connected in series to a 600 V d.c. supply. Find (*i*) charge on each capacitor. (*ii*) p.d. across each capacitor.**

**Solution.** (*i*) Equivalent capacitance, $C_S = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{15 \times 20}{15 + 20} = 8.57\,\mu F$

In series connection, charge on each capacitor is the same.

$\therefore$ Charge on each capacitor, $q = C_S V = (8.57 \times 10^{-6}) \times 600 = \mathbf{5.14 \times 10^{-3}\ C}$

(*ii*) P.D. across 15 μF capacitor $= \dfrac{q}{C_1} = \dfrac{5.14 \times 10^{-3}}{15 \times 10^{-6}} = \mathbf{342.7\ V}$

P.D. across 20 μF capacitor $= \dfrac{q}{C_2} = \dfrac{5.14 \times 10^{-3}}{20 \times 10^{-6}} = \mathbf{257\ V}$

**Example 4.13. The total capacitance of two capacitors is 4μF when connected in series and 18 μF when connected in parallel. Find the capacitance of each capacitor.**

**Solution.** Let $C_1$ and $C_2$ be the unknown capacitances. Then,

$$C_1 + C_2 = 18 \qquad ...(i) \qquad \text{when in parallel}$$

$$\frac{C_1 C_2}{C_1 + C_2} = 4 \qquad ...(ii) \qquad \text{when in series}$$

Multiplying eqs. (*i*) and (*ii*), $C_1 C_2 = 72$

Now $$C_1 - C_2 = \sqrt{(C_1 + C_2)^2 - 4C_1 C_2} = \sqrt{(18)^2 - 4 \times 72} = \pm 6 \qquad ...(iii)$$

Solving eqs. (*i*) and (*iii*), we get, $C_1 = \mathbf{12\ \mu F\ or\ 6\ \mu F;}$ $C_2 = \mathbf{6\ \mu F\ or\ 12\ \mu F}$

**Example 4.14. In the circuit shown in Fig. 4.11, the total charge is 750 μC. Find the values of $V_1$, $V$ and $C_2$.**

**Solution.** Voltage across $C_1$, $V_1 = \dfrac{q}{C_1} = \dfrac{(750 \times 10^{-6})}{15 \times 10^{-6}} = \mathbf{50\ V}$

Applied voltage, $$V = V_1 + V_2 = 50 + 20 = \mathbf{70\ V}$$

$$\text{Charge on } C_3 = C_3 V_2 = (8 \times 10^{-6})\,20 = 160 \times 10^{-6} = 160\ \mu C$$

$\therefore$ $$\text{Charge on } C_2 = 750 - 160 = 590\ \mu C$$

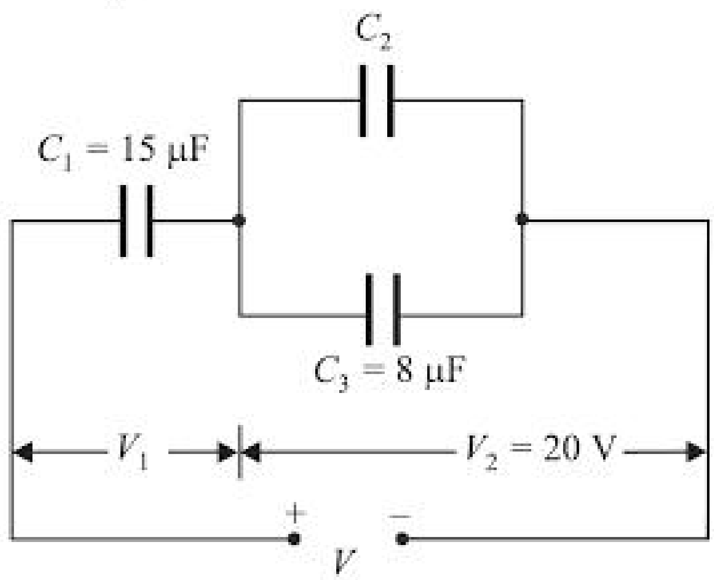



**Fig. 4.11**

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$$

Therefore, $C_S = 10/3\ \mu F$. Now $C_S$ is in parallel with $C_4$ so that :

$$C_{AB} = C_S + C_4 = 10/3 + 10 = \mathbf{13.3\ \mu F}$$

(*b*) Voltage across each series-connected capacitors is 500/3 volts.

$\therefore$ Charge on each series connected capacitor ($C_1$, $C_2$ and $C_3$) is

$$Q = C_1\ (500/3) = 10\ \mu F \times (500/3) V = 1.7 \times 10^{-3}\ C$$

Charge on capacitor $C_4$ is given by;

$$Q' = C_4 \times V = 10\ \mu F \times 500 V = 5 \times 10^{-3}\ C$$

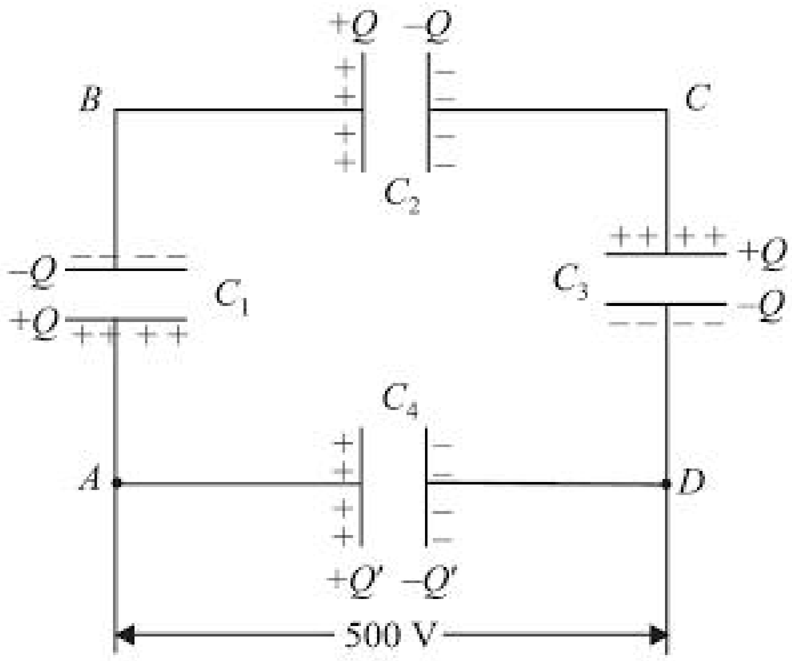



Fig. 4.20

**Example 4.23. Fig. 4.21 shows a network of capacitors where the numbers indicate the capacitances in μF. Find the value of capacitance $C$ if the equivalent capacitance between points $A$ and $B$ is to be 1 μF.**

**Solution.** Capacitors $C_2$ and $C_3$ are in parallel and their equivalent capacitance $C_P = C_2 + C_3 = 2 + 2 = 4\ \mu F$. The capacitors $C_4$ and $C_5$ are in series and their equivalent capacitance is

$$C_S = \frac{C_4 \times C_5}{C_4 + C_5} = \frac{6 \times 12}{6 + 12} = 4\ \mu F$$

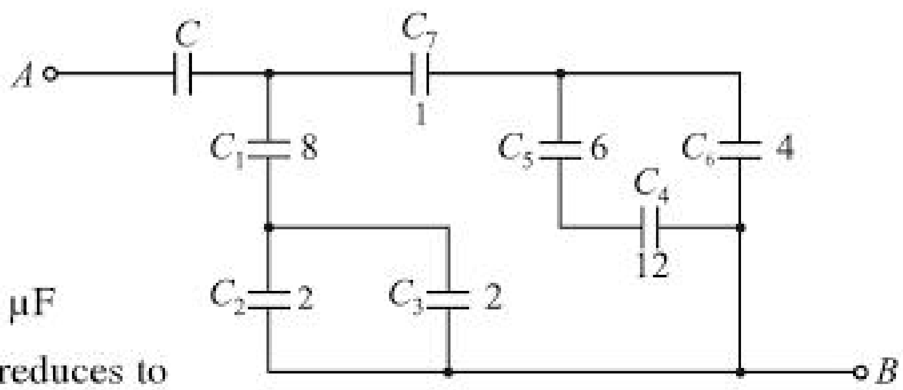



Fig. 4.21

Therefore, the circuit shown in Fig. 4.21 reduces to the one shown in Fig. 4.22 (*i*).

Now in Fig. 4.22 (*i*), $C_1$ and $C_P$ are in series and their equivalent capacitance is

$$C_S' = \frac{C_1 \times C_P}{C_1 + C_P} = \frac{8 \times 4}{8 + 4} = \frac{32}{12} = \frac{8}{3}\ \mu F$$

Also $C_S$ and $C_6$ are in parallel and their equivalent capacitance is $C_P' = 4 + 4 = 8\ \mu F$. Therefore, the circuit shown in Fig. 4.22 (*i*) reduces to the circuit shown in Fig. 4.22. (*ii*).

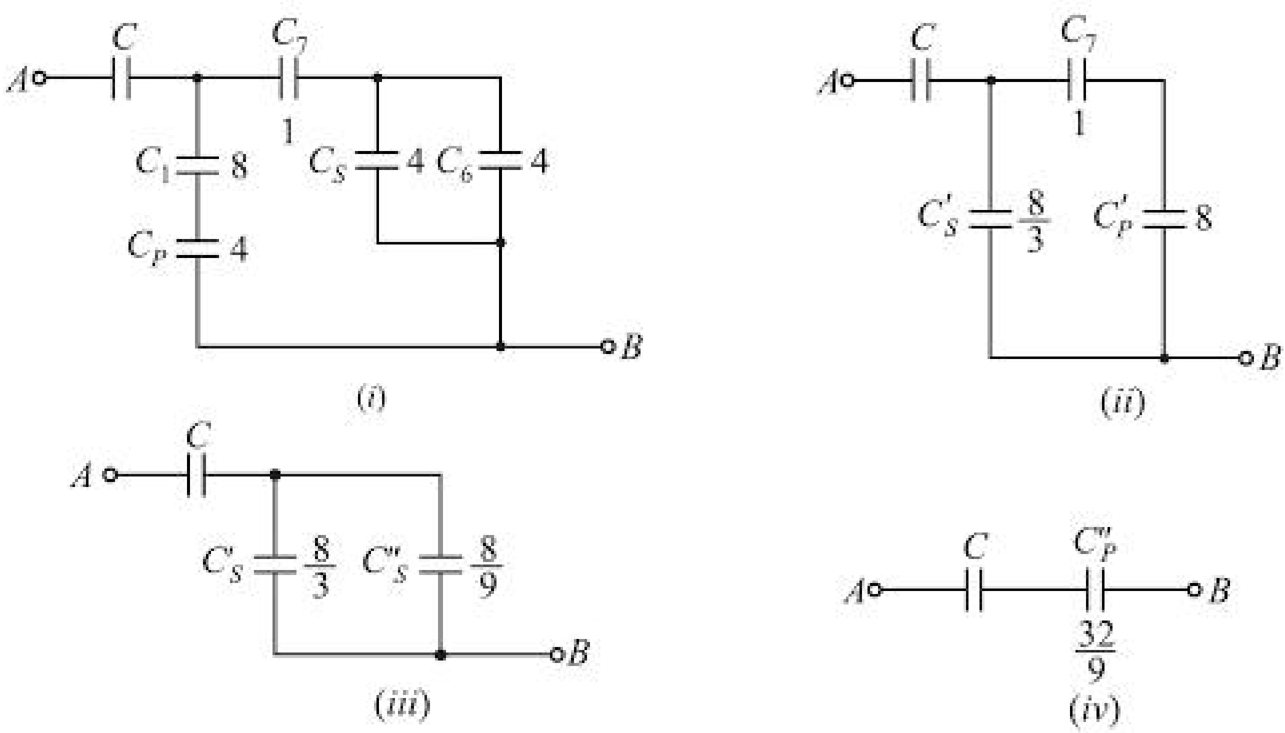



Fig. 4.22

2. Calculate the capacitance per unit length of a cylindrical condenser having inner conductor of 3.5 cm and outer of radius 4.5 cm. The space between the conductors has a dielectric of relative permittivity 6. **[1.33 × $10^{-9}$ F]**

## 4.13. ENERGY STORED IN A CHARGED CAPACITOR

Charging a capacitor means transferring electrons from one plate of the capacitor to the other. Therefore, work will have to be done (by battery) because electrons are to be moved against the *opposing forces. This work done is stored in the form of electric potential energy in the electric field between the plates. You can recover this energy by discharging the capacitor in a circuit.

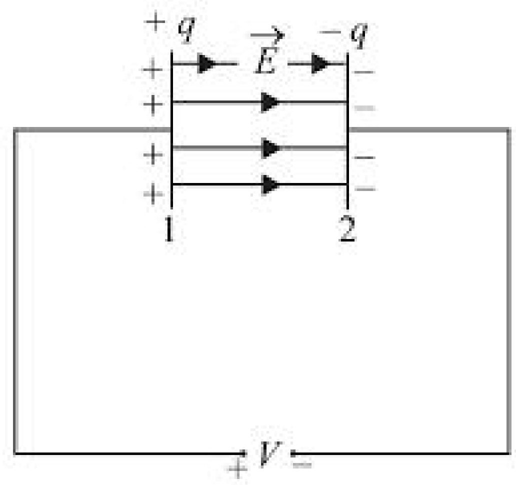


**Fig. 4.28**

Consider a capacitor of capacitance $C$ being charged from a d.c. source of $V$ volts as shown in Fig. 4.28. Initially, its two plates (1 and 2) are uncharged. The positive charge is transferred from plate 2 to plate 1 in very small instalments of $dq$ till plate 1 acquires the final charge $+q$ and plate 2 acquires charge $-q$. Suppose at any stage of charging, the charge on the capacitor is $q'$ and p.d. between the plates is $v$. Then,

$$C = \frac{q'}{v}$$

If a small charge $dq$ is further transferred, then small amount of work done is

$$dW = v\,dq = Cv\,dv \qquad [\because dq = C\,dv]$$

The total work done in raising the potential of uncharged capacitor to $V$ is

$$W = \int_0^V C\,v\,dv = C\left[\frac{V^2}{2}\right] = \frac{1}{2}CV^2$$

This work done is stored as electric potential energy in the electric field between the plates of the capacitor.

$\therefore$ Energy stored in the capacitor, $U = \frac{1}{2}CV^2 \overset{**}{=} \frac{1}{2}qV = \frac{q^2}{2C}$ joules

Note that $q$ is the final charge on the capacitor. Further, energy stored will be in joules if $q$, $C$ and $V$ are taken in coulomb, farad and volts respectively.

**Discussion.** The following points are worth noting :

(*i*) The formula for energy stored is a general expression (although we derived it considering parallel plate capacitor) and is applicable to any capacitor. It is because geometry of the capacitor plays no role in this formula.

(*ii*) The potential energy of the capacitor is stored in the electric field between the plates.

(*iii*) The potential energy of the capacitor is obtained at the expense of chemical energy of the battery.

(*iv*) The energy supplied by the battery is $qV$ but energy stored in the electric field is $qV/2$. The rest half ($qV/2$) of energy is wasted as heat in the connecting wires and battery itself.

---

* Electrons are being pushed to the negative plate which tends to repel them. Similarly, electrons are removed from the positive plate which tends to attract them. In either case, forces oppose the transfer of electrons from one plate to the other. This opposition increases as the charge on the plates increases.

** Putting $C = q/V$, $U = \frac{1}{2}qV$; putting $V = q/C$, $U = \frac{q^2}{2C}$

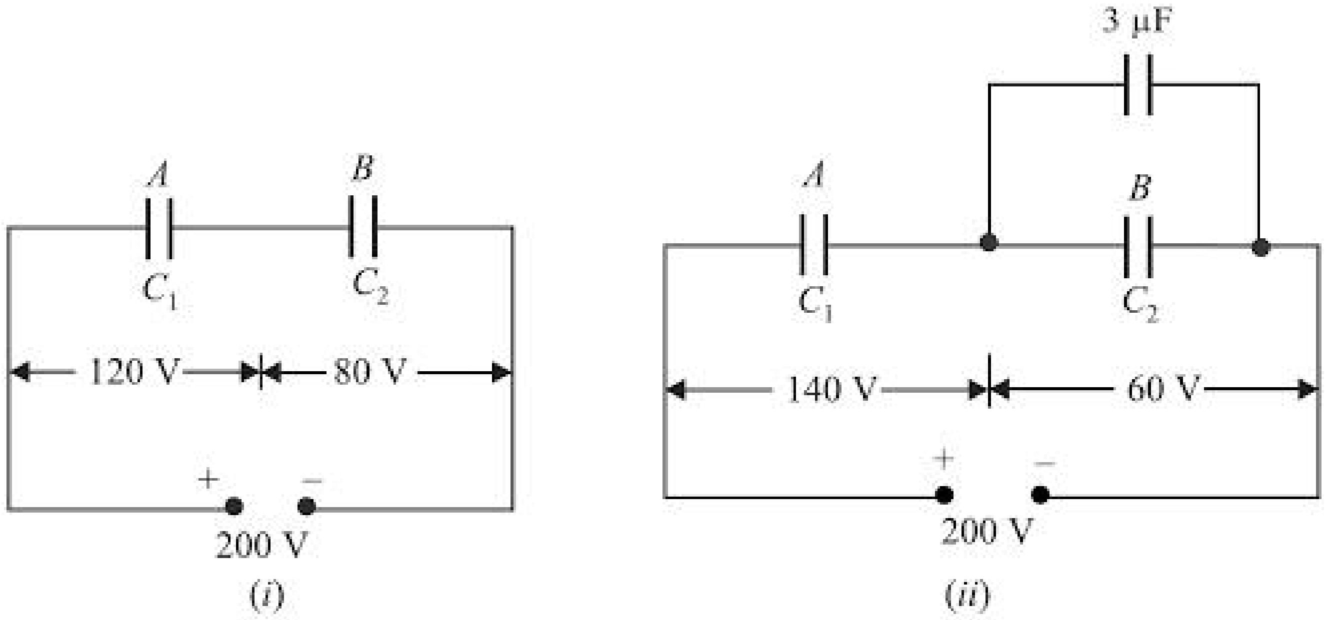



**Fig. 4.31**

Solving eqs. (*i*) and (*ii*), we have, $C_1$ = 3.6 μF; $C_2$ = 5.4 μF

Example 4.28. **A 16 μF capacitor is charged to 100V. After being disconnected, it is immediately connected to an uncharged capacitor of 4 μF. Determine (*i*) the p.d. across the combination (*ii*) the electrostatic energies before and after the capacitors are connected.**

**Solution.** $C_1 = 16\ \mu\text{F}$; $C_2 = 4\ \mu\text{F}$

**Before joining**

Charge on 16 μF capacitor, $q = C_1V_1 = (16 \times 10^{-6}) \times 100 = 1.6 \times 10^{-3}$ C

$$\text{Energy stored, } U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(16\times10^{-6})\times100^2 = 0.08 \text{ J}$$

**After joining.** When the capacitors are connected through a wire, the total capacitance, $C_P = C_1 + C_2 = 16 + 4 = 20$ μF. The charge $1.6 \times 10^{-3}$ C distributes between the two capacitors to have a common p.d. of $V$ volts.

$$\therefore \quad \text{P.D. across capacitors, } V = \frac{q}{C_P} = \frac{1.6\times10^{-3}}{20\times10^{-6}} = 80 \text{ V}$$

$$\text{Energy stored, } U_2 = \frac{1}{2}C_PV^2 = \frac{1}{2}\ (20 \times 10^{-6}) \times (80)^2 = 0.064 \text{ J}$$

It may be noted that there is a loss of energy. This is primarily due to the heat dissipated in the conductor connecting the capacitors.

Example 4.29. **A metal sphere 4 m in diameter is charged to a potential of 3 MV. Calculate the heat generated when the sphere is earthed through a long resistance wire.**

**Solution:** Potential at the surface of sphere, $V = 9\times10^9\frac{q}{r}$

$$\therefore \quad \text{Charge on sphere, } q = \frac{V\times r}{9\times10^9} = \frac{(3\times10^6)\times2}{9\times10^9} = 0.67\times10^{-3}\,\text{C}$$

$$\text{Energy stored in sphere} = \frac{1}{2}qV = \frac{1}{2}(0.67\times10^{-3})\times(3\times10^6) = 1005 \text{ J}$$

When the sphere is earthed, stored energy will be dissipated as heat in the resistance wire.

Example 4.30. **A parallel plate capacitor is connected to a 12 V battery. The charge on the capacitor is $1.35 \times 10^{-10}$ C. If the plate separation is decreased to half, find the extra charge given by the battery.**

## 4.20. SOME DEFINITIONS

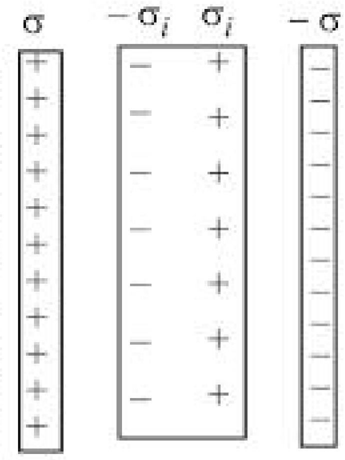



Fig. 4.36

Fig. 4.36 shows a slab of dielectric material placed between the plates of a charged capacitor. The capacitor sets up a uniform electric field $\vec{E_0}$ so that the dielectric slab is in this uniform field. The net effect on the dielectric is the formation of an "induced" positive charge density $\sigma_i$ on the right face and an equal negative charge density on the left face as shown in Fig. 4.36. These induced surface charges on the dielectric give rise to an induced electric field $\vec{E_i}$ which *opposes* the external field $\vec{E_0}$. Therefore, the net electric field $\vec{E}$ in the dielectric has a magnitude given by ;

$$E = E_0 - E_i \qquad \text{... } (i)$$

Note that the induced charge density ($\sigma_i$) on the dielectric is *less* than the free charge density ($\sigma$) on the plates.

**(i) Relative permittivity.** *The ratio of the strength of the applied electric field ($E_0$) to the strength of the reduced electric field (E) on introducing the dielectric between the plates of the capacitor is called* **relative permittivity** *of the dielectric medium i.e.*

$$\text{Relative permittivity, } K = \frac{E_0}{E} = \frac{\text{Applied Electric field}}{\text{Net electric field in dielectric}}$$

Now, $E_0 = \sigma/\varepsilon_0$ ; $E_i = \sigma_i/\varepsilon_0$ and $E = E_0/K = \sigma/K\varepsilon_0$. Putting these values in eq. (*i*), we have,

$$\frac{\sigma}{K\varepsilon_0} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_i}{\varepsilon_0}$$

$$\therefore \qquad \sigma_i = \left(\frac{K-1}{K}\right)\sigma$$

Since the value of $K$ is *always* greater than 1, *the induced charge density $\sigma_i$ on the dielectric is less than the free charge density $\sigma$ on the plates.* For instance if $K = 3$, then induced charge density on the dielectric is $\sigma_i = (3 - 1/3)\ \sigma = (2/3)\ \sigma$ *i.e.* induced charge density is two-third the free charge density on the plates. If there is no dielectric present, $K = 1$ so that $\sigma_i = (1 - 1/1)\sigma = 0$ as expected. However, if the dielectric is replaced by a *conductor*, for which $K = \infty$, then $E \to 0$ so that $\sigma_i \to \sigma$. This means the surface charge induced on the conductor will be equal and opposite to that on the plates, resulting in a net field of zero on the conductor.

**(ii) Polarisation density.** *The induced dipole moment developed per unit volume in a dielectric slab on placing it inside the electric field is called* **polarisation density** ($P$).

The induced dipole moment $p$ acquired by an atom (or molecule) is found to be

$$p = \alpha\, \varepsilon_0\, E_0$$

where $\alpha$ is a constant of proportionality and is called *atomic/molecular polarizability*. The dimensions of $\alpha$ are that of volume ($m^3$). If $N$ is the number of atoms per unit volume, then polarisation density $P$ is given by ;

$$P = N\, \alpha\, \varepsilon_0\, E_0$$

If the dielectric slab has length $l$ and area $A$, then,

$$\text{Volume of slab} = Al$$

If $\sigma_i$ is the induced charge density (*i.e.* induced charge per unit area), then,

$$\text{Dipole moment of slab} = (\sigma_i\, A)\, l$$

$$\therefore \qquad \text{Polarisation density, } P = \frac{(\sigma_i\, A)l}{Al} = \sigma_i$$

Thus the induced surface charge density is equal in magnitude to the polarisation density $P$.

Net electric field $E$ in the dielectric slab is given by ;

$$E = E_0 - E_i = E_0 - \frac{\sigma_i}{\varepsilon} = E_0 - \frac{P}{\varepsilon}$$

**Solution.** Capacitance of the capacitor, $C = \dfrac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{K}\right)}$

Here $A = 2 \times 10^{-3}$ m$^2$; $d = 0.01$ m; $t = 6 \times 10^{-3}$ m; $K = 3$

$$\therefore \quad C = \frac{(8.854 \times 10^{-12}) \times 2 \times 10^{-3}}{0.01 - 6 \times 10^{-3}(1 - 1/3)} = \mathbf{2.95 \times 10^{-12}\ F}$$

Example 4.33. **A parallel plate capacitor has plate area of 2 m$^2$ spaced by three layers of different dielectric materials. The relative permittivities are 2, 4, 6 and thicknesses are 0.5, 1.5 and 0.3 mm respectively. Calculate the capacitance of the capacitor.**

**Solution.** Capacitance of capacitor, $C = \dfrac{\varepsilon_0 A}{\dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} + \dfrac{t_3}{K_3}}$

$$= \frac{(8.854 \times 10^{-12}) \times 2}{\dfrac{0.5 \times 10^{-3}}{2} + \dfrac{1.5 \times 10^{-3}}{4} + \dfrac{0.3 \times 10^{-3}}{6}} = \mathbf{0.026 \times 10^{-6}\ F}$$

Example 4.34. **A capacitor is composed of two plates separated by 3 mm dielectric of relative permittivity 4. An additional piece of insulation 5 mm thick is now inserted between the plates. If the capacitor has now capacitance one-third of its original capacitance, find the relative permittivity of the additional dielectric.**

**Solution.** Fig. 4.39 (*i*) and Fig. 4.39 (*ii*) respectively show the two cases.

For the first case, $C = \dfrac{\varepsilon_0 K_1 A}{d} = \dfrac{\varepsilon_0 \times 4 \times A}{d}$ ...(*i*)

For the second case, $\dfrac{C}{3} = \dfrac{\varepsilon_0 A}{\dfrac{t_1}{K_1} + \dfrac{t_2}{K_2}} = \dfrac{\varepsilon_0 A}{\dfrac{3 \times 10^{-3}}{4} + \dfrac{5 \times 10^{-3}}{K_2}}$ ...(*ii*)

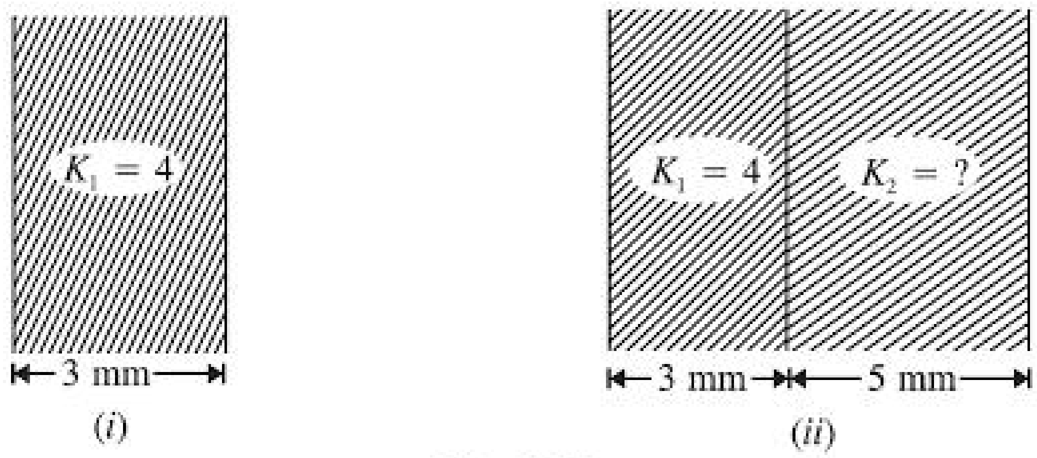



**Fig. 4.39**

Dividing eq. (*i*) by eq. (*ii*), $3 = \dfrac{4}{3}\left(\dfrac{3}{4} + \dfrac{5}{K_2}\right)$

$\therefore \quad K_2 = \mathbf{3.33}$

Example 4.35. **An air capacitor has two parallel plates of 1500 cm$^2$ area and held 5 mm apart. If a dielectric slab of area 1500 cm$^2$, thickness 2 mm and relative permittivity 3 is now introduced between the plates, what must be the new separation between the plates to bring the capacitance to the original value?**

positively charged, the negative ions (*i.e.*, electrons) are attracted to the pointed end and neutralise the positive charge on the pointed end. Thus, the conductor discharges quickly. At the same time, the charged conductor repels positive ions. Thus, an "*electric wind*" (consisting of positive ions in this case) is set up. The fact that pointed ends of a charged conductor lose charge rapidly is of great importance in the design of lightning conductors and electrostatic generators. The reader may note that the process of losing charge rapidly at pointed ends of a charged conductor is known as *corona discharge*.

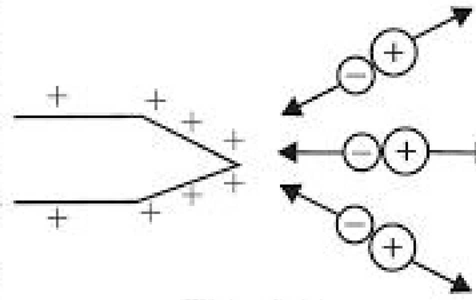


Fig. 4.41

## 4.25. LIGHTNING CONDUCTORS

*A lightning conductor is a device which is used to save big buildings from the lightning strokes.* A lightning conductor is a flat thick strip of copper with several sharp spikes (*i.e.*, pointed conductors), projecting above the highest part of the building as shown in Fig. 4.42. The lower end of the lightning conductor is connected to copper plate buried deep into the earth to provide good earth connection.

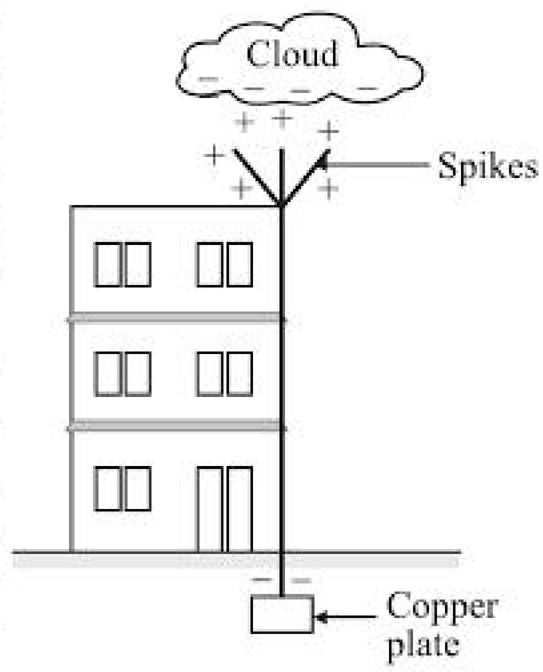



Fig. 4.42

(*i*) When a negatively charged cloud passes over the building, it induces a positive charge on the spikes and a negative charge on metal plate as shown in Fig. 4.42. The discharging action of pointed ends sets up an "electric wind" of positive ions which partly neutralises the negative charge on the cloud. This decreases the potential of the cloud, thereby preventing the lightning stroke. It may be noted that negative charge induced on the metal plate is neutralised in the earth.

(*ii*) If a lightning stroke does strike the lightning conductor, the electric discharge passes down the copper strip into the earth. This prevents the high current from passing through the building, thus ensuring the safety of the building and its occupants.

## 4.26. VAN DE GRAAFF GENERATOR

*The Van de Graaff generator is an electrostatic device that is capable of producing potential differences of the order of* $10^7$ *volts.* Such high voltages are required to accelerate charged particles (*e.g.*, $\alpha$–particles, electrons *etc.*) in several experiments pertaining to nuclear physics.

**Principle.** Designed by R.J. Van de Graaff in 1931, it is based on the following two electrostatic phenomena :

(*i*) The discharging action of pointed ends sets up an "electric wind".

(*ii*) A charge given to a hollow conductor is transferred to the outer surface and spreads uniformly over it.

**Construction.** Fig. 4.43 shows the various parts of Van de Graaff generator. It consists of a large spherical metal shell $S$ mounted on two insulating supports. A belt made of insulating material is run at high speed over pulleys $P_1$ and $P_2$. The pulley $P_1$ is in the base of the machine (run by a motor) while pulley $P_2$ is at the centre of the spherical shell $S$. The metal combs $C_1$ and $C_2$ having sharp ends are mounted on the generator as shown. The comb $C_1$ (called *emitter comb*) is held near the lower end of the belt and is given a high positive potential (about $10^4$ V) w.r.t. the ground. The comb $C_2$ (called *collector comb*) is placed near the upper end of the belt such that its pointed ends touch the belt and the other end is in contact with the inner surface of the metal sphere $S$.

$\therefore$ Potential difference $V'$ across each capacitor is

$$V' = \frac{q}{2C} = \frac{CV}{2C} = \frac{V}{2}$$

*i.e.* P.D. across $P = V/2$

*(ii) When P is connected to R in series*

When $P$ is connected to $R$ in series, the circuit is incomplete and no charge is transferred from capacitor $P$. Hence potential difference across the plates of $P$ remains $V$.

**Q.13. The safest way to protect yourself from lightning is to be inside a car. Comment.**

**Ans.** We know that electric field inside a conductor is zero. Since the body of the car is a metal, the electric field inside it is zero. The discharging current due to lightning passes to earth through the metallic body of the car.

**Q.14. Five identical capacitor plates each of area $A$ are arranged such that the adjacent plates are at a distance $d$ apart. The plates are connected to a battery of $V$ volts as shown in Fig. 4.46. What is the magnitude and nature of charge on plates 1 and 4 ?**

**Ans.** The system constitutes 4 capacitors in parallel across $V$ volts. Since area of each plate is $A$ and the separation between adjacent plates is $d$, all the four capacitors have the same capacitance $C$. The plate 1 acts as a positive plate of capacitor $C_1$ (of capacitance $C$) formed between plates 1 and 2.

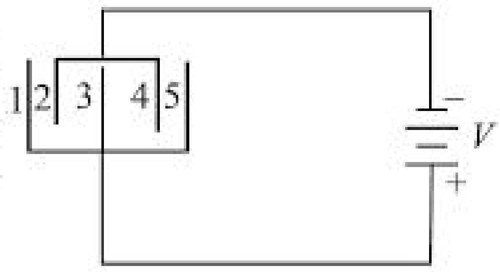


**Fig. 4.46**

$\therefore$ Charge on plate $1 = +CV = \frac{\varepsilon_0 A}{d} V$

The plate 4 acts as a negative plate of both the capacitors $C_3$ (between plates 3 and 4) and $C_4$ (between plates 4 and 5).

$\therefore$ Charge on plate $4 = -(C + C)V = \frac{-2\varepsilon_0 A}{d} V$

## VERY SHORT ANSWER QUESTIONS

**Q.1. What is a capacitor?**

**Ans.** A capacitor consists of two conducting surfaces separated by an insulating medium.

**Q.2. What is the purpose of a capacitor?**

**Ans.** A capacitor is a device that is capable of storing electric charge.

**Q.3. What do you mean by capacitance of a capacitor?**

**Ans.** The capacitance of a capacitor is defined as the ratio of charge on the plates to the potential difference across the plates *i.e.*

$$\text{Capacitance, } C = \frac{\text{Charge on either plate}}{\text{P.D. across plates}} = \frac{q}{V}$$

**Q.4. What is the SI unit of capacitance?**

**Ans.** Capacitance $= \frac{\text{Charge}}{\text{P.D.}}$. Therefore, SI unit of capacitance is $CV^{-1}$ which is also called farad.

**Q.5. What is one picofarad?**

**Ans.** 1 picofarad (pF) $= 10^{-12}$ F or 1F $= 10^{12}$ pF.

**Q.6. Draw a graph showing how charge $q$ on a capacitor of capacitance $C$ varies with potential difference $V$ across it?**

**Ans.** $q \propto V$ so that graph between $q$ and $V$ is a straight line passing through the origin as shown in Fig. 4.47.

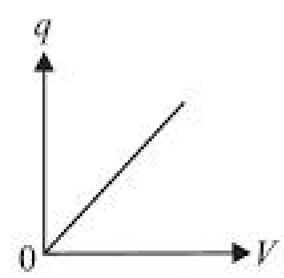


**Fig. 4.47**

**Q.7. A single conductor behaves as a capacitor. Where is the second plate?**

**Ans.** Earth is the second plate.

As the capacitor plates are separated, $C$ decreases. Since charge on the plates remains the same, value of $V$ increases. Hence, the reading of the voltmeter will increase.

**Q.3. Can you place a parallel plate capacitor (consisting of two plates) of 1 farad in your almirah?**

**Ans.** No. Suppose the two plates of the capacitor are separated by as small a distance as 1 mm.

$$C = \frac{\varepsilon_0 A}{d} \quad \text{or} \quad A = \frac{Cd}{\varepsilon_0} = (1) \times \frac{(1 \times 10^{-3})}{8.854 \times 10^{-12}} = 1.1 \times 10^8 \text{ m}^2$$

This area is equal to the area of a square having each side more than 10 km. Modern technology, however, has permitted the construction of 1F capacitors of very moderate size.

**Q.4. The distance between the plates of a parallel plate air capacitor is doubled and the area of the plates is halved. What is the new value of the capacitance of the capacitor?**

**Ans.** $$C_{air} = \frac{\varepsilon_0 A}{d} \;;\; C'_{air} = \frac{\varepsilon_0 (A/2)}{2d} \quad \therefore \quad C'_{air} = \frac{C_{air}}{4}$$

Therefore, the capacitance becomes one-fourth of its initial value.

**Q.5. Fig. 4.48 shows the variation of charge $q$ versus potential difference $V$ for two capacitors $C_1$ and $C_2$. The two capacitors have the same plate separation but plate area of $C_2$ is double that of $C_1$. Which of the lines in Fig. 4.48 correspond to $C_1$ and $C_2$ and why?**

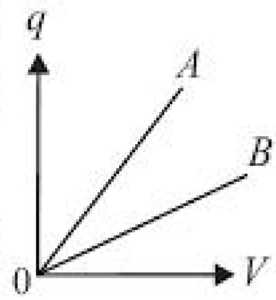


**Fig. 4.48**

**Ans.** $C = \varepsilon_0 A/d$. Since plate area ($A$) of $C_2$ is double that of $C_1$, $C_2 > C_1$. Now slope of the lines ($= q/V$) represents the capacitance of the capacitors. Since slope of line $A$ is greater than that of line $B$, line $A$ corresponds to $C_2$ and line $B$ corresponds to $C_1$.

**Q.6. Why is it not possible to make a spherical conductor of capacity one farad?**

**Ans.** $$C = 4\pi\varepsilon_0 r \quad \therefore \quad r = \frac{C}{4\pi\varepsilon_0} = 9 \times 10^9 \times 1\text{m} = 9 \times 10^6 \text{ km}$$

It is not possible to have a spherical conductor of such a big radius.

**Q.7. $n$ small drops of the same size are charged to $V$ volt each. They coalesce to form a bigger drop. Calculate the potential of the bigger drop.**

**Ans.** Let $r$ be the radius of each small drop and $R$ the radius of the bigger drop. Since the volume of bigger drop is equal to that of $n$ small drops,

$$\therefore \quad \frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$$

$$\text{or} \quad R = n^{1/3} r$$

If $q$ is the charge on each small drop, then,

$$\text{Potential of bigger drop} = \frac{\text{Total charge}}{\text{Capacitance}} = \frac{nq}{4\pi\varepsilon_0 R} = \frac{nq}{4\pi\varepsilon_0 n^{1/3} r}$$

$$= n^{2/3} \times \frac{q}{4\pi\varepsilon_0 r} = n^{2/3} \times \text{potential on each small drop}$$

**Q.8. Should two capacitors $C_1$ and $C_2$ be connected in series or in parallel to a battery of $V$ volts to store greater total charge and energy?**

**Ans.** Total charge, $q = CV$; Total energy stored $= \frac{1}{2}CV^2$. For a given $V$, total charge as well as total stored energy is greater for that combination which has larger value of resultant capacitance. Clearly, the choice is parallel combination of capacitors.

**Q.9. The battery remains connected to a parallel plate capacitor and a dielectric slab is inserted between the plates. What will be the effect on its (*i*) capacitance (*ii*) charge (*iii*) potential difference (*iv*) electric field (*v*) energy stored?**

(*c*) $E = V/d$. Since $V$ decreases and $d$ remains the same, the electric field within the dielectric decreases.

(*d*) $C = q/V$. Since $q$ remains same but $V$ decreases, the capacitance increases.

(*e*) The stored energy decreases.

(*ii*) When the **capacitor is connected to a battery** and dielectric slab is introduced between the plates as shown in Fig. 4.51 (*ii*), this results in the following effects:

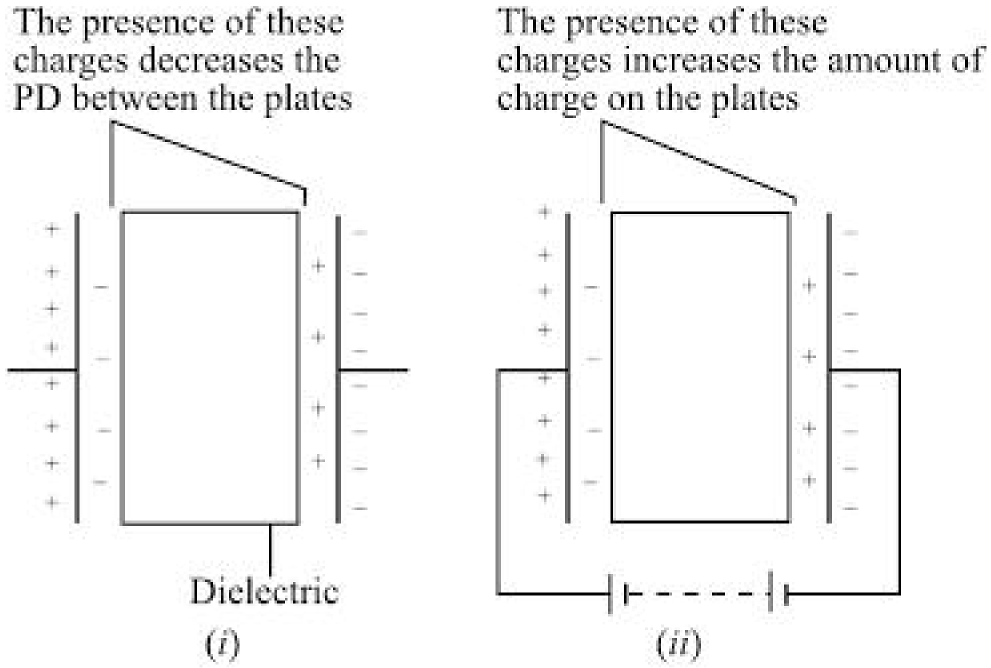



**Fig. 4.51**

(*a*) The potential difference ($V$) between plates cannot change since battery is connected.

(*b*) The charge on the plates is increased.

(*c*) The electric field is unchanged.

(*d*) $C = q/V$. Since $q$ is increased but $V$ remains same, $C$ is increased.

(*e*) The stored energy increases.

**9.** If the space between the plates is **entirely** filled with a dielectric of dielectric constant $K$, the effects are illustrated in the table below :

| S. No. | Without dielectric | With dielectric (Isolated Capacitor) | With dielectric (Capacitor connected to battery) |
|---|---|---|---|
| 1. | $E$ | $\frac{E}{K}$ | $E$ |
| 2. | $q$ | $q$ | $Kq$ |
| 3. | $V$ | $\frac{V}{K}$ | $V$ |
| 4. | $C = \frac{q}{V}$ | $C = \frac{q}{V/K} = KC$ | $C = \frac{Kq}{V} = KC$ |
| 5. | Energy, $U_0 = \frac{q^2}{2C}$ | $U = \frac{U_0}{K}$ | $U = KU_0$ |

**10.** Dielectric constant, $K = \dfrac{\text{Field in vacuum}}{\text{Field in dielectric}}$

(*i*) Value of $K$ is equal to or greater than 1.

(*ii*) Materials which are easily polarised have larger values of $K$.

(*a*) 1.5 μF (*b*) 1 μF

(*c*) 3 μF (*d*) 2 μF

**20.** A capacitor $C_1$ is charged to a potential difference $V_0$ as shown in Fig. 4.58. It is then connected to an uncharged capacitor $C_2$. What is the final potential ($V$) across the combination ? **[AIIMS 1990]**

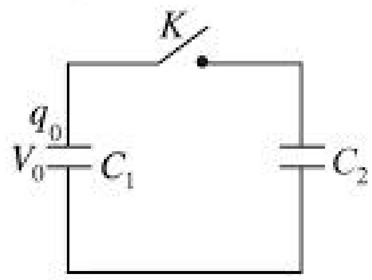



**Fig. 4.58**

(*a*) $V_0 \times \dfrac{C_1}{C_1 + C_2}$

(*b*) $V_0 \times \dfrac{C_1 + C_2}{C_1}$

(*c*) $V_0 \times \dfrac{C_1 - C_2}{C_1}$

(*d*) $V_0 \times \dfrac{C_1}{C_1 - C_2}$

## ANSWERS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

| | | | | |
|---|---|---|---|---|
| **1.** (*a*) | **2.** (*b*) | **3.** (*c*) | **4.** (*a*) | **5.** (*d*) |
| **6.** (*c*) | **7.** (*b*) | **8.** (*d*) | **9.** (*a*) | **10.** (*c*) |
| **11.** (*b*) | **12.** (*d*) | **13.** (*d*) | **14.** (*c*) | **15.** (*d*) |
| **16.** (*d*) | **17.** (*b*) | **18.** (*a*) | **19.** (*b*) | **20.** (*a*) |

## HINTS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

**1.** Energy stored, $U = \dfrac{1}{2}CV^2 = \dfrac{1}{2} \times (12 \times 10^{-12}) \times (50)^2 = \mathbf{1.5 \times 10^{-8}\ J}$

**2.** Energy stored per unit volume, $u = \dfrac{1}{2}\varepsilon_0 E^2$. Now $E = \dfrac{V}{d}$. Therefore, $u = \mathbf{\dfrac{1}{2}\varepsilon_0 \dfrac{V^2}{d^2}}$.

**3.** A soap bubble is an isolated sphere and capacitance of isolated sphere in air $= 4\pi\, \varepsilon_0 r$ *i.e.* capacitance is directly proportional to radius. If $C_1$ is the capacitance in the first case, then capacitance in the second case $C_2 = 2C_1$.

Now $V = \dfrac{q}{C}$. Since $q$ remains the same, $V \propto \dfrac{1}{C}$.

$$\therefore \quad \frac{V_2}{V_1} = \frac{C_1}{C_2} = \frac{C_1}{2C_1} = \frac{1}{2} \quad \therefore \quad V_2 = \frac{V_1}{2} = \frac{16}{2} = \mathbf{8V}$$

**4.** Capacity of first capacitor, $C_1 = \dfrac{\varepsilon_0 K_1 A}{2d}$; Capacity of second capacitor, $C_2 = \dfrac{\varepsilon_0 K_2 A}{2d}$.

Since the two capacitors are connected in parallel, the equivalent capacitance $C$ is

$$C = C_1 + C_2 = \frac{\varepsilon_0 K_1 A}{2d} + \frac{\varepsilon_0 K_2 A}{2d} = \mathbf{\frac{A\, \varepsilon_0 (K_1 + K_2)}{2d}}$$

**5.** Capacitance of parallel-plate capacitor, $C = \dfrac{\varepsilon_0 K A}{d}$ $\quad \therefore \ C \propto \dfrac{K}{d}$ ($\because$ area $A$ remains same).

$$\therefore \quad \frac{C_2}{C_1} = \frac{K_2}{d_2} \times \frac{d_1}{K_1} = \frac{2K_1}{d_1/2} \times \frac{d_1}{K_1} = 2 \times 2 = 4 \quad \therefore \quad C_2 = \mathbf{4\ C_1}$$

6. The capacitors $C_1$ and $C_2$ are connected in parallel and their equivalent capacitance is $C_P = C_1 + C_2 = 5 + 10 = 15\mu F$. Now $C_P$ and $C_3$ are in series and their equivalent capacitance $C_{AB}$ is

$$\frac{1}{C_{AB}} = \frac{1}{C_P} + \frac{1}{C_3} = \frac{1}{15} + \frac{1}{4} = \frac{19}{60} \quad \therefore \quad C_{AB} = \frac{60}{19} = \mathbf{3.2\ \mu F}$$

7. Suppose $E_0$ is the electric field between the plates of an air capacitor. When a dielectric material of dielectric constant $K$ is introduced between the plates, the electric field in the dielectric is reduced to $E_0/K$. As a result, the electric field between the plates becomes **less than $E_0$.**

8. Initial stored energy, $U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

$$= \frac{1}{2}\times(3\times10^{-6})\times(300)^2 + \frac{1}{2}\times(5\times10^{-6})\times(500)^2 = 76\times10^{-2}\ \text{J}$$

When the capacitors are joined by a wire, they are in parallel. There is a redistribution of charge till they acquire a common potential $V$.

$$\text{Common potential, } V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{3\times10^{-6}\times300 + 5\times10^{-6}\times500}{(3+5)\times10^{-6}} = 425\text{V}$$

Total capacitance, $C = C_1 + C_2 = 3 + 5 = 8\mu F = 8 \times 10^{-6}$ F

$$\text{Final stored energy, } U_f = \frac{1}{2}CV^2 = \frac{1}{2}\times(8\times10^{-6})\times(425)^2 = 72.25\times10^{-2}\ \text{J}$$

Loss of energy, $U = U_i - U_f = (76 - 72.25) \times 10^{-2} =$ **0.0375 J**

9. $q = CV$. Since the capacitor is disconnected from the battery, the charge on capacitor plates remains the same *i.e.* $CV$ = constant. When the distance between capacitor plates is increased, the capacitance of the capacitor decreases ($\because\ C = \varepsilon_0\ A/d$). Therefore, **potential difference must increase** to keep $CV$ = constant.

10. $C = \frac{\varepsilon_0 A}{d}$. For the same $A$, $C \propto \frac{1}{d}$.

$$\therefore \quad \frac{C_2}{C_1} = \frac{d_1}{d_2} \quad \text{or} \quad C_2 = C_1 \times \frac{d_1}{d_2} = 15 \times \frac{6}{2} = \mathbf{45\ \mu F}$$

11. When the two capacitors are connected in parallel, there is redistribution of charge till the two capacitors acquire a common potential $V$ given by ;

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{(20\times10^{-6}\times500) + (10\times10^{-6}\times200)}{(20\times10^{-6}) + (10\times10^{-6})},$$

$$= \mathbf{400\ V}$$

12. The equivalent capacitance $C_S$ of the series combination is

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{10} + \frac{1}{15} = \frac{1}{2} \quad \therefore C_S = 2\mu\text{F}$$

When the capacitors are connected in series, the charge on each capacitor is the same and $= C_S V = 2 \times 100 =$ **200 μC.**

13. The circuit can be redrawn as shown in Fig. 4.59. Since the product of capacitances of the opposite arms of this Wheatstone bridge are equal, the bridge is balanced.

Therefore, there will be no charge on capacitor $C_5$ connected between $B$ and $C$. As a result, $C_5$ is ineffective and may be removed from the circuit. The capacitors $C_1$ and $C_2$ in branch $ABD$ are in series and their equivalent capacitance $C'$ is

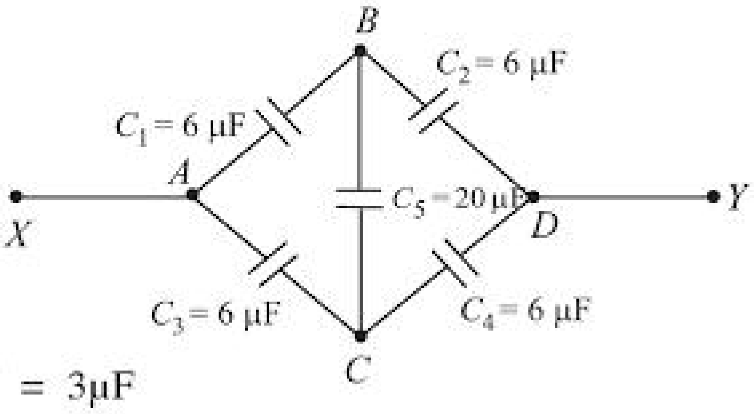


**Fig. 4.59**

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \quad \therefore \quad C' = 3\mu F$$

The capacitors $C_3$ and $C_4$ in branch $ACD$ are in series and their equivalent capacitance $C''$ is

$$\frac{1}{C''} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \quad \therefore \quad C'' = 3\mu F$$

Now capacitances $C'$ (= 3 μF) and $C''$ (= 3 μF) are in parallel. Therefore, $C_{XY} = C' + C'' = 3 + 3 =$ **6 μF.**

**14.** $C = \frac{q}{V}$ where $q$ is the charge on the capacitor and $V$ is the voltage. Clearly, the capacity **does not depend upon the nature of the material.**

**15.** Let $C_0$ be the capacitance of the capacitor when there is vacuum/air between the plates. When vacuum/air is replaced by a dielectric, then capacitance $C_m$ is

$$C_m = KC_0 \quad \text{or} \quad C_0 = \frac{C_m}{K} = \frac{C}{2}$$

**16.** The charge on $C_1$ is equal to charges on both $C_2$ and $C_3$. Therefore, charge on $C_1$ is more than the charge on $C_2$ or $C_3$. Now we find the value of charge on $C_1$. The capacitors $C_2$ and $C_3$ are in parallel and their equivalent capacitance is $C_P = C_2 + C_3 = 3 + 6 =$ 9F. Now $C_1$ and $C_P$ are in series and their equivalent capacitance is

$$\frac{1}{C_S} = \frac{1}{C_P} + \frac{1}{C_1} = \frac{1}{9} + \frac{1}{3} = \frac{4}{9} \quad \therefore \quad C_S = \frac{9}{4}\text{F}$$

$$\therefore \quad \text{Charge on } C_1 = C_S V = \frac{9}{4} \times 4 = 9\text{C}$$

**17.** We have identical capacitors marked 8μF, 250V. For 1000V supply, we should put 4 capacitors in series. The equivalent capacitance of this series combination = 8/4 = 2μF. To get 16 μF, 8 such series-connected capacitors should be connected in parallel. Therefore, number of capacitors = 8 × 4 = **32.**

**18.** Let $E_0$ be the electric field with vacuum/air between the plates. When vacuum/air is replaced by a dielectric of dielectric constant $K$, then electric field $E_m$ is given by ;

$$E_m = \frac{E_0}{K} \quad \text{or} \quad K = \frac{E_0}{E_m} = \frac{2 \times 10^5}{1 \times 10^5} = 2$$

**19.** The branch $HIJ$ has three 3 μF capacitors in series and their equivalent capacitance = 3/3 = 1 μF. This 1 μF capacitance is in parallel with 2μF capacitance. Therefore, $C_{HK} = 1 + 2 = 3\mu F$. Now branch $GHKL$ has three 3μF capacitors in series and their equivalent capacitance = 3/3 = 1 μF. Therefore, $C_{GL} = 1 + 2 = 3$ μF. Now we have three 3 μF capacitors in the branch $AGLB$ and, therefore, $C_{AB} = 3/3 =$ **1μF.**

20. The original charge on $C_1$, $q_0 = C_1 V_0$. When key $K$ is closed, the uncharged capacitor $C_2$ is in parallel with $C_1$. There is a redistribution of charge till the two capacitors acquire a common potential $V$. Since charge remains the same,

$$C_1V_0 = C_1V + C_2V \text{ or } C_1V_0 = V(C_1 + C_2) \therefore V = V_0 \times \frac{C_1}{C_1 + C_2}$$

## MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. If $n$ capacitors each of capacitance $C$ are connected in series with a battery of $V$ volts, then energy stored in all the capacitors will be **[CBSE AIEEE 2002]**

(*a*) $nCV^2$ (*b*) $\frac{1}{4}nCV^2$

(*c*) $\frac{1}{2}nCV^2$ (*d*) $\frac{1}{2n}CV^2$

2. Four capacitors, each of capacitance 50 μF are connected as shown in Fig. 4.60. If the

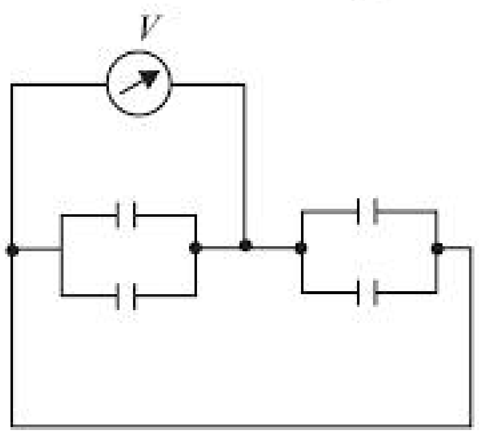



**Fig. 4.60**

voltmeter reads 100V, the charge on each capacitor is **[CEEE Delhi 2002]**

(*a*) $5 \times 10^{-3}$ C (*b*) $2 \times 10^{-3}$ C

(*c*) 0.5 C (*d*) 0.2 C

3. While a capacitor remains connected to a battery and a dielectric slab is slipped between the plates, then, **[Karnataka CET 2001]**

(*a*) energy stored in the capacitor decreases

(*b*) electric field between the plates increases

(*c*) charge from the battery flows to the capacitor

(*d*) potential difference between the plates is changed

4. A metal foil of negligible thickness is introduced between two plates of a capacitor at the centre. The capacitance of capacitor will be **[IIT 1992]**

(*a*) same (*b*) doubled

(*c*) halved (*d*) none of above

5. Two capacitors $C_1$ and $C_2$ are joined as shown in Fig. 4.61. The potential of point $A$ is $V_1$ and that of $B$ is $V_2$. The potential of point $D$ will be **[MP PET 1997]**

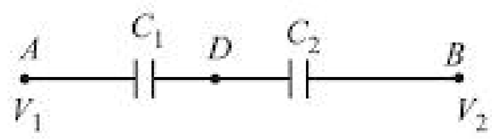



**Fig. 4.61**

(*a*) $\frac{1}{2}(V_1 + V_2)$ (*b*) $\frac{C_1V_2 + C_2V_1}{C_1 + C_2}$

(*c*) $\frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ (*d*) $\frac{C_2V_1 - C_1V_2}{C_1 + C_2}$

6. The effective capacitance between points $A$ and $B$ in the circuit shown in Fig. 4.62 is (capacitance of each capacitor is 1μF) **[Karnataka CET 1997]**

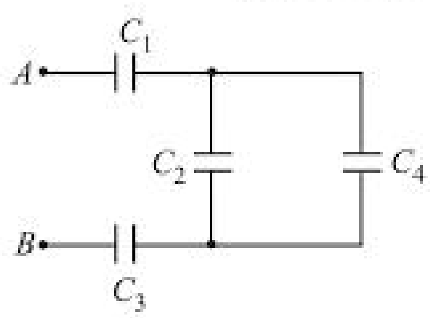



**Fig. 4.62**

(*a*) 2 μF (*b*) 4 μF

(*c*) 3 μF (*d*) 0.4 μF

7. A 10μF capacitor and 20μF capacitor are connected in series across 200 V supply line. The charged capacitors are then disconnected from the supply and reconnected with their positive plates together and negative plates together and no external voltage is applied. What is the potential difference across each capacitor? **[MP PET 1997]**

(*a*) $\frac{800}{9}$V (*b*) $\frac{800}{3}$V

(*c*) 400 V (*d*) 200 V

8. To obtain 3μF capacity from three capacitors 2μF each, they will be arranged **[MP PET 1998]**

(*a*) all three in series

(*b*) all three in parallel

(*c*) two capacitors in series and the third in parallel with the combination of the first two

(*d*) two capacitors in parallel and the third in series with the combination of the two

**9.** Equal charges are given to two conducting spheres of different radii. The potential will **[MP PET 1998]**

(*a*) be more on the smaller sphere

(*b*) be more on the bigger sphere

(*c*) be equal on both the spheres

(*d*) depend on the nature of the material of the sphere

**10.** Minimum number of capacitors of 2μF each required to obtain a capacitance of 5μF will be **[Haryana CET 2001]**

(*a*) 6 (*b*) 4

(*c*) 3 (*d*) 5

**11.** Five capacitors of 10 μF capacitance each are connected to a d.c. potential of 100 V as shown in Fig. 4.63. The equivalent capacitance between points *A* and *B* will be **[KCET 2000]**

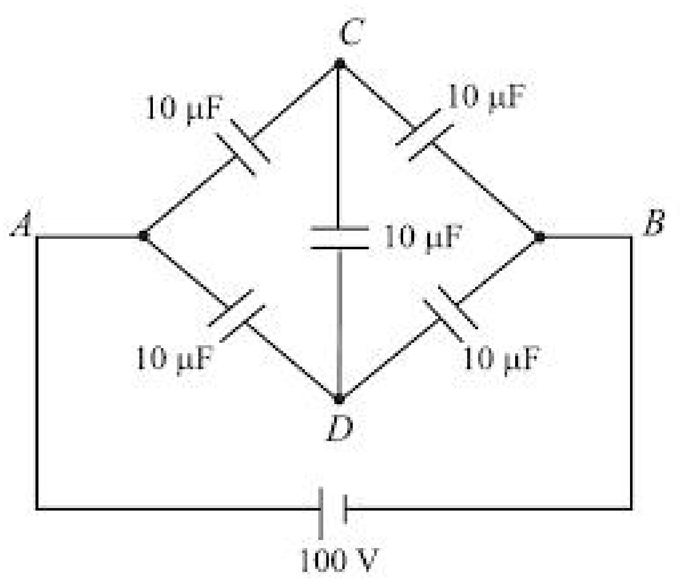



**Fig. 4.63**

(*a*) 40 μF (*b*) 20 μF

(*c*) 30 μF (*d*) 10 μF

**12.** If a slab of insulating material $4 \times 10^{-5}$ m thick is introduced between the plates of a parallel-plate capacitor, the distance between the plates has to be increased by $3.5 \times 10^{-5}$ m to restore the capacity to the original value. Then dielectric constant of the material of the slab is **[AMU 1999]**

(*a*) 8 (*b*) 6

(*c*) 12 (*d*) 10

**13.** A 2 μF capacitor is charged to 100 V and then its plates are connected by a conducting wire. The heat produced is **[MP CEE 1999]**

(*a*) 0.001 J (*b*) 0.01 J

(*c*) 0.1 J (*d*) 1 J

**14.** A parallel-plate capacitor has the space between its plates filled by two slabs of thickness *d*/2 and dielectric constants $K_1$ and $K_2$. Here *d* is the plate separation of the capacitor. The capacitance of the capacitor is **[MP CEE 1999]**

(*a*) $\frac{2\varepsilon_0 d}{A}\left(\frac{K_1 + K_2}{K_1 + K}\right)$

(*b*) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$

(*c*) $\frac{2\varepsilon_0 A}{d}(K_1 + K_2)$

(*d*) $\frac{2\varepsilon_0 A}{d}(K_1 - K_2)$

**15.** A 10 μF capacitor is charged to a potential difference of 50V and is connected to another uncharged capacitor in parallel. Now the common potential difference becomes 20V. The capacitance of the second capacitor is **[MP CEE 1999]**

(*a*) 15 μF (*b*) 30 μF

(*c*) 20 μF (*d*) 10 μF

**16.** A capacitor is charged to store an energy of *U*. The charging battery is disconnected. An identical capacitor is now connected in parallel with this capacitor. The energy stored in each of the capacitors is **[Karnataka CET 1998]**

(*a*) 3*U*/2 (*b*) *U*

(*c*) *U*/4 (*d*) *U*/2

**17.** Two capacitors *A* and *B* are connected in series with a battery as shown in Fig. 4.64. When the switch *S* is closed and the two capacitors get charged fully, then, **[MP PET 2000]**

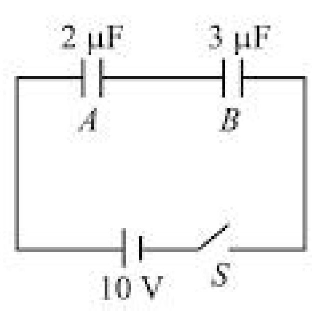



**Fig. 4.64**

(*a*) potential difference across the plates of $A$ is 4V and across the plates of $B$ is 6V

(*b*) potential difference across the plates of $A$ is 6V and across the plates of $B$ is 4V

(*c*) the ratio of electric energies stored in $A$ and $B$ is 2 : 3

(*d*) the ratio of charges on $A$ and $B$ is 3 : 2

18. A parallel-plate capacitor is connected to a battery. The distance between the plates is 6mm. If a glass plate (dielectric constant $K$ = 9) of 4.5 mm is introduced between the plates, then capacitance will become

**[Haryana CET 1997]**

(*a*) 4 times (*b*) the same

(*c*) 2 times (*d*) 3 times

19. The total capacitance between points $A$ and $B$ in the circuit shown in Fig. 4.65 is

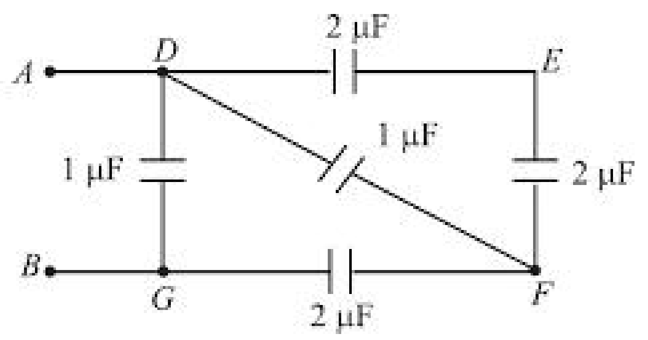



**Fig. 4.65**

**[CEEE Delhi 1998]**

(*a*) 4 μF (*b*) 3 μF

(*c*) 2 μF (*d*) 1 μF

20. A parallel-plate capacitor has area of each plate $A$ and the separation between the plates $d$. It is charged to a potential $V$ and then disconnected from the battery. How much work will be done in filling the capacitor completely with a dielectric of dielectric constant $K$ ?

**[IIT 1991]**

(*a*) $\frac{1}{2}\frac{\varepsilon_0 AV^2}{d}\left(1-\frac{1}{K^2}\right)$

(*b*) $\frac{1}{2}\frac{\varepsilon_0 AV^2}{Kd}$

(*c*) $\frac{1}{2}\frac{\varepsilon_0 AV^2}{K^2 d}$

(*d*) $\frac{1}{2}\frac{\varepsilon_0 AV^2}{d}\left(1-\frac{1}{K}\right)$

## ANSWERS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

| | | | | |
|---|---|---|---|---|
| **1.** (*d*) | **2.** (*a*) | **3.** (*c*) | **4.** (*a*) | **5.** (*b*) |
| **6.** (*d*) | **7.** (*a*) | **8.** (*c*) | **9.** (*a*) | **10.** (*b*) |
| **11.** (*d*) | **12.** (*a*) | **13.** (*b*) | **14.** (*b*) | **15.** (*a*) |
| **16.** (*c*) | **17.** (*b*) | **18.** (*d*) | **19.** (*c*) | **20.** (*d*) |

## HINTS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. The equivalent capacitance $C_S$ of these series-connected capacitors is

$$\frac{1}{C_S} = \frac{1}{C}+\frac{1}{C}+\frac{1}{C}.....n \text{ times} = \frac{n}{C} \qquad \therefore \quad C_S = \frac{C}{n}$$

Energy stored in all capacitors $= \frac{1}{2}C_S V^2 = \frac{1}{2}\frac{C}{n}V^2 = \frac{1}{2n}CV^2$

2. One plate of each capacitor is connected to one point and the other plate of each capacitor is connected to another point. Therefore, all four capacitors are in parallel and voltage across them is 100 V. Hence, charge on each capacitor is

$$q = CV = 50 \times 10^{-6} \times 100 = 5 \times 10^{-3} \text{ C}$$

3. When a capacitor remains connected to the battery and a dielectric slab is introduced between the plates, the p.d. across the plates cannot change. The capacitance of the capacitor increases. Since $C = q/V$, the charge on the plates must increase. This **charge is supplied by the battery.**

4. Suppose the capacitance of the capacitor is $C$ prior to the introduction of the metal foil. When metal foil of negligible thickness is introduced at the centre, the system is equivalent to series combination of two capacitors each of capacitance $2C$ ($= \varepsilon_0 A/d/2 = 2\varepsilon_0 A/d = 2C$).

$$\therefore \qquad \frac{1}{C_S} = \frac{1}{2C} + \frac{1}{2C} \quad \text{or} \quad C_S = C$$

5. Let the potential of point $D$ be $V$. If $q$ is the charge on each capacitor, then,

$$V_1 - V = qC_1 \quad \text{and} \quad V - V_2 = qC_2$$

$$\therefore \qquad \frac{V_1 - V}{V - V_2} = \frac{C_1}{C_2} \quad \text{or} \quad VC_1 - V_2C_1 = V_1C_2 - VC_2$$

$$\therefore \qquad V(C_1 + C_2) = C_1V_2 + C_2V_1 \quad \text{or} \quad V = \frac{C_1V_2 + C_2V_1}{C_1 + C_2}$$

6. The capacitors $C_2$ and $C_4$ are in parallel and their equivalent capacitance = 1 + 1 = 2 μF. We now have three capacitors of capacitance 1 μF, 2 μF and 1 μF in series.

$$\therefore \qquad \frac{1}{C_{AB}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{1} = \frac{5}{2} \quad \therefore \quad C_{AB} = \frac{2}{5} = 0.4\ \mu\text{F}$$

7. When capacitors are connected in series, $C_S = \dfrac{C_1C_2}{C_1 + C_2} = \dfrac{10 \times 20}{10 + 20} = \dfrac{20}{3}\mu\text{F}$

Charge on each capacitor $= C_S V = \dfrac{20}{3} \times 200 = \dfrac{4000}{3}\mu\text{C}$

When capacitors are disconnected and reconnected, they are in parallel. Their equivalent capacitance = $C_1 + C_2$ = 10 + 20 = 30 μF. Since charge remains same, total charge = 2 × 4000/3 μC.

$$\therefore \text{Common potential} = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{2 \times 4000/3}{30} = \frac{800}{9}\text{V}$$

8. When the two capacitors are connected in series, their equivalent capacitance $= \dfrac{2 \times 2}{2 + 2} = 1\mu\text{F}$. When the third capacitor is connected in parallel with this series combination, the total capacitance = 1 + 2 = 3 μF.

As shown in Fig. 4.66, $C_{AB}$ = 3 μF.

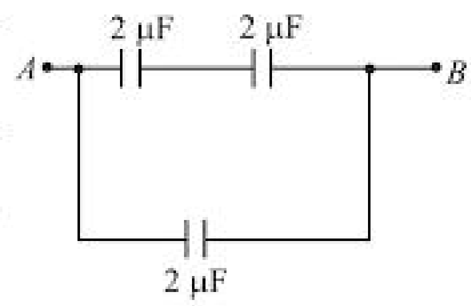



**Fig. 4.66**

9. $V = \dfrac{q}{4\pi\varepsilon_0 R}$. Therefore, potential will be more on the smaller sphere.

10. When two capacitors are connected in parallel, the total capacitance = 2 + 2 = 4 μF. When two series connected capacitors are connected in parallel with the first combination, the total capacitance = 4 + 1 = 5 μF . As shown in Fig. 4.67, $C_{AB}$ = 5μF. Therefore, the minimum number of capacitors required is 4.

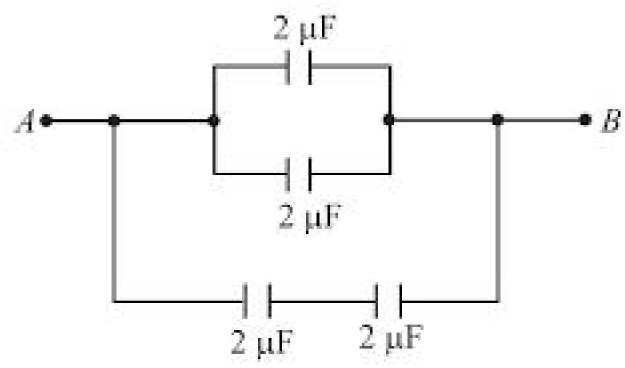



**Fig. 4.67**

11. Since the products of capacitances of opposite arms of Wheatstone bridge are equal, the bridge is balanced. Therefore, there is no charge on the capacitor connected between $C$ and $D$. This capacitor is ineffective and

may be considered as removed. Now branch $ACB$ has two 10 μF capacitances in series and their equivalent capacitance $C'$ is given by ;

$$C' = \frac{10 \times 10}{10 + 10} = \frac{100}{20} = 5\mu F$$

Again in branch $ADB$, two 10 μF capacitances are in series and their equivalent capacitance $C''$ is given by ;

$$C'' = \frac{10 \times 10}{10 + 10} = 5\mu F$$

Now $C'$ and $C''$ are in parallel so that $C_{AB} = C' + C'' = 5 + 5 =$ **10 μF.**

**12.** Capacitance of parallel plate air capacitor $C = \varepsilon_0 A/d$. When a dielectric slab of thickness $t$ $(t < d)$ and of dielectric constant $K$ is introduced between the plates of the capacitor, its capacitance is increased and is given by ;

$$C' = \frac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{K}\right)}$$

In order to restore capacitance to original value $(= \varepsilon_0 A/d)$, the distance between the plates must be increased by $x$ given by ;

$$x = t\left(1 - \frac{1}{K}\right) \quad \text{or} \quad 3.5 \times 10^{-5} = 4 \times 10^{-5}\left(1 - \frac{1}{K}\right) \quad \therefore \quad K = \mathbf{8}$$

**13.** When the plates of the charged capacitor are connected by a conducting wire, the energy stored in the capacitor is dissipated as heat in the connecting wire.

$$\therefore \text{ Heat produced } = \frac{1}{2}CV^2 = \frac{1}{2} \times (2 \times 10^{-6}) \times (100)^2$$

$$= \mathbf{0.01\ J}$$

**14.** Fig. 4.68 shows the conditions of the problem.

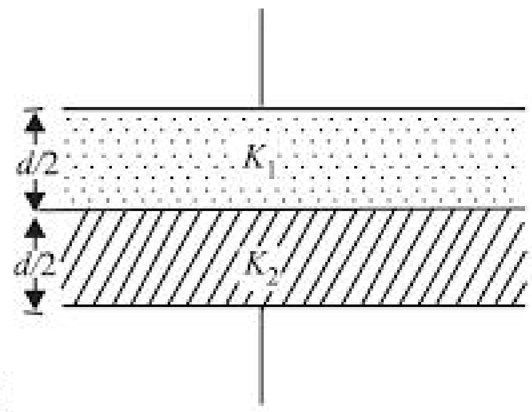



**Fig. 4.68**

$$C_1 = \frac{K_1 \varepsilon_0 A}{d/2} = \frac{2K_1 \varepsilon_0 A}{d}; \qquad C_2 = \frac{2K_2 \varepsilon_0 A}{d}$$

The arrangement constitutes two capacitors $C_1$ and $C_2$ in series. Therefore total capacitance $C_S$ of the composite capacitor is

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1 \varepsilon_0 A} + \frac{d}{2K_2 \varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$$

$$\therefore \qquad C_S = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$$

**15.** $C_1 = 10\mu F$ ; Initial charge on $C_1$, $q_1 = C_1 V_1 = 10 \times 50 = 500\ \mu C$ ; $C_2 = ?$

When the uncharged capacitor $C_2$ is connected in parallel with $C_1$, the total capacitance $= C_1 + C_2$. However, the charge remains the same *i.e.* 500 μC.

$$\therefore \text{ Common potential, } V = \frac{\text{Total charge}}{\text{Total capacitance}} \quad \text{or} \quad 20 = \frac{500}{10 + C_2} \quad \therefore \quad C_2 = \mathbf{15\ \mu F}$$

**16.** $U = q^2/2C$ where $q$ is the charge on the capacitor. When the battery is disconnected and an identical uncharged capacitor (*i.e.* of capacitance $C$) is connected in parallel with the first capacitor, the charge $q$ is shared equally by the two capacitors.

$\therefore$ Energy stored in each capacitor $= \dfrac{(q/2)^2}{2C} = \dfrac{1}{4} \times \dfrac{q^2}{2C} = \dfrac{U}{4}$

**17.** Since the capacitors are connected in series, charge on each capacitor is the same.

$C_S = \dfrac{C_A C_B}{C_A + C_B} = \dfrac{2 \times 3}{2+3} = \dfrac{6}{5}\mu F$; Charge on each capacitor $= \dfrac{6}{5}\mu F \times 10 V = 12 \mu C$

P.D. across $A = \dfrac{12\mu C}{2\mu F} = 6V$ ; P.D. across $B = \dfrac{12\mu C}{3\mu F} = 4V$

Therefore, option ($b$) is correct.

**18.** Capacitance of parallel-plate air capacitor, $C_0 = \dfrac{\varepsilon_0 A}{d}$

Capacitance of parallel-plate capacitor with slab, $C_m = \dfrac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{K}\right)}$

$$\therefore \quad \frac{C_m}{C_0} = \frac{d}{d - t\left(1 - \dfrac{1}{K}\right)} = \frac{6}{6 - 4.5\left(1 - \dfrac{1}{9}\right)} = 3 \quad \therefore \quad C_m = 3C_0$$

**19.** In the branch $DEF$, two 2μF capacitances are in series and their equivalent capacitance = 2/2 = 1μF. Now this 1 μF capacitance is in parallel with 1μF capacitance in the branch $DF$. Therefore, $C_{DF} = 1 + 1 = 2\mu F$. Now $C_{DF}$ ( = 2μF) is in series with 2μF in branch $FG$ and the equivalent capacitance = 2/2 = 1μF. This 1μF is in parallel with 1μF in branch $DG$.

$$\therefore \quad C_{AB} = 1 + 1 = 2\mu F$$

**20.** Energy stored in parallel-plate air capacitor, $E = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\dfrac{\varepsilon_0 A}{d}V^2$. On filling the space between the plates completely with dielectric,

$$C' = KC; V' = \frac{V}{K}$$

$\therefore$ Energy stored, $E' = \dfrac{1}{2}C'V'^2 = \dfrac{1}{2}\dfrac{K\varepsilon_0 A}{d} \times \left(\dfrac{V}{K}\right)^2 = \dfrac{\varepsilon_0 AV^2}{2dK}$

Work done = Decrease in energy $= E - E' = \dfrac{1}{2}\dfrac{\varepsilon_0 AV^2}{d} - \dfrac{\varepsilon_0 AV^2}{2dK} = \dfrac{\varepsilon_0 AV^2}{2d}\left(1 - \dfrac{1}{K}\right)$

## NUMERICAL PROBLEMS FOR COMPETITIVE EXAMINATIONS

**1. A capacitor of 20 μF and charged to 500 V is connected in parallel with another capacitor of 10 μF charged to 200 V. Find the common potential. [Roorkee]**

**Hint.** Charge on one capacitor, $q_1 = C_1 V_1 = (20 \times 10^{-6}) \times 500 = 0.01$ C

Charge on second capacitor, $q_2 = C_2 V_2 = (10 \times 10^{-6}) \times 200 = 0.002$ C

Total charge on capacitors, $q = q_1 + q_2 = 0.01 + 0.002 = 0.012$ C

Total capacitance, $C = C_1 + C_2 = (20 \times 10^{-6}) + (10 \times 10^{-6}) = 30 \times 10^{-6}$ F

$\therefore$ Common potential, $V = \dfrac{q}{C} = \dfrac{0.012}{30 \times 10^{-6}} = $ **400 V**

**2. Five equal capacitors connected in series have a resultant capacitance of 4 μF. What is the total energy stored in these when connected in parallel and charged to 400 V ? [E.A.M.C.E.T.]**

**Hint.** Suppose $C$ μF is the capacitance of each capacitor. Since 5 (= $n$) capacitors are connected in series,

$$\frac{C}{n} = 4 \quad \text{or} \quad C = 4\,n = 4 \times 5 = 20\ \mu\text{F}$$

When the capacitors are connected in parallel, then equivalent capacitance $C'$ is

$$C' = 5 \times 20 = 100\ \mu\text{F} = 100 \times 10^{-6}\ \text{F}$$

Energy stored is given by ;

$$U = \frac{1}{2}\,C'\,V^2 = \frac{1}{2} \times (100 \times 10^{-6}) \times (400)^2 = \mathbf{8\ J}$$

**3. Find the length of the paper used in a capacitor of capacitance 2 μF if the dielectric constant of the paper is 2.5 and its width and thickness are 50 mm and 0.05 mm respectively.**

**Hint.** $$C = \frac{\varepsilon_0 K A}{d}$$

Here $C = 2\ \mu\text{F} = 2 \times 10^{-6}$ F; $K = 2.5$; $d = 0.05$ mm $= 0.05 \times 10^{-3}$ m

$$\therefore \quad A = \frac{Cd}{\varepsilon_0 K} = \frac{(2 \times 10^{-6}) \times (0.05 \times 10^{-3})}{8.85 \times 10^{-12} \times 2.5} = 4.5\text{m}^2$$

$$\therefore \quad \text{Length} = \frac{\text{Area}}{\text{Width}} = \frac{4.5}{50 \times 10^{-3}}\ \text{m} = \mathbf{90\ m}$$

**4. A 5 μF capacitor is fully charged across a 12 V battery and connected to an uncharged capacitor. The voltage across it is found to be 3 V. What is the capacity of the uncharged capacitor ? [E.A.M.C.E.T.]**

**Hint.** The common potential $V$ after connection is

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Here $C_1 = 5$ μF; $V = 3$ volts; $V_1 = 12$ volts; $V_2 = 0$; $C_2 = ?$

$$\therefore \quad 3 = \frac{5 \times 12 + C_2 \times 0}{5 + C_2}$$

$$\therefore \quad C_2 = \mathbf{15\mu F}$$

**5. A parallel-plate capacitor has plates of dimensions 2 cm × 3 cm. The plates are separated by a 1 mm thickness of paper.**

**(*i*) Find the capacitance of the paper capacitor. The dielectric constant of paper is 3.7.**

**(*ii*) What is the maximum charge that can be placed on the capacitor ? The dielectric strength of paper is 16 × 10⁶ V/m.**

**Hint.** (*i*) $$C = \frac{\varepsilon_0 K A}{d}$$

Here $\varepsilon_0 = 8.85 \times 10^{-12}$ C$^2$ N$^{-1}$ m$^{-2}$; $K = 3.7$; $A = 6 \times 10^{-4}$ m$^2$; $d = 1 \times 10^{-3}$ m

$$C = \frac{(8.85\times10^{-12})\times(3.7)\times(6\times10^{-4})}{1\times10^{-3}} = \mathbf{19.6 \times 10^{-12}\ F}$$

(*ii*) Since the thickness of the paper is 1 mm, the maximum voltage that can be applied before breakdown occurs is

$$V_{max} = E_{max} \times d$$

Here $E_{max} = 16 \times 10^6$ V/m; $d = 1$ mm $= 1 \times 10^{-3}$ m

$\therefore$ $V_{max} = (16 \times 10^6) \times (1 \times 10^{-3}) = 16 \times 10^3$ V

$\therefore$ Maximum charge that can be placed on capacitor is

$$q_{max} = C\,V_{max} = (19.6 \times 10^{-12}) \times (16 \times 10^3) = 0.31 \times 10^{-6}\ \text{C} = \mathbf{0.31\ \mu C}$$

**6. A parallel plate capacitor is maintained at a certain potential difference. When a 3mm slab is introduced between the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab. [M.N.R.]**

**Hint.** The capacitance of parallel-plate capacitor in air is

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{... }(i)$$

With the introduction of slab of thickness $t$, the new capacitance is

$$C' = \frac{\varepsilon_0 A}{d' - t(1 - 1/K)} \qquad \text{... }(ii)$$

Now the charge ($q = CV$) remains the same in the two cases.

$$\therefore \quad \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t(1 - 1/K)}$$

or $\quad d = d' - t\,(1 - 1/K\,)$

Here $\quad d' = d + 2.4 \times 10^{-3}$ m ; $t = 3$ mm $= 3 \times 10^{-3}$ m

$$\therefore \quad d = d + 2.4 \times 10^{-3} - 3 \times 10^{-3}\left(1 - \frac{1}{K}\right)$$

$$\text{or} \quad 2.4 \times 10^{-3} = 3 \times 10^{-3}\left(1 - \frac{1}{K}\right)$$

$\therefore \quad K = \mathbf{5}$

**7. The capacitance of a variable radio capacitor can be changed from 50 pF to 950 pF by turning the dial from 0° to 180°. With dial set at 180°, the capacitor is connected to 400 V battery. After charging, the capacitor is disconnected from the battery and the dial is turned at 0°.**

**(*i*) What is the potential difference across the capacitor when the dial reads 0° ?**

**(*ii*) How much work is required to turn the dial if friction is neglected ? [M.N.R.]**

**Hint.** (*i*) With dial at 0°, the capacitance of the capacitor is

$$C_1 = 50\ \text{pF} = 50 \times 10^{-12}\ \text{F}$$

With dial at 180°, the capacitance of the capacitor is

$$C_2 = 950\ \text{pF} = 950 \times 10^{-12}\ \text{F}$$

P.D. across $C_2$, $V_2 = 400$ V

$\therefore$ Charge on $C_2$, $q = C_2V_2 = (950 \times 10^{-12}) \times 400 = 380 \times 10^{-9}$ C

When the battery is disconnected, charge $q$ remains the same. Suppose $V_1$ is the potential difference across the capacitor when the dial reads 0°.

$$\therefore \quad q = C_1V_1$$

$$\text{or} \quad V_1 = \frac{q}{C_1} = \frac{380 \times 10^{-9}}{50 \times 10^{-12}} = \mathbf{7600\ V}$$

(*ii*) Work required = Gain in energy of the capacitor

$$= \frac{1}{2}C_1V_1^2 - \frac{1}{2}C_2V_2^2$$

$$= \frac{1}{2} \times 50 \times 10^{-12} \times (7600)^2 - \frac{1}{2} \times 950 \times 10^{-12} \times (400)^2$$

$$= \mathbf{1.37 \times 10^{-3}\ J}$$

**8. A 90 pF capacitor is connected to a 12 V battery and is charged to 12 V. How many electrons are transferred from one plate to the other?**

**Hint.** $q = CV = (90 \times 10^{-12}) \times (12) = 1.1 \times 10^{-9}$ C

Now $q = ne$

$\therefore$ Number of electrons transferred is

$$n = \frac{q}{e} = \frac{1.1 \times 10^{-9}}{1.6 \times 10^{-19}} = \mathbf{6.9 \times 10^9}$$

**9. *N* drops of mercury of equal radii and possessing equal charges combine to form a big drop. What is the charge, capacitance and potential of the bigger drop?**

**Hint.** Let $q$, $v$ and $c$ be the charge, potential and capacitance of the individual small drop. The corresponding quantities for the bigger drop are $Q$, $V$ and $C$.

Charge on bigger drop = $N$ × charge on small drop

$$\therefore \quad Q = \mathbf{Nq}$$

The capacitance of a spherical drop is proportional to the radius.

$$\therefore \quad \frac{C}{c} = \frac{R}{r}$$

Since mass is a conserved quantity,

$$\therefore \quad \frac{4}{3}\pi R^3 \rho = N \times \frac{4}{3}\pi r^3 \rho$$

$$\text{or} \quad R = r \times N^{1/3}$$

$$\text{or} \quad \frac{R}{r} = N^{1/3}$$

$$\therefore \quad \frac{C}{c} = N^{1/3} \quad \text{or} \quad C = \mathbf{c \times N^{1/3}}$$

$$\text{Now} \quad V = \frac{Q}{C} \text{ and } v = \frac{q}{c}$$

$$\therefore \quad \frac{V}{v} = \left(\frac{Q}{q}\right) \times \left(\frac{c}{C}\right) = (N) \times \left(\frac{1}{N^{1/3}}\right) = N^{2/3}$$

or $$V = v \times N^{2/3}$$

**10. An infinite identical capacitors each of capacitance 1 μF are connected as shown in Fig. 4.69. What is the equivalent capacitance between terminals $A$ and $B$ ?**

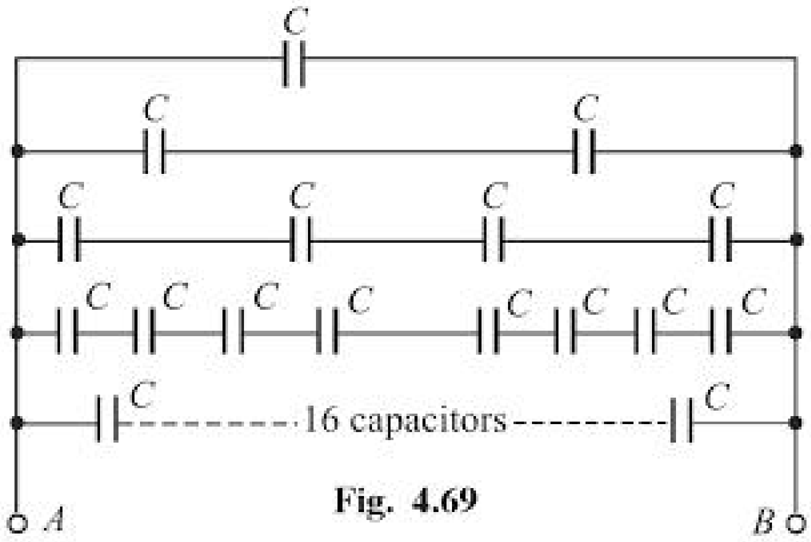



Fig. 4.69

**Hint.** It is clear from the figure that the rows of capacitors are connected in parallel. The capacitance of first row, second row, third row, fourth row ... . is $C$, $C/2$, $C/4$, $C/8$ ... .

The equivalent capacitance of the arrangement is

$$C_{AB} = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + .... = C\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right)$$

$$= C\left[\frac{1}{1-1/2}\right] = 2C = 2 \times 1 = 2\ \mu F$$

**11. A spherical capacitor has 10 cm and 12 cm as the radii of inner and outer spheres. The space between the two is filled with a dielectric of dielectric constant 5. Find the capacitance when (*i*) the outer sphere is earthed and (*ii*) inner sphere is earthed.**

**Hint.** (*i*) When the outer sphere is earthed, the capacitance of the spherical capacitor is

$$C = 4\pi\varepsilon_0 K \frac{r_A r_B}{r_B - r_A}$$

Here $4\pi\varepsilon_0 = \frac{1}{9\times10^9}$ $C^2\ N^{-1}\ m^{-2}$; $K = 5$; $r_A = 0.1$ m; $r_B = 0.12$ m

$$\therefore \quad C = \frac{1}{9\times10^9} \times 5 \times \frac{0.1\times0.12}{0.12-0.1} = 3.33 \times 10^{-10}\ F$$

(*ii*) When the inner sphere is earthed, the system is equivalent to capacitors in parallel. One capacitor is between outer sphere and earth and the other capacitor is between outer sphere and inner earthed sphere. The equivalent capacitance is

$$C = 4\pi\varepsilon_0\ b + 4\pi\varepsilon_0\ K\ \frac{r_A r_B}{r_B - r_A}$$

$$= \frac{1}{9\times10^9} \times 0.12 + \frac{1}{9\times10^9} \times 5 \times \frac{0.1\times0.12}{0.12-0.1}$$

$$= 0.13 \times 10^{-10} + 3.33 \times 10^{-10}$$

$$= 3.46 \times 10^{-10}\ F$$

**12. Find the charge on 5 μF capacitor in the circuit shown in Fig. 4.70.**

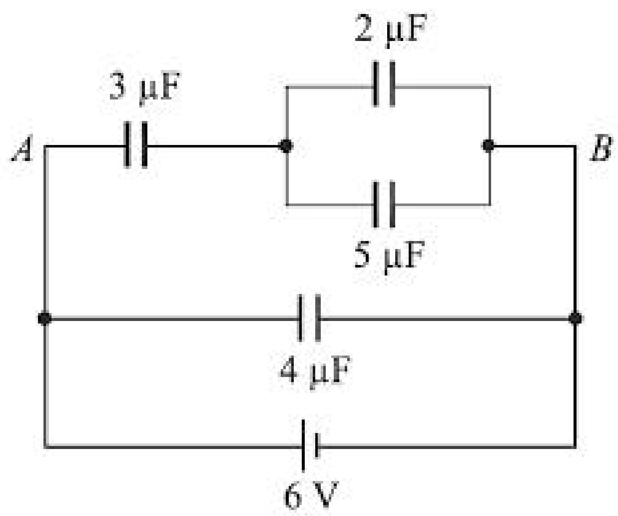



Fig. 4.70

**Hint.** The p.d. between $AB$ is 6 V. Considering the branch $AB$, the capacitors 2 μF and 5 μF are in parallel and their equivalent capacitance = 2 + 5 = 7 μF. The branch $AB$ then has 7 μF and 3 μF in series. Therefore, the effective capacitance of branch $AB$ is

$$C_{AB} = \frac{7\times 3}{7+3} = \frac{21}{10}\mu F$$

Total charge in branch $AB$, $q = C_{AB}V = \frac{21}{10} \times 6 = \frac{63}{5}$ μC

P.D. across 3 μF capacitor $= \frac{q}{3} = \frac{63}{5} \times \frac{1}{3} = \frac{21}{5}$ volts

∴ P.D. across parallel combination $= 6 - \frac{21}{5} = \frac{9}{5}$ volts

Charge on 5 μF capacitor $= (5 \times 10^{-6}) \times \frac{9}{5} = 9 \times 10^{-6}$ C = **9 μC**

**13. Two parallel plate capacitors $A$ and $B$ having capacitance of 1 μF and 5 μF are charged separately to the same potential of 100 V. Now positive plate of $A$ is connected to the negative plate of $B$ and the negative plate of $A$ is connected to the positive of $B$. Find the final charge on each capacitor.**

**Hint.** Initial charge on $A$, $q_1 = C_1V = (1 \times 10^{-6}) \times 100 = 100$ μC

Initial charge on $B$, $q_2 = C_2V = (5 \times 10^{-6}) \times 100 = 500$ μC

When the oppositely charged plates of $A$ and $B$ are connected together, the net charge is

$$q = q_2 - q_1 = 500 - 100 = 400 \text{ μC}$$

Final potential difference $= \frac{\text{Net charge}}{\text{Net capacitance}} = \frac{400\times 10^{-6}}{(1+5)10^{-6}} = \frac{200}{3}$ V

Final charge on $A = C_1 \times \frac{200}{3} = (1 \times 10^{-6}) \times \frac{200}{3} = \frac{200}{3}$ μC

Final charge on $B = C_2 \times \frac{200}{3} = (5 \times 10^{-6}) \times \frac{200}{3} = \frac{1000}{3}$ μC

**14. A capacitor is filled with two dielectrics of the same dimensions but of dielectric constants $K_1$ and $K_2$ respectively. Find the capacitances in two possible arrangements** **[M.N.R.]**

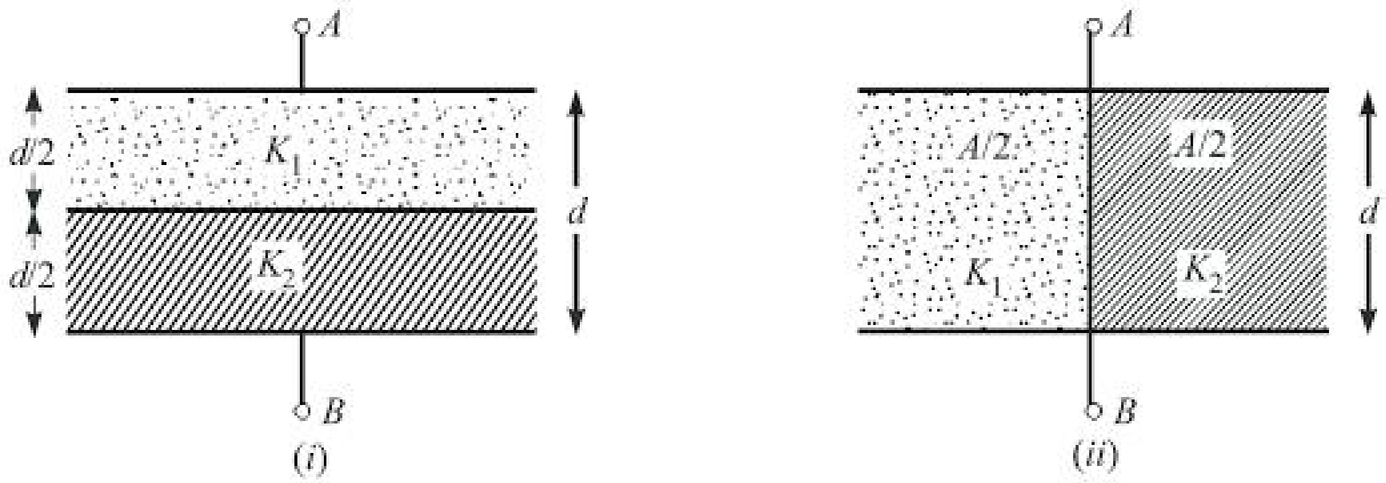



**Fig. 4.71**

**Hint.** The two possible arrangements are shown in Fig. 4.71.

(*i*) The arrangement shown in Fig. 4.71(*i*) is equivalent to two capacitors in series, each with plate area $A$ and plate separation $d/2$ *i.e.*,

$$C_1 = \frac{K_1 \varepsilon_0 A}{d/2} = \frac{2K_1 \varepsilon_0 A}{d}$$

$$C_2 = \frac{K_2 \varepsilon_0 A}{d/2} = \frac{2K_2 \varepsilon_0 A}{d}$$

The equivalent capacitance $C'$ is given by ;

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1 \varepsilon_0 A} + \frac{d}{2K_2 \varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$$

$$\therefore \quad C' = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$$

(*ii*) The arrangement shown in Fig. 4.71 (*ii*) is equivalent to two capacitors in parallel, each with plate area $A/2$ and plate separation $d$ *i.e.*,

$$C_1 = \frac{K_1 \varepsilon_0 (A/2)}{d} = \frac{K_1 \varepsilon_0 A}{2d}$$

$$C_2 = \frac{K_2 \varepsilon_0 (A/2)}{d} = \frac{K_2 \varepsilon_0 A}{2d}$$

The equivalent capacitance $C''$ is given by ;

$$C'' = C_1 + C_2 = \frac{K_1 \varepsilon_0 A}{2d} + \frac{K_2 \varepsilon_0 A}{2d} = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

$$\therefore \quad C'' = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

# N.C.E.R.T. TEXTBOOK EXERCISES

## NCERT CHAPTER 1 : ELECTRIC CHARGES AND FIELDS

**Q.1 What is the force between two small charged spheres having charges of $2 \times 10^{-7}$ C and $3 \times 10^{-7}$ C placed 30 cm apart in air?**

**Ans.** Here, $q_1 = 2 \times 10^{-7}$ C; $q_2 = 3 \times 10^{-7}$ C ; $r = 30$ cm $= 30 \times 10^{-2}$ m

$$\therefore \quad \text{Force, } F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2} = 9\times10^9 \times \frac{(2\times10^{-7})\times(3\times10^{-7})}{(30\times10^{-2})^2} = 6 \times 10^{-3} \text{ N}$$

The positive sign with force means that the force is repulsive.

**Q.2 The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge –0.8 μC in air is 0.2 N. (*a*) What is the distance between the two spheres? (*b*) What is the force on the second sphere due to the first?**

**Ans.** Here, $q_1 = 0.4$ μC $= 0.4 \times 10^{-6}$ C ; $q_2 = -0.8$ μC $= -0.8 \times 10^{-6}$ C ; $F = 0.2$ N

(*a*) Now
$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2} \quad \therefore \quad r^2 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{F}$$

or
$$r^2 = 9 \times 10^9 \times \frac{(0.4\times10^{-6})\times(0.8\times10^{-6})}{0.2} = 144 \times 10^{-4}$$

$$\therefore \quad r = \sqrt{144\times10^{-4}} = 12\times10^{-2} \text{ m} = 0.12 \text{ m}$$

(*b*) The force on the second sphere due to the first is the same *i.e.* 0.2N

**Q.3 Check that the ratio $ke^2/G\, m_e m_p$ is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?**

**Ans.**
$$\frac{ke^2}{G\, m_e m_p} = \frac{[\text{N m}^2\, \text{C}^{-2}]\,[\text{C}^2]}{[\text{N m}^2\, \text{kg}^{-2}]\,[\text{kg}]\,[\text{kg}]} = 1 = [\text{M}^\circ\ \text{L}^\circ\ \text{T}^\circ] = \text{Dimensionless}$$

Now
$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2} \ ; \ e = 1.6 \times 10^{-19} \text{ C} \ ;$$
$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \ ; \ m_e = 9.1 \times 10^{-31} \text{ kg} \ ;$$
$$m_p = 1.66 \times 10^{-27} \text{ kg}$$

$$\therefore \quad \frac{ke^2}{G\, m_e m_p} = \frac{9\times10^9 \times (1.6\times10^{-19})^2}{6.67\times10^{-11} \times (9.1\times10^{-31}) \times (1.66\times10^{-27})} = 2.29 \times 10^{39}$$

This is the ratio of electrostatic force to gravitational force between an electron and a proton.

**Q.4 (*a*) Explain the meaning of the statement 'Electric charge of a body is quantized'.**

**(*b*) Why can one ignore quantization of electric charge when dealing with macroscopic *i.e.*, large scale charges?**

**Ans.** (*a*) Refer to Art. 1.12

(*b*) The charge on a body is $q = \pm ne$ where $n = 1, 2, 3$ .... and $e = 1.6 \times 10^{-19}$C. At macroscopic level, we deal with charges that are of very large magnitude as compared to magnitude of charge $e$. For example, the number ($n$) of electrons in 1 μC charge is

$$n = \frac{q}{e} = \frac{1\mu\text{C}}{1.6\times10^{-19}\text{C}} = \frac{1\times10^{-6}\text{ C}}{1.6\times10^{-19}\text{ C}} \simeq 6.2\times10^{12}$$

Thus a charge of 1 μC contains $6.2 \times 10^{12}$ electrons—an extremely large number. Thus if electrons are added or removed from such a body, it hardly makes any change in the charge. Therefore, the quantization of the charge can be ignored while dealing with large scale charges.

**Q.5 When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.**

**Ans.** Refer to Art. 1.13.

**Q.6 Four point charges $q_A$ = 2 μC, $q_B$ = –5 μC, $q_C$ = 2 μC, and $q_D$ = –5 μC are located at the corners of a square *ABCD* of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the square?**

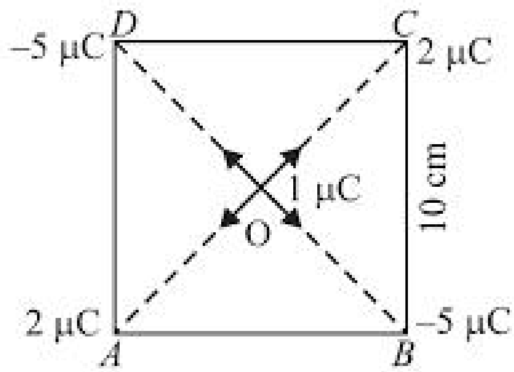



**Fig. 1.01**

**Ans.** Fig. 1.01 shows the conditions of the problem. By geometry, *OA* = *OC*. Therefore, charge of 2 μC at *A* and 2 μC at C exert equal and opposite replusion forces on 1 μC charge placed at centre *O* of the square. Similarly, charge of –5 μC at *B* and –5 μC at *D* exert equal and opposite attraction forces on charge 1 μC placed at *O*. Consequently, the net force on charge of 1 μC at *O* is **zero.**

**Q.7** (*a*) **An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?**

(*b*) **Explain why two field lines never cross each other at any point?**

**Ans.** (*a*) An electric line of force represents the actual path of a unit positive charge. The tangent to the field line at any point represents the direction of electric field at that point. The electric lines of force are generally curved lines because electric field varies from point to point. Note that electric lines of force are continuous curves and cannot have sudden breaks. It is because break points will indicate absence of electric field.

(*b*) Refer to Art. 2.10.

**Q.8 Two point charges $q_A$ = 3 μC and $q_B$ = –3 μC are located 20 cm apart in vacuum.**

**(*a*) What is the electric field at the mid point *O* of the line *AB* joining the two charges?**

**(*b*) If a negative test charge of magnitude 1.5 × $10^{-9}$ C is placed at this point, what is the force experienced by the test charge?**

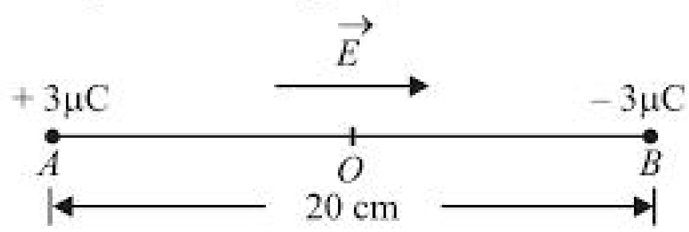



**Fig. 1.02**

**Ans.** Fig. 1.02 shows the conditions of the problem. Here *O* is the mid-point *i.e.* *AO* = *OB* = 10 cm = 0.1 m.

(*a*) Electric field at *O* due to charge + 3 μC is

$$\vec{E_1} = 9 \times 10^9 \times \frac{3\times10^{-6}}{(0.1)^2} = 2.7 \times 10^6 \text{ N/C along } AO$$

Electric field at *O* due to charge –3 μC is

$$\vec{E_2} = 9 \times 10^9 \times \frac{3\times10^{-6}}{(0.1)^2} = 2.7 \times 10^6 \text{ N/C along OB}$$

∴Resultant field at *O* is given by ;

$$\vec{E} = \vec{E_1} + \vec{E_2} = \mathbf{5.4 \times 10^6 \text{ N/C } \textit{along OB}}$$

(*b*) Force on charge *q* (= 1.5 × $10^{-9}$ C) placed at *O* is

$$\vec{F} = q\vec{E} = (1.5 \times 10^{-9}) \times (5.4 \times 10^6) = \mathbf{8.1 \times 10^{-3} \text{ N } \textit{along OA}}$$

**Q.9 A system has two charges $q_A$ = 2.5 × $10^{-7}$ C and $q_B$ = –2.5 × $10^{-7}$ C located at points A: (0, 0, –15 cm) and B : (0, 0, + 15 cm), respectively. What are the total charge and electric dipole moment of the system?**

**Ans.** Fig. 1.03 shows the conditions of the problem. It is clear that the two charges constitute an electric dipole.

Total charge, $q = q_A + q_B$

$= (2.5 \times 10^{-7}) + (-2.5 \times 10^{-7}) = 0\ C$

Separation of charges,

$AB = 15 + 15 = 30$ cm

$= 30 \times 10^{-2}$ m

Electric dipole moment,

$p$ = either charge $\times AB$

$= (2.5 \times 10^{-7}) \times 30 \times 10^{-2}$

$= 7.5 \times 10^{-8}$ Cm

Note that the direction of $\vec{p}$ is from $q_B$ ($-ve$ charge) to $q_A$ ($+ve$ charge) *i.e.* along negative $Z$-axis.

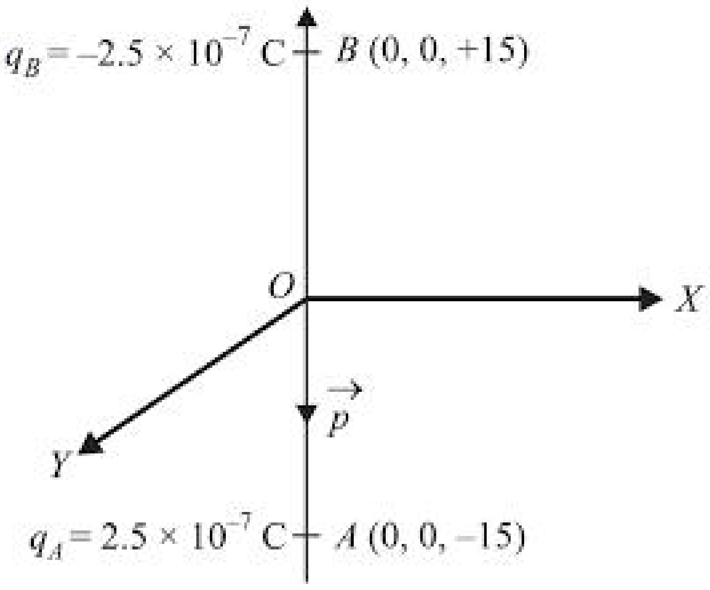



Fig. 1.03

**Q.10 An electric dipole with dipole moment $4 \times 10^{-9}$Cm is aligned at 30° with the direction of a uniform electric field of magnitude $5 \times 10^4$ NC$^{-1}$. Calculate the magnitude of the torque acting on the dipole.**

**Ans.** Here, $p = 4 \times 10^{-9}$ Cm ; $\theta = 30°$; $E = 5 \times 10^4$ NC$^{-1}$

Now, Torque, $\tau = pE \sin\theta = 4 \times 10^{-9} \times 5 \times 10^4 \sin 30° = 10^{-4}$ Nm

**Q.11 A polythene piece rubbed with wool is found to have a negative charge of $3 \times 10^{-7}$ C.**

**(*a*) Estimate the number of electrons transferred (from which to which?).**

**(*b*) Is there a transfer of mass from wool to polythene?**

**Ans.** (*a*) Here, $q = -3 \times 10^{-7}$ C; Charge on electron, $e = -1.6 \times 10^{-19}$ C

Number of electrons transferred to polythene piece from wool is

$$n = \frac{q}{e} = \frac{-3\times10^{-7}}{-1.6\times10^{-19}} = 1.875 \times 10^{12}$$

(*b*) Since an electron has a finite mass ($m_e = 9 \times 10^{-31}$ kg), there is transfer of mass from wool to polythene.

Mass transferred to polythene $= 2 \times 10^{12} \times 9 \times 10^{-31} = 1.8 \times 10^{-18}$ kg

**Q.12 (*a*) Two insulated charged copper spheres $A$ and $B$ have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is $6.5 \times 10^{-7}$ C? The radii of $A$ and $B$ are negligible compared to the distance of separation.**

**(*b*) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?**

**Ans.** (*a*) Here, $q_1 = q_2 = 6.5 \times 10^{-7}$ C ; $r = 50$ cm $= 0.5$ m

$$\therefore \quad F = 9 \times 10^9 \times \frac{q_1 q_2}{r^2} = 9\times10^9 \times \frac{6.5\times10^{-7}\times6.5\times10^{-7}}{(0.5)^2}$$

$$= 1.521 \times 10^{-2} \text{ N}$$

(*b*) Here, $q_1 = q_2 = 2 \times 6.5 \times 10^{-7}$ C ; $r = 50/2 = 25$ cm $= 0.25$ m

$$\therefore \quad F = 9\times10^9 \times \frac{q_1 q_2}{r^2} = 9\times10^9 \times \frac{2\times6.5\times10^{-7}\times2\times6.5\times10^{-7}}{(0.25)^2}$$

$$= 0.24 \text{ N}$$

**Q.13 Suppose the spheres $A$ and $B$ in Q 12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between $A$ and $B$?**

**Ans.** Charge on sphere $A = 6.5 \times 10^{-7}$ C; Charge on sphere $B = 6.5 \times 10^{-7}$ C

When a third uncharged sphere $C$ of identical size is brought in contact with sphere $A$, their charges are shared equally.

$\therefore$ Charge on sphere $A$, $q_A = \dfrac{6.5 \times 10^{-7} + 0}{2} = 3.25 \times 10^{-7}$ C

Now the charge on sphere $C$ is $3.25 \times 10^{-7}$ C and when it is brought in contact with sphere $B$, their charges are shared equally.

$\therefore$ Charge on sphere $B$, $q_B = \dfrac{(6.5 + 3.25) \times 10^{-7}}{2} = 4.875 \times 10^{-7}$ C

$\therefore$ New force between spheres $A$ and $B$ is

$$F = 9\times10^9 \times \frac{q_A q_B}{r^2} = 9 \times 10^9 \times \frac{3.25\times10^{-7} \times 4.875\times10^{-7}}{(0.5)^2}$$

$$= \mathbf{5.7 \times 10^{-3}\ N}$$

**Q.14 Figure 1.04 shows tracks of three charged particles crossing a uniform electrostatic field with the same velocities along the horizontal. Give the signs of the three charges. Which particle has the highest charge to mass ratio?**

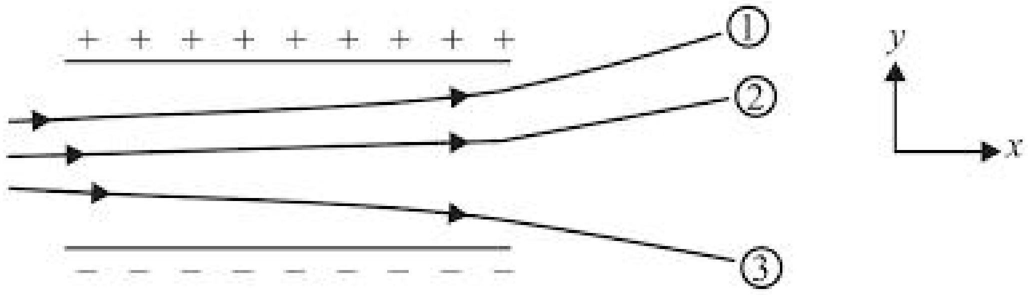



**Fig. 1.04**

**Ans.** Since particles 1 and 2 are attracted towards the positive plate, they are negatively charged. However, particle 3 is attracted towards the negative plate and hence it is positively charged. Since the particles enter the field with the same horizontal velocity, their initial velocity along the vertical is zero.

$$\text{Acceleration, } a = \frac{F}{m} = \frac{qE}{m}$$

$$\text{Vertical deflection, } y = v_0 t + \frac{1}{2}at^2 = 0 \times t + \frac{1}{2}\frac{qE}{m}t^2 = \frac{1}{2}\frac{qE}{m}t^2$$

Since the particles enter the field with the same horizontal velocity, they remain in the field for the same time $t$. Therefore $E$ and $t$ are constant in the expression for $y$.

$$\therefore \qquad y \propto \frac{q}{m}$$

Since vertical deflection is maximum for **particle 3,** its charge to mass ratio is the highest.

**Q.15 Consider a uniform electric field $\vec{E} = 3 \times 10^3\ \hat{i}$ N/C. (*a*) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the *yz* plane? (*b*) What is the flux through the same square if the normal to its plane makes a 60° angle with the *x*-axis?**

**Ans.** $\vec{E} = 3 \times 10^3\ \hat{i}$ N/C *i.e.* electric field is along + direction of $x$-axis.

Surface area of square, $\Delta S = 10 \times 10 = 100\ \text{cm}^2 = 100 \times 10^{-4}\ \text{m}^2 = 10^{-2}\ \text{m}^2$

Electric flux through square, $\phi_E = \vec{E} \cdot \vec{\Delta S} = E\Delta S \cos\theta$

Here $\theta$ is the angle between outward drawn normal to the square and the direction of electric field.

(*a*) In this case, $\theta = 0°$ so that electric flux through the square is

$$\phi_E = E\,\Delta S \cos 0° = 3 \times 10^3 \times 10^{-2} \times 1 = \mathbf{30\ NC^{-1}\ m^2}$$

(*b*) In this case, $\theta = 60°$ so that electric flux through the square is

$$\phi_E = E\,\Delta S \cos 60° = 3 \times 10^3 \times 10^{-2} \times 0.5 = \mathbf{15\ NC^{-1}\ m^2}$$

**Q.16 What is the net flux of the uniform electric field of Q 15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?**

**Ans.** The net flux through the cube is zero. It is because the number of electric field lines entering the cube is equal to the number of electric field lines leaving the cube.

**Q.17 Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is $8.0 \times 10^3$ Nm$^2$/C. (*a*) What is the net charge inside the box? (*b*) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?**

**Ans.** (*a*) $\phi_E = 8.0 \times 10^3$ Nm$^2$/C. According to Gauss's law,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\therefore \quad q = \varepsilon_0\,\phi_E = (8.85 \times 10^{-12}) \times 8.0 \times 10^3 = 0.07 \times 10^{-6}\text{ C} = 0.07\ \mu\text{C}$$

(*b*) When $\phi_E = 0$, $q = 0$. This means that either there is no charge inside the box or the algebraic sum of charges (*i.e.* net charge) inside the box is zero.

**Q.18 A point charge +10 μC is at a distance 5 cm directly above the centre of a square of side 10 cm. as shown in Fig. 1.05. What is the magnitude of the electric flux through the square? (*Hint*: Think of the square as one face of a cube with edge 10 cm.)**

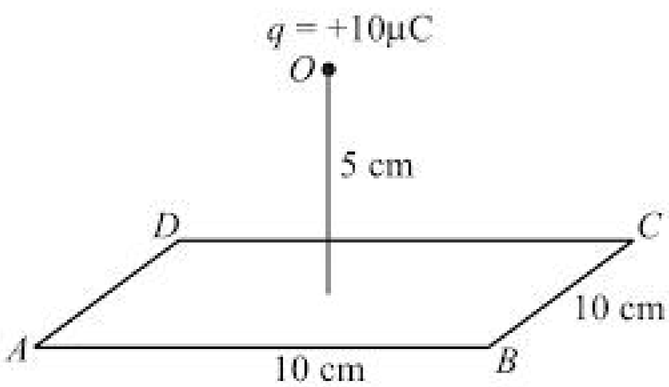



**Fig. 1.05**

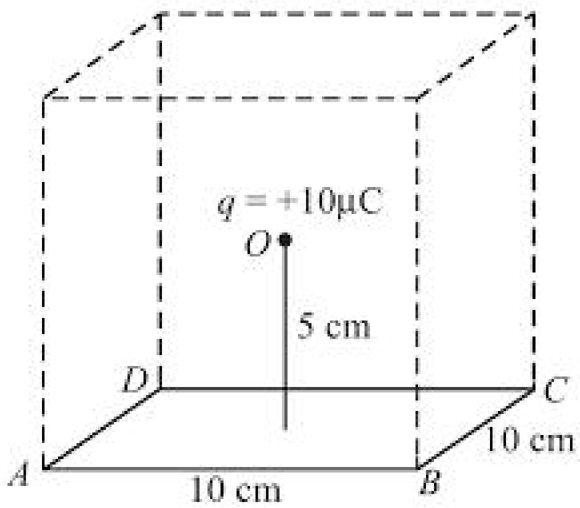



**Fig. 1.06**

**Ans.** Construct a cube with square as one of the faces such that charge is enclosed in this cube [See fig. 1.06]. Then cube is the Gaussian surface. According to Gauss's law, the electric flux passing through the cube $\phi_E = q/\varepsilon_0$. By symmetry, the electric flux passing through each face of the cube is the same.

$\therefore$ Electric flux through the square *ABCD*

$$= \frac{\phi_E}{6} = \frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.85 \times 10^{-12}} = 1.88 \times 10^5\ \text{Nm}^2\text{C}^{-1}$$

**Q.19 A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?**

**Ans.** $q = 2.0\ \mu\text{C} = 2.0 \times 10^{-6}$ C. According to Gauss's law,

$$\phi_E = \frac{q}{\varepsilon_0} = \frac{2.0 \times 10^{-6}}{8.85 \times 10^{-12}} = 2.26 \times 10^5\ \text{Nm}^2\text{C}^{-1}$$

**Q.20 A point charge causes an electric flux of $-1.0 \times 10^3$ Nm$^2$/C to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (*a*) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (*b*) What is the value of the point charge?**

**Ans.** $\phi_E = -1.0 \times 10^3$ Nm$^2$/C; Radius of Gaussian surface, $r$ = 10 cm = 0.1 m

(*a*) If the radius of the Gaussian surface is doubled, the electric flux through the new surface remains the same *i.e.* $-1.0 \times 10^3$ Nm$^2$/C. If is because electric flux $\phi_E$ $(=q/\varepsilon_0)$ depends only on charge ($q$) present inside the surface and the charge enclosed in the two cases is the same.

(*b*) Now $\phi_E = q/\varepsilon_0$

$\therefore$ $q = \varepsilon_0 \, \phi_E = (8.85 \times 10^{-12}) \times (-1.0 \times 10^3) = \mathbf{-8.85 \times 10^{-9}\ C}$

**Q.21 A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5 × 10³ N/C and points radially inward, what is the net charge on the sphere?**

**Ans.** Electric field at a point outside the charged sphere is

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2} = 9 \times 10^9 \times \frac{q}{r^2}$$

Here, $E = -1.5 \times 10^3$ N/C ....negative sign for inward field

$r$ = distance of the point from the centre of sphere = 20 cm = 0.2 m

$$\therefore \quad q = \frac{E\,r^2}{9 \times 10^9} = \frac{(-1.5 \times 10^3) \times (0.2)^2}{9 \times 10^9} = \mathbf{-6.67 \times 10^{-9}\ C}$$

**Q.22 A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC/m². (*a*) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?**

**Ans.** Radius of sphere, $r = 2.4/2 = 1.2$ m; $\sigma = 80.0\ \mu C/m^2 = 80.0 \times 10^{-6}\ C/m^2$

Surface area of sphere $= 4\pi r^2$

(*a*) Charge on sphere, $q = \sigma \times 4\pi r^2 = (80 \times 10^{-6}) \times 4\pi \times (1.2)^2 = \mathbf{1.45 \times 10^{-3}\ C}$

(*b*) Electric flux, $\phi_E = \dfrac{q}{\varepsilon_0} = \dfrac{1.45 \times 10^{-3}}{8.85 \times 10^{-12}} = \mathbf{1.6 \times 10^8\ Nm^2C^{-1}}$

**Q.23 An infinite line charge produces a field of 9 × 10⁴ N/C at a distance of 2 cm. Calculate the linear charge density.**

**Ans.** Here, $E = 9 \times 10^4$ N/C; $r = 2$ cm $= 2 \times 10^{-2}$ m

Electric field at a distance $r$ (perpendicular) from a linear charge having linear charge density $\lambda$ is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

or

$$\lambda = 2\pi\varepsilon_0 rE$$

$$= 2\pi \times 8.85 \times 10^{-12} \times 2 \times 10^{-2} \times 9 \times 10^4 = \mathbf{10^{-7}\ Cm^{-1}}$$

**Q.24 Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0 × 10⁻²² C/m². What is $\vec{E}$ (*a*) in the outer region of the first plate, (*b*) in the outer region of the second plate, and (*c*) between the plates?**

**Ans.** Here, $\sigma = 17.0 \times 10^{-22}\ Cm^{-2}$

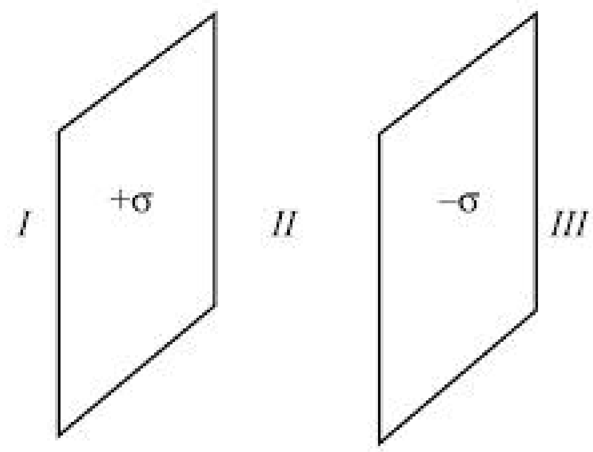



**Fig. 1.07**

(*a*) To the left of the plates (region I in Fig. 1.07), the electric fields of the two plates are equal and opposite. Therefore, electric field ($E$) to the left of plates is **zero.**

(*b*) To the right of the plates (region III in Fig. 1.07), the electric fields of the two plates are equal and opposite. Therefore, electric field to the right of plates is **zero.**

(*c*) Between the plates (region II in Fig. 1.07), electric fields due to the two plates are in the same direction. Therefore, the resultant field $E$ is

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{17.0 \times 10^{-22}}{8.85 \times 10^{-12}} = \mathbf{1.92 \times 10^{-10}\ NC^{-1}}$$

## ADDITIONAL EXERCISES

**Q.25 An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55 × $10^4$ $NC^{-1}$ in Millikan's oil drop experiment. The density of the oil is 1.26 g $cm^{-3}$. Estimate the radius of the drop. ($g$ = 9.81 m $s^{-2}$; $e$ = 1.60 × $10^{-19}$ C).**

**Ans.** Charge on oil drop, $q = 12\,e = 12 \times 1.6 \times 10^{-19} = 1.92 \times 10^{-18}$ C;

Mass of oil drop, $m = \frac{4}{3}\pi r^3 \rho$ ; $E = 2.55 \times 10^4\ NC^{-1}$

As the oil drop is held stationary under the electric field,

$$\therefore \quad mg = qE$$

$$\text{or} \quad \frac{4}{3}\pi r^3 \rho \times g = qE$$

$$\therefore \quad r^3 = \frac{3}{4} \times \frac{qE}{\pi \rho g} = \frac{3}{4} \times \frac{1.92 \times 10^{-18} \times 2.55 \times 10^4}{\pi \times 1.26 \times 10^3 \times 9.81} = 0.94 \times 10^{-18}$$

$\therefore$ Radius of drop, $r = (0.94 \times 10^{-18})^{1/3} = 9.82 \times 10^{-7}$ m = $\mathbf{9.82 \times 10^{-4}}$ **mm**

**Q.26 Which among the curves shown in Fig. 1.08 cannot possibly represent electrostatic field lines?**

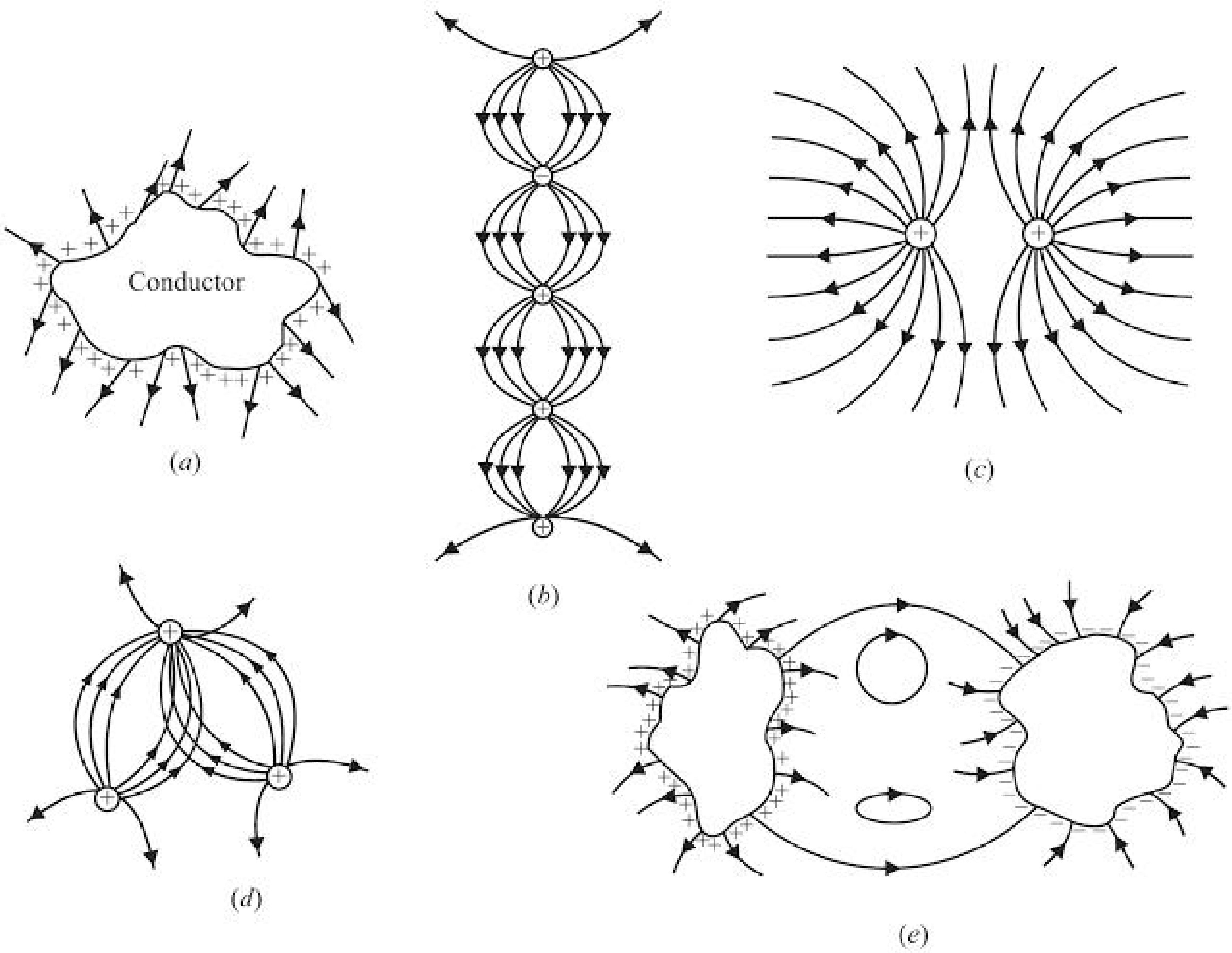



**Fig. 1.08**

**Ans.** (*a*) The electrostatic field lines always start or end normally (at 90°) at the surface of the conductor. This is not the case in Fig, 1.08 (*a*), so the field line representation is not possible.

(*b*) Field lines shown in Fig. 1.08 (*b*) are wrong because field lines cannot start from a negative charge.

(*c*) Field lines shown in Fig. 1.08 (*c*) are correct because the representation shows all the properties of field lines.

(*d*) Field lines shown in Fig. 1.08 (*d*) are wrong because lines of force cannot intersect each other.

(*e*) Field lines shown in Fig. 1.08 (*e*) are wrong because electrostatic field lines do not form closed loops.

**Q.27. In a certain region of space, electric field is along the Z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive Z-direction, at the rate of $10^5$ NC$^{-1}$ per metre. What are the force and torque experienced by a system having a total dipole moment equal to $10^{-7}$ Cm in the negative Z-direction?**

**Ans.** Fig 1.09 shows the conditions of the problem. The dipole consisting of charge $-q$ at $A$ and $+q$ at $B$ is placed along $Z$-axis such that its dipole moment is along negative $Z$-direction *i.e.*,

$$p_z = -10^{-7} \text{ Cm}$$

The negative sign has been taken because the dipole moment is along negative $Z$-axis. The electric field is along + direction of Z-axis such that :

$$\frac{dE}{dz} = 10^5 \text{ NC}^{-1}\text{m}^{-1}$$

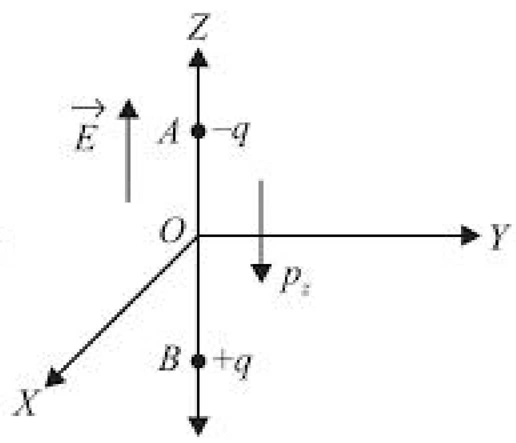



**Fig. 1.09**

Now $F = q\,dE = q \times \frac{dE}{dz} \times dz = (q \times dz) \times \frac{dE}{dz} = p_z \frac{dE}{dz}$

$\therefore \quad F = (-10^{-7}) \times (10^5) = \mathbf{-\ 10^{-2}\ N}$

Now the dipole moment acts in the negative $Z$-direction while the electric field acts in the positive $Z$-direction. Therefore, angle between the two is $\theta = 180°$.

$\therefore$ Torque on the dipole, $\tau = p_z E \sin\theta = p_z E \sin 180° = \mathbf{0}$

**Q.28.** (*a*) **A conductor *A* with a cavity as shown in Fig. 1.010(*a*) is given a charge *Q*. Show that the entire charge must appear on the outer surface of the conductor. (*b*) Another conductor *B* with charge *q* is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of *A* is $Q + q$ [Fig. 1.010 (*b*)]. (*c*) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.**

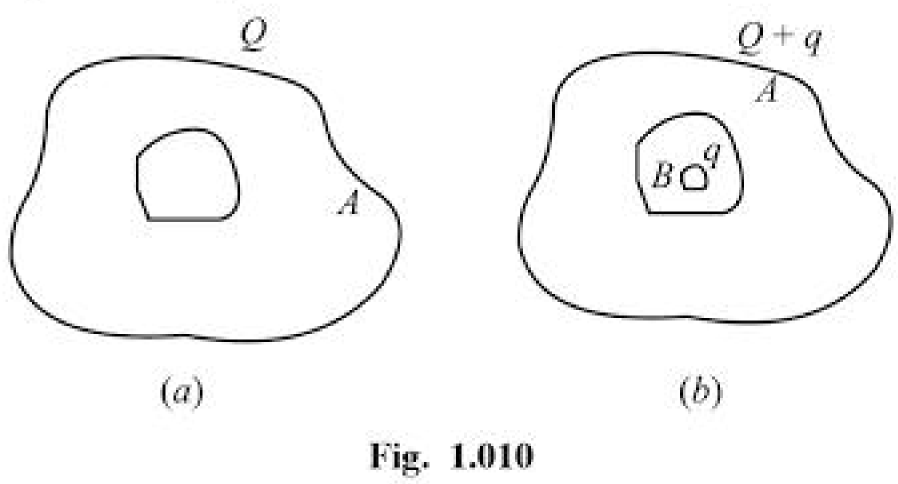



**Fig. 1.010**

**Ans.** (*a*) We know that electric field ($\vec{E}$) inside a charged conductor is zero. To show that charge resides on the outer surface of conductor $A$, consider the Gaussian surface (dotted) enclosing the cavity and lying wholly within the conductor as shown in Fig. 1.011. According to Gauss's law

$$\oint \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0}$$

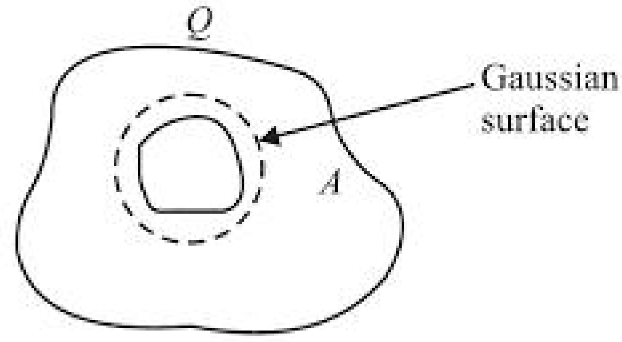



Fig. 1.011

Since $\vec{E} = 0$, $\oint \vec{E}.\vec{dS} = 0$.

Therefore, $q/\varepsilon_0 = 0$ or $q = 0$

Therefore, charge inside the cavity is zero. Also $\vec{E} = 0$ within the conductor so that no charge exists inside the conductor. Hence the entire charge $Q$ appears on the outer surface of conductor $A$.

(*b*) When conductor $B$ carrying a charge $+q$ is inserted in the cavity, a charge $-q$ is induced on the inner metal surface of the cavity and $+q$ on the outer surface of the conductor as shown in Fig. 1.012. As the outer surface of conductor $A$ already has a charge $Q$, the total charge on the surface of conductor $A = Q + q$.

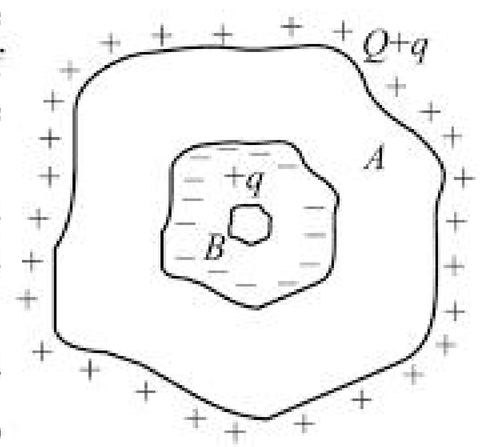



Fig. 1.012

(*c*) The electric field inside a metallic conductor is zero. Therefore, the instrument can be shielded from the strong electrostatic fields by enclosing it within a hollow metallic cover.

**Q.29. A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $(\sigma/2\varepsilon_0)\hat{n}$, where $\hat{n}$ is the unit vector in the outward normal direction, and $\sigma$ is the surface charge density near the hole.**

**Ans.** Consider that the hole in the hollow charged conductor is filled up as shown by the shaded portion in Fig. 1.013. Then applying Gauss's law, the electric field just outside the surface is $\frac{\sigma}{\varepsilon_0}\hat{n}$ and is zero inside. This field $(\sigma/\varepsilon_0\hat{n})$ can be viewed as superposition of two fields *viz.*

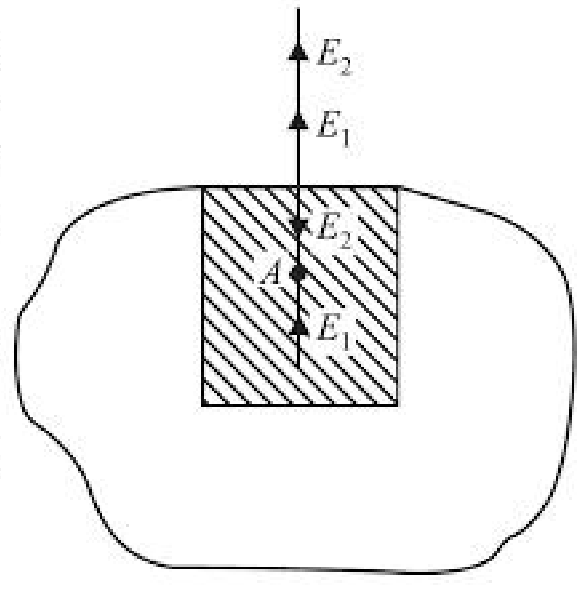



Fig. 1.013

(*i*) Field $E_2$ due to filled up hole. (*ii*) Field $E_1$ in the hole due to the rest of the charged conductor.

Since field inside the conductor is zero, the two fields ($E_1$ and $E_2$) must be equal and opposite, *i.e.*,

$$E_1 - E_2 = 0 \quad \text{... (i)}$$

But outside the conductor, the two fields add up so that :

$$E_1 + E_2 = \frac{\sigma}{\varepsilon_0} \quad \text{... (ii)}$$

From eqs. (*i*) and (*ii*), $E_1 = \frac{\sigma}{2\varepsilon_0}$

$\therefore$ Electric field inside the hole $= (\sigma/2\varepsilon_0)\hat{n}$ where $\hat{n}$ is a unit vector in the outward normal direction.

**Q.30. Obtain the formula for the electric field due to a long thin wire of uniform linear charge density $\lambda$ without using Gauss's law. [Hint: Use Coulomb's law directly and evaluate the necessary integral.]**

**Ans.** Fig. 1.014 shows a long thin wire $AB$ of uniform linear charge density $\lambda$. It is desired to find the formula for electric field due to this wire at a point $P$ at a perpendicular distance $PC = r$ from the wire. Consider a small element $dx$ of the wire with centre $O$ such that $OC = x$. Let $\angle CPO = \theta$.

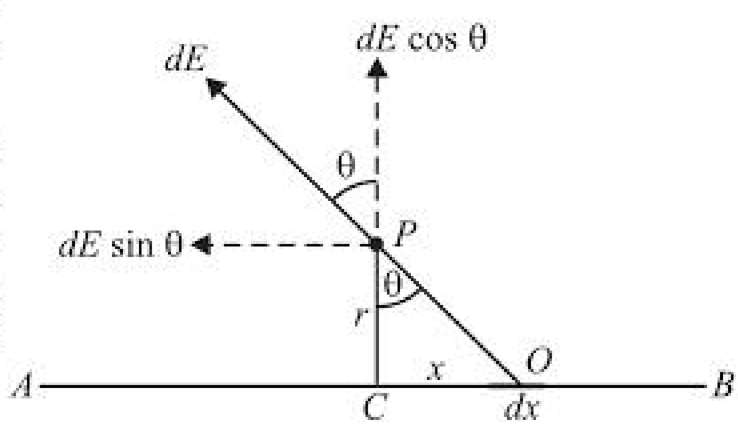



Fig. 1.014

Charge on the element $dx$, $dq = \lambda dx$

$\therefore$ Electric field at point $P$ due to element $dx$ is

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda\, dx}{OP^2} = \frac{\lambda\, dx}{4\pi\varepsilon_0 (r^2 + x^2)} \qquad (\because OP^2 = r^2 + x^2)$$

The electric field $dE$ (See Fig. 1.014) can be resolved into two rectangular components *viz*

(*i*) $dE \cos\theta$ along normal to the wire

(*ii*) $dE \sin\theta$ acting parallel to $BA$

The component $dE \sin\theta$ is cancelled by the component $dE \sin\theta$ of the field due to corresponding element $dx$ on the left of point $C$. Therefore, $dE \sin\theta$ component's net contribution is zero. However, the components $dE \cos\theta$ are added up.

$\therefore$ Effective electric field $dE'$ at point $P$ due to element $dx$ is

$$dE' = \frac{\lambda\, dx \cos\theta}{4\pi\varepsilon_0 (r^2 + x^2)} \qquad \ldots (i)$$

From $\Delta\, OCP$, $x = r\tan\theta \quad \therefore \quad dx = r\sec^2\theta\, d\theta$

Also, $\qquad r^2 + x^2 = r^2 + r^2\tan^2\theta = r^2(1 + \tan^2\theta) = r^2\sec^2\theta$

Putting the values of $dx$ $(= r\sec^2\theta\, d\theta)$ and $r^2 + x^2$ $(= r^2\sec^2\theta)$ in eq. (*i*),

$$dE' = \frac{\lambda\, r\sec^2\theta\, d\theta}{4\pi\varepsilon_0\, r^2\sec^2\theta}\cos\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\cos\theta\, d\theta$$

As the wire is very long, therefore, its ends $A$ and $B$ may be considered infinite apart. As a result, $\theta$ varies from $-\pi/2$ to $\pi/2$.

$\therefore$ Electric field at point $P$ due to the whole wire is

$$E' = \int_{-\pi/2}^{\pi/2} \frac{\lambda}{4\pi\varepsilon_0 r}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}[\sin\theta]_{-\pi/2}^{\pi/2}$$

or
$$E' = \frac{\lambda}{4\pi\varepsilon_0 r} \times 2 = \frac{\lambda}{2\pi\varepsilon_0 r}$$

**Q.31. It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by *u*) of charge + (2/3) *e*, and the 'down' quark (denoted by *d*) of charge (–1/3) *e*, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.**

**Ans.** $u = +\frac{2}{3}e$ ; $d = -\frac{1}{3}e$ ; charge on proton $= e$; charge on neutron $= 0$

**In a proton.** Let a proton contain $x$ 'up' quarks and $(3 - x)$ 'down' quarks**.** Then total charge on proton is

$$ux + d(3 - x) = e \quad \text{or} \quad \frac{2}{3}ex - \frac{1}{3}e(3 - x) = e$$

$$\therefore \qquad x = 2 \quad \text{and} \quad 3 - x = 1$$

Therefore, a proton contains 2 'up' quarks and 1 'down' quark. The quark composition of proton is : **uud.**

**In a neutron.** Let a neutron contain $y$ 'up' quarks is and $(3 - y)$ 'down' quarks. Then total charge on neutron is

$$uy + d(3 - y) = 0 \quad \text{or} \quad \frac{2}{3}ey - \frac{1}{3}e(3 - y) = 0$$

$$\therefore \qquad y = 1 \quad \text{and} \quad 3 - y = 3 - 1 = 2$$

Therefore, a neutron contains 1 'up' quark and 2 'down' quarks. The quark composition of neutron is : **udd.**

**Q.32.** (*a*) **Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (*i.e.,* where $\vec{E} = 0$) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.**

(*b*) **Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.**

**Ans.** (*a*) To prove this, let us suppose that the test charge placed at the null point is in stable equilibrium. If it is so, then on being displaced from the null point in any direction, it must experience a restoring force towards the null point. This means that there is net inward flux of electric field through a closed surface around the null point. But by Gauss's law, the flux of electric field through a surface enclosing no charge is zero. Hence our assumption is wrong and the test charge cannot be in stable equilibrium.

(*b*) In this case, the null point lies at the mid-point of the line joining the two charges. If the test charge is displaced slightly from the null point along the line, it will experience a restoring force that tends to bring the test charge to the mid-point. But if we displace the test charge normal to the line, the net force on the charge takes it further away from the null point *i.e.,* no restoring force acts. Therefore, the test charge placed at the null point is not in stable equilibrium.

**Q.33. A particle of mass $m$ and charge $(-q)$ enters the region between the two charged plates initially moving along $x$-axis with speed $v_x$ (like particle 1 in Fig. 1.04). The length of plate is $L$ and a uniform electric field $E$ is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2m\,v_x^2)$. Compare this motion with motion of a projectile in a gravitational field.**

**Ans.** The initial velocity of the charged particle in the vertical direction is zero, *i.e.,* $v_0 = 0$. Force $(= qE)$ on the charged particle is in the upward direction so that :

$$\text{Acceleration, } a = \frac{qE}{m}\text{; Deflection, } y = s = ?$$

Time taken to cross the field, $t = \dfrac{L}{v_x}$

Now $$s = v_0\, t + \frac{1}{2}\, a\, t^2$$

or $$y = 0 \times t + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v_x}\right)^2$$

$\therefore$ $$y = \frac{q\,E\,L^2}{2m\,v_x^2} \qquad \dots (i)$$

Eq. (*i*) is the equation of a parabola. Thus like the motion of the projectile in gravitational field, the path of a charged particle in an electric field is a parabola.

**Q.34. Suppose that the particle in Q.33 is an electron projected with velocity $v_x = 2.0 \times 10^6$ ms$^{-1}$. If $E$ between the plates separated by 0.5 cm is $9.1 \times 10^2$ N/C, where will the electron strike the upper plate? ($|e| = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg.)**

**Ans.** $$y = \frac{q\,E\,L^2}{2m\,v_x^2} \quad \text{or} \quad L^2 = \frac{2ym\,v_x^2}{q\,E}$$

Here $$y = \frac{5 \times 10^{-3}}{2} = 2.5 \times 10^{-3}\text{ m; } m = m_e = 9.1 \times 10^{-31}\text{ kg;}$$

$$v_x = 2.0 \times 10^6\text{ ms}^{-1}\text{; } q = e = 1.6 \times 10^{-19}\text{ C; } E = 9.1 \times 10^2\text{ N/C}$$

$\therefore$ $$L^2 = \frac{2 \times (2.5 \times 10^{-3}) \times (9.1 \times 10^{-31}) \times (2.0 \times 10^6)^2}{1.6 \times 10^{-19} \times 9.1 \times 10^2} = 1.254 \times 10^{-4}$$

or $$L = \sqrt{1.254 \times 10^{-4}} = 1.12 \times 10^{-2}\text{ m} = \mathbf{1.12\ cm}$$

## NCERT CHAPTER 2 : ELECTROSTATIC POTENTIAL AND CAPACITANCE

**Q.1. Two charges $5 \times 10^{-8}$ C and $-3 \times 10^{-8}$ C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.**

**Ans.** Fig. 2.01 shows the conditions of the problem. Let the potential be zero at point $C$ at a distance $x$ from the charge $q_1$.


**Fig. 2.01**

$$\text{Potential at } C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{x} + \frac{q_2}{0.16 - x}\right]$$

or $$0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{5\times10^{-8}}{x} + \frac{-3\times10^{-8}}{0.16 - x}\right]$$

or $$0 = \frac{5\times10^{-8}}{x} - \frac{3\times10^{-8}}{0.16 - x} \quad \text{or} \quad \frac{5}{x} = \frac{3}{0.16 - x} \quad \therefore\ x = \mathbf{10\ cm}$$

Therefore, the point of zero potential lies 10 cm from charge $q_1$ ($= 5 \times 10^{-8}$ C).

The other possibility is that point of zero potential $C$ may lie on $AB$ produced at a distance $x$ from $q_1$ as shown in Fig. 2.02.

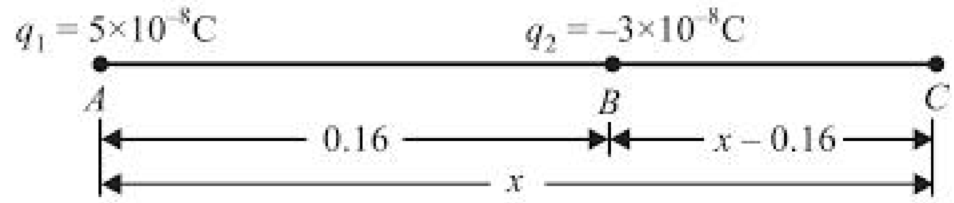



**Fig. 2.02**

$\therefore$ $$\text{Potential at } C = \frac{1}{4\pi\varepsilon_0}\left[\frac{5\times10^{-8}}{x} - \frac{3\times10^{-8}}{x - 0.16}\right]$$

or $$0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{5\times10^{-8}}{x} - \frac{3\times10^{-8}}{x - 0.16}\right]$$

or $$0 = \frac{5\times10^{-8}}{x} - \frac{3\times10^{-8}}{x - 0.16}$$

or $$\frac{5}{x} = \frac{3}{x - 0.16} \quad \therefore\ x = 0.40 \text{ m} = \mathbf{40\ cm}$$

Therefore, the point of zero potential $C$ also lies on $AB$ produced at a distance 40 cm from the charge $q_1$ ($= 5 \times 10^{-8}$ C).

**Q.2. A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.**

**Ans.** Fig. 2.03 shows the conditions of the problem. Here $O$ is the centre of the hexagon. By geometry, the distance of each charge from centre $O$ is $r$ = 10 cm = 0.1 m. Since potential is a scalar, total potential due to six equal charges at the centre

$O$ is $$V_0 = 6\times\frac{q}{4\pi\varepsilon_0 r} = 6 \times 9 \times 10^9 \times \frac{q}{r}$$

$$= 54 \times 10^9 \times \frac{5\times10^{-6}}{0.1} = \mathbf{2.7 \times 10^6\ V}$$

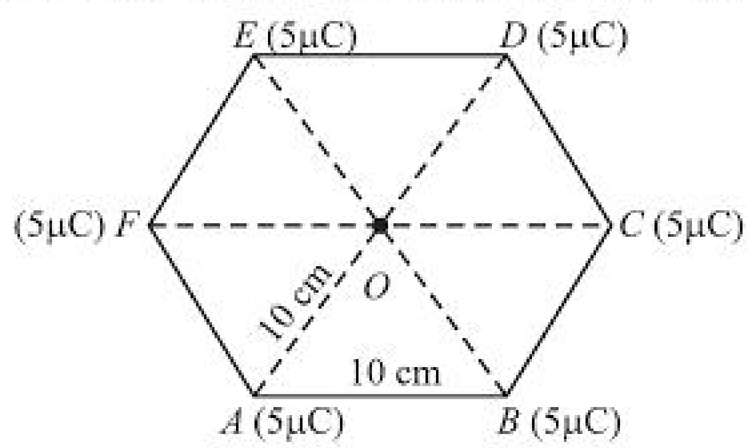



**Fig. 2.03**

**Q.3. Two charges 2 μC and –2 μC are placed at points *A* and *B* 6 cm apart.**

**(*a*) Identify an equipotential surface of the system.**

**(*b*) What is the direction of the electric field at every point on this surface?**

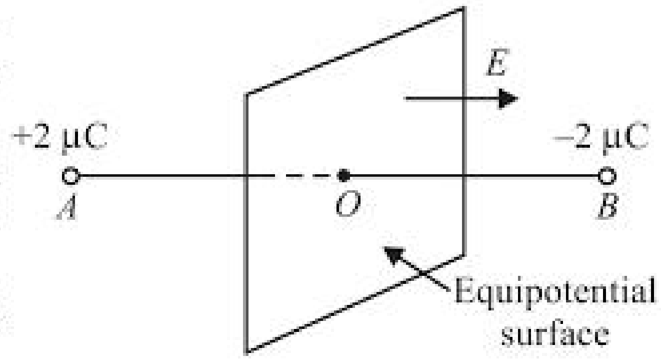



**Fig. 2.04**

**Ans.** (*a*) For the given system of charges, the equipotential surface will be a plane normal to the line *AB* and passing through the mid-point *O* as shown in Fig. 2.04. It is because on any point on this plane, the potential is zero (*i.e.*, the same).

(*b*) The direction of electric field $(\vec{E})$ is normal to the plane in the direction of *AB* (*i.e.*, from positive charge to negative charge) as shown in Fig. 2.04.

**Q.4. A spherical conductor of radius 12 cm has a charge of $1.6 \times 10^{-7}$ C distributed uniformly on its surface. What is the electric field**

**(*a*) inside the sphere**

**(*b*) just outside the sphere**

**(*c*) At a point 18 cm from the centre of the sphere?**

**Ans.** Here, $q = 1.6 \times 10^{-7}$ C; $r = 12$ cm $= 0.12$ m

(*a*) Inside the sphere, $E = \mathbf{0}$. It is because charge resides on the outer surface of a conductor.

(*b*) Just outside the sphere, $r = 0.12$ m. For a point on the surface of charged sphere (or outside), the charge ($q$) may be assumed to be concentrated at the centre of the sphere.

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2} = 9\times10^9\times\frac{1.6\times10^{-7}}{(0.12)^2} = \mathbf{10^5\ NC^{-1}}$$

(*c*) At a point 18 cm from the centre, $r = 18$ cm $= 0.18$ m.

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2} = 9\times10^9\times\frac{1.6\times10^{-7}}{(0.18)^2} = \mathbf{4.4\times10^4\ NC^{-1}}$$

**Q.5. A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1 pF = $10^{-12}$ F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?**

**Ans.** For the first case, $C = \dfrac{\varepsilon_0 A}{d} = 8$ pF

For the second case, $C' = \dfrac{\varepsilon_0 K A}{d/2} = 2K\left(\dfrac{\varepsilon_0 A}{d}\right)$

$$\therefore \quad C' = 2\times6\times C = 2\times6\times8 = \mathbf{96\ pF}$$

**Q.6. Three capacitors each of capacitance 9 pF are connected in series.**

**(*a*) What is the total capacitance of the combination?**

**(*b*) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?**

**Ans.** (*a*)
$$\frac{1}{C_s} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

or
$$\frac{1}{C_s} = \frac{1}{9}+\frac{1}{9}+\frac{1}{9}=\frac{3}{9} \quad \therefore \quad C_s = \frac{9}{3} = \mathbf{3\ pF}$$

(*b*) Since the three capacitors of the same capacitance are in series, potential difference ($V = q/C$) across each capacitor is the same.

∴ P.D. across each capacitor = 120/3 = **40 V**

**Q.7. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.**

(*a*) **What is the total capacitance of the combination?**

(*b*) **Determine the charge on each capacitor if the combination is connected to a 100 V supply.**

**Ans.** (*a*) $$C_p = C_1 + C_2 + C_3 = 2 + 3 + 4 = \mathbf{9\ pF}$$

(*b*) Since the capacitors are in parallel, voltage across each capacitor, $V$ = 100 volts.

Charge on 2 pF capacitor, $q_1 = C_1 V = 2 \times 10^{-12} \times 100 = \mathbf{2 \times 10^{-10}\ C}$

Charge on 3 pF capacitor, $q_2 = C_2 V = 3 \times 10^{-12} \times 100 = \mathbf{3 \times 10^{-10}\ C}$

Charge on 4 pF capacitor, $q_3 = C_3 V = 4 \times 10^{-12} \times 100 = \mathbf{4 \times 10^{-10}\ C}$

**Q.8. In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3}$ m$^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?**

**Ans.** Capacitance $C$ of a parallel plate air capacitor is

$$C = \frac{\varepsilon_0 A}{d}$$

Here $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m ; $A = 6 \times 10^{-3}$ m$^2$ ; $d$ = 3 mm = $3 \times 10^{-3}$ m

$$\therefore \qquad C = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}} = \mathbf{1.77 \times 10^{-11}\ F}$$

When the capacitor is connected to a 100 volt supply, charge on each plate is

$$q = C\,V = 1.77 \times 10^{-11} \times 100 = \mathbf{1.77 \times 10^{-9}\ C}$$

**Q.9. Explain what would happen if in the capacitor given in Q. 8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates.**

(*a*) **while the voltage supply remained connected.**

(*b*) **after the supply was disconnected.**

**Ans.** Capacitance of capacitor, $C$ = 17.7 pF ; Dielectric constant of mica, $K$ = 6

(*a*) **When the voltage supply remains connected.**

When mica sheet is inserted between the plates of the capacitor, the capacitance becomes $K$ times *i.e.*,

$$C' = KC = 6 \times 17.7 = \mathbf{106.2\ pF}$$

The p.d. between the plates of the capacitor remains equal to **100 V.**

Since $C = q/V$ and $V$ is same but $C$ has increased, the charge on capacitor must increase *i.e.*, charge on capacitor, $q' = C'\,V = 106.2 \times 10^{-12} \times 100 = \mathbf{1.06 \times 10^{-8}\ C}$

The extra charge is supplied by the battery.

(*b*) **After the voltage supply is disconnected.**

$$C' = KC = 6 \times 17.7 = \mathbf{106.2\ pF}$$

Charge on capacitor, $q = CV = 17.7 \times 10^{-12} \times 100 = \mathbf{1.77 \times 10^{-9}\ C}$

Since the supply is disconnected, the charge on the plates remains the same. Because the capacitance ($C = q/V$) has increased, the potential difference across the plates must decrease to maintain the same charge.

$$\therefore \qquad V' = \frac{q}{C'} = \frac{1.77 \times 10^{-9}}{106.2 \times 10^{-12}} = \mathbf{16.67\ V}$$

**Q.10. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?**

**Ans.** Here, $C$ = 12 pF = $12 \times 10^{-12}$ F ; $V$ = 50 volt.

Energy stored in capacitor is given by ;

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times (12 \times 10^{-12}) \times (50)^2 = \mathbf{1.5 \times 10^{-8}\ J}$$

**Q.11. A 600 pF capactior is charged by a 200 V supply. It is then disconnected from the supply**

**and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process ?**

**Ans.** $C_1 = 600$ pF ; $C_2 = 600$ pF

**Before joining.** $C_1 = 600 \text{ pF} = 600 \times 10^{-12}$ F ; $V_1 = 200$ V

$\therefore$ Energy stored, $U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times (600 \times 10^{-12}) \times (200)^2 = 12 \times 10^{-6}$ J

Charge on $C_1$, $q_1 = C_1 V_1 = (600 \times 10^{-12}) \times 200 = 12 \times 10^{-8}$ C

**After joining.** When uncharged capacitor $C_2$ (= 600 pF) is joined to the charged capacitor $C_1$, the total capacitance $C_P = C_1 + C_2 = 600 + 600 = 1200 \text{ pF} = 1200 \times 10^{-12}$ F.

The charge of $12 \times 10^{-8}$ C on $C_1$ is distributed between $C_1$ and $C_2$ till they have a common p.d. $V$ given by ;

Common p.d. across capacitors, $V = \frac{q_1}{C_P} = \frac{12\times10^{-8}}{1200\times10^{-12}} = 100$ volt

$\therefore$ Energy stored in the combination of capacitors is

$$U_2 = \frac{1}{2}C_P V^2 = \frac{1}{2}\times(1200\times10^{-12})\times(100)^2 = 6 \times 10^{-6} \text{ J}$$

$\therefore$ Loss of energy $= U_1 - U_2 = (12 \times 10^{-6}) - (6 \times 10^{-6}) =$ **$6 \times 10^{-6}$ J**

## ADDITIONAL EXERCISES

**Q.12. A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of $-2 \times 10^{-9}$ C from point $P$ (0, 0, 3 cm) to a point $Q$ (0, 4 cm, 0), via a point $R$ (0, 6 cm, 9 cm).**

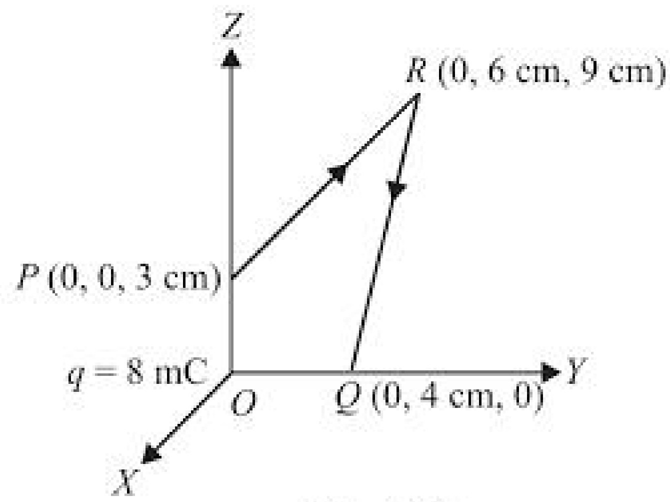


**Fig. 2.05**

**Ans.** Fig. 2.05 shows the conditions of the problem. The charge $q = 8$ mC ($= 8 \times 10^{-3}$ C) is located at the origin $O$. A charge $q_0$ ($= -2 \times 10^{-9}$ C) is to be carried from point $P$ to point $Q$ via the point $R$.

Here $r_P = 3 \text{ cm} = 0.03$ m ; $r_Q = 4 \text{ cm} = 0.04$ m

Since electrostatic force is conservative, the *work done in taking $q_0$ from $P$ to $Q$ is path independent; it depends only upon the initial and final positions (*i.e.*, $r_P$ and $r_Q$).

$$\therefore \quad W_{PQ} = q_0 (V_Q - V_P) = \frac{q\,q_0}{4\pi\varepsilon_0}\left[\frac{1}{r_Q} - \frac{1}{r_P}\right]$$

$$= (9 \times 10^9) \times (8 \times 10^{-3} \times -2 \times 10^{-9}) \left[\frac{1}{0.04} - \frac{1}{0.03}\right]$$

$$= -144 \times 10^{-3} \left[-\frac{100}{12}\right] = \mathbf{1.2\ J}$$

**Q.13. A cube of side $b$ has a charge $q$ at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.**

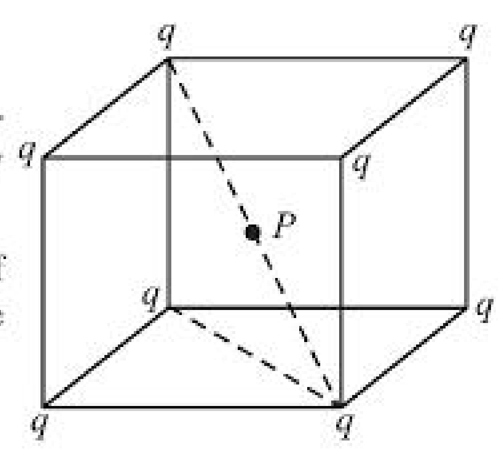


**Fig. 2.06**

**Ans.** Fig. 2.06 shows the conditions of the problem. The distance of each vertex from the centre $P$ of the cube is equal to half of the diagonal of the cube.

Diagonal of the cube $= \sqrt{b^2 + b^2 + b^2} = \sqrt{3}\,b$

Distance of each vertex from $P$, $r = \frac{\sqrt{3}\,b}{2}$

* Point $P$ is at higher potential than point $Q$. Since negative charge ($q_0$) is to be moved from higher potential to lower potential, external work has to be done.

$\therefore$ Potential at $P$ due to charges at its 8 vertices is

$$V_P = 8\times\frac{1}{4\pi\varepsilon_0}\times\frac{q}{r} = 8\times\frac{1}{4\pi\varepsilon_0}\times\frac{q}{\frac{\sqrt{3}\,b}{2}}$$

$$\therefore \quad V_P = \frac{4q}{\sqrt{3}\,\pi\varepsilon_0 b}$$

The electric field intensity at $P$ due to charges on opposite corners of the cube cancel out. Hence, total electric field intensity at $P$ is **zero.**

**Q.14. Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field :**

**(*a*) at the mid-point of the line joining the two charges, and**

**(*b*) at a point 10 cm from this mid-point in a plane normal to the line and passing through the mid-point.**

**Ans.** Fig. 2.07 shows two tiny spheres carrying charges $q_1$ (= 1.5 μC) and $q_2$ (= 2.5 μC) separated by a distance = 30 cm = 0.3 m.

(*a*) The distance of each charge from mid-point $M$ is $r$ = 0.3/2 = 0.15 m.

$\therefore$ Potential at mid-point $M$ is given by ;

$$V_M = 9\times10^9\left[\frac{q_1}{r}+\frac{q_2}{r}\right] = 9\times10^9\left[\frac{1.5\times10^{-6}}{0.15}+\frac{2.5\times10^{-6}}{0.15}\right]$$

$$= \mathbf{2.4\times10^5\ V}$$

Net electric field at mid-point $M$ due to two charges is

$$E_M = E_2 - E_1 = \frac{1}{4\pi\varepsilon_0 r^2}(q_2 - q_1)$$

$$= \frac{9\times10^9}{(0.15)^2}(2.5\times10^{-6} - 1.5\times10^{-6})$$

$$= \mathbf{4.0\times10^5\ Vm^{-1}\ towards\ } q_1$$

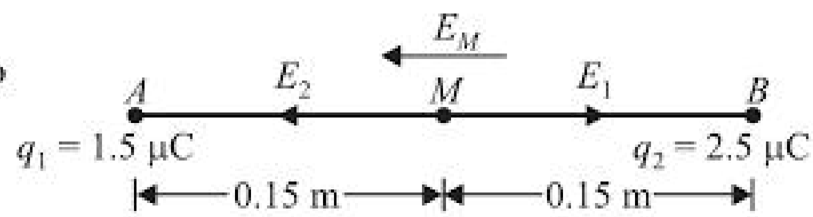



**Fig. 2.07**

(*b*) Let $P$ be the point 10 cm from the mid-point $M$ in a plane normal to the line $AB$ and passing through the mid-point (See Fig. 2.08).

$$PA = PB = \sqrt{(0.15)^2+(0.1)^2} = 0.18\ \text{m}$$

$\therefore$ Potential at $P$, $V_P = 9\times10^9\left[\frac{q_1}{PA}+\frac{q_2}{PB}\right]$

$$= 9\times10^9\left[\frac{1.5\times10^{-6}}{0.18}+\frac{2.5\times10^{-6}}{0.18}\right] = \mathbf{2\times10^5\ V}$$

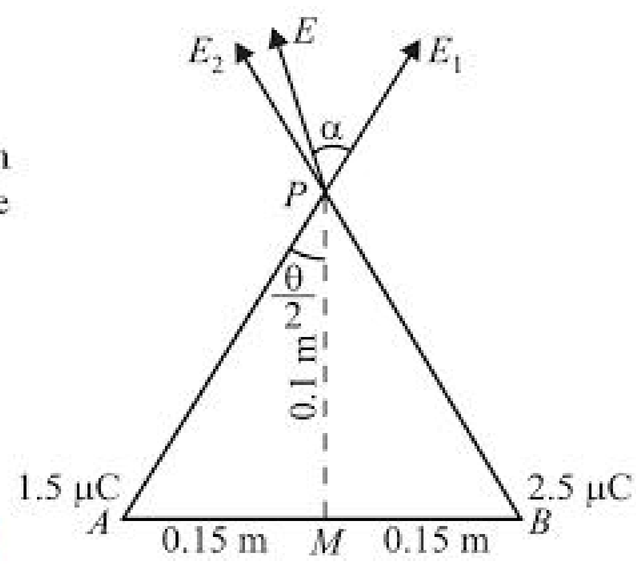



**Fig. 2.08**

From Fig. 2.08, $\cos\frac{\theta}{2} = \frac{0.1}{0.18} = 0.5556$

or $\frac{\theta}{2} = 56.25°$ so that $\theta = 112.5°$.

Electric field at point $P$ due to charge $q_1$ is

$$E_1 = 9\times10^9\times\frac{q_1}{(PA)^2} = 9\times10^9\times\frac{1.5\times10^{-6}}{(0.18)^2}$$

$$= 0.42\times10^6\ \text{Vm}^{-1}$$

Electric field at point $P$ due to charge $q_2$ is

$$E_2 = 9\times10^9\times\frac{q_2}{(PB)^2} = 9\times10^9\times\frac{2.5\times10^{-6}}{(0.18)^2} = 0.69\times10^6\ \text{Vm}^{-1}$$

It is clear from Fig. 2.08 that angle between $E_1$ and $E_2$ is $\theta$ (= 112.5°).

∴ Resultant electric field at point $P$ is

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1 E_2 \cos\theta}$$

Putting the values of $E_1$, $E_2$ and $\theta$, we get, $E$ = **$6.58 \times 10^5$ Vm$^{-1}$**

Let the resultant field $\vec{E}$ make an angle $\alpha$ with $\vec{E_1}$.

$$\therefore \qquad \tan\alpha = \frac{E_2 \sin\theta}{E_1 + E_2\cos\theta} = \frac{0.69\times10^6 \times \sin 112.5°}{0.42\times10^6 + 0.69\times10^6 \cos 112.5°} = 4.3$$

$$\therefore \qquad \alpha = \tan^{-1} 4.3 = \mathbf{76.9°}$$

**Q.15. A spherical conducting shell of inner radius $r_1$ and outer radius $r_2$ has a charge $Q$.**

**(a) A charge $q$ is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?**

**(b) Is the electric field inisde a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.**

**Ans.** (a) The charge $+Q$ resides on the outer surface of the shell. When charge $q$ is placed at the centre ($O$) of the shell, it induces $-q$ charge on the inner surface and charge $+q$ on the outer surface of the shell as shown in Fig. 2.09. Therefore, total charge on the outer surface of the shell is $Q + q$ and total charge on the inner surface of shell is $-q$.

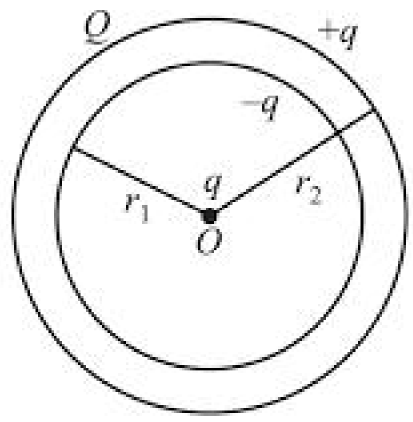


**Fig. 2.09**

∴ Charge density on inner surface, $\sigma_1 = \dfrac{-q}{4\pi r_1^2}$

Charge density on outer surface, $\sigma_2 = \dfrac{Q+q}{4\pi r_2^2}$

(b) Electric field inside a cavity with no charge is zero, even when the shell has any irregular shape. It is because there can be no charge on the inner surface of the cavity and hence there is no electric field inside the cavity.

**Q. 16. (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by**

$$(\vec{E_2} - \vec{E_1}).\hat{n} = \frac{\sigma}{\varepsilon_0}$$

**where $\hat{n}$ is a unit vector normal to the surface at a point and $\sigma$ is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2.) Hence show that just outside a conductor, the electric field is $\sigma\ \hat{n}/\varepsilon_0$.**

**(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. [Hint: For (a), use Gauss's law. For, (b) use the fact that work done by electrostatic field on a closed loop is zero.]**

**Ans.** (a) Refer to Art. 3.30. Let $\vec{E_1}$ and $\vec{E_2}$ be the electric fields due to left and right sides of infinite sheet of charge as shown in Fig. 3.41. Then normal components of the field due to its two sides are :

$$\vec{E_1} = -\frac{\sigma}{2\varepsilon_0}\hat{n} \quad \text{and} \quad \vec{E_2} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

Discontinuity in the normal components of electric field from one side of the charged conductor to the other is

$$\vec{E_2} - \vec{E_1} = \frac{\sigma}{2\varepsilon_0}\hat{n} - \left(-\frac{\sigma}{2\varepsilon_0}\hat{n}\right) = \frac{\sigma}{\varepsilon_0}\hat{n} \qquad \text{... }(i)$$

$$\therefore \qquad \left(\vec{E}_2 - \vec{E}_1\right).\hat{n} = \frac{\sigma}{\varepsilon_0}\hat{n}.\hat{n} = \frac{\sigma}{\varepsilon_0}$$

Outside the conductor, the electric field is given by eq. (*i*).

(*b*) Since work done by electrostatic field along a closed path is zero, the tangential component of electric field is continuous from one side of a charged conductor to another.

**Q.17. A long charged cylinder of linear charge density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?**

**Ans.** $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. Refer to Art. 4.12.

**Q.18. In a hydrogen atom. the electron and proton are bound at a distance of about 0.53 Å:**

**(*a*) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.**

**(*b*) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (*a*)?**

**(*c*) What are the answers to (*a*) and (*b*) above if the zero of potential energy is taken at 1.06 Å separation?**

**Ans.** Here, $q_1 = -1.6 \times 10^{-19}$ C; $q_2 = +1.6 \times 10^{-19}$ C; $r = 0.53$ Å $= 0.53 \times 10^{-10}$ m

(*a*) P.E. of proton-electron system is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = 9\times10^9 \times \frac{(-1.6\times10^{-19})\times1.6\times10^{-19}}{(0.53\times10^{-10})}$$

$$= -43.47 \times 10^{-19} \text{ J}$$

$$= -\frac{43.47\times10^{-19}}{1.6\times10^{-19}} \text{ eV} = -27.16 \text{ eV}$$

(*b*) K.E. of electron $= \dfrac{1}{2}\times 27.16 = 13.58$ eV

Total energy of electron = K.E. + P.E. = 13.58 − 27.16 = − 13.58 eV

∴ Work required to free the electron = 13.58 eV

(*c*) Potential energy of the system at separation of 1.06 Å (= $1.06 \times 10^{-10}$ m) is

$$U' = 9\times10^9 \times \frac{(-1.6\times10^{-19})\times1.6\times10^{-19}}{(1.06\times10^{-10})} = -21.73 \times 10^{-19} \text{ J}$$

$$= -\frac{21.73\times10^{-19}}{1.6\times10^{-19}} \text{ eV} = -13.58 \text{ eV}$$

If −13.58 eV is taken as zero potential energy, then potential energy of the system = −27.16 − (−13.58) = **−13.58 eV.**

Work required to free the electron = 0− (−13.58) = **13.58 eV**

**Q.19. If one of the two electrons of a $H_2$ molecule is removed, we get a hydrogen molecular ion $H_2^+$. In the ground state of an $H_2^+$, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.**

**Ans.** Fig. 2.010 shows the system of charges.

Charge on electron, $q_1 = -e = -1.6 \times 10^{-19}$ C

Charge on proton, $q_2 = q_3 = +e = +1.6 \times 10^{-19}$ C

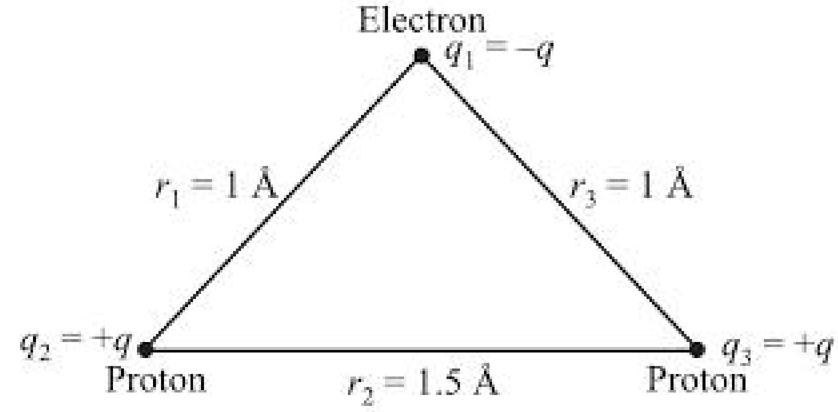



**Fig. 2.010**

Taking zero of potential energy at **infinity,** the potential energy ($U$) of the system is

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_1} + \frac{q_2 q_3}{r_2} + \frac{q_1 q_3}{r_3}\right]$$

$$= 9\times10^9\left[\frac{(-q)\,q}{1\times10^{-10}} + \frac{q\times q}{1.5\times10^{-10}} + \frac{(-q)\,q}{1\times10^{-10}}\right]$$

$$= \frac{9\times10^9\times e^2}{10^{-10}}\left[-1+\frac{1}{1.5}-1\right]$$

$$= \frac{9\times10^9\times(1.6\times10^{-19})^2}{10^{-10}}\left[-\frac{4}{3}\right] = -30.72\times10^{-19}\text{ J}$$

$$= \frac{-30.72\times10^{-19}}{1.6\times10^{-19}}\text{ eV} = \mathbf{-19.2\ eV}$$

**Q.20. Two charged conducting spheres of radii $a$ and $b$ are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.**

**Ans.** When charged conducting spheres are connected by a wire (See Fig. 2.011), charge flows from a sphere at higher potential to that at lower potential till their potentials become equal ($V$). If now $q_1$ and $q_2$ be the charges on the spheres, then,

$$V = k\frac{q_1}{a} = k\frac{q_2}{b} \quad\text{or}\quad \frac{q_1}{q_2} = \frac{a}{b} \qquad \ldots (i)$$

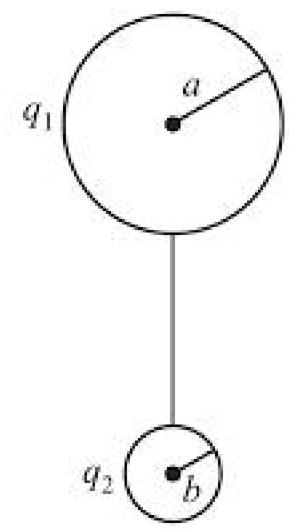



**Fig. 2.011**

We can express electric fields at the surface of the spheres as :

$$E_1 = k\frac{q_1}{a^2} \quad\text{and}\quad E_2 = k\frac{q_2}{b^2}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{q_1 b^2}{q_2 a^2} = \left(\frac{a}{b}\right)\left(\frac{b^2}{a^2}\right)$$

or

$$\frac{E_1}{E_2} = \frac{b}{a}$$

Thus the electric field (and hence surface charge density) at the surface of a sphere is inversely proportional to its radius. A sharp and pointed end of a conductor may be considered as a sphere of negligibly small radius. For this reason, the surface charge density on the sharp and pointed ends of a conductor is much higher than that on its flat portion.

**Q.21. Two charges $-q$ and $+q$ are located at points (0, 0, $-a$) and (0, 0, $a$), respectively.**

**(*a*) What is the electrostatic potential at the points (0, 0, $z$) and ($x$, $y$, 0)?**

**(*b*) Obtain the dependence of potential on the distance $r$ of a point from the origin when $r/a$ >> 1.**

**(*c*) How much work is done in moving a small test charge from the point (5, 0, 0) to (–7, 0, 0) along the *x*-axis? Does the answer change if the path of the test charge between the same points is not along the *x*-axis?**

**Ans.** Fig. 2.012 shows two charges $-q$ and $+q$ placed at the points $A$ (0, 0, $-a$) and $B$ (0, 0, $a$) respectively. It is clear that the two charges form an electric dipole of length $2a$ with its centre at the origin. The magnitude of dipole moment of the dipole is $p = q\ (2a)$.

(*a*) Potential at (0, 0, $z$) *i.e.*, point $P$ in fig. 2.012 will be :

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_1} - \frac{q}{r_2}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{z-a} - \frac{q}{z-(-a)}\right] = \frac{q}{4\pi\varepsilon_0}\cdot\frac{2a}{z^2-a^2}$$

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{z^2-a^2} \qquad (\because p = q \times 2a)$$

Potential at any point ($x$, $y$, 0) is **zero** because it is at the same distance from the two charges ($+q$ and $-q$).

(*b*) We have proved that electric potential due to electric dipole at any point $P$ at a distance $r$ from the centre of dipole is

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0\,(r^2 - a^2\cos^2\theta)}$$

If $r/a >> 1$, then $a << r$ so that $V = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

or $$V \propto \frac{1}{r^2}$$

*i.e.*, electric potential is inversely proprotional to the square of the distance ($r$).

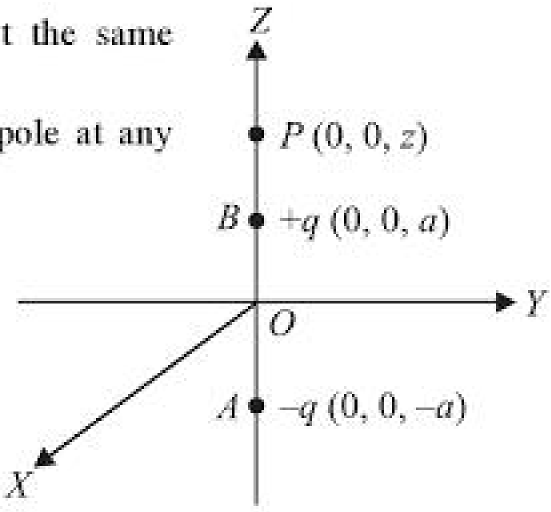


**Fig. 2.012**

(*c*) (5, 0, 0) and (–7, 0, 0) points lie on the $x$-axis and lie on the right bisector of the dipole. Each point is at the same distance from the two charges so that electric potential at each of these points is zero. Therefore, work done in moving a test charge $q_0$ from point (5, 0, 0) to point (–7, 0,0) is

$$W = q_0\,(V_1 - V_2) = q_0\,(0 - 0) = 0$$

Since work done by electrostatic field in moving a charge between two points is independent of the path followed, the answer **will not change** *i.e.*, $W = 0$.

**Q.22. Figure 2.013 shows a charge array known as an electric quadrupole. For a point on the axis of the quadrupole, obtain the dependence of potential on *r* for *r*/*a* >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (*i.e.*, a single charge).**

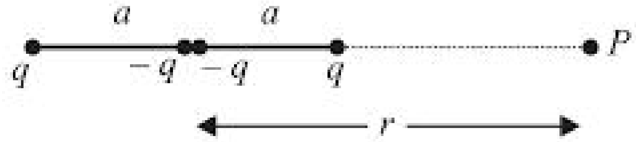


**Fig. 2.013**

**Ans.** Since electric potential is a scalar, the electric potential $V$ at point $P$ due to electric quadrupole is equal to the algebraic sum of the potentials at point $P$ due to the four charges forming the quadrupole.

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r-a} - \frac{q}{r} - \frac{q}{r} + \frac{q}{r+a}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r(r+a) - 2(r-a)(r+a) + r(r-a)}{r(r-a)(r+a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2 + ar - 2r^2 + 2a^2 + r^2 - ar}{r(r^2-a^2)}\right]$$

$$= \frac{q.\,2a^2}{4\pi\varepsilon_0\, r\,(r^2 - a^2)}$$

$$\therefore \qquad V = \frac{q.\,2a^2}{4\pi\varepsilon_0\, r^3\,(1 - a^2/r^2)}$$

When $r/a >> 1$, then $a << r$ so that $a^2/r^2$ can be neglected.

$$\therefore \qquad V = \frac{q.\,2a^2}{4\pi\varepsilon_0\, r^3} \quad \text{or} \quad V \propto \frac{1}{r^3}$$

For electric dipole, $V \propto 1/r^2$; for electric monopole, $V \propto 1/r$.

Therefore, for quadrupole, electric potential varies as $1/r^3$ while for electric dipole, potential varies as $1/r^2$. However, for an electric monopole (*i.e.*, a single charge), potential varies as $1/r$.

**Q.23. An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.**

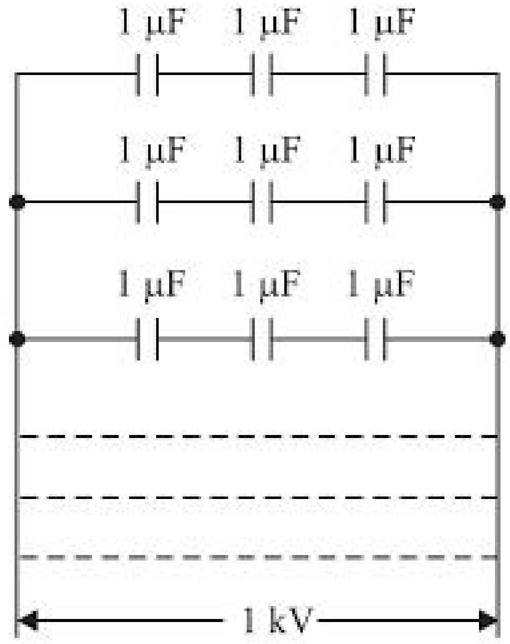


**Fig. 2.014**

**Ans.** Capacitance of each capacitor, $C$ = 1 μF

Voltage rating of each capacitor, $V_C$ = 400 V

Supply voltage, $V$ = 1 kV = 1000 volt

Since each capacitor can withstand 400 V, the number of capacitors to be connected in series = 1000/400 = 2.5. But the number of capacitors cannot be a fraction. Therefore, 3 capacitors will be connected in series. Capacitance of 3 series connected capacitors is $C_S$ = 1/3 μF. Since it is desired to have a total capacitance of 2 μF, number of such rows in parallel = 2 ÷ 1/3 = 6.

$\therefore$ Total number of capacitors = 6 × 3 = 18

Fig. 2.014 shows the arrangement of capacitors.

**Q.24. What is the area of the plates of a 2F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realise from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductros.]**

**Ans.**
$$C = \frac{\varepsilon_0 A}{d} \quad \text{or} \quad A = \frac{C d}{\varepsilon_0}$$

Here $\qquad C$ = 2F; $d$ = 0.5 cm = $0.5 \times 10^{-2}$ m

$$\therefore \qquad A = \frac{2 \times 0.5 \times 10^{-2}}{8.85 \times 10^{-12}} = 1.13 \times 10^9 \text{ m}^2$$

Note that the value of area is very large. This shows that farad (F) is a very large unit of capacitance. In practice, the capacitors are generally available in μF range.

**Q.25. Obtain the equivalent capacitance of the network in Fig. 2.015. For a 300 V supply, determine the charge and voltage across each capacitor.**

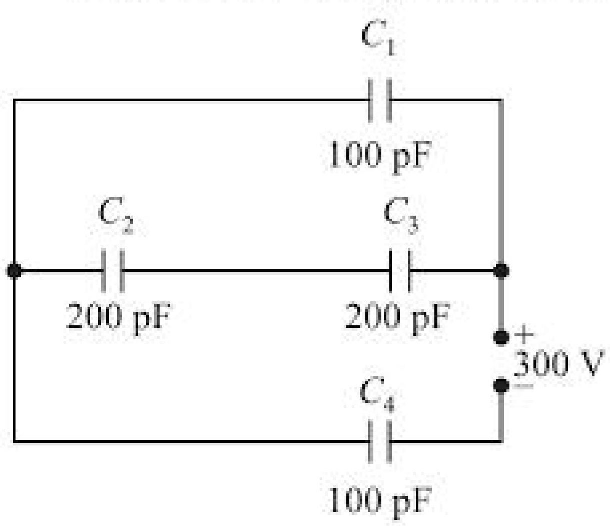


**Fig. 2.015**

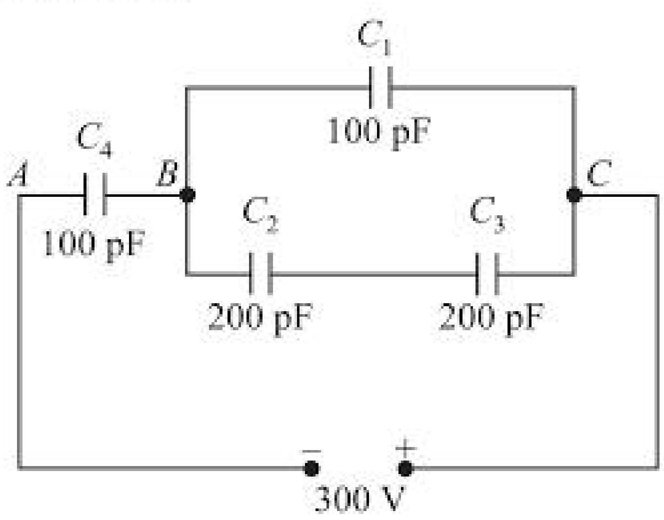


**Fig. 2.016**

**Ans. Equivalent Capacitance.** The above network can be redrawn as shown in Fig. 2.016. The equivalent capacitance $C'$ of series-connected capacitors $C_2$ and $C_3$ is

$$C' = \frac{C_2 \times C_3}{C_2 + C_3} = \frac{200 \times 200}{200 + 200} = 100 \text{ pF}$$

The equivalent capacitance of parallel combination of $C'$ (= 100 pF) and $C_1$ is

$$C_{BC} = C' + C_1 = 100 + 100 = 200 \text{ pF}$$

The entire circuit now reduces to two capacitors $C_4$ and $C_{BC}$ (= 200 pF) in series.

∴ Equivalent capacitance of the network is

$$C = \frac{C_4 \times C_{BC}}{C_4 + C_{BC}} = \frac{100 \times 200}{100 + 200} = \frac{200}{3} \text{ pF}$$

**Charges and p.d. on various capacitors**

$$\text{Total charge, } q = C\,V = \left(\frac{200}{3} \times 10^{-12}\right) \times 300 = 2 \times 10^{-8} \text{ C}$$

$$\therefore \quad \text{Charge on } C_4 = 2 \times 10^{-8} \text{ C}$$

$$\therefore \quad \text{P.D. across } C_4,\ V_4 = \frac{q}{C_4} = \frac{2 \times 10^{-8}}{100 \times 10^{-12}} = 200 \text{ V}$$

$$\text{P.D. between } B \text{ and } C,\ V_{BC} = 300 - 200 = 100 \text{ V}$$

$$\text{Charge on } C_1,\ q_1 = C_1\,V_{BC} = (100 \times 10^{-12}) \times 100 = 10^{-8} \text{ C}$$

$$\text{P.D. across } C_1,\ V_1 = V_{BC} = 100 \text{ V}$$

$$\text{P.D. across } C_2 = \text{P.D. across } C_3 = 100/2 = 50 \text{ V}$$

$$\text{Charge on } C_2 = \text{Charge on } C_3 = \text{Total charge} - \text{Charge on } C_1$$

$$= (2 \times 10^{-8}) - (10^{-8}) = 10^{-8} \text{ C}$$

**Q.26. The plates of a parallel plate capacitor have an area of 90 cm$^2$ each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.**

**(*a*) How much electrostatic energy is stored by the capacitor?**

**(*b*) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume *u*. Hence arrive at a relation between *u* and the magnitude of electric field *E* between the plates.**

**Ans.** Here, $A = 90 \text{ cm}^2 = 90 \times 10^{-4} \text{ m}^2$ ; $d = 2.5 \text{ mm} = 2.5 \times 10^{-3}$ m ; $V = 400$ volt

(*a*)

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 90 \times 10^{-4}}{2.5 \times 10^{-3}} = 3.187 \times 10^{-11} \text{ F}$$

$$\text{Energy stored, } U = \frac{1}{2} C V^2 = \frac{1}{2} \times (3.187 \times 10^{-11}) \times (400)^2 = 2.55 \times 10^{-6} \text{ J}$$

(*b*) Volume of capacitor = $A\,d = 90 \times 10^{-4} \times 2.5 \times 10^{-3} = 2.25 \times 10^{-5} \text{ m}^3$

∴ Energy stored per unit volume is

$$u = \frac{U}{Ad} = \frac{2.55 \times 10^{-6}}{2.25 \times 10^{-5}} = 0.113 \text{ Jm}^{-3}$$

**Relation between *u* and *E*.** It is derived as under :

$$u = \frac{U}{Ad} = \frac{\frac{1}{2} C V^2}{A\,d} = \frac{\frac{1}{2} (\varepsilon_0 A/d)\, V^2}{A\,d} = \frac{1}{2} \varepsilon_0 \left(\frac{V}{d}\right)^2$$

But $\dfrac{V}{d} = E$ = Electric field between capacitor plates

$$\therefore \quad u = \frac{1}{2} \varepsilon_0 E^2$$

**Q.27. A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capactior is lost in the form of heat and electromagnetic radiation?**

**Ans.** Initial energy stored in 4 μF capacitor is

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}\times(4\times10^{-6})\times(200)^2 = 8\times10^{-2}\text{ J}$$

Charge on 4 μF capacitor, $q = CV = 4\times10^{-6}\times200 = 8\times10^{-4}$ C. When this charged capacitor is connected to another uncharged capacitor of 2 μF, the total capacitance $C_p = 4 + 2 = 6$ μF. The charge $8\times10^{-4}$ C is distributed between the two capacitors to have a common p.d. of $V'$.

$$\therefore \text{ P.D. across capacitors, } V' = \frac{q}{C_P} = \frac{8\times10^{-4}}{6\times10^{-6}} = \frac{400}{3}\text{ volt}$$

Final energy stored in the combination of capacitors is

$$U_f = \frac{1}{2}C_P V'^2 = \frac{1}{2}\times(6\times10^{-6})\times\left(\frac{400}{3}\right)^2 = 5.33\times10^{-2}\text{ J}$$

$\therefore$ Energy dissipated in the form of heat and electromagnetic radiations

$$= U_i - U_f = 8\times10^{-2} - 5.33\times10^{-2} = \mathbf{2.67\times10^{-2}\ J}$$

**Q.28. Show that the force on each plate of a parallel plate capacitor has a magnitude equal to (1/2) $QE$, where $Q$ is the charge on the capacitor, and $E$ is the magnitude of electric field between the plates. Explain the origin of the factor 1/2.**

**Ans.** If $F$ is the force on each plate of a parallel plate capacitor, then work done in increasing the separation of the plates by $\Delta x$ is $dW = F.\,\Delta x$.

$\therefore$ Increase in P.E. of capacitor $= F.\,\Delta x$ ... (*i*)

If $u$ is the energy stored per unit volume of capacitor, then,

Increase in P.E. of capacitor $= u \times$ Increase in volume $= u.\,A.\,\Delta x$ ... (*ii*)

From eqs. (*i*) and (*ii*), $F.\,\Delta x = uA\,\Delta x$

or
$$F = uA = \left(\frac{1}{2}\varepsilon_0 E^2\right)A \qquad \left(\because\ u = \frac{1}{2}\varepsilon_0 E^2\right)$$

$$= \frac{1}{2}(\varepsilon_0 AE)\,E = \frac{1}{2}\left(\text{E}_0 A\frac{V}{d}\right)E \qquad \left(\because\ E = \frac{V}{d}\right)$$

$$= \frac{1}{2}(CV)\,E \qquad \left(\because\ \frac{\varepsilon_0 A}{d} = C\right)$$

$$\therefore \quad F = \frac{1}{2}QE \qquad (\because\ Q = CV)$$

The origin of the factor 1/2 in the force formula lies in the fact that just inside the capacitor, the field is $E$ and outside the capacitor, $E = 0$. Therefore, the average value of field [$= (E + 0)/2 = E/2$] contributes to the force.

**Q.29. A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports. Show that the capacitance of a spherical capacitor is given by**

$$C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

**where $r_1$ and $r_2$ are the radii of outer and inner spheres, respectively.**

**Ans.** Refer to Art. 4.8.

**Q.30. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.**

**(*a*) Determine the capacitance of the capacitor.**

**(*b*) What is the potential of the inner sphere?**

(*c*) **Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.**

**Ans.** (*a*) Capacitance of the spherical capacitor is

$$C = 4\pi\varepsilon_0 \frac{K\, r_A\, r_B}{r_B - r_A}$$

Here, $q = 2.5\ \mu C = 2.5 \times 10^{-6}$ C ; $r_B = 13$ cm $= 13 \times 10^{-2}$ m ; $r_A = 12$ cm $= 12 \times 10^{-2}$ m ; $K = 32$,

$$\therefore \qquad C = \frac{1}{9 \times 10^9} \times \frac{32 \times 12 \times 10^{-2} \times 13 \times 10^{-2}}{(13 - 12) \times 10^{-2}} = \mathbf{5.5 \times 10^{-9}\ F}$$

(*b*) Since the outer sphere is earthed, the potential of the inner sphere is equal to the potential difference between two spheres of the capacitor.

$$\therefore \text{ Potenital of inner sphere, } V = \frac{q}{C} = \frac{2.5 \times 10^{-6}}{5.5 \times 10^{-9}} = \mathbf{4.5 \times 10^2\ V}$$

(*c*) Capacitance of isolated sphere of radius $r$ (= 12 cm) is

$$C' = 4\pi\varepsilon_0 r = \frac{1}{9 \times 10^9} \times 12 \times 10^{-2} = \mathbf{1.33 \times 10^{-11}\ F}$$

The capacitance of the isolated sphere is much smaller. The reason is simple. In case of spherical capacitor, the outer sphere is earthed and the potential of the charged shell decreases considerably. As a result, capacitance (= $q/V$) increases by a large amount.

**Q.31. Answer carefully :**

(*a*) **Two large conducting spheres carrying charges $Q_1$ and $Q_2$ are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1 Q_2/4\pi\varepsilon_0 r^2$, where $r$ is the distance between their centres?**

(*b*) **If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?**

(*c*) **A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?**

(*d*) **What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?**

(*e*) **We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?**

(*f*) **What meaning would you give to the capacitance of a single conductor?**

(*g*) **Guess a possible reason why water has a much greater dielectric constant (= 80) than, say, mica (= 6).**

**Ans.** (*a*) No. It is because when two large conducting spheres are brought close to each other, their charge distribution do not remain uniform and they will not act as point charges.

(*b*) No. Gauss's law will not be true if Coulomb's law involved $1/r^3$ dependence instead of $1/r^2$ dependence.

(*c*) Not necessarily. The small test charge will travel along the line of force only if the line of force is a straight line. If the line of force is curved (*i.e.*, nonlinear), the test charge will not move along the line of force. It is because the line of force gives the direction of acceleration of charge and not that of velocity.

(*d*) In either case, the work done by the field is zero. It is because electric force is a conservative force and work done by it over a closed path is zero regardless of the shape of the closed path.

(*e*) No, electric potential is continuous as it is constant everywhere and is a scalar quantity.

(*f*) The capacitance of single conductor implies that the other plate is earth.

(*g*) A water molecule has permanent dipole moment while a molecule of mica does not have permanent dipole moment. For this reason, water has much greater dielectric constant than mica.

**Q.32. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (*i.e.*, bending of field lines at the ends).**

**Ans.** Capacitance of a cylindrical capacitor is given by ;

$$C = \frac{2\pi\varepsilon_0 l}{2.303\log_{10}\left(\frac{b}{a}\right)}$$

Here, $l$ = 15 cm = 0.15 m ; $a$ = 1.4 cm = 0.014 m ; $b$ = 1.5 cm = 0.015 m ; $q = 3.5 \times 10^{-6}$ C

$$\therefore \quad C = \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.303\log_{10}\left(\frac{0.015}{0.014}\right)} = \mathbf{1.2 \times 10^{-10}\ F}$$

Since the outer cylinder is earthed, the potential of the inner cylinder is equal to the potential difference between them.

$$\therefore \text{ Potential of inner cylinder, } V = \frac{q}{C} = \frac{3.5 \times 10^{-6}}{1.2 \times 10^{-10}} = \mathbf{2.9 \times 10^{4}\ volt}$$

**Q.33. A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about $10^7$ Vm$^{-1}$. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, *i.e.*, without starting to conduct electrictiy through partial ionisation.) For safety, we should like the field never to exceed, say, 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?**

**Ans.**

$$E = 10\% \text{ of } 10^7 = 10^6 \text{ Vm}^{-1}$$

Now $$E = \frac{V}{d} \quad \therefore \quad d = \frac{V}{E} = \frac{1 \times 10^3}{10^6} = 10^{-3} \text{ m}$$

Now $$C = \frac{\varepsilon_0 K A}{d} \quad \therefore \quad A = \frac{C d}{\varepsilon_0 K}$$

$$\therefore \quad \text{Plate area, } A = \frac{(50 \times 10^{-12}) \times 10^{-3}}{8.85 \times 10^{-12} \times 3} = \mathbf{1.9 \times 10^{-3}\ m^2}$$

**Q.34. Describe schematically the equipotential surfaces corresponding to**

**(*a*) a constant electric field in the Z-direction,**

**(*b*) a field that uniformly increases in magnitude but remains in a constant (say, Z) direction,**

**(*c*) a single positive charge at the origin, and**

**(*d*) a uniform grid consisting of long equally spaced parallel charged wires in a plane.**

**Ans.** (*a*) For a constant electric field in the $Z$-direction, the equipotential surfaces will be planes parallel to $X$-$Y$ plane as shown in Fig. 2.017

(*b*) In this case also, the equipotential surfaces will be planes parallel to $XY$ plane. Since the field increases uniformly, distance between the planes decreases.

(*c*) In this case, the equipotential surfaces will be concentric spheres with origin as their common centre.

(*d*) Near the grid, the equipotential surfaces will have varying shapes. At far off distances from the grid, the

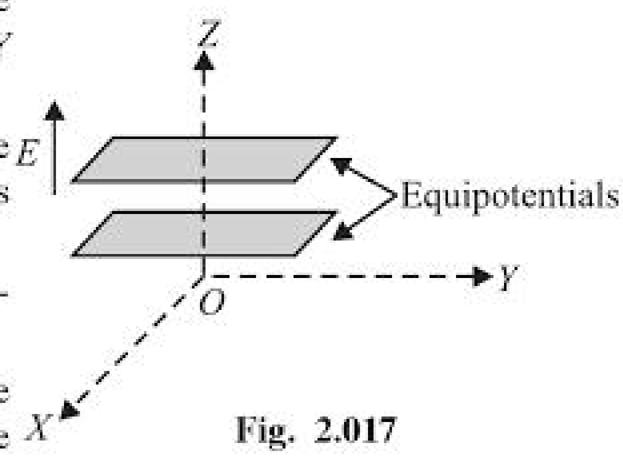



**Fig. 2.017**

equipotential surfaces will be planes parallel to the grid.

**Q.35. In a Van de Graaff type generator a spherical metal shell is to be a $15 \times 10^6$ V electrode. The dielectric strength of the gas surrounding the electrode is $5 \times 10^7$ Vm$^{-1}$. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)**

**Ans.** Electric potential $V$ of a charged spherical shell is

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$$

Electric intensity at the surface of a charged spherical shell is

$$E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2}$$

$$\therefore \quad \frac{V}{E} = r$$

For safety, $E$ = 10% of dielectric strength = 10% of $5 \times 10^7 = 5 \times 10^6$ Vm$^{-1}$

$$\therefore \quad \text{Minimum radius, } r = \frac{V}{E} = \frac{15 \times 10^6}{5 \times 10^6} = \mathbf{3m}$$

**Q.36. A small sphere of radius $r_1$ and charge $q_1$ is enclosed by a spherical shell of radius $r_2$ and charge $q_2$. Show that if $q_1$ is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge $q_2$ on the shell is.**

**Ans.** Fig. 2.018 shows the conditions of the problem. We know that charge always resides on the outer surface of a conductor. Therefore, when the sphere and shell are connected by a wire, the charge will flow from the sphere to the shell. We can also explain above action mathematically.

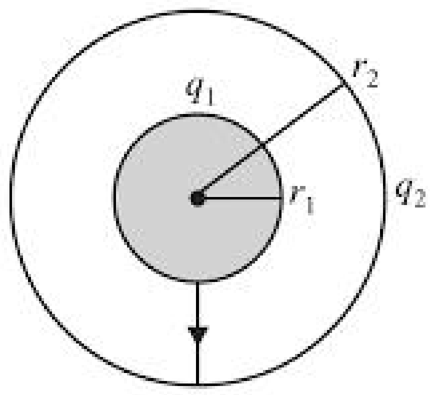



**Fig. 2.018**

Total potential on the outer sphere is

$$V_{outer} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{r_2} + \frac{q_1}{r_2}\right)$$

Total potential on the inner sphere is

$$V_{inner} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$\therefore \quad V_{inner} - V_{outer} = \frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

It is clear that the potential of inner sphere will always be higher than that of the outer sphere. Therefore, if the two are connected by a wire, the charge will always flow from the inner sphere to the outer spherical shell.

**Q.37. Answer the following :**

(*a*) **The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm$^{-1}$. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)**

(*b*) **A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1m$^2$. Will he get an electric shock if he touches the metal sheet next morning?**

(*c*) **The discharging current in the atmosphere due to the small conductivtiy of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?**

(*d*) **What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?**

**(Hint: The earth has an electric field of about 100 $Vm^{-1}$ at its surface in the downward direction, corresponding to a surface charge density = $-10^{-9}$ $Cm^{-2}$. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)**

**Ans.** (*a*) Since our body and the surface of earth are both conducting, they form an equipotential surface *i.e.*, our body and earth are always at the same potential. Normally, the equipotential surfaces of open air are parallel to the surface of earth as shown in Fig. 2.019. As we step out into open from our house, the original equipotential surfaces of open air get modified, keeping our body and ground at the same potential. Since there is no potential difference between our body and the ground, we do not get any electric shock.

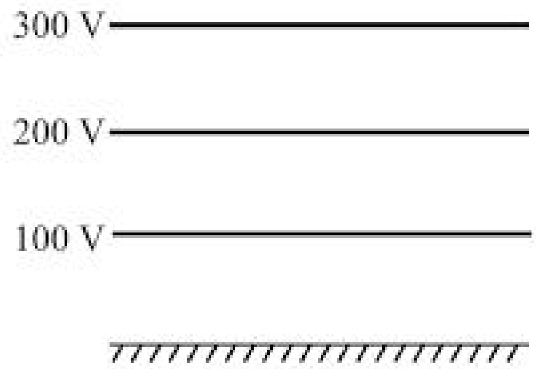



**Fig. 2.019**

(*b*) Yes. The reason is simple. The aluminium sheet and the ground form a capacitor with insulating slab acting as dielectric. The discharging current in the atmosphere will charge the capacitor gradually and raise its voltage. The potential to which the capacitor gets raised depends upon factors like capacitance of the capacitor formed, time duration etc. Next morning, if the man touches the metal sheet, he will receive a shock to the extent depending on how much voltage is developed across the capacitor.

(*c*) The atmosphere keeps on charging continuously by thunderstorms and lightning occurring all over the globe. It is also discharging due to small conductivity of earth. The two processes oppose each other. Since the conductivtiy of air is small, the atmosphere remains charged.

(*d*) The electrical energy of the atmosphere is lost as (*i*) light energy in lightning, (*ii*) heat and sound energy in the accompanying thunder.

# Unit II

## CURRENT ELECTRICITY

- Electric Current and Resistance
- Electrical Measurements
- Heating Effect of Electric Current
- N.C.E.R.T. Textbook Exercises

# 5
# Electric Current and Resistance

## INTRODUCTION

In the previous chapters, we dealt with electrostatics *i.e.* we studied the behaviour of charges at rest. In this chapter, we shall discuss current electricity *i.e.* charges in motion. We know that force acts on a charged particle placed in an electric field. If the charge on the particle is positive, it will move in the direction of electric field. On the other hand, if charge on the particle is negative, it will move in a direction opposite to the electric field. This flow of charge in a definite direction is called electric current and is important in many ways. For example, it is the electric current by means of which electrical energy is transferred from one place to another for utilisation. The operation of all electrical appliances we use such as heater, electric iron, incandescent lamp, etc., depends upon the flow of electric current through them. In this chapter, we shall deal with various aspects of current electricity.

## 5.1. CURRENT ELECTRICITY

Consider two bodies $A$ and $B$ having potentials of +5V and +3V respectively as shown in Fig. 5.1 (*i*). Clearly, body $A$ is at higher potential than the body $B$. Since the bodies are separated by air, the charges on the bodies are stationary or static. *The branch of physics which deals with charges at rest is called electrostatics.*

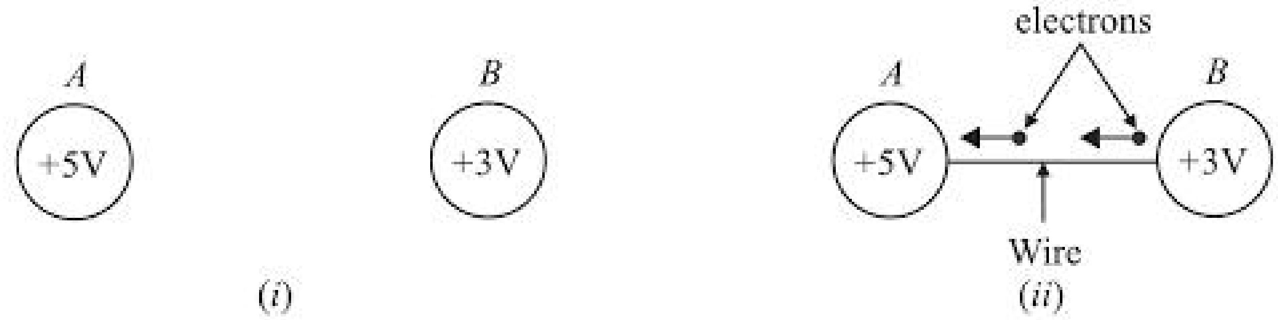



**Fig. 5.1**

If the two bodies are connected through a wire as shown in Fig. 5.1 (*ii*), then free electrons will *flow from body $B$ to body $A$ (conventional current is from $A$ to $B$). This flow of current (*i.e.*, charges moving in a definite direction) will continue till the two bodies attain the same potential. Once the two bodies attain the same potential, current flow ceases. *Therefore, we arrive at a very important conclusion that current will flow in a conductor or circuit if potential difference*

* Because body $B$ has excess of electrons as compared to body $A$.

*exists*. If no p.d. exists between two points in a circuit, no current will flow between these points. A device that maintains potential difference between two points is said to develop electromotive force (e.m.f.) *e.g.*, battery, generator, etc. *The branch of physics which deals with charges in motion is called current electricity.*

## 5.2. ELECTROMOTIVE FORCE (E.M.F.)

If we want that electric current should flow through a conductor, we must maintain potential difference (p.d.) across its ends.

*A device that maintains potential difference between two points in a circuit is said to develop an* **electromotive force** *(e.m.f.)*.

The simple example of such a device is that of a cell. A cell consists of two dissimilar metal plates (say copper and zinc) immersed in an electrolyte (say dil. $H_2SO_4$) as shown in Fig. 5.2 (*i*). The chemical action in the cell causes copper plate to become positively charged and zinc plate negatively charged. Note that chemical energy has *separated the positive and negative charges, causing a potential difference to exist between the terminals of the cell.

*The maximum p.d. (e) between the two plates of a cell on open-circuit (i.e. cell delivering no current) is called* **electromotive force (e.m.f.) of the cell** [See Fig. 5.2 (*i*)].

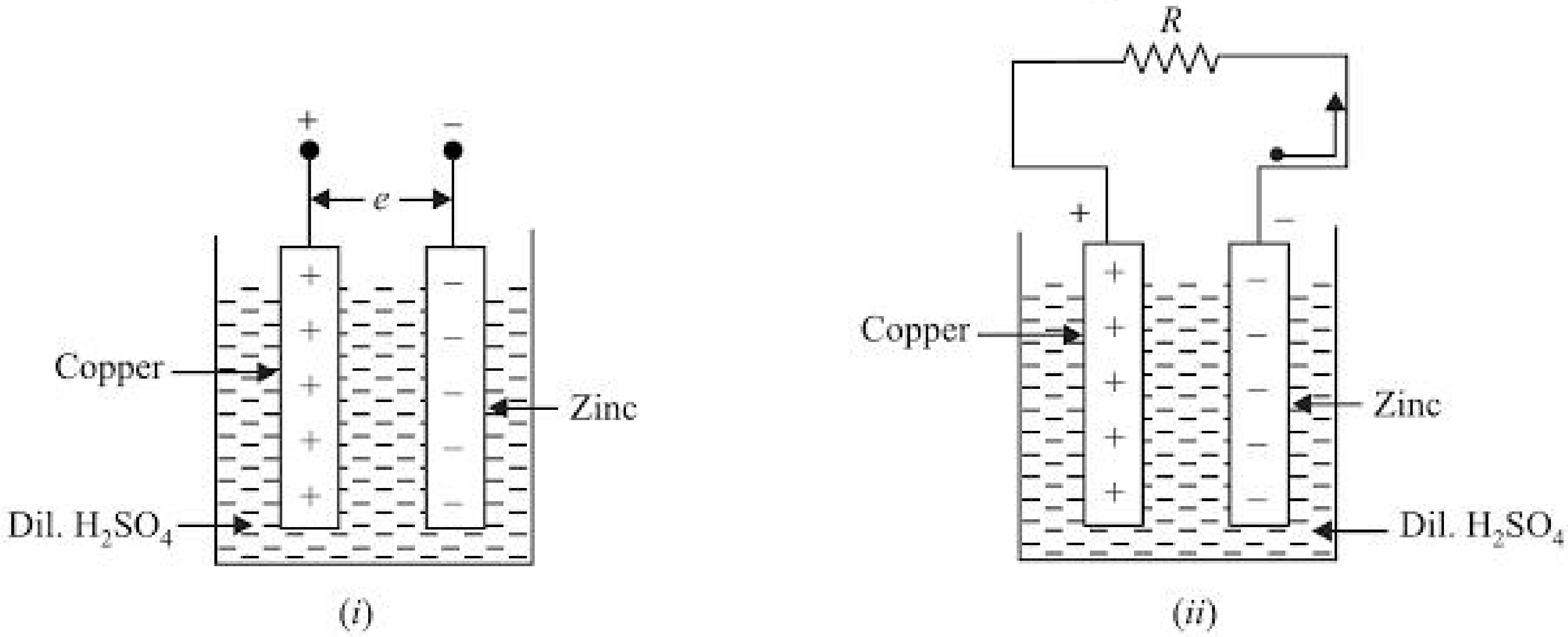



**Fig. 5.2**

If the two plates are joined through a wire as shown in Fig. 5.2 (*ii*), an electron is attracted from zinc plate through the wire to copper plate. The charges on the plates are reduced by one electron. The chemical action of the cell now transfers one electron from copper plate to zinc plate internally through the cell. Thus, plates acquire original charge to maintain original p.d. This process goes on so long as the circuit is complete or so long as there is chemical energy available.

The e.m.f. of a cell can also be defined as the energy supplied by the cell to drive a unit charge once around the complete circuit and is given by ;

$$e = \oint \vec{E} \cdot \vec{dl}$$

where $\vec{E}$ = electric field *i.e.* force per unit charge

$\vec{dl}$ = small path segment vector

Since e.m.f. of a cell is the potential difference between its plates on open circuit, one can expect that SI unit of e.m.f. is **volt.**

*The e.m.f. of a cell is said to be* **1V** *if 1 joule of energy is supplied by the cell to drive 1 coulomb of charge once around the whole circuit.*

* In order to separate positive and negative charges, energy is needed. This required energy is supplied by the chemical action.Thus a cell converts chemical energy into electrical energy.

**Note.** Unfortunately, the early scientists called e.m.f. a force instead of incorporating the word 'work' into the terminology. However, e.m.f. has unit of work and not unit of force.

## 5.3. ELECTRIC CURRENT

The flow of charge in a definite direction in a conductor is called *electric current. The* **electric current is measured** *by the flow of charge through any cross-section of the conductor in a unit time.*

Thus if $q$ is the charge flowing through any cross-section of the conductor in time $t$, then,

$$\text{Electric current, } I = \frac{q}{t}.$$

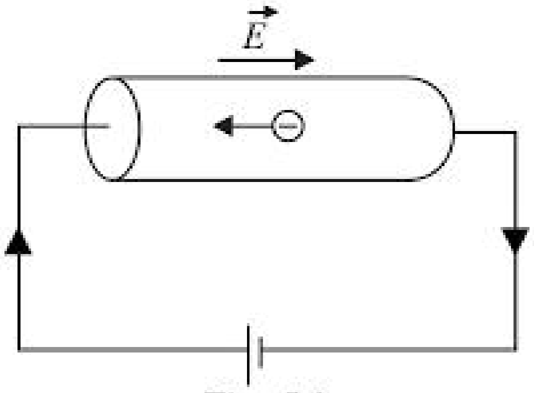


Fig. 5.3

When a cell is connected across the ends of a conductor [See Fig. 5.3], electric field is set up in the conductor. The electric field exerts force on the free electrons, causing the free electrons to drift towards the positive terminal of the cell. This constitutes electric current in the conductor. *As a convention, the direction of flow of positive charge is taken as the direction of current flow.* If moving charges are negative, as with free electrons in a metal, then direction of current is opposite to the flow of negative charges. This convention does not make any difference in the actual operation of the electrical appliances.

If the charge flowing through a conductor is not steady but varying with time, then electric current at any time (*i.e.,* instantaneous current) is given by ;

$$\text{Instantaneous current, } i = \frac{dq}{dt}$$

**Unit of electric current** Electric current, $I = q/t$

The SI unit of charge is one coulomb and that of time is one second. Therefore, SI unit of current is 1 coulomb/sec which has been given a special name of *ampere*. If $q$ = 1C and $t$ = 1s, then $I$ = 1/1 = 1 ampere.

**One ampere** *of current is said to flow through a wire if at any cross-section, one coulomb of charge flows in one second.*

Thus, if 5 ampere current is flowing through a wire, it means that 5 C per second flows past any cross-section of the wire.

**Note.** 1C = charge on $6.25 \times 10^{18}$ electrons. Thus when we say that current through a wire is 1A, it means that $6.25 \times 10^{18}$ electrons per second flow past any cross-section of the wire.

$$\therefore \qquad I = \frac{q}{t} = \frac{ne}{t} \quad \text{where } e = -1.6 \times 10^{-19}\text{ C}$$

## 5.4. CURRENT CARRIERS

The flow of charged particles in a definite direction is called electric current. Those substances which permit the flow of charges through them are called *conductors, e.g.*, metals. However, those substances which do not permit the flow of charges are called *insulators, e.g.*, glass, mica, etc. It is reminded that current will only flow through a conductor if p.d. is maintained across its ends.

**(*i*) Current carriers in solid conductors.** In solid conductors (*e.g.*, metals), there are a large number of free electrons. When electric field (*i.e.*, p.d) is applied to the conductor, the free electrons start drifting in a particular direction (in a direction opposite to the field) to constitute electric current. *Hence, free electrons are the current carriers in solid conductors.*

**(*ii*) Current carriers in liquids.** Some liquids are conductors of electricity. A conducting liquid is called an electrolyte (*e.g.*, solution of $CuSO_4$). The electrolytic solution provides positive ions (*e.g.*, $Cu^{++}$) and negative ions (*e.g.*, $SO_4^{--}$). When external electric field (*i.e.*, p.d.) is applied, the positive ions move in one direction and negative ions in the opposite direction to constitute electric current. *Hence, in conducting liquids, ions (positive and negative) are the current carriers.*

**(iii) Current carriers in gases.** Under ordinary conditions, gases are insulators. However, when a gas under low pressure is subjected to high electric field (*i.e.*, high p.d.), ionisation of gas molecules takes place, *i.e.*, electrons and positive ions are formed. *Hence, current carriers in gases are free electrons and positive ions.*

**Note.** It is possible to have a stream of electrons move across empty space, as from cathode to plate in a vacuum tube. This is also current. *In general, the flow of charged particles (positive or negative) in a definite direction constitutes electric current.*

## 5.5. ELECTRIC CURRENT IS A SCALAR QUANTITY

(*i*) Electric current, $I = q/t$. As both charge and time are scalars, therefore, electric current is a scalar quantity.

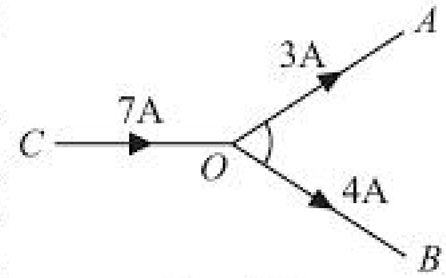



**Fig. 5.4**

(*ii*) We show electric current in a wire by an arrow to indicate the direction of flow of positive charge. But such arrows are not vectors because they do not obey the laws of vector algebra. This point can be explained by referring to Fig. 5.4. The wires $OA$ and $OB$ carry currents of 3A and 4A respectively. The total current in the wire $CO$ is $3 + 4 =$ 7A irrespective of the angle between the wires $OA$ and $OB$. This is not surprising because the charge is conserved so that magnitudes of currents in wires $OA$ and $OB$ must add to give the magnitude of current in the wire $CO$.

## 5.6. TYPES OF ELECTRIC CURRENT

The electric current may be classified into three main classes: (*i*) steady current (*ii*) varying current and (*iii*) alternating current.

**(i) Steady current.** *When the magnitude of current does not change with time, it is called a steady current.* Fig. 5.5 (*i*) shows the graph between steady current and time. Note that value of current remains the same as the time changes. The current provided by a battery is almost a steady current.

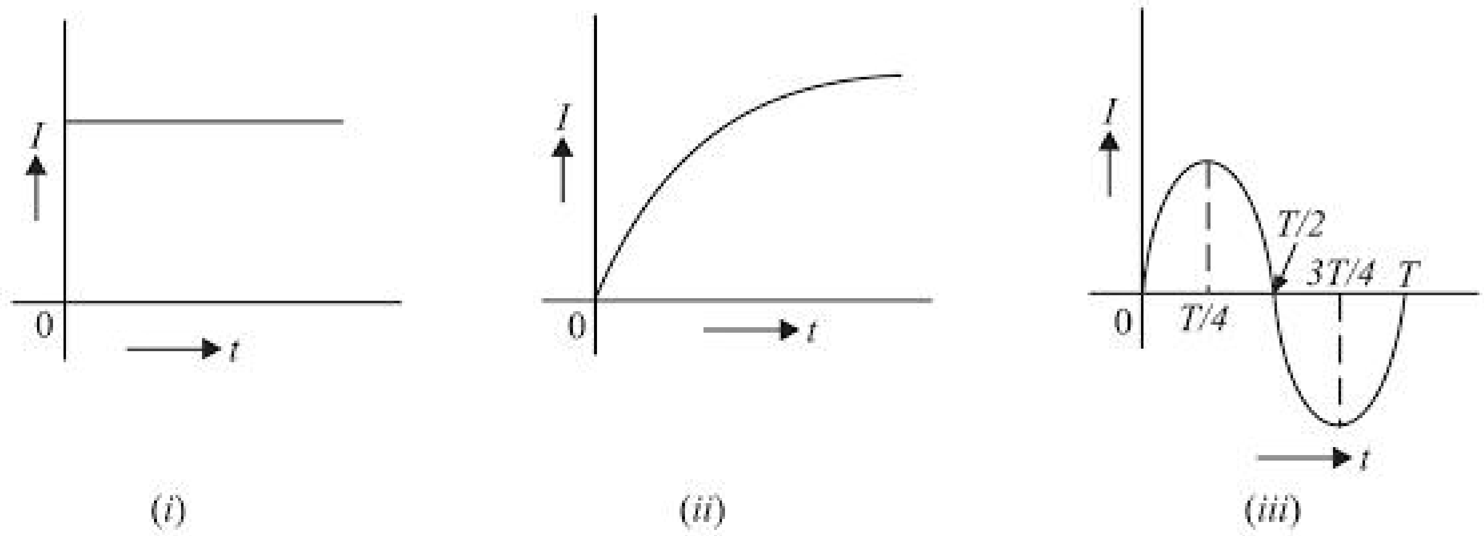



**Fig. 5.5**

**(ii) Varying current.** *When the magnitude of current changes with time, it is called a varying current.* Fig. 5.5 (*ii*) shows the graph between varying current and time. Note that value of current varies with time.

**(iii) Alternating current.** *An alternating current is one whose magnitude changes continuously with time and direction changes periodically.* Due to technical and economical reasons, we produce alternating currents that have sine waveform (or cosine waveform) as shown in Fig. 5.5 (*iii*). It is called *alternating current* because current flows in alternate directions in the circuit, *i.e.*, from 0 to $T/2$ second ($T$ is the time period of the wave) in one direction and from $T/2$ to $T$ second in the opposite direction.

**Example 5.1.** **Each second $10^{17}$ electrons flow from right to left across a cross-section of a wire attached to the two terminals of a battery. Calculate the magnitude and direction of current in the wire.**

**Solution.** Electric current, $I = \frac{q}{t} = \frac{ne}{t}$

Here $n = 10^{17}$; $e = 1.6 \times 10^{-19}$ C; $t = 1$s

$$\therefore \quad I = \frac{(10^{17}) \times (1.6 \times 10^{-19})}{1} = \mathbf{1.6 \times 10^{-2}\ A}$$

The direction of current is from left to right, *i.e.*, opposite to the direction of electron flow.

**Example 5.2.** **A 60 W light bulb has a current of 0.5A passing through it. Calculate (*i*) the number of electrons passing a cross-section of the bulb, (*ii*) the number of electrons that pass that cross-section in one hour.**

**Solution.** (*i*) $I = \frac{q}{t} = \frac{ne}{t}$

$$\therefore \quad n = \frac{It}{e} = \frac{0.5 \times 1}{1.6 \times 10^{-19}} = \mathbf{3.1 \times 10^{18}\ electrons/s}$$

(*ii*) Charge passing the cross-section in one hour is

$$q = It = (0.5) \times (60 \times 60) = 1800 \text{ C}$$

Now $q = ne$

$$\therefore \quad n = \frac{q}{e} = \frac{1800}{1.6 \times 10^{-19}} = \mathbf{1.1 \times 10^{22}\ electrons/hour}$$

**Example 5.3.** **In hydrogen atom, electron revolves around the nucleus along a path of radius 0.51 Å making $6.8 \times 10^{15}$ revolutions per second. Calculate the equivalent current. Given charge on electron = $1.6 \times 10^{-19}$ C.**

**Solution.** Here $e = 1.6 \times 10^{-19}$ C; $f = 6.8 \times 10^{15}$ rev./sec; $r = 0.51$ Å $= 0.51 \times 10^{-10}$ m.

If $T$ is the time period of the electron, then it crosses a point on its circular path once in $T$ seconds.

$$\therefore \quad \text{Equivalent current, } I = \frac{\text{Charge}}{\text{Time}} = \frac{e}{T}$$

$$= ef = 1.6 \times 10^{-19} \times 6.8 \times 10^{15} = \mathbf{1.088 \times 10^{-3}\ A}$$

**Example 5.4.** **The charge flowing in a conductor varies with time as :**

$$q = at - \frac{1}{2}bt^2 + \frac{1}{6}ct^3$$

**where a, b and c are positive constants. Find (*i*) initial current (*ii*) the time after which the current reaches the maximum value and (*iii*) the maximum or minimum value of current.**

**Solution.** Current, $i = \frac{dq}{dt} = \frac{d}{dt}\left(at - \frac{1}{2}bt^2 + \frac{1}{6}ct^3\right) = a - bt + \frac{1}{2}ct^2$. ... (*i*)

(*i*) At $t = 0, i = a - b \times 0 + \frac{1}{2}c \times (0)^2 = \mathbf{a}$

(*ii*) For $i$ to be maximum or minimum $di/dt = 0$.

$$\therefore \quad \frac{di}{dt} = -b + ct \quad \text{or} \quad 0 = -b + ct \quad \therefore \quad t = \mathbf{b/c}$$

(*iii*) Putting the value of $t$ ( = $b/c$) in eq. (*i*), we have,

$$i = a - b \times \frac{b}{c} + \frac{1}{2}c\,\frac{b^2}{c^2} = a - \frac{b^2}{2c}$$

Since the value of $i$ is less than at $t = 0$, it must be minimum.

$\therefore$ Minimum value of current $= a - \dfrac{b^2}{2c}$

## PROBLEMS FOR PRACTICE

1. One billion electrons pass from a point $P$ to another point $Q$ in $10^{-3}$ s . What is the current in amperes ? What is its direction ?

   **[1.6 × $10^{-7}$ A; direction of current is from $Q$ to $P$]**

2. In the electrolysis of silver chloride, a charge of $4 \times 10^5$ C is flowing through the electrolyte. Calculate the number of silver ($Ag^+$) ions flowing through it. **[2.5 × $10^{24}$]**

3. The current through a wire depends on time as $I = I_0 + \alpha t$ where $I_0 = 10$ A and $\alpha = 4$ $As^{-1}$. Find the charge that flows across a section of the wire in 10 seconds. **[300 C]**

4. An electric current of 2.0 μA exists in a discharge tube. How much charge flows across a cross-section of the tube in 5 minutes? **[6 × $10^{-4}$ C]**

5. An electron moves in a circle of radius 10 cm with a constant speed $5 \times 10^6$ $ms^{-1}$. Find the electric current at a point on the circle. Given $e = 1.6 \times 10^{-19}$ C. **[1.27 × $10^{-12}$ A]**

## 5.7. CURRENT CONDUCTION IN METALLIC CONDUCTORS

Metals have a large number of free electrons, about $10^{28}$ per $m^3$. In the absence of electric field, these electrons are in a state of random motion due to thermal energy. The average speed of these electrons is sufficiently high ($\simeq 10^5$ $ms^{-1}$) at room temperature. However, these velocities are distributed randomly in all directions so that there is no net movement of charge in any particular direction. Consequently, no current is established in the conductor in the absence of electric field.

When potential difference is applied across the ends of a conductor (say copper wire) as shown in Fig. 5.6, electric field is applied at every point of the copper wire. The electric field exerts force on the free electrons which start accelerating towards the positive terminal (*i.e.*, opposite to the direction of the field). As the free electrons move, they *collide again and again with positive ions of the metal. Each collision destroys the extra velocity gained by the free electrons.

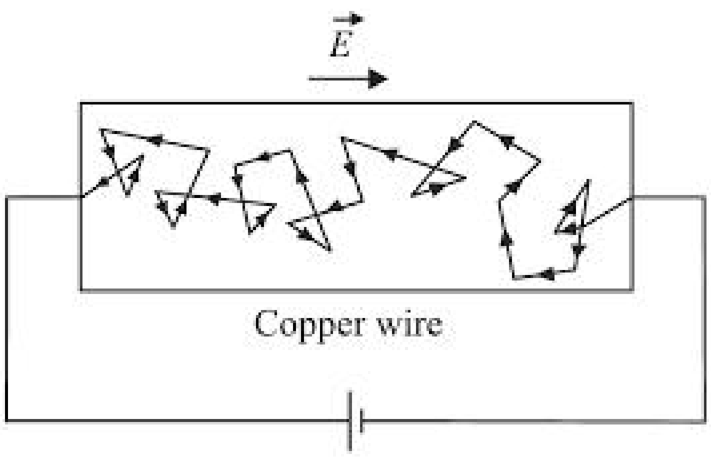



**Fig. 5.6**

*The average time that an electron spends between two collisions is called the* ***relaxation time*** ($\tau$). Its value is of the order of $10^{-14}$ second.

Although the free electrons are continuously accelerated by the electric field, collisions prevent their velocity from becoming large. The result is that electric field provides a small constant velocity towards positive terminal which is superimposed on the random motion of the electrons. This constant velocity is called the drift velocity.

*The average velocity with which free electrons get drifted in a metallic conductor under the influence of electric field is called* ***drift velocity*** $(\vec{v}_d)$. The drift velocity of free electrons is of the order of $10^{-5}$$ms^{-1}$.

---

* What happens to an electron after collision with an ion? It moves off in some new and quite random direction. However, it still experiences the applied electric field, so it continues to accelerate again, gaining a velocity in the direction of the positive terminal. It again encounters an ion and loses its directed motion. This situation is repeated again and again for every free electron in a metal.

Thus when a metallic conductor is subjected to electric field (or potential difference), free electrons move towards the positive terminal of the source with drift velocity $\vec{v_d}$ . Small though it is, the drift velocity is entirely responsible for electric current in the metal.

**Note.** The reader may wonder that if electrons drift so slowly, how room light turns on quickly when switch is closed? The answer is that propagation of electric field takes place with the speed of light. When we apply electric field (*i.e.*, potential difference) to a wire, the free electrons everywhere in the wire begin drifting almost at once.

## 5.8. RELATION BETWEEN ELECTRIC FIELD AND DRIFT VELOCITY

Consider a metallic conductor connected to a battery as shown in Fig. 5.7.

Let $l$ = length of the conductor

$V$ = p.d. across the conductor

$m$ = mass of electron

$e$ = charge on electron

$\vec{v_d}$ = drift velocity of the free electrons

$\tau$ = relaxation time

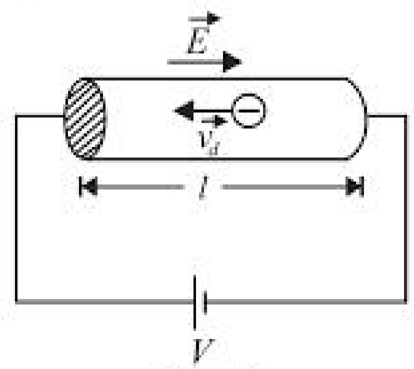



Fig. 5.7

Magnitude of electric field, $E = V/l$

Under the influence of electric field, each free electron experiences a force of $- e\ E$. The acceleration $a$ of the electron is given by ;

$$\vec{a} = -\frac{e\vec{E}}{m}$$

Since relaxation time is $\tau$, the drift velocity of the free electron is given by ;

$$\vec{v_d} = \vec{a}\tau = -\frac{e\vec{E}\tau}{m}$$

$$\therefore \quad \vec{v_d} = -\frac{e\vec{E}\tau}{m}$$

The negative sign shows that direction of drift velocity is opposite to that of the electric field. Note that drift velocity is directly proportional to the applied electric field.

$$\text{Drift speed, } v_d = \frac{eE}{m}\tau$$

## 5.9. RELATION BETWEEN CURRENT AND DRIFT VELOCITY

Consider a portion of a copper wire through which current $I$ is flowing as shown in Fig. 5.8. Clearly, copper wire is under the influence of electric field.

Let $A$ = area of X-section of the wire

$n$ = electron density, *i.e.*, number of free electrons per unit volume

$e$ = charge on each electron

$v_d$ = drift velocity of free electrons

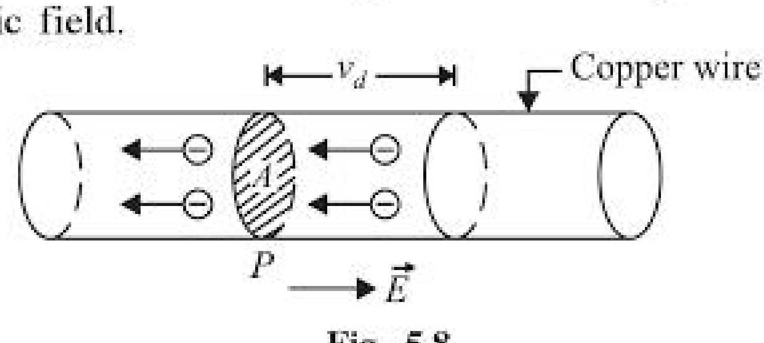



Fig. 5.8

In one second, all those free electrons within a distance $v_d$ to the right of cross-section at $P$ (*i.e.*, in a volume $Av_d$) will flow through the cross-section at $P$ as shown in Fig. 5.8. This volume contains $n\ Av_d$ electrons and, hence, a charge $(n\ A\ v_d)e$. Therefore, a charge of $n\ eA\ v_d$ per second passes the cross-section at $P$.

$$\therefore \quad I = n\ e\ A\ v_d$$

Since $A$, $n$ and $e$ are constant, $I \propto v_d$

*Hence, current flowing through a conductor is directly proportional to the drift velocity.*

(*i*) The drift velocity of free electrons is very small. Since the number of free electrons in a metallic conductor is very large, even small drift velocity of free electrons gives rise to sufficient current.

(*ii*)
$$I = neAv_d = neA\left(\frac{eE}{m}\tau\right) \qquad \left(\because \ v_d = \frac{eE}{m}\tau\right)$$

$$\therefore \qquad I = \frac{ne^2 A\tau E}{m}$$

## 5.10. ELECTRON MOBILITY

*The mobility of free electron is defined as the drift velocity of electron per unit electric field applied.* It is denoted by $\mu_e$.

$$\text{Electron mobility, } \mu_e = \frac{\text{drift velocity}}{\text{electric field}} = \frac{v_d}{E}$$

$$\therefore \qquad v_d = \mu_e E$$

The SI unit of electron mobility is $m^2 s^{-1} V^{-1}$ or $ms^{-1} N^{-1}C$.

(*i*) We can express electron mobility ($\mu_e$) in terms of relaxation time ($\tau$).

$$\text{Elecron mobility, } \mu_e = \frac{v_d}{E} = \frac{eE\tau/m}{E} = \frac{e\tau}{m}$$

where $e$ is the charge on electron, $m$ is the mass of electron and $\tau$ is the relaxation time of electron.

(*ii*) We can also express electric current in terms of electron mobility.

$$I = neAv_d = neA(\mu_e E) \qquad (\because \ v_d = \mu_e E)$$

$$\therefore \qquad I = neA\mu_e E$$

## 5.11. CURRENT DENSITY

*Current density is defined as the electric current per unit cross-sectional area (area normal to current) at any point in a conductor.*

Consider a wire of cross-sectional area $A$ and carrying current $I$. In this case, the current density is constant for all points within the conductor. The magnitude of current density is given by ;

$$\text{Current density, } J = \frac{I}{A} \qquad ...(i)$$

Obviously, the SI unit of current density is $Am^{-2}$.

Current density ($\vec{J}$) is a vector quantity whose magnitude is given by eq. (*i*) above and whose direction is in the direction of electric field.

$$I = neAv_d$$

$$\text{or} \qquad I/A = nev_d$$

$$\therefore \qquad J = nev_d$$

Thus, current density is a fundamental quantity and is related to the charge density ($n$), charge on current carriers ($e$) and drift velocity ($v_d$) of current carriers.

**Notes.** (*i*)
$$J = nev_d = ne\left(e\frac{E}{m}\tau\right) \qquad \left(\because \ v_d = \frac{eE}{m}\tau\right)$$

$$\therefore \qquad J = \frac{ne^2 E\tau}{m}$$

(*ii*)
$$I = \frac{V}{R} = \frac{V}{\rho\,(l/A)} = \frac{VA}{\rho\, l}$$

$$\therefore \qquad \frac{I}{A} = \frac{1}{\rho}\frac{V}{l} = \sigma E \qquad \left(\because \quad \sigma = \frac{1}{\rho} \text{ and } E = V/l\right)$$

or
$$J = \sigma E$$

*This shows that conductivity (σ) is the ratio of the magnitude of current density to the magnitude of applied electric field.*

**Note.** If current density is not uniform, then general relation is

$$I = \int \vec{J} \cdot d\vec{A}$$

where $\vec{J} \cdot d\vec{A}$ is the current through small area $d\vec{A}$.

## 5.12. OHM'S LAW

The relationship between voltage across and current through a conductor was first discovered by German scientist George Simon Ohm. This relationship is called Ohm's law and may be stated as under :

*The current (I) flowing through a conductor is directly proportional to the potential difference (V) across its ends provided the physical conditions (temperature, strain, etc.) do not change, i.e.,*

$$I \propto V$$

or
$$\frac{V}{I} = \text{Constant} = R$$

where $R$ is a constant of proportionality and is called *resistance* of the conductor.

For example, if in Fig. 5.9 (*i*), the p.d. between points $A$ and $B$ of the conductor is $V$ and current flowing is $I$, then $V/I$ will be constant and equal to $R$, resistance of the conductor between points $A$ and $B$. If $V$ is doubled up, current will also be doubled up so that ratio $V/I$ is constant.

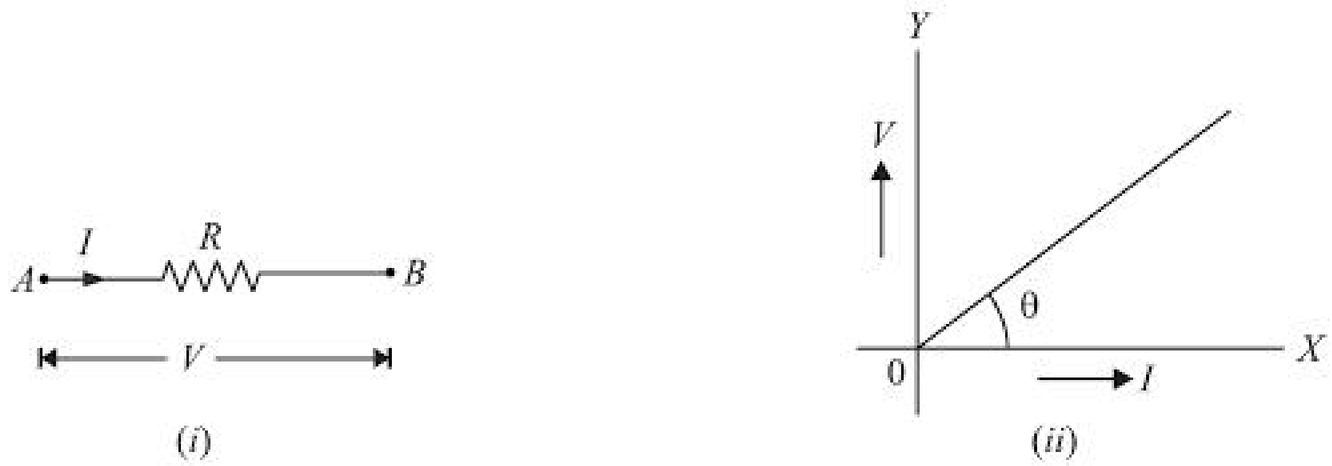



**Fig. 5.9**

If a *graph is drawn between applied potential difference ($V$) and current ($I$) flowing through the conductor, it will be a straight line passing through the origin as shown in Fig. 5.9 (*ii*). Note that slope of the graph gives the resistance of the conductor ($\tan\theta = V/I = R$).

Those conductors (*e.g.*, metals) which obey Ohm's law are called **ohmic conductors**. It may be noted that Ohm's law (a linear $V-I$ graph or constant resistance) is true only for ohmic conductors.

**Note.** Ohm's law ($R = V/I$ = constant) is a statement about the electrical behaviour of an ohmic conductor. If the material is not ohmic, relation between $V$ and $I$ will not be linear so that $R$ is not constant. Nevertheless, $R = V/I$ serves as the definition of resistance whether the material is ohmic or non-ohmic.

* Taking $V$ along $Y$-axis and $I$ along $X$-axis.

## 5.13. VALIDITY OF OHM'S LAW

Consider a metallic wire of length $l$ and uniform area of cross-section $A$ as shown in Fig. 5.10. Let $V$ be the applied p.d. across the ends of the conductor and $I$ the resulting current.

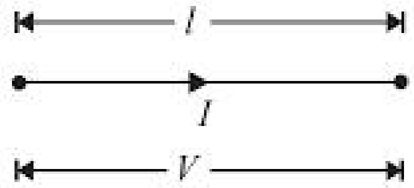


**Fig. 5.10**

$$\text{Drift velocity, } v_d = -\frac{eE}{m}\tau$$

Here $e$ ( $= 1.6 \times 10^{-19}$C) is the charge on electron and $m$ is the mass of electron. Now $E$ ( $= V/l$) is the magnitude of the electric field established in the conductor and $\tau$ is the relaxation time.

$$\therefore \qquad v_d = \frac{e}{m}\tau\left(\frac{V}{l}\right)$$

$$\text{We know that } I = n\,e\,A\,v_d$$

$$\text{or} \qquad I = neA\left[\frac{e}{m}\tau\left(\frac{V}{l}\right)\right] = \frac{nAe^2\tau}{ml}V$$

$$\text{or} \qquad \frac{V}{I} = \frac{ml}{nAe^2\tau}$$

The quantity $ml/n\,Ae^2\tau$ is constant at a given temperature for a given conductor. It is called resistance $R$ of the conductor.

$$\therefore \qquad \frac{V}{I} = \frac{ml}{nAe^2\tau} = \text{Constant} = R$$

This proves the validity of Ohm's law.

## 5.14. RESISTANCE OF A CONDUCTOR

*Resistance of a conductor is defined as the ratio of p.d. applied across its ends to the resulting current through the conductor, i.e.,*

$$R = \frac{V}{I}$$

If we look at the above relation, we find that "resistance" is properly named. For a given potential difference, the greater the resistance to current flow, the smaller is the current ($I = V/R$). In fact, resistance is opposition offered by the substance to the flow of electric current. This opposition occurs because atoms and molecules of the substance obstruct the flow of charge carriers, *i.e.*, free electrons in this case. Certain substances (*e.g.*, metals such as silver, copper, aluminium, etc.) offer very little opposition to the flow of electric current and are called *conductors*. On the other hand, those substances which offer very high opposition to the flow of electric current are called *insulators*, *e.g.*, glass, rubber, mica, etc.

### Unit of Resistance $\qquad R = V/I$

The SI unit of p.d. is 1V and that of current is 1A. Therefore, SI unit of resistance is V/A which has been given the special name *Ohm* (symbol Ω).

$$1 \text{ Ohm} = 1\,\Omega = 1 \text{ V/A}$$

*A conductor is said to have a resistance of* ***1 Ohm*** *if a p.d. of 1V across its ends causes a current of 1A to flow through it.*

$$\text{Dimensions of resistance} = \frac{\text{Dimensions of } V}{\text{Dimensions of } I}$$

$$= \frac{\text{Energy / Charge}}{I} = \frac{ML^2T^{-2}}{IC}$$

$$= \frac{ML^2T^{-2}}{A \times AT} = [ML^2T^{-3}A^{-2}]$$

**International ohm.** *It is defined as the resistance of 106.3 cm long mercury column of 1 mm$^2$ cross-sectional area and mass 14.4521g at 0°C.*

**Note.** It may be noted that resistance is the electric friction offered by the conductor and causes production of heat with the flow of electric current. The moving free electrons collide with atoms or molecules of the conductor; each collision resulting in the liberation of a minute quantity of heat.

## 5.15. FACTORS UPON WHICH RESISTANCE DEPENDS

The resistance $R$ of a conductor

(*i*) is directly proportional to its length, *i.e.*, $R \propto l$

(*ii*) is inversely proportional to its area of $X$-section, *i.e.*, $R \propto 1/A$

(*iii*) depends upon the nature of the material

(*iv*) changes with temperature.

From the first three points (leaving temperature for the time being), we have,

$$R \propto \frac{l}{A}$$

or $$R = \rho \frac{l}{A}$$

where $\rho$ (Greek letter 'Rho') is a constant of proportionality and is known as *resistivity* or *specific resistance* of the conductor. Its value depends upon the nature of the material and temperature.

**Resistivity or Specific Resistance.**


**Fig. 5.11**

We have seen above that $R = \rho \dfrac{l}{A}$

If $l$ = 1 m; $A$ = 1 m$^2$, then $R = \rho$

*Hence* ***specific resistance*** *(or* ***resistivity****) of a material is the resistance offered by 1 m length of wire of the material having area of X-section of 1 m$^2$* [See Fig. 5.11].

For example, the resistivity of copper is $1.7 \times 10^{-8}$ Ω-m. It means that if you take a copper wire 1 m long and having an area of $X$-section of 1m$^2$, then resistance of this piece of copper wire will be $1.7 \times 10^{-8}$ Ω.

**Another definition of ρ.** If we take a cube of the material of each side 1m, then area of cross-section of each face is 1m$^2$ and length between opposite faces is 1m.

*Hence* ***resistivity*** *may be defined as the resistance between the opposite faces of a metre cube of the material.*

**Unit of Resistivity.** We know that $R = \rho \dfrac{l}{A}$

or $$\rho = \frac{RA}{l}$$

The SI unit of length is 1m and that of area is 1m$^2$.

$$\therefore \quad \text{Unit of } \rho = \frac{\text{ohm} \times \text{m}^2}{\text{m}} = \text{Ohm-m}$$

The resistivity of substances varies over a wide range. To give an idea to the reader, the following table may be referred :

| S.No. | Material | Nature | Resistivity (Ω-m) at room temperature |
|---|---|---|---|
| 1. | Copper | metal | $1.7 \times 10^{-8}$ |
| 2. | Iron | metal | $9.68 \times 10^{-8}$ |
| 3. | Manganin | alloy | $48 \times 10^{-8}$ |
| 4. | Nichrome | alloy | $100 \times 10^{-8}$ |
| 5. | Pure silicon | semiconductor | $2.5 \times 10^{3}$ |
| 6. | Pure germanium | semiconductor | 0.6 |
| 7. | Glass | insulator | $10^{10}$ to $10^{14}$ |
| 8. | Mica | insulator | $10^{11}$ to $10^{15}$ |

The reader may note that resistivity of metals and alloys is very small. Therefore, these materials are good conductors of electric current. On the other hand, resistivity of insulators is extremely large. As a result, these materials hardly conduct any current. There is also an intermediate class of semiconductors. The resistivity of these substances lies between conductors and insulators.

**Relation between resistivity and electron mobility.** There is a simple relation between resistivity ($\rho$) and electron mobility ($\mu_e$).

We know that : $I = ne\,A\,v_d$

or $$\frac{I}{A} = ne\,v_d = ne\,\mu_e\,E \qquad (\because v_d = \mu_e E)$$

Now $$\frac{I}{A} = J = \sigma E = \frac{E}{\rho}$$

$$\therefore \quad \frac{E}{\rho} = n\,e\,\mu_e\,E$$

or $$\rho = \frac{1}{ne\mu_e}$$

## 5.16. CONDUCTANCE

*The reciprocal of resistance of a conductor is called its* ***conductance*** *(G).* If a conductor has resistance $R$, then its conductance $G$ is given by ;

$$G = 1/R$$

The SI unit of conductance is mho (*i.e.*, ohm spelt backward). These days, it is a usual practice to use **siemen** as the unit of conductance. It is denoted by the symbol S.

**Electrical conductivity.** *The reciprocal of resistivity of a conductor is called its* ***electrical conductivity.*** It is denoted by the symbol $\sigma$. If a conductor has resistivity $\rho$, then its conductivity is given by ;

$$\sigma = \frac{1}{\rho}$$

We know that $G = \frac{1}{R} = \frac{A}{\rho l} = \sigma \frac{A}{l}$. Clearly, the SI unit of electrical conductivity is *siemen metre*$^{-1}$ ($Sm^{-1}$).

## 5.17. CLASSIFICATION OF MATERIALS ON ELECTRICAL CONDUCTIVITY

On the basis of electrical conductivity, the materials are classified as (*i*) insulators, (*ii*) conductors and (*iii*) semiconductors.

**(*i*) Insulators.** *Those materials whose electrical conductivity is negligible are called* **insulators** *e.g.* mica, glass, wood, rubber etc. When a small potential difference is applied across an insulator, practically no current flows through it. There are practically no free electrons in an insulator. For this reason, they are poor conductors of electric current as well as heat.

*(ii)* **Conductors.** *Those materials whose electrical conductivity is very high are called* **conductors** *e.g.* copper, silver, aluminium etc. Metals are generally good conductors. When a small potential difference is applied across a conductor, a large current flows through it. There are a large number of free electrons in a conductor. For this reason, they are good conductors of electric current as well as heat.

*(iii)* **Semiconductors.** *Those materials whose electrical conductivity lies inbetween conductors and insulators are called* **semiconductors** *e.g.* germanium, silicon etc. When a small potential difference is applied across a semiconductor, a very weak current flows through it. The conductivity of a semiconductor can be increased by adding controlled amount of suitable impurities. Semiconductors are being widely used in the manufacture of a variety of electronic devices.

## 5.18. VARIATION OF RESISTIVITY WITH TEMPERATURE

$$R = \frac{V}{I} = \frac{ml}{nAe^2\tau} = \frac{m}{ne^2\tau}\left(\frac{l}{A}\right)$$

$$\therefore \text{ Resistivity of a material, } \rho = \frac{m}{ne^2\tau}$$

$$\text{Since } m \text{ and } e \text{ are constants, } \rho \propto \frac{1}{n\tau}$$

Therefore, resistivity of the material depends upon the following factors :

(*i*) *It is inversely proportional to the number of free electrons per unit volume (n) of the material*. Since the value of $n$ depends upon the nature of the material, *the resistivity of a material depends upon the nature of material and not on its dimensions.*

(*ii*) *It is inversely proportional to the average relaxation time* ($\tau$). The value of $\tau$ decreases with the increase in temperature and vice-versa.

**1. Metals.** In most of the metals, the value of $n$ does not change with temperature so that

$$\rho \propto \frac{1}{\tau}$$

Now relaxation time ($\tau$) is the average time between two successive collisions of a free electron with positive ions of metallic conductor. As the temperature increases, the amplitude of vibrations of the positive ions also increases. Consequently, the relaxation time decreases. This in turn increases the value of $\rho$. *Hence, in case of metals, the resistivity (and hence resistance $R = \rho l/A$) increases with the increase in temperature and vice-versa.*

It is found that resistivity of a conductor increases linearly with temperature and is given by ;

$$\rho = \rho_0\,[1 + \alpha_r\,(t - t_0)]$$

where $\rho$ and $\rho_0$ are the resistivity at temperatures $t$°C and $t_0$°C respectively. The term $\alpha_r$ is called temperature co-efficient of resistivity.

Clearly,
$$\alpha_r = \frac{\rho - \rho_0}{\rho_0\,(t - t_0)} = \frac{d\rho}{\rho_0} \cdot \frac{1}{dt}$$

*Therefore, $\alpha_r$ is defined as the fractional change in resistivity ($d\rho\,/\rho_0$) per unit change in temperature ($dt$).*

Note that $\alpha_r$ is positive for metals.

**2. Semiconductors.** In case of semiconductors, the value of $n$ (*i.e.*, free electron density) is very small as compared to metals. When the temperature of a semiconductor increases, the value of $n$ increases and that of $\tau$ decreases. But the increase in the value of $n$ is greater than the decrease in the value of $\tau$. The net result is that the resistivity of a semiconductor decreases with the increase in temperature. *Therefore, the resistivity (and hence resistance) of a semiconductor decreases with the increase in temperature and vice-versa.* The reader may note that effect of temperature on semiconductors is opposite to that on metallic conductors. Note that $\alpha_r$ for semiconductors is negative.

**3. Insulators.** The resistivity (and hence resistance) of an insulator decreases exponentially with the rise in temperature. A temperature rise in insulators creates many more free electrons than that existed in the cooler state. Often this increase in the number of free electrons more than offsets the interference to the drift movement caused by the increased molecular activity. *Hence the resistivity of an insulator decreases with the increase in temperature and vice-versa.*

**Note.** In case of a semiconductor and an insulator, the temperature dependence of resistivity is given by ;

$$\rho = \rho_0 \, e^{E_g/2kT}$$

where $k$ = Boltzmann constant ($= 1.381 \times 10^{-23}$ J mol$^{-1}$ K$^{-1}$ )
$T$ = Absolute temperature
$E_g$ = Energy gap between conduction band and valence band in the atoms of the material

The value of $E_g$ decides whether a material is a semiconductor or insulator. If $E_g \simeq 1$ eV, the material is a semiconductor and the resistivity at room temperature is not high. However, if $E_g > 1$ eV, the material is an insulator and the value of resistivity is very high. Note that at 0K, both the semiconductor and insulator have infinite value of resistivity.

## 5.19. EFFECT OF TEMPERATURE ON RESISTANCE

It has been found that in the normal range of temperatures, the resistance of a metallic conductor increases linearly with the rise in temperature. Therefore, resistance/temperature graph is a straight line as shown in Fig. 5.12.

Consider a metallic conductor having resistance $R_0$ at 0°C and $R_1$ at $t_1$°C. Then in the normal range of temperatures, the increase in resistance (*i.e.*, $R_1 - R_0$)

(*i*) is directly proportional to the initial resistance, *i.e.*,

$$R_1 - R_0 \propto R_0$$

(*ii*) is directly proportional to the rise in temperature, *i.e.*,

$$R_1 - R_0 \propto t_1$$

(*iii*) depends upon the nature of the material.

Combining the first two, we get,

$$R_1 - R_0 \propto R_0 t_1$$

or $$R_1 - R_0 = \alpha R_0 t_1 \quad \text{.... } (i)$$

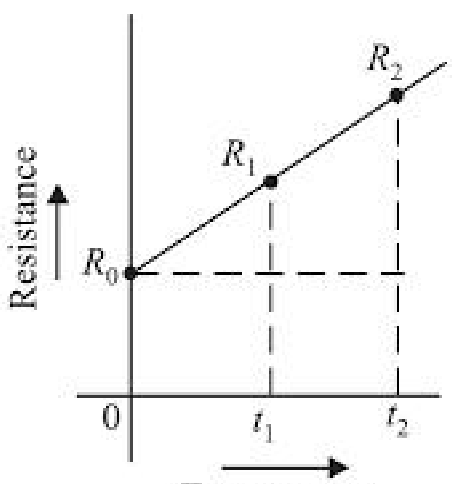


**Fig. 5.12**

where $\alpha$ is a constant of proportionality and is called *temperature co-efficient of resistance*. Its value depends upon the nature of the material and temperature.

Rearranging eq. (*i*), we get, $R_1 = R_0 (1 + \alpha t_1)$

**Definition of $\alpha$.** From eq. (*i*), we get,

$$\alpha = \frac{R_1 - R_0}{R_0 \times t_1} = \text{Increase in resistance/ohm original resistance/°C rise in temperature}$$

Hence, **temperature co-efficient of resistance** *of a conductor is the increase in resistance per ohm original resistance per °C rise in temperature.*

A little reflection shows that unit of $\alpha$ will be ohms/ohms/ °C *i.e.*, / °C. Thus copper has a temperature co-efficient of resistance of 0.004/ °C. It means that if a copper wire has a resistance of 1Ω at 0°C, then it will increase by 0.004 Ω for every 1°C rise in temperature, *i.e.*, it will become 1.004 Ω at 1°C. It may be noted that metals have positive temperature co-efficient of resistance while semiconductors and insulators have negative temperature co-efficient of resistance. It may be noted that value of $\alpha$ is very small for alloys. Since alloys have high value of resistivity and low value of $\alpha$, they are used for making standard resistance coils.

Note. If the resistance of a conductor is $R_2$ at $t^0{}_2$C and $R_1$ at $t^0{}_1$C ($t_1 < t_2$), then,

$$R_2 = R_1 [1 + \alpha (t_2 - t_1)]$$

Example 5.5. **A copper wire of area of *X*-section 1 mm$^2$ is carrying a current of 10A. If the number density of conduction electrons is $10^{28}$m$^{-3}$, find the drift velocity of the conduction electrons.**

**Solution.** $I = n\,A\,e\,v_d$

Here $I = 10$ A; $A = 1$ mm$^2$ $= 10^{-6}$ m$^2$; $e = 1.6 \times 10^{-19}$ C; $n = 10^{28}$ m$^{-3}$

$$\therefore \quad \text{Drift velocity, } v_d = \frac{I}{nAe} = \frac{10}{10^{28} \times 10^{-6} \times 1.6 \times 10^{-19}} = \mathbf{1/160 \text{ ms}^{-1}}$$

Note that drift speed of conduction electrons is surprisingly small as compared to their thermal speeds.

Example 5.6. **A copper wire of area of *X*-section 4 mm$^2$ is 4 m long and carries a current of 10A. The number density of free electrons is $8 \times 10^{28}$ m$^{-3}$. How much time is required by an electron to travel the length of wire?**

**Solution.** $I = n\,A\,e\,v_d$

Here $I = 10$A; $A = 4$ mm$^2$ $= 4 \times 10^{-6}$ m$^2$; $e = 1.6 \times 10^{-19}$ C; $n = 8 \times 10^{28}$ m$^{-3}$

$$\therefore \quad \text{Drift velocity, } v_d = \frac{I}{nAe} = \frac{10}{8 \times 10^{28} \times (4 \times 10^{-6}) \times 1.6 \times 10^{-19}} = 1.95 \times 10^{-4} \text{ ms}^{-1}$$

$\therefore$ Time taken by the electron to travel the length of the wire is

$$t = \frac{l}{v_d} = \frac{4}{1.95 \times 10^{-4}} = 2.05 \times 10^4 \text{ s} = \mathbf{5.7 \text{ hours}}$$

Example 5.7. **(a) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area $1.0 \times 10^{-7}$ m$^2$ carrying a current of 1.5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is $9.0 \times 10^3$ kg/m$^3$, and its atomic mass is 63.5 u. (*b*) Compare the drift speed obtained above with, (*i*) thermal speeds of copper atoms at ordinary temperatures, (*ii*) speed of propagation of electric field along the conductor which causes the drift motion.**

**Solution.** (*a*) Density of conduction electrons, $n = \dfrac{\text{Avogadro's number}}{\text{Atomic mass}} \times \text{density}$

$$= \frac{6.023 \times 10^{23}}{63.5 \times 10^{-3}} \times 9 \times 10^3 = 8.54 \times 10^{28} \text{ m}^{-3}$$

We know that, $I = n\,A\,ev_d$

Here $n = 8.54 \times 10^{28}$ m$^{-3}$; $A = 10^{-7}$ m$^2$; $e = 1.6 \times 10^{-19}$ C; $I = 1.5$ A

$$\therefore \quad \text{Drift velocity, } v_d = \frac{I}{nAe} = \frac{1.5}{(8.54 \times 10^{28}) \times 10^{-7} \times (1.6 \times 10^{-19})}$$

$$= \mathbf{1.1 \times 10^{-3} \text{ ms}^{-1}}$$

(*b*) (*i*) At any temperature $T$, the thermal speed of copper atom of mass $m$ is given by ;

$$v = \sqrt{\frac{3kT}{m}} \quad \text{... where } k \text{ is Boltzmann constant}$$

Here $T = 300\ K$; $k = 1.38 \times 10^{-23}$ J K$^{-1}$; Mass of copper atom,

$$m = \frac{63.5}{6.0 \times 10^{23}}\ g = \frac{63.5 \times 10^{-3}}{6.0 \times 10^{23}}\ kg$$

$$\therefore \quad v = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300 \times 6.0 \times 10^{23}}{63.5 \times 10^{-3}}} = 342.57 \text{ ms}^{-1}$$

$$\therefore \quad \frac{v_d}{v} = \frac{1.1 \times 10^{-3}}{342.57} = \mathbf{3.21 \times 10^{-6}}$$

(*ii*) As electric field travelling along the conductor has a speed of an electromagnetic wave (*i.e.*, $3 \times 10^8$ ms$^{-1}$),

$$\therefore \quad \frac{v_d}{c} = \frac{1.1 \times 10^{-3}}{3 \times 10^8} = \mathbf{3.6 \times 10^{-12}}$$

Example 5.8. **A potential difference of 5V is applied across a conductor of length 0.1 m. If drift velocity of electrons is $2.5 \times 10^{-4}$ ms$^{-1}$, calculate the electron mobility.**

**Solution.** Electric field set up across the conductor, $E = \frac{V}{l} = \frac{5}{0.1} = 50 \text{ Vm}^{-1}$

$$\therefore \text{ Electron mobility,} \quad \mu_e = \frac{v_d}{E} = \frac{2.5 \times 10^{-4}}{50} = \mathbf{5 \times 10^{-6}\ m^2\ V^{-1}\ s^{-1}}$$

Example 5.9. **When a potential difference of 1.5 V is applied across a wire of length 0.2 m and area of cross-section 0.30 mm$^2$, a current of 2.4 A flows through the wire. If the number density of free electrons in the wire is $8.4 \times 10^{28}$ m$^{-3}$, calculate the average relaxation time.**

**Solution.** Electric field set up across the conductor, $E = V/l = 1.5/0.2 = 7.5 \text{ Vm}^{-1}$

$$\text{Current density in the wire, } J = \frac{I}{A} = \frac{2.4}{0.30 \times 10^{-6}} = 8 \times 10^6 \text{ Am}^{-2}$$

$$\text{Now, Current density, } J = \frac{ne^2\tau}{m} E$$

$$\therefore \text{ Average relaxation time, } \tau = \frac{mJ}{ne^2 E} = \frac{9.1 \times 10^{-31} \times 8 \times 10^6}{(8.4 \times 10^{28}) \times (1.6 \times 10^{-19})^2 \times 7.5}$$

$$= \mathbf{4.51 \times 10^{-16}\ s}$$

Example 5.10. **An aluminium wire of diameter 0.24 cm is connected in series to a copper wire of diameter 0.16 cm. The wires carry an electric current of 10 A. Find (*i*) current density in the aluminium wire, (*ii*) drift velocity of electrons in the copper wire. Given: Number of electrons per cubic metre volume of copper = $8.4 \times 10^{28}$.**

**Solution.** (*i*) **Aluminium wire.** $r = 0.24/2 = 0.12$ cm $= 0.12 \times 10^{-2}$ m

$\therefore$ Area of X-section of Al wire, $A = \pi r^2 = \pi \times (0.12 \times 10^{-2})^2 = 4.5 \times 10^{-6}$ m$^2$

$\therefore$ Current density in the aluminium wire is

$$J = \frac{I}{A} = \frac{10}{4.5 \times 10^{-6}} = \mathbf{2.2 \times 10^6\ Am^{-2}}$$

(*ii*) **Copper wire.** $r = 0.16/2 = 0.08$ cm $= 0.08 \times 10^{-2}$ m

$\therefore$ Area of X-section of Cu wire, $A = \pi r^2 = \pi \times (0.08 \times 10^{-2})^2 = 2.0 \times 10^{-6}$ m$^2$

Also $n = 8.4 \times 10^{28}$ m$^{-3}$; $e = 1.6 \times 10^{-19}$ C; $I = 10$ A

$\therefore$ Drift velocity of electrons in copper wire is

$$v_d = \frac{I}{enA} = \frac{10}{1.6\times10^{-19}\times8.4\times10^{28}\times2.0\times10^{-6}}$$

$$= \mathbf{3.7 \times 10^{-4}\ ms^{-1}}$$

Example 5.11. **If 10m long manganin wire, 0.13cm in diameter has a resistance of 3.4 ohms, find the resistivity of the material. What is the conductivity of the material?**

**Solution.** Length of the wire, $l = 10$ m

Area of $X$-section of the wire, $A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.13\times10^{-2})^2 = 132.8\times10^{-8}\ \text{m}^2$

Now, $$R = \frac{\rho l}{A}$$

or $$\rho = \frac{RA}{l} = \frac{3.4\times132.8\times10^{-8}}{10} = \mathbf{45 \times 10^{-8}\ \Omega m}$$

Conductivity, $$\sigma = \frac{1}{\rho} = \frac{1}{45\times10^{-8}} = \mathbf{2.2 \times 10^{6}\ Sm^{-1}}$$

Example 5.12. **Find the resistance of 1000 metres of copper wire 25 sq. mm in cross-section. The resistance of copper is 1/58 ohm per metre length and 1 sq. mm cross-section.**

**Solution.** For the first case, $R_1 = ?;\ A_1 = 25\ \text{mm}^2;\ l_1 = 1000$ m

For the second case, $R_2 = 1/58\ \Omega;\ A_2 = 1\ \text{mm}^2;\ l_2 = 1\text{m}$

Now $R_1 = \rho\,(l_1/A_1);\ R_2 = \rho\,(l_2/A_2)$

$$\therefore \quad \frac{R_1}{R_2} = \frac{l_1}{l_2}\times\frac{A_2}{A_1} = \left(\frac{1000}{1}\right)\times\left(\frac{1}{25}\right) = 40$$

or $R_1 = 40\,R_2 = 40 \times 1/58 = \mathbf{20/29\ \Omega}$

Example 5.13. **A length of wire has a resistance of 4.5 ohms. Find the resistance of another wire of the same material three times as long and twice the cross-sectional area.**

**Solution.** For the first case, $R_1 = 4.5\ \Omega;\quad l_1 = l;\quad A_1 = A$

For the second case, $R_2 = ?;\ l_2 = 3l;\quad A_2 = 2A$

Now $$R_1 = \rho\frac{l_1}{A_1};\qquad R_2 = \rho\frac{l_2}{A_2}$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{l_2}{l_1}\times\frac{A_1}{A_2} = \left(\frac{3l}{l}\right)\times\left(\frac{A}{2A}\right) = 1.5$$

or $R_2 = 1.5\,R_1 = 1.5 \times 4.5 = \mathbf{6.75\ \Omega}$

Example 5.14. **A copper wire of diameter 1 cm has a resistance of 0.15Ω. It is drawn under pressure so that its diameter is reduced to 50%. What is the new resistance of the wire?**

**Solution.** Area of the wire before drawing, $A_1 = \frac{\pi}{4}(1)^2 = 0.785\ \text{cm}^2$

Area of the wire after drawing, $A_2 = \frac{\pi}{4}(0.5)^2 = 0.196\ \text{cm}^2$

As the volume of the wire remains the same before and after drawing,

$$A_1 l_1 = A_2 l_2 \quad \text{or} \quad \frac{l_2}{l_1} = \frac{A_1}{A_2} = \frac{0.785}{0.196} = 4$$

For the first case, $R_1 = 0.15\ \Omega;\ A_1 = 0.785\ \text{cm}^2;\ l_1 = l$

For the second case, $R_2 = ?;\ A_2 = 0.196\ \text{cm}^2;\ l_2 = 4l$

Now, $$R_1 = \rho\frac{l_1}{A_1};\quad R_2 = \rho\frac{l_2}{A_2}$$

$\therefore$ $$\frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right)\times\left(\frac{A_1}{A_2}\right) = (4)\times(4) = 16$$

or $$R_2 = 16\ R_1 = 16 \times 0.15 = \mathbf{2.4\ \Omega}$$

**Example 5.15. A potential difference of 3V is applied across a conductor of resistance 1.5 Ω. Calculate the number of electrons flowing through it in one second, charge on electron, $e = 1.6 \times 10^{-19}$ C.**

**Solution.** Here, $V = 3$ volt; $R = 1.5\ \Omega$; $e = 1.6 \times 10^{-19}$ C; $t = 1$s

Now $$I = \frac{V}{R} = \frac{3}{1.5} = 2\text{A}$$

Also $$I = \frac{q}{t} = \frac{ne}{t}$$

$\therefore$ $$n = \frac{It}{e} = \frac{2\times1}{1.6\times10^{-19}} = \mathbf{1.25 \times 10^{19}}$$

**Example 5.16. In a discharge tube, the number of hydrogen ions (*i.e.*, protons) drifting across a cross-section per second is $1.2 \times 10^{18}$ while the number of electrons drifting in opposite direction is $2.8 \times 10^{18}$ per second. If the supply voltage is 220V, what is the effective resistance of the tube?**

**Solution.** The electrons and protons move in the opposite directions. Therefore, total charge $q$ passing through the tube is

$$q = (n_e + n_p)\ e = (2.8 \times 10^{18} + 1.2 \times 10^{18})\ 1.6 \times 10^{-19} = 0.64\ \text{C}$$

$\therefore$ Current, $I = q/t = 0.64/1 = 0.64$ A

Effective resistance of tube, $$R = \frac{V}{I} = \frac{220}{0.64} = \mathbf{344\ \Omega}$$

**Example 5.17. A solid cube of silver has a mass of 84g. What is the resistance between the opposite faces? Given that density of silver is 10.5g/cm³ and resistivity is $1.6 \times 10^{-6}\ \Omega$ cm.**

**Solution.** Volume of the cube, $$V = \frac{\text{mass}}{\text{density}} = \frac{84}{10.5} = 8\ \text{cm}^3$$

Each side of the cube $= (8)^{1/3} = 2$ cm

$\therefore$ $$R = \rho\frac{l}{A} = 1.6 \times 10^{-6} = \frac{2}{2\times2} = \mathbf{0.8 \times 10^{-6}\ \Omega}$$

**Example 5.18. A copper wire is stretched so that its length is increased by 0.1%. What is the percentage change in its resistance?**

**Solution.** Suppose the initial length and area of copper wire are $l$ and $A$. Let, after stretching, these values be $l'$ and $A'$ respectively.

$\therefore$ $$l' = l + \frac{0.1}{100}\times l = 1.001\,l$$

Since volume of the wire remains the same before and after stretching,

$\therefore$ $$Al = A'\ l'$$

or $$A' = A\frac{l}{l'} = \frac{A}{1.001}\qquad\left(\because\ \frac{l}{l'} = \frac{1}{1.001}\right)$$

Let $R$ and $R'$ be the resistances of the wire before and after stretching respectively.

$$\therefore \qquad R = \rho\frac{l}{A}; \qquad R' = \rho\frac{l'}{A'}$$

$$\therefore \qquad \frac{R'}{R} = \left(\frac{l'}{l}\right)\times\left(\frac{A}{A'}\right) = (1.001)\,(1.001) = 1.002$$

$$\text{or} \qquad \frac{R'-R}{R} = 0.002 \qquad \text{[subtracting 1 from both sides]}$$

$$\therefore \qquad \text{Percentage increase} = \frac{R'-R}{R}\times 100 = (0.002)\times 100 = 0.2\%$$

**Example 5.19. A wire having a mass of 0.45 kg possesses a resistance of 0.0014 Ω . If the resistivity of the material of wire is $1.78 \times 10^{-8}$ Ω m , calculate its length and radius. Given that density of the material of wire is $8.93 \times 10^3$ kg/m$^3$**

**Solution.** Here $m = 0.45$ kg; $R = 0.0014\ \Omega$; $\rho = 1.78 \times 10^{-8}\ \Omega$ m; $d = 8.93 \times 10^3$ kg/m$^3$

Let $l$ and $r$ be the length and radius of the wire respectively.

$$\text{Resistance of wire, } R = \rho\frac{l}{A} = \rho\frac{l}{\pi r^2} \qquad \text{..... }(i)$$

$$\text{Mass of wire, } m = \text{volume} \times \text{density} = \pi\, r^2 l \times d \qquad \text{..... }(ii)$$

Multiplying eqs. (*i*) and (*ii*), we have,

$$R \times m = \rho\frac{l}{\pi r^2}\times \pi r^2 l \times d$$

$$\text{or} \qquad l = \sqrt{\frac{Rm}{\rho d}} = \sqrt{\frac{0.0014\times 0.45}{1.78\times 10^{-8}\times 8.93\times 10^3}} = 1.99\text{ m}$$

$$\text{From eq. }(ii), \qquad r = \sqrt{\frac{m}{\pi l d}} = \sqrt{\frac{0.45}{\pi\times 1.99\times 8.93\times 10^3}} = 2.84\times 10^{-3}\text{ m} = 2.84\text{ mm}$$

**Example 5.20. The winding of a motor has a resistance of 80 Ω at 15°C. Find its resistance at 50°C. The temperature co-efficient of resistance is 0.004/°C.**

**Solution.** $R_{15} = R_0\,(1 + \alpha \times 15)$

$$\therefore \qquad R_0 = \frac{R_{15}}{1+15\alpha} = \frac{80}{1+15\times 0.004} = 75.47\ \Omega$$

$$\text{Now} \qquad R_{50} = R_0\,(1 + \alpha\times 50) = 75.47\,(1 + 0.004 \times 50) = 90.56\ \Omega$$

**Example 5.21. The base of an incandescent lamp with a tungsten filament is marked 120 V, 60 W. The resistance of the filament when not burning is 20 Ω; the room temperature being 20°C. The lamp is then switched on to 120*V* supply. What is the normal working temperature of the lamp? The temperature co-efficient of resistance of tungsten is $5 \times 10^{-3}$/°C.**

**Solution.** Let $t$°C be the normal working temperature of the lamp.

$$\text{Resistance at 20°C, } R_{20} = 20\ \Omega$$

$$\text{Working current of the lamp} = 60/120 = 0.5\text{ A}$$

$$\text{Hot resistance of the lamp, } R_t = 120/0.5 = 240\ \Omega$$

$$R_t = R_{20}\,[1 + \alpha\,(t - 20)]$$

$$240 = 20\,[1 + 5 \times 10^{-3}\,(t - 20)]$$

$$\therefore \qquad t = 2220°\text{C}$$

**Example 5.22. The resistance of the platinum wire of a platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5.23Ω. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.795 Ω. Calculate the temperature of the bath.**

**Solution.** Here, $R_0 = 5\ \Omega$; $R_{100} = 5.23\ \Omega$; $R_t = 5.795\ \Omega$; $t = ?$

Now $$\alpha = \frac{R_{100} - R_0}{R_0 \times 100} = \frac{R_t - R_0}{R_0 \times t}$$

$\therefore$ $$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{5.795 - 5}{5.23 - 5} \times 100 = 345.65°\text{C}$$

**Example 5.23.** **A conductor has a cross-section of 15 cm$^2$ and a specific resistance of 7.6 μΩ cm at 0°C. If the temperature co-efficient of resistance of the material is 0.005°C$^{-1}$, estimate its resistance in ohm per 2 km, when its temperature is 50°C.**

**Solution.** Here, $A$ = 15 cm$^2$ = $15 \times 10^{-4}$ m$^2$; $\rho_0$ = 7.6 μΩ cm = $7.6 \times 10^{-8}$ Ωm; $l$ = 2 km = 2000 m; $\alpha$ = 0.005°C$^{-1}$; $R_{50}$ = ?

Resistance at 0°C, $$R_0 = \rho_0 \frac{l}{A} = 7.6 \times 10^{-8} \times \frac{2000}{15 \times 10^{-4}} = 0.101\ \Omega$$

$\therefore$ Resistance at 50°C, $$R_{50} = R_0 (1 + \alpha \times 50) = 0.101 (1 + 0.005 \times 50) = 0.126\ \Omega$$

**Example 5.24.** **A carbon filament has a resistance of 100 Ω at 0°C. What must be the resistance of a copper filament placed in series with carbon filament so that the combination has the same resistance at all temperatures? Temperature co-efficient of resistance of carbon = – 0.0007°C$^{-1}$ and that of copper is 0.004°C$^{-1}$.**

**Solution.** As $$\alpha = \frac{R_t - R_0}{R_0 t}$$

$\therefore$ Change in resistance $= \alpha R_0 t$

Let the resistance of copper filament placed in series with carbon filament be $R$ at 0°C so that the combination has the same resistance at all temperatures. For this to happen, the increase in resistance of copper filament should be equal to the decrease in resistance of the carbon filament *i.e.,*

$\therefore$ $$\alpha_{Cu} R t = \alpha_C R_0 t$$

or $$0.004 \times R \times t = 0.0007 \times 100 \times t$$

$\therefore$ $$R = \frac{0.0007 \times 100}{0.004} = 17.5\ \Omega$$

## PROBLEMS FOR PRACTICE

1. The density of conduction electrons in a wire is $10^{22}$ m$^{-3}$. If the radius of the wire is 0.6 mm and it is carrying a current of 2A, what will be the average drift velocity? **[$1.1 \times 10^{-3}$ ms$^{-1}$]**
2. A copper conductor has a uniform area of X-section of $5 \times 10^{-6}$ m$^2$ and carries a current of 10A. If the density of conduction electrons is $8 \times 10^{28}$ m$^{-3}$, what is the drift velocity of conduction electrons? **[$1.56 \times 10^{-4}$ ms$^{-1}$]**
3. In a hydrogen atom, the electron makes $6 \times 10^{15}$ revolutions per second around the nucleus. What will be the current developed in the orbit? **[0.96 mA]**
4. In the Bohr's model of hydrogen atom, the electron circulates around the nucleus in a path of radius $5.1 \times 10^{-11}$ m at a frequency of $6.8 \times 10^{15}$ revolutions per second. What is the equivalent current? **[$1.088 \times 10^{-3}$A]**
   [**Hint.** $I = e \times$ frequency $= (1.6 \times 10^{-19}) \times 6.8 \times 10^{15} = 1.088 \times 10^{-3}$A]
5. The earth's surface has a negative surface charge density of $10^{-9}$ Cm$^{-2}$. The potential difference of 400 kV between the top of atmosphere and the surface results in a current of only 1800A over the entire globe. If there were no mechanism of sustaining atmospheric electric field, how much time would be required to neutralise earth's surface? (This never happens in practice because there is a mechanism to replenish charge *viz.* the continual thunderstorms and lightening in different parts of the world.) Radius of earth is $6.37 \times 10^6$ m. **[283 s]**

[**Hint.** Total charge on entire earth, $q$ = Surface area × Surface charge density

$$= (4\pi R^2) \times \sigma = 4\pi \times (6.37 \times 10^6)^2 \times 10^{-9} = 5.1 \times 10^5 \text{ C}$$

$$I = \frac{q}{t} \quad \text{or} \quad t = \frac{q}{I} = \frac{5.1\times 10^5}{1800} = 283 \text{ s}]$$

**6.** $10^{22}$ conduction electrons each having a charge of $1.6 \times 10^{-19}$ C pass from a point $P$ towards point $Q$ in 10 s. What is the magnitude of current and its direction?

**[160 A; from $Q$ to $P$]**

[**Hint.** $$I = \frac{ne}{t} = \frac{10^{22}\times 1.6\times 10^{-19}}{10} = 160 \text{ A}]$$

**7.** The density of conduction electrons in a copper wire is $8.4 \times 10^{28}$ m$^{-3}$. What is the relaxation time for conduction electrons? The resistivity of copper is $1.7 \times 10^{-8}$ Ωm at room temperature. The mass and charge on an electron are $9.1 \times 10^{-31}$ kg and $1.6 \times 10^{-19}$ C respectively. **[$2.49 \times 10^{-14}$ s]**

**8.** The resistance of a coil is 135 Ω at 15°C. If current is passed through it, its resistance becomes 155 Ω. What is the rise in temperature? Given that temperature co-efficient of resistance of coil material is $4.28 \times 10^{-3}$/°C. **[37°C]**

**9.** A certain aluminium wire has a resistance of 28.3 Ω at 20°C. What is its resistance at 60°C? The temperature co-efficient of resistance of aluminium is 0.00403/°C.

[**Hint.** $R_{60} = R_{20}\,[1 + \alpha\,(60 - 20)]$ **[32.86 Ω]**

**10.** A copper coil has a resistance of 40 Ω at 0°C. Find the resistance of this coil at 50°C. The temperature co-efficient of resistance of coil material is 0.0043/°C. **[48.6 Ω]**

**11.** Calculate the resistance of a manganin wire 100m long having a uniform area of X-section of 0.1 mm$^2$. Given that resistivity of manganin is $50 \times 10^{-8}$ Ω m. **[500Ω]**

**12.** The resistance of a wire 15m long having a uniform area of X-section of $6 \times 10^{-7}$m$^2$ is 5 Ω. What is the resistivity of the material of the wire? **[$2 \times 10^{-7}$ Ω m]**

**13.** A cube of a material of side 1 cm has a resistance of 0.001Ω between the opposite faces. If the same material has a length of 9 cm and a uniform X-sectional area of 1 cm$^2$, what will be the resistance of this length? **[0.009 Ω]**

**14.** A wire of length 1m has a resistance of 2 ohms. What is the resistance of a second wire, whose resistance is double of the first, if length of wire is 3m and diameter is double of the first? **[3 Ω]**

**15.** The resistance of a wire is $R$. What will be the new resistance if it is stretched $n$ times to its original length? **[$n^2R$]**

**16.** When same potential difference is applied across the ends of wires of iron and copper of same length, the same current flows through them. Find the ratio of their radii. The resistivities of iron and copper are $1.0 \times 10^{-7}$ Ω m and $1.6 \times 10^{-8}$ Ω m respectively.

**Hint.** $R = \rho\dfrac{l}{A} = \rho\dfrac{l}{\pi r^2}$ or $r = \sqrt{\rho\dfrac{l}{\pi R}}$ or $r \propto \sqrt{\rho}$ [∵ $l$ and $R$ are constant]

**17.** A current of 1.6 A is flowing through a copper voltameter. Determine the number of copper ions deposited on cathode in one minute ($e = 1.6 \times 10^{-19}$ C)

**[$3.0 \times 10^{20}$]**

**18.** A current of 0.5 A is flowing through a manganin wire. Calculate the number of electrons passing per minute through the cross-section of the wire. **[$18.75 \times 10^{19}$]**

## 5.20. CARBON RESISTORS

A component whose function in a circuit is to provide a specified value of resistance is called a *resistor*. The most commonly used resistors in electrical and electronic circuits are the carbon resistors. A carbon resistor is made from powdered carbon mixed with a binding material and baked into a small tube with a wire attached to each end. These small-sized resistors are manufactured in values from a fraction of an ohm to several million ohms.

**Colour code for carbon resistors.** Since a carbon resistor is physically quite small, it is more convenient to use a *colour code* indicating the resistance value than to imprint the numerical value on the case. *In this scheme, there are generally four colour bands A, B, C and D printed on the body of the resistor as shown in Fig. 5.13.* The first three colour bands ($A$, $B$ and $C$) give the value of the resistance while the fourth band ($D$) tells about the *tolerance in percentage. The table below shows the colour code for resistance values and colour code for tolerance.

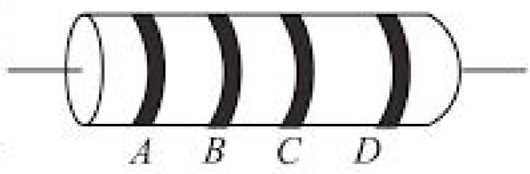



**Fig. 5.13**

**Colour Code for Resistance Values**

| | | | |
|---|---|---|---|
| Black | 0 | Green | 5 |
| Brown | 1 | Blue | 6 |
| Red | 2 | Violet | 7 |
| Orange | 3 | Grey | 8 |
| Yellow | 4 | White | 9 |

**Colour Code for Tolerance**

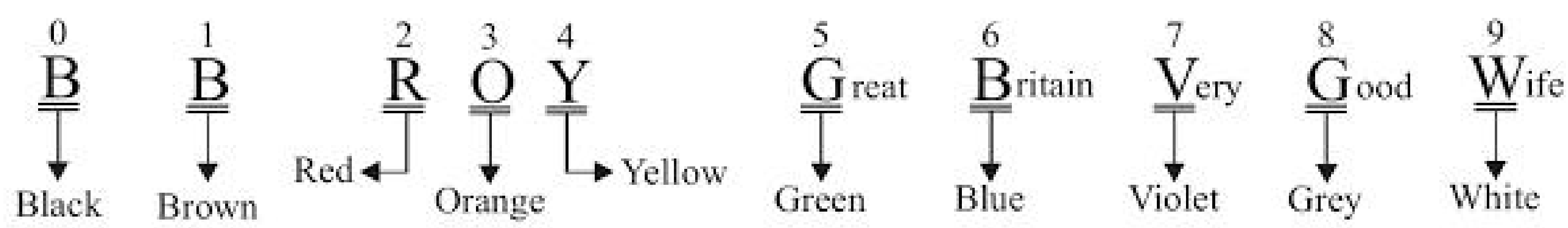
| | |
|---|---|
| Gold | ± 5% |
| Silver | ± 10% |
| No colour | ± 20% |

(*i*) To read the resistance value, we refer to the first three colour bands ($A$, $B$ and $C$). **The first two colour bands ($A$, $B$) specify the first two digits of the resistance value and the third colour band ($C$) gives the number of zeros that follow the first two digits.** Suppose the first three colour bands ($A$, $B$, $C$) on the resistor are red, brown, orange respectively. Then value of the resistance is 21, 000 Ω.

| | | |
|---|---|---|
| Red | : | 2 |
| Brown | : | 1 |
| Orange | : | 000 |

$\therefore$ Value = 21,000 Ω

(*ii*) **The fourth band $D$ gives the value of tolerance in percentage.** If colour of the fourth band is gold, tolerance is ± 5 per cent and if silver, then tolerance is ± 10 per cent. If the fourth band is omitted, the tolerance is assumed to be ± 20 per cent.




**Note.** In order to remember the colour code, the above sentence may be helpful.

**Example 5.25. The colour coded carbon resistors are shown in Fig. 5.19. Find their resistance values.**

**Solution.** Refer to Fig. 5.14 (*i*). The first colour represents the digit 5. The second colour represents the digit 6. The third colour represents the digit 4, *i.e.*, four zeros. Therefore, the value of the resistance is 560000 Ω. The fourth gold strip indicates ±5% tolerance. Hence, resistance specification of the resistor is

**560000 Ω ;** ± 5%

* Due to manufacturing variations, the resistance value may not be the same as indicated by colour code. Thus, a resistor marked 100 Ω; ± 10% tolerance means that resistance value is between 90 Ω and 110 Ω.

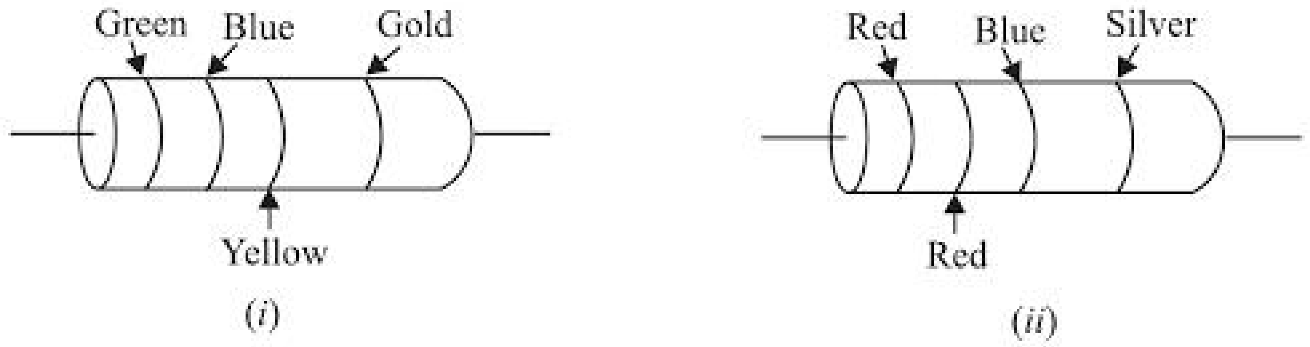



**Fig. 5.14**

Refer to Fig. 5.14 (*ii*). Following above procedure, the resistance specification of this resistor is

**22,000000 Ω; ± 10%**

**Example 5.26. How will you represent a resistance of 3700 Ω ± 10% by colour code?**

**Solution.** The value of the carbon resistance = 3700 Ω ± 10% $= 37 \times 10^2$ Ω ± 10%

The colours of bands corresponding to numbers 3 and 7 are orange and violet respectively. The colour band corresponding to multiplier $10^2$ is red. For 10% tolerance, colour band is silver. Thus the sequence of colour bands on this carbon resistor is orange, violet, red and silver.

## PROBLEMS FOR PRACTICE

1. A voltage of 30 V is applied across a colour coded carbo resistor with first, second and third rings of blue, black and yellow colours. What is the current flowing through the resistor? **[$0.5 \times 10^{-4}$ A]**
2. The carbon resistor has coloured strips with sequence brown, black, brown and gold. What is the value of the resistor? **[(100 ± 5%) Ω]**
3. A carbon resistor is marked in coloured bands in the sequence blue, green, orange and gold. What is the resistance and tolerance value of the resistor? **[$65 \times 10^3$ Ω; ± 5%]**
4. Draw the colour code scheme of 42 kΩ + 10% carbon resistance **[yellow, red, orange and silver]**

## 5.21. NON-OHMIC CONDUCTORS

*Those conductors which do not obey Ohm's law ($I \propto V$) are called non-ohmic conductors.* *e.g.*, vacuum tubes, transistors, electrolytes, etc. A non-ohmic conductor may have one or more of the following properties :

(*i*) The *V–I* graph is non-linear *i.e.* *V/I* is variable.

(*ii*) The *V–I* graph may not pass through the origin as in case of an ohmic conductor.

(*iii*) A non-ohmic conductor may conduct poorly or not at all when the p.d. is reversed.

The non-linear circuit problems are generally solved by graphical methods.

Fig. 5.15 illustrates the graphs of non-ohmic conductors. Note that *V-I* graphs for these non-ohmic conductors are not a straight line.

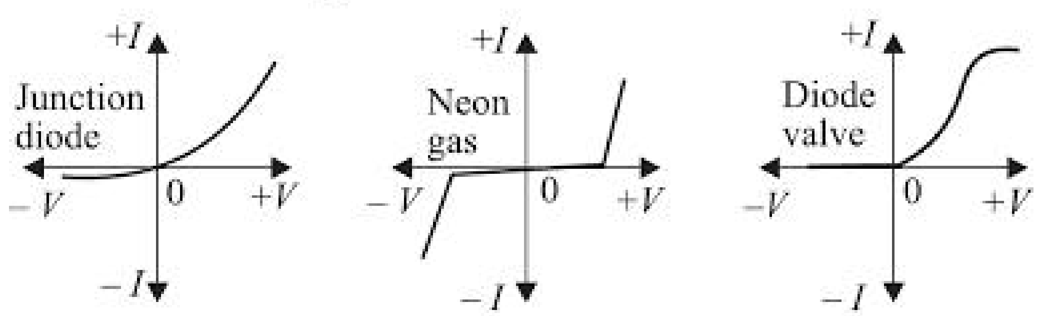



**Fig. 5.15**

## 5.22. THERMISTORS

*A **thermistor** is a heat sensitive device usually made of a semiconductor material whose resistance changes very rapidly with change of temperature.* A thermistor has the following important properties :

(*i*) The resistance of a thermistor changes very rapidly with change of temperature.

(*ii*) The temperature co-efficient of a thermistor is very high.

(*iii*) The temperature co-efficient of a thermistor can be both positive and negative.

**Construction.** Thermistors are made from semiconductor oxides of iron, nickel and cobalt. They are generally in the form of beads, discs or rods (See Fig. 5.16). A pair of platinum leads are attached at the two ends for electrical connections. The arrangement is enclosed in a very small glass bulb and sealed.

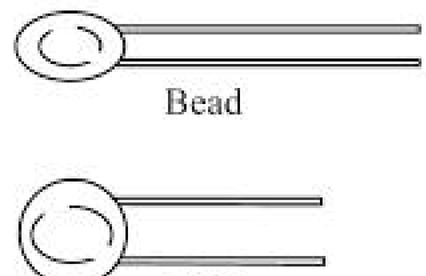



**Fig. 5.16**

**Applications**

(*i*) A thermistor with negative temperature co-efficient of resistance may be used to safeguard against current surges in a circuit where this could be harmful *e.g.* in a circuit where the heaters of the radio valves are in series (See Fig. 5.17).

A thermistor $T$ is included in the circuit. When the supply voltage is switched on, the thermistor has a high resistance at first because it is cold. It thus limits the current to a moderate value. As it warms up, the thermistor resistance drops appreciably and an increased current then flows through the heaters.

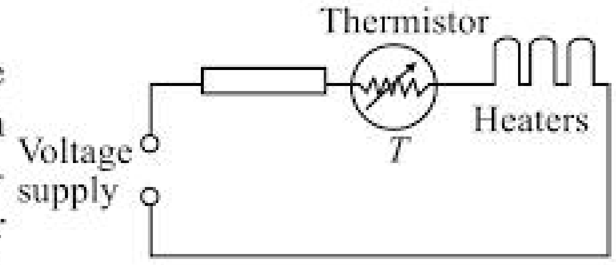



**Fig. 5.17**

(*ii*) A thermistor with a negative temperature coefficient can be used to issue an alarm for excessive temperature of winding of motors, transformers and generators [See Fig. 5.18].

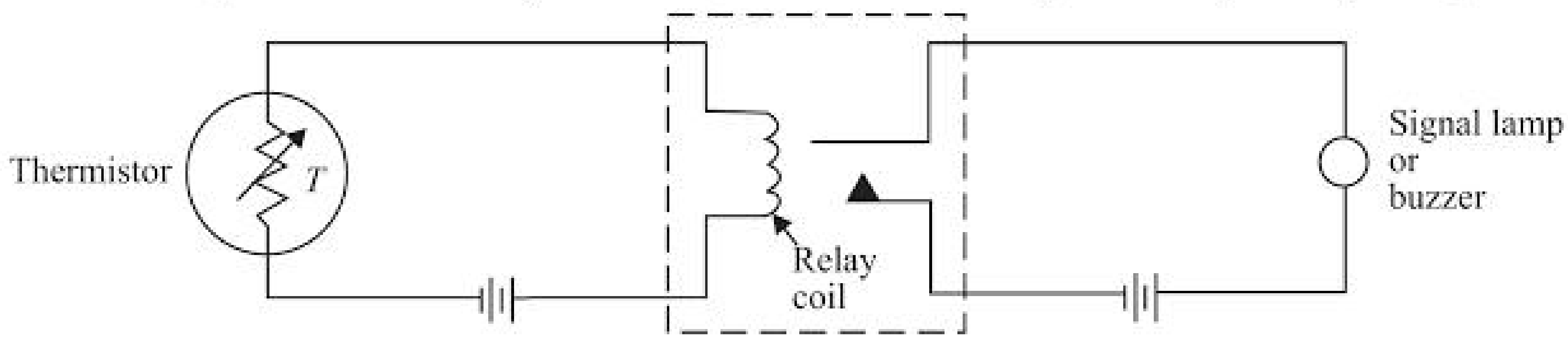



**Fig. 5.18**

When the temperature of windings is low, the thermistor is cool and its resistance is high. Therefore, only a small current flows through the thermistor and the relay coil. When the temperature of the windings is high, the thermistor is hot and its resistance is low. Therefore, a large current flows in the relay coil to close the contacts. This completes the circuit for the signal lamp or buzzer.

(*iii*) Thermistors are used for voltage stabilisation, temperature control and remote sensing.

(*iv*) Thermistors are used to measure very low temperatures of the order of 10K.

## 5.23. SUPERCONDUCTIVITY

There are metals and compounds whose resistivity goes to zero below a certain temperature $T_C$ called **critical temperature** (or transition temperature). These materials are known as **superconductors.**

| *Material* | $T_C$ (K) |
|---|---|
| Al | 1.20 |
| Pb | 7.19 |
| Hg | 4.15 |
| Nb | 9.26 |
| Sn | 3.72 |
| Zn | 0.85 |
| $Nb_3Ge$ | 23.2 |
| $Nb_3Sn$ | 18.05 |
| $NbSn_2$ | 2.60 |
| $PbTi_{0.27}$ | 6.43 |

*This phenomenon of zero resistance of some metals and compounds when cooled to critical temperature is called* **superconductivity**.

The critical temperatures for several metals and alloys are shown in the adjoining table. At present, the problem with using the materials listed in the table for practical applications is that we must use liquid helium to cool them into the superconducting state. Liquid helium is expensive and supplies are limited.

Fig. 5.19 (*i*) shows resistance-temperature graph for normal metals whereas Fig. 5.19 (*ii*) shows resistance-temperature graph for a superconductor. Note that resistance-temperature graph for a superconductor follows that for a normal metal at temperature above $T_C$ (critical temperature). When the temperature is at or below $T_C$, the resistance drops suddenly to zero [See Fig. 5.19 (*ii*)]. Recent measurements have shown that resistivities of superconductors below $T_C$ are less than $4 \times 10^{-25}$ Ω m which is around $10^{17}$ times smaller than the resistivity of copper !

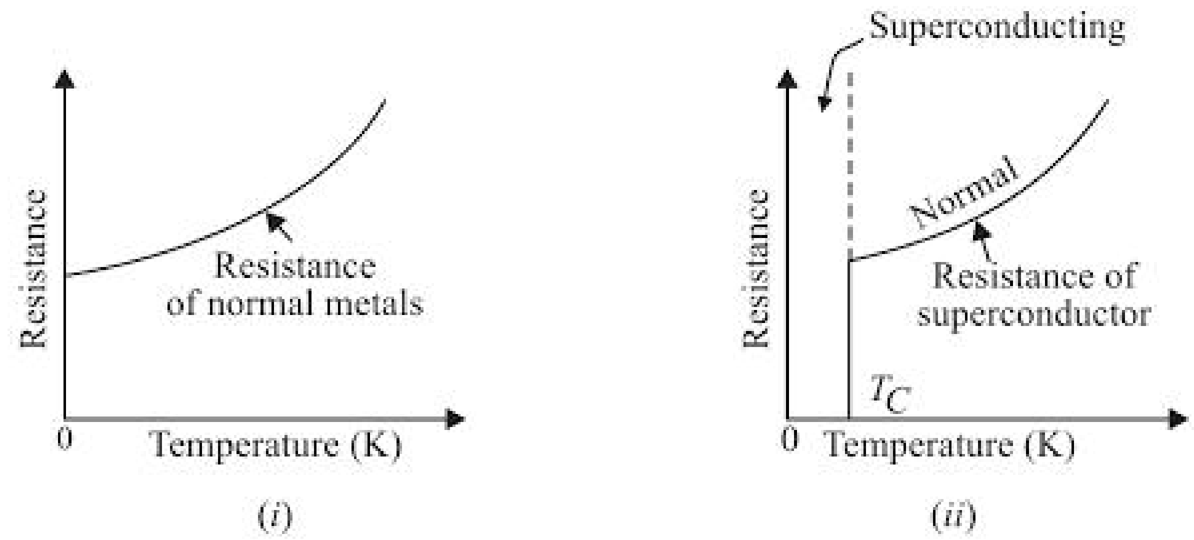



**Fig. 5.19**

One of the remarkable features of superconductors is that once a current is set up in the material, the current will persist *without any applied voltage* (since $R = 0$). In fact, steady currents have been observed to persist in superconducting loops for several years with no apparent decay. Today there are thousands of known superconductors with critical temperatures as high as 23K for one alloy made of a combination of niobium, aluminium and germanium. The critical temperature is sensitive to chemical composition, pressure and crystalline structure.

**Applications (or uses) of superconductors.** The scientists are making continuous efforts to produce room-temperature superconductors. Once this goal is achieved, the superconductivity will offer the following uses :

(*i*) It will offer the possibility of loss-free transmission of electric power. It is because superconducting transmission lines would have zero resistance.

(*ii*) It will help for the construction of superconducting magnets in which the magnetic field intensities would be about 10 times greater than those of the best normal electromagnets.

(*iii*) A superconducting computer would be smaller and faster than today's computers.

(*iv*) Superconducting electromagnets offer substantial savings in large particle accelerators for high-energy physics research.

(*v*) Superconducting transmission lines would permit the transmission of electric power at lower voltage, making underground power lines possible.

## 5.24. D.C. CIRCUIT

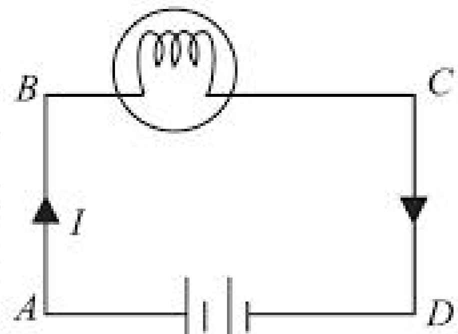



Fig. 5.20

The closed path followed by direct current (d.c.) is called a *d.c. circuit*. A d.c. circuit essentially consists of a source of direct voltage (*e.g.*, battery), the conductors used to carry current and the load. Fig. 5.20 shows a torch bulb (*i.e.*, load) connected to a battery through conducting wires. The direct current *starts from the positive terminal of the battery and comes back to the starting point via the load. The direct current follows the closed path $ABCDA$ and hence $ABCDA$ is a d.c. circuit. The load for a d.c. circuit is usually a **resistance. In a d.c. circuit, loads (*i.e.*, resistances) may be connected in series or parallel or series-parallel.

## 5.25. RESISTORS IN SERIES

*A number of resistors are said to be connected in series if the same current flows through each resistor and there is only one path for the current flow throughout.*

Consider three resistors of resistances $R_1$, $R_2$ and $R_3$ connected in series across a battery of e.m.f. $E$ volts as shown in Fig. 5.21 (*i*). Let $I$ be the circuit current. By Ohm's law,

$$V_1 = I R_1; \quad V_2 = I R_2; \; V_3 = I R_3$$

Now, $$E = V_1 + V_2 + V_3 = I R_1 + I R_2 + I R_3$$

or $$E = I (R_1 + R_2 + R_3)$$

or $$E/I = R_1 + R_2 + R_3$$

But $E/I$ is the †total or equivalent resistance $R_S$ between points $A$ and $B$.

$\therefore$ $$R_S = R_1 + R_2 + R_3$$

*Hence, when a number of resistances are connected in series, the total resistance is equal to the sum of the individual resistances.*

Thus, we can replace the series connected resistors shown in Fig. 5.21 (*i*) by a single resistor $R_S$ ( $= R_1 + R_2 + R_3$) as shown in Fig. 5.21 (*ii*). This will enable us to calculate the circuit current easily ($I = E/R_S$).

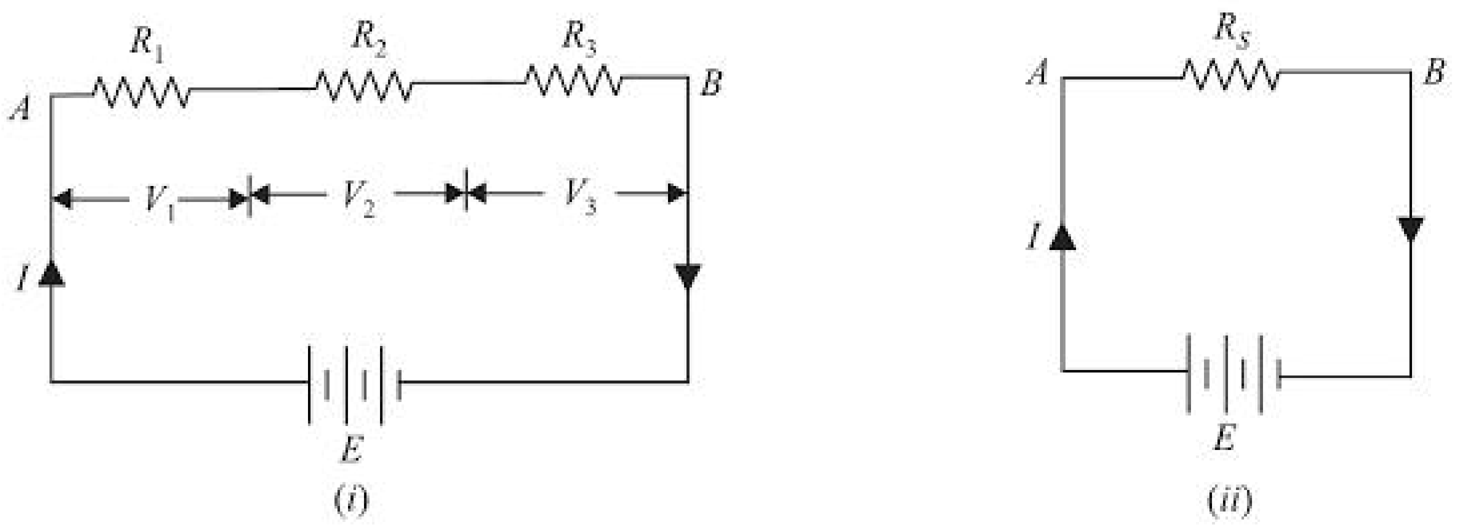



Fig. 5.21

**Characteristics.** The following are the characteristics of a series circuit :

(*i*) The current in each resistor is the same.

(*ii*) The total resistance in the circuit is equal to the sum of individual resistances plus internal resistance of the cell if any.

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* This is the direction of conventional current. However, the electron flow will be in the opposite direction.

** Other passive elements *viz.* inductance ($L$) and capacitance ($C$) are relavant only in a.c. circuits.

† Total or equivalent resistance is the single resistance, which if substituted for the series resistances, would provide same current in the circuit.

(*iii*) The voltage drop across any resistor is directly proportional to the resistance of that resistor.

(*iv*) The current in the circuit is independent of the relative positions of the resistors in the circuit.

(*v*) The total resistance in the series circuit is more than the largest resistance in the circuit.

**Note.** *The main disadvantage of a series circuit is that if one device (or resistor) fails, the current in the whole circuit ceases.*

## 5.26. RESISTORS IN PARALLEL

*A number of resistors are said to be connected in parallel if potential difference (i.e., voltage) across each resistor is the same and there are as many parallel paths as the number of resistors.*

Consider three resistors of resistances $R_1$, $R_2$ and $R_3$ connected in parallel across a battery of $E$ volts as shown in Fig. 5.22 (*i*). Note that voltage across each resistor is the same (*i.e.*, $E$ volts) and there are as many paths for current flow as the number of resistors. By Ohm's law,

$$I_1 = \frac{E}{R_1};\ I_2 = \frac{E}{R_2};\ I_3 = \frac{E}{R_3}$$

Now $$I = I_1 + I_2 + I_3 = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3}$$

or $$I = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

or $$\frac{I}{E} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

But $E/I$ is the total or equivalent resistance $R_P$ of the parallel connected resistors so that $I/E = 1/R_P$.

$$\therefore \qquad \frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

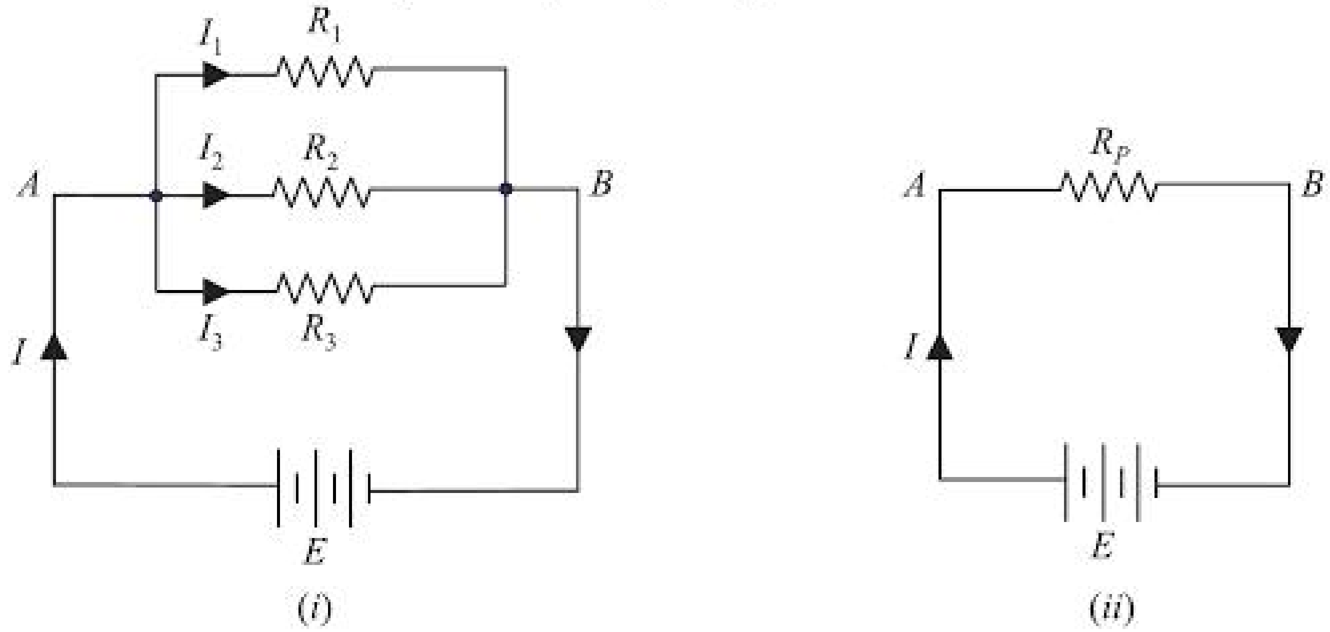



**Fig. 5.22**

*Hence, when a number of resistances are connected in parallel, the reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.*

Again we can replace parallel connected resistors shown in Fig. 5.22 (*i*) by a single resistor $R_P$ as shown in Fig. 5.22 (*ii*). The circuit current is $I = E/R_P$.

**Characteristics.** The following are the characteristics of a parallel circuit :

(*i*) The voltage across each resistor is the same.

(*ii*) The current through any resistor is inversely proportional to its resistance.

(*iii*) The total current in the circuit is equal to the sum of currents in its parallel branches.

(*iv*) The reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.

(*v*) As the number of parallel branches is increased, the total resistance of the circuit is decreased.
(*vi*) The total resistance of the circuit is always less than the smallest of resistances.
(*vii*) If $n$ resistors, each of resistance $R$, are connected in parallel, then total resistance, $R_p = R/n$.

**Note.** *In homes, electrical devices are connected in parallel (not in series). Any electrical device can be turned on or off without affecting the operation of other electrical devices.*

## 5.27. TWO RESISTORS IN PARALLEL

A frequent special case of parallel resistors is a circuit that contains two resistors in parallel as shown in Fig. 5.23. The total circuit current $I$ divides into two parts; $I_1$ flowing through $R_1$ and $I_2$ flowing through $R_2$.

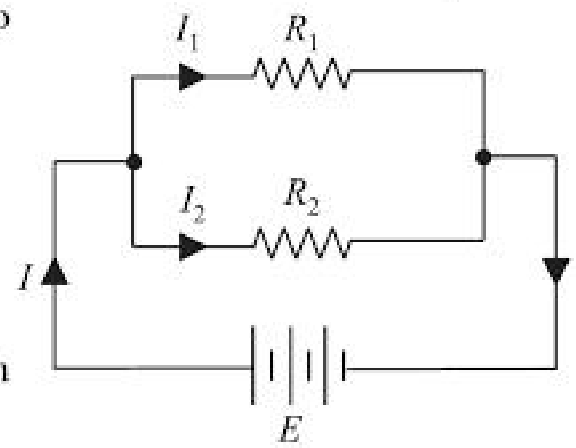


**Fig. 5.23**

**(*i*) Total Resistance.**
$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

or
$$R_P = \frac{R_1 R_2}{R_1 + R_2} \quad i.e., \quad \frac{\text{Product}}{\text{Sum}}$$

Thus, if two resistances of $3\Omega$ and $6\ \Omega$ are connected in parallel, then their total or equivalent resistance $R$ is
$$R = \frac{3 \times 6}{3 + 6} = \frac{18}{9} = 2\Omega$$

**(*ii*) Branch Currents.**
$$E = I\, R_P = I\, \frac{R_1 R_2}{R_1 + R_2}$$

Now
$$I_1 = \frac{E}{R_1} = \left( I\, \frac{R_1 R_2}{R_1 + R_2} \right) \frac{1}{R_1} = I\, \frac{R_2}{R_1 + R_2}$$

$\therefore$
$$I_1 = I \times \frac{R_2}{R_1 + R_2}$$

Similarly,
$$I_2 = I \times \frac{R_1}{R_1 + R_2}$$

*i.e.*, Current in any of the two branches = Total current $\times \dfrac{\text{Other resistance}}{\text{Sum of the two resistances}}$

Thus referring to Fig. 5.24, the currents in the two branches are :
$$I_1 = 9 \times \frac{6}{3 + 6} = 6\,\text{A}$$
$$I_2 = 9 \times \frac{3}{3 + 6} = 3\,\text{A}$$

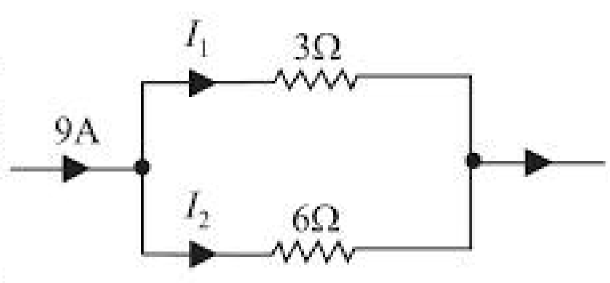


**Fig. 5.24**

**Example 5.27.** **Two coils connected in series have a resistance of 18 Ω and when connected in parallel have a resistance of 4 Ω. Find the value of individual resistances of the coils.**

**Solution.** Let $R_1$ and $R_2$ be the resistances of the coils.
$$R_1 + R_2 = 18 \qquad ...(i) \text{ series connection}$$
$$\frac{R_1 R_2}{R_1 + R_2} = 4 \qquad ...(ii) \text{ parallel connection}$$

Multiplying eqs. (*i*) and (*ii*), we have, $R_1 R_2 = 18 \times 4 = 72$

Now $(R_1 - R_2)^2 = (R_1 + R_2)^2 - 4R_1R_2 = 18^2 - 4\ (72) = 36$

$\therefore$ $R_1 - R_2 = \pm 6$ ...(*iii*)

Solving eqs. (*i*) and (*iii*), we get, $R_1 = 12\Omega$ or $6\Omega$; $R_2 = 6\Omega$ or $12\Omega$

**Example 5.28. Five equal resistors each of 2Ω are connected in a network as shown in Fig. 5.25. Find the equivalent resistance between points *A* and *B*.**

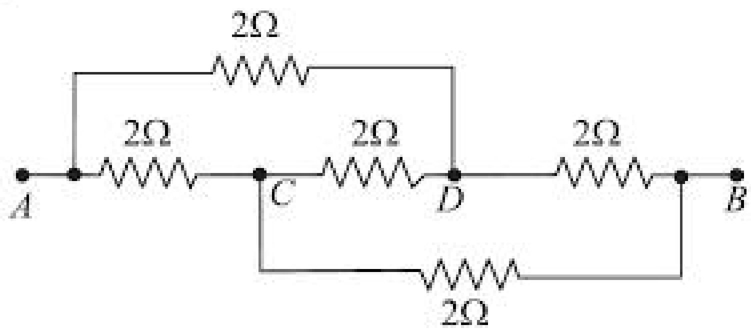



Fig. 5.25

**Solution.** The given network can be redrawn as shown in Fig. 5.26 (*i*). Clearly, it is balanced Wheatstone bridge. Therefore, point *C* and *D* are at the same potential. Since no current flows in the branch *CD*, this branch is ineffective in determining the equivalent resistance between *A* and *B* and can be removed. The circuit then reduces to that shown in Fig. 5.26 (*ii*).

$$\therefore \qquad \frac{1}{R_{AB}} = \frac{1}{4}+\frac{1}{4}=\frac{1+1}{4}=\frac{1}{2}$$

$$\text{or} \qquad R_{AB} = 2\Omega$$

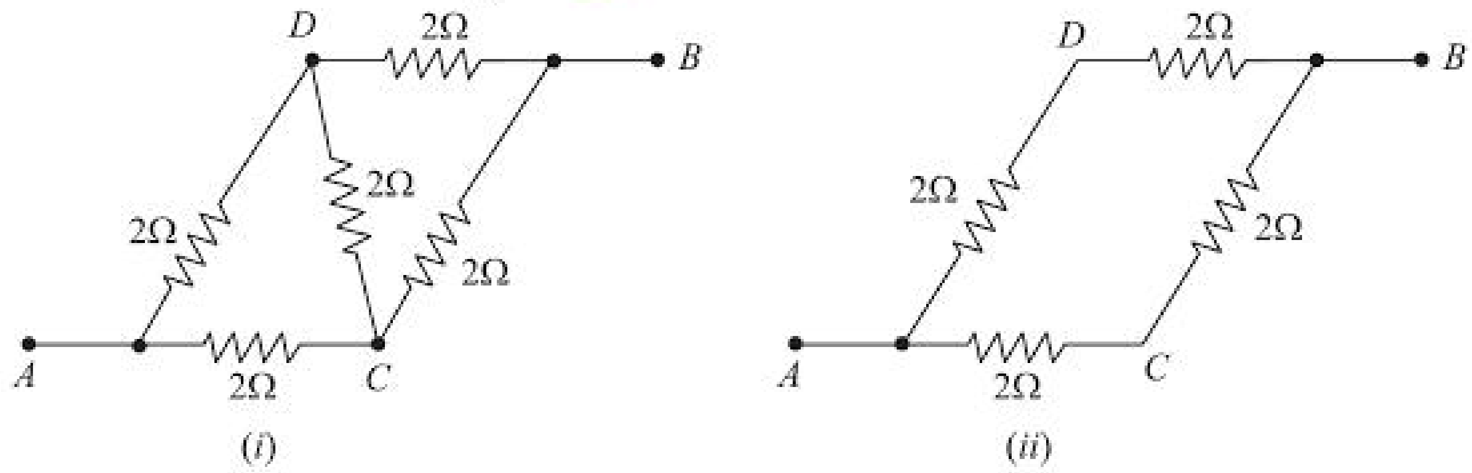



Fig. 5.26

**Example 5.29. Find the equivalent resistance between points *A* and *B* in the circuit shown in Fig. 5.27.**

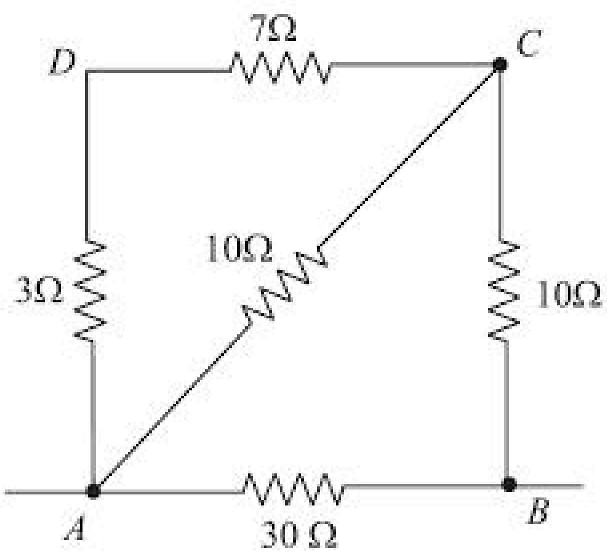



Fig. 5.27

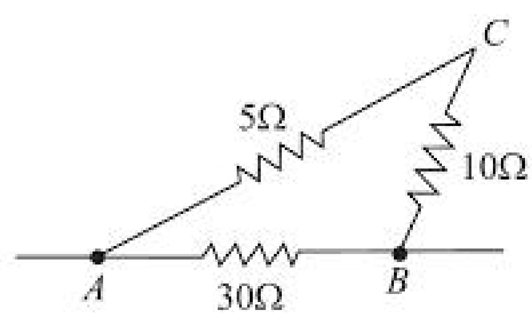



Fig. 5.28

**Solution.** The resistors in the branch *ADC* are in series ( = 3 + 7 = 10 Ω) and this branch is in parallel with *AC* ( = 10 Ω). The two 10 Ω resistors in parallel will give a resistance of 5 Ω. The circuit then reduces to as shown in Fig. 5.28. The resistors in the branch *ACB* are in series ( = 5 + 10 = 15 Ω) and this branch is in parallel with 30Ω resistor.

$$\therefore \qquad R_{AB} = \frac{15\times 30}{15+30}=\frac{15\times 30}{45} = 10\ \Omega$$

**Example 5.30. What is the equivalent resistance between the terminals *A* and *B* in Fig. 5.29?**

**Solution.** The network shown in Fig. 5.29 can be redrawn as shown in Fig. 5.30 (*i*). It is a balanced Wheatstone bridge. Therefore, points *C* and *D* are at the same potential. Since no current flows in the branch $CD$, this branch is ineffective in determining the equivalent resistance between terminals $A$ and $B$ and can be removed. The circuit then reduces to that shown in Fig. 5.30 (*ii*).

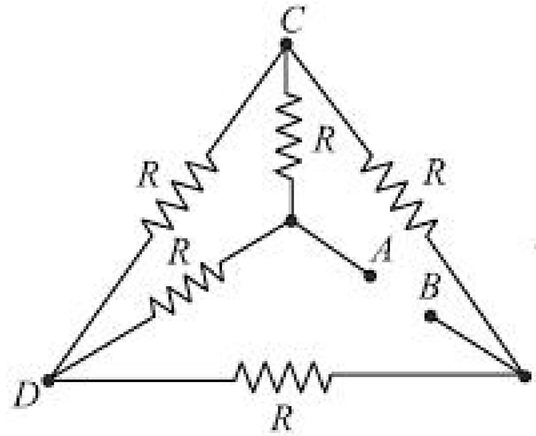



Fig. 5.29

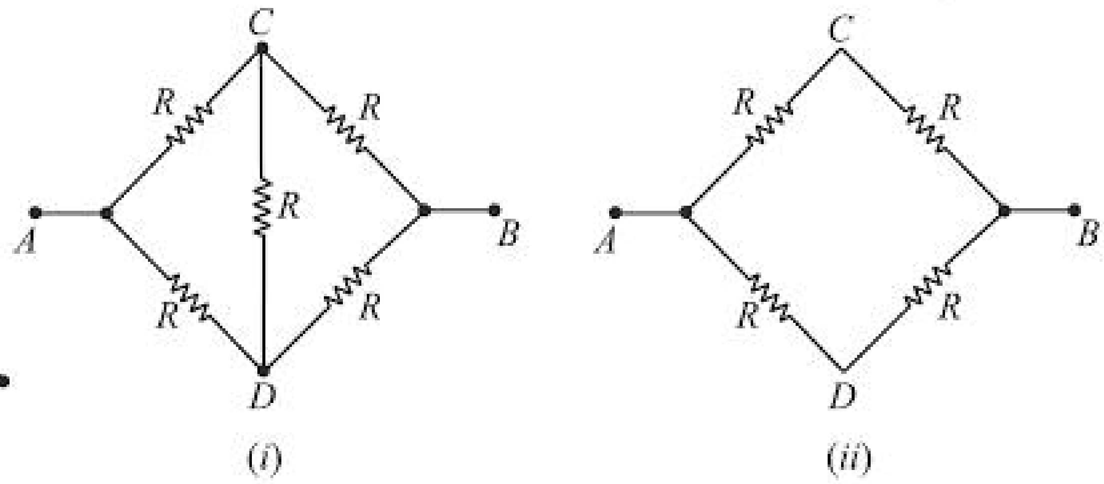



Fig. 5.30

The branch $ACB$ ( $= R + R = 2R$ ) is in parallel with branch $ADB$ ($= R + R = 2R$ ).

$$\therefore \qquad R_{AB} = \frac{(2R)\times(2R)}{2R+2R} = R$$

**Example 5.31. An electrical network is arranged as shown in Fig. 5.31. Find the value of current in the branch $AF$.**

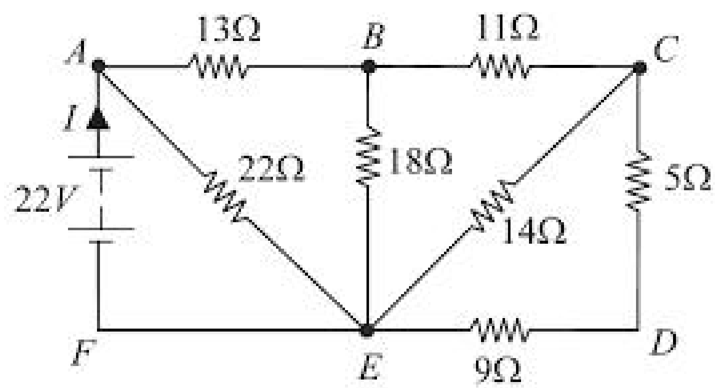



Fig. 5.31

**Solution.** Resistance between $E$ and $C$,

$$R_{EC} = \frac{(5+9)\times 14}{(5+9)+14} = 7\,\Omega$$

Resistance between $B$ and $E$,

$$R_{BE} = \frac{(11+7)\times 18}{(11+7)+18} = 9\,\Omega$$

Resistance between $A$ and $E$,

$$R_{AE} = \frac{(13+9)\times 22}{(13+9)+22} = 11\,\Omega$$

*i.e.*, Total circuit resistance, $R_T = 11\Omega$

$\therefore$ Current in branch $AF$, $I = V/R_T = 22/11 = 2$ **A**

**Example 5.32. Determine the equivalent resistance between terminals $A$ and $B$ of the network shown in Fig. 5.32.**

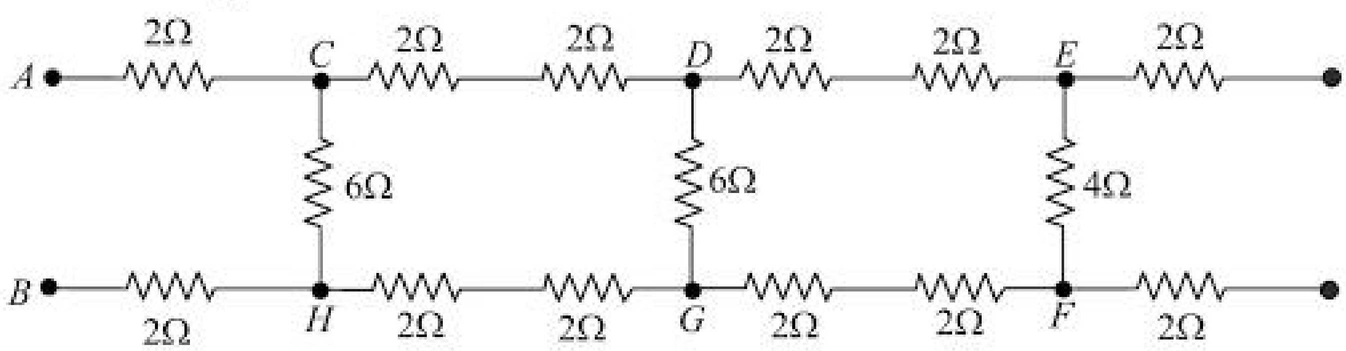



Fig.5.32

**Solution.** Two 2-ohm resistors at the extreme right are ineffective in determining the equivalent resistance and may be assumed as absent. The resistors in branch $DEFG$ are in series ( $= 2 + 2 + 4 + 2 + 2 = 12\ \Omega$) and this branch is in parallel with branch $DG$ ( $= 6\ \Omega$).

$\therefore$ Resistance between $D$ and $G$, $R_{DG} = \dfrac{12\times 6}{12+6} = 4\,\Omega$

The circuit shown in Fig. 5.32 reduces to the circuit shown in Fig. 5.33.

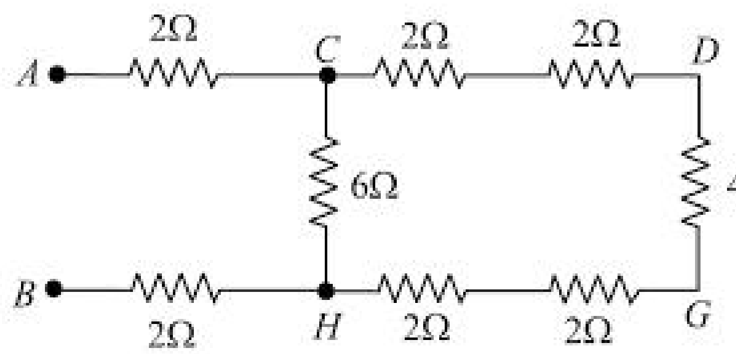



**Fig. 5.33**

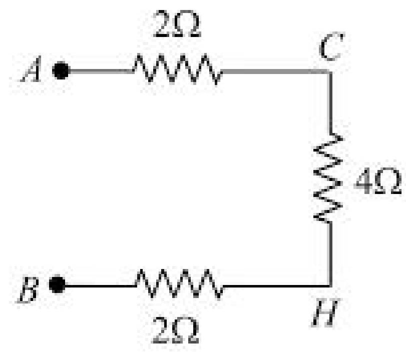



**Fig. 5.34**

Resistance between $C$ and $H$, $R_{CH} = \dfrac{12 \times 6}{12 + 6} = 4\,\Omega$

The circuit then reduces to that shown in Fig. 5.34.

$\therefore \quad R_{AB} = 2 + 4 + 2 = $ **8 Ω**

Example 5.33. **Fig. 5.35 shows a variable rheostat of 2*k* Ω used to control the potential difference across a 500 Ω load.**

(*i*) **If the resistance *AB* is 500 Ω, what is the potential difference across the load?**

(*ii*) **If the load is removed, what would be the resistance at *BC* to get 40V between *B* and *C*?**

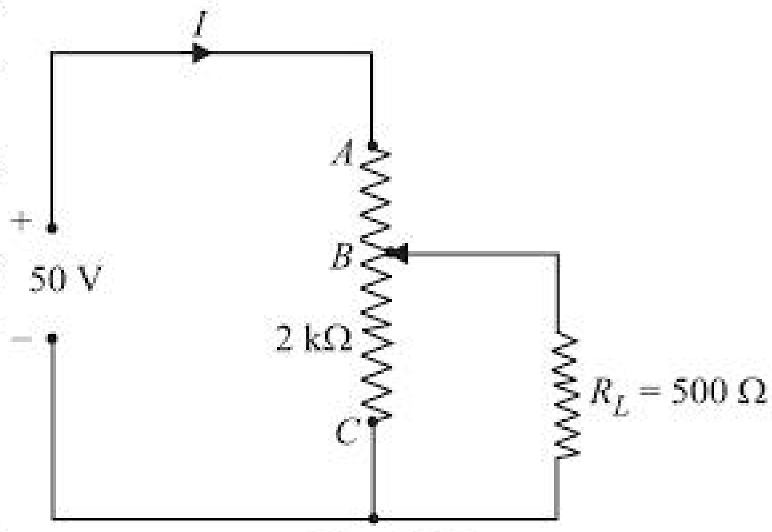



**Fig. 5.35**

**Solution.**

(*i*) $R_{AB} = 500\ \Omega$; $\quad R_L = 500\ \Omega$;

$R_{BC} = 1500\ \Omega$. Now $R_{BC}$ (=1500 Ω); and $R_L$ (= 500 Ω) are in parallel and their total resistance $R'$ is

$$R' = \frac{R_{BC} \times R_L}{R_{BC} + R_L} = \frac{1500 \times 500}{1500 + 500} = 375\ \Omega$$

Also $R'$ and $R_{AB}$ are in series and the circuit current $I$ is given by:

$$I = \frac{E}{R_{AB} + R'} = \frac{50}{500 + 375} = \frac{2}{35}\ \text{A}$$

Potential difference across $AB = I \times R_{AB} = \dfrac{2}{35} \times 500 = 28.57$ V

$\therefore$ P.D. across $R_L = 50 - 28.57 = $ **21.43 V**

(*ii*) When load $R_L$ (= 500 Ω) is removed, the total circuit resistance is $R_{AC} = 2\ k\Omega = 2000\ \Omega$.
Therefore circuit current, $I' = E/R_{AC} = 50/2000 = (1/40)$ A.

Voltage across $BC = 40$ V

$\therefore$ Resistance of $BC = 40 \div \dfrac{1}{40} = $ **1600 Ω**

Example 5.34. **A uniform wire of resistance *R* is shaped into a regular polygon of *n* sides where *n* is even. Find the equivalent resistance between (*i*) opposite corners of the polygon, (*ii*) adjacent corners of the polygon.**

**Solution.** The resistance of each side of polygon = $R/n$.

(*i*) Between opposite corners of the polygon, we have two resistances in parallel, each of value $R/2$.

$\therefore$ Total resistance $R'$ between opposite corners is

$$R' = \frac{(R/2)\times(R/2)}{\frac{R}{2}+\frac{R}{2}} = \frac{R}{4}$$

(*ii*) Between adjacent corners of the polygon, we have two resistances $R/n$ and $(n-1)R/n$ in parallel. Therefore total resistance $R''$ between adjacent corners is

$$R'' = \frac{(R/n)\,(n-1)\,R/n}{(R/n)+(n-1)\,R/n} = \frac{(n-1)\,R}{n^2}$$

Example 5.35. **A resistor of 5 Ω is connected in series with a parallel combination of a number of resistors each of 5 Ω. If the total resistance of the combination is 6 Ω, how many resistors are in parallel?**

**Solution.** Let $n$ be the required number of 5 Ω resistors to be connected in parallel. The equivalent resistance of this parallel combination is

$$\frac{1}{R_P} = \frac{1}{5}+\frac{1}{5}+\frac{1}{5}\ldots n \text{ times} = \frac{n}{5}$$

Therefore, $R_P = 5/n$

Now $R_P$ (= $5/n$) in series with 5 Ω is equal to 6 Ω *i.e.*,

$$\frac{5}{n} + 5 = 6 \quad \therefore \quad n = 5$$

Example 5.36. **A letter A consists of a uniform wire of resistance 1 Ω per cm. The sides of the letter are each 20 cm long and the cross-piece in the middle is 10 cm long while the apex angle is 60°. Find the resistance of the letter between the two ends of the legs.**

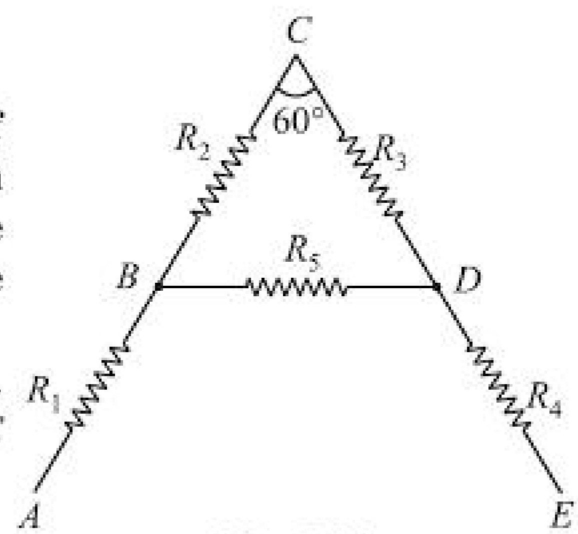



Fig. 5.36

**Solution.** Fig. 5.36 shows the conditions of the problem. Point $B$ is the mid-point of $AC$, point $D$ is the mid-ponit of $EC$ and $BD$ = 10 cm.

$\therefore$ $AB = BC = CD = DE = BD = 10$ cm

or $R_1 = R_2 = R_3 = R_4 = R_5 = 10\ \Omega$ ($\because$ 1 cm = 1 Ω)

Now $R_2$ ad $R_3$ are in series and their total resistance = 10 + 10 = 20 Ω. This 20 Ω resistance is in parallel with $R_5$.

$$\therefore \quad R_{BD} = 20\ \Omega \parallel R_5 = 20\ \Omega \parallel 10\ \Omega = \frac{20\times 10}{20+10} = \frac{20}{3}\ \Omega$$

Now $R_1$, $R_{BD}$ and $R_4$ are in series so that:

$$R_{AE} = R_1 + R_{BD} + R_4 = 10 + \frac{20}{3} + 10 = 26.67\ \Omega$$

Example 5.37. **Calculate the steady-state current in the 2 Ω resistor shown in Fig 5.37. The internal resistance of the battery is negligible and $C$ = 2 μF.**

**Solution.** In the steady state, the capacitor offers infinite reactance to d.c. so that no current flows in the branch containing $C$. Therefore, branch containing $C$ may be considered as removed.

$$\therefore \quad R_{AB} = 2\ \Omega \parallel 3\ \Omega = \frac{2\times 3}{2+3} = \frac{6}{5} = 1.2\ \Omega$$

$\therefore$ Total circuit resistance is given by ;

$$R_T = R_{AB} + 2.8 = 1.2 + 2.8 = 4\ \Omega$$

Current drawn from the battery is

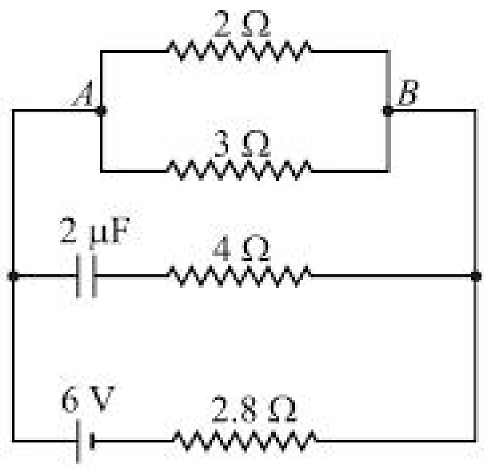



**Fig. 5.37**

$$I = E/R_T = 6/4 = 1.5 \text{ A}$$

Potential difference between points $A$ and $B$ is

$$V_{AB} = I \times R_{AB} = 1.5 \times 1.2 = 1.8 \text{ V}$$

$\therefore$ Current through 2 Ω resistor is

$$I_{2\Omega} = V_{AB}/2 = 1.8/2 = 0.9\text{A}$$

**Example 5.38. All the resistances in Fig. 5.38 are in ohms. Find the effective resistance between the points *A* and *B*.**

**Solution.** Between points $A$ and $D$,

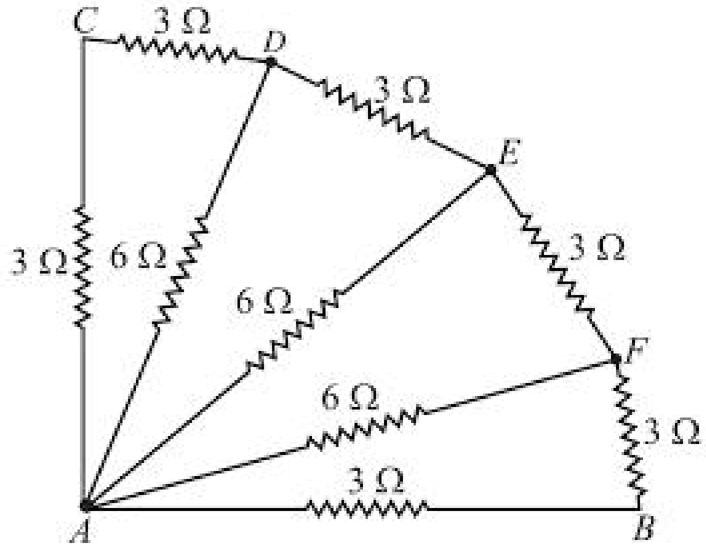



**Fig. 5.38**

$$R_{AD} = (3 + 3)\ \Omega \parallel 6\ \Omega = \frac{6 \times 6}{6+6} = 3\ \Omega$$

$$R_{AE} = (R_{AD} + 3)\ \Omega \parallel 6\ \Omega = \frac{6 \times 6}{6+6} = 3\ \Omega$$

$$R_{AF} = (R_{AE} + 3)\ \Omega \parallel 6\ \Omega = \frac{6 \times 6}{6+6} = 3\ \Omega$$

$\therefore$ Resistance between points $A$ and $B$ is

$$R_{AB} = (R_{AF} + 3)\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6+3} = \frac{18}{9} = 2\ \Omega$$

**Example 5.39. In the circuit shown in Fig. 5.39, $R_1$ = 100 Ω, $R_2 = R_3$ = 50 Ω, $R_4$ = 75 Ω and $E$ = 4.75 V. Work out the equivalent resistance of the circuit and the current in each resistance.**

**Solution.** It is clear from the figure that between points $B$ and $C$, $R_2$, $R_4$ ad $R_3$ are in parallel so that:

$$\frac{1}{R_{BC}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{50} + \frac{1}{50} + \frac{1}{75} = \frac{4}{75}$$

$$\therefore \quad R_{BC} = 75/4\ \Omega$$

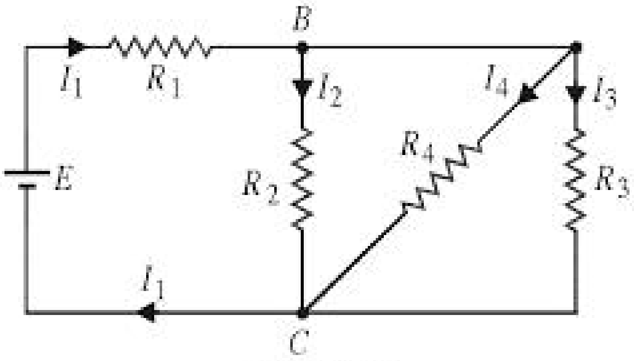



**Fig. 5.39**

Total circuit resistance of the circuit = $R_1$ + 75/4 = 100 + 75/4 = **475/4 Ω**

$$\text{Circuit current,} \quad I_1 = \frac{E}{R_T} = \frac{4.75}{475/4} = \mathbf{0.04\ A}$$

$$\text{Drop across} \quad R_1 = I_1 R_1 = 0.04 \times 100 = 4\ \text{V}$$

$$\therefore \quad V_{BC} = E - I_1 R_1 = 4.75 - 4 = 0.75\ \text{V}$$

$$\therefore \quad \text{Current in } R_2 = \frac{V_{BC}}{R_2} = \frac{0.75}{50} = \mathbf{0.015\ A}$$

$$\text{Current in} \quad R_3 = \frac{V_{BC}}{R_3} = \frac{0.75}{50} = \mathbf{0.015\ A}$$

$$\text{Current in} \quad R_4 = \frac{V_{BC}}{R_4} = \frac{0.75}{75} = \mathbf{0.01\ A}$$

## PROBLEMS FOR PRACTICE

1. Find the equivalent resistance between terminals $A$ and $B$ in the network shown in Fig. 5.40. **[2 Ω]**

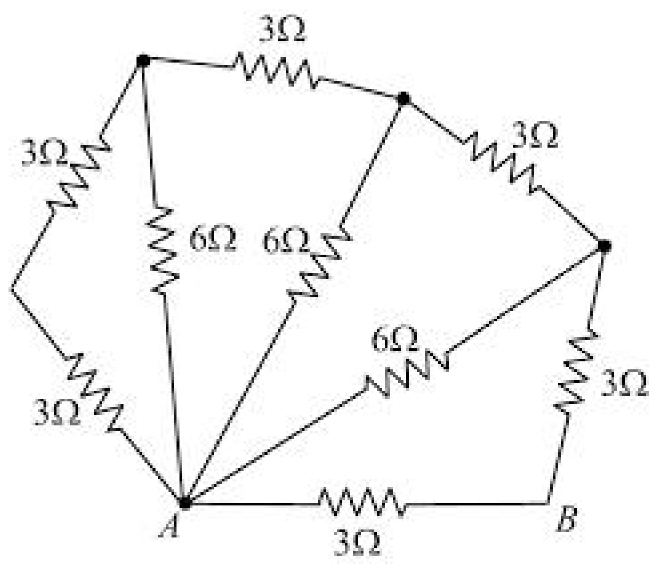



**Fig. 5.40**

2. Determine the equivalent resistance between terminals $A$ and $B$ of the network shown in Fig. 5.41. **[40/3 Ω]**

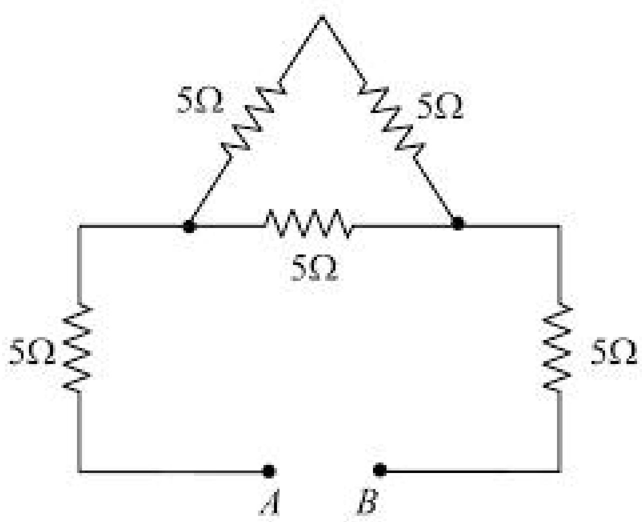



**Fig. 5.41**

3. Find the equivalent resistance between terminals $A$ and $B$ of the network shown in Fig. 5.42. **[7Ω]**
4. Find the value of $R$ in Fig. 5.43 if equivalent resistance between terminals $A$ and $B$ is 2Ω. **[6Ω]**

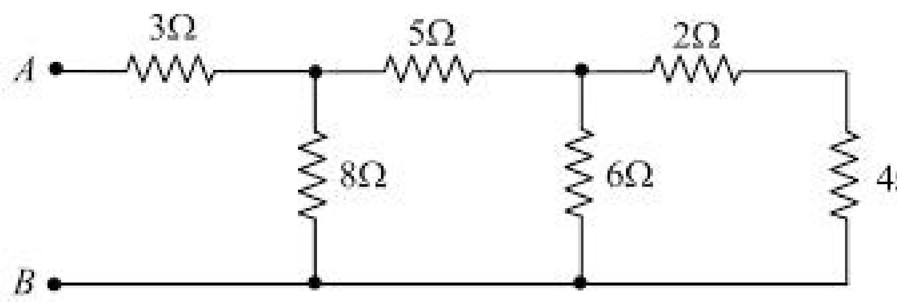



**Fig. 5.42**

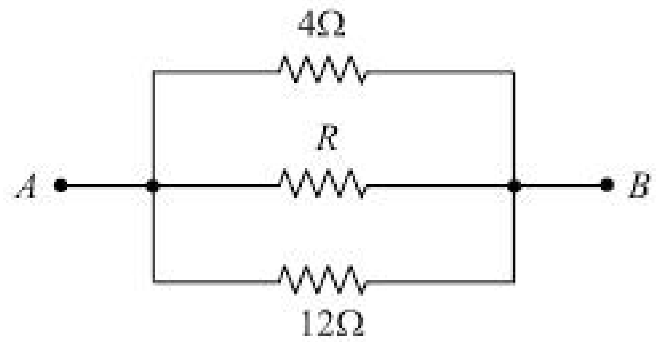



**Fig. 5.43**

5. Find the equivalent resistance between terminals $A$ and $E$ of the network shown in Fig. 5.44. **[16/3 Ω]**

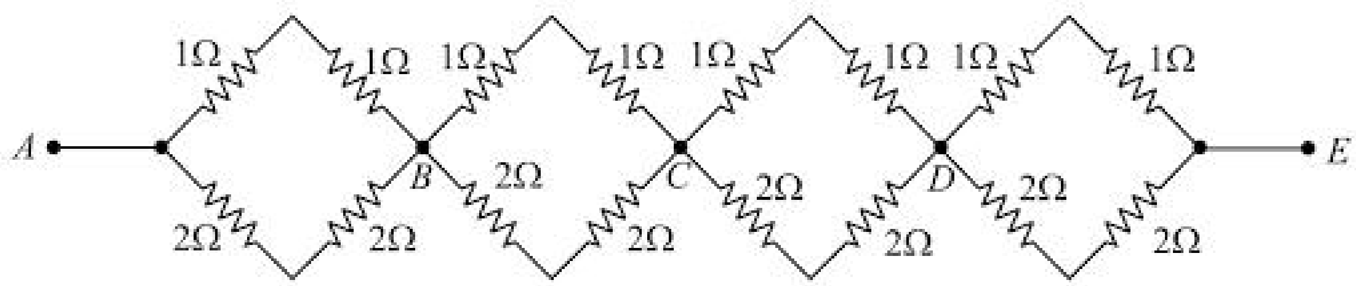



**Fig. 5.44**

6. Calculate the current in the 3Ω resistor shown in Fig. 5.45. **[2.5 A]**

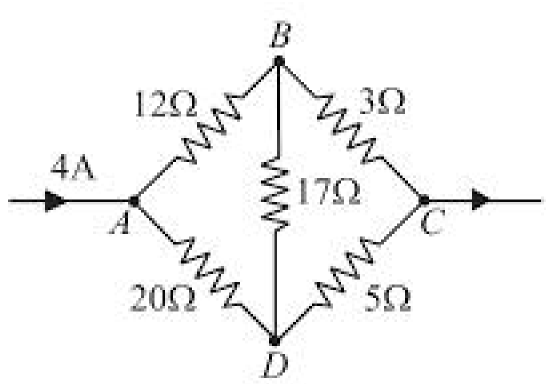



**Fig. 5.45**

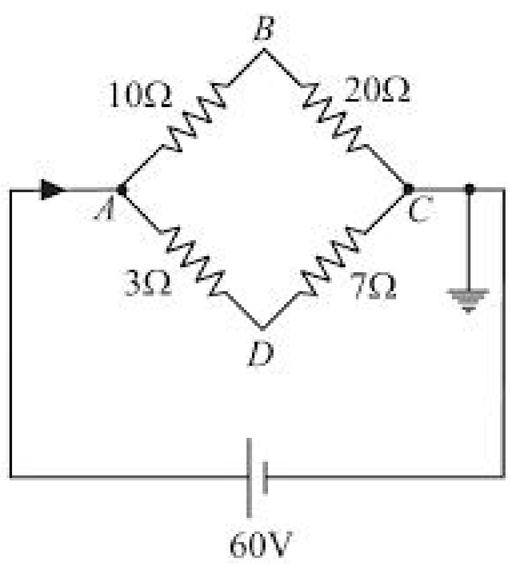



**Fig. 5.46**

7. Find the potentials of points $B$ and $D$ in Fig. 5.46. **[+40V, +42V]**

[**Hint:** Since point $C$ is earthed, $V_C = 0$. Now current in branch $ABC$ is 2 A and that in branch $ADC$ is 6 A.

$$V_B - V_C = 2 \times 20 \quad \text{or} \quad V_B - V_C = 40 \quad \text{or} \quad V_B = 40 \text{ V}$$
$$V_D - V_C = 6 \times 7 \quad \text{or} \quad V_D - V_C = 42 \quad \text{or} \quad V_D = 42 \text{ V}$$

8. A uniform wire of resistance 2.2 Ω has a length of 2m. Find the length of the similar wire which, connected in parallel with the 2m long wire, will give a resistance of 2Ω. **[20 m]**
9. Two resistors are in the ratio 1 : 4. If these are connected in parallel, their total resistance becomes 20 Ω. Find the value of each resistance. **[25 Ω; 100 Ω]**

10. A set of $n$ identical resistors, each of resistance $R$ ohm, when connected in series have an effective resistance $X$ ohm and when the resistors are connected in parallel, their effective resistance is $Y$ ohm. Find the relation between $R$, $X$ and $Y$. **[$XY = R^2$]**

11. Calculate the total resistances that can be obtained by combining three 100 Ω resistances in all possible ways. **[300 Ω; 33.3 Ω; 66.7 Ω; 150 Ω]**

12. A parallel combination of three resistors takes a current of 7.5 A from a 30 V supply. If the two resistors are 10 Ω and 12 Ω, find the third one. **[15 Ω]**

## 5.28. INTERNAL RESISTANCE OF A CELL

*The resistance offered by a cell to the current flow is called internal resistance r of the cell.* The internal resistance ($r$) of a fresh cell is generally low and as the cell is used, its internal resistance goes on increasing. The internal resistance of a cell depends upon the following factors :

(*i*) distance between the plates – increases with the increase in distance between the plates

(*ii*) nature of the electrolyte

(*iii*) concentration of the electrolyte – increases with the increase in concentration of electrolyte.

(*iv*) nature of the electrodes

(*v*) area of the plates – decreases with the increase in plate area.

**Fig. 5.47**

It is a usual practice to show internal resistance of a cell as a series resistor external to the cell as shown in Fig. 5.47.

## 5.29. TERMINAL POTENTIAL DIFFERENCE OF A CELL

*The terminal potential difference of a cell is the potential difference between the electrodes of the cell when the cell is delivering current (i.e., cell circuit is closed).* It is denoted by the symbol $V$.

(*i*) When the cell is delivering no current (*i.e.*, on open-circuit), the p.d. across the terminals of the cell is equal to e.m.f. $E$ of the cell as shown in Fig. 5.48.

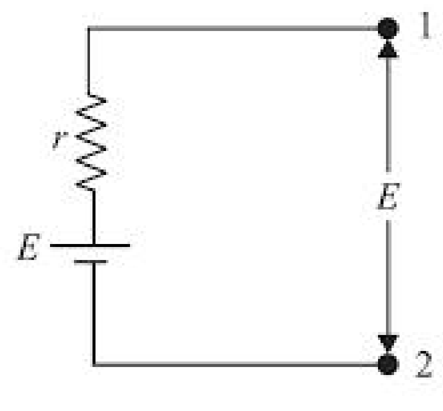


**Fig. 5.48**

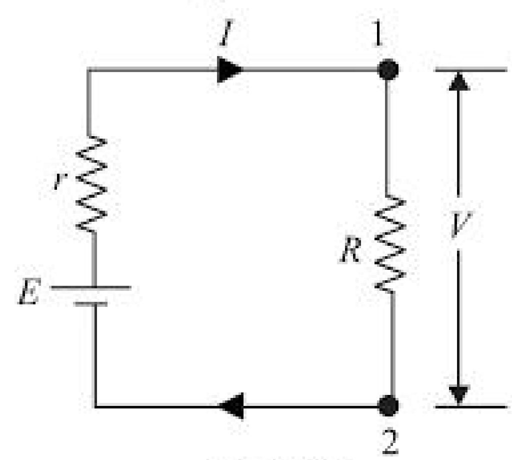


**Fig. 5.49**

(*ii*) When a resistance $R$ is connected across the cell [See Fig. 5.49], current $I$ starts flowing in the circuit. This current causes a voltage drop ( = $Ir$) across internal resistance of the cell so that terminal voltage $V$ is less than the e.m.f. $E$ of the cell.

$$\therefore \qquad I = \frac{E}{R + r}$$

or $$IR + Ir = E$$

But $$IR = V = \text{Terminal p.d. of the cell.}$$

$$\therefore \qquad V + Ir = E$$

$$\therefore \qquad \text{Terminal p.d. of cell, } V = E - Ir$$

Due to voltage drop in the internal resistance ($= I\,r$) of the cell, the terminal voltage $V$ of the cell is *less* than its *e.m.f.* $E$.

Internal resistance of the cell is

$$r = \frac{E - V}{I} = \left(\frac{E - V}{V}\right) R \qquad \left(\because I = \frac{V}{R}\right)$$

**Discussion.** The following points may be noted :

(*a*) $V = E - I\,r$. When the cell is delivering no current (*i.e.*, cell on open-circuit), $I = 0$.

$\therefore \qquad V = E - (0) \times r \quad \text{or} \quad V = E$

Hence when the cell is delivering no current, its terminal voltage $V$ is equal to *e.m.f.E* of the cell.

(*b*) When the cell is delivering current, the voltmeter connected across its terminals measures the terminal voltage $V$ of the cell.

**Note.** When the cell is delivering current to a load (*e.g.*, resistor $R$), it is said to be **discharging.** In this case, the terminal voltage $V$ of the cell is *less* than its *e.m.f.E i.e.*,

$$V = E - I\,r$$

During ***charging of the cell,** direct current (d.c.) is supplied to the cell from the d.c. charging source. In this case, the terminal voltage $V$ of the cell is *more* than its *e.m.f.E i.e.*,

$$V = E + I\,r$$

## 5.30. DIFFERENCE BETWEEN E.M.F. AND POTENTIAL DIFFERENCE

| | *E.M.F. (E)* | | *Potential Difference (V)* |
|---|---|---|---|
| 1. | It is the potential difference across the terminal of the cell when it is delivering no current *i.e.* when the cell is in the open circuit. | 1. | It is the potential difference across the terminal of the cell when it is delivering current *i.e.* the cell curcuit is closed. |
| 2. | It is independent of circuit resistance and depends upon the nature of the electrodes and the electrolyte. | 2. | It depends upon circuit resistance and circuit current. |
| 3. | It maintains potential difference. | 3. | It causes current to flow in the circuit. |
| 4. | It is greater than $V$. | 4. | It is less than $E$. |
| 5. | The term e.m.f. is used for a source of voltage. | 5. | The p.d. ($V$) is the voltage between two points in a closed circuit. |

## 5.31. BATTERY AND ITS NEED

*When a number of cells are suitably connected, the resulting arrangement is called a battery.*

The *e.m.f.* and current obtained from a single cell are generally small. For instance, an ordinary dry cell has an e.m.f of 1.5V and can deliver about 1/8 ampere continuously. Such a cell can, therefore, supply electrical energy to a circuit requiring 1.5V and not more than 1/8A. Many occasions arise when higher voltage or higher current or both are required. To meet these needs, a number of cells are suitably connected or grouped. The combination of cells thus obtained is called a *battery*. Depending upon voltage and current requirements, cells may be connected in three ways to form a battery *viz.*

(*i*) Series grouping (*ii*) Parallel grouping (*iii*) Series-parallel grouping.

* When a cell is to be charged, the positive terminal of the charging d.c. source is connected to the positive terminal of the cell and negative terminal of d.c. source is connected to the negative terminal of the cell.

## 5.32. CELLS IN SERIES (SERIES GROUPING)

*The cells are said to be connected in series if the negative terminal of one cell is connected to the positive terminal of the next cell and so on.*

Consider $n$ cells, each of *e.m.f.* $E$ and internal resistance $r$ connected in series across an external resistance $R$ as shown in Fig. 5.50.

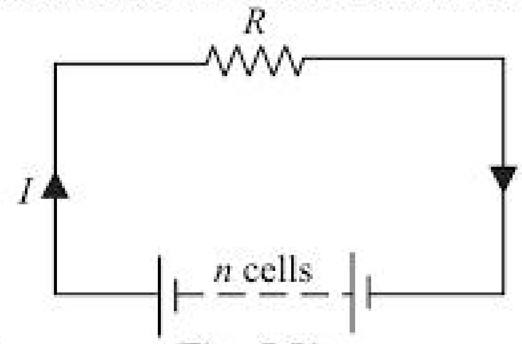



**Fig. 5.50**

$$\text{Total battery e.m.f.} = nE$$

$$\text{Internal resistance of the battery} = nr$$

$$\text{Total circuit resistance} = R + nr$$

$$\therefore \quad \text{Circuit current, } I = \frac{nE}{R+nr}$$

**Special Cases.** (*i*) If $R >> nr$, then $nr$ can be neglected as compared to $R$.

$$\therefore \quad I = n\frac{E}{R} = n \times \text{current due to one cell}$$

(*ii*) If $R << nr$, then $R$ can be neglected as compared to $nr$.

$$\therefore \quad I = \frac{nE}{nr} = \frac{E}{r} = \text{current due to one cell}$$

*Hence in order to get maximum current in a series grouping of cells, the external resistance (R) should be very high as compared to the internal resistance of the battery (nr).*

## 5.33. CELLS IN PARALLEL (PARALLEL GROUPING)

*The cells are said to be connected in parallel if the positive terminals of all the cells are joined together and negative terminals of all the cells are joined together.*

Consider $m$ rows of cells in parallel, each row containing one cell. Let $E$ and $r$ be the e.m.f. and internal resistance respectively of each cell. Further, let this battery be connected across an external resistance $R$ as shown in Fig. 5.51.

$$\text{E.M.F. of the battery} = E$$

Since the cells are connected in parallel, their internal resistances are also in parallel. If $r_p$ is the total resistance of the battery, then,

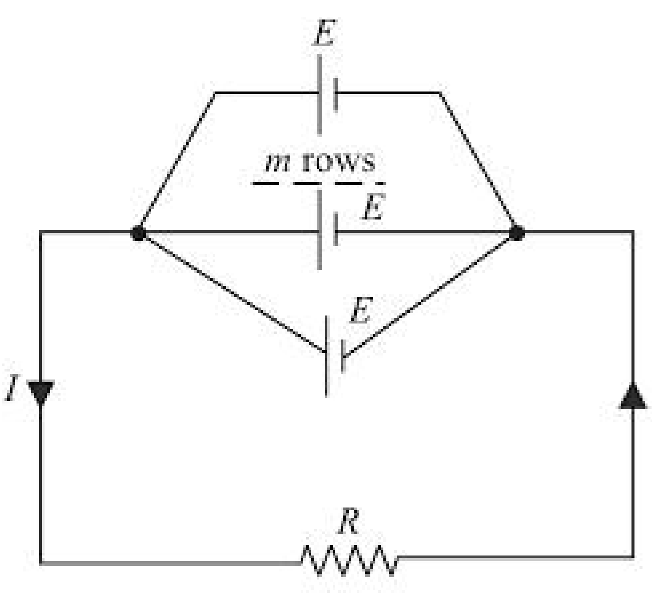



**Fig. 5.51**

$$\frac{1}{r_p} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \ldots m \text{ terms}$$

$$= \frac{1+1+1+\ldots m \text{ terms}}{r} = \frac{m}{r}$$

$$\therefore \quad r_p = r/m$$

$$\text{Total circuit resistance} = R + r_p = R + \frac{r}{m}$$

$$\therefore \quad \text{Circuit current, } I = \frac{E}{R + (r/m)} = \frac{mE}{mR + r}$$

or

$$I = \frac{mE}{mR + r}$$

**Special Cases.** (*i*) If $R << r$, then $mR$ may be neglected as compared to $r$.

$$\therefore \quad I = m\frac{E}{r} = m \times \text{current due to one cell}$$

(*ii*) If $r << R$, then $r$ may be neglected as compared to $mR$.

$$\therefore \qquad I = \frac{mE}{mR} = \frac{E}{R} = \text{current due to one cell}$$

*Hence in order to get maximum current in parallel grouping of cells, the external resistance (R) should be very low as compared to the internal resistance of each cell.*

## 5.34. SERIES-PARALLEL GROUPING OF CELLS (MIXED GROUPING)

*When a number of cells are connected in series and a number of sets of such series-connected cells are connected in parallel, the cells are said to be connected in series-parallel grouping.*

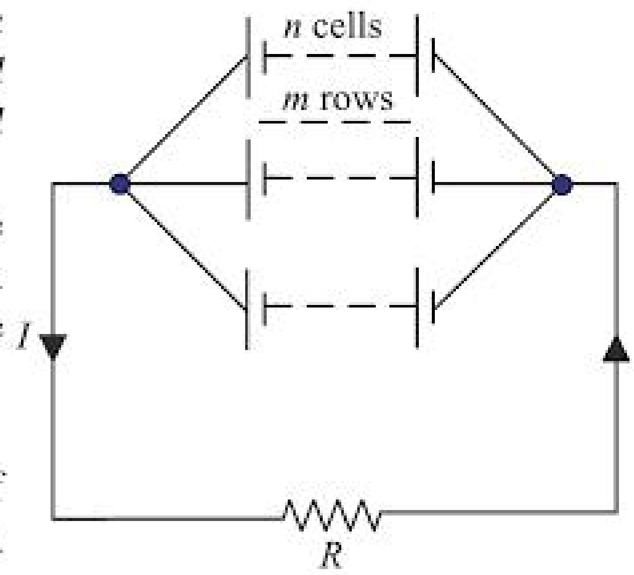



**Fig. 5.52**

Fig. 5.52 shows series-parallel grouping of cells. Let there be $n$ cells connected in series and $m$ such rows connected in parallel across an external resistance $R$. Further, let $E$ and $r$ be the e.m.f. and internal resistance of each cell respectively.

Resistance of one row of cells = $n\,r$

There are $m$ rows in parallel, each having a resistance of $n\,r$. If $r_T$ is the total internal resistance of the battery, then,

$$\frac{1}{r_T} = \frac{1}{nr} + \frac{1}{nr} + \frac{1}{nr} + \ldots m \text{ terms}$$

$$= \frac{1 + 1 + 1 + \ldots m \text{ terms}}{n\,r} = \frac{m}{n\,r}$$

$$\therefore \qquad r_T = \frac{n\,r}{m}$$

Total circuit resistance = $R + (n\,r/\,m)$

Total battery e.m.f. = E.M.F. due to one row = $n\,E$

$$\therefore \qquad \text{Circuit current, } I = \frac{n\,E}{R + (n\,r\,/\,m)} = \frac{mnE}{mR + nr}$$

or
$$I = \frac{mnE}{mR + nr}$$

### Condition for maximum current

$$I = \frac{mnE}{mR + nr}$$

or
$$I = \frac{mnE}{(\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mR}\,\sqrt{nr}} \qquad \ldots (i)$$

As $m$, $n$ and $E$ are fixed quantities, therefore, numerator of eq. (*i*) is constant. Hence, current will be maximum when the denominator of eq. (*i*) is minimum. The denominator will be minimum when quantity $(\sqrt{mR} - \sqrt{nr})^2$ is minimum. Now the minimum value of a squared quantity is zero *i.e.*,

$$(\sqrt{mR} - \sqrt{nr})^2 = 0$$

or
$$mR = n\,r$$

or
$$R = \frac{n\,r}{m}$$

*i.e.*, External resistance = Total internal resistance of the battery

*Hence, in order to get maximum current in series-parallel grouping of cells, the external resistance (R) should be equal to the total internal resistance of the battery (nr/m).*

## 5.35. EFFICIENCY OF A CELL

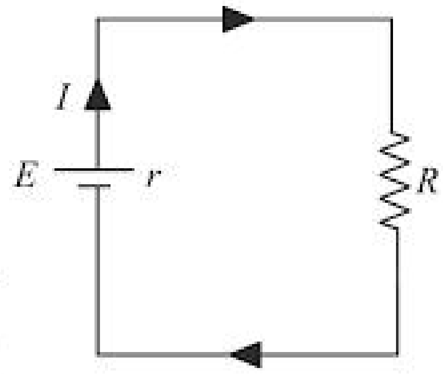


**Fig. 5.53**

In general, the efficiency of any system is given by :

$$\text{Efficiency, } \eta = \frac{\text{Output power}}{\text{Input power}}$$

Consider a cell of e.m.f. $E$ and internal resistance $r$ delivering power to a resistance $R$ as shown in Fig. 5.53. If $I$ is the circuit current, then useful power developed is $I^2R$ and power wasted in the internal resistance of the cell is $I^2r$. Total power developed (input power) by the cell is $I^2R + I^2r$ (= $E I$).

$$\therefore \qquad \eta = \frac{\text{Useful power}}{\text{Total power developed}} = \frac{I^2 R}{I^2 R + I^2 r}$$

$$\text{or} \qquad \eta = \frac{R}{R+r} = \frac{1}{1+(r/R)}$$

It is clear that greater the value of external resistance ($R$), the higher is the efficiency of the cell.

Example 5.40. **When a current of 0.5 A is drawn from a battery, then p.d. between its terminals is 19 V and when a current of 2 A is drawn, then p.d. across terminals drops to 16 V. Determine the e.m.f. and internal resistance of the battery.**

**Solution.** $E = V + I\,r$

For the first case, $E = 19 + 0.5r$ ...(*i*)

For the second case, $E = 16 + 2r$ ...(*ii*)

Solving eqs. (*i*) and (*ii*), we get, $E$ = **20V**; $r$ = **2Ω**

Example 5.41. **Two cells *A* and *B* are connected in series, each having an e.m.f. of 1.5V. The internal resistances of *A* and *B* are 0.5Ω and 0.25Ω respectively. The combination is connected across a resistance of 2.25Ω. Determine (*i*) the circuit current, (*ii*) p.d. across the terminals of each cell.**

**Solution.** E.M.F. of the battery, $E = 1.5 + 1.5 = 3$ V

Internal resistance of the battery, $r = 0.5 + 0.25 = 0.75\Omega$

Total circuit resistance, $R_T = 2.25 + 0.75 = 3\Omega$

(*i*) ∴ Circuit current, $I = E/R_T = 3/3 =$ **1A**

(*ii*) P.D. across the terminals of cell $A = E - I\,r_A = 1.5 - 1 \times 0.5 =$ **1 V**

P.D. across the terminals of cell $B = E - I\,r_B = 1.5 - 1 \times 0.25 =$ **1.25 V**

Example 5.42. **A battery consisting of 10 cells in series has two cells reverse connected by mistake. E.M.F. of each cell is 1.5V and internal resistance 0.1Ω. The value of external resistance is 4Ω. Find the reduction in current due to the two cells being reverse connected.**

**Solution. When the 10 cells are properly connected**

Total e.m.f. of the battery $= n\,E = 10 \times 1.5 = 15$ V

Total circuit resistance $= R + n\,r = 4 + 10 \times 0.1 = 5\ \Omega$

∴ Circuit current $= 15/5 = 3$ A

**When two of the cells are reverse connected**

Total e.m.f. of the battery $= 8 \times 1.5 - 2 \times 1.5 = 9$ V

Total circuit resistance = 5 Ω, *i.e.*, the same as before

∴ Circuit current $= 9/5 = 1.8$ A

Reduction in current $= 3 - 1.8 =$ **1.2 A**

Example 5.43. **Three cells of e.m.f. 2V, 1.9 V and 1.8V are connected in series. The internal resistances of the cells are 0.05Ω, 0.06Ω and 0.07Ω respectively. The battery is connected to an external resistance of 5Ω via a very low resistance ammeter. What will be**

**the reading of the ammeter? If the above three cells were connected in parallel, would they be characterised by a definite e.m.f. and internal resistance (independent of the external resistance)? If not, how will you obtain currents in different branches of the circuit?**

**Solution.** Since ammeter has very low resistance, its introduction in the circuit will not affect the circuit resistance. In the first case, the cells are in series.

$\therefore$ Total e.m.f. $= 2 + 1.9 + 1.8 = 5.7\text{V}$

Internal resistance of the battery $= 0.05 + 0.06 + 0.07 = 0.18\ \Omega$

Total circuit resistance $= 5 + 0.18 = 5.18\ \Omega$

$\therefore$ Circuit current, $I = 5.7/5.18 =$ **1.1A**

Therefore, the reading of the ammeter will be 1.1A.

There is no formula for the total e.m.f. and the internal resistance of non-similar cells connected in parallel. Hence, when the above three non-similar cells are connected in parallel, they will not be characterised by definite e.m.f. and internal resistance. However, currents in different branches can be found out by using Kirchhoff's laws.

Example 5.44. **A battery of e.m.f. 24 V and internal resistance *r* is connected in a circuit having two parallel resistors of 3Ω and 6Ω in series with 8Ω resistor as shown in Fig. 5.54 (*i*). The current flowing in 3Ω resistor is 0.8A. Calculate (*i*) current in the 6 Ω resistor, (*ii*) internal resistance *r* of the battery, (*iii*) the terminal p.d. of the battery.**

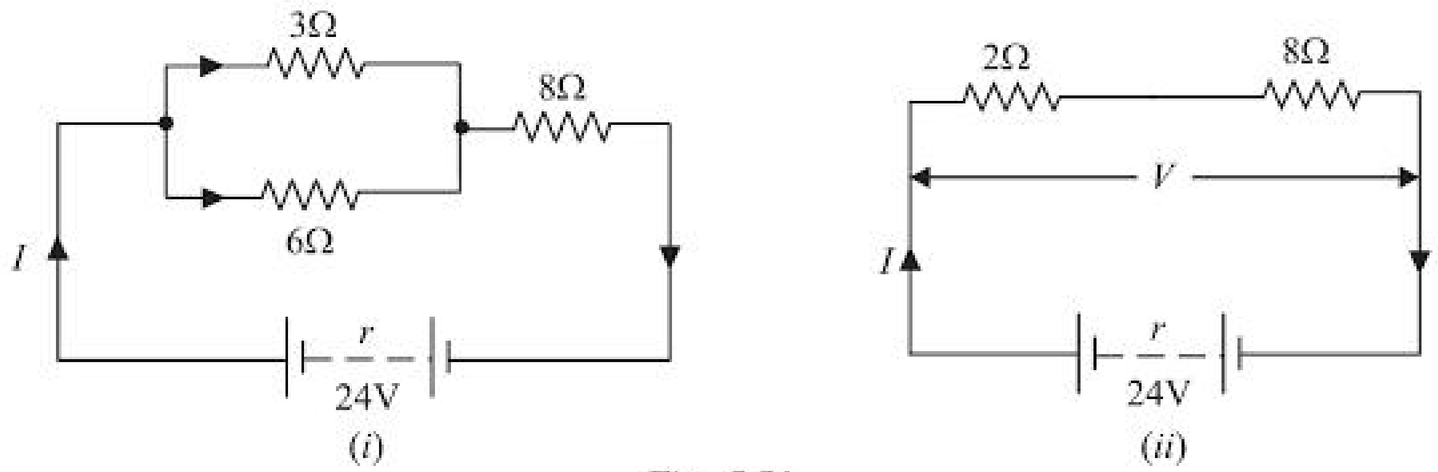



**Fig. 5.54**

**Solution.** (*i*) Current in 3Ω resistor = Total current $\times \dfrac{\text{Other resistance}}{\text{Sum of the two}}$

$$\text{or} \quad 0.8 = I \times \frac{6}{3+6}$$

$$\text{or} \quad I = 1.2\ \text{A}$$

$\therefore$ Current in 6Ω resistor $= 1.2 - 0.8 =$ **0.4 A**

(*ii*) The equivalent resistance $R_P$ of parallel resistors is given by ;

$$R_P = \frac{3 \times 6}{3+6} = 2\Omega$$

The circuit then reduces to the circuit shown in Fig. 5.54 (*ii*).

Total circuit resistance, $R_T = R_P + 8 = 2 + 8 = 10\ \Omega$

$$\therefore \quad \text{Circuit current, } I = \frac{24}{10+r}$$

$$\text{or} \quad 1.2 = \frac{24}{10+r}$$

$\therefore$ $r =$ **10 Ω**

(*iii*) Terminal p.d. of battery, $V = I\, R_T = 1.2 \times 10 =$ **12 V**

Alternatively; $V = \text{E} - I\, r = 24 - 1.2 \times 10 =$ **12 V**

**Example 5.45. A uniform wire of resistance 12Ω is cut into three pieces in the ratio 1 : 2 : 3 and the three pieces are connected to form a triangle. A cell of e.m.f. 8 V and internal resistance 1Ω is connected across the highest of the three resistors. Calculate the current through each part of the circuit.**

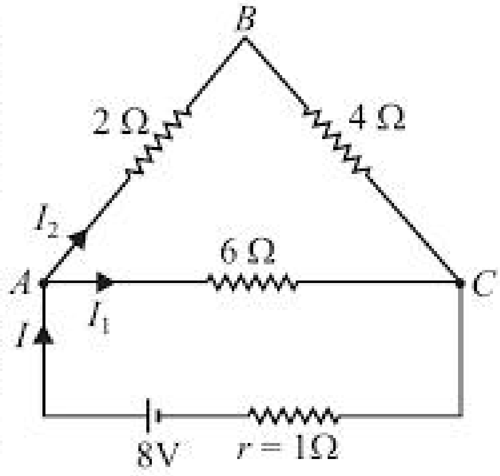


**Fig. 5.55**

**Solution.** Fig. 5.55 shows the conditions of the problem. Here uniform wire is cut into three pieces *AB*, *BC* and *CA* in the ratio 1: 2 : 3. Therefore, resistances of *AB*, *BC* and *CA* are 2 Ω, 4 Ω and 6 Ω respectively. As per the statement, a cell of *e.m.f.* 8 V and internal resistance 1 Ω is connected across *AC* (highest resistor).

The series combination of *AB* and *BC* is in parallel with *AC*. Therefore, effective resistance between points *A* and *C* is

$$R_{AC} = (AB + BC) \parallel AC = (2 + 4)\ \Omega \parallel 6\ \Omega = \frac{6 \times 6}{6 + 6} = 3\ \Omega$$

∴ Current drawn from the battery is

$$I = \frac{E}{R_{AC} + r} = \frac{8}{3 + 1} = 2\ \text{A}$$

Since the resistances of the parallel paths *ABC* and *AC* are equal, the current *I* divides equally between them.

$$\therefore \quad I_1 = I_2 = I/2 = 2/2 = 1\ \text{A}$$

**Example 5.46. Two identical cells, whether joined together in series or in parallel give the same current when connected to an external resistance of 1 Ω. Find the internal resistance of each cell.**

**Solution.** Let each cell be of *e.m.f.E* and internal resistance *r*. It is given that external resistance $R = 1\ \Omega$.

**When the two cells are connected in series**

Total *e.m.f.* = $E + E = 2E$; Total resistance = $1 + r + r = 1 + 2r$

$$\therefore \quad \text{Circuit current, } I_1 = \frac{2E}{1 + 2r}$$

**When the two cells are connected in parallel**

Totel *e.m.f.* = $E$; Total resistance = $1 + r/2$

$$\therefore \quad \text{Circuit current, } I_2 = \frac{E}{1 + r/2}$$

$$\text{As } I_1 = I_2,$$

$$\therefore \quad \frac{2E}{1 + 2r} = \frac{E}{1 + r/2} \quad \text{or} \quad r = 1\ \Omega$$

**Example 5.47. Four identical cells, each of e.m.f. 2 V, are joined in parallel providing supply of current to external circuit consisting of two 15Ω resistors joined in parallel. The terminal voltage of the cells as read by an ideal voltmeter is 1.6 V. Calculate the internal resistance of each cell.**

**Solution.** Fig. 5.56 shows the conditions of the problem.

Here, *e.m.f.* of each cell, $E = 2$ V; Terminal voltage, $V = 1.6$ volt

$$\text{External resistance, } R = 15\ \Omega \parallel 15\ \Omega = \frac{15 \times 15}{15 + 15} = 7.5\ \Omega$$

Let *r* be the internal resistance of each cell. Therefore, total internal resistance $r'$ of all the cells is given by ;

$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{4}{r} \qquad \therefore r' = \frac{r}{4}$$

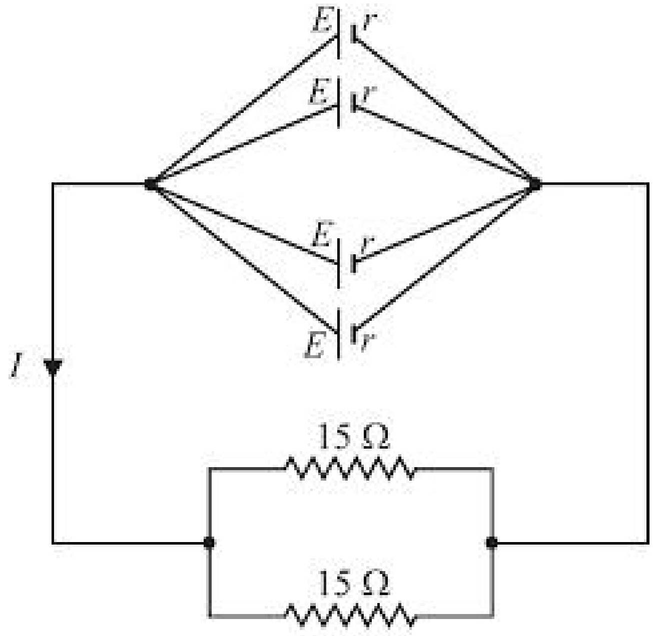



**Fig. 5.56**

But internal resistance $r'$ of the parallel combination of cells is also given by;

$$r' = \left(\frac{E}{V} - 1\right) R$$

or $$\frac{r}{4} = \left(\frac{2}{1.6} - 1\right) \times 7.5 \qquad \therefore \; r = \mathbf{7.5\ \Omega}$$

**Example 5.48. A galvanometer together with an unknown resistance in series is connected across two identical batteries each of 1.5 V. When the batteries are connected in series, the galvanometer records a current of 1 A and when the batteries are connected in parallel, the current is 0.6 A. What is the internal resistance of each battery?**

**Solution.** Suppose $R$ is the combined resistance of the galvanometer and unknown series resistance and $r$ is the internal resistance of each battery.

(*i*) When the batteries (each of e.m.f. $E$) are connected in series, the net e.m.f. = 2 $E$ and net internal resistance = $2r$.

$\therefore$ Circuit current, $$I_1 = \frac{2E}{R + 2r}$$

or $$1 = \frac{2 \times 1.5}{R + 2r}$$

or $$R + 2r = 3 \qquad \text{... } (i)$$

(*ii*) When the batteries are connected in parallel, the net e.m.f. = $E$ and the net internal resistance = $r/2$.

$\therefore$ Circuit current, $$I_2 = \frac{E}{R + r/2} = \frac{2E}{2R + r}$$

or $$0.6 = \frac{2 \times 1.5}{2R + r}$$

or $$2R + r = 5 \qquad \text{... } (ii)$$

Solving eqs. (*i*) and (*ii*), we get, $r = \mathbf{\frac{1}{3}\ \Omega}$

**Example 5.49. A network of resistances is connected to a 16 V battery with internal resistance of 1Ω as shown in Fig. 5.57.**

**(*a*) Compute equivalent resistance of the network,**

**(*b*) Obtain the current in each resistor, and**

**(*c*) Obtain the voltage drops $V_{AB}$, $V_{BC}$ and $V_{CD}$.**

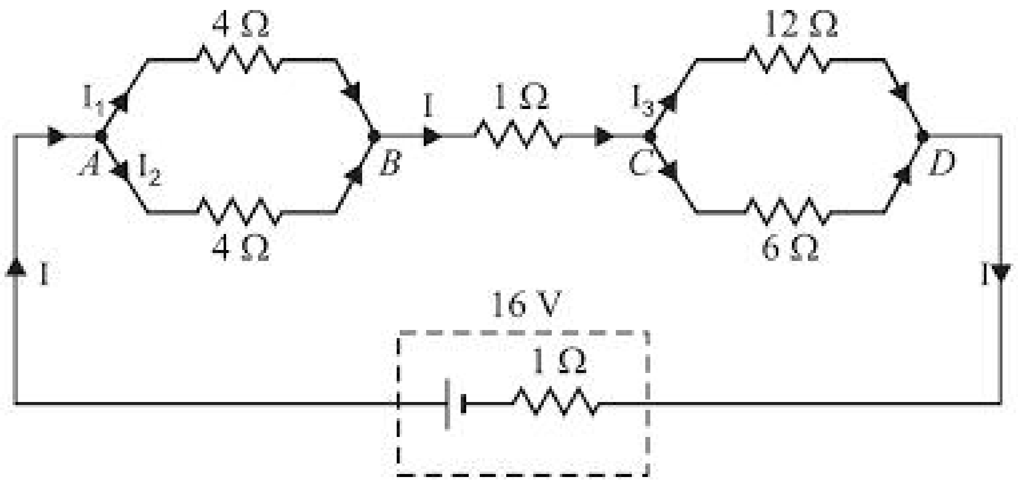



**Fig. 5.57**

**Solution.** (*a*) Referring to Fig. 5.57, we have,

$$R_{AB} = 4\ \Omega \parallel 4\ \Omega = \frac{4\times 4}{4+4} = 2\ \Omega$$

$$R_{CD} = 12\ \Omega \parallel 6\ \Omega = \frac{12\times 6}{12+6} = 4\ \Omega$$

Now resistances $R_{AB}$, $R_{CD}$ and 1 Ω are in series.

∴ Equivalent resistance of the network is

$$R = R_{AB} + R_{CD} + 1 = 2 + 4 + 1 = 7\ \Omega$$

(*b*) Circuit current, $$I = \frac{E}{R+r} = \frac{16}{7+1} = 2\text{ A}$$

$$I_1 = I_2 = I/2 = 2/2 = 1\text{ A}$$

Now $$I_3 = I\times\frac{6}{6+12} = 2\times\frac{6}{18} = \frac{2}{3}\text{ A}$$

$$I_4 = I\times\frac{12}{6+12} = 2\times\frac{12}{18} = \frac{4}{3}\text{ A}$$

(*c*) $$V_{AB} = 4\ I_1 = 4\times 1 = 4\text{ V}$$

$$V_{BC} = I\times 1 = 2\times 1 = 2\text{ V}$$

$$V_{CD} = 12\ I_3 = 12\times\frac{2}{3} = 8\text{ V}$$

## PROBLEMS FOR PRACTICE

1. The p.d. of a cell on open circuit is 6V which falls to 4V when a current of 4A is drawn from the cell. Find the internal resistance of the cell. **[1Ω]**

2. A secondary cell after a long use has an e.m.f. of 1.8V and internal resistance 350Ω. What maximum current can be drawn from the cell? Can the cell drive the starting motor of a car? **[0.005 A ; No]**

   [Hint. Maximum current is delivered by the cell when external resistance is zero (*i.e.*, $R = 0$).

   ∴ $I_{max} = E/r = 1.8/350 = 0.005\text{A}$

   The starting motor of a car requires more than 100A at start.]

3. A battery of e.m.f. 4V and internal resistance 2Ω is joined to a resistor of 8Ω. What additional resistance in series with 8Ω resistor would produce a terminal p.d. of 3.6V? **[10Ω]**

   [Hint. Let $R$ ohms be the required resistance.

   Drop in battery, $I\ r = 4 - 3.6 = 0.4$ or $I = 0.4/r = 0.4/2 = 0.2\text{A}$

Now terminal p.d. $= I\ (8 + R)$

or $3.6 = 0.2\ (8 + R) \quad \therefore R = 10\Omega$]

4. Three cells of e.m.f. 1.1V, 1.3V and 1.5V are connected in series to supply current to an external resistance of 1.6Ω. The internal resistances of the three cells are 0.25Ω, 0.35Ω and 0.4Ω respectively. Determine the current through external resistance. **[1.5A]**

5. Six cells each of e.m.f. $E$ and internal resistance of 2Ω are connected in parallel across an external resistance of 5Ω. What is the current delivered by the battery? **[0.53A]**

6. In the circuit shown in Fig. 5.58, find the terminal p.d. across each battery. **[5.8V; 4.8V]**

[Hint. Net e.m.f. in the circuit $= 6 - 4 = 2\text{V}$

Now Total circuit resistance $= r_1 + r_2 + R = 2 + 8 + 10 = 20\Omega$

Therefore, circuit current, $I = 2/20 = 0.1\text{A}$

The first battery (with e.m.f. $E_1$) is discharging.

Therefore, its terminal p.d., $V_1 = E_1 - I\,r_1 = 6 - 0.1 \times 2 = 5.8\text{V}$

The second battery (with e.m.f. $E_2$) is charging.

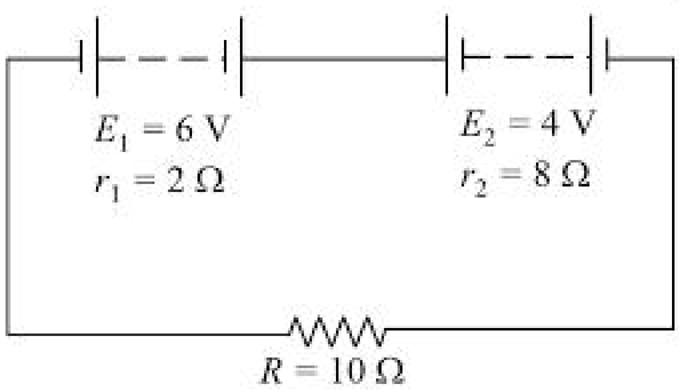



**Fig. 5.58**

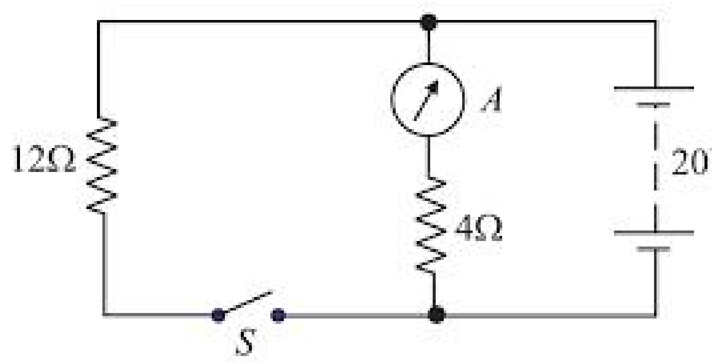



**Fig. 5.59**

Therefore, its p.d., $V_2 = E_2 + I\,r_2 = 4 + 0.1 \times 8 = 4.8\text{V}$

7. In the circuit shown in Fig. 5.59, what will be the reading of ammeter $A$ when switch $S$ is ($i$) open ($ii$) closed? The internal resistance of battery is negligible. **[($i$) 5A ($ii$) 5A]**

[Hint. The resistance of ammeter is negligible and may be assumed zero.]

8. In the circuit shown in Fig. 5.60, what is the current through 2-ohm resistor? **[0.6A]**

[Hint. Under steady state conditions, no direct current can flow through capacitor. Hence the branch containing capacitor is ineffective.]

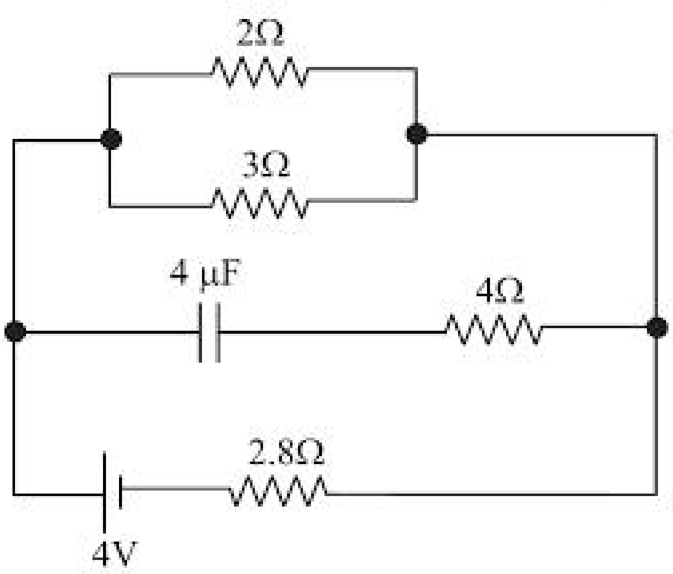



**Fig. 5.60**

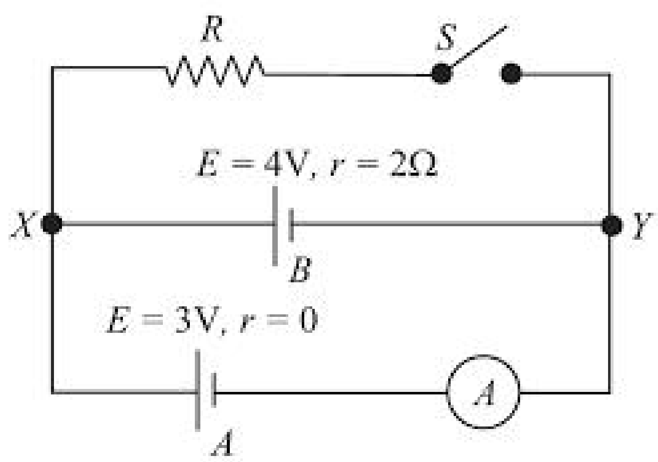



**Fig. 5.61**

9. In the circuit shown in Fig. 5.61, when switch $S$ is closed, no current flows through ammeter $A$. What is the value of $R$? **[6Ω]**

[Hint. When no current flows in $A$, then P.D. across $XY$ = P.D. across cell $A$ = 3 V This means that p.d. across $B$ = 3V. Therefore, drop in the internal resistance, $I\,r$ = 4 – 3= 1V. That is, current through $R$ is $I = 1/r = 1/2 = 0.5$A. Now p.d. across $R$ = 3V. Therefore, $R = 3/0.5 = 6\Omega$.]

10. Four cells, each of e.m.f. 1.5V and internal resistance 0.5Ω, are connected in series parallel across an external resistance of 2.5Ω. Find the current in the external resistance. **[1A]**

[Hint. Two cells in series and two such rows in parallel.]

11. A storage battery has e.m.f. 25 V and internal resistance 0.2 Ω. Calculate the terminal voltage of the battery (*i*) when it is delivering 8 A and (*ii*) when it is being charged with 8 A. **[(*i*) 23.4 V (*ii*) 26.6 V]**

12. The potential difference across a cell is 1.8 V when a current of 0.5 A is drawn from it. The p.d. falls to 1.6 V when a current of 1.0 A is drawn. Find the e.m.f. and internal resistance of the cell. **[2.0 V; 0.4 Ω]**

## CONCEPTUAL QUESTIONS

**Q.1. Two wires of equal length, one of copper and the other of manganin, have the same resistance. Which wire is thicker ?**

**Ans.**
$$\rho_c \frac{l}{A_c} = \rho_m \frac{l}{A_m} \quad \text{or} \quad \frac{A_m}{A_c} = \frac{\rho_m}{\rho_c}$$

Since the resistivity of copper ($\rho_c$) is less than the resistivity of manganin ($\rho_m$), the area of X-section of manganin wire ($A_m$) will be more than the area of X-section of copper wire ($A_c$). Hence, manganin wire will be thicker.

**Q.2. We know that a large number of free electrons are present in metals. Why is no current established in the absence of electric field?**

**Ans.** In the absence of electric field, the free electrons in the metal have random motions, *i.e.*, free electrons move in all directions haphazardly. During motion, they collide with positive ions of the metal again and again and after each collision, their direction changes. The result is that the net motion in any particular direction is zero. However, when an electric field is applied, the free electrons experience a force and start drifting towards the positive terminal of the source with a small velocity (drift velocity).

**Q.3. The charge on an electron is extremely small and also the drift velocity is very small. Even then we get sufficient amount of current in a wire. Why?**

**Ans.**
$$I = n\,e\,A\,v_d$$

Although the values of $e$ and $v_d$ are small, the value of $n$ (density of electrons) is very large, *e.g.*, in metals its value is about $10^{28}\,\text{m}^{-3}$. For this reason, we get a sizable current in the wire.

**Q.4. If the length of a conductor is doubled keeping the p.d. across it unchanged, what will be the effect on the drift velocity of free electrons?**

**Ans.**
$$\text{Drift velocity, } v_d = \frac{eE}{m}\tau$$

$$\text{Since } E = \frac{V}{l}; \quad v_d = \frac{e}{m}\frac{V}{l}\tau$$

This means that $v_d \propto 1/l$. Hence when length is doubled (for same p.d.), the drift velocity will be halved.

**Q.5. Can we verify Ohm's law by using a filament lamp?**

**Ans.** No. As the voltage across the lamp is varied, the temperature of the filament also changes. This in turn changes the resistance of the filament. The essence of Ohm's law is that relation between $V$ and $I$ is linear if $R$ is independent of the magnitude of $V$.

**Q.6. You are given $n$ wires, each of resistance $R$. What is the ratio of maximum to minimum resistance obtainable from these wires?**

**Ans.** The resistance obtained will be maximum when the wires are connected in series. However, when the wires are connected in parallel, the resistance will be minimum.

$$\therefore \quad R_s = n\,R \quad \text{and} \quad R_P = R/n$$

$$\therefore \quad \frac{R_s}{R_P} = \frac{n\,R}{R/n} = n^2$$

**Q.7. A 100-ohm carbon resistor is marked ±10% tolerance. What do you mean by it?**

**Ans.** Tolerance means maximum possible error expected in the value. Thus, a 100-ohm resistor with ± 10% tolerance means that resistance will be within ± 10% of 100Ω *i.e.*, somewhere between 90Ω and 110Ω.

**Q.8. A low voltage supply should have low internal resistance. Why?**

**Ans.**

$$I = \frac{E}{R+r}$$

The maximum current that can be drawn is $I_{max} = E/r$ ... when $R = 0$

We can obtain a large maximum current from a low voltage supply only if its internal resistance is small.

**Q.9. A high voltage supply should have high internal resistance. Why?**

**Ans.** If the internal resistance of a high voltage supply is small, then on accidental short circuit, a damaging large current will flow through the source of e.m.f. This may damage the high voltage source. For this reason, a high voltage supply should have high internal resistance.

**Q.10. Is current density a vector quantity?**

**Ans.** Yes. Sometimes we are interested in the flow of charge at a particular point within a conductor. A positive charge carrier at a given point will flow in the direction of the electric field $\vec{E}$ at the point. To describe this flow, we use the term current density $\vec{j}$, a vector quantity. The direction of current density is that of electric field $\vec{E}$ regardless of the sign of the charge carriers.

**Q.11. What are the dimensions of electrical conductivity ?**

**Ans.** The reciprocal of resistivity ($\rho$) of a conductor is called its electrical conductivity ($\sigma$) *i.e.* $\sigma = 1/\rho$.

$$\text{Electrical conductivity, } \sigma = \frac{1}{\rho} = \frac{n\,e^2\,\tau}{m}$$

$$\therefore \quad \text{Dimensions of } \sigma = \frac{L^{-3} \times (AT)^2 \times T}{M} = M^{-1}L^{-3}T^3A^2$$

**Q.12. Two wires $A$ and $B$ of the same material and of the same length have their cross-sectional areas in the ratio 2 : 1. If the same potential difference is applied across each wire in turn, what will be the ratio of currents flowing in $A$ and $B$ ?**

**Ans.**

$$R = \rho\frac{l}{A}. \text{ For same } \rho \text{ and } l,\ R \propto \frac{1}{A}.$$

For wire $A$, $R_A \propto \frac{1}{2A}$; For wire $B$, $R_B \propto \frac{1}{A}$ $\quad \therefore \quad \frac{R_B}{R_A} = 2$

Now $\quad I_A = \frac{V}{R_A};\ I_B = \frac{V}{R_B} \quad \therefore \quad \frac{I_A}{I_B} = \frac{R_B}{R_A}$ or $I_A : I_B = 2 : 1$

**Q.13. A student obtains resistances of 3Ω , 4Ω , 12Ω and 16Ω using two metallic wires, either separated or joined together. What is the value of resistance of each of these wires ?**

**Ans.** Let the values of the two resistances be $R_1$ and $R_2$. When they are connected in series, the total resistance is always greater than the individual resistance *i.e.* $R_1 + R_2 = 16$ . When the resistances are connected in parallel, the total resistance is always less than either of them *i.e.*

$$\frac{R_1R_2}{R_1 + R_2} = 3 \quad \text{or} \quad \frac{R_1R_2}{16} = 3 \quad \therefore \quad R_1R_2 = 3 \times 16 = 48$$

Now $(R_1 - R_2)^2 = (R_1 + R_2)^2 - 4\,R_1R_2 = (16)^2 - 4 \times 48 = 64 \quad \therefore \quad R_1 - R_2 = \pm 8$

$$R_1 + R_2 = 16 \quad \text{and} \quad R_1 - R_2 = \pm 8$$

$\therefore \quad R_1 = 4\ \Omega\ ;\ R_2 = 12\ \Omega \quad \text{or} \quad R_1 = 12\ \Omega,\ R_2 = 4$

**Q.14. Light from bathroom bulb gets dimmer for a moment when the geyser is switched on. Explain.**

**Ans.** When the geyser is switched on, it draws a large current due to its higher wattage. This increases the line current and there is a greater voltage drop in the line. Therefore, the voltage across the bulb decreases and it gets dim. Soon the system stabilises and restores the original voltage.

**Q.15. Explain why bending a wire does not affect its electrical resistance.**

**Ans.** The drift velocity of electron in a wire is very small. Therefore, drifting electrons have a low value of inertia of motion. For this reason, they are able to go around the bends easily.

## VERY SHORT ANSWER QUESTIONS

**Q.1. What do you mean by electromotive force (e.m.f.)?**

**Ans.** The maximum potential difference between the two plates (or electrodes) of a cell on open circuit (*i.e.*, cell delivering no current) is called electromotive force (*e.m.f.*) of the cell.

**Q.2. What is the SI unit of e.m.f.?**

**Ans.** The SI unit of e.m.f. is volt.

**Q.3. What is the energy concept of e.m.f.?**

**Ans.** The e.m.f. of a cell is defined as the energy supplied by the cell to drive a unit charge (*i.e.*, 1C) once around the complete circuit.

**Q.4. The e.m.f. of a cell is 1V. What does it mean?**

**Ans.** It means that 1 joule of energy is supplied by the cell to drive 1C of charge once around the complete circuit.

**Q.5. On what factors does the e.m.f. of a cell depend?**

**Ans.** The *e.m.f.* of a cell depends upon (*i*) the nature of electrodes, (*ii*) the nature and concentration of electrolyte and (*iii*) temperature.

**Q.6. What is electric current?**

**Ans.** It is the flow of charge (positive or negative) through a conductor (or material) in a particular direction.

**Q.7. How is electric current measured?**

**Ans.** It is measured by the flow of charge through any cross-section of the conductor in one second. If $q$ is the charge flowing through any cross-section of the conductor in time $t$, then,

$$\text{Electric current, } I = \frac{q}{t}$$

**Q.8. What is the SI unit of electric current?**

**Ans.** Electric current, $I = q/t$. Therefore, SI unit of electric current is coulomb/sec which is called *ampere*.

**Q.9. Define the SI unit of electric current.**

**Ans.** The current through a wire is 1A if one coulomb of charge (1C) flows through any cross-section of the wire in one second ($I = q/t$ = 1C/1s = 1A).

**Q.10. Is electric current a scalar or a vector quantity?**

**Ans.** Electric current, $I = q/t$. Since charge ($q$) and time ($t$) are scalars, electric current is a scalar quantity.

**Q.11. What is the direction of electric current?**

**Ans.** The direction of flow of positive charge is the direction of electric current.

**Q.12. What do you mean by electronic current?**

**Ans.** The flow of electrons constitutes the electronic current.

**Q.13. What is the significance of direction of electric current?**

**Ans.** By convention, the direction of flow of positive charge in a wire is considered as the direction of electric current. However, electrons flow in the opposite direction. Therefore, the direction of conventional current is opposite to the direction of electronic current.

**Q.14. Name the current carriers in (*i*) solid conductors, (*ii*) liquids and (*iii*) gases.**

**Ans.** (*i*) Free electrons; (*ii*) Positive ions and negative ions; (*iii*) Free electrons and positive ions.

**Q.15. What is (*i*) steady current; (*ii*) varying current?**

**Ans.** (*i*) Current which does not change with time (*ii*) Current which changes with time.

**Q.16. What is drift velocity?**

**Ans.** The average velocity with which free electrons get drifted in a conductor under the influence of electric field applied across the conductor is called drift velocity.

**Q.17. What is the order of the thermal speed of an electron in a conductor?**

**Ans.** It is of the order of $10^5$ ms$^{-1}$ at room temperature.

**Q.18. What is the order of the drift velocity of an electron in a conductor?**

**Ans.** It is of the order of $10^{-5}$ ms$^{-1}$.

**Q.19. How does the drift velocity of an electron in a conductor vary with the magnitude of current?**

**Ans.** Drift velocity, $v_d = \dfrac{I}{neA}$. Since $n$, $e$ and $A$ are constant, $v_d \propto I$.

**Q.20. How does the drift velocity of electrons in a conductor vary with increase in temperature?**

**Ans.** The drift velocity of electrons in a conductor decreases with the increase in temperature of the conductor. It is because resistance of a conductor increases with the increase in temperature.

**Q.21. If potential difference $V$ applied across a conductor is increased to $2V$, how will the drift velocity of the electrons change?**

**Ans.** Drift velocity, $v_d = \dfrac{eE}{m}\tau = \dfrac{e}{m}\tau\left(\dfrac{V}{l}\right)$. Therefore, $v_d \propto V$. Thus when potential difference is doubled, the drift velocity will also be doubled.

**Q.22. What is a non-ohmic conductor or device?**

**Ans.** A conductor or device which does not obey Ohm's law is called a non-ohmic conductor or device *e.g.*, vacuum tubes, transistors, electrolytes etc.

**Q.23. Define resistivity or specific resistance of a material.**

**Ans.** The resistivity of a material is defined as the resistance between the opposite faces of a metre cube of the material.

**Q.24. On what factors the resistivity of a material depends?**

**Ans.** The resistivity of a material depends upon (*i*) nature of the material and (*ii*) temperature of the material.

**Q.25. A wire of resistivity ρ is stretched to double its length. What will be its new resistivity?**

**Ans.** It will remain the same (*i.e.*, ρ). It is because resistivity depends *only* on the nature of material and its temperature.

**Q.26. Resistivities of copper, silver and manganin are $1.7 \times 10^{-8}$ Ωm, $1.0 \times 10^{-8}$ Ωm and $44 \times 10^{-8}$ Ωm respectively. Which of these is the best conductor?**

**Ans.** The best conductor is that which has least value of resistivity. Therefore, out of the given three materials, silver is the best conductor.

**Q.27. How does the resistivity of a conductor depend upon (*i*) number density (*n*) of free electrons and (*ii*) relexation time (τ)?**

**Ans.** Resistivity, $\rho \propto \frac{1}{n\tau}$. Therefore, resistivity of a conductor is (*i*) inversely proportional to number density (*n*) of electrons (*ii*) inversely proportional to relaxation time (τ).

**Q.28. What is conductance of a conductor?**

**Ans.** The reciprocal of resistance of a conductor is called its conductance ($G$). If a conductor has resistance $R$, then its conductance is $G = 1/R$. The unit of conductance is siemen.

**Q.29. What do you mean by conductivity of a material?**

**Ans.** Conductivity ($\sigma$) of a material is the reciprocal of its resistivity ($\rho$) *i.e.*, $\sigma = 1/\rho$.

**Q.30. What is the SI unit of conductivity?**

**Ans.** $G = \frac{1}{R} = \frac{A}{\rho l} = \sigma \frac{A}{l}$ $\therefore$ $\sigma = \frac{G \times l}{A}$. Clearly, the SI unit of conductivity ($\sigma$) is siemen metre$^{-1}$ ($Sm^{-1}$).

**Q.31. Why are connecting wires made of copper?**

**Ans.** Because copper has very low resistivity (or very high conductivity). As a result, circuit resistance is not affected and circuit current practically remains unaffected.

**Q.32. How does the conductance of a semiconductor material change with rise in temperature?**

**Ans.** Conductance, $G = 1/R$. As the temperature increases, the resistance ($R$) of a semiconductor decreases. Therefore, the conductance ($G$) of a semiconductor increases with the rise in temperature.

**Q.33. How does the change of temperature affect the resistance of (*i*) metals (*ii*) semiconductors (*iii*) insulators?**

**Ans.** (*i*) The resistance of metals increases with increase in temperature and *vice-versa*.

(*ii*) The resistance of semiconductors decreases with the increase in temperature and *vice-versa*.

(*iii*) The resistance of insulators decreases with the increase in temperature and *vice-versa*.

**Q.34. Why is constantan or manganin used for making standard resistors?**

**Ans.** This is due to two prinicpal reasons *viz* (*i*) very high resistivity and (*ii*) very low temperature co-efficient of resistance.

**Q.35. Is temperature co-efficient of resistance positive or negative for (*i*) metals, (*ii*) semiconductors and (*iii*) insulators?**

**Ans.** (*i*) Positive (*ii*) Negative (*iii*) Negative.

**Q.36. What is a thermistor?**

**Ans.** A thermistor is a heat sensitive device whose resistance changes very rapidly with change in temperature.

**Q.37. What is a superconductor?**

**Ans.** A material (metal or alloy) whose resistance becomes zero when cooled to its critical temperature is called a superconductor.

**Q.38. Is ohm's law true for all conductors?**

**Ans.** No, it is true only for metallic conductors provided the physical conditions do not change.

**Q.39. The temperature co-efficient of resistance of copper is 0.004/°C. What does it mean?**

**Ans.** It means that if a copper wire has a resistance of 1 Ω at 0°C, then it will increase by 0.004 Ω for every 1°C rise in temperature.

**Q.40. Give reason why the electrical conductivity of electrolytes is less than that of metals?**

**Ans.** In electrolytes, the charge carriers are positive and negative ions while in metals, the charge carriers are free electrons. Since the mass of ions is very large compared to that of electrons, the ions move slowly than the free electrons. For this reason, the electrical conductivity of electrolytes is less than that of metals.

**Q.41. How will you connect three resistances to get (*i*) maximum resistance and (*ii*) minimum resistance?**

**Ans.** (*i*) In series; (*ii*) In parallel.

**Q.42. When resistances are connected in series, the total resistance increases. Why?**

**Ans.** When resistances are connected in series, the effective length of the conductor increases. For this reason, the total resistance increases ($\because$ $R \propto l$).

**Q.43. When resistances are connected in parallel, the total resistance decreases. Why?**

**Ans.** When resistances are connected in parallel, the effective area of cross-section of the conductor increases. For this reason, the total resistance decreases ($\because R \propto 1/A$).

**Q.44. A wire has a resistance of 90 Ω and it is cut into three pieces having equal lengths. If these are now connected in parallel, what is the resistance of the combination so formed?**

**Ans.** When the wire is cut into three equal pieces, the resistance of each piece = 90/3 = 30 Ω. When the three pieces are connected in parallel, the resistance $R_P$ of this parallel combination is given by:

$$\frac{1}{R_P} = \frac{1}{30}+\frac{1}{30}+\frac{1}{30} \quad \text{or} \quad \frac{1}{R_P}=\frac{1}{10} \qquad \therefore R_P = 10\ \Omega$$

**Q.45. Join three resistances each of 2 Ω to get a total resistance of 3 Ω.**

**Ans.** Two resistances in parallel and third resistance in series with this parallel combination.

**Q.46. A wire of resistance 4*R* is bent in the form of a circle. What is the effective resistance between the ends of the diameter?**

**Ans.** Between the ends of the diameter, we have two resistances, each of value $2R$, in parallel. Therefore, effective resistance between the ends of the diameter is $R$.

**Q.47. How does the resistance of a wire vary with diameter?**

**Ans.** $R = \rho\frac{l}{A}=\rho\frac{l}{\pi D^2/4} \qquad \therefore R \propto \frac{1}{D^2}$

**Q.48. In homes, electrical devices are connected in parallel. Why?**

**Ans.** It is because any electrical device can be turned on or off without affecting the operation of other devices.

**Q.49. The order of coloured rings on the carbon resistor is red, yellow, blue and gold. What is the resistance of the carbon resistor?**

**Ans.** $24 \times 10^6\ \Omega \pm 5\%$

**Q.50. What is the colour coding of rings on the carbon resistor of resistance 200 Ω ± 20%?**

**Ans.** Red, black and brown.

**Q.51. On what factors does the internal resistance of a cell depend?**

**Ans.** The internal resistance of a cell depends upon (*i*) nature of electrolyte, (*ii*) nature of material of electrodes (*iii*) distance between electrodes and (*iv*) area of electrodes inside the electrolyte.

**Q.52. What is the relation between e.m.f. ($E$) and potential difference ($V$) of a cell when it is (*i*) discharging; (*ii*) charging (*iii*) on open-circuit?**

**Ans.** (*i*) $V = E - Ir$, (*ii*) $V = E + Ir$ (*iii*) $V = E$. Here $I$ is the discharging/charging current and $r$ is the internal resistance of the cell.

**Q.53. What is a battery?**

**Ans.** A number of cells grouped suitably constitutes a battery.

**Q.54. When cells are connected in parallel, what will be the effect on (*i*) current capacity, (*ii*) e.m.f. of cells?**

**Ans.** (*i*) Current capacity increases. (*ii*) E.M.F. of the battery is equal to the e.m.f. of one cell.

**Q.55. Why are cells connected (*i*) in series, (*ii*) in parallel and (*iii*) in series-parallel?**

**Ans.** (*i*) When higher voltage is required. (*ii*) When higher current is required. (*iii*) When higher voltage as well as higher current is required.

**Q.56. When do you get maximum current in series grouping of cells?**

**Ans.** When the external resistance is very large as compared to the internal resistance of the series-connected cells.

**Q.57. When do you get maximum current in parallel grouping of cells?**

**Ans.** When the external resistance is very small as compared to the internal resistance of a single cell.

**Q.58. When is the efficiency of a cell high?**

**Ans.** Efficiency of cell, $\eta = \frac{R}{R+r}=\frac{1}{1+(r/R)}$

When the external resistance ($R$) is high compared to the internal resistance ($r$) of the cell, the efficiency of the cell is high.

**Q.59. What is the condition to obtain maximum current from series-parallel grouping of cells?**

**Ans.** The external resistance should be equal to the total internal resistance of series-parallel grouping of cells.

**Q.60. How many electrons pass the cross-section of a wire when current is 1A?**

**Ans.** $6.25 \times 10^{18}$ electrons.

## SHORT ANSWER QUESTIONS

**Q.1. A wire is carrying current. Is it charged?**

**Ans.** No. The current in a wire is due to the drifting of free electrons in a definite direction. But the number of electrons in the wire at any instant is equal to the number of protons. Hence, the net charge on the wire is zero.

**Q.2. How many electrons pass through a wire in 2 minutes if the current passing through the wire is 300 mA?**

**Ans.** Here, $I = 300 \text{ mA} = 300 \times 10^{-3}$ A ; $t = 2$ min $= 120$s ; $n = ?$

Now
$$I = \frac{ne}{t} \text{ or } n = \frac{It}{e} = \frac{300 \times 10^{-3} \times 120}{1.6 \times 10^{-19}} = 2.25 \times 10^{20} \text{ electrons}$$

**Q.3. One million electrons are passing through the cross-section of a conductor in 1 millisecond. What is the current through the conductor?**

**Ans.** Here, $n = 10^6$ electrons ; $t = 1 \text{ ms} = 1 \times 10^{-3}$ s

Now
$$I = \frac{ne}{t} = \frac{10^6 \times 1.6 \times 10^{-19}}{1 \times 10^{-3}} = 1.6 \times 10^{-10} \text{ A}$$

**Q.4. Name the charge carriers in (*i*) platinum, (*ii*) a primary cell, (*iii*) germanium (*iv*) superconductor.**

**Ans.** (*i*) Electrons, (*ii*) Positive and negative ions, (*iii*) holes and electrons, (*iv*) electrons.

**Q.5. The thermal speeds of free electrons are very large. Inspite of these high speeds, why they fail to escape from the surface of a conductor?**

**Ans.** The free electrons in a conductor are free only to the extent that they may transfer from one atom to another within the conductor. It is because the free electrons that start at the surface of a conductor find behind them the positive ions pulling them back and none pulling forward. Thus, at the surface of a conductor, a free electron encounters forces that prevent it to leave the conductor surface. External energy is required for the free electrons to escape the conductor surface.

**Q.6. The drift velocity of free electrons is very small. Why does room light turn on at once as the switch is closed?**

**Ans.** The electric field is transmitted with the speed of light. As soon as the switch is closed, the electric field is at once established in the whole circuit. As a result, free electrons everywhere in the wire begin drifting at once.

**Q.7. A current of 4A is flowing in a copper wire. If the same current is allowed to flow through another copper wire of double the radius, what is the effect on the drift velocity of free electrons?**

**Ans.**
$$I = n\,e\,A\,v_d$$

Since $e$ is a constant and $n$ and $I$ are the same, $v_d \propto 1/A$. As the area of X-section is increased four times, therefore, the drift velocity will be one-fourth that in the first wire.

**Q.8. Two wires, one of copper and other of iron, have the same diameter and carry the same current. In which wire the drift velocity of free electrons will be more?**

**Ans.**
$$I = n\,e\,A\,v_d$$

Since $I$, $e$ and $A$ in the two cases are the same, $v_d \propto 1/n$. The value of $n$ (*i.e.*, number of electrons per unit volume) in iron is less than that of copper. Hence, drift velocity of free electrons will be more in iron wire as compared to copper wire.

**Q.9. If the temperature of a conductor increases, how does the relaxation time of electrons in the conductor change?**

**Ans.** Resistivity, $\rho \propto 1/n\tau$. Since $n$ for a conductor is independent of temperature, $\rho \propto 1/\tau$ or $\tau \propto 1/\rho$. For a conductor, $\rho$ increases with temperature. Therefore, relaxation time ($\tau$) of electrons decreases with the increase in temperature.

**Q.10. A potential difference of $V$ is applied to a copper wire of length $l$ and diameter $D$. What is effect on the drift velocity of electrons by (*i*) doubling $V$, (*b*) doubling $l$ (*iii*) doubling $D$?**

**Ans.** Drift velocity, $$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml} \quad \left(\because\ E = \frac{V}{l}\right)$$

(*i*) When $V$ is doubled, the drift velocity is doubled.

(*ii*) When length is doubled, drift velocity becomes half.

(*iii*) Now $v_d \propto 1/A$ or $v_d \propto 1/D^2$. Therefore, when diameter is doubled, drift velocity becomes one-fourth.

**Q.11. Define Ohm's law.**

**Ans.** The current ($I$) flowing through a conductor is directly proportional to the potential difference ($V$) applied across its ends provided the physical conditions (temperature, strain *etc*) do not change *i.e.*, $I \propto V$ or $V/I$ = Constant.

**Q.12. Does the formula $V = IR$ define Ohm's law?**

**Ans.** No. This formula defines resistance and can be applied to any conducting device whether or not it obeys Ohm's law. The essence of Ohm's law is that the graph between $V$ and $I$ is linear, *i.e.*, the value of $R$ is independent of the value of $V$.

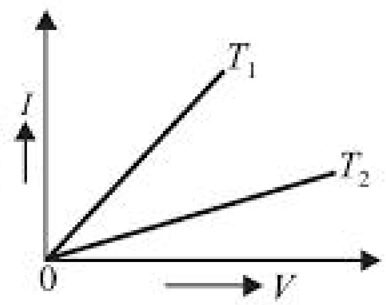


**Fig. 5.62**

**Q.13. The $V$–$I$ graph for a metallic wire at two different temperatures $T_1$ and $T_2$ is shown in Fig. 5.62. Which of the two temperatures is higher and why?**

**Ans.** The resistance of a conductor = $V/I$. Now the resistance of a conductor increases with the increase in temperature. It is clear from Fig. 5.62 that $V_2/I_2$ is greater than $V_1/I_1$. Therefore, $T_2$ is the higher temperature.

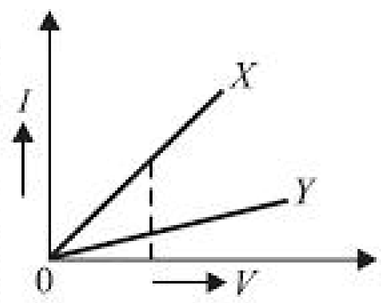


**Fig. 5.63**

**Q.14. The voltage-current variations of two metallic wires $X$ and $Y$ at constant temperature are shown in Fig. 5.63. Assuming that the wires have the same length and same diameter, which of the two wires will have larger resistivity?**

**Ans.** The resistance of a wire = $V/I$. It is clear from Fig. 5.63 that for the same $V$ current in $Y$ is less than that in $X$. Therefore, resistance of wire $Y$ is more than the wire $X$. Now $R = \rho\, l/A$. Since $l$ and $A$ are the same for the two wires, wire $Y$ has larger resistvity.

**Q.15. The resistivtiy of a wire is $1.963 \times 10^{-8}$ Ωm. What is its conductivity?**

**Ans.** Conductivity, $$\sigma = \frac{1}{\rho} = \frac{1}{1.963 \times 10^{-8}} = 5.094 \times 10^{7}\ \text{Sm}^{-1}$$

**Q.16. What will be the change in the resistance of a circular wire if its radius is halved and length is reduced to 1/4th of its original length?**

**Ans.** $R = \rho\frac{l}{A} = \rho\frac{l}{\pi r^2}$. When radius is halved and length is reduced to 1/4th of the original length, the new resistance $R'$ becomes :

$$R' = \rho\frac{l/4}{(\pi r^2)/4} = \rho\frac{l}{\pi r^2} = R$$

Therefore, the resistance value remains the same.

**Q.17. A 4Ω non-insulated wire is bent in the middle by 180° and both the wires are twisted with each other. What will be its new resistance?**

**Ans.** $R = \rho\, l/A$. When the wire is bent in the middle by 180° and both wires twisted, the length becomes $l/2$ and area of cross-section $2A$. Therefore, new resistance $R'$ becomes :

$$R' = \rho\frac{l/2}{2A} = \frac{1}{4}\rho\frac{l}{A} = \frac{R}{4} = \frac{4}{4} = 1\ \Omega$$

**Q.18. A wire of resistance 1 Ω is stretched to double its length. What is the new resistance?**

**Ans.** When a wire is stretched to double its length, its area of cross-section becomes half of the initial value.

For first case: $R = \rho\frac{l}{A}$; For second case, $R' = \rho\frac{2l}{A/2}$

$$\therefore \quad \frac{R'}{R} = 4 \quad \text{or} \quad R' = 4R = 4 \times 1 = 4\ \Omega$$

**Q.19. A wire of resistance *R* is streched so as to reduce its diameter to half of its previous value. What will be its new resistance?**

**Ans.** When the diameter of the wire is reduced to half, its area of cross-section becomes one-fourth of initial value and length becomes 4 times the initial value ($\because$ volume is same).

For first case : $R = \rho\frac{l}{A}$; For second case, $R' = \rho\frac{4l}{A/4}$

$$\therefore \quad \frac{R'}{R} = 4 \times 4 = 16 \quad \text{or} \quad R' = 16\,R$$

**Q.20. A carbon resistor has a value of 62 kΩ with a tolerance of ± 5%. Give colour code for this resistor.**

**Ans.** Blue, red, orange, gold.

**Q.21. The resistivity of a material depends upon nature of the material and temperature. Explain.**

**Ans.** Resistivity of a material, $\rho = \dfrac{m}{n e^2 \tau}$

The value of $n$ (electron density) depends upon the nature of material and the value of relaxation time ($\tau$) depends upon temperature of the material.

**Q.22. A voltage of 30V is applied across carbon resistor with first, second and third rings of blue, black and yellow colours respectively. Find the value of current through the resistor.**

**Ans.**
$$R = 60 \times 10^4\ \Omega \quad \therefore\ I = \frac{V}{R} = \frac{30}{60 \times 10^4} = 0.5 \times 10^{-4}\ \text{A}$$

**Q.23. When two resistances are in series, they have value 25 Ω and in parallel 4 Ω. Find the value of each.**

**Ans.** $R_1 + R_2 = 25$ and $\dfrac{R_1 R_2}{R_1 + R_2} = 4$. On solving, $R_1 = 5\ \Omega$ or $20\ \Omega$; $R_2 = 20\ \Omega$ or $5\ \Omega$.

**Q.24. In how many equal parts a wire having resistance of 100 Ω be cut so that we may obtain a resistance of 1 Ω by connecting them in parallel?**

**Ans.** Let $n$ be the number of equal parts. Then resistance of each part is $100/n$. When $n$ resistances each of $100/n\ \Omega$ are connected in parallel, then effective resistance $R_p$ is

$$R_p = \frac{R}{n} \quad \text{Here } R_p = 1\ \Omega\ ;\ R = 100/n\ \Omega$$

$$\therefore \quad 1 = \frac{100/n}{n} \quad \text{or } n^2 = 100 \ \therefore\ n = 10$$

**Q.25. Three identical cells, each of e.m.f. 2V and internal resistance of 0.2 Ω, are connected in series to an external resistance of 7.4 Ω. What is the current in the circuit?**

**Ans.** Here, $n = 3$ ; $r = 0.2\ \Omega$ ; $R = 7.4\ \Omega$ ; $E = 2\text{V}$

$\therefore$ Circuit current, $I = \dfrac{nE}{R + nr} = \dfrac{3 \times 2}{7.4 + 3 \times 0.2} = 0.75$ A

**Q.26. A battery of e.m.f. 6V is connected to a resistor of 100 Ω through an ammeter of resistance 2 Ω. If the circuit current is 50 mA, find the internal resistance of the battery.**

**Ans.** Here, $E = 6$V ; $R = 100\ \Omega$ ; $R_A = 2\ \Omega$ ; $I = 0.05$ A ; $r = ?$

Now $I = \dfrac{E}{R + R_A + r}$ or $r = \dfrac{E}{I} - (R + R_A) = \dfrac{6}{0.05} - (100 + 2) = 18\ \Omega$

**Q.27. The graph of variation of potential difference across a combination of three identical cells in series versus current is shown in Fig. 5.64. What is the e.m.f. of each cell?**

**Ans.** When the battery is delivering no current ($I = 0$), the terminal voltage of the battery is equal to its e.m.f. It is clear from Fig. 5.64 that when $I = 0$, the terminal voltage of the battery is 6V. This is the e.m.f. of the battery. Since the battery consists of three identical cells in series, e.m.f. of each cell $= 6/3 = 2$V.

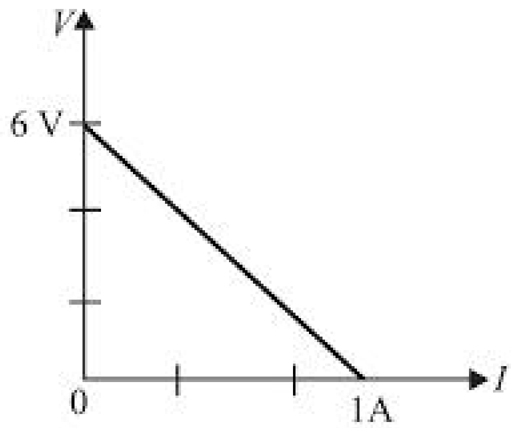


**Fig. 5.64**

**Q.28. A cell of e.m.f. $E$ and internal resistance $r$ is connected across a variable external resistance $R$. Plot graphs to show the variation of (*i*) $E$ with $R$, (*ii*) Terminal voltage of the cell $V$ with $R$.**

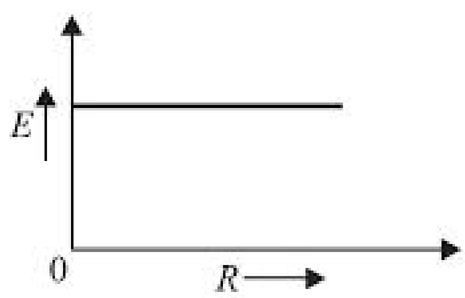


**Fig. 5.65**

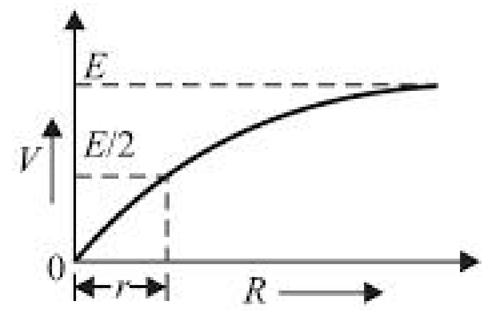


**Fig. 5.66**

**Ans.** (*i*) The value of e.m.f $E$ of the cell does not depend upon $R$. Therefore, the plot of $E$ versus $R$ is a straight line parallel to $R$ axis as shown in Fig. 5.65.

(*ii*) When the value of $R$ changes, the circuit current $I$ and hence the terminal voltage $V$ of the cell changes.

Now $V = E - Ir = E - \dfrac{E}{R + r} r = E - \dfrac{E}{1 + R/r}$

For $R = 0$, $V = 0$; For $R = r$, $V = E/2$; For $R \to \infty$, $V = E$. Therefore, the graph of $V$ versus $R$ will be as shown in Fig. 5.66.

**Q.29. Two cells of same e.m.f. $E$ but of different internal resistances $r_1$ and $r_2$ are connected in series with an external resistance $R$. The potential drop across the first cell is found to be zero. What is the value of $R$?**

**Ans.** Circuit current, $I = \dfrac{2E}{R + r_1 + r_2}$ ; Potential drop across first cell $= E - I r_1$

$\therefore$ $0 = E - \dfrac{2Er_1}{R + r_1 + r_2}$ or $R + r_1 + r_2 = 2r_1$ $\therefore$ $R = r_1 - r_2$

**Q.30. The resistance of a coil is 4.2 Ω at 100°C and temperature co-efficient of resistance is 0.004/ °C. What is its resistance at 0°C?**

**Ans.** $R_{100} = R_0\,[1 + \alpha\,(t_2 - t_1)]$ or $4.2 = R_0\,[1 + 0.004\,(100 - 0)]$ $\therefore$ $R_0 = 3\ \Omega$

**Q.31. A metallic wire has a resistance of 1.1 Ω at 20°C and 1.7 Ω at 100°C. What is the temperature co-efficient of resistance of the metal?**

**Ans.** $$\alpha = \frac{R_{100} - R_{20}}{R_{20}(t_2 - t_1)} = \frac{1.7 - 1.1}{1.1\,(100 - 20)} = 0.0068\ ^\circ C^{-1}$$

**Q.32. A carbon resistor of 74 kΩ is to be marked with rings of different colours for its identification. Write the sequence of colours.**

**Ans.** Here $R = 74\ k\Omega = 74 \times 10^3\ \Omega$. Number 7 corresponds to violet colour, number 4 corresponds to yellow colour and multiplier $10^3$ corresponds to orange colour. Therefore, sequence of colours is violet, yellow and orange.

**Q.33. A parallel combination of 3 resistances takes a current of 7.5 A from a 30V battery. If the two resistances are 10 Ω and 12 Ω, what is the value of third resistance?**

**Ans.** $$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{or} \quad 7.5 = \frac{30}{10} + \frac{30}{12} + \frac{30}{R_3}$$

$$\therefore \quad 7.5 = 3 + 2.5 + \frac{30}{R_3} \quad \text{or} \quad R_3 = 15\ \Omega$$

**Q.34. What is a resistor? What is its function?**

**Ans.** A component whose function in a circuit is to provide a specified value of resistance is called a *resistor*. Its function is to limit current, divide voltage and in certain cases generate heat.

**Q.35. Why is carbon resistor given a colour code?**

**Ans.** The physical size of a carbon resistor is very small. Therefore, it is more convenient to use colour code indicating the resistance value than to imprint the numerical value on the body of the resistor.

**Q.36. A resistor of 24 Ω resistance is bent in the form of a circle as shown in Fig. 5.67. What is the effective resistance between points *A* and *B*?**

**Ans.** The resistance of the portion of resistor subtending an angle of 60° at the centre of the circle = 24 × (60°/360°) = 4 Ω. The resistance of the remaining portion of resistor = 24 – 4 = 20 Ω. It is clear from the figure that between points *A* and *B*, effective resistance is

$$R_{AB} = 20\ \Omega \parallel 4\ \Omega = \frac{20 \times 4}{20 + 4} = \frac{10}{3}\Omega$$

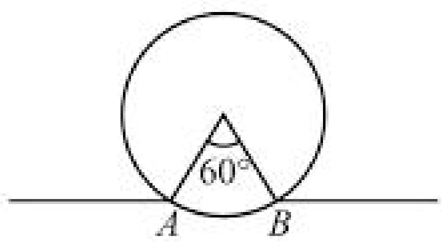


**Fig. 5.67**

## LONG ANSWER QUESTIONS

1. Explain (*i*) electric current and (*ii*) electromotive force. **[Refer to Arts. 5.3 and 5.2]**
2. Explain the mechanism of current flow in a metallic conductor. **[Refer to Art. 5.7]**
3. Explain (*i*) drift velocity, (*ii*) relaxation time of free electrons in a metallic conductor carrying current. Establish a relation between them. **[Refer to Arts. 5.7 and 5.8]**
4. Derive the relation between electric current and drift velocity of free electrons in a metallic conductor. **[Refer to Art. 5.9]**
5. Expalin Ohm's law. How will you prove its validity? **[Refer to Arts. 5.12 and 5.13]**
6. Explain the terms (*i*) resistance, (*ii*) resistivity and (*iii*) conductance. **[Refer to Arts. 5.14, 5.15 and 5.16]**
7. Find the total resistance when three resistors of resistances $R_1$, $R_2$ and $R_3$ are connected (*i*) in series (*ii*) in parallel. **[Refer to Arts. 5.25 and 5.26]**
8. How does resistivity vary with temperature in case of (*i*) conductors, (*ii*) semiconductors (*iii*) insulators? **[Refer to Art. 5.18]**
9. Derive an expression for the circuit current when a number of cells are connected (*i*) in series (*ii*) in parallel. **[Refer to Arts. 5.32 and 5.33]**
10. Find the condition for maximum current in the external resistor in case of (*i*) series grouping of cells (*ii*) parallel grouping of cells. **[Refer to Arts. 5.32 and 5.33]**
11. Find the condition for maximum current in the external resistor for mixed grouping of cells. **[Refer to Art. 5.34]**

# COMPETITION SUCCESS MATERIAL

## Useful Concepts/Information

1. There are two types of charge—positive charge and negative charge.
2. In any process in which charge is transferred from one body to another, the total charge is constant throughout, *i.e., charge is conserved.*
3. An electric current consists of flow of charged particles in a definite direction. The direction of current flow is conventionally taken to be in the direction of positive charge flow.
   (*i*) In solid conductors (*e.g.* metals), free electrons (negative charges) are the current carriers.
   (*ii*) In conducting liquids, ions (positive and negative) are the current carriers.
   (*iii*) In gases, free electrons and positive ions are the current carriers.
   (*iv*) In semiconductors, free electrons and holes (positive charges) are the current carriers.
4. The magnitude of current is the rate of flow of charge. For a steady current $I$,
$$I = \frac{q}{t} \text{ amperes; } q = \text{charge in C; } t = \text{time in sec.}$$
When the current is not steady, the instantaneous current $i = dq/dt$.
5. Electric current is a *scalar quantity.*
6. Those conductors which obey ohm's law ($I \propto V$) are called *ohmic conductors* (*e.g.* metals). The $V$-$I$ graph ($V$ on $Y$-axis and $I$ on $X$-axis) is a straight line passing through the origin. The slope of the graph gives the *resistance* $R$ ($= V/I$) of the conductor. The reciprocal of the slope of the graph gives *conductance* $G$ ($= 1/R$) of the conductor.
7. Those conductors which do not obey ohm's law (*i.e.* $V/I$ graph is not a straight line) are called *non-ohmic conductors e.g.* filament lamp, gas discharge tube, junction diode, thermistor etc.
8. The resistance $R$ and conductance $G$ of a conductor are given by ;
$$R = \rho\frac{l}{A} \quad ; \quad G = \frac{1}{R} = \frac{1}{\rho}\frac{A}{l} = \sigma\frac{A}{l}$$
where $\sigma = \frac{1}{\rho}$ = electrical conductivity. The unit of $G$ is *siemen* (S) and that of $\sigma$ is S m$^{-1}$ ($= \Omega^{-1}$ m$^{-1}$).
Resistance ($R$) as well as conductance ($G$) of a conductor depends upon (*i*) the length ($l$) (*ii*) the area of cross-section ($A$) (*iii*) nature of material of the conductor and (*iv*) temperature of the conductor.
9. The relation between current $I$ flowing through a conductor and drift velocity $v_d$ of free electrons is
$$I = neAv_d$$
Here $n$ = free electron density *i.e.* number of free electrons per unit volume
$e$ = charge on each electron
$A$ = area of $X$-section of conductor
$v_d$ = drift velocity of free electrons
Since $n$, $A$ and $e$ are constant, $I \propto v_d$.

10. Free electron density ($n$) in a metal is given by ;

$$n = N_A\, x\, d/M$$

Here $N_A$ = Avogadro's number; $x$ = No. of free electrons per atom
$d$ = density of metal; $M$ = atomic mass of metal

11. Current density ($J$) at a point in a conductor is the current per unit area at that point *i.e.* area normal to current. Its unit is $Am^{-2}$.

$$J = \frac{I}{A} = \frac{neAv_d}{A} = nev_d$$

12. When a wire is drawn under pressure, its length increases and diameter decreases. However, the volume of the wire remains the same before and after drawing.

13. The resistance of pure metals and metallic alloys increases with increase in temperature and vice-versa. However, the resistance of insulators and semiconductors decreases with increase in temperature and vice-versa.

14. The temperature co-efficient of resistance $\alpha$ is given by ;

$$\alpha = \frac{\text{Increase in resistance per °C}}{\text{Resistance at 0°C}}$$

For any given conductor, the value of $\alpha$ depends on temperature but the variation is slight. Therefore, average value of $\alpha$ between temperatures $t_1$°C and $t_2$°C ($t_2 > t_1$) is given by ;

$$\alpha = \frac{R_2 - R_1}{R_1\,(t_2 - t_1)}$$

where $R_2$ and $R_1$ are the resistances of the conductor at $t_2$°C and $t_1$°C respectively.

$$\therefore \quad R_2 = R_1\,[\,1 + \alpha\,(t_2 - t_1)]$$

15. When $n$ equal resistors, each of resistance $R$, are connected in series, the equivalent resistance $R_S = nR$. When these resistors are connected in parallel, the equivalent resistance $R_P = R/n$. Therefore, $R_S/R_P = nR/R/n = n^2$.

16. When two resistances $R_1$ and $R_2$ are connected in parallel, the equivalent resistance $R_P$ is given by ;

$$R_P = \frac{R_1 R_2}{R_1 + R_2} \quad i.e. \quad \frac{\text{Product}}{\text{Sum}}$$

Current through $R_1, I_1 = \text{Total current} \times \dfrac{\text{Other resistance}}{\text{Sum}} = I \times \dfrac{R_2}{R_1 + R_2}$

Current through $R_2, I_2 = \text{Total current} \times \dfrac{\text{Other resistance}}{\text{Sum}} = I \times \dfrac{R_1}{R_1 + R_2}$

17. When the cell is delivering no current, the potential difference across the terminals of the cell is equal to e.m.f. of the cell.

18. When a cell of e.m.f. $E$ and internal resistance $r$ is delivering current $I$ to an external resistance $R$, then,

$$E = V + I\,r = IR + Ir = I\,(R + r)$$

where $V$ (= $IR$) is the terminal voltage.

19. Depending upon voltage and current requirements, cells may be connected in three ways to form a battery *viz* (*i*) series grouping (*ii*) parallel grouping (*iii*) series-parallel grouping.

20. If $n$ cells, each of e.m.f. $E$ and internal resistance $r$, are connected in series across an external resistance $R$, then total e.m.f. = $nE$ and total circuit resistance = $R + nr$ where $nr$ = internal resistance of the battery.

$$\therefore \quad \text{Circuit current, } I = \frac{\text{Total e.m.f.}}{\text{Total circuit resistance}} = \frac{nE}{R + nr}$$

In order to get maximum current in a series grouping of cells, the external resistance ($R$) should be very large as compared to the internal resistance of the battery ($nr$).

21. If $m$ rows of cells, each row containing one cell of e.m.f. $E$ and internal resistance $r$, are connected in parallel across an external resistance $R$, then total e.m.f. = $E$ and total circuit resistance = $R + r/m$ where $r/m$ = internal resistance of the battery.

$$\therefore \quad \text{Circuit current, } I = \frac{\text{Total e.m.f.}}{\text{Total circuit resistance}} = \frac{E}{R + (r/m)} = \frac{mE}{mR + r}$$

In order to get maximum current in this parallel grouping of cells, the external resistance ($R$) should be very small as compared to the internal resistance of each cell.

22. If $n$ cells, each of e.m.f. $E$ and internal resistance $r$, are connected in series and $m$ such rows are connected in parallel across an external resistance $R$, then total e.m.f. = $nE$. The resistance of $n$ series connected cells = $nr$. Therefore, internal resistance $r_T$ of the battery is

$$\frac{1}{r_T} = \frac{1}{nr} + \frac{1}{nr} + .... m \text{ terms} = \frac{m}{nr} \quad \therefore \; r_T = \frac{nr}{m}$$

Total circuit resistance = $R + (nr / m)$

$$\text{Circuit current, } I = \frac{nE}{R + (nr/m)} = \frac{mnE}{mR + nr}$$

In order to get maximum current in this series-parallel grouping of cells, the external resistance ($R$) should be equal to the total internal resistance of the battery ($nr/m$).

23. If a battery is receiving current $I$ (*i.e.* being charged), then terminal voltage $V$ is $V = E + Ir$ ; $E$ = battery e.m.f. ; $r$ = internal resistance of battery.

24. The internal resistance of a lead-acid cell is very low (typically 0.01Ω ).

25. When unlike cells are connected in parallel, Kirchhoff's laws have to be used to solve the problem.

## MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. When a wire is stretched and its radius becomes $r/2$, then its resistance will be **[CBSE PMT 2002]**
   (*a*) 16 $R$ (*b*) 4 $R$
   (*c*) 2 $R$ (*d*) $R/2$

2. A 100W, 220 V bulb is connected across 110 V supply. The actual power consumed by the bulb will be **[AFMC 2000]**
   (*a*) 25W (*b*) 50W
   (*c*) 100W (*d*) 200W

3. The resistance of each arm of the Wheatstone bridge is 10 Ω. A resistance of 10 Ω is connected in series with the galvanometer. Then equivalent resistance across the battery will be **[CBSE PMT 2000]**
   (*a*) 20 Ω (*b*) 40 Ω
   (*c*) 15 Ω (*d*) 10 Ω

4. In the circuit shown in Fig. 5.68, each cell has an e.m.f. of 2 V and internal resistance of 1Ω. The external resistance is 2Ω. The value of current $I$ is **[Pb PMT 2001]**
   (*a*) 4 A (*b*) 2 A
   (*c*) 0.4 A (*d*) 1A

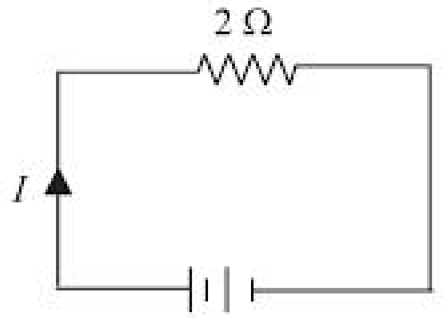



**Fig. 5.68**

5. In Fig. 5.69, the equivalent resistance between points $A$ and $B$ is **[AIIMS 1999]**

(a) 8 Ω (b) 6 Ω

(c) 4 Ω (d) 2 Ω

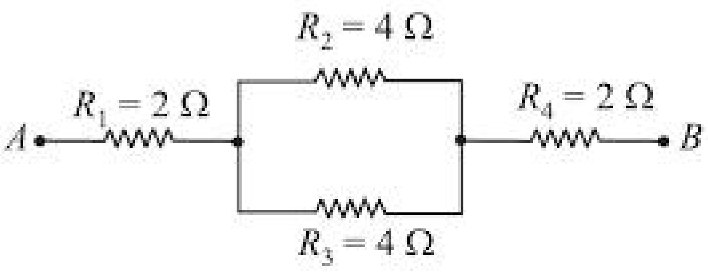



**Fig. 5.69**

6. Two batteries of e.m.f. 4V and 8 V and internal resistances of 1 Ω and 2 Ω respectively are connected in a circuit with a resistance of 9 Ω as shown in Fig. 5.70. The current and potential difference between points $P$ and $Q$ is **[AFMC 1999]**

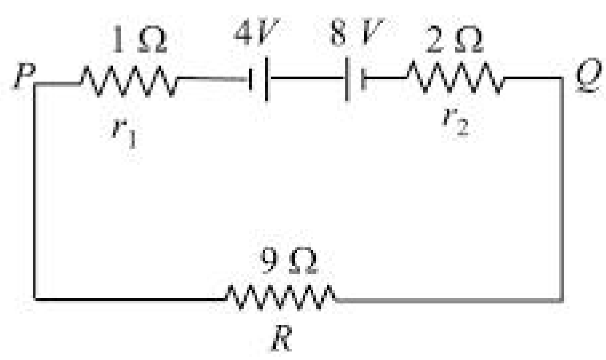



**Fig. 5.70**

(a) $\frac{1}{3}$ A and 3 V (b) $\frac{1}{6}$ A and 4 V

(c) $\frac{1}{9}$ A and 9 V (d) $\frac{1}{12}$ A and 12 V

7. The internal resistance of a car battery of e.m.f. 12 V is $5 \times 10^{-2}$ Ω. It is connected across an unknown resistance. Voltage across the battery when it draws a current of 60 A is **[CBSE PMT 2000]**

(a) 15 V (b) 12 V

(c) 6 V (d) 9 V

8. The current in 8 Ω resistance in Fig. 5.71 is **[CBSE PMT 1999]**

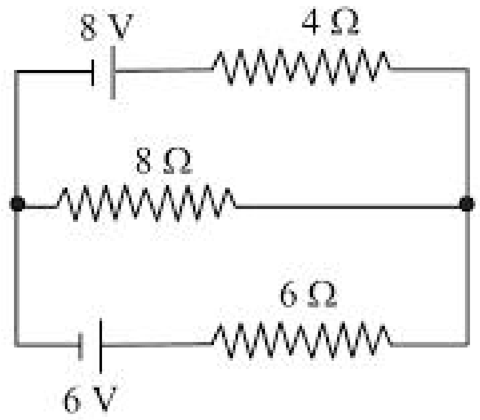



**Fig. 5.71**

(a) 0.92 A (b) 1.3 A

(c) $\frac{9}{13}$ A (d) 1.6 A

9. The current $I$ in the circuit shown in Fig. 5.72 is **[CBSE PMT 1999]**

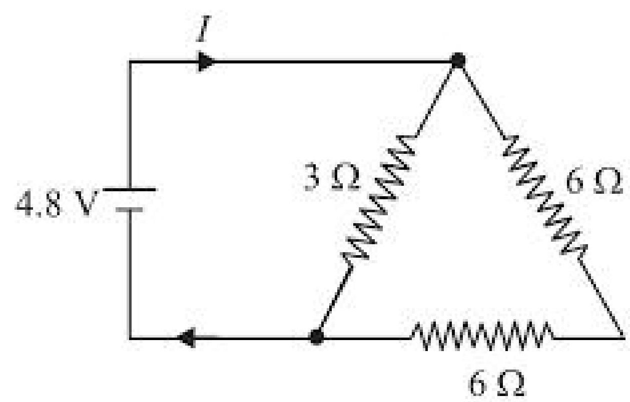



**Fig. 5.72**

(a) 8.31 A (b) 2 A

(c) 4.92 A (d) 1.33 A

10. A battery of e.m.f. 10 V and internal resistance 3 Ω is connected to a resistor $R$ as shown in Fig. 5.73. If current in the circuit is 0.5A, what is the resistance of the resistor ? **[Pb PMT 2000]**

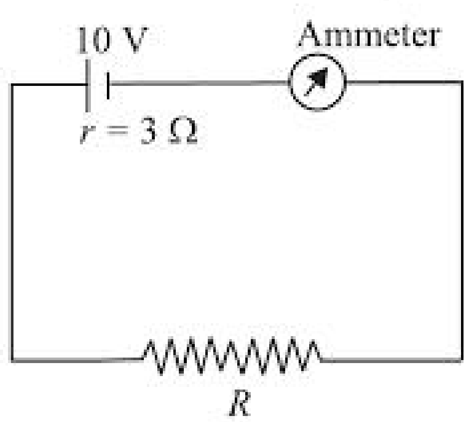



**Fig. 5.73**

(a) 13 Ω (b) 15 Ω

(c) 17 Ω (d) 19 Ω

11. If current through 3Ω resistor in Fig. 5.74 is 0.8A, then potential drop across 4 Ω resistor is **[AFMC 1999]**

(a) 9.6 V (b) 2.6 V

(c) 1.2 V (d) 4.8 V

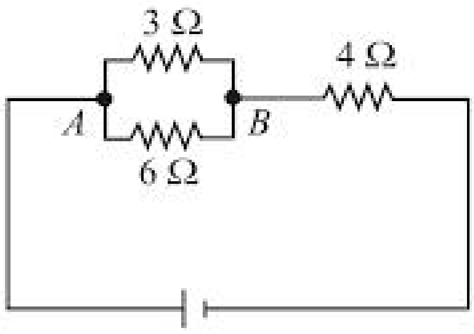



**Fig. 5.74**

**12.** What will be the equivalent resistance between points $A$ and $D$ in Fig. 5.75 ? **[CBSE PMT 1996]**

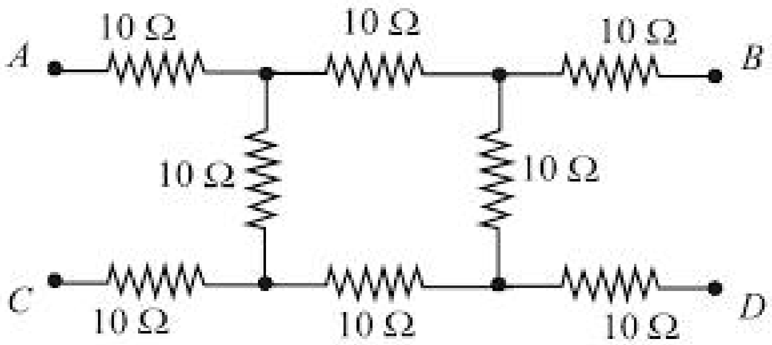



**Fig. 5.75**

(*a*) 27.5 Ω (*b*) 30 Ω
(*c*) 10 Ω (*d*) 20 Ω

**13.** In Fig. 5.76, the resistance between points $A$ and $B$ is **[CBSE PMT 1995]**

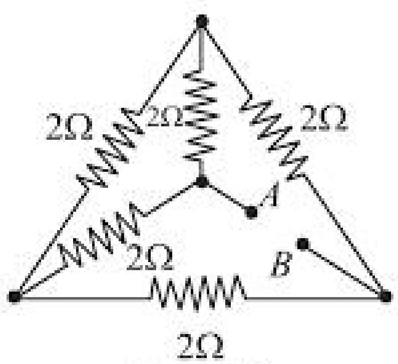



**Fig. 5.76**

(*a*) 2 Ω (*b*) 1 Ω (*c*) 3 Ω (*d*) 4 Ω

**14.** The equivalent resistance between $A$ and $B$ in Fig. 5.77 is **[CBSE PMT 1994]**

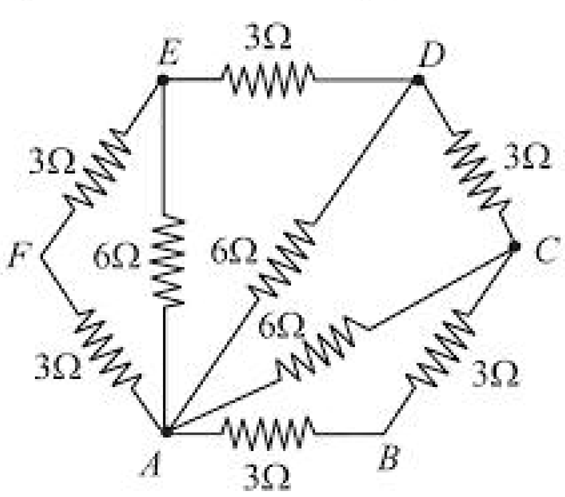



**Fig. 5.77**

(*a*) 3 Ω (*b*) 9 Ω
(*c*) 2 Ω (*d*) 6 Ω

**15.** A flow of $10^7$ electrons per second in a conducting wire constitutes a current of **[CBSE PMT 1994]**

(*a*) $1.6 \times 10^{-26}$ A (*b*) $1.6 \times 10^{12}$ A
(*c*) $1.6 \times 10^{-12}$ A (*d*) $1.6 \times 10^{26}$ A

**16.** Two wires of the same metal have the same length but their cross-sectional areas are in the ratio 3 : 1. They are joined in series. The resistance of the thicker wire is 10 Ω. The total resistance of the combination is **[CBSE PMT 1995]**

(*a*) 5/2 Ω (*b*) 40/3 Ω
(*c*) 100 Ω (*d*) 40 Ω

**17.** Two cells of same e.m.f. $E$ but of different internal resistances $r_1$ and $r_2$ are connected in series with an external resistance $R$. The potential drop across the first cell is found to be zero. The external resistance $R$ is **[CPMT 1996]**

(*a*) $r_1 + r_2$ (*b*) $r_1 - r_2$
(*c*) $r_2 - r_1$ (*d*) $r_1\ r_2$

**18.** In the circuit shown in Fig. 5.78, the resistance $R_2$ in decreased., **[JK PMT 1997]**

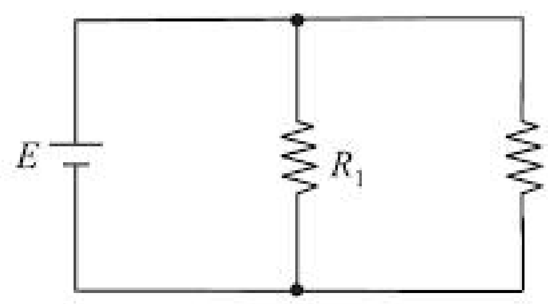



**Fig. 5.78**

(*a*) the current through $R_1$ increases
(*b*) the voltage drop across $R_1$ increases
(*c*) the voltage drop across $R_2$ decreases
(*d*) the power dissipated by $R_2$ increases

**19.** The resistance between the points $A$ and $B$ of the given infinitely long circuit will be [See Fig. 5.79] **[MP PMT 1998]**

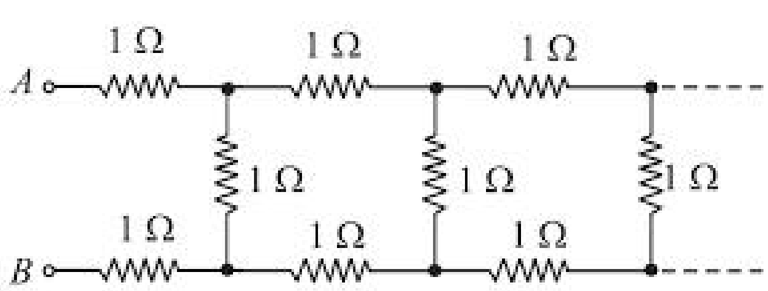



**Fig. 5.79**

(*a*) $(\sqrt{3} - 1)$ Ω (*b*) $(1 - \sqrt{3})$ Ω
(*c*) $(1 + \sqrt{3})$ Ω (*d*) $(2 + \sqrt{3})$ Ω

20. A wire of resistance $R$ is cut into $n$ equal parts. These parts are then connected in parallel. The equivalent resistance of the combination will be **[MP PMT 1998]**

(a) $nR$ (b) $R/n$
(c) $n/R$ (d) $R/n^2$

## ANSWERS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

| | | | | |
|---|---|---|---|---|
| **1.** (a) | **2.** (a) | **3.** (d) | **4.** (c) | **5.** (b) |
| **6.** (a) | **7.** (d) | **8.** (c) | **9.** (b) | **10.** (c) |
| **11.** (d) | **12.** (b) | **13.** (a) | **14.** (c) | **15.** (c) |
| **16.** (d) | **17.** (b) | **18.** (d) | **19.** (c) | **20.** (d) |

## HINTS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. $R = \rho\frac{l}{A}$ and new resistance $R' = \rho\frac{l'}{A'}$ .

Since volume remains same, $Al = A'l'$ or $\frac{l'}{l} = \frac{A}{A'} = \frac{(r)^2}{(r/2)^2} = 4$

$$\therefore \quad \frac{R'}{R} = \frac{l'}{l} \times \frac{A}{A'} = 4 \times 4 = 16 \quad \therefore \quad R' = \mathbf{16\ R}$$

2. Resistance of bulb, $R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\,\Omega$ . Therefore, $P' = \frac{(V')^2}{R}$

$$= \frac{(110)^2}{484} = \mathbf{25\ W}$$

3. Fig. 5.80 shows the conditions of the problem. Since the products of resistances of the opposite arms of the bridge are equal, the bridge is balanced. Therefore, 10 Ω resistance in the branch $QS$ is ineffective and may be considered as removed. In branch $PQR$, two 10 Ω resistances are in series and their equivalent resistance $R' = 10 + 10 = 20\ \Omega$. In branch $PSR$, two 10 Ω resistances are in series and their equivalent resistance $R'' = 10 + 10 = 20\ \Omega$. Now $R'$ and $R''$ are in parallel so that equivalent resistance across battery is

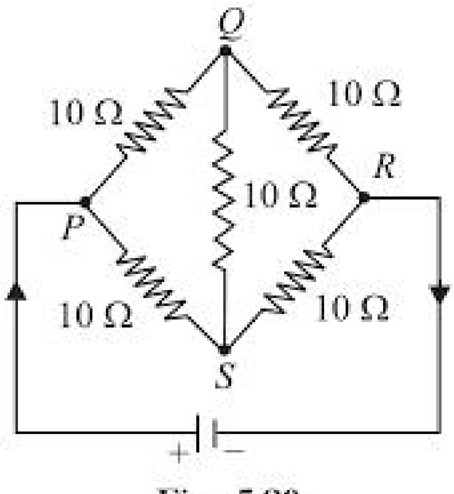



**Fig. 5.80**

$$R_{PR} = R' \| R'' = 20\Omega \| 20\Omega = \frac{20 \times 20}{20 + 20} = \mathbf{10\ \Omega}$$

4. Two cells aid each other and one cell opposes them. Therefore, net e.m.f. is $E = 2 + 2 - 2 = 2\text{V}$. Total internal resistance, $r = 3 \times 1 = 3\ \Omega$.

$\therefore$ Circuit current, $I = \frac{E}{R + r} = \frac{2}{2 + 3} = \mathbf{0.4A}$

5. Resistances $R_2$ and $R_3$ are in parallel and their equivalent resistance $R'$ is

$$R' = \frac{R_2 R_3}{R_2 + R_3} = \frac{4 \times 4}{4 + 4} = 2\ \Omega$$

Now $R_1$, $R'$ and $R_4$ are in series and their equivalent resistance is

$$R_{AB} = R_1 + R' + R_4 = 2 + 2 + 2 = \mathbf{6\ \Omega}$$

6. Net e.m.f. in the circuit $E = 8 - 4 = 4$ V. Total circuit resistance, $R_T = r_1 + r_2 + R$ $= 1 + 2 + 9 = 12\ \Omega$. Therefore, circuit current $I$ is

$$I = E/R_T = 4/12 = \mathbf{1/3\ A}$$

$$P.D.\ \text{across}\ PQ = P.D.\ \text{across}\ 9\Omega = \frac{1}{3}\times 9 = \mathbf{3\ V}$$

7. $V = E - Ir = 12 - 60 \times 5 \times 10^{-2} = 12 - 3 = \mathbf{9\ V}$

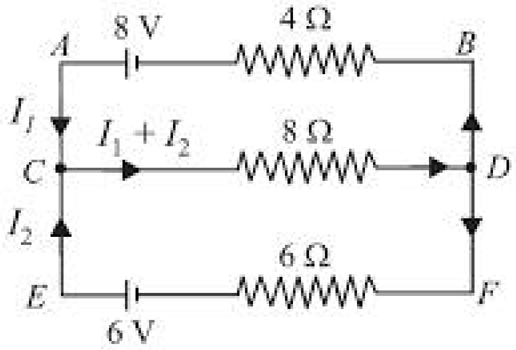


**Fig. 5.81**

8. Let the current supplied by battery of 8 V be $I_1$ and that supplied by battery of 6V be $I_2$. Then currents in the various branches of the circuit will be as shown in Fig. 5.81. Applying Kirchhoff's voltage law to the loop $ABDCA$, we have,

$$-8 + 4I_1 + 8(I_1 + I_2) = 0$$

or $$3I_1 + 2I_2 = 2 \quad \text{... } (i)$$

Applying Kirchhoff's voltage law to the loop $CDFEC$, we have,

$$-8(I_1 + I_2) - 6I_2 + 6 = 0$$

or $$4I_1 + 7I_2 = 3 \quad \text{... } (ii)$$

Solving eqs. ($i$) and ($ii$), $I_1 = 8/13$ A and $I_2 = 1/13$ A.

$$\therefore \quad \text{Current in } 8\Omega = I_1 + I_2 = \frac{8}{13} + \frac{1}{13} = \mathbf{\frac{9}{13}\ A}$$

9. The resistances 6 Ω and 6Ω are in series and their equivalent resistance $R' = 6 + 6$ $= 12\ \Omega$. Now $R'$ and 3 Ω resistances are in parallel. Therefore, total resistance $R_T$ across the battery is

$$R_T = \frac{R' \times 3}{R' + 3} = \frac{12 \times 3}{12 + 3} = \frac{12}{5}\ \Omega$$

$$\therefore \quad I = \frac{E}{R_T} = \frac{4.8}{12/5} = \mathbf{2\ A}$$

10. Battery e.m.f., $E = 10$ V. Total circuit resistance $= R + r$ (It is assumed that ammeter has zero resistance).

$$\therefore \quad I = \frac{E}{R + r} \quad \text{or} \quad 0.5 = \frac{10}{R + 3} \quad \therefore \quad R = \mathbf{17\Omega}$$

11. $V_{AB}$ = current in 3 Ω × 3 Ω = 0.8 × 3 = 2.4 V . Therefore, current in 6 Ω = 2.4/6 = 0.4 A . As a result, current in 4 Ω = 0.8 + 0.4 = 1.2 A. Therefore potential drop across 4 Ω = 1.2 × 4 = **4.8 V**.

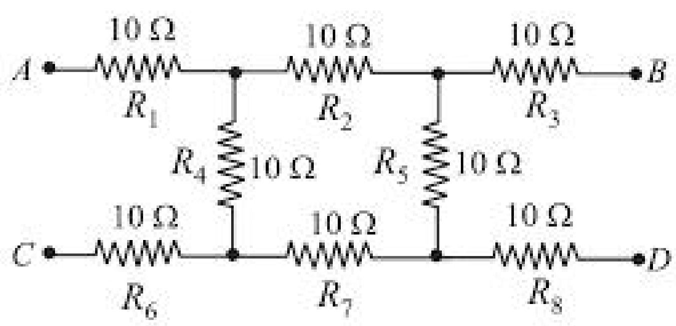


**Fig. 5.82**

12. Refer to Fig. 5.82. In order to find the resistance between points $A$ and $D$, imagine a battery connected between $A$ and $D$. No current will flow in resistances $R_3$ and $R_6$ and, therefore, these resistances are ineffective. Resistances $R_2$ and $R_5$ are in series and their equivalent resistance $R' = R_2 + R_5 = 10 + 10 = 20\ \Omega$. Again resistances $R_4$ and $R_7$ are in series and their equivalent resistance $R'' = R_4 + R_7 = 10 + 10 = 20\ \Omega$. Now $R'$ and $R''$ are in

parallel and their equivalent resistance $R''' = 10\ \Omega$.

$$\therefore \quad R_{AD} = R_1 + R''' + R_8 = 10 + 10 + 10 = \mathbf{30\ \Omega}$$

**13.** The circuit can be redrawn as shown in Fig. 5.83. Since the products of resistances of the opposite arms of the bridge circuit are equal, the bridge is balanced. Therefore resistance in the branch $PQ$ is ineffective and may be considered as removed. In branch $APB$, two 2 Ω resistances are in series and their equivalent resistance $R' = 2 + 2 = 4\ \Omega$. In branch $AQB$, two 2 Ω resistances are in series and their equivalent resistance $R'' = 2 + 2 = 4\ \Omega$ . Now $R'$ and $R''$ are in parallel.

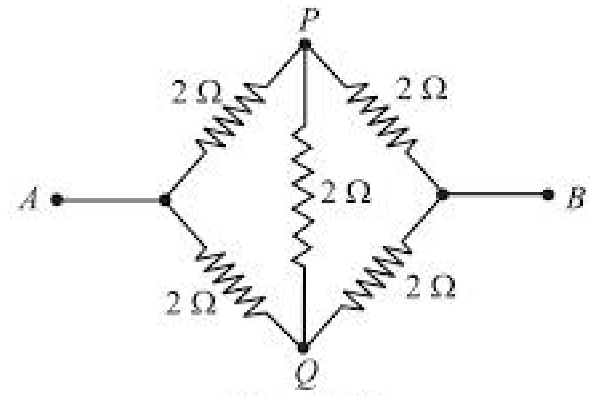


**Fig. 5.83**

$$\therefore \quad R_{AB} = R' \parallel R'' = 4\Omega \parallel 4\Omega = \frac{4\times 4}{4+4} = \mathbf{2\ \Omega}$$

**14.** In order to find resistance between $A$ and $B$, imagine a battery is connected between $A$ and $B$. The resistances in the branch $AFE$ are in series and their equivalent resistance $R_1 = 3 + 3 = 6\ \Omega$. The resistance $R_1$ is in parallel with 6 Ω resistance in branch $EA$ and their equivalent resistance $R_2 = 6 \times 6/\ (6 + 6) = 3\ \Omega$. Now $R_2$ is in series with 3Ω resistance in the branch $ED$ and their equivalent resistance $R_3 = 3 + 3 = 6\ \Omega$. The resistance $R_3$ is in parallel with 6 Ω resistance in the branch $AD$ and their equivalent resistance $R_4 = 6 \times 6/\ (6 + 6) = 3\ \Omega$. The resistance $R_4$ is in series with 3 Ω resistance in the branch $DC$ and their equivalent resistance $R_5 = 3 + 3 = 6\ \Omega$. Now $R_5$ is in parallel with 6 Ω resistance in the branch $CA$ and their equivalent resistance $R_6 = 6 \times 6/(6 + 6) = 3\ \Omega$. The resistance $R_6$ is in series with 3Ω resistance in the branch $CB$ and their equivalent resistance $R_7 = 3 + 3 = 6\ \Omega$. Now resistance $R_7$ is in parallel with 3 Ω resistance in the branch $AB$.

$$\therefore \quad R_{AB} = R_7 \parallel 3\Omega = 6\Omega \parallel 3\Omega = \frac{6\times 3}{6+3} = \mathbf{2\ \Omega}$$

**15.**
$$I = \frac{ne}{t} = \frac{10^7 \times 1.6 \times 10^{-19}}{1} = \mathbf{1.6 \times 10^{-12}\ A}$$

**16.** $R_1 = \rho\dfrac{l}{3A};\ R_2 = \rho\dfrac{l}{A} \quad \therefore \quad \dfrac{R_2}{R_1} = \dfrac{3A}{A} = 3 \quad$ or $\quad R_2 = 3\ R_1 = 3 \times 10 = 30\ \Omega$

Therefore, total resistance $= R_1 + R_2 = 10 + 30 = \mathbf{40\ \Omega}$

**17.** Circuit current, $I = \dfrac{2E}{R + r_1 + r_2}$ ; Potential drop across first cell $= E - Ir_1$

$$\therefore 0 = E - \frac{2Er_1}{R + r_1 + r_2} \quad \text{or} \quad R + r_1 + r_2 = 2r_1 \quad \therefore \quad R = \mathbf{r_1 - r_2}$$

**18.** Since $R_1$ and $R_2$ are connected in parallel across $E$, the voltage drop across the resistances remains the same irrespective of the values of $R_1$ and $R_2$. Now power dissipated in $R_2 = E^2/R_2$. As $R_2$ is decreased, **power dissipated in $R_2$ will increase.**

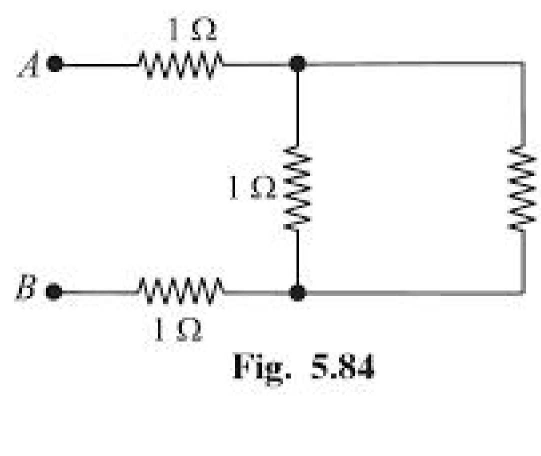


**Fig. 5.84**

**19.** Let $x$ ohms be the resistance of the infinitely long network. Since the network is infinitely long, the addition of one set of three resistances, each of 1 Ω will not change the total resistance *i.e.* it will remain $x$. The network would then become as shown in Fig. 5.84. Then effective resistance between terminals $A$ and $B$ is

$$x = 1 + \frac{1 \times x}{1 + x} + 1 = 2 + \frac{x}{1 + x}$$

$$\therefore \quad x^2 - 2x - 2 = 0 \quad \text{or} \quad x = (1 \pm \sqrt{3})\ \Omega$$

As the value of resistance cannot be negative, $x = \mathbf{(1 + \sqrt{3})\Omega}$

**20.** Resistance of each equal part, $r = R/n$. When these $n$ equal resistances are connected in parallel, equivalent resistance $= r/n = \frac{R/n}{n} = \mathbf{R/n^2}$

## MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

**1.** The resistance of a coil is 4.2 Ω at 100°C and temperature co-efficient of resistance is 0.004/°C. Its resistance at 0°C is **[CBSE Engineering 2002]**

(*a*) 6.5 Ω (*b*) 5 Ω
(*c*) 3 Ω (*d*) 2.5 Ω

**2.** If a wire is melted and recasted to half of its length, then the new resistance is **[CBSE Engineering 2002]**

(*a*) $R/4$ (*b*) $R/2$
(*c*) $R$ (*d*) $2R$

**3.** In the circuit shown in Fig. 5.85, the current through **[CEE Delhi 2002]**

(*a*) 3 Ω resistor is 0.5 A
(*b*) 3 Ω resistor is 0.25 A

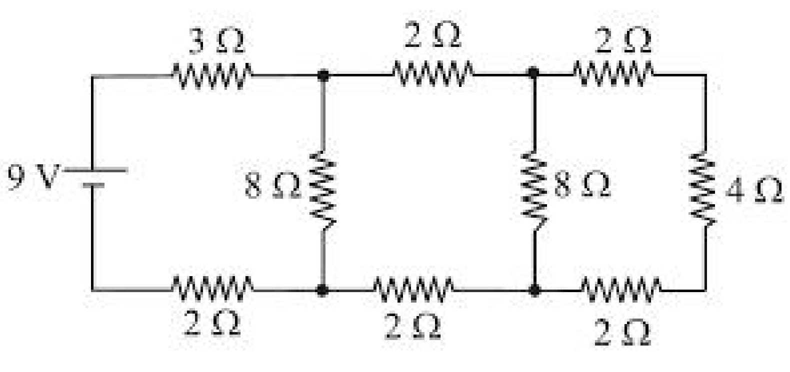



**Fig. 5.85**

(*c*) 4 Ω resistor is 0.5 A
(*d*) 4 Ω resistor is 0.25 A

**4.** Iron and silicon wires are heated from 30°C to 50°C. The correct statement is that **[MP PET 1995]**

(*a*) resistance of both the wires increases
(*b*) resistance of both the wires decreases
(*c*) resistance of iron wire increases and that of silicon wire decreases
(*d*) resistance of iron wire decreases and that of silicon wire increases

**5.** The voltage $V$ and current $I$ graph for a conductor at two different temperatures $T_1$ and $T_2$ are shown in Fig. 5.86. The relation between $T_1$ and $T_2$ is **[MP PET 1996]**

(*a*) $T_1 = T_2$ (*b*) $T_1 > T_2$

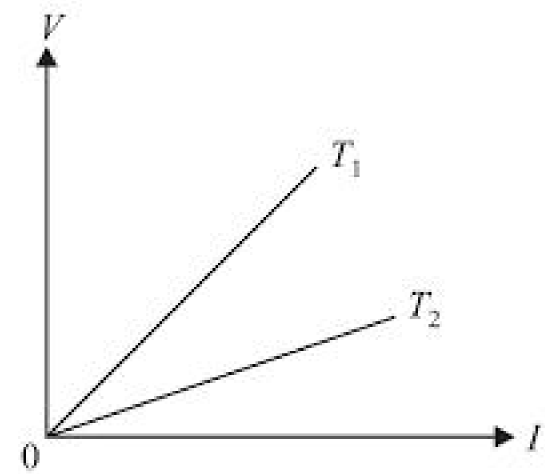



**Fig. 5.86**

(*c*) $T_1 < T_2$ (*d*) none of the above

**6.** Two electric bulbs have tungsten filament of same length. If one of them gives 60W and the other 100 W, then, **[CEE Delhi 2000]**

(*a*) 60 W bulb has thicker filament
(*b*) both filaments are of same thickness
(*c*) 100 W bulb has thicker filament
(*d*) none of above

**7.** In the circuit shown in Fig. 5.87, the current drawn from the battery is 4A. If 10 Ω resistor is replaced by 20 Ω resistor, the current drawn from the battery will be **[Karnataka CET 2000]**

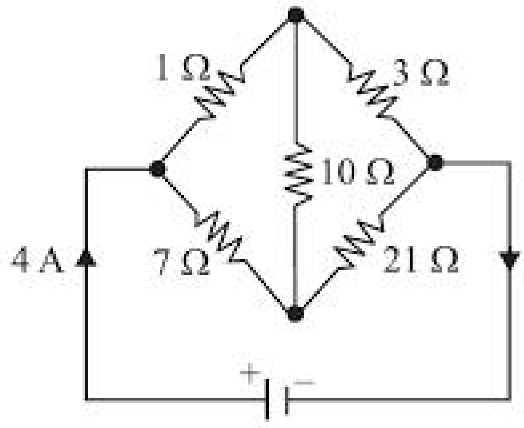



**Fig. 5.87**

(*a*) 1 A (*b*) 2 A

(*c*) 3 A (*d*) 4 A

8. What is the total resistance of the circuit shown in Fig. 5.88 ?

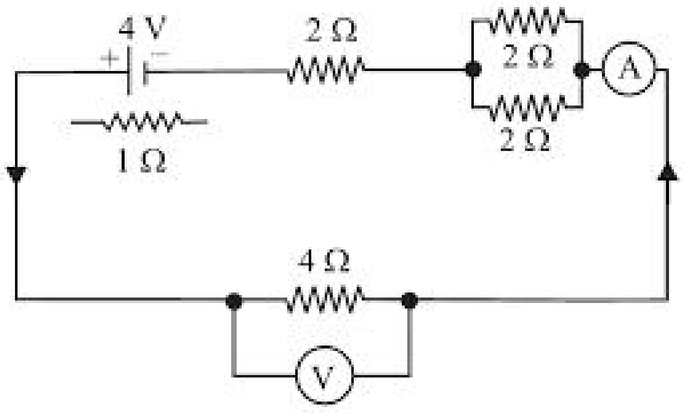



**Fig 5.88**

(*a*) 6 Ω (*b*) 7 Ω

(*c*) 8 Ω (*d*) 9 Ω

9. In the circuit shown in Fig. 5.89, the internal resistance of the battery is 1.5 Ω and $V_P$ and $V_Q$ are the potentials at points *P* and *Q* respectively. What is the potential difference between the points *P* and *Q* ? **[MP PET 2000]**

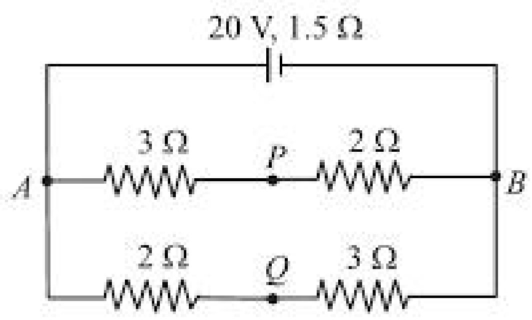



**Fig. 5.89**

(*a*) $V_P - V_Q = 4$ V (*b*) $V_P - V_Q = -4$ V

(*c*) $V_P - V_Q = 0$ (*d*) $V_P - V_Q = -2.5$ V

10. There are two electric bulbs of 40 W and 100 W. Which one will be brighter when they are first connected in series and then in parallel ?

**[CEE Delhi 1998]**

(*a*) 100 W both in series and parallel

(*b*) 40 W both in series and parallel

(*c*) 100 W in series and 40 W in parallel

(*d*) 40 W in series and 100 W in parallel

11. Potential difference between points *P* and *Q* in the circuit shown in Fig. 5.90 is

**[KCET 2000]**

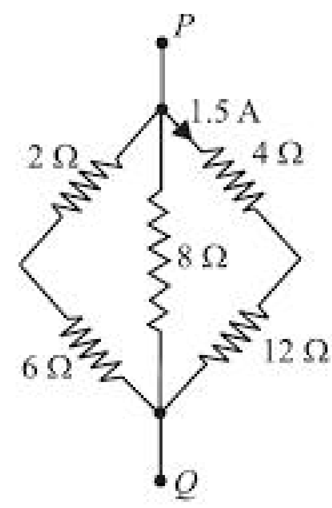



**Fig. 5.90**

(*a*) 8 V (*b*) 24 V

(*c*) 4.8 V (*d*) 12 V

12. In the circuit shown in Fig 5.91 with steady current, the potential drop across the capacitor will be **[IIT Screening 2001]**

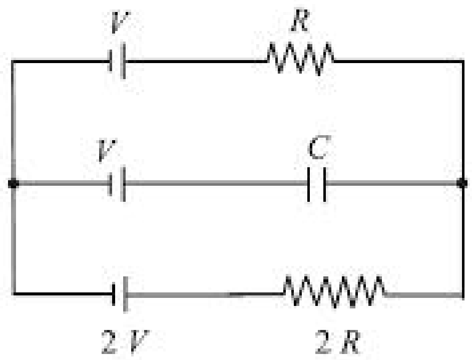



**Fig. 5.91**

(*a*) *V*/3 (*b*) *V*/2

(*c*) *V* (*d*) 2*V*/3

13. A uniform wire of resistance *R* is uniformly compressed along its length until its radius becomes *n* times the original radius. Now resistance of the wire becomes **[KCET 2000]**

(*a*) $\frac{R}{n^4}$ (*b*) $\frac{R}{n^2}$

(*c*) $\frac{R}{n}$ (*d*) $nR$

14. A rod of certain metal is 1.0 m long and 0.6 cm in diameter. Its resistance is $3.0 \times 10^{-3}$ Ω. Another disc made of the same metal is 2.0 cm in diameter and 1.0 mm thick. What is the resistance between the round faces of the disc ?

**[MP PET 2000]**

(*a*) $1.35 \times 10^{-8}$ Ω (*b*) $2.70 \times 10^{-7}$ Ω

(*c*) $4.05 \times 10^{-6}$ Ω (*d*) $8.10 \times 10^{-5}$ Ω

15. The equivalent resistance between points $A$ and $B$ in the circuit shown in Fig. 5.92 is **[SCRA 2000]**

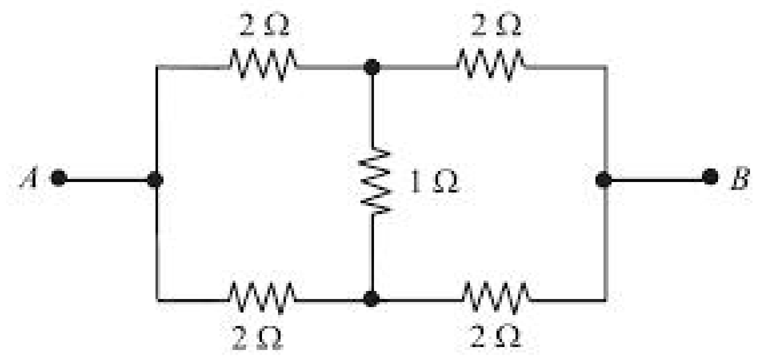



**Fig. 5.92**

(a) 9 Ω (b) 4 Ω
(c) 2 Ω (d) 1 Ω

16. If there is an increase in length by 0.1% due to stretching, the percentage increase in its resistance will be **[MP PET 1996]**

(a) 0.1 % (b) 2 %
(c) 1 % (d) 0.2 %

17. The potential difference across 8 Ω resistance in the circuit shown in Fig. 5.93 is 48 V. The value of potential difference across points $A$ and $B$ will be **[MP PET 2000]**

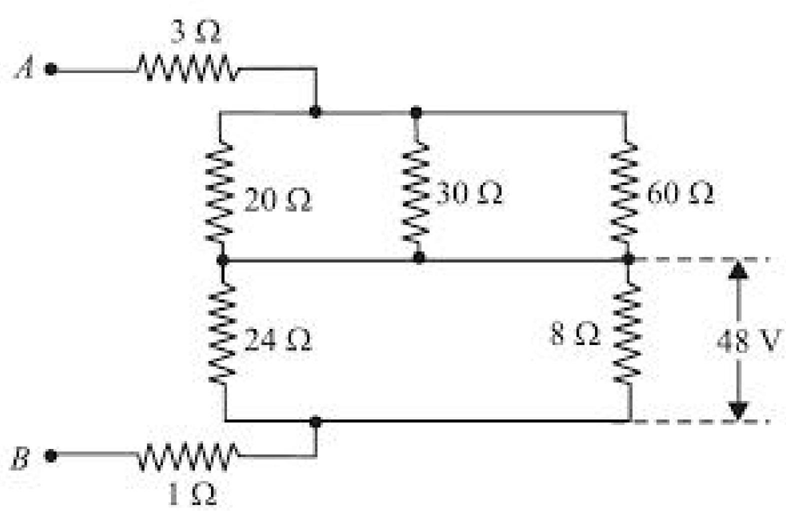



**Fig. 5.93**

(a) 62 V (b) 80 V
(c) 128 V (d) 160 V

18. In the circuit shown in Fig. 5.94, it is observed that the current $I$ is independent of the value of resistance $R_6$. Then the resistance values must satisfy the condition **[IIT Screening 2001]**

(a) $R_1 R_2 R_3 = R_3 R_4 R_5$

(b) $\frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$

(c) $R_1 R_4 = R_2 R_3$

(d) $R_1 R_3 = R_2 R_4 = R_5 R_6$

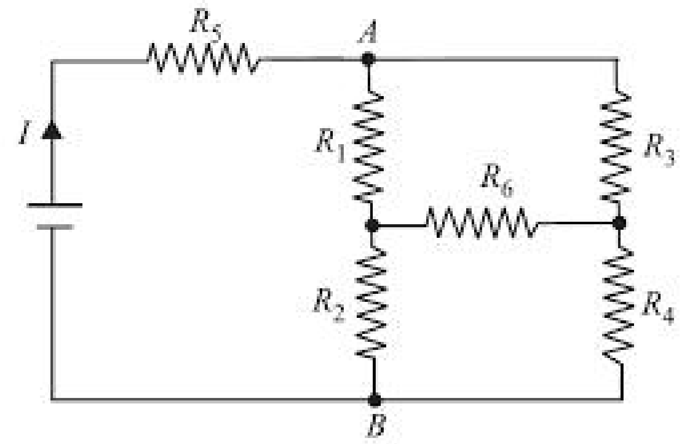



**Fig. 5.94**

19. An electron (charge = $1.6 \times 10^{-19}$C) is moving in a circle of radius $5.1\times 10^{-11}$m at a frequency of $6.8 \times 10^{15}$ revolutions per second. The equivalent current is approximately

(a) $5.1 \times 10^{-3}$ A (b) $6.8 \times 10^{-3}$ A
(c) $1.1 \times 10^{-3}$ A (d) $2.2 \times 10^{-3}$ A

20. At what temperature, the resistance of a copper wire will become three times its value at 0°C ? Temperature co-efficient of resistance for copper = $4 \times 10^{-3}$ per °C. **[MP PET 2000]**

(a) 400 °C (b) 450 °C
(c) 500 °C (d) 550 °C

## ANSWERS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

| | | | | |
|---|---|---|---|---|
| **1.** (c) | **2.** (a) | **3.** (d) | **4.** (c) | **5.** (b) |
| **6.** (c) | **7.** (d) | **8.** (c) | **9.** (d) | **10.** (d) |
| **11.** (b) | **12.** (a) | **13.** (a) | **14.** (b) | **15.** (c) |
| **16.** (d) | **17.** (d) | **18.** (c) | **19.** (c) | **20.** (c) |

## HINTS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. $R_{100} = R_0\,[1 + \alpha\,(t_2 - t_1)]$ or $4.2 = R_0\,[1 + 0.004\,(100 - 0)]$ $\therefore$ $R_0 = \mathbf{3\ \Omega}$

2. $R = \rho\frac{l}{A}$ and new resistance $R' = \rho\frac{l'}{A'}$.

Since volume of wire remains the same, $lA = l'A'$ or $\frac{A}{A'} = \frac{l'}{l} = \frac{0.5l}{l} = \frac{1}{2}$

$$\therefore \quad \frac{R'}{R} = \frac{l'}{l} \times \frac{A}{A'} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \quad \therefore \quad R' = \mathbf{R/4}$$

**3.** It is easy to see that circuit reduces to the one shown in Fig. 5.95 (*i*). The total circuit resistance across the battery, $R_T = 3 + 4 + 2 = 9\ \Omega$ and current supplied by battery $= V/R_T = 9/9 = 1$ A.

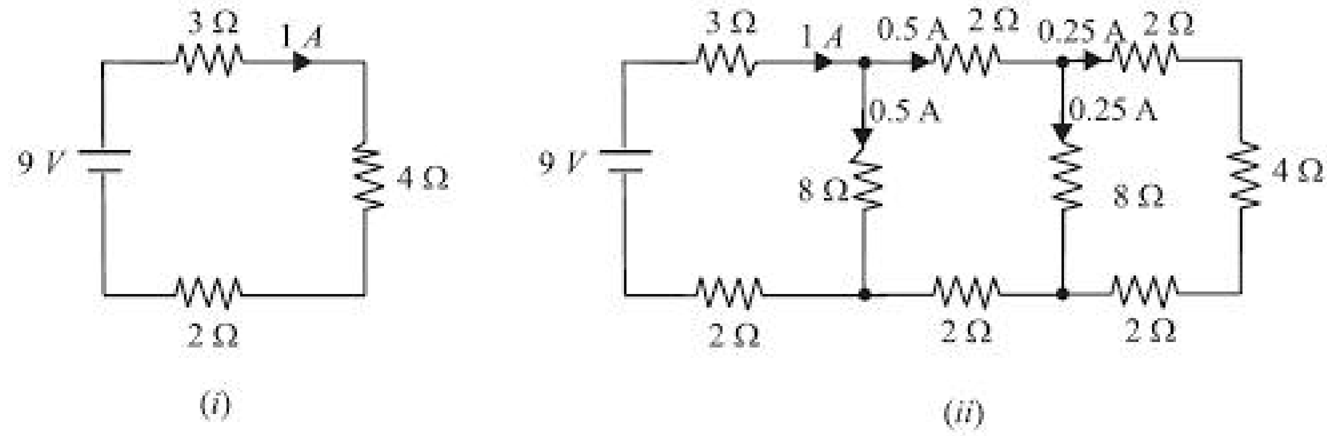



**Fig. 5.95**

Due to symmetry of the circuit, the current distribution will be as shown in Fig. 5.95 (*ii*). Therefore, **current through 4 Ω resistor is 0.25 A**.

**4.** The temperature co-efficient of resistance of iron is positive and that of silicon is negative. Therefore, with the increase in temperature, the **resistance of iron wire increases and that of silicon wire decreases.**

**5.** The slope (= $V/I$) of the graph gives the resistance of the conductor. Since the slope of the graph of $T_1$ is greater, it represents larger resistance. As the resistance of a conductor increases with rise in temperature, therefore, $\mathbf{T_1 > T_2}$.

**6.** $P = \frac{V^2}{R}$. For the same $V$, $P \propto \frac{1}{R}$ or $P \propto \frac{A}{\rho l}$. For the same length, $P \propto A$. Therefore area of $x$-section of the bulb with greater power will be more *i.e.* **100 W bulb will have thicker filament**.

**7.** Since the products of the resistances of the opposite arms of the bridge are equal, the bridge is balanced. Therefore, 10 Ω resistor is ineffective and may be considered as removed. If we replace 10 Ω resistor by 20 Ω resistor, the bridge is still balanced and 20 Ω resistor is ineffective.

**8.** An ideal ammeter has zero resistance while an ideal voltmeter has infinite resistance. Therefore, we can replace ammeter by a conductor and voltmeter may be considered as removed from the circuit.

$$\therefore \quad \text{Total circuit resistance} = 1 + 4 + \frac{2 \times 2}{2 + 2} + 2 = \mathbf{8\ \Omega}$$

**9.** Total circuit resistance $= 1.5 + \frac{5 \times 5}{5 + 5} = 4\Omega$; Current supplied by battery = 20/4 = 5A

Each branch has 5 Ω resistance so that 5/2 = 2.5 A current flows in each branch.

$$V_A - V_P = 3 \times 2.5 = 7.5 \text{ V and } V_A - V_Q = 2 \times 2.5 = 5 \text{ V}$$

$$\therefore \quad V_P - V_Q = (V_A - V_Q) - (V_A - V_P) = 5 - 7.5 = \mathbf{-\,2.5\ V}$$

**10.** Resistance of electric bulb, $R = V^2/P$. Therefore, 40 W bulb will have more resistance than 100 W bulb. When lamps are connected in series, $P = I^2R$. Since both bulbs carry the same current, 40 W bulb will be brighter. When bulbs are connected in parallel, $P = V^2/R$. Since $R$ is less for 100 W bulb, it will be brighter. Therefore, **40W bulb will be brighter in series connection and 100W bulb will be brighter in parallel connection**.

**11.** Potential difference between $P$ and $Q = I\,(4 + 12) = 1.5\,(16) =$ **24 V**

**12.** Under steady state conditions, there will be no current in the branch containing capacitance as shown in Fig. 5.96. Therefore, current $I$ will flow only in the loop $ABCDEFA$.

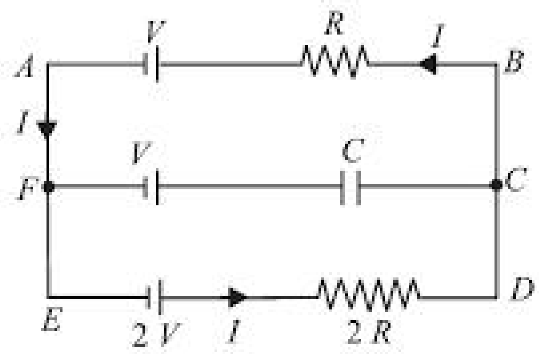



**Fig. 5.96**

$$I = \frac{\text{Net e.m.f.}}{\text{Total ckt. resistance}} = \frac{2V - V}{2R + R} = \frac{V}{3R}$$

$$\text{P.D. across } 2R = I \times 2R = \frac{V}{3R} \times 2R = \frac{2}{3}V$$

$$\therefore \quad \text{P.D. between } F \text{ and } C = 2V - \frac{2}{3}V = \frac{4}{3}V$$

$$\therefore \quad \text{P.D. across capacitor} = \frac{4}{3}V - V = \mathbf{\frac{V}{3}}$$

**13.** $R = \rho\dfrac{l}{A}$ and new resistance $R' = \rho\dfrac{l'}{A'}$.

Since volume remains the same, $lA = l'A'$ or $\dfrac{l'}{l} = \dfrac{A}{A'} = \dfrac{\pi r^2}{\pi (nr)^2} = \dfrac{1}{n^2}$

$$\therefore \quad \frac{R'}{R} = \frac{l'}{l} \times \frac{A}{A'} = \frac{1}{n^2} \times \frac{1}{n^2} = \frac{1}{n^4} \quad \therefore \quad R' = \mathbf{\frac{R}{n^4}}$$

**14.** For metal rod, $R_1 = \rho\dfrac{l_1}{A_1}$; For disc, $R_2 = \rho\dfrac{l_2}{A_2}$ $\therefore$ $\dfrac{R_2}{R_1} = \dfrac{l_2}{l_1} \times \dfrac{A_1}{A_2}$

$$\therefore \quad R_2 = R_1 \times \frac{l_2}{l_1} \times \frac{A_1}{A_2} = (3.0 \times 10^{-3}) \times \left(\frac{1 \times 10^{-3}}{1}\right) \times \frac{\pi \times (0.003)^2}{\pi \times (0.01)^2}$$

$$= \mathbf{2.7 \times 10^{-7}\ \Omega}$$

**15.** It is a Wheatstone bridge. Since the products of resistances of the opposite arms of the bridge are equal, the bridge is balanced. Therefore 1 Ω resistance is ineffective and may be considered as removed from the circuit.

$$\therefore \quad R_{AB} = (2 + 2)\,\Omega \,\|\, (2 + 2)\Omega = 4\Omega \,\|\, 4\Omega = \frac{4 \times 4}{4 + 4} = \mathbf{2\ \Omega}$$

**16.** $R = \rho\dfrac{l}{A}$ and new resistance $R' = \rho\dfrac{l'}{A'}$.

Since the volume remains the same, $lA = l'A'$ or $\dfrac{A}{A'} = \dfrac{l'}{l} = \dfrac{(1.001)\,l}{l} = 1.001$

$$\therefore \quad \frac{R'}{R} = \frac{l'}{l} \times \frac{A}{A'} = 1.001 \times 1.001 = 1.002$$

$$\therefore \quad \frac{R' - R}{R} = 1.002 - 1 = 0.002 = \mathbf{0.2\%}$$

**17.** The resistances of 20 Ω , 30 Ω and 60 Ω are in parallel and their equivalent resistance $R'$ is

$$\frac{1}{R'} = \frac{1}{20}+\frac{1}{30}+\frac{1}{60} \quad \therefore \; R' = 10\Omega$$

The resistances of 24 Ω and 8 Ω are in parallel and their equivalent resistance $R''$ is

$$R'' = \frac{24\times 8}{24+8} = 6\Omega$$

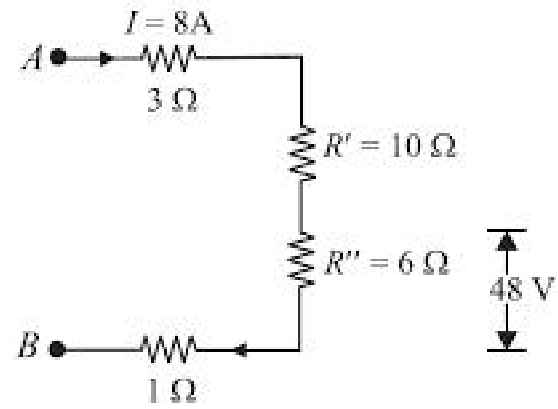


**Fig. 5.97**

The circuit then reduces to the one shown in Fig. 5.97. Now p.d. across $R''$ (= 6 Ω) is 48 V. Therefore, circuit current $I$ = 48/6 = 8 A.

∴ P.D. across $AB$ = 8 × 3 + 8 × 10 + 8 × 6 + 8 × 1 = **160 V**

**18.** This is a Wheatstone bridge with resistance $R_5$ in series with the battery. When Wheatstone bridge is balanced, the resistance $R_6$ is ineffective and may be considered as removed from the circuit. Under this condition the resistance between terminals $A$ and $B$ is the same irrespective of the value of $R_6$. This means that current $I$ supplied by the battery is independent of the value of $R_6$. For Wheatstone bridge to be balanced, the products of resistances of the opposite arms of the bridge should be equal *i.e* $\boldsymbol{R_1R_4 = R_2R_3}$.

**19.** $$\text{Current} = \frac{\text{Charge}}{\text{Time period}} = \text{Charge} \times \text{Frequency} = 1.6 \times 10^{-19} \times 6.8 \times 10^{15}$$

$$= \mathbf{1.1 \times 10^{-3}\ A}$$

**20.** Let $t$°C be the required temperature. Now $R_t = 3\ R_0$.

$$\therefore \quad 3\ R_0 = R_0\ [1 + \alpha\ (t - 0)] \quad \text{or} \quad 3 = 1 + \alpha\ t \quad \therefore \quad t = \frac{3-1}{\alpha} = \frac{3-1}{4\times 10^{-3}} = \mathbf{500\ °C}$$

## NUMERICAL PROBLEMS FOR COMPETITIVE EXAMINATIONS

**1. How many turns of a nichrome wire 1 mm in diameter should be wound around a porcelain cylinder with radius 2.5 cm to obtain a furnace with a resistance of 20 Ω ? Given ρ (for nichrome) = 1.0 × 10⁻⁶ Ω m.**

**Hint.** If $r_1$ is the radius of the porcelain cylinder and $N$ is the required number of turns, then length of the wire is

$$l = 2\pi r_1 N$$

Let $r_2$ be the radius of the wire. Therefore, $A = \pi r_2^2$.

We know that $$R = \rho\frac{l}{A} = \rho \times \frac{2\pi r_1 N}{\pi r_2^2}$$

$$\therefore \quad N = \frac{R r_2^2}{2\rho r_1} = \frac{20\times(0.5\times10^{-3})^2}{2\times(1.0\times10^{-6})\times 2.5\times10^{-2}} = \mathbf{100}$$

**2. Find the resistance of a hollow cylindrical pipe of length 1 m whose inner and outer radii are 10 cm and 20 cm respectively. The specific resistance of the material is 2 × $10^{-8}$ Ω m.**

**Hint.** Outer radius, $r_2 = 0.2$ m; Inner radius, $r_1 = 0.1$ m

$\therefore$ Cross-sectional area, $A = \pi\left[r_2^2 - r_1^2\right] = \pi\,[(0.2)^2 - (0.1)^2] = 9.42 \times 10^{-2}\ m^2$

Length of pipe, $l = 1$ m

Now $$R = \rho\frac{l}{A} = 2\times10^{-8}\times\frac{1}{9.42\times10^{-2}} = 2.1 \times 10^{-7}\ \Omega$$

**3. Find the equivalent resistance between *A* and *B* in Fig. 5.98 (*i*).**

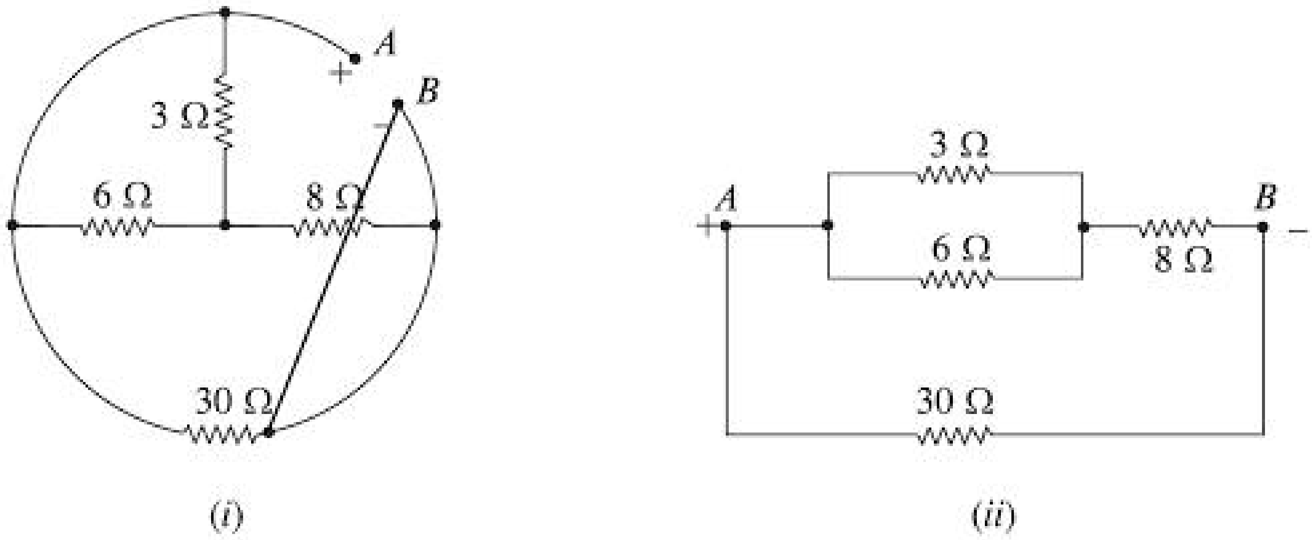



**Fig. 5.98**

**Hint.** The circuit shown in Fig. 5.98 (*i*) is equivalent to the circuit shown in Fig. 5.98 (*ii*). Resistance of upper branch in Fig. 5.98 (*ii*)

$$= \frac{3\times6}{3+6} + 8 = 10\Omega$$

Now between points *A* and *B*, we have resistances of 10 Ω and 30 Ω in parallel.

$$\therefore \quad R_{AB} = \frac{10\times30}{10+30} = 7.5\ \Omega$$

**4. Two cells with the same e.m.f. *E* and different internal resistances $r_1$ and $r_2$ are connected in series to an external resistance *R*. Can a value of *R* be selected such that the p.d. across the first cell should be zero?**

**Hint.** Fig. 5.99 shows the conditions of the problem.

Circuit current, $$I = \frac{2E}{r_1 + r_2 + R}$$

P.D. across first cell, $$V_1 = E - Ir_1 = E - \frac{2Er_1}{r_1 + r_2 + R}$$

It is given that $V_1 = 0$.

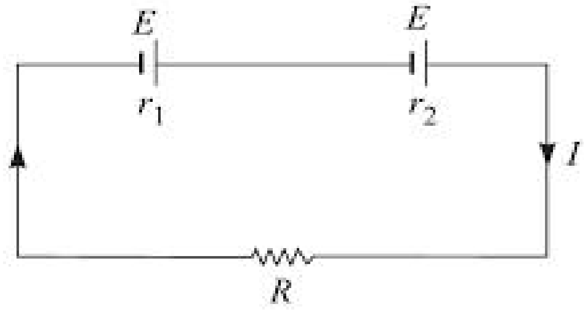



**Fig. 5.99**

$$\therefore \quad 0 = E - \frac{2Er_1}{r_1 + r_2 + R}$$

or $$E = \frac{2Er_1}{r_1 + r_2 + R}$$

or $$r_1 + r_2 + R = 2r_1$$

$$\therefore \quad R = r_1 - r_2$$

Hence p.d. at the terminals of first cell will be zero if $R = r_1 - r_2$.

**5. 12 cells each having the same e.m.f. are connected in series and are kept in a closed box. Some of the cells are wrongly connected. The battery is connected in series with an ammeter and two cells identical with the others. The current is 3 A when the cells and battery aid each other and 2 A when the cells and battery oppose each other. How many cells in the battery are wrongly connected ? [Roorkee]**

**Hint.** Suppose $x$ cells are connected correctly and $y$ cells are connected wrongly. Then,

$$x + y = 12 \qquad \text{... } (i)$$

If $E$ is the e.m.f. of each cell, then,

Net battery e.m.f. $= (x - y)\, E$

Let $R$ be the resistance of the circuit. Note that circuit resistance remains constant.

($i$) When the cells aid the battery, net e.m.f. in the circuit

$$= (x - y)\, E + 2\, E$$

$$\therefore \quad \text{Current} = \frac{\text{Net e.m.f.}}{\text{Resistance}} = \frac{(x-y)\, E + 2E}{R}$$

$$\text{or} \quad 3 = \frac{(x-y)\, E + 2E}{R} \qquad \text{... } (ii)$$

($ii$) When the cells oppose the battery, the net e.m.f.

$$= (x - y)\, E - 2\, E$$

$$\therefore \quad \text{Current} = \frac{(x-y)\, E - 2E}{R}$$

$$\text{or} \quad 2 = \frac{(x-y)\, E - 2E}{R} \qquad \text{... } (iii)$$

Dividing eq. ($ii$) by eq. ($iii$), we get,

$$\frac{3}{2} = \frac{(x-y)\, E + 2E}{(x-y)\, E - 2E}$$

$$\text{or} \quad \frac{3}{2} = \frac{(x-y) + 2}{(x-y) - 2} \qquad \text{... } (iv)$$

From eqs. ($i$) and ($iv$), $x = 11$ and $y = 1$.

**6. Find the potential difference between the plates $A$ and $B$ of the capacitor $C$ in the circuit shown in Fig. 5.100. Internal resistances of the cells are negligible.**

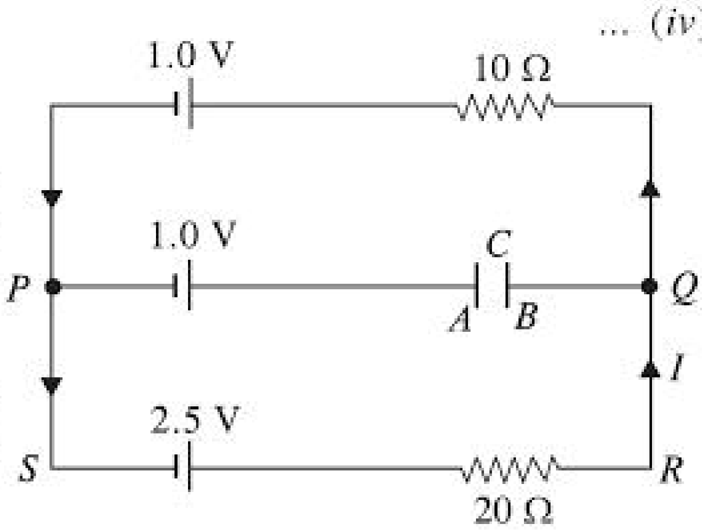



**Fig. 5.100**

**Hint.** In the steady state, the branch containing capacitor $C$ behaves as an open circuit and may be considered as removed from the circuit.

$$\therefore \quad \text{Effective circuit resistance} = 10 + 20 = 30\ \Omega$$

$$\text{Net circuit e.m.f.} = 2.5 - 1.0 = 1.5 \text{ volt}$$

$$\therefore \quad \text{Circuit current, } I = \frac{1.5}{30} \text{ A} = 0.05 \text{ A}$$

Applying Kirchhoff's Voltage Law (KVL) to the loop $PQRSP$, we have,

$$1.0 + (V_B - V_A) + 20 \times (0.05) - 2.5 = 0$$

$$1 + (V_B - V_A) + 1 - 2.5 = 0$$

$$\therefore \qquad V_B - V_A = \mathbf{0.5\ volt}$$

Note that no current flows in the branch $PQ$ of the circuit.

**7. Two coils connected in series have resistances of 600 Ω and 300 Ω and temperature co-efficients of 0.1% and 0.4% respectively at 20° C. Find the resistance of the combination at a temperature of 50°C. What is the effective temperature co-efficient of the combination ?**

**Hint.** Resistance of 600 Ω resistor at 50° C = 600 [1 + 0.001 (50 – 20)] = 618 Ω

Resistance of 300 Ω resistor at 50° C = 300 [1 + 0.004 (50 – 20)] = 336 Ω

∴ Total resistance of the combination at 50° $C$ = 618 + 336 = **954 Ω**

Let β be the effective temperature coefficient of the combination at 20°C.

$$\therefore \qquad 954 = 900\,[1 + \beta\,(50 - 20)]$$

$$\text{or} \qquad \beta = \mathbf{0.002\ per\ {}^\circ C}$$

**8. Two resistances 400 Ω and 800 Ω are connected in series with a battery of 6 V. To measure current in the circuit, an ammeter of 10 Ω resistance is used. What will be the reading of the ammeter? Similarly, if a voltmeter of 10 kΩ resistance is used to measure the potential difference across 400 Ω resistance, find out the reading of voltmeter.**

**Hint.** Fig. 5.101 (*i*) shows the conditions of the problem for current measurement in the circuit.

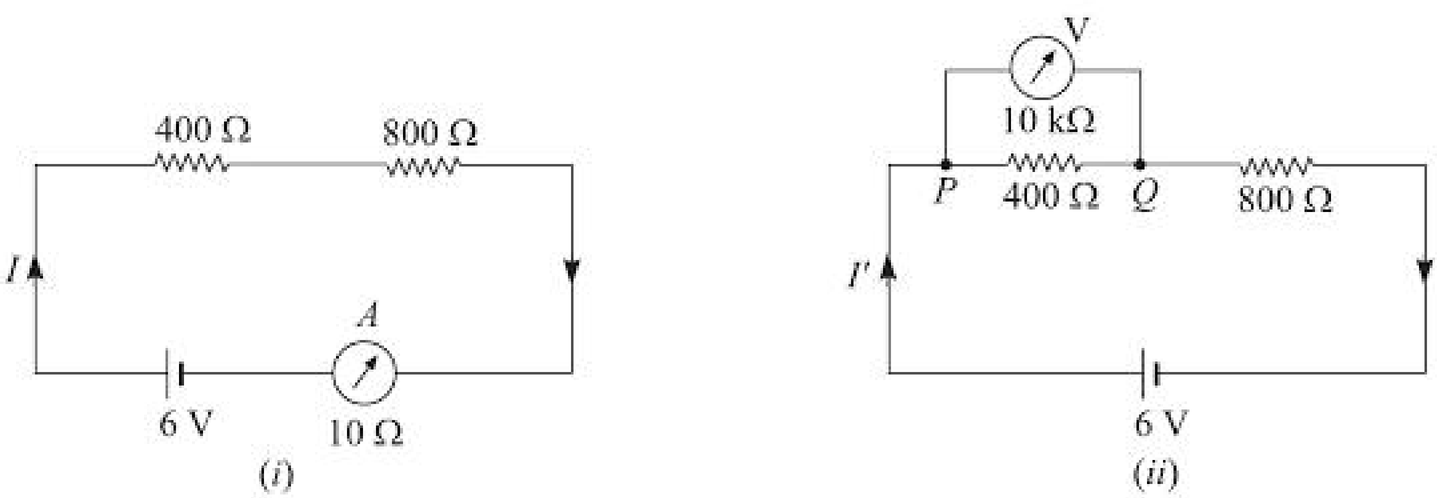



**Fig. 5.101**

Net resistance of the circuit, $R$ = 400 + 800 + 10 = 1210 Ω

$$\therefore \quad \text{Circuit current, } I = \frac{E}{R} = \frac{6}{1210} = 0.00496\text{ A} = \mathbf{4.96\ mA}$$

Fig. 5.101 (*ii*) shows the conditions of the problem for the measurement of potential difference across 400 Ω resistor. Referring to Fig. 5.101 (*ii*),

$$\text{Net circuit resistance, } R' = \frac{400 \times 10{,}000}{400 + 10{,}000} + 800 = 384.6 + 800 = 1184.6\,\Omega$$

$$\therefore \quad \text{Circuit current, } I' = \frac{E}{R'} = \frac{6}{1184.6}$$

$$\text{P.D. across } PQ = I' \times 384.6 = \frac{6}{1184.6} \times 384.6 = 1.95\text{ V}$$

$\therefore$ Reading of voltmeter = **1.95 V**

**9. A current of 2 A flows in a system of conductors shown in Fig. 5.102. What is the potential difference $V_A - V_B$ ? [C.P.M.T.]**

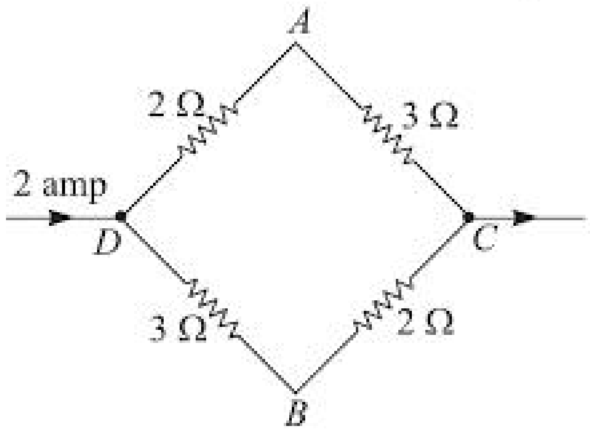



**Fig. 5.102**

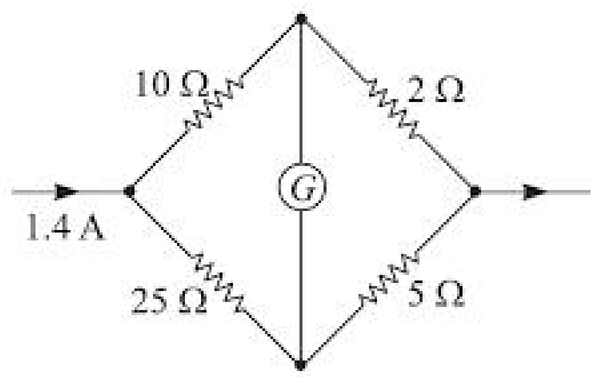



**Fig. 5.103**

**Hint.** Current in branch $DA = 2 \times \dfrac{(3+2)}{(2+3)+(3+2)} = 1\text{ A}$

$\therefore$ Current in branch $DB = 2 - 1 = 1$ A

$$V_D - V_A = 2 \times 1 = 2\text{ V}$$

$$V_D - V_B = 3 \times 1 = 3\text{ V}$$

$\therefore$ $(V_D - V_B) - (V_D - V_A) = 3 - 2 = 1$

or $V_A - V_B =$ **1 V**

**10. What is the value of current in 2 Ω resistor in the circuit shown in Fig. 5.103. [C.P.M.T.]**

**Hint.** The conditions of the problem suggest that it is a balanced Wheatstone bridge. Therefore, current through the galvanometer is zero. Consequently, the branch containing galvanometer may be considered as removed. Note that current in 2 Ω resistor is the same as in 10 Ω resistor.

$\therefore$ Current in 2 Ω resistor $= 1.4 \times \dfrac{(25+5)}{(10+2)+(25+5)} =$ **1 A**

**11. In the circuit shown in Fig. 5.104, find the voltage drop across capacitor $C$.**

**Hint.** In the steady state, no current flows in the branch $PQR$. Therefore, net circuit resistance $= r_1 + r_2$.

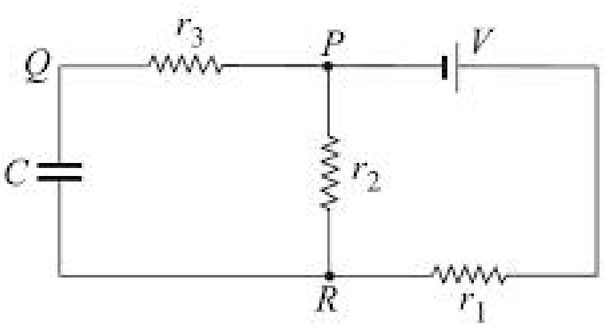



**Fig. 5.104**

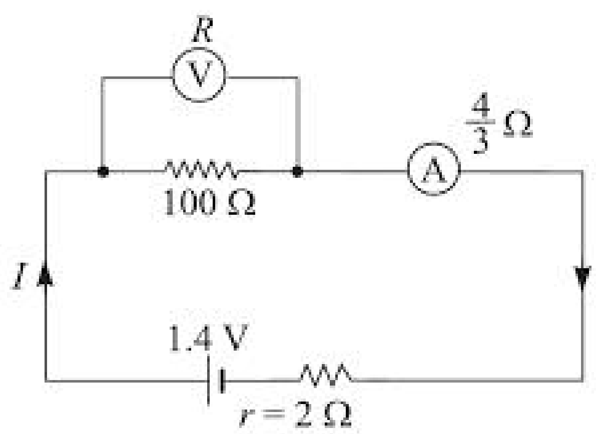



**Fig. 5.105**

Circuit current, $I = \dfrac{V}{r_1 + r_2}$; Voltage across $r_2 = I\, r_2 = \dfrac{V r_2}{r_1 + r_2}$

This is also the voltage across the capacitor.

**12. A battery of e.m.f. 1.4 V and internal resistance 2 Ω is connected to a resistor**

**of 100 Ω through an ammeter. The resistance of ammeter is (4/3) Ω. A voltmeter has also been connected to find the p.d. across the resistor.**

**(*i*) The ammeter reads 0.02 A. What is the resistance of the voltmeter ?**

**(*ii*) The voltmeter reads 1.1 V. What is the error in the reading ?** **[I.I.T.]**

**Hint.** (*i*) Fig. 5.105 shows the desired circuit. Suppose $R$ is the resistance of the voltmeter. Since voltmeter is connected in parallel with 100 Ω resistor, effective resistance $R'$ of 100 Ω and voltmeter is

$$R' = \frac{100 \times R}{100 + R} = \frac{100R}{100 + R}$$

$$\text{Total circuit resistance, } R_T = 2 + \frac{4}{3} + \frac{100R}{100 + R} = \frac{10}{3} + \frac{100R}{100 + R}$$

$$\therefore \quad \text{Circuit current, } I = \frac{E}{R_T} = \frac{1.4}{(10/3) + (100R/100 + R)}$$

Since reading of ammeter is 0.02 A, $I = 0.02$ A.

$$\therefore \quad 0.02 = \frac{1.4}{(10/3) + (100R/100 + R)}$$

On solving, $R$ = **200 Ω**

(*ii*) $$\therefore \quad R' = \frac{100R}{100 + R} = \frac{100 \times 200}{100 + 200} = \frac{200}{3}\,\Omega$$

$$\text{P.D. across voltmeter, } V = IR' = 0.02 \times \frac{200}{3} = 1.33 \text{ V}$$

$\therefore$ Error in voltmeter reading = 1.33 − 1.1 = **0.23 V**

# 6

# Electrical Measurements

## INTRODUCTION

So far we have come across many electrical quantities, *e.g.*, resistance, electric current, voltage, etc. In order to have complete understanding about the circuits, it is important that we should be able to measure the electrical quantities involved. For this purpose, electrical instruments are used. The accuracy, convenience and reliability are mainly responsible for the widespread use of electrical methods of measurements. In this chapter, we shall confine our attention to the measurement of a few electrical quantities.

## 6.1. COMPLEX CIRCUITS

Sometimes we encounter circuits where simplification by series and parallel combinations is impossible. Consequently, Ohm's law cannot be applied to solve such circuits. This happens when there is more than one source of e.m.f. in the circuit or when resistors are connected in a complicated manner. Such circuits are called *complex circuits*. We shall discuss two such circuits by way of illustration.

(*i*) Fig. 6.1 shows a circuit containing two sources of e.m.f. $E_1$ and $E_2$ and three resistors. This circuit cannot be solved by series-parallel combinations. Are resistors $R_1$ and $R_3$ in parallel? Not quite, because there is an e.m.f. source $E_1$ between them. Are they in series? Not quite, because same current does not flow in them.

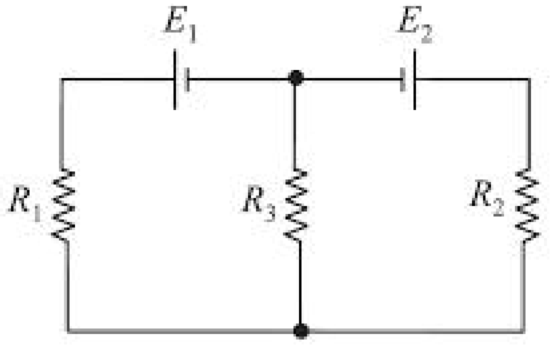



**Fig. 6.1**

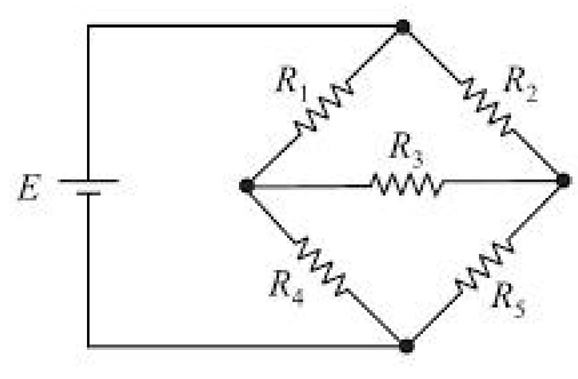



**Fig. 6.2**

(*ii*) Fig. 6.2 shows another circuit where we cannot solve the circuit by using series and parallel combinations. Though this circuit contains one source of e.m.f. ($E$), it cannot be solved by using series and parallel combinations. Thus, resistors $R_1$ and $R_2$ are neither in series nor in parallel, same is true for other pair of resistors.

In order to solve such complex circuits, Gustav Kirchhoff gave two laws, known as Kirchhoff's laws.

## 6.2. KIRCHHOFF'S LAWS

To solve complicated circuits, Kirchhoff gave two simple laws, called Kirchhoff's laws. These laws are simply the applications of the laws of conservation of charge and energy.

1. Kirchhoff's Current Law (KCL) 2. Kirchhoff's Voltage Law (KVL)

**1. Kirchhoff's Current Law (KCL) or Kirchhoff's Junction Rule.** This law is based on the conservation of charge and may be stated as under :

*The algebraic sum of the currents meeting at a *junction in an electrical circuit is zero.*

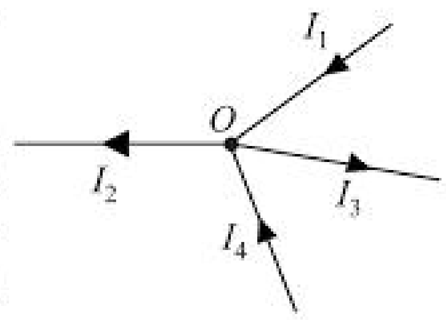


Fig. 6.3

An algebraic sum is one in which the sign of the quantity is taken into account. For example, consider four conductors carrying currents $I_1$, $I_2$, $I_3$ and $I_4$ and meeting at point $O$ as shown in Fig. 6.3. If we take the signs of currents flowing towards point $O$ as positive, then currents flowing away from point $O$ will be assigned negative sign. Thus, applying Kirchhoff's current law to the junction $O$ in Fig. 6.3, we have,

$$(I_1) + (I_4) + (-I_2) + (-I_3) = 0$$

or
$$I_1 + I_4 = I_2 + I_3$$

*i.e.*, Sum of incoming currents = Sum of outgoing currents

Hence, Kirchhoff's current law may also be stated as under :

*The sum of currents flowing towards any junction in an electrical circuit is equal to the sum of currents flowing away from that junction.* Kirchhoff's current law is rightly called the **junction rule**.

Kirchhoff's current law is true because electric current is merely the flow of free electrons and they cannot accumulate at any point in the circuit. This is in accordance with the law of conservation of charge. Hence, Kirchhoff's current law is based on the law of conservation of charge.

**2. Kirchhoff's Voltage Law (KVL) or Kirchhoff's Loop Rule.** This law is based on the conservation of energy and may be stated as under :

*In any closed electrical circuit or loop, the algebraic sum of all the electromotive forces (e.m.f.s) and voltage drops in resistors is equal to zero, i.e.*, in any closed circuit or loop,

*Algebraic sum of e.m.f.s + Algebraic sum of voltage drops* = 0

Note that a closed electrical circuit is called a **loop.**

The validity of Kirchhoff's voltage law can be easily established by referring to the loop $ABCDA$ shown in Fig. 6.4. If we start from any point (say point $A$) in this closed circuit and go back to this point (*i.e.*, point $A$) after going round the circuit, then there is no increase or decrease in potential. This means that algebraic sum of the e.m.f.s of all the sources (here only one e.m.f. source is considered) met on the way *plus* the algebraic sum of the voltage drops in the resistances must be zero. Kirchhoff's voltage law is based on the law of **conservation of energy, *i.e.*, net change in the energy of a charge after completing the closed path is zero. Kirchhoff's voltage law is also called **loop rule.**

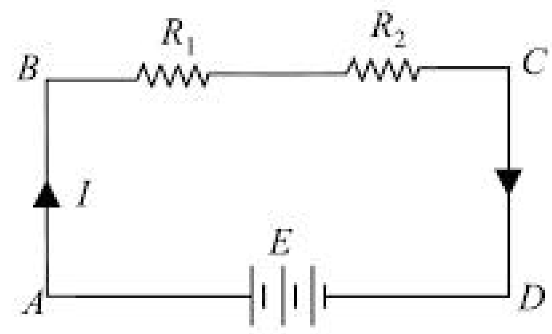


Fig. 6.4

* A junction is that point in an electrical circuit where *three or more* circuit elements meet.

** As a charge traverses a loop and returns to the starting point, the sum of rises of electric potential energy associated with e.m.f.s in the loop must be equal to the sum of the drops of electric potential energy associated with resistors.

## 6.3. SIGN CONVENTION

While applying Kirchhoff's voltage law to a closed circuit, algebraic sums are considered. Therefore, it is very important to assign proper signs to e.m.f.s and voltage drops in the closed circuit. The following sign convention may be followed :

*A **rise in potential** should be considered **positive** and **fall in potential** should be considered **negative**.*

(*i*) Thus in Fig. 6.5 (*i*), *as we go from A to B (i.e., from negative terminal of the cell to the positive terminal), there is a rise in potential.* In Fig. 6.5 (*ii*), as we go from *A* to *B*, there is also a rise in potential.

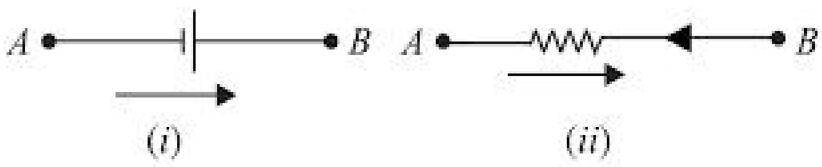



**Fig. 6.5**

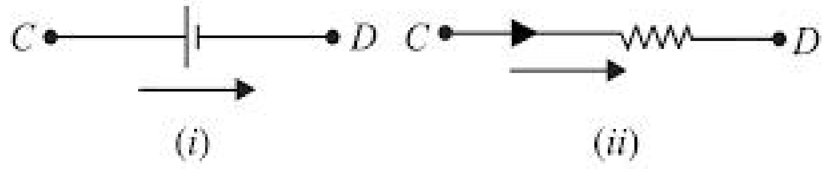



**Fig. 6.6**

(*ii*) *In Fig. 6.6 (i), as we go from C to D, there is a fall in potential.* In Fig. 6.6 (*ii*), as we go from *C* to *D*, there is again a fall in potential.

## 6.4. ILLUSTRATION OF KIRCHHOFF'S LAWS

Kirchhoff's laws can be beautifully explained by referring to Fig. 6.7. Mark the directions of currents as indicated. The directions in which currents are assumed to flow is unimportant, since if wrong direction is chosen, it will be indicated by the negative sign in the final result.

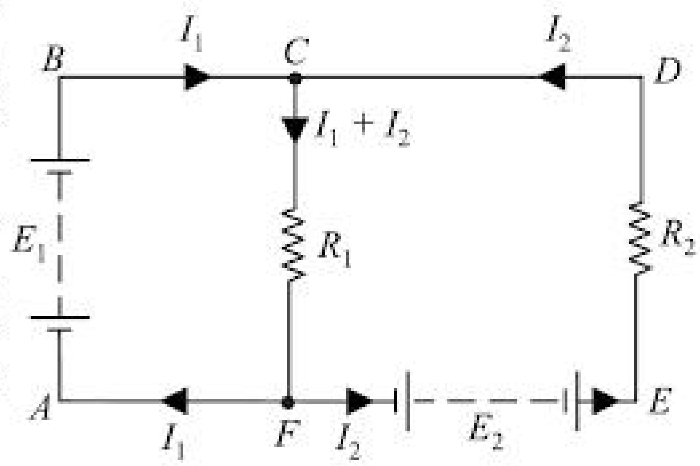



**Fig. 6.7**

(*i*) The magnitude of current in any branch of the circuit can be found by applying Kirchhoff's current law. Thus at point *C* in Fig. 6.7, the incoming currents to the junction *C* are $I_1$ and $I_2$. Obviously, the current in branch *CF* will be $I_1 + I_2$.

(*ii*) There are three loops in Fig. 6.7, *viz*., *ABCFA*, *CDEFC* and *ABCDEFA*. Since there are only two unknown quantities (*i.e.*, $I_1$ and $I_2$), we need only two equations in terms of $I_1$ and $I_2$. This can be achieved by applying Kirchhoff's voltage law to any two loops.

**Loop *ABCFA*.** In this loop, e.m.f. $E_1$ will be given *positive* sign. It is because as we consider the loop in the order *ABCFA*, we go from the negative terminal to the positive terminal of the battery in the branch *AB* and, hence, there is rise in potential. The voltage drop in branch *CF* is $(I_1 + I_2)\, R_1$ and shall bear *negative* sign. It is because as we consider the loop in the order *ABCFA*, we go with the current in the branch *CF* and there is a fall in potential. Applying Kirchhoff's voltage law to the loop *ABCFA*, we have,

$$E_1 - (I_1 + I_2)\, R_1 = 0$$

or
$$E_1 = (I_1 + I_2)\, R_1 \qquad ...(i)$$

**Loop *CDEFC*.** As we go round the loop in the order *CDEFC*, drop $I_2 R_2$ is *positive*, e.m.f. $E_2$ is *negative* and drop $(I_1 + I_2)\, R_1$ is *positive*. Therefore, applying Kirchhoff's voltage law to this loop, we get,

$$I_2 R_2 - E_2 + (I_1 + I_2)\, R_1 = 0$$

or
$$I_2 R_2 + (I_1 + I_2)\, R_1 = E_2 \qquad ...(ii)$$

Since $E_1$, $E_2$, $R_1$ and $R_2$ are known, we can find the values of $I_1$ and $I_2$ from the above two equations. Hence current in all the branches can be determined.

## 6.5. STEPS TO SOLVE CIRCUITS BY KIRCHHOFF'S LAWS

(*i*) Assume unknown currents in the given circuit and show their directions by arrows.

(*ii*) Choose any loop and find the algebraic sum of voltage drops plus the algebraic sum of e.m.f.s in that loop and put it equal to zero.

(*iii*) Write equations for as many loops as the number of unknown quantities. Solve the equations to find the unknown quantities.

(*iv*) If the value of assumed current comes to be negative, it means that actual direction of current is opposite to that of assumed direction.

## 6.6. WHEATSTONE BRIDGE

*The Wheatstone bridge is a circuit which is used to measure accurately an unknown resistance.* It was first proposed by Wheatstone.

Wheatstone bridge consists of four resistors (two fixed known resistances $P$ and $Q$, a known variable resistance $R$ and the unknown resistance $X$ whose value is to be found out) connected to form a diamond-shaped circuit $ABCDA$ as shown in Fig. 6.8. Across one pair of opposite junctions ($A$ and $C$), a battery is connected through key $K_1$ and across the other opposite pair of junctions ($B$ and $D$), a galvanometer is connected through a key $K$. The circuit is called a bridge because galvanometer bridges the opposite junctions $B$ and $D$. The keys $K_1$ and $K$ are closed and the value of $R$ (variable) is varied till galvanometer shows no *deflection (null point). Under such conditions, the bridge is said to be **balanced.** *Note that when no current flows through the galvanometer* ($G$), *the bridge is said to be balanced.* This method is also called *null method.*

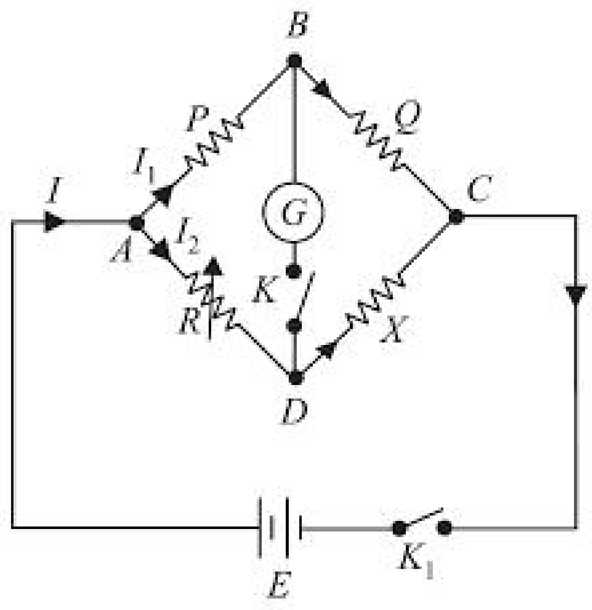



**Fig. 6.8**

**Principle.** *Wheatstone bridge principle states that when the bridge is balanced, the products of the resistances of the opposite arms are equal, i.e.,*

$$PX = QR \quad \text{or} \quad X = \frac{Q}{P} \times R$$

Since the values of $Q$, $P$ and $R$ are known, the value of the unknown resistance $X$ can be calculated.

**Proof.** Let under balanced conditions of the bridge :

$I_1$ = current through $P$ ; $I_2$ = current through $R$

Since there is no current in the galvanometer,

$\therefore$ Current through $Q = I_1$ and Current through $X = I_2$

Since points $B$ and $D$ are at the same potential (because there is no current in the galvanometer),

$$\text{P.D. across } P = \text{P.D. across } R$$

or $$I_1 P = I_2 R \qquad \ldots(i)$$

Also $$\text{P.D. across } Q = \text{P.D. across } X$$

or $$I_1 Q = I_2 X \qquad \ldots(ii)$$

Dividing $\left[\frac{(i)}{(ii)}\right]$, we get, $$\frac{P}{Q} = \frac{R}{X}$$

or $$PX = QR \qquad \ldots(iii)$$

---

* *i.e.*, current in the galvanometer is zero.

Note that exp. (*iii*) is true only under balanced conditions of the bridge *i.e.* when the galvanometer current is zero.

**Note.** The voltmeter-ammeter method is not recommended for accurate measurement of resistance because most meters have limited accuracy. However, Wheatstone bridge requires no calibrated meters.

## 6.7. MEASUREMENT OF TEMPERATURE BY WHEATSTONE BRIDGE

We know that resistance of a conductor increases with increase in temperature. A platinum wire wound non-inductively on a mica former is connected in the arm *CD* of Wheatstone bridge as shown in Fig. 6.9.

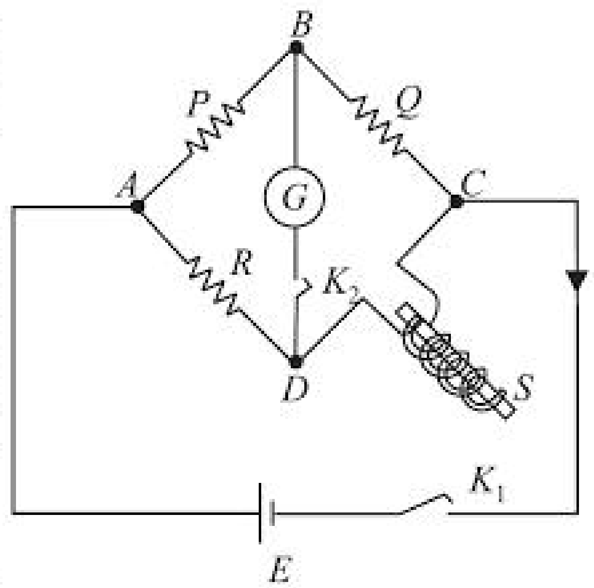



**Fig. 6.9**

(*i*) The platinum wire is immersed in ice at 0°C. Making $P = Q$, the value of $R$ is adjusted so that on closing keys $K_1$ and $K_2$, the galvanometer gives zero reading. Since the values of resistances in the three arms *AB*, *BC* and *AD* are known, the value $R_0$ of the platinum wire at 0°C can be found using Wheatstone bridge formula.

(*ii*) The platinum wire is then maintained at 100°C by placing the wire in steam. Making $P = Q$, the value of $R$ is so adjusted that on closing the keys $K_1$ and $K_2$, the galvanometer gives zero reading. Using Wheatstone bridge formula, we can find the resistance $R_{100}$ of the platinum wire at 100°C.

(*iii*) The platinum wire is then placed in the bath whose temperature $t$°C is to be determined. Following the above procedure, the resistance $R_t$ of the platinum wire at $t$°C can be found.

**Calculations.** If $\alpha$ is the temperature coefficient of resistance of platinum wire, then,

$$R_{100} = R_0\,(1 + \alpha \times 100) \text{ or } \alpha = \frac{R_{100} - R_0}{R_0 \times 100} \qquad \text{... }(i)$$

Also

$$R_t = R_0\,(1 + \alpha \times t) \text{ or } \alpha = \frac{R_t - R_0}{R_0 \times t} \qquad \text{... }(ii)$$

From eqs. (*i*) and (*ii*), we have,

$$\frac{R_t - R_0}{R_0 \times t} = \frac{R_{100} - R_0}{R_0 \times 100}$$

or

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100\,\text{M}$$

Since the values of $R_0$, $R_t$ and $R_{100}$ are known, the unknown temperature $t$ can be determined.

**Example 6.1. Two batteries $E_1$ and $E_2$ having e.m.f.s of 6V and 2V respectively and internal resistances of 2Ω and 3Ω respectively are connected in parallel across a 5Ω resistor. Calculate (*i*) current through each battery and (*ii*) terminal voltage.**

**Solution.** Fig. 6.10 shows the conditions of the problem. Mark the currents in the various branches according to Kirchhoff's current law. Since there are two unknown quantities $I_1$ and $I_2$, two loops will be considered.

**(*i*) Loop *HBCDEFH*.** Applying Kirchhoff's voltage law to loop *HBCDEFH*, we get,

$$2I_1 - 6 + 2 - 3I_2 = 0$$

or

$$2I_1 - 3I_2 = 4 \qquad \text{...}(i)$$

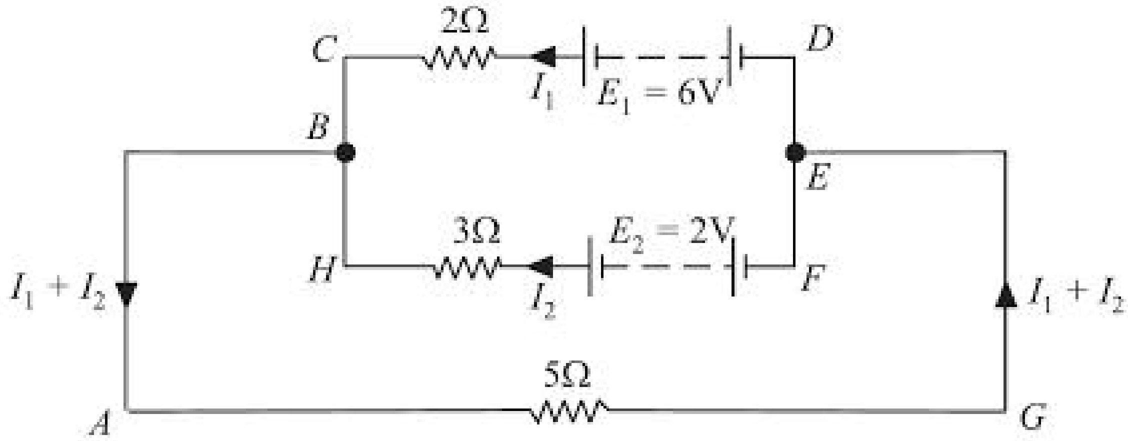



**Fig. 6.10**

**Loop *ABHFEGA*.** Applying Kirchhoff's voltage law to the loop *ABHFEGA*, we get,

$$3I_2 - 2 + 5\,(I_1 + I_2) = 0$$

or $$5I_1 + 8I_2 = 2 \qquad ...(ii)$$

Multiplying eq. (*i*) by 8 and eq. (*ii*) by 3 and then adding them, we get,

$$31I_1 = 38 \quad \text{or} \quad I_1 = 38/31 = \mathbf{1.23\ A}$$

*i.e.*, battery $E_1$ is being discharged at 1.23A. Substituting $I_1$ = 1.23A in eq. (*i*), we get, $I_2$ = **– 0.52 A** *i.e.*, battery $E_2$ is being charged at 0.52A.

(*ii*) Terminal voltage = $(I_1 + I_2)\,5 = (1.23 - 0.52) \times 5 =$ **3.55 V**

**Example 6.2. The Wheatstone bridge circuit have the resistances in various arms as shown in Fig. 6.11. Calculate the current through the galvanometer.**

**Solution.** Mark the currents in the various branches according to Kirchhoff's current law. Since there are three unknown quantities $I_1$, $I_2$ and $I_g$, three loops will be considered.

**Loop *ABDA*.** Applying Kirchhoff's voltage law to the loop *ABDA*, we get,

$$-100\,I_1 - 15\,I_g + 60\,I_2 = 0$$

or $$20\,I_1 + 3\,I_g - 12\,I_2 = 0 \qquad ...(i)$$

**Loop *BCDB*.** Applying Kirchhoff's voltage law to the loop *BCDB*, we get,

$$-10\,(I_1 - I_g) + 5\,(I_2 + I_g) + 15\,I_g = 0$$

or $$2\,I_1 - 6\,I_g - I_2 = 0 \qquad ...(ii)$$

**Loop *ADCEA*.** Applying Kirchhoff's voltage law to the loop *ADCEA*, we get,

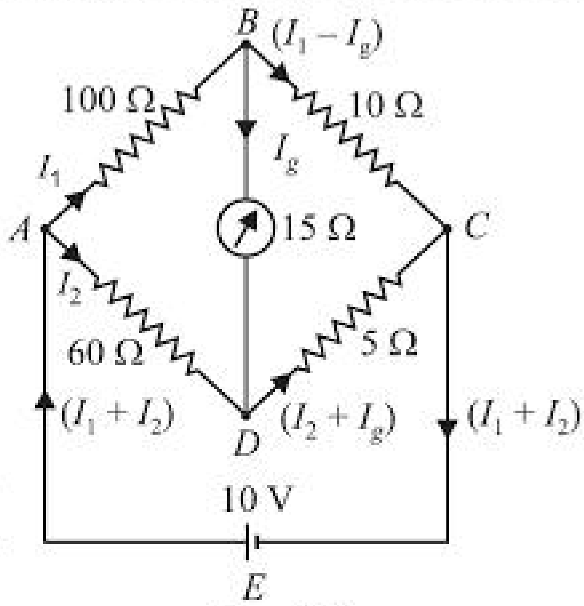



**Fig. 6.11**

$$-60\,I_2 - 5\,(I_2 + I_g) + 10 = 0$$

or $$13\,I_2 + I_g = 2 \qquad ...(iii)$$

On solving eqs. (*i*), (*ii*) and (*iii*), we get, $I_g$ = **4.87 mA**

**Example 6.3. Twelve wires, each of resistance *r*, are connected to form a skeleton cube. Find the equivalent resistance between the two diagonally opposite corners of the cube.**

**Solution.** Let *ABCDEFGH* be the skeleton cube formed by joining 12 wires, each of resistance *r* as shown in Fig. 6.12. Suppose a current of 6*I* enters the cube at the corner *A*. Since the resistance of each wire is the same, the current at corner *A* is divided into three equal parts; 2*I* flowing in *AE*, 2*I* flowing in *AB* and 2*I* flowing in *AD*. At points *B*, *D* and *E*, these currents are divided into equal parts, each part being equal to *I*. Applying Kirchhoff's current law, 2*I* current flows in each of the wires *CG*, *HG* and *FG*. These three currents add up at the corner *G* so that current flowing out of this corner is 6*I*.

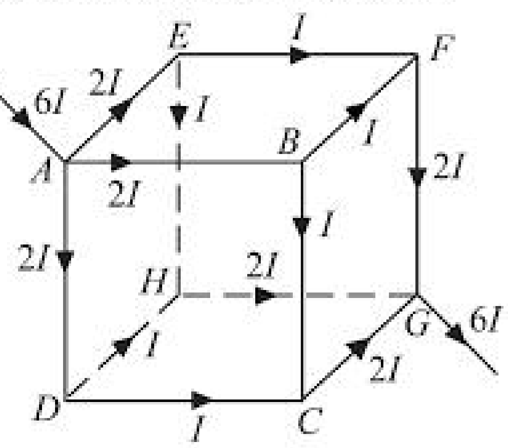



**Fig. 6.12**

Let *E* = e.m.f. of the battery connected to corners *A* and *G* of the cube; corner *A* being connected to the +ve terminal. Now consider any closed circuit between corners *A* and *G*, say the closed circuit $AEFGA$. Applying Kirchhoff's voltage law to the closed circuit $AEFGA$, we have,

$$-2I\,r - Ir - 2Ir = -E \text{ or } 5Ir = E \qquad ...(i)$$

Let $R$ be the equivalent resistance between the diagonally opposite corners $A$ and $G$.

$$\text{Then, } E = 6I\,R \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), $6I\,R = 5I\,r$ or $R = \mathbf{(5/6)\,r}$

**Example 6.4. Two cells of e.m.f. 2V and 1 V and internal resistance 1 Ω and 2 Ω respectively have their positive terminals connected by a wire of 10 Ω resistance, and their negative terminals by a wire of 4 Ω resistance. A resistance of 10 Ω joins the mid-points of these two wires. Calculate the current through each cell and the potential difference between the ends of the wire.**

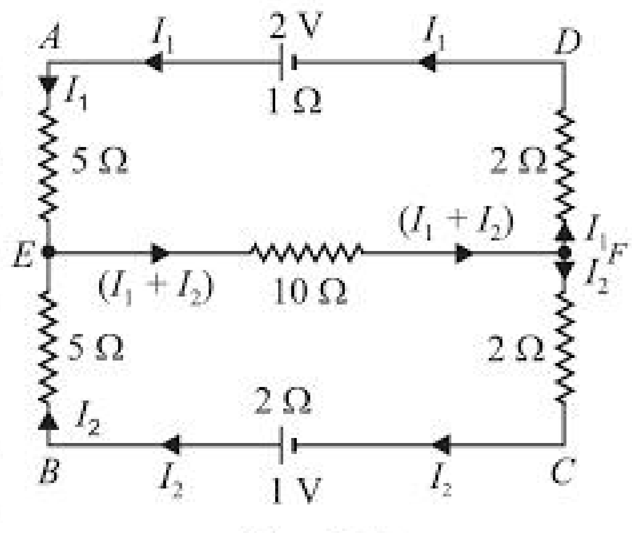



**Fig. 6.13**

**Solution.** Fig. 6.13 shows the conditions of the problem. Mark the currents in the various branches according to Kirchhoff's current law. Since there are two unknown quantities $I_1$ and $I_2$, two loops will be considered.

**Loop *AEFDA*.** Applying Kirchhoff's voltage law to the loop $AEFDA$, we get,

$$-5\,I_1 - 10\,(I_1 + I_2) - 2\,I_1 - 1\,I_1 + 2 = 0$$

or

$$9\,I_1 + 5\,I_2 = 1 \qquad ...(i)$$

**Loop *EFCBE*.** Applying Kirchhoff's voltage law to the loop $EFCBE$, we get,

$$-10\,(I_1 + I_2) - 2\,I_2 - 2\,I_2 + 1 - 5\,I_2 = 0$$

or

$$10\,I_1 + 19\,I_2 = 1 \qquad ...(ii)$$

Solving eqs. (*i*) and (*ii*), we get, $I_2 = \mathbf{-\frac{1}{121}\,A}$ and $I_1 = \mathbf{\frac{14}{121}\,A}$

∴ Total current through 10 Ω wire is

$$I = I_1 + I_2 = \frac{14}{121} + \left(-\frac{1}{121}\right) = \frac{13}{121}\text{ A}$$

∴ Potential difference across 10 Ω wire

$$= I \times 10 = \frac{13}{121} \times 10 = \mathbf{1.074\ V}$$

**Example 6.5. Twelve wires, each of resistance *r*, are connected to form a skeleton cube. Find the resistance between the two corners of same edge of the cube.**

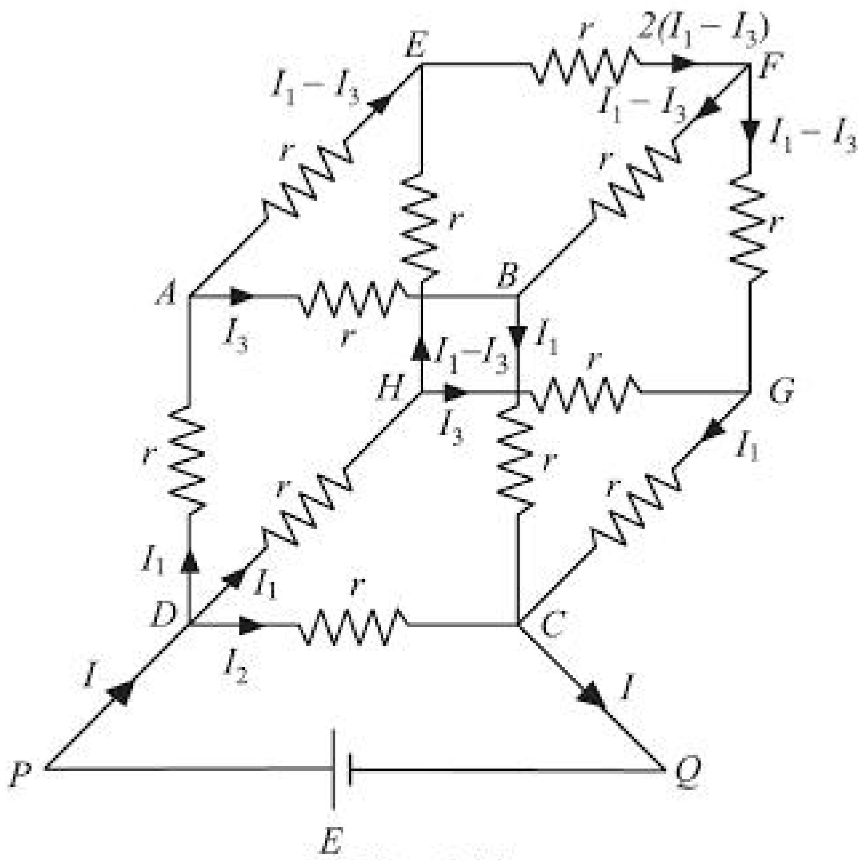



**Fig. 6.14**

**Solution.** Fig. 6.15 shows the conditions of the problem. Let $I_1$, $I_2$ and $I_3$ be the currents through the cells $E_1$, $E_2$ and $E_3$ respectively. Applying Kirchhoff's current law to the junction A, we get,

$$I_1 + I_2 + I_3 = 0$$

or $$I_3 = -(I_1 + I_2) \qquad \text{... }(i)$$

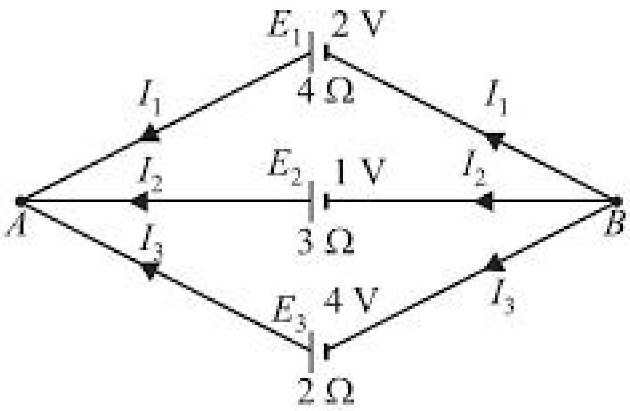



**Fig. 6.15**

**Loop $BE_1AE_2B$.** Applying Kirchhoff's voltage law to the loop $B\,E_1\,A\,E_2\,B$, we get,

$$-4\,I_1 + 2 + 3\,I_2 - 1 = 0$$

or $$4\,I_1 - 3\,I_2 = 1 \qquad \text{... }(ii)$$

**Loop $BE_1AE_3B$.** Applying Kirchhoff's voltage law to the loop $BE_1\,AE_3\,B$, we get,

$$-4\,I_1 + 2 + 2\,I_3 - 4 = 0$$

or $$3\,I_1 + I_2 = -1 \qquad \text{...}(iii)$$

Solving eqs. (*i*), (*ii*) and (*iii*), we get, $I_1 = -\dfrac{2}{13}$ **A** ; $I_2 = -\dfrac{7}{13}$ **A** ; $I_3 = \dfrac{9}{13}$ **A**

**Example 6.7. Four resistances of 15Ω, 12Ω, 4Ω and 10Ω respectively are connected in cyclic order to form a Wheatstone bridge network. Is the network balanced? If not, calculate the resistance to be connected in parallel with the resistance of 10Ω to balance the network.**

**Solution.** Fig. 6.16 shows the conditions of the problem. The bridge will be balanced if the products of opposite arms of the bridge are equal. Since this condition is not satisfied in the given network, the bridge is **unbalanced**.

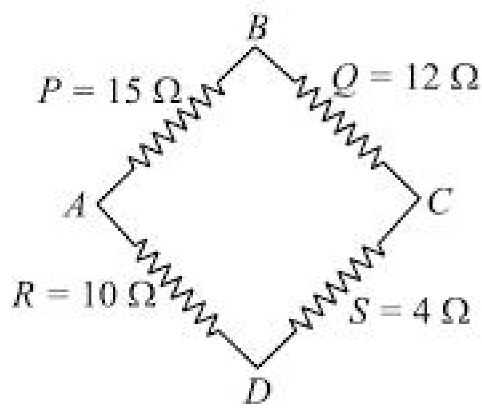



**Fig. 6.16**

Let $X$ be the resistance to be connected in parallel with 10Ω resistance to balance the bridge. The total resistance $R'$ between points $A$ and $D$ is

$$R' = \frac{10X}{10 + X}$$

Since the bridge is balanced, we have,

$$R' \times 12 = 15 \times 4 \quad \text{or} \quad R' = (15 \times 4)/12 = 5\Omega$$

$$\therefore \qquad 5 = \frac{10X}{10 + X} \quad \text{or} \quad 50 + 5X = 10\,X \quad \therefore \quad X = 50/5 = \mathbf{10\Omega}$$

**Example 6.8. Find the value of unknown resistance *X* in the circuit shown in Fig. 6.17 if no current flows through the section *AO*. Also calculate the current drawn by the circuit from the battery of e.m.f. 6V and negligible internal resistance.**

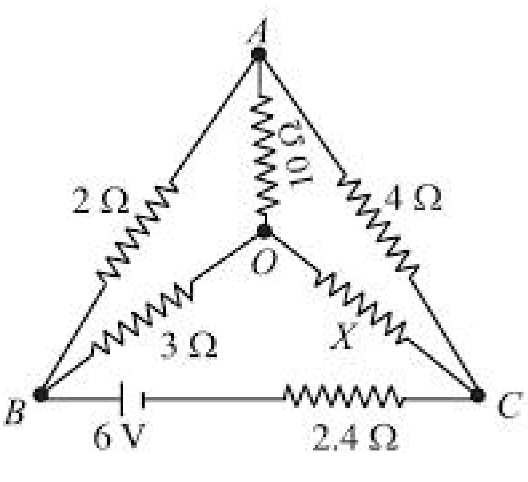



**Fig. 6.17**

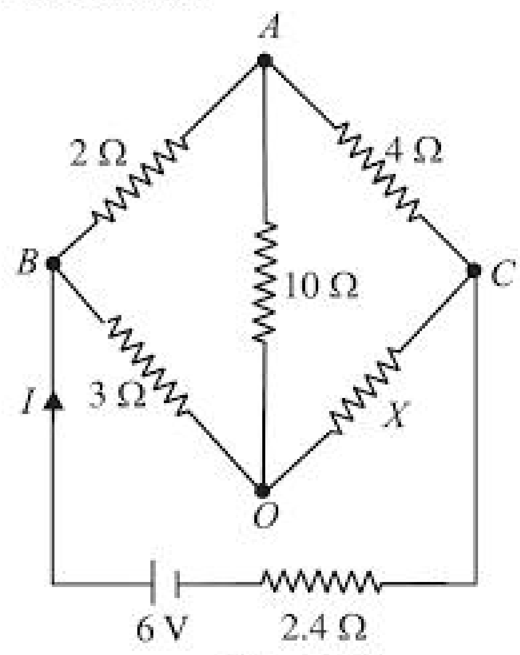



**Fig. 6.18**

**Solution.** The circuit shown in Fig. 6.17 can be redrawn as shown in Fig. 6.18. Since no current flows through the section $AO$, the bridge is balanced. Therefore, the products of the opposite arms of the bridge are equal *i.e.*

$$2 \times X = 4 \times 3 \quad \text{or} \quad X = \frac{4 \times 3}{2} = 6\Omega$$

Since no current flows through 10Ω resistance, the 10Ω resistance is ineffective and can be removed. The circuit then becomes as shown in Fig. 6.19. Referring to Fig. 6.19, the resistance of branch $BAC$ (= 2 + 4 = 6Ω) is in parallel with resistance of branch $BOC$ (3 + 6 = 9Ω). Therefore, the total resistance $R_T$ of the circuit is

$$R_T = \frac{6 \times 9}{6 + 9} + 2.4 = 6\Omega$$

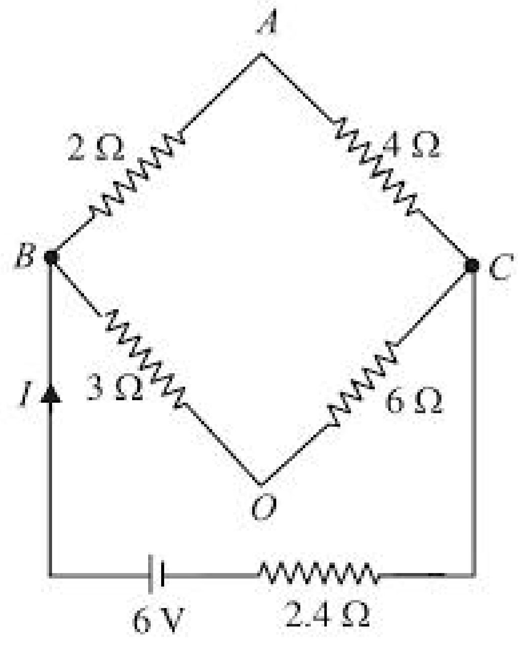



**Fig. 6.19**

$$\therefore \quad \text{Current drawn from battery, } I = \frac{E}{R_T} = \frac{6}{6} = 1\text{A}$$

**Example 6.9. Determine current in each branch of the network shown in Fig. 6.20.**

**Solution.** Mark the currents in the various branches according to Kirchhoff's current law. Since there are three unknown quantities $I_1$, $I_2$ and $I_3$, three loops will be considered.

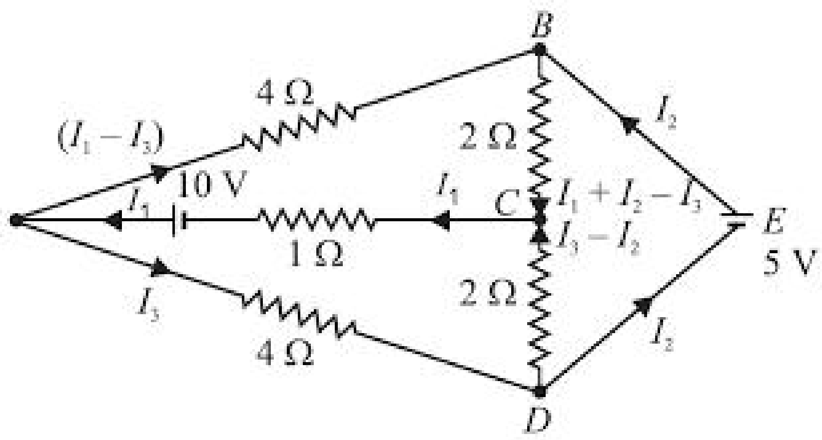



**Fig. 6.20**

**Loop *ABCA*.** Applying Kirchhoff's voltage law to the loop $ABCA$, we get,

$$-4(I_1 - I_3) - 2(I_1 + I_2 - I_3) - 1\, I_1 + 10 = 0$$

$$\text{or} \qquad 7 I_1 + 2 I_2 - 6 I_3 = 10 \qquad \ldots (i)$$

**Loop *ADCA*.** Applying Kirchhoff's voltage law to the loop $ADCA$, we get,

$$-4 I_3 - 2(I_3 - I_2) - 1\, I_1 + 10 = 0$$

$$\text{or} \qquad I_1 - 2 I_2 + 6 I_3 = 10 \qquad \ldots (ii)$$

**Loop *ABEDA*.** Applying Kirchhoff's voltage law to the loop $ABEDA$, we get,

$$-4(I_1 - I_3) - 5 + 4 I_3 = 0$$

$$\text{or} \qquad 4 I_1 - 8 I_3 = -5 \qquad \ldots (iii)$$

Solving eqs. (*i*), (*ii*) and (*iii*), we get, $I_1$ = 2.5 A ; $I_2$ = 1.875 A ; $I_3$ = 1.875 A

**Example 6.10. Find the equivalent resistance between points *X* and *Y* of the network of resistors shown in Fig. 6.21. Each resistor is of resistance *r*.**

**Solution.** In order to find the equivalent resistance between points $X$ and $Y$, we connect a battery of e.m.f. $E$ between points $X$ and $Y$ as shown in Fig. 6.22. Let the current drawn from the battery be $I$. At point $H$, the current $I$ is divided into three paths *viz.*, $HAB$, $HO$ and $HGF$. By symmetry, the same current $I_1$ flows through the paths $HAB$ and $HGF$ (See Fig. 6.22). Therefore, according to Kirchhoff's current law, the current through branch $HO$ is $I–2I_1$. By symmetry, the

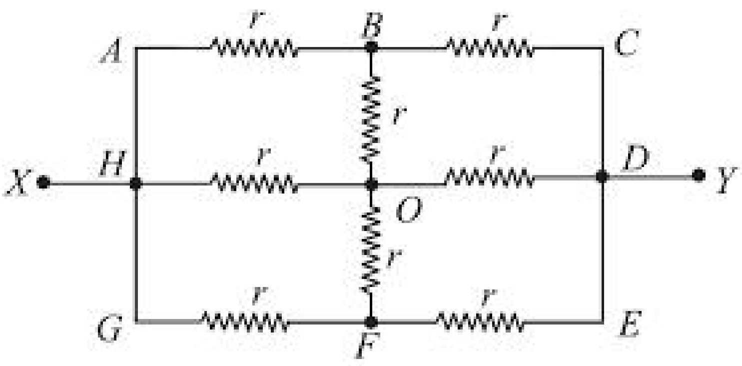



Fig.6.21

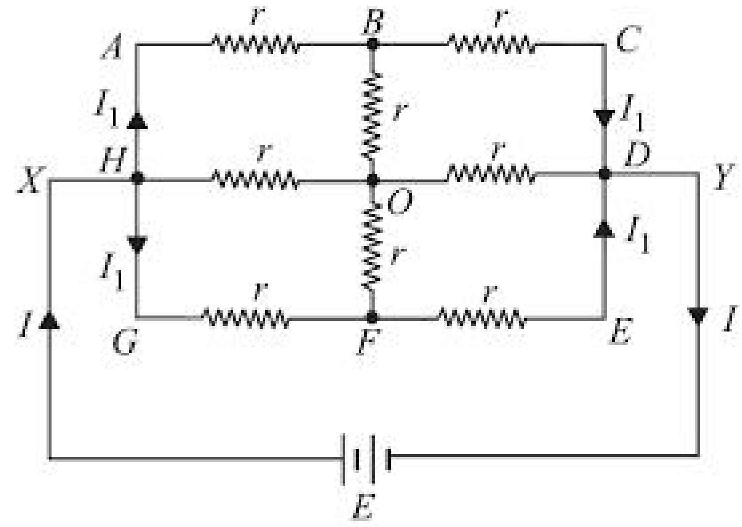



Fig. 6.22

current $I$ flowing out of point $D$ consists of current $I_1$ flowing in path $BCD$, current $I_1$ flowing in path $FED$ and current flowing in path $OD$. According to Kirchhoff's current law, current in path $OD$ must be $I–2I_1$. Since current flowing in path $HO$ as well as $OD$ is $I–2I_1$, no current flows in path $OB$ as well as $OF$ and resistance of these paths are ineffective and may be considered as removed. The circuit then becomes as shown in Fig. 6.23. Referring to Fig. 6.23, the resistance between points $X$ and $Y$ is equivalent to three $2r$ resistances in parallel.

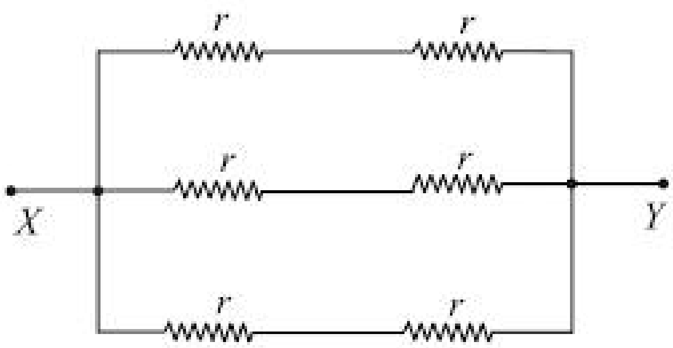



Fig. 6.23

$$\therefore \qquad R_{XY} = \frac{2r}{3}$$

**Example 6.11. Eleven equal wires each of resistance 2Ω form the edges of an incomplete cube. Find the total resistance from one end of vacant edge to the other end.**

**Solution.** Fig. 6.24 shows the conditions of the problem. We are to find the resistance between points $A$ and $K$. To do so, we connect a battery of e.m.f. $E$ between points $A$ and $K$. Let the current supplied by the battery be $2I$. At point $A$, due to symmetry, current $2I$ divides equally so that current in branch $AD$ as well as $AB$ is $I$. At point $B$, the current $I$ divides into two parts ; $I_1$ flows along branch $BH$ and $I–I_1$ flows along branch $BC$. At point $D$, current divides into two parts; $I_1$ along $DF$ and $I–I_1$ along $DC$. The currents in the various branches can be determined by applying Kirchhoff's current law. Applying Kirchhoff's voltage law to the loop $EABHKE$, we have,

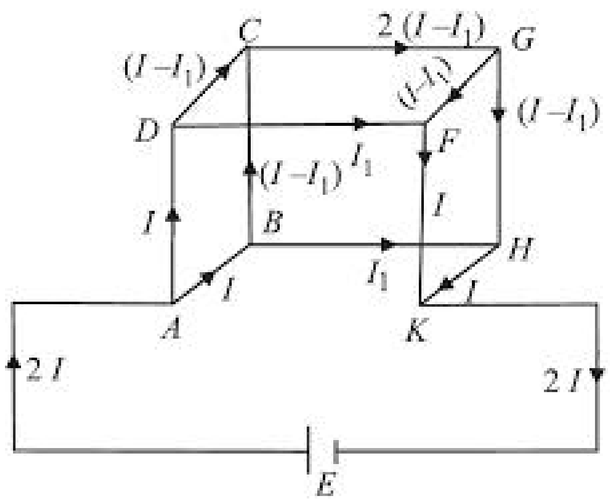



Fig. 6.24

$$E - I \times 2 - 2I_1 - 2I = 0$$

$$\therefore \qquad E = 4I + 2I_1 \qquad \text{... } (i)$$

Applying Kirchhoff's voltage law to loop $DFGCD$, we have,

$$-2I_1 + 2(I - I_1) + 4(I - I_1) + 2(I - I_1) = 0$$

or $\qquad 10\,I_1 - 8\,I = 0 \quad \therefore \quad I_1 = \frac{8}{10} I = \frac{4}{5} I$

From eq. ($i$), we have, $E = 4\,I + 2\left(\frac{4}{5}I\right) = \frac{28}{5} I$ $\qquad$ ... ($ii$)

If $R_{AK}$ is the resistance between points $A$ and $K$, then,

$$E = 2\,I \times R_{AK} \qquad \text{... } (iii)$$

From eqs. ($ii$) and ($iii$), $2I \times R_{AK} = \frac{28}{5} I \quad \therefore \quad R_{AK} = \frac{28}{5} \times \frac{1}{2} = 2.8\Omega$

$$V = kl \quad \text{... } (i)$$

If $L$ is the length of potentiometer wire $AB$ and $r$ its resistance, then,

$$\text{Current in } AB,\ I = \frac{E}{R+r} \quad \text{.... } R \text{ is rheostat resistance}$$

Now,
$$k = \frac{Ir}{L} = (I)\frac{r}{L} = \left(\frac{E}{R+r}\right)\frac{r}{L}$$

Putting the value of potential gradient $k$ in eq. $(i)$, we have,

$$V = \left(\frac{E}{R+r}\right)\frac{r}{L}l$$

Thus potential difference $V$ across $R_1$ can be determined.

## 6.10. COMPARISON OF E.M.F.S OF TWO CELLS BY POTENTIOMETER

Fig. 6.34 shows the arrangement for comparing the e.m.f.s $E_1$ and $E_2$ of two cells with the help of a potentiometer. The positive terminals of the cells ($E_1$ and $E_2$) are connected to the positive terminal of the driver battery $E$. The negative terminals of the cells are connected to terminals 1 and 2 of a two-way key. The common terminal 3 of the two-way key is connected to the jockey through a galvanometer $G$.

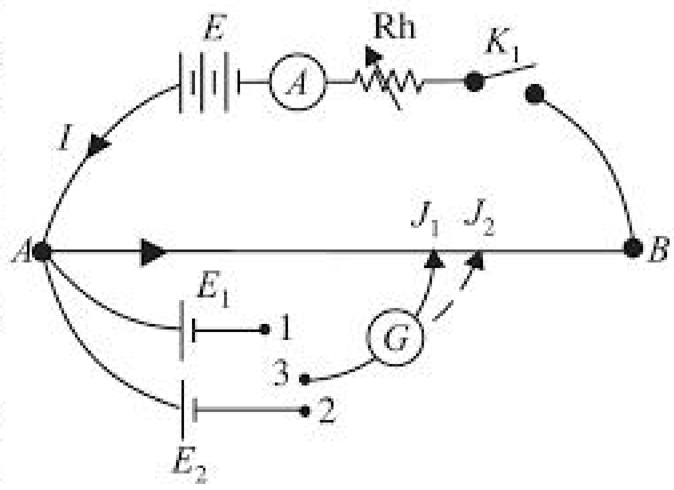



**Fig. 6.34**

(*i*) The key $K_1$ is closed and current in potentiometer wire is adjusted to a suitable constant value (say $I$) with the help of the rheostat. *The rheostat setting is not to be disturbed throughout the experiment.*

(*ii*) The terminals 1 and 3 are closed by a plug so that *only* cell of e.m.f. $E_1$ is put in the circuit. The jockey is moved on the potentiometer wire till galvanometer reads zero (null point). Let this point be $J_1$ on the wire and let distance $AJ_1 = l_1$.

$$\therefore \quad E_1 \propto l_1$$

(*iii*) Remove the plug between the terminals and insert it between terminals 2 and 3. This puts the cell of e.m.f. $E_2$ in the circuit. Repeat the procedure as in para (*ii*) above till null point is obtained at point $J_2$ on the potentiometer wire. Let distance $AJ_2 = l_2$.

$$\therefore \quad E_2 \propto l_2$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{l_1}{l_2}$$

**Note.** The e.m.f. $E$ of the driver battery should be greater than the e.m.f. $E_1$ or $E_2$ otherwise null point will *not be obtained. Further, a shunt must be used with the galvanometer initially and near the null point, it should be removed.

## 6.11. TO FIND INTERNAL RESISTANCE OF CELL BY POTENTIOMETER

Fig. 6.35 shows the arrangement for determining the internal resistance ($r$) of a cell of e.m.f. $E$ with the help of a potentiometer. The positive terminal of the cell is connected to the positive terminal of the battery. The negative terminal of the cell is connected to the jockey through a galvanometer $G$. A resistance box $R$ is connected across the cell through key $K_2$.

* It is because the null point can only be obtained if the fall of potential across the potentiometer wire due to the driving cell $E$ is greater than the e.m.f. of either cell ($E_1$ or $E_2$).

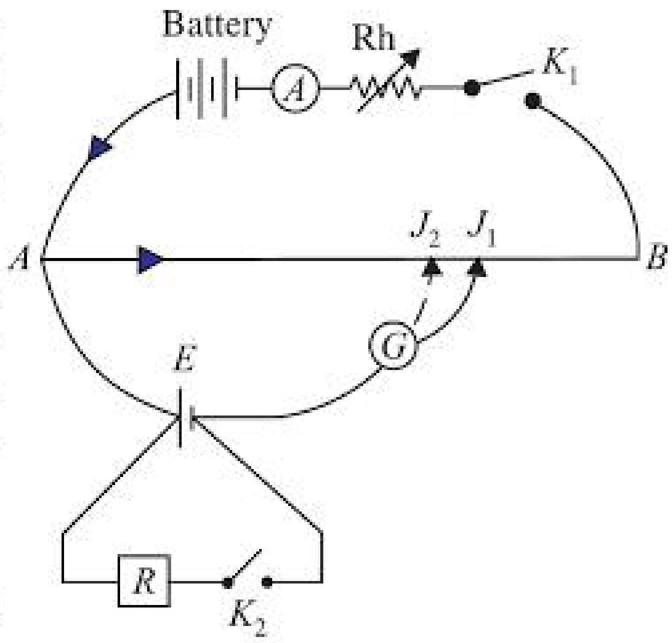



**Fig. 6.35**

(*i*) The key $K_1$ is closed and current in potentiometer wire $AB$ is adjusted to a suitable constant value with the help of rheostat. *The setting of the rheostat is not to be disturbed throughout the experiment.*

(*ii*) Keeping key $K_2$ open, the position of jockey is adjusted till null point is obtained, *i.e.*, galvanometer reads zero. Let the null point be obtained at point $J_1$ on the potentiometer wire and let distance $AJ_1 = l_1$.

$\therefore$ E.M.F. of cell, $E \propto l_1$

(*iii*) Now suitable resistance $R$ from the resistance box is inserted and key $K_2$ is closed. Again the position of the null point is obtained on the potentiometer wire. Now the null point corresponds to the p.d.$V$ across the terminals of the cell. If the null point is obtained at point $J_2$ and distance $AJ_2 = l_2$, then,

P.D. across the cell, $V \propto l_2$

$$\therefore \qquad \frac{E}{V} = \frac{l_1}{l_2}$$

Now, Internal resistance of the cell,

$$r = \left(\frac{E-V}{V}\right)R = \left(\frac{E}{V} - 1\right)R$$

$$= \left(\frac{l_1}{l_2} - 1\right)R = \left(\frac{l_1 - l_2}{l_2}\right)R$$

$$\therefore \qquad r = \left(\frac{l_1 - l_2}{l_2}\right)R$$

Since the values of $l_1$, $l_2$ and $R$ are known, the value of the internal resistance $r$ of the cell can be determined.

## 6.12. SENSITIVITY OF POTENTIOMETER

*The* **sensitivity** *of a potentiometer indicates the smallest potential difference that can be measured with it.*

The sensitivity of a potentiometer depends upon the potential gradient of the potentiometer wire (*i.e.* fall in potential per unit length along the potentiometer wire). The smaller the potential gradient, the more is the sensitivity of the potentiometer. Therefore, the sensitivity of a potentiometer can be increased by decreasing the potential gradient *i.e.*

(*i*) by increasing the length of potentiometer wire.

(*ii*) by reducing the current in the potentiometer wire with a resistance in series with potentiometer wire. This is equivalent to increasing the length of potentiometer wire.

## 6.13. POTENTIOMETER VERSUS VOLTMETER

We now discuss advantages and disadvantages of a potentiometer compared to a voltmeter.

**Advantages.**

(*i*) It is a null-deflection method and, therefore, the balance condition can be found with a high degree of sensitivity.

(*ii*) No current is drawn from the circuit under test. Therefore, it can measure the e.m.f. of a cell accurately.

(*iii*) The scale can be made long for maximum accuracy.

(*iv*) Results are dependent only on measurements of lengths and the values of standard resistances and standard e.m.f.s.

(*v*) The potentiometer is more accurate than the moving-coil voltmeter for measuring the e.m.f. The moving coil voltmeter has finite resistance and while connected across a cell, the voltmeter draws some current. Therefore, the voltmeter measures the e.m.f. of a cell approximately.

(*vi*) A potentiometer is a voltmeter with infinite resistance. Therefore, it is an *ideal voltmeter*.

(*vii*) Apart from measuring potential difference, a potentiometer can be used for various purposes *e.g.* to measure the internal resistance of a cell etc. However, voltmeter can measure only potential difference.

**Disadvantages.**

(*i*) It is slow in operation.

(*ii*) The potentiometer wire must be of uniform thickness.

(*iii*) The temperature of potentiometer wire must remain constant.

## 6.14. SLIDE WIRE BRIDGE (OR METRE BRIDGE)

A slide wire bridge is a sensitive device used for measuring the unknown resistance. *Its operation is based on the principle of Wheatstone bridge.*

**Construction.** Fig. 6.36 shows the various parts of a slide wire bridge.

(*i*) $AC$ is 1m long wire made of manganin (or constantan) and having uniform area of cross-section. A graduated scale runs parallel to this wire. Since wire $AC$ is 1m long, the device is also called metre bridge.

(*ii*) The resistance $R$ is a known resistance (resistance box) and $X$ is unknown resistance whose value is to be determined. Note that copper strips are used for making connections. This ensures negligible resistance of the connections.

(*iii*) A battery $E$ is connected between points $A$ and $C$ through key $K$; the positive terminal of $E$ being connected to point $A$.

(*iv*) One end of galvanometer $G$ is connected to point $D$ (junction point of $R$ and $X$) and the other end is connected to jockey $B$ which can slide over the wire $AC$.

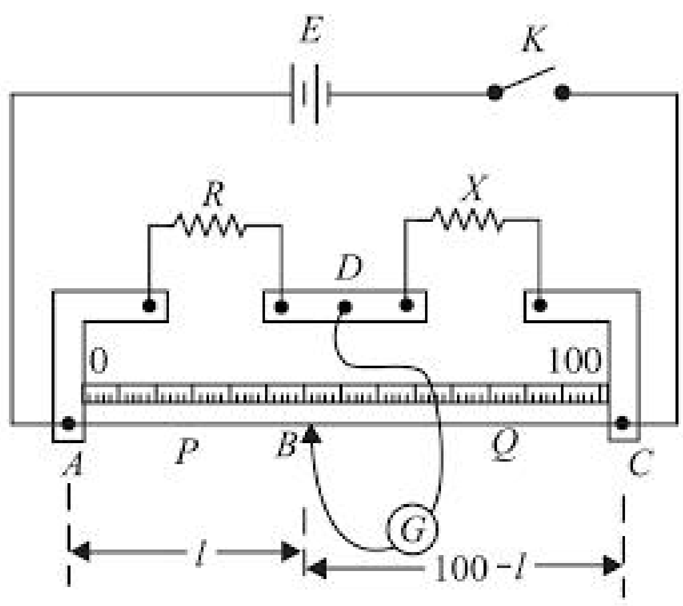



Fig. 6.36

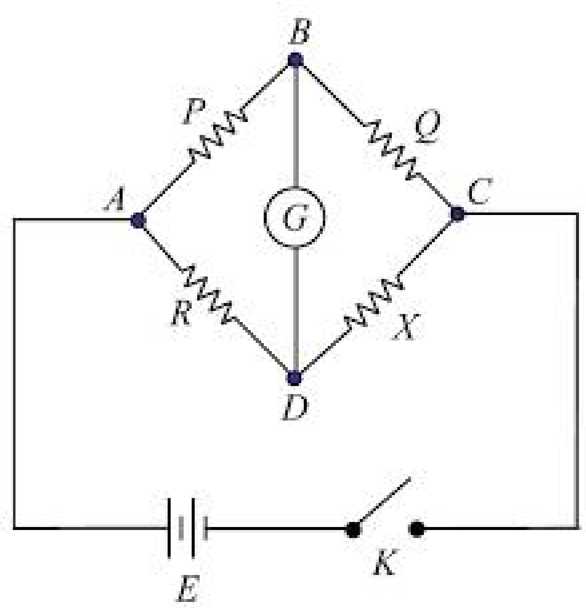



Fig. 6.37

Fig. 6.37 shows the representation of slide wire bridge in the familiar form of Wheatstone bridge.

**Theory.**

(*i*) Introduce a suitable value of $R$ and close the key $K$.

(*ii*) Move the jockey on the wire $AC$ to obtain the null point (*i.e.*, zero reading of the galvanometer). Let point $B$ be the null point on the wire $AC$. Suppose the resistance of portion $AB$ of the wire is $P$ and that of portion $BC$ is $Q$. Let length $AB = l$ cm. Then length $BC = (100 - l)$ cm.

Now $\quad *P \propto l \quad$ and $Q \propto (100 - l)$

$$\therefore \quad \frac{P}{Q} = \frac{l}{100 - l}$$

(*iii*) According to Wheatstone bridge principle, the relation between four resistances ($P$, $Q$, $X$ and $R$) at null point is given by ;

$$PX = R\,Q \quad \text{or} \quad \frac{P}{Q} = \frac{R}{X} \quad \text{or} \quad \frac{l}{100 - l} = \frac{R}{X}$$

$$\therefore \quad X = \left(\frac{100 - l}{l}\right) R$$

Since the values of $l$ and $R$ are known, the value of unknown resistance $X$ can be determined.

**Note.** The resistance $R$ should be so selected that null point is obtained near the middle of the wire $AC$. This will result in minimum percentage of error in the measurement.

**Advantages of Slide Wire Bridge**

(*i*) It is a null-deflection method and, therefore, the balance condition can be found with a high degree of sensitivity.

(*ii*) At balance, the galvanometer takes no current and, therefore, the result is not affected by the faulty calibration of the galvanometer.

(*iii*) Variation in the e.m.f. of the supply does not affect the balance point.

(*iv*) The error inherent in the voltmeter-ammeter method of measuring resistance is not present.

**Example 6.12. Fig. 6.38 shows a potentiometer circuit for comparing two resistances. The balance point with a standard resistance $R = 10.0\ \Omega$ is found to be at 58.3 cm while that with the unknown resistance $X$ is at 68.5 cm. Determine the value of $X$. What would you do if you fail to find a balance point with the given cell?**

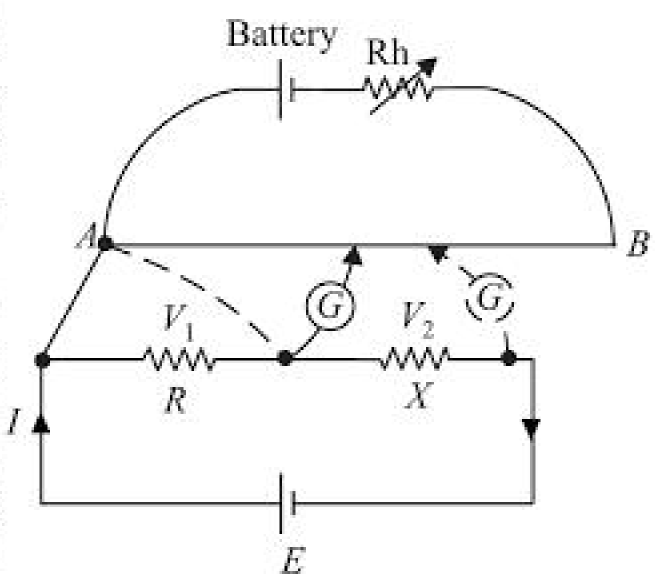



**Fig. 6.38**

**Solution.** We can find the value of low resistances by potentiometer. In this method, instead of comparing the two e.m.f.s, potential drops across an unknown resistance $X$ and a standard resistance $R$ are compared. Referring to Fig. 6.38, suppose the cell $E$ sends a current $I$ through $R$ and $X$. The p.d. across the standard resistance $R$ ($V_1 = IR$) is balanced against a length $l_1$ (= 58.3 cm) of the potentiometer wire. Similarly, p.d. across unknown resistance $X$ ($V_2 = IX$) is balanced against a length $l_2$ (= 68.5 cm) of the potentiometer wire.

$$\therefore \quad \frac{l_1}{l_2} = \frac{V_1}{V_2} = \frac{IR}{IX} = \frac{R}{X} \quad \text{or} \quad \frac{R}{X} = \frac{l_1}{l_2}$$

$$\text{or} \quad X = R\left(\frac{l_2}{l_1}\right) = 10\left(\frac{68.5}{58.3}\right) = 11.7\Omega$$

If there is no balance point, it means that the potential drop across $R$ or $X$ is greater than the p.d. across the potentiometer wire $AB$. The obvious remedy is to reduce current through $R$–$X$

* Resistance $P = \rho(l/A)$. Since $\rho$ and $A$ are constant, $P \propto l$.

$$= \frac{100\,(l_2 - l_1)}{l_1\,(100 - l_2)}$$

$$\therefore \qquad X = \frac{l_1\,(100 - l_2)}{100\,(l_2 - l_1)}\,S$$

**Example 6.15. In a metre bridge in Fig. 6.41, the null point is found at a distance of 33.7 cm from *A*. If now a resistance of 12 Ω is connected in parallel with *S*, the null point occurs at 51.9 cm. Determine the values of *R* and *S*.**

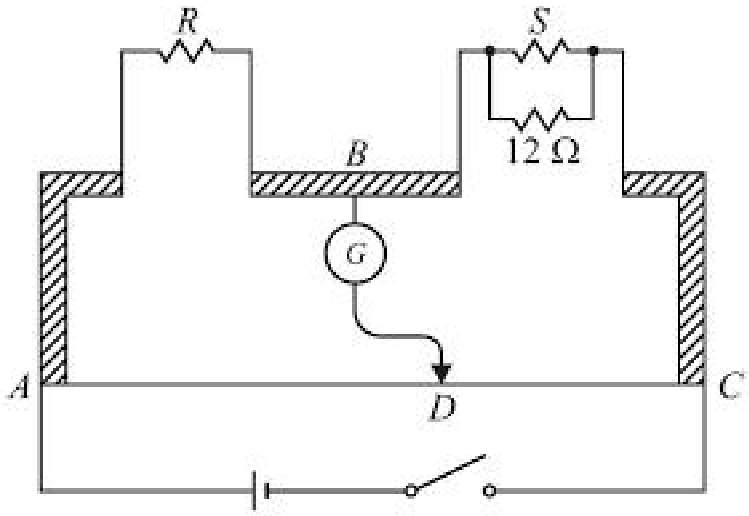



**Fig. 6.41**

**Solution.** In the first case, balance point is obtained at $l_1$ = 33.7 cm from *A*.

$$\therefore \qquad \frac{R}{S} = \frac{l_1}{100 - l_1} = \frac{33.7}{100 - 33.7}$$

$$\text{or} \qquad \frac{R}{S} = \frac{33.7}{66.3} \qquad \text{... }(i)$$

In the second case, *S* is shunted by 12 Ω so that the effective resistance of the right gap is

$$S' = \frac{S \times 12}{S + 12}$$

Now balance point is obtained at $l_2$ = 51.9 cm from *A*.

$$\therefore \qquad \frac{R}{S'} = \frac{l_2}{100 - l_2} = \frac{51.9}{100 - 51.9} = \frac{51.9}{48.1}$$

$$\text{or} \qquad \frac{R\,(S + 12)}{12S} = \frac{51.9}{48.1} \quad \text{or} \quad \frac{R}{S} \times \frac{S + 12}{12} = \frac{51.9}{48.1}$$

$$\therefore \qquad \frac{33.7}{66.3} \times \frac{S + 12}{12} = \frac{51.9}{48.1}$$

On solving, we get, $S$ = **13.1 Ω**

From eq. (*i*), $R = S \times \frac{33.7}{66.3} = 13.1 \times \frac{33.7}{66.3}$ = **6.64 Ω**

**Example 6.16. The total length of a potentiometer wire is 10m. The distance between the null points on the potentiometer wire for two cells is 60 cm. If the difference between the e.m.f.s of the cells be 0.4V, calculate (*i*) the potential gradient along the wire, (*ii*) p.d. between the ends of this wire.**

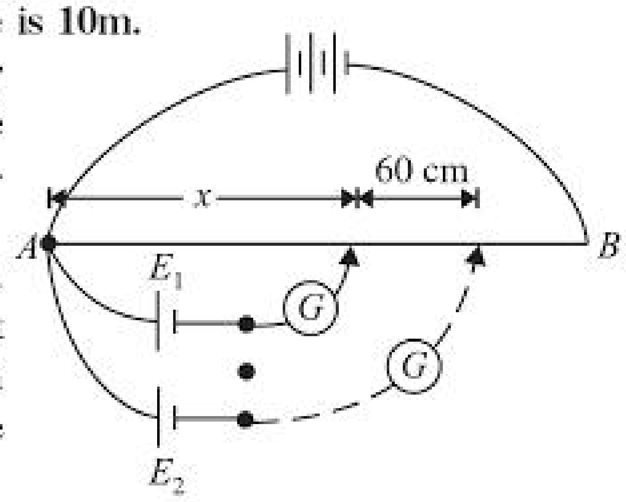



**Fig. 6.42**

**Solution.** Fig. 6.42 shows the conditions of the problem. Let the e.m.f.s of the two cells be $E_1$ and $E_2$ ($E_2 > E_1$). Let the null point for cell $E_1$ be at $x$ metre from the end *A*. Then distance of null point for cell $E_2$ will be = $x$ + 0.6. If the potential gradient along the wire *AB* is $k$ volts/m, then,

$$E_1 = kx \quad \text{and} \quad E_2 = k\,(x + 0.6)$$

Now, $$E_2 - E_1 = k\,[(x + 0.6) - x] \quad \text{or} \quad 0.4 = k\,(0.6)$$

(*i*) $\therefore$ Potential gradient along $AB$, $k = 0.4/0.6 =$ **2/3 V/m**

(*ii*) P.D. between points $A$ and $B$, $V_{AB} = k \times l_{AB} = (2/3) \times 10 =$ **(20/3)V**

**Example 6.17. The circuit diagram in Fig. 6.43 shows the set-up for measurement of e.m.f. generated in a thermocouple connected between *X* and *Y*. The cell *E* of e.m.f. 2 V has negligible internal resistance. The potentiometer wire of length 1 m has a resistance of 10 Ω. The balance point *S* is found to be 400 mm from point *P*. Calculate the e.m.f. generated by the thermocouple.**

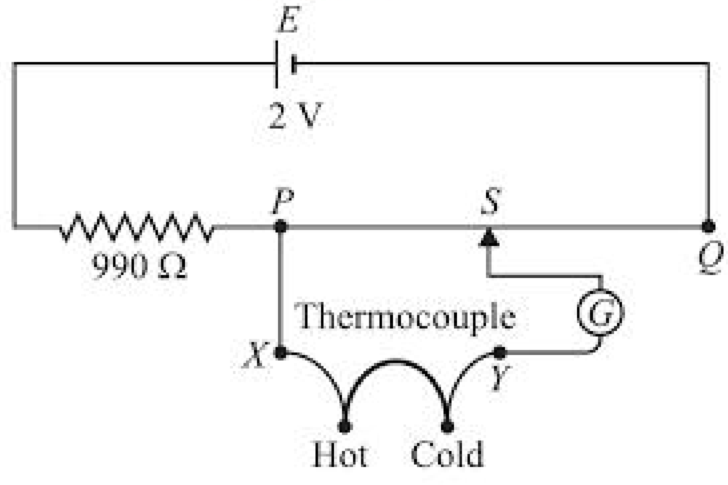



**Fig. 6.43**

**Solution.** Total circuit resistance is

$$R_T = 990 + 10 = 1000\ \Omega$$

Current through wire $PQ$ is

$$I = \frac{E}{R_T} = \frac{2}{1000} = 2 \times 10^{-3}\ \text{A}$$

Potential drop across wire $PQ = I\,r = 2 \times 10^{-3} \times 10 = 0.02$ V

$\therefore$ Potential gradient along the wire $PQ$ is

$$k = \frac{I\,r}{l} = \frac{0.02}{1\ \text{m}} = \frac{0.02}{1000\ \text{mm}}$$

$$\therefore \text{ P.D. across } PS = \frac{0.02}{1000\ \text{mm}} \times 400\ \text{mm} = \mathbf{0.008\ V}$$

Therefore, e.m.f. generated by the thermocouple is 0.008V.

**Example 6.18. The length of a potentiometer wire is 600 cm and it carries 40 mA current. For a cell of emf 2 volt and internal resistance 10 ohm, the null-point is found at 500 cm. If a voltmeter is connected across the cell, the balancing length of the wire is decreased by 10 cm. Find (*i*) the resistance of the whole wire (*ii*) reading of voltmeter (*iii*) resistance of voltmeter.**

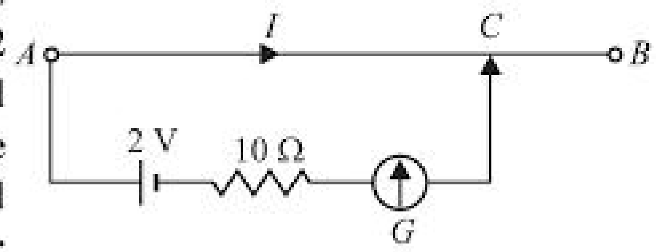



**Fig. 6.44**

**Solution.** (*i*) With cell of e.m.f. 2 V in the circuit, the balance point is at $C$ as shown in Fig. 6.44. Here, $AB = 600$ cm ; $AC = 500$ cm ; $I = 40$ mA $= 40 \times 10^{-3}$ A.

Let $R$ be the resistance of wire $AB$.

$$\text{Potential gradient, } k = \frac{IR}{AB} = \frac{2}{AC}$$

$$\therefore \qquad R = \frac{2}{AC} \times \frac{AB}{I} = \frac{2}{500} \times \frac{600}{40 \times 10^{-3}} = \mathbf{60\ \Omega}$$

(*ii*) When voltmeter is connected across the cell, the balance point is now at $C'$ (See Fig. 6.45) so that $AC' = 500 - 10 = 490$ cm.

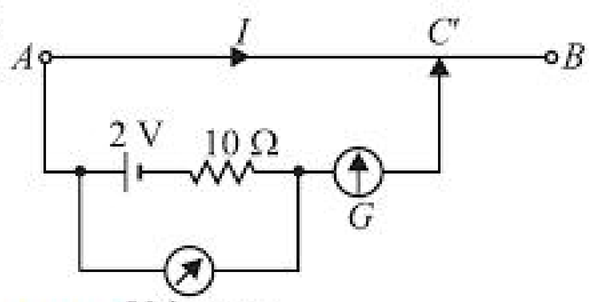



**Fig. 6.45**

From part (*i*), $$k = \frac{2}{AC} = \frac{2}{500}\ \text{V/cm}$$

$$\therefore \text{ P.D. between } A \text{ and } C' = k \times AC' = \frac{2}{500} \times 490 = \mathbf{1.96V}$$

Since there is no current in the galvanometer $G$, p.d. across votlmeter is 1.96 V, *i.e.*, voltmeter reading is 1.96 V.

(*iii*) The cell is sending current $I_v$ to the voltmeter given by ;

$$I_v = \frac{E-V}{r} = \frac{2-1.96}{10} = 0.004 \text{ A}$$

$$\therefore \text{ Resistance of voltmeter } = \frac{V}{I_v} = \frac{1.96}{0.004} = \mathbf{490\ \Omega}$$

**Example 6.19. A resistance of $R$ Ω draws current from a potentiometer. The potentiometer has a total resistance $R_0$ Ω (Fig. 6.46). A voltage $V$ is supplied to the potentiometer. Derive an expression for the voltage across $R$ when the sliding contact is in the middle of the potentiometer.**

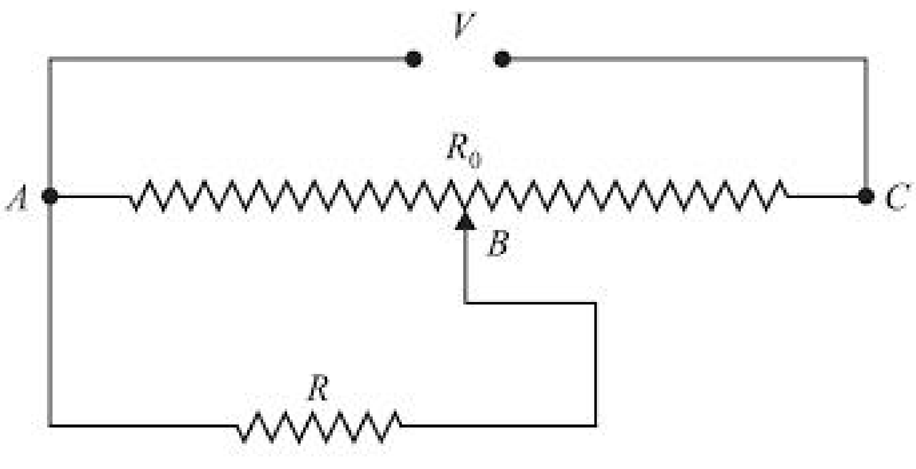



**Fig. 6.46**

**Solution.** When the slide is in the middle of the potentiometer wire, only half the resistance of the potentiometer wire ($R_0/2$) will be between the points $A$ and $B$. Therefore, the effective resistance $R_1$ between points $A$ and $B$ is given by ;

$$\frac{1}{R_1} = \frac{1}{R} + \frac{1}{R_0/2} \quad \text{or} \quad R_1 = \frac{R_0 R}{R_0 + 2R}$$

Total resistance between points $A$ and $C = R_1 + R_0/2$

$\therefore$ Current through the potentiometer wire is

$$I = \frac{V}{R_1 + R_0/2} = \frac{2V}{2R_1 + R_0}$$

The voltage $V_1$ taken from the potentiometer will be the product of current $I$ and resistance $R_1$.

$$\therefore \qquad V_1 = I R_1 = \left(\frac{2V}{2R_1 + R_0}\right) \times R_1$$

$$= \frac{2V}{2\left(\frac{R_0 R}{R_0 + 2R}\right) + R_0} \times \frac{R_0 R}{R_0 + 2R} = \frac{2VR}{2R + R_0 + 2R}$$

$$\therefore \qquad V_1 = \mathbf{\frac{2VR}{R_0 + 4R}}$$

**Example 6.20. With two resistance wires in the two gaps of a metre bridge, the balance point was found to be 1/3 m from the zero end. When a 6 Ω coil is connected in series with the smaller of the two resistances, the balance point is shifted to 2/3 m from the same end. Find the resistance of the two wires.**

**Solution.** Refer to Fig. 6.36. Let $R_1$ (left gap) and $R_2$ (right gap) be the resistances of the two wires. For balance point, we have,

$$R_1 \times \frac{2}{3} = R_2 \times \frac{1}{3} \quad \therefore \quad R_2 = 2\ R_1 \qquad \text{... (i)}$$

When 6 Ω resistance in connected in series with $R_1$ (smaller resistance), the total resistance in the left gap is $R' = (R_1 + 6)$ Ω. For balance point, we have,

$$(R_1 + 6) \times \frac{1}{3} = R_2 \times \frac{2}{3} \quad \therefore \quad \frac{R_1 + 6}{R_2} = 2 \qquad \text{... (ii)}$$

Solving eqs. (*i*) and (*ii*), $R_1 = 2\ \Omega$ ; $R_2 = 4\ \Omega$

Example 6.21. **The resistance of a potentiometer wire of length 10m is 20Ω. A resistance box and a 2 volt accumulator are connected in series with it. What resistance should be introduced in the box to have a potential drop of 1μV per millimetre of the potentiometer wire ?**

**Solution.** Resistance of potentiometer wire, $R = 20\Omega$

Length of potentiometer wire, $L = 10\text{m}$

Required potential gradient $= 1\mu\text{V/mm} = 10^{-6} \times 10^{3} = 10^{-3}$ V/m

P.D. across potentiometer wire, $V = 10 \times 10^{-3} = 10^{-2}$ volt

Current through potentiometer wire, $I = \dfrac{V}{R} = \dfrac{10^{-2}}{20} = 5 \times 10^{-4}$ A

Let $R'$ be the resistance to be introduced in the resistance box. Then,

$$I = \frac{E}{R + R'} \quad \text{or} \quad 5 \times 10^{-4} = \frac{2}{20 + R'} \quad \therefore \quad R' = 3980\ \Omega$$

## PROBLEMS FOR PRACTICE

1. The e.m.f. of a battery is balanced by a length of 75.0 cm on a potentiometer wire. The e.m.f. of a standard cell, 1.02V is balanced by a length of 50.0 cm. What is the e.m.f. of the battery? **[1.53V]**

2. A 10 m long wire of uniform cross-section of 20Ω resistance is fitted on the board. The wire is connected in series with a battery of 5V along with an external resistance of 480Ω. If an unknown e.m.f. $E$ is balanced at 600 cm of this wire, calculate (*i*) the potential gradient of wire, (*ii*) value of unknown e.m.f. [(*i*) **0.02V/m;** (*ii*) **0.12V**]

   [Hint. Total resistance of primary current = 480 + 20 = 500Ω

   Therefore, Current in potentiometer wire = 5/500 = 0.01A

   The p.d. across wire = 0.01 × 20 = 0.2 V

   Potential gradient along wire, $k$ = 0.2/10 = 0.02 V/m

   The value of $E = k\ l = 0.02 \times 6 = 0.12$V]

3. A standard cell of e.m.f. 1.0185V when used in one metre long potentiometer balances at 60 cm. Calculate the percentage error in a voltmeter which balances at 65 cm when reading is 1.1V. **[0.31%]**

   [Hint. Potential gradient along potentiometer wire, $k$ = 1.0185/60 volts/cm. Voltage drop across 65 cm of wire = $kl$ = (1.0185/60)65 = 1.1034 V. But voltmeter reads 1.1V. Clearly the voltmeter reading is low by 0.0034 V.

   $$\therefore \quad \%\text{age error} = \frac{0.0034}{1.1034} \times 100 = 0.31\%\ ]$$

4. In Fig. 6.47, the p.d. between points $A$ and $B$ balances at 40 cm length of potentiometer wire $PQ$. To balance the p.d. between points $B$ and $C$, at what distance from $P$, the jockey $J$ should be pressed? **[32 cm.]**

**Q.13. What happens if the galvanometer and cell are interchanged at the balance point in a metre bridge? Would the galvanometer show any current?**

**Ans.** It will have no effect because the bridge is still balanced. The galvanometer will show zero current because the bridge is balanced.

**Q.14. Give reasons for the following :**

**(*i*) The area of cross-section of wire of metre bridge should be uniform.**

**(*ii*) The length of wire of the metre bridge need not be necessarily 1 m.**

**Ans.** (*i*) If area of cross-section of metre bridge wire is not constant, the resistance per unit length of the wire will be different over different lengths of the metre bridge wire. Therefore, principle of potentiometer will not be obeyed.

(*ii*) It is not necessary that the length of metre bridge wire should be 1m. Since the wire forms two arms of Wheatstone bridge, any length of wire can be used.

**Q.15. With a cell of e.m.f. $E$, the balance point is obtained at a distance of 180 cm from one end of a potentiometer wire. If potential gradient along the wire is 0.006 V/cm, find the e.m.f. $E$ of the cell.**

**Ans.** Fig. 6.53 shows the conditions of the problem. Potential gradient, $k$ = 0.006 V/cm.

$\therefore$ E.M.F. of cell, $E = k\,l = 0.006 \times 180 = 1.08$V

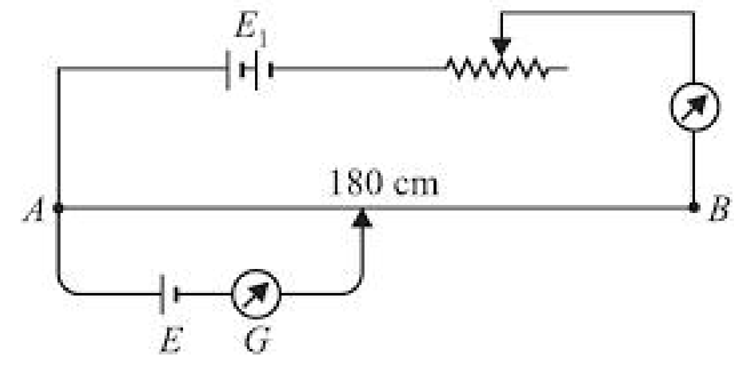



**Fig. 6.53**

**Q.16. If the e.m.f. $E_1$ of the driving cell in Fig. 6.53 is decreased, what will be the effect on the position of balance point (zero deflection)?**

**Ans.** If the e.m.f. $E_1$ of the driving cell is decreased, the potential gradient along the wire will decrease. Therefore, the position of zero deflection (*i.e.*, balance point) will occur at longer length. Reverse would happen should e.m.f. $E_1$ of driving cell increase.

**Q.17. What will be the effect on the position of zero deflection in a potentiometer if the length of the potentiometer wire is (*i*) increased (*ii*) decreased?**

**Ans.** (*i*) The potential gradient along the potentiometer wire will decrease. Therefore, the zero deflection position will occur at longer length.

(*ii*) The potential gradient along the potentiometer wire will increase. Therefore, the zero deflection position will occur at shorter length.

**Q.18. What will be the effect on the position of zero deflection in a potentiometer if the current flowing through the potentiometer wire is (*i*) increased (*ii*) decreased?**

**Ans.** (*i*) The potential gradient along the potentiometer wire will increase and the zero deflection position will occur at shorter length.

(*ii*) The potential gradient along the potentiometer wire will decrease and the zero deflection position will occur at longer length.

**Q.19. How can you increase the sensitivity of a given potentiometer by using a resistance box?**

**Ans.** The smaller the potential gradient along the potentiometer wire, the greater is the sensitivity of the potentiometer *i.e.*, the device will be able to measure smaller potential differences. The smaller potential gradient can be obtained by decreasing the current flowing through the potentiometer wire. This can be achieved by adding resistance from resistance box in series with potentiometer circuit.

**Q.20. Fig. 6.54 shows the potentiometer circuit for measuring the e.m.f. $E$ of a cell.**

**(*i*) Will the circuit work if $E > E_1$?**

**(*ii*) Why is very high resistance (= 0.5 MΩ) put in series with the circuit? Will it affect the balance point?**

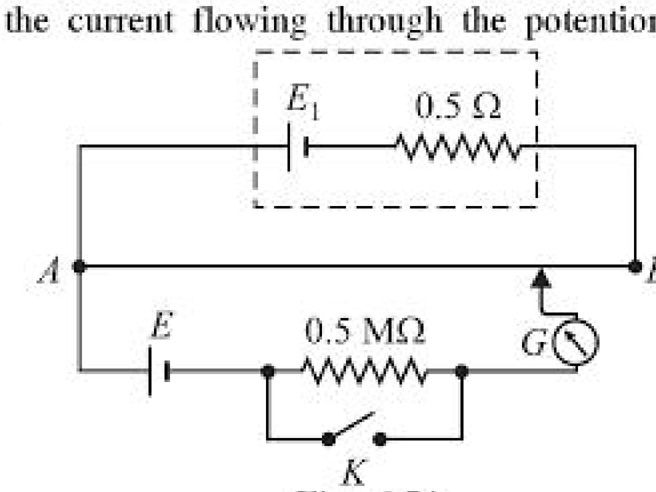



**Fig. 6.54**

**(*iii*) Is the balance point affected by the internal resistance of the driving cell $E_1$?**

**Ans.** (*i*) If $E > E_1$, the balance point will not be obtained and the circuit will not work.

(*ii*) The high resistance (= 0.5 M$\Omega$) in series with galvanometer will ensure that current through the galvanometer does not increase the permitted value. No, it will not affect the balance point.

(*iii*) No, the balance point is not affected by the internal resistance (= 0.5 $\Omega$) of the driving cell $E_1$.

**Q.21. Two cells of e.m.f. $E_1$ and $E_2$ ($E_1 > E_2$) are connected to a potentiometer wire *AB* as shown in Fig. 6.55. If the balancing length for the two combinations of cells are 250 cm and 400 cm, find the ratio $E_1/E_2$.**

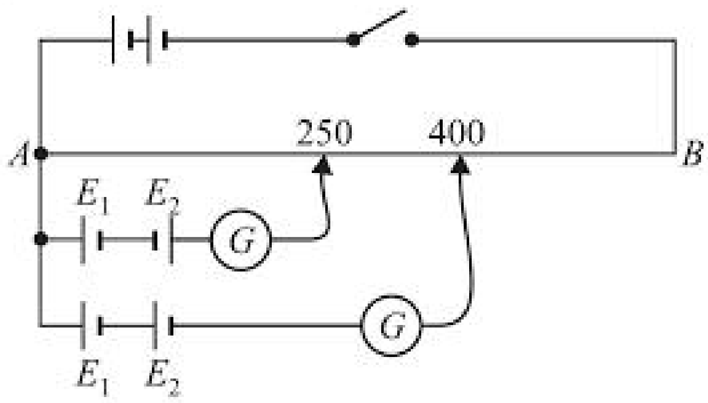



**Fig. 6.55**

**Ans.** Let $k$ volts per cm be the potential gradient along the potentiometer wire *AB*. Then,

$$E_1 + E_2 = 400 \times k \quad \text{and} \quad E_1 - E_2 = 250 \times k$$

$$\therefore \quad \frac{E_1 + E_2}{E_1 - E_2} = \frac{400}{250} = \frac{8}{5} \quad \text{or} \quad 5\,(E_1 + E_2) = 8\,(E_1 - E_2) \quad \therefore \quad \frac{E_1}{E_2} = \frac{13}{3}$$

## LONG ANSWER QUESTIONS

1. State and explain Kirchhoff's laws of electric circuits. **[Refer to Art. 6.2]**
2. With a neat diagram, explain the working of Wheatstone bridge. **[Refer to Art. 6.6]**
3. Explain the principle and working of potentiometer. **[Refer to Art. 6.8]**
4. How will you measure the e.m.f. of a cell by a potentiometer? **[Refer to Art. 6.10]**
5. Explain how is potential difference across a resistor measured by a potentiometer? **[Refer to Art. 6.9]**
6. How will you compare the e.m.f.s of two cells by a potentiometer? **[Refer to Art. 6.10]**
7. Dercribe the method to measure the internal resistance of a cell by a potentiometer. **[Refer to Art. 6.11]**
8. Describe the construction and working of slide wire bridge. **[Refer to Art. 6.14]**
9. Write a short note on (*i*) principle of Wheatstone bridge (*ii*) principle of potentiometer. **[Refer to Arts. 6.6 and 6.8]**

# COMPETITION SUCCESS MATERIAL

## Useful Concepts/Information

1. We use Kirchhoff's laws where the circuit cannot be simplified by using series and parallel combinations. Such circuits are called *complex circuits*. There are two Kirchhoff's laws to solve complex circuits.
2. Kirchhoff's current law (KCL) is based on the principle of conservation of charge. It states that *sum of the currents flowing into a junction is equal to the sum of currents leaving the junction*. If this were not so, then the junction would be either creating or destroying the charge.
3. Kirchhoff's voltage law (KVL) is based on the principle of conservation of energy. It states that in any closed circuit or loop, *the algebraic sum of all the electromotive forces (e.m.f.s) and voltage drops in resistors is equal to zero i.e.*

   Algebraic sum of e.m.f.s + Algebraic sum of voltage drops = 0 ...... in a loop.

   This means that if a charge is taken from some point in the circuit and moved completely around the loop to its starting point, the net work done on the charge is zero. This must be so because the charge has returned to the same energy level.
4. While applying Kirchhoff's voltage law, the following sign convention should be followed:

   (*i*) Batteries or e.m.f.s are **positive** when going from the negative to the positive terminal and vice-versa.

   (*ii*) Voltage drops are **negative** if there is a fall in potential and vice-versa.
5. The d.c. current in a branch containing a capacitor is zero.
6. The Wheatstone bridge circuit provides an accurate method of measuring unknown resistance.
7. When Wheatstone bridge is balanced (*i.e.* galvanometer reads zero), the products of circuit parameters (*e.g.* resistance, capacitive reactance etc.) of opposite arms of the bridge are equal.
8. When Wheatstone bridge is balanced, the branch containing galvanometer or other circuit parameters (*e.g.* resistance, capacitive reactance etc.) is ineffective and may be considered as removed.
9. A potentiometer primarily measures potential difference, but on this basis it can be used to :

   (*i*) compare the e.m.f.s of two cells

   (*ii*) measure the e.m.f. of a cell

   (*iii*) compare two resistances

   (*iv*) measure current in a circuit

   (*v*) measure the internal resistance of a cell
10. The potentiometer method of measurement is a *null-deflection method i.e.* operator looks for zero deflection and not for a reading of zero. Therefore, this method does not depend on the calibration of the galvanometer.
11. If a potentiometer is to be used to measure a potential difference which is larger than that of the driver cell of the potentiometer, the potential difference first be reduced in a *known* ratio by using a potential divider circuit.

12. A slide wire bridge is a sensitive device which is used for measuring unknown resistance. Its operation is based on the principle of Wheatstone bridge.
13. The slide wire bridge method is a null-deflection method.
14. In a slide wire bridge variation in the e.m.f. of the supply does not affect the balance point.
15. A standard cell is one which provides a constant and accurately known e.m.f. The e.m.f. of the standard Weston cell is 1.0183V at 20°C.

## MCQ FROM COMPETITIVE ENTRANCE EXAMINATIONS

1. Kirchhoff's current law at a junction deals with conservation of **[AIIMS 2000]**
   (*a*) charge (*b*) energy
   (*c*) momentum (*d*) all of these
2. If the specific resistance of a potentiometer wire is $10^{-7}\Omega$m, the current flowing through it is 0.1 A and cross-sectional area of the wire is $10^{-6}$ m$^2$, then potential gradient is **[CBSE PMT 2001]**
   (*a*) $10^{-2}$ V/m (*b*) $10^{-4}$V/m
   (*c*) $10^{-6}$ V/m (*d*) $10^{-8}$ V/m
3. Fig. 6.56 shows a network carrying currents. The current $I$ will be **[PMT 1995]**
   (*a*) –3A (*b*) 3A

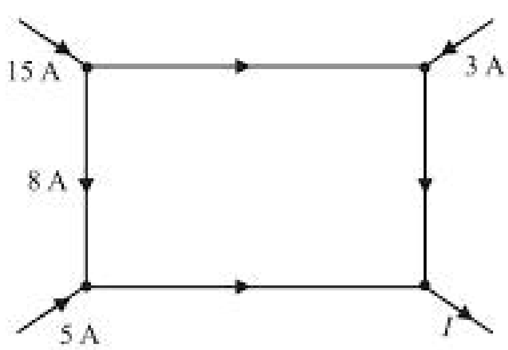



**Fig. 6.56**

   (*c*) 13A (*d*) 23A
4. In an experiment to measure the internal resistance of a cell by a potentiometer, it is found that balance point is at a length of 2m when the cell is shunted by a 5Ω resistance and is at a length 3m when the cell is shunted by a 10Ω resistance. The internal resistance of the cell is **[Haryana CEET 1996]**
   (*a*) 1Ω (*b*) 1.5Ω
   (*c*) 10Ω (*d*) 15Ω
5. A 2Ω resistor is connected in series with $R\Omega$ resistor. The combination is connected across a cell. When potential difference across 2Ω resistor is balanced on a potentiometer wire, the null point is obtained at a length of 300 cm. When the same procedure is repeated for $R$ Ω resistor, the null point is obtained at a length of 350 cm. The value of $R$ is **[CPMT 1996]**
   (*a*) 5Ω (*b*) 2.33Ω
   (*c*) 4.66Ω (*d*) 3.33Ω
6. In a potentiometer experiment, two primary cells are connected in series to support each other and then connected to oppose each other. The balance points are obtained at 4m and 1m respectively. The ratio of the e.m.f.s of the two cells is **[AMU 1997]**
   (*a*) 5 : 3 (*b*) 4 : 1
   (*c*) 2 : 1 (*d*) 5 : 4
7. For a cell of e.m.f. 2V, the balance point is obtained for 50 cm of potentiometer wire. If the cell is shunted by a 2Ω resistor and balance is obtained across 40 cm of the wire, then internal resistance of the cell is **[SCRA – 1992]**
   (*a*) 0.25Ω (*b*) 0.80Ω
   (*c*) 1.00Ω (*d*) 0.50Ω
8. In a metre bridge the balancing length from the left (standard resistance of 1Ω is in the right gap) is found to be 20 cm. The value of the unknown resistance is **[CBSE PMT 1999]**
   (*a*) 0.8Ω (*b*) 0.5Ω
   (*c*) 0.25Ω (*d*) 0.4Ω
9. A potentiometer consists of a wire of length 4m and resistance 10Ω. It is connected to a cell of e.m.f. 2V. The p.d. per unit length of the wire will be **[CBSE PMT 1999]**
   (*a*) 2V/m (*b*) 0.5V/m
   (*c*) 5V/m (*d*) 10V/m
10. Fig. 6.57 shows a part of a closed electrical circuit. Then $V_A - V_B$ is **[KCET 1997]**

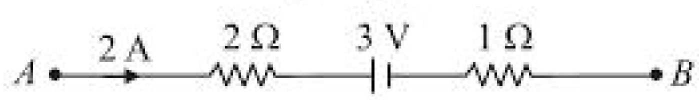



**Fig. 6.57**

   (*a*) 10V (*b*) –8V
   (*c*) 6V (*d*) 3V

$l$ = length of wire = 10 m

$E$ = e.m.f. of the cell = 2 V

$$\text{Potential gradient } = I\,r = \frac{E\,r}{R + l\,r}$$

$$\therefore \qquad 0.1 = \frac{2\times 4}{R+10\times 4}$$

or $\qquad R = $ **40 Ω**

12. **A standard cell of 1.0185 V, when used in a one metre long slide-wire potentiometer, balances at 60 cm. Calculate the percentage error in a voltmeter which balances at 65 cm when reading is 1.1 V.**

**Hint.** It is clear that 1.0185 V drops over 60 cm length of wire.

$$\therefore \qquad \text{Potential gradient} = \frac{1.0185}{60}\ \text{volt/cm}$$

$$\text{Voltage drop across 65 cm of slide-wire} = \frac{1.0185}{60}\times 65 = 1.1035\ \text{V}$$

Reading indicated by voltmeter = 1.1 V

Obviously the metre reads low by 0.0035 V.

$$\therefore \qquad \%\ \text{error} = \frac{0.0035}{1.1035}\times 100 = \mathbf{0.31\%}$$

# 7
# Heating Effect of Electric Current

## INTRODUCTION

Heat is an important form of energy in the study of electricity, not only because it affects the electrical properties of the materials but also because it is liberated when current flows. This liberation of heat is in fact the conversion of electrical energy into heat energy. Joule performed a number of experiments on heating effect of electric current. He summed up his conclusions into a law, known as *Joule's law of heating*. In this chapter, we shall discuss the various aspects of heating effect of electric current.

## 7.1. HEATING EFFECT OF ELECTRIC CURRENT

*When electric current is passed through a conductor, heat is produced in the conductor. This effect is called* **heating effect of electric current.**

It is a matter of common experience that when electric current is passed through the element of an electric heater, the element becomes red hot. It is because electrical energy is converted into heat energy. This is called heating effect of electric current and is utilised in the manufacture of many heating appliances, *e.g.*, electric iron, electric kettle, etc. The basic principle of all these devices is the same. Electric current is passed through a high resistance (called *heating element*), thus producing the required heat.

## 7.2. CAUSE OF HEATING EFFECT OF ELECTRIC CURRENT

When potential difference is applied across the ends of a conductor, the free electrons move with drift velocity and current is established in the conductor. As the free electrons move through the conductor, they collide with positive ions of the conductor. On collision, the kinetic energy of an electron is transferred to the ion with which it has collided. As a result, the kinetic energy of vibration of the positive ion increases, *i.e.*, temperature of the conductor increases. Therefore, as current flows through a conductor, the free electrons lose energy which is converted into heat. Since the source of e.m.f. (*e.g.*, a battery) is maintaining current in the conductor, it is clear that electrical energy supplied by the battery is converted into heat in the conductor.

## 7.3. HEAT PRODUCED IN A CONDUCTOR BY ELECTRIC CURRENT

On the basis of his experimental results, Joule found that the amount of heat produced ($H$) when current $I$ amperes flows through a conductor of resistance $R$ ohms for time $t$ seconds is $H = I^2Rt$ Joules. This equation is known as Joule's law of heating.

Suppose a battery maintains a potential difference of $V$ volts across the ends of a conductor $AB$ of resistance $R$ ohms as shown in Fig. 7.1. Let the steady current that passes from $A$ to

$B$ be $I$ amperes. If this current flows for $t$ seconds, then charge transferred from $A$ to $B$ in $t$ seconds is

$$q = It$$

The electric potential energy lost ($W$) by the charge $q$ as it moves from $A$ to $B$ is given by ;

$$W = \text{Charge} \times \text{P.D. between } A \text{ and } B$$

$$= qV = (It)\ V = I^2R\ t \quad (\because\ V = IR)$$

or $\quad W = I^2R\ t$

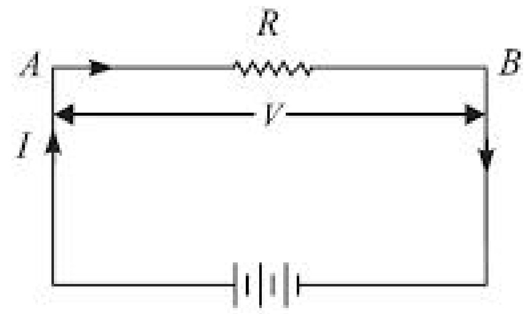




**Fig. 7.1**

This loss of electric potential energy of charge is converted into heat ($H$) because the conductor $AB$ has resistance only.

$$\therefore \quad H = W = I^2\ R\ t \text{ joules} = \frac{I^2\ Rt}{4.18} \text{ calories} \qquad \text{... } (i)$$

It is found experimentally that 1 cal = 4.18 J.

Eq. ($i$) is known as **Joule's law of heating.** It is because Joule was the first scientist who studied the heating effect of electric current through a resistor. Thus according to Joule, heat produced in a conductor is directly proportional to

($i$) square of current through the conductor

($ii$) resistance of the conductor

($iii$) time for which current is passed through the conductor.

**Note.** $$H = V\ I\ t = I^2R\ t = \frac{V^2}{R}\ t \text{ joules}$$

$$= \frac{VIt}{4.18} = \frac{I^2Rt}{4.18} = \frac{V^2t}{R \times 4.18} \text{calories}$$

## 7.4. ELECTRIC POWER

When voltage is applied in a circuit, it causes current (*i.e.*, free electrons) to flow through it. Clearly, work is being done by the voltage source in moving the electrons in the circuit. This electric work done in a unit time is called *electric power.*

*The rate at which electric work is done by the voltage source (e.g., a battery) in maintaining electric current in the electric circuit is called* **electric power** *of the circuit.*

Consider that a current $I$ amperes flows through a conductor $AB$ of resistance $R$ ohms for $t$ seconds under the potential difference of $V$ volts as shown in Fig. 7.2. The charge flowing from $A$ to $B$ in $t$ seconds is

$$q = It$$

As the charge $q$ $(= It)$ moves from $A$ to $B$, it loses electric potential energy $= qV$. In other words, electric work done in the circuit for $t$ seconds is

$$W = qV = (It)\ V = VIt \text{ joules}$$

$$\therefore \text{ Electric power, } P = \frac{W}{t} = \frac{V\ I\ t}{t} = V\ I$$

or $\quad P = VI$ J/s or watt

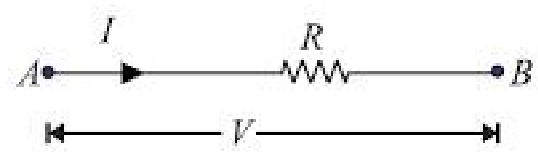




**Fig. 7.2**

If $V = 1$ volt and $I = 1$ A, then, $P = 1 \times 1 = 1$W.

*Hence electric power of a circuit or device is* **one watt** *if a current of 1A flows through it when a p.d. of 1V is maintained across it.*

The bigger units of electric power are kilowatt (kW) and megawatt (MW).

$$1\text{kW} = 1000\text{W} \ ; \quad 1\text{MW} = 10^3 \text{ kW} = 10^6 \text{ W}$$

The commercial unit of electric power is horse power (H.P.) where 1 H.P. = 746 W

**Discussion.** The following points may be noted :

(*i*) As $V = IR$ or $I = V/R$, therefore, electric power $P$ can be expressed as :

$$P = VI = I^2R = V^2/R$$

Any one of the three formulas can be used to calculate electric power.

(*ii*) Electric appliances are rated in terms of electric power. The faster the appliance converts electrical energy into some other form of energy, the greater the electric power it has. Thus, in 1 second, a 100 W bulb converts more electrical energy into heat and light than a 60 W bulb.

## 7.5. ELECTRICAL ENERGY

*The total electric work done or energy supplied by a voltage source in maintaining current in an electric circuit for a given time is called* **electrical energy consumed** *in the circuit during that time, i.e.,*

Electrical energy, $E$ = Electric Power × Time = $V\,I \times t$

$$\therefore \quad E = V\,I\,t$$

Unit of $E$ = Unit of $P$ × unit of $t$ = 1 W × 1 s = 1 Ws or 1 J

*The electrical energy consumed in a circuit is* **1 Ws** (or **1 J**) *if a power of 1W is supplied to the circuit for 1s.*

The commercial unit of electrical energy is **kilowatt-hour (kWh).**

*The electrical energy consumed in a circuit is* **1 kWh** *if a power of 1 kW is supplied to the circuit for 1 hour.*

Electrical energy in kWh = Power in kW × Time in hours

The electricity bills are made on the basis of total electrical energy consumed by the consumer. The unit for billing of electrical energy is 1kWh. Thus when we say that a consumer has consumed 100 units, it means that electrical energy consumption is 100 kWh. Note that 1kWh is also called **Board of Trade Unit (B.O.T.U.)** or unit of electricity.

Electrical energy can be expressed as :

$$E = VIt = I^2Rt = \frac{V^2}{R}t$$

Any one of the three formulas can be used to calculate electrical energy.

## 7.6. APPLICATIONS OF HEATING EFFECT OF ELECTRIC CURRENT

The heating effect of electric current has a large number of applications in everyday life. A few of them are given below by way of illustration.

**(*i*) Incandescent bulb.** It consists of a metal filament of fine wire (generally tungsten) enclosed in a glass bulb [See Fig. 7.3]. Tungsten is used because it has a large value of resistivity and high melting point (3650 K). The bulb also contains a small amount of inert gas such as nitrogen or argon to prevent oxidation and evaporation of the filament. When voltage is applied across the bulb, current flows through the filament. The filament gets heated to a high temperature and soon reaches the stage of incandescence. Note that electrical energy supplied to the electric bulb is converted into heat as well as light. Filament lamps are inefficient because the greater part of energy is emitted as heat and not as light.

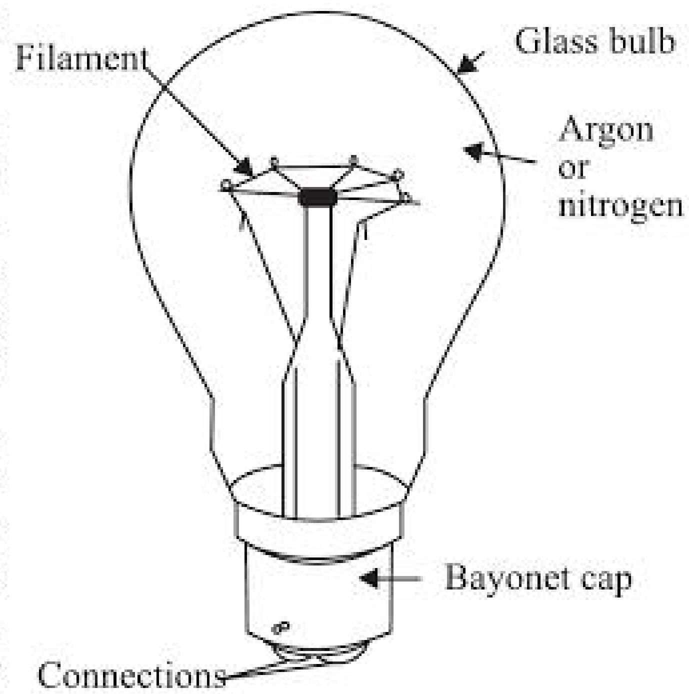



**Fig. 7.3**

**Note.** *Fluorescent tubes are about three times as efficient as filament lamps.* Although initially more expensive than tungsten filament lamps, the running costs are far less because the current is less and they also last longer.

***(ii)* Electric heating appliances.** The heating effect of electric current is utilised in the manufacture of many heating appliances such as electric heater, electric toaster, electric kettle, soldering iron, electric iron etc. The basic principle of all these appliances is the same. Electric current is passed through a high resistance (called heating element), thus producing the required heat. The heating element is generally made of **nichrome** (an alloy of nickel and chromium) due to the following reasons :

(*a*) It has high resistivity ; more than 50 times that of copper.

(*b*) It has high melting point.

(*c*) It is not oxidised easily when heated in air.

The electric power dissipated ($P$) and energy consumed ($W$) by electric heating appliances are :

$$P = VI = I^2 R = \frac{V^2}{R}$$

$$W = VIt = I^2 Rt = \frac{V^2}{R} t$$

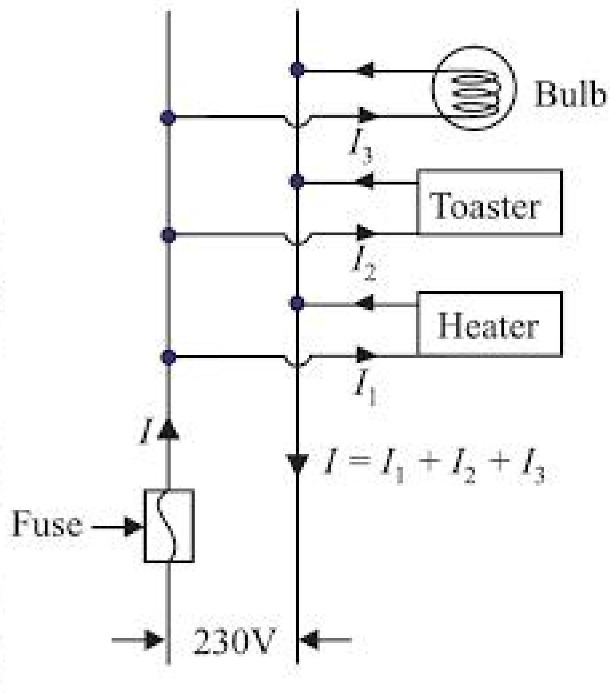


**Fig. 7.4**

***(iii)* Arc lamp.** It consists of two carbon rods separated by a small air-gap. When high voltage is applied across the electrodes, a spark jumps across the gap and a very bright light is emitted.

***(iv)* Fuse.** *A **fuse** is a short piece of metal, inserted in the circuit, which melts when excessive current flows through it and thus breaks the circuit.*

A fuse is made of materials having low melting point and high resistivity. Generally, it is made of tin-lead alloy (63% tin and 37% lead). It is inserted in series with the circuit to be protected and carries the total circuit current as shown in Fig. 7.4. Under normal conditions, the fuse element is at a temperature below its melting point. Therefore, it carries the normal current without overheating. However, when a short-circuit or overload occurs, the current through the fuse increases beyond its normal value. This raises the temperature and fuse element melts (or blows out), disconnecting the circuit protected by it. In this way, fuse protects the electrical appliances from damage due to excessive currents.

**Note.** When a fuse blows, it should be replaced by fuse wire of the same thickness. On no account should it be replaced by a fuse wire of greater thickness.

## 7.7. EFFICIENCY OF ELECTRIC DEVICE

*The efficiency of an electric device is the ratio of useful output power to the input power, i.e.*

$$\text{Efficiency, } \eta = \frac{\text{Useful output power}}{\text{Input power}} = \frac{\text{Useful output Energy}}{\text{Input energy}}$$

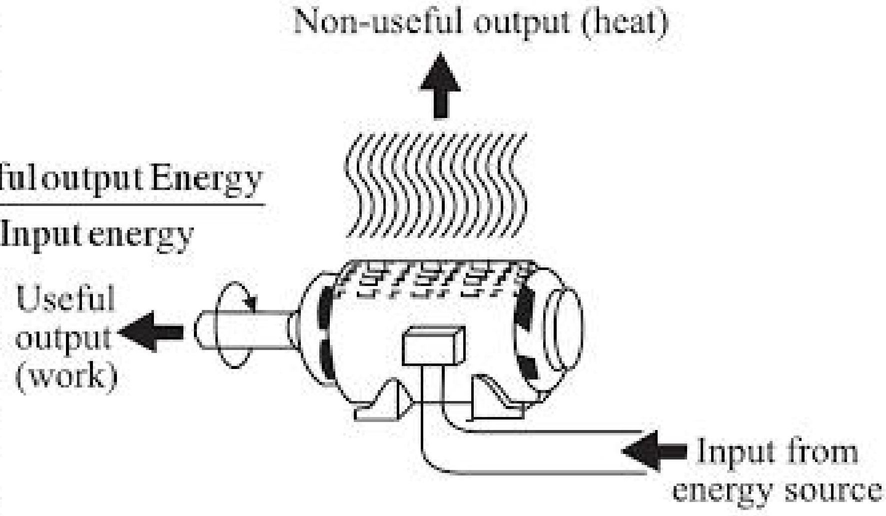


**Fig. 7.5**

The law of conservation of energy states that "energy cannot be created or destroyed but can be converted from one form to another." Some of the input energy to an electric device may be converted into a form that is not useful. For example, consider an electric motor shown in Fig. 7.5. The purpose of the motor is to convert electric energy into mechanical energy. It does this but it also converts a part of input energy into heat. The heat produced is not useful. Therefore, the useful output energy is less than the input energy. In other words, the efficiency of motor is less than 100%.

Some electric devices are nearly 100% efficient. An electric heater is an example. In a heater, the heat is useful output energy and practically all the input electric energy is converted into heat energy.

## 7.8. MAXIMUM POWER TRANSFER THEOREM

When load is connected across a d.c. voltage source, power is transferred from the source to the load. The amount of power transferred will depend upon the load resistance. If load resistance $R_L$ is made equal to the internal resistance $R_i$ of the source, then maximum power is transferred to the load $R_L$. This is known as *maximum power transfer theorem* and can be stated as follows :

*Maximum power is transferred from a d.c. source to a load when the load resistance is made equal to the internal resistance of the source.*

## 7.9. IMPORTANT POINTS

While dealing with problems on heating appliances, the following points should be kept in mind :

(*i*) The electrical energy in kWh can be converted to joules by the following relation :

$$*1 \text{ kWh} = 36 \times 10^5 \text{ joules}$$

(*ii*) The heat energy in calories can be converted into joules by the following relation :

$$1 \text{ cal} = 4.18 \text{ J} \ ; \ 1 \text{ kcal} = 4180 \text{ J}$$

(*iii*) The electrical energy in kWh can be converted into calories (or kilocalories) by the following relation :

$$1 \text{ kWh} = 36 \times 10^5 \text{ J} = \frac{36 \times 10^5}{4.18} \text{cal} = 860 \times 10^3 \text{ cal}$$

$$\therefore \qquad 1 \text{ kWh} = 860 \text{ kcal}$$

(*iv*) The electrical energy supplied to the heating appliance forms the *input energy*. The heat obtained from the device is the *output energy*. The difference between the two, if any, represents the loss of energy during conversion from electrical to heat energy.

Example 7.1. **One electric bulb is marked 60W, 240V and the other is marked 100W, 240V. Which electric bulb has higher resistance?**

**Solution.** Resistance of electric bulb, $R = V^2/P$ [$\because$ $P = V^2/R$ ]

For 60W, 240V bulb, $R_1 = (240)^2/60 = 960\Omega$

For 100W, 240V bulb, $R_2 = (240)^2/100 = 576\Omega$

Therefore, the resistance of 60W, 240V bulb is more than that of 100W, 240V bulb. Note that for the same voltage rating, the resistance of that bulb is more which has lower wattage.

Example 7.2. **An electric bulb is marked 100W, 230V. If the supply voltage drops to 115V, what is the heat and light energy produced by the bulb in 20 minutes?**

**Solution.** Resistane of the bulb, $R = \dfrac{V^2}{P} = \dfrac{(230)^2}{100} = 529\Omega$

The supply voltage drops to 115V, *i.e.*, $V' = 115$ volts

$\therefore$ Energy dissipated by bulb $= \dfrac{V'^2}{R}t = \dfrac{(115)^2}{529} \times (20 \times 60) = 30{,}000\text{J}$

Example 7.3. **An electric iron marked 500W, 200V is to be operated by 240V supply. How will you do it?**

* 1 kWh = 1 kW × 1 hour = 1000 W × 3600 s = $36 \times 10^5$ Ws or joules.

**Solution.** Current rating of electric iron, $I = \frac{P}{V} = (500/200) = 2.5\,A$

For proper operation of electric iron, p.d. across it should be 200V and current through it is to be 2.5A. To operate the electric iron on 240V supply, we will have to connect a resistance $R$ in series with the iron.

Voltage to be dropped across $R = 240 - 200 = 40V$

Current in the circuit $= 2.5A$ $\therefore$ Value of $R = 40/2.5 =$ **16Ω**

Example 7.4. **A 100W, 250V bulb is connected in parallel with an unknown resistance *R* across a 250V supply. The total power dissipated in the circuit is 1100W. Find the value of the unknown resistance. Assume that resistance of lamp remains unaltered.**

**Solution.** The total power dissipated in the circuit is equal to the *sum of the powers dissipated by the bulb and the unknown resistance $R$.

$\therefore$ Power dissipated by $R = 1100 - 100 = 1000W$

$$\text{Value of resistance } R = \frac{V^2}{1000} = \frac{(250)^2}{1000} = \mathbf{62.5\ \Omega}$$

Example 7.5. **Forty similar electric bulbs are connected in series across 220V supply. After one bulb is fused, the remaining 39 bulbs are again connected in series across the same supply. In which case will there be more illumination?**

**Solution.** Suppose the resistance of each bulb is $R$.

First Case: Total resistance of 40 bulbs $= 40R$

$$\text{Circuit current } I_1 = \frac{220}{40R}$$

$$\text{Power dissipated by 40 bulbs, } P_1 = 40\left[I_1^2 R\right] = 40\left(\frac{220}{40R}\right)^2 \times R$$

$$\therefore \quad P_1 = \frac{(220)^2}{40R}\text{ watts} \quad ...(i)$$

Second Case: Total resistance of 39 bulbs $= 39R$

$$\text{Circuit current } I_2 = \frac{220}{39R}$$

$$\text{Power dissipated by 39 bulbs, } P_2 = 39\left[I_2^2 R\right] = 39\left(\frac{220}{39R}\right)^2 \times R$$

$$\therefore \quad P_2 = \frac{(220)^2}{39R}\text{ watts} \quad ...(ii)$$

A look at eqs. (*i*) and (*ii*) shows that $P_2 > P_1$. **Hence, more illumination is produced in the second case.**

Example 7.6. **Two bulbs are marked 220V, 200W and 220V, 100W respectively. They are connected in series to 220V supply. Find the ratio of heats generated in them.**

**Solution.** Resistance of 220 V, 200W bulb, $R_1 = (220)^2/200 = 242\Omega$

Resistance of 220 V, 100W bulb, $R_2 = (220)^2/100 = 484\ \Omega$

When the two bulbs are connected in series across 220V supply, the same current flows through them. Let it be $I$ amperes.

---

* Suppose the resistance of lamp is $R_1$. Since the two are connected in parallel, the total circuit resistance $R_T$ is given by ;

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R} \quad \text{or} \quad \frac{V^2}{R_T} = \frac{V^2}{R_1} + \frac{V^2}{R} \quad \text{or} \quad P_T = P_1 + P$$

Hence when appliances are connected in parallel, the total power dissipated is equal to the sum of the powers dissipated by the individual appliances.

Rate of heat produced by 200W bulb, $H_1 = I^2 R_1 \text{ Js}^{-1}$

Rate of heat produced by 100W bulb, $H_2 = I^2 R_2 \text{ Js}^{-1}$

$$\therefore \qquad \frac{H_1}{H_2} = \frac{R_1}{R_2} = \frac{242}{484} = \frac{1}{2}$$

*Alternatively,* since the bulbs are connected in series, the rate of heats generated in them are directly proportional to their resistances ($\because$ current in them is the same).

$$\therefore \qquad \frac{H_1}{H_2} = \frac{R_1}{R_2} = \frac{242}{484} = \frac{1}{2}$$

Example 7.7. **An electric kettle marked 1kW, 230V takes 7.5 minutes to bring 1 kg of water at 15°C to boiling point (100°C). Find the efficiency of the kettle.**

**Solution.** Heat received by water (*i.e.*, output energy)

$$= mc\theta = 1 \times 1 \times (100 - 15) = 85 \text{ kcal}$$

Electrical energy supplied by heater (*i.e.*, input energy)

$$= \text{Wattage} \times \text{time} = 1 \text{ kW} \times \frac{7.5}{60} \text{hr} = 0.125 \text{kWh}$$

$$= 0.125 \times 860 \text{ kcal} = 107.5 \text{ kcal} \quad (\because \ 1\text{kWh} = 860 \text{ kcal})$$

$$\therefore \qquad \text{Efficiency of kettle} = \frac{85}{107.5} \times 100 = 79.1\%$$

Example 7.8. **In Fig. 7.6 power dissipated in 8Ω resistor is 9W. What is the power dissipated in 12Ω resistor?**

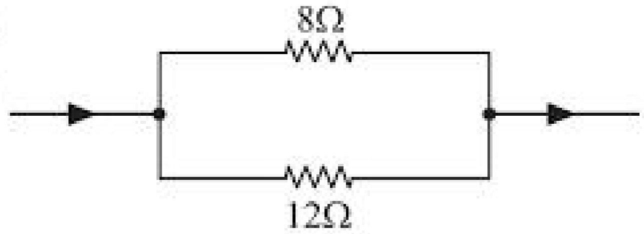


**Fig. 7.6**

**Solution.** $P = \dfrac{V^2}{R}$ or $V = \sqrt{PR}$

Since voltage is same in a parallel circuit,

$$P_1 R_1 = P_2 R_2 \quad \text{or} \quad 9 \times 8 = P_2 \times 12 \quad \therefore \quad P_2 = 6\text{W}$$

Example 7.9. **In Fig. 7.7, the heat produced in 5Ω resistor due to current flowing through it is 10 calories per second. Calculate the heat generated in 4Ω resistor.**

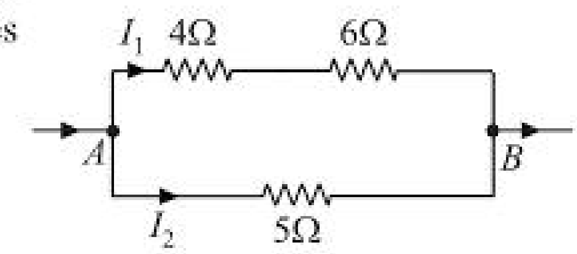


**Fig. 7.7**

**Solution.** Let $I_1$ and $I_2$ be the currents in the two parallel branches as shown in Fig. 7.7. The p.d. across parallel branches is the same, *i.e.*,

$$I_1 (4 + 6) = I_2 \times 5 \quad \therefore \quad I_2 = 2I_1$$

Heat produced per second in 5Ω resistor

$$H_1 = \frac{I_2^2 \times 5}{4.2} = \frac{(2I_1)^2 \times 5}{4.2}$$

or

$$10 = \frac{(2I_1)^2 \times 5}{4.2}$$

$$\therefore \qquad I_1^2 = 2.1$$

Heat produced in 4Ω resistor per second

$$= \frac{I_1^2 \times 4}{4.2} = \frac{2.1 \times 4}{4.2} = 2 \text{ cal/sec}$$

Example 7.10. **A battery of e.m.f. 12V and internal resistance 0.5Ω is connected across a resistor in series with a low resistance ammeter. When the circuit is switched on, the steady reading of the ammeter is 1A. Calculate : (*i*) rate of consumption of chemical energy of the battery, (*ii*) rate of dissipation of energy inside the battery (*iii*) rate of dissipation of energy in the resistor, (*iv*) power output of the source.**

**Solution:** Here $E = 12$ V; $I = 1$A ; $r = 0.5\Omega$

(*i*) Rate of consumption of chemical energy, $P_1 = EI = 12 \times 1 = $ **12W**

(*ii*) Rate of energy dissipation inside the battery, $P_2 = I^2 r = (1)^2 \times 0.5 = $ **0.5W**

(*iii*) Rate of energy dissipation in the resistor $= P_1 - P_2 = 12 - 0.5 = $ **11.5W**

(*iv*) Power output of the source $= P_1 - P_2 = $ **11.5W**

Example 7.11. **A house is fitted with 5 electric bulbs of 100W each, one electric press of 220V drawing 2A of current, 4 fans of 110W each and a heater of 1120W. If all the appliances work for 2 hours a day, find the electricity bill for the month of September. Electrical energy is supplied at Rs. 1.5 per kWh.**

**Solution.**

$$\begin{aligned} \text{Total wattage of the bulbs} &= 5 \times 100 = 500 \text{ W} \\ \text{Wattage of electric press} &= 220 \times 2 = 440 \text{ W} \\ \text{Total wattage of fans} &= 4 \times 110 = 440 \text{ W} \\ \text{Wattage of heater} &= 1120 \text{ W} \\ \text{Total wattage of all the appliances} &= 500 + 440 + 440 + 1120 = 2500 \text{ W} \end{aligned}$$

Energy consumed by all the appliances per day = $2500 \times 2 = 5000$ Wh = 5 kWh

Energy consumed by all the appliances in the month of September

$$= 5 \times 30 = 150 \text{ kWh}$$

$$\text{Bill for the month of September} = \text{Rs. } 1.5 \times 150 = \textbf{Rs. 225}$$

Example 7.12. **An electric bulb marked 60W, 220V will get fused if electric power fed to it becomes 85W or more. What maximum voltage can the bulb withstand ?**

**Solution.** Resistance of bulb, $R = \dfrac{V^2}{P} = \dfrac{(220)^2}{60} = 806.67\,\Omega$

Suppose $V'$ is the supply voltage at which the power consumed by the bulb becomes 85W.

$$\therefore \quad 85 = \frac{(V')^2}{R} \text{ or } V' = \sqrt{85 \times R} = \sqrt{85 \times 806.67} = \textbf{261.8V}$$

Example 7.13. **A 500 W heating unit is designed to operate from a 200 volts line. By what percentage will its heat output drop if the line voltage drops to 160 volts?**

**Solution.** Resistance of heating unit, $R = \dfrac{V^2}{P} = \dfrac{(200)^2}{500} = 80\ \Omega$

When voltage drops to 160 volts, then,

$$\begin{aligned} \text{Heat produced/second} &= \frac{(160)^2}{R} = \frac{(160)^2}{80} = 320 \text{ W} \\ \text{Dorp in heat production} &= 500 - 320 = 180 \text{ W} \\ \therefore \ \% \text{ drop in heat production} &= \frac{180}{500} \times 100 = \textbf{36\%} \end{aligned}$$

Example 7.14. **A 100 W and a 500 W bulbs are joined in series and connected to the mains. Which bulb will glow brighter?**

**Solution.** $P_1 = 100$ W ; $P_2 = 500$ W ; Mains voltage = $V$

Now $$P_1 = \frac{V^2}{R_1} \quad \text{and} \quad P_2 = \frac{V^2}{R_2}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{P_2}{P_1} = \frac{500}{100} = 5$$

When the bulbs are connected in series with the mains, the same current $I$ flows through them. Therefore, new powers are :

$$P_1' = I^2 R_1 \quad \text{and} \quad P_2' = I^2 R_2$$

$$\therefore \quad \frac{P_1'}{P_2'} = \frac{I^2 R_1}{I^2 R_2} = \frac{R_1}{R_2} = 5 \qquad \left(\because \frac{R_1}{R_2} = 5\right)$$

Since $P_1' = 5\, P_2'$, **100 W bulb will glow brighter.**

**Example 7.15. A copper electric kettle weighing 1 kg contains 900 g of water at 20°C. It takes 12 minutes to raise the temperature to 100°C. If electric energy is supplied at 210 V, calculate the strength of the current, assuming that 10% heat is wasted. Specific heat of copper is 0.1.**

**Solution.** Water equivalent of copper kettle, $w = 1000 \times 0.1 = 100$ g; Mass of water, $m = 900$ g; Rise of temperature, $\theta = \theta_2 - \theta_1 = 100 - 20 = 80$°C. Specific heat of water, $c = 1$; $V = 210$ volt ; $t = 12$ min $= 12 \times 60$ sec ; $I = ?$

Heat required to raise temperature of water to 100°C is

$$H_{out} = (m + w)\, c\theta = (900 + 100) \times 1 \times 80 = 80{,}000 \text{ cal} \qquad ...(i)$$

Total heat produced by electric current is

$$H_{in} = \frac{VIt}{4.2} = \frac{210 \times I \times 12 \times 60}{4.2} = 36{,}000\, I \text{ cal}$$

$$\text{Heat utilised} = 90\% \text{ of } H_{in} = \frac{90}{100} \times 36{,}000\, I = 32{,}400\, I \text{ cal} \qquad ...(ii)$$

Since $(ii) = (i)$ so that , we have,

$$32,\,400\, I = 80{,}000 \quad \text{or} \quad I = \frac{80{,}000}{32{,}400} = \mathbf{2.469\ A}$$

**Example 7.16. Determine the percentage by which the illumination of a lamp will decrease if current drops by 20%.**

**Solution.** Illumination of the lamp is directly proportional to the heat produced by electric current.

Now, Heat produced, $H = I^2Rt$

When current drops by 20%, the new circuit current is $0.8I$.

$\therefore$ Heat produced, $H' = (0.8I)^2\, Rt = 0.64\, I^2\, Rt = 0.64\, H$

$\therefore$ %age decrease in illumination of the lamp

$$= \frac{H - H'}{H} \times 100 = \frac{H - 0.64H}{H} \times 100 = \mathbf{36\%}$$

**Example. 7.17. An electric kettle has two heating coils. When one of the coils is switched on, the kettle begins to boil in 6 minutes. When the other is switched on, the boiling begins in 8 minutes. In what time will the boiling begin if both coils are switched on simultaneously (*i*) in series, (*ii*) in parallel?**

**Solution.** Let $R_1$ and $R_2$ be the resistances of the first and second coil respectively. Let $V$ be the supply voltage.

Heat required to begin boiling by first coil is

$$H_1 = \frac{V^2 t_1}{J\, R_1} = \frac{V^2 \times (6 \times 60)}{4.2 \times R_1} \qquad ...\ (i)$$

Heat required to begin boiling by second coil is

$$H_2 = \frac{V^2 t_2}{J\, R_2} = \frac{V^2 \times (8 \times 60)}{4.2 \times R_2} \qquad ...\ (ii)$$

But $H_1 = H_2$

$$\therefore \quad \frac{V^2 \times (6 \times 60)}{4.2 \times R_1} = \frac{V^2 \times (8 \times 60)}{4.2 \times R_2}$$

$$\therefore \qquad R_2 = \frac{4}{3}R_1$$

**(*i*) When coils are in series.** Equivalent resistance, $R_S = R_1 + R_2 = R_1 + \frac{4}{3}R_1 = \frac{7}{3}R_1$

If $t'$ is the time taken to begin boiling, then,

$$H_l = \frac{V^2 t'}{J R_S} = \frac{V^2 t'}{4.2 \times (7R_1/3)} \qquad \text{... } (iii)$$

Comparing eqs. (*i*) and (*iii*), we have,

$$\frac{V^2 t'}{4.2 \times (7R_1/3)} = \frac{V^2 \times 6 \times 60}{4.2 \times R_1}$$

$$\therefore \qquad t' = 14 \times 60 \text{ s} = \textbf{14 minutes}$$

**(*ii*) When coils are in parallel.** Equivalent resistance, $R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 \times (4R_1/3)}{R_1 + (4R_1/3)} = \frac{4}{7}R_1$

It $t''$ is the time taken to begin boiling, then,

$$H_l = \frac{V^2 t''}{J R_P} = \frac{V^2 t''}{4.2 \times 4R_1/7} \qquad \text{... } (iv)$$

Comparing eqs. (*i*) and (*iv*), we have,

$$\frac{V^2 t''}{4.2 \times (4R_1/7)} = \frac{V^2 \times 6 \times 60}{4.2 \times R_1}$$

$$\text{or} \qquad t'' = \frac{24}{7} \times 60 \text{ s} = \frac{24}{7} \text{ min} = \textbf{3.43 min}$$

**Example 7.18. Find out the ratio of heat produced in the four arms of Wheatstone bridge shown in Fig. 7.8.**

**Solution.** Since the product of resistances of the opposite arms are equal, the Wheatstone bridge is balanced.

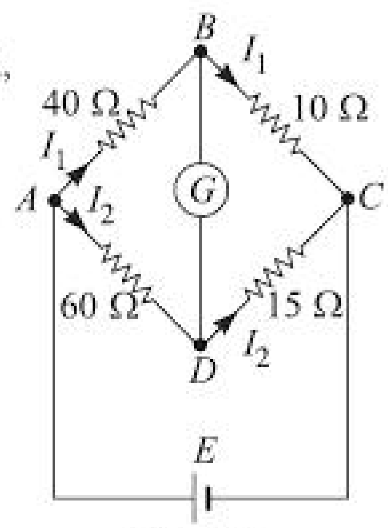


Fig. 7.8

$$\therefore \qquad \text{P.D. across } AB = \text{P.D. across } AD$$

$$\text{or} \qquad 40\, I_1 = 60\, I_2 \quad \text{or} \quad I_1 = 1.5\, I_2$$

$$\text{Heat produced in arm } AB,\ H_1 = I_1^2\, Rt$$
$$= (1.5\, I_2)^2 \times 40 \times t = 90\, I_2^2\, t$$

$$\text{Heat produced in arm } BC,\ H_2 = I_1^2\, Rt$$
$$= (1.5\, I_2)^2 \times 10 \times t = 22.5\, I_2^2\, t$$

$$\text{Heat produced in arm } AD,\ H_3 = I_2^2\, Rt = I_2^2 \times 60 \times t = 60\, I_2^2\, t$$

$$\text{Heat produced in arm } DC,\ H_4 = I_2^2\, Rt = I_2^2 \times 15 \times t = 15\, I_2^2\, t$$

$$\therefore \qquad H_1 : H_2 : H_3 : H_4 = 90\, I_2^2\, t : 22.5\, I_2^2\, t : 60\, I_2^2\, t : 15\, I_2^2\, t$$
$$= 90 : 22.5 : 60 : 15 = \textbf{6 : 1.5 : 4 : 1}$$

**Example 7.19. A thin metallic wire of resistance 100 Ω is immersed in a calorimeter containing 200 g of water at 10° C and a current of 0.5 A is passed through it for half an hour. If water equivalent of the calorimeter be 10 g, find the rise of temperature.**

**Solution.** Heat generated by the current in half hour ($t = 30 \times 60$ s) is

$$H = \frac{I^2 Rt}{4.2} = \frac{(0.5)^2 \times 100 \times (30 \times 60)}{4.2} = 10714 \text{ cal}$$

This heat is absorbed by water and calorimeter.

$$\therefore \qquad H = (m + w)\, c\, \theta \text{ or } \theta = \frac{H}{(m + w)\, c}$$

Here $H = 10714$ cal; $m = 200$ g; $c = 1$ cal g$^{-1}$ °C$^{-1}$; $w = 10$ g

$$\therefore \quad \theta = \frac{10714}{(200+10)\times 1} = \mathbf{51.02°C}$$

**Example 7.20. It is desired to heat a cup of coffee (200 mL) by use of immersion heater from 20° C to 90°C in 0.5 minute. How much current would the heater draw from 230 V supply?**

**Solution.** The specific heat and mass of coffee can be taken as that of water.

The energy needed is $H = cm\,\theta$

Here $c = 4.2$ kJ kg$^{-1}$ °C$^{-1}$; $m = 0.2$ kg; $\theta = 90 - 20 = 70$°C

$$\therefore \quad H = 4.2 \times 0.2 \times 70 = 58.8 \text{ kJ}$$

$$\text{Power needed, } P = \frac{\text{Energy}}{t} = \frac{58.8}{30} = 1.96 \text{ kW} = 1.96\times 10^3 \text{ W}$$

$$\text{Now} \quad P = VI \text{ or } I = \frac{P}{V} = \frac{1.96\times 10^3}{230} = \mathbf{8.52\ A}$$

## PROBLEMS FOR PRACTICE

1. A steady current of 4A flows through a resistor of resistance 10Ω for 5 minutes. Calculate the heat produced. **[11428 calories]**

   [Hint. $H = \dfrac{I^2Rt}{J}$]

2. A 200 W, 220 V bulb is connected in series with a 40 W, 220 V bulb and the combination is connected across 220 V. Which will glow brighter? **[40W bulb]**

   [Hint. Resistance of 40 W bulb is 5 times that of 200 W bulb. Now power dissipated $= I^2R$].

3. What will be the voltage drop across a 1kW electric heater whose resistance when hot is 40Ω? **[200V]**

   [Hint. $P = V^2/R$ or $V = \sqrt{PR} = \sqrt{1000\times 40} = 200$ V ]

4. Two heaters, each marked 500 W, 220 V, are connected in series across 220V supply. What is the total power consumed? Assume that resistance of each heater remains unchanged. **[250 W]**

   [Hint. Let resistance of each heater be $R$. Then,

   $R_T = R + R$ or $\dfrac{R_T}{(230)^2} = \dfrac{R}{(230)^2} + \dfrac{R}{(230)^2}$ or $\dfrac{1}{P_T} = \dfrac{1}{P} + \dfrac{1}{P}$ ]

5. The resistance of each wire shown in Fig. 7.9 is 2Ω and each can have a maximum power of 36 W (otherwise it will melt). What is the maximum power the whole circuit may take? **[54 W]**

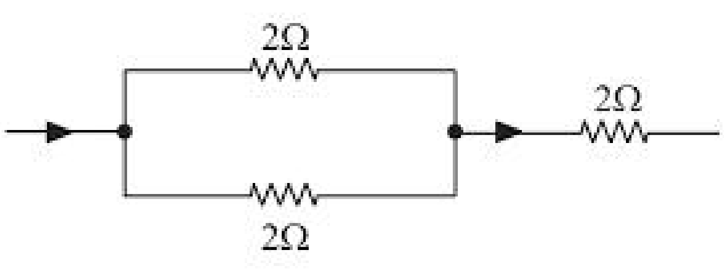



**Fig. 7.9**

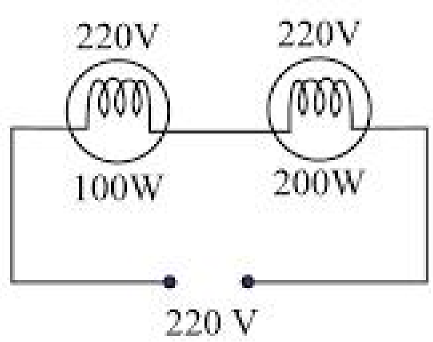



**Fig. 7.10**

[**Hint.** Let $I$ be the current rating of resistor. Then $I^2 R = 36$ or $I^2 \times 2 = 36$ $\therefore$ $I^2 = 18$. The effective resistance of parallel combination = $1\Omega$. Therefore, power dissipation = $I^2 \times 1 = 18 \times 1 = 18$ W. Power dissipation in series resistor = $I^2 R = 18 \times 2 = 36$ W. Therefore, total power dissipation = 18 + 36 = 54 W.]

6. What is the power consumed in the circuit shown in Fig. 7.10? **[66.7 W]**

[**Hint.** $P = \dfrac{P_1 P_2}{P_1 + P_2} = \dfrac{100 \times 200}{100 + 200} = 66.7\,\text{W}$]

7. The heater coil of an electric kettle is rated at 2000 W at 200 V. How much time will it take to heat one litre of water from 20°C to 100°C, assuming that entire energy liberated by heater is utilised by water? **[168 sec]**

[**Hint.** Input energy $= \dfrac{P \times t}{J}$ cals ; Output energy $= m\ c\ \theta$

$$\therefore \quad \frac{P \times t}{J} = mc\theta \text{ or } \frac{2000 \times t}{4.2} = 1000 \times 1 \times (100 - 20) \therefore t = 168\text{s}]$$

8. Two lamps, one rated at 100 W, 240 V and the other 60 W at 240 V, are connected in parallel across 240 V supply. What current is drawn from supply line? **[0.67A]**

9. A 100 MW power station transmits power to a distant city through long cables. Which mode of transmission would you prefer, power transmission at 20,000 V or 200 V? **[20,000 V]**

[**Hint.** $I = P/V$. For 20,000 V transmission, current will be less.]

10. A battery of 6 accumulators, each of e.m.f. 2V and internal resistance 0.5Ω, is charged by 100V d.c. supply. What series resistance is required to limit the charging current to 8A? Using this resistor, find (*i*) power supplied by d.c. source (*ii*) power dissipated as heat, (*iii*) chemical energy stored in the battery in 15 minutes.

**[8Ω, (*i*) 800 W, (*ii*) 704 W, (*iii*) 86400 J]**

[**Hint.** E.M.F. of battery = 6 × 2 = 12 V. Therefore, net charging voltage = 100 –12 = 88V. If $R$ is the required series resistance, then total circuit resistance, $R_T = R + n\ r = R + 6 \times 0.5 = R + 3$.

Charging current, $I = 88/R_T$ or $8 = 88/R + 3$ $\therefore$ $R = 8\Omega$

Power supplied by d.c. source = $100 \times I = 100 \times 8 = 800$ W

Power dissipated as heat = $I^2 R_T = (8)^2 \times (8 + 3) = 704$ W

The chemical energy stored in the battery is equal to total energy supplied by d.c. source *minus* the energy dissipated as heat and = (800 – 704) × 15 × 60 = 86400J.]

11. Two bulbs marked 25W, 220V and 100W, 220V are connected in series across 400V supply. What will happen? **[25W bulb will fuse]**

[**Hint.** Voltage across 25W exceeds 220V.]

12. A fuse wire of circular cross-section has a radius of 0.8 mm. This wire blows at a current of 8A. Calculate the radius of fuse wire that will blow off at a current of 1A.

[**Hint.** $I^2 \propto r^3$] **[0.2 mm]**

13. A 6V battery is being charged at the rate of 20 C/s. Calculate (*i*) the power used to charge the battery, (*ii*) the energy stored in the battery in 1hr.

**[(*i*) 120W, (*ii*) $4.32 \times 10^5$J]**

14. The resistance of an electric heater is 150 ohms and it can take a maximum current of 1.1A. If it is to be operated on 220V supply, how will you do it?

**[50Ω in series]**

15. The following are the details of load on a circuit connected through a supply meter:

(i) Six lamps of 40W each working for 4 hours per day.

(ii) Two fluorescent tubes 125W each working for 2 hours per day.

(iii) One 1000W heater working for 3 hours per day.

If each unit of energy costs Rs. 1.50, what will be the electricity bill for the month of June? **[Rs. 200.7]**

## CONCEPTUAL QUESTIONS

**Q.1. What precaution is taken to protect an electric circuit from becoming overloaded?**

**Ans.** A fuse is inserted in series with the circuit. When current in the circuit exceeds the safe value, the heat produced melts the fuse wire. Consequently, the circuit is broken. This saves the circuit from damage.

**Q.2. What do you mean by safe value of fuse wire current?**

**Ans.** It is the maximum value of current that a fuse wire can carry without getting melted. Under such conditions, the rate of production of heat in the fuse wire is equal to the rate of loss of heat.

**Q.3. What are the factors upon which the safe current of a fuse wire depends?**

**Ans.** The safe current of a fuse wire is given by; $I \propto r^{3/2}/\rho$. Therefore, safe current of a fuse wire depends upon (i) radius of the wire, (ii) resistivity of the material of wire. It also depends upon the surrounding conditions such as temperature etc.

**Q.4. Two bulbs of resistances 75 Ω and 250 Ω are connected in a room. Which will glow brighter? If one of the bulbs is turned off, what will be the effect on illumination?**

**Ans.** All domestic appliances are connected in parallel so that voltage across each bulb is the same. The rate of dissipation of energy, $P = V^2/R$. Therefore, the bulb of smaller resistance will glow brighter. When one of the bulbs is switched off, there will be no effect on the illumination of the other because in a parallel circuit, the operation of each appliance is independent of the other. Therefore, total light in the room will decrease.

**Q.5. Two bulbs, one of 60 W and the other of 100 W, having same voltage rating are connected in series across the supply in a room. Which bulb will glow brighter? If one of the bulbs is switched off, what will be the effect on illumination?**

**Ans.** The 60 W bulb has more resistance ($R = V^2/P$) than that of 100 W bulb. Therefore, when the two bulbs are connected in series, the rate of dissipation of energy $P$ ( $= I^2 R$; $I$ is the same) is more in that bulb which has larger resistance. Consequently, 60 W bulb will glow brighter. When one of the bulbs is switched off, the circuit resistance decreases and hence current increases. As a result, rate of dissipation of energy $P$ (= $I^2R$) increases. Hence, total light in the room will *increase.

**Q.6. Water boils in an electric kettle in 15 minutes after being switched on. Using the same supply, what should be the length of heating element if water is to be boiled in 10 minutes?**

**Ans.** Heat produced is $H = (V^2/R)t$. Now heat produced in the two cases is the same.

For the first case, $H = \dfrac{V^2 A\, t_1}{\rho\, l_1}$; For the second case, $H = \dfrac{V^2 A\, t_2}{\rho\, l_2}$

Dividing first by second, we have, $1 = \dfrac{t_1 l_2}{l_1 t_2}$ or $1 = \dfrac{15 \times l_2}{l_1 \times 10}$ $\therefore\ l_2 = \dfrac{2}{3} l_1$

Hence, the length ($l_2$) of the heating element should be 2/3 of its initial value ($l_1$).

* When bulbs are in series, total power, $P_T = \dfrac{P_1 P_2}{P_1 + P_2} = \dfrac{60 \times 100}{60 + 100} = 37.5$ W

**Q.7. The nichrome heater consumes 1.5 kW and heats up to a temperature of 750°C. A tungsten bulb operating at same voltage operates at a much higher temperature of 1600°C in order to emit light. Does it mean that tungsten bulb consumes more power?**

**Ans.** Not necessarily. The steady temperature attained by a resistor depends not only on the power consumed but also on its characteristics (*e.g.*, surface area, emissivity, etc.) which decides heat loss due to radiation.

**Q.8. A heater in series with a 60W bulb is connected to the supply mains. If 60W bulb is replaced by a 100W bulb, will the heater now give more heat, less heat or same heat?**

**Ans.** In series combination, the rate of loss of heat is $P\ (= I^2R\ )$. When 60W bulb is replaced by a 100W bulb, the circuit resistance decreases ($\because$ resistance of 100W bulb is less than that of 60W bulb). This increases circuit current and hence the rate of loss of heat in the heater. Consequently, the heater will give more heat.

**Q.9. A heater in parallel with a 60W bulb is connected to supply mains. If 60W bulb is replaced by a 100 W bulb, will heater now give more heat, less heat or same heat?**

**Ans.** When the appliances are connected in parallel, they operate independent of each other. Therefore, by replacing 60W bulb by 100W bulb, the heat given by the heater will be unchanged.

**Q.10. A 100W bulb and 60W bulb both rated at 220V are connected in series across 220V supply. If 60W bulb is removed and only 100W bulb remains in the circuit, will illumination increase or decrease?**

**Ans.** The greater the rate of dissipation of energy (*i.e.*, power), the greater is the illumination.

$$\text{For the first case, } P_T = \frac{P_1 P_2}{P_1 + P_2} = \frac{100 \times 60}{100 + 60} = 37.5\text{ W}$$

For the second case, $P_T$ = 100W. Hence, the illumination will increase.

**Q.11. A toaster produces more heat as compared to an electric bulb when these are connected in parallel. Which of the two has larger resistance?**

**Ans.** Since the two are connected in parallel, the rate of production of heat is $P = V^2/R$. As the toaster produces more heat, its resistance must be less than that of the bulb ($\because$ $V$ is the same in the two cases). Hence, electric bulb has larger resistance.

**Q.12. A heater is designed to operate with a power of 1000W on 100V line. It is connected to two resistances of 10Ω and 5Ω as shown in Fig. 7.11. What is the power output of the heater?**

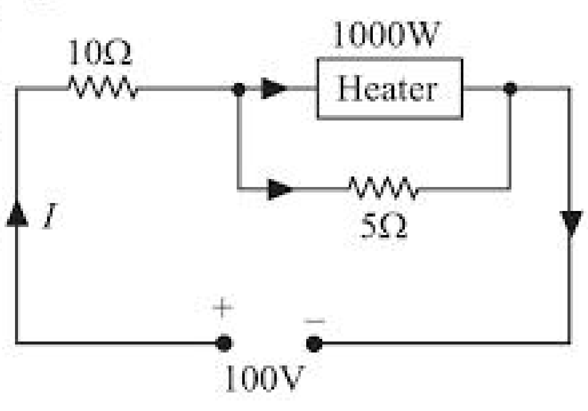



**Fig. 7.11**

**Ans.** Resistance of the heater = $V^2/P = (100)^2/1000 = 10\Omega$

The equivalent resistance of parallel circuit is

$$= \frac{10 \times 5}{10 + 5} = \frac{50}{15} = \frac{10}{3}\ \Omega$$

Total circuit resistance, $R_T$ = $10 + 10/3 = 40/3\Omega$

$$\text{Circuit current, } I = \frac{V}{R_T} = \frac{100 \times 3}{40} = 7.5\text{ A}$$

$$\text{Current in the heater} = 7.5 \times \frac{5}{10 + 5} = 2.5\text{ A}$$

Power output of the heater $= (2.5)^2 \times 10 = 62.5$ W

**Q.13. The wires supplying current to an electric bulb are not heated whereas the filament of the bulb becomes white hot. Explain.**

**Ans.** The resistance of the wires per unit length is negligible as compared to that of the filament of the bulb. As a result, the rate of production of heat in the wires is so small to cause any noticeable rise of temperature. On the other hand, the rate of heat production in the bulb is very high. Consequently, the filament becomes white hot.

**Q.14. What do you mean by power dissipation ?**

**Ans.** Power dissipation usually refers to the rate at which electric energy is converted to heat energy. The resistor dissipates the heat energy by transferring it to surrounding structures and to the atmosphere.

**Q.15. By what percentage will the illumination of the lamp decrease if the current drops by 20% ?**

**Ans.** Heat produced, $H = I^2 R t$

If the current drops by 20%, the circuit current becomes $(80/100)I = (4/5)I$.

$$\therefore \quad \text{Heat produced, } H' = \left(\frac{4}{5}I\right)^2 Rt = \frac{16}{25}I^2 Rt = \frac{16}{25}H$$

$$\therefore \quad \% \text{ decrease in illumination} = \frac{H-H'}{H}\times 100 = \frac{H-\frac{16}{25}H}{H}\times 100$$

$$= \left(1-\frac{16}{25}\right)\times 100 = 36\%$$

## VERY SHORT ANSWER QUESTIONS

**Q.1. What is Joule's law of heating?**

**Ans.** According to Joule's law, when current flows through a conductor, heat produced is directly proportional to (*i*) square of the current through the conductor, (*ii*) resistance of the conductor and (*iii*) time for which current flows in the conductor.

**Q.2. Express Joule's law of heating in (*i*) joules and (*ii*) calories.**

**Ans.** (*i*) Heat produced, $H = I^2 R t$ joules, (*ii*) Heat produced, $H = \frac{I^2 R t}{4.18}$ cal. Here $I$, $R$ and $t$ are in amperes, ohms and seconds respectively.

**Q.3. What is electric power?**

**Ans.** The power of an electric appliance is the time rate at which electrical energy is converted into other form/forms of energy (*e.g.*, heat, light etc).

**Q.4. Give the various formulas for electric power.**

**Ans.** Electric Power, $P = VI = I^2R = V^2/R$.

**Q.5. What is the SI unit of power? Define it.**

**Ans.** The SI unit of power is watt (W) or $Js^{-1}$. The electric power of a circuit or device is 1 watt if a current of 1 A flows through it when a potential difference of one volt is maintained across it ($P = VI = 1\text{ V} \times 1\text{ A} = 1\text{ W}$).

**Q.6. The power of an electric bulb is 60 W. What does it mean?**

**Ans.** It means that the bulb converts 60 J of electrical energy into light and heat in one second.

**Q.7. What is the resistance of 100 W, 220 V bulb?**

**Ans.** $P = \frac{V^2}{R}$ or $R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$

**Q.8. A coil develops heat of 800 cal/sec when 20 V is applied across it. What is the resistance of the coil? Given 1 cal = 4.2 J.**

**Ans.** $P = 800$ cal/sec $= 800 \times 4.2$ J/sec $= 800 \times 4.2$ W

Now $R = \frac{V^2}{P} = \frac{(20)^2}{800 \times 4.2} = 0.12\ \Omega$

**Q.9. The current in a circuit having constant resistance is doubled. How will power dissipation change?**

**Ans.** Power, $P = I^2 R$. Since $R$ is constant, $P \propto I^2$. Therefore, when current is doubled, power dissipation will become 4 times.

**Q.10. Which bulb has greater resistance: 60 W or 100 W bulb when connected to the same supply?**

**Ans.** $P = V^2/R$ or $R = V^2/P$. Since $V$ is same, $R \propto 1/P$. This means that the bulb which has lesser power will have greater resistance. Therefore, 60 W bulb has greater resistance.

**Q.11. Three electric appliances of 100 W, 60 W and 200 W are connected in a house. What is the total power?**

**Ans.** Since electric appliances in a house are always in parallel, the total power is equal to the sum of the powers of the individual appliances, *i.e.*, Total power = 100 + 60 + 200 = 360 W.

**Q.12. A 100 W, 220 V bulb is connected across 110 V supply. What is the power consumed by the bulb?**

**Ans.** Resistance of the bulb, $R = \dfrac{V^2}{P} = \dfrac{(220)^2}{100} = 484\ \Omega$

When connected to 110 V, $P' = \dfrac{(110)^2}{R} = \dfrac{(110)^2}{484} = 25$ W

**Q.13. What is electrical energy?**

**Ans.** The total work done (or energy supplied) by a source of e.m.f. in maintaining electric current in an electrical circuit for a given time is called electrical energy consumed in the circuit during that time.

**Q.14. Give various formulas for electrical energy.**

**Ans.** Electrical energy, $W = VIt = I^2Rt = V^2t/R$

**Q.15. What is the SI unit of electrical energy? Define it.**

**Ans.** The SI unit of electrical energy is joule (J) or watt-sec (W.s). The electrical energy consumed is 1J or 1Ws if an electric appliance works for 1 second ($W = VIt = Pt = 1$ W × 1 s = 1 Ws or 1J).

**Q.16. What is the commercial unit of electrical energy? Define it.**

**Ans.** The commercial unit of electrical energy is *kilowatt-hour* (kWh). It is also called Board of Trade Unit (B.O.T.U.). The electrical energy consumed is one kilowatt-hour (*i.e.*, 1 kWh) if an electrical appliance of power 1 kilowatt (= 1000 W) works for one hour.

Electrical energy consumed in kWh = Power in kW × Time in hours

**Q.17. One kilowatt-hour is equal to how many joules?**

**Ans.** 1 kWh = 1 kW × 1h = (1000 W) × (60 × 60 s) = $3.6 \times 10^6$ J

**Q.18. Why are electrical appliances connected in parallel in homes?**

**Ans.** When electrical appliances are connected in parallel, we can switch on or off any appliance without affecting the operation of other appliances.

**Q.19. A 10 Ω electric heater operates on a 110 V line. Find the rate of heat production.**

**Ans.** Rate of heat production = $\dfrac{V^2}{R} = \dfrac{(110)^2}{10} = 1210$ W or J/s

**Q.20. Find the rate of heat production in calories when a heating coil of resistance 200 Ω is connected to a supply of 100 V.**

**Ans.** Rate of heat production = $\dfrac{V^2}{R}$ J/s = $\dfrac{V^2}{R} \times \dfrac{1}{4.2}$ cal/s = $\dfrac{(100)^2}{200} \times \dfrac{1}{4.2} = 11.9$ cal/s

**Q.21. An electric heater of 1kW operates at 250V for 6 hours. What is the electrical energy consumed?**

**Ans.** Electrical energy consumed = 1 kW × 6 h = 6 kWh

**Q.22. A house consumes 100 units of electrical energy. What does it mean?**

**Ans.** It means that consumption of electrical energy in the house is 100 kWh.

**Q.23. What is the material of the element used in an electric heater?**

**Ans.** Nichrome (an alloy of nickel and chromium).

**Q.24. Why is nichrome used for the element of an electric heater?**

**Ans.** (*i*) It has high resistivity (*ii*) It has high melting point; and (*iii*) It is not oxidised easily when heated in air.

**Q.25. What is a fuse wire?**

**Ans.** It is a short metallic wire inserted in the circuit which melts when excessive current flows through it, thus disconnecting the circuit from the supply.

**Q.26. What should be the characteristics of a fuse?**

**Ans.** The material of the fuse should have high resistivity and low melting point.

**Q.27. What is the composition of the material used in a fuse wire?**

**Ans.** It is made of tin-lead alloy (63% tin and 37% lead).

**Q.28. What is the difference between heating wire and fuse wire?**

**Ans.** (*i*) A heating wire has high resistivity and high melting point while a fuse wire has high resistivity and low melting point, (*ii*) A heating wire is generally made of nichrome while a fuse wire is an alloy of tin and lead.

**Q.29. An electric bulb and a heater are marked 100 W, 220 V and 1000 W, 220 V respectively. Which of the two has greater resistance?**

**Ans.** $R = V^2/P$. For the same $V$ (= 220 V), $R \propto 1/P$. Therefore, 100 W bulb has greater resistance.

**Q.30. If current flowing in a conductor increases by 1%, by what percentage the power will increase?**

**Ans.** $P = I^2R$ and $P' = (1.01I)^2\ R = 1.02\ I^2R = 1.02\ P$

$$\therefore \quad \%\text{ increase in power} = \frac{P'-P}{P}\times 100 = \frac{1.02\,P-P}{P}\times 100 = 2\%$$

**Q.31. A 25 W, 220 V bulb and a 100 W, 220 V bulb are connected in series across 220 V supply. Which bulb will glow brighter?**

**Ans.** When connected in series, the two bulbs will carry the same current $I$. Now $P = I^2\ R$. Since $R$ for 25 W bulb is more, it will glow brighter.

**Q.32. Two bulbs, one 60 W, 220 V and the other 100 W, 220 V, are connected in (*i*) parallel, (*ii*) series across 220 V supply. What is the total power in each case?**

**Ans.** (*i*) Total power, $P_T = P_1 + P_2 = 60 + 100 = 160$ W (*ii*) $\frac{1}{P_T} = \frac{1}{P_1}+\frac{1}{P_2}$ or $\frac{1}{P_T} = \frac{1}{60}+\frac{1}{100}$

$$\therefore \qquad P_T = 37.5\text{ W}$$

**Q.33. In the above question, what is the current drawn from the supply in each case?**

**Ans.** (*i*) $I = \frac{P_T}{V} = \frac{160}{220} = 0.727$ A (*ii*) $I = \frac{P_T}{V} = \frac{37.5}{220} = 0.17$ A

**Q.34. A 60 W bulb is connected to 240 V supply. Find the total charge passing through it in one hour.**

**Ans.** $I = 60/240 = 0.25$ A. Now $q = It = 0.25 \times (60 \times 60) = 900$ coulombs

**Q.35. Equal lengths of silver wire and iron wire having the same diameters are connected in series to a dry cell. Which wire becomes hotter?**

**Ans.** In series connection, the same current flows through the two wires. The rate of production of heat, $P = I^2R$. For a given value of $I$, $P \propto R$. Since the resistance of iron wire is more than that of silver wire, the iron wire becomes hotter.

**Q.36. One length of platinum wire is connected across the terminals of a dry cell. The second platinum wire, identical with the first, is connected across a battery of two dry cells. Which wire becomes hotter?**

**Ans.** The rate of production of heat $P = I^2R$. For the same $R$, $P \propto I^2$. Since current in the second wire is larger, it will become hotter than the first wire.

**Q.37. A wire is connected across a source of constant potential difference. After some time, the wire becomes hot. If cold water is poured on half of its portion, what will happen?**

**Ans.** The half portion of the wire on which cold water is not poured becomes more hot. It is because as cold water is poured, the resistance of cooled portion decreases and hence current through the wire increases. As a result, rate of production of heat $P$ $(= I^2R)$ increases.

**Q.38. Nichrome and copper wires of same lengths and diameters are connected one by one between two points of constant potential difference. Which wire will become hotter?**

**Ans.** The rate of production of heat $P = V^2/R$. For a given $V$, $P \propto 1/R$. Since the resistivity of nichrome is greater than that of copper, the copper wire will have lesser resistance. Hence, rate of production of heat will be more in copper wire, *i.e.*, copper wire becomes hotter.

**Q.39. What do you mean by an overloaded circuit?**

**Ans.** An overloaded circuit is that in which there is more current than the conductors can carry safely.

**Q.40. What specifications are given for electrical appliances?**

**Ans.** (*i*) Power rating, (*ii*) Voltage rating and (*iii*) Current rating.

## SHORT ANSWER QUESTIONS

**Q.1. Prove that when electrical appliances are connected in parallel, total power is equal to the sum of the powers of the individual appliances.**

**Ans.** Consider three electric bulbs of powers $P_1$, $P_2$ and $P_3$ having resistances of $R_1$, $R_2$ and $R_3$ respectively connected in parallel to a supply of voltage $V$. The total resistance $R_T$ of the three appliances is given by ;

$$\frac{1}{R_T} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} \quad \text{or} \quad \frac{V^2}{R_T} = \frac{V^2}{R_1}+\frac{V^2}{R_2}+\frac{V^2}{R_3}$$

$\therefore$ Total power, $P_T = P_1 + P_2 + P_3$ $\quad (\because P = V^2/R)$

**Q.2. Prove that when electrical appliances are connected in series, the reciprocal of total power is equal to the sum of the reciprocal of the powers of the individual appliances.**

**Ans.** Consider three electric bulbs of powers $P_1$, $P_2$ and $P_3$ having resistances $R_1$, $R_2$ and $R_3$ respectively connected in series to a supply of voltage $V$.

Now, $\quad R_1 = V^2/P_1$ ; $R_2 = V^2/P_2$ ; $R_3 = V^2/P_3$

When the three appliances are connected in series, their total resistance $R_T$ is

$$R_T = R_1 + R_2 + R_3 \quad \text{or} \quad \frac{R_T}{V^2} = \frac{R_1}{V^2}+\frac{R_2}{V^2}+\frac{R_3}{V^2}$$

$$\therefore \quad \frac{1}{P_T} = \frac{1}{P_1}+\frac{1}{P_2}+\frac{1}{P_3} \qquad \left(\because \frac{R}{V^2}=\frac{1}{P}\right)$$

**Q.3. Two electric bulbs marked 100 W, 220 V and 200 W, 220 V have tungsten filament of the same length. Which bulb will have thicker filament?**

**Ans.** $R = V^2/P$. Since $V$ is constant, $R \propto 1/P$. Therefore, 100 W bulb will have more resistance than that of 200 W bulb. Now $R = \rho l/A$. Since $\rho$ and $l$ are the same in the two cases, $R \propto 1/A$. Therefore, filament will be thicker (more $A$) for that bulb which has less resistance. Obviously, 200 W bulb will have thicker filament.

**Q.4. The coil of a heater is cut into two equal parts and one of them is used as heating element. What is the ratio of heat produced by this half coil to that of the original coil?**

**Ans.** Suppose the resistance of the original coil is $R$. Then the resistance of the half coil = $R/2$.

For first case ; $H = \dfrac{V^2}{R}t$ ; For second case, $H' = \dfrac{V^2}{R/2}t$

$$\therefore \quad \frac{H}{H'} = \frac{1}{2} \quad \text{or} \quad H' = 2H \text{ or } H' : H = 2 : 1$$

**Q.5. Two identical heater wires are first connected in series and then in parallel. What is the ratio of heat produced in the two cases?**

**Ans.** Suppose the resistance of each heater wire is $R$. When connected in series, total resistance = $R + R = 2R$ and when connected in parallel, total resistance = $R/2$.

For series connection, $H = \dfrac{V^2}{2R}t$ ; For parallel connection; $H' = \dfrac{V^2 t}{R/2}$

$$\therefore \qquad \frac{H}{H'} = \frac{1}{4} \quad \text{or} \quad H : H' = 1 : 4$$

**Q.6. How does a fuse protect electrical installations?**

**Ans.** A fuse is connected in series with the circuit to be protected and carries the total circuit current. When fault occurs (e.g. short circuit), current through the fuse exceeds the permitted value. This raises the temperature and the fuse element melts or blows out, disconnecting the circuit protected by it.

**Q.7. If a high power heater is connected to supply mains, then bulbs in the house become dim. Why?**

**Ans.** When a high power heater is connected to the supply, the heater draws a large current from the supply and there is appreciable voltage drop in the line. As a result, the voltage across the bulbs decreases and they become dim ($P = V^2/R$).

**Q.8. Two bulbs when connected in parallel to 230 V supply take 60 W each. What is the total power consumed when they are connected in series with the same supply?**

**Ans.** $P_1 = P_2 = 60$ W. When they are connected in series, the total power $P_T$ is

$$P_T = \frac{P_1 P_2}{P_1 + P_2} = \frac{60 \times 60}{60 + 60} = 30 \text{ W}$$

**Q.9. Two bulbs of 25 W and 30 W, each rated at 220 V, are connected in series with 440 V supply. Which bulb will fuse and why?**

**Ans.** Voltage across 25 W bulb + Voltage across 30 W bulb = 440 V. The bulbs carry the same current and resistance of 25 W bulb is more than that of 30 W bulb ($\because R = V^2/P$). Therefore, voltage across 25 W bulb will be more than 220 V and that across 30W bulb will be less than 220V. As as result, 25 W bulb will fuse.

**Q.10. Under which conditions is the heat produced in an electric circuit (*i*) directly proportional to resistance and (*ii*) inversely proportional to resistance?**

**Ans.** (*i*) $H = I^2 R t = (V^2/R)\, t$. Therefore, when $I$ is constant in a circuit (*e.g.*, as in a series circuit), $H \propto R$. (*ii*) However, when voltage $V$ in a circuit is constant (*e.g.*, as in a parallel circuit), $H \propto 1/R$.

**Q.11. An electric motor operating at 100 V d.c. draws a current of 10 A. Calculate the resistance of the windings of the motor if its efficiency is 25%.**

**Ans.** Input power = 100 × 10 = 1000 W. Power loss = 0.75 × 1000 = 750 W

Now, $\quad I^2 R = 750 \quad \text{or} \quad (10)^2 \times R = 750 \;\therefore\; R = 750/100 = 7.5\ \Omega$

**Q.12. A potential difference of 10 V is applied across a 2.5 Ω resistor. Calculate (*i*) current, (*ii*) power dissipated and (*iii*) energy converted into heat in 5 minutes.**

**Ans.** (*i*) $I = \frac{V}{R} = \frac{10}{2.5} = 4$ A; (*ii*) $P = VI = 10 \times 4 = 40$ W; (*iii*) $H = \frac{Pt}{J} = \frac{40 \times 5 \times 60}{4.2} = 2860$ cal

**Q.13. The maximum power rating of a 20 Ω resistor is 2 kW. Would you connect this resistor directly across a 300 V d.c. supply of negligible internal resistance?**

**Ans.** No. When this resistor is connected to 300 V d.c. source, the power dissipated or rate of heat production

$$= \frac{V^2}{R} = \frac{(300)^2}{20} = 4500 \text{ W} = 4.5 \text{ kW}$$

Since power dissipation exceeds the maximum power rating (= 2 kW) of the resistor, the resistor will be burnt due to excessive heat. Therefore, the resistor should not be connected directly across 300 V d.c. source.

**Q.14. How much electrical energy is consumed in operating 1 h.p. motor for 5 hours?**

**Ans.** 1 h.p. = 746 W = 0.746 kW

$\therefore$ Electrical energy consumed = 0.746 kW × 5 hr = 3.73 kWh

**Q.15. An electric kettle has two heating elements. One brings water to boiling point in 10 minutes and the other in 15 minutes. If the two elements are connected in parallel, find the time taken to boil the water in the kettle.**

**Ans.** Let $P_1$ and $P_2$ be the dissipating power of the first and second heating elements respectively.

$$\therefore \qquad H = P_1 t_1 = P_2 t_2 = (P_1 + P_2)\, t$$

$$\therefore \qquad t = \frac{H}{P_1 + P_2} = \frac{H}{H/t_1 + H/t_2} = \frac{t_1 t_2}{t_1 + t_2} = \frac{10 \times 15}{10 + 15} = 6 \text{ minutes}$$

**Q.16. In the above question if the two elements are connected in series, find the time taken to boil water in the kettle.**

**Ans.**
$$H = P_1 t_1 = P_2 t_2 = \left(\frac{P_1 P_2}{P_1 + P_2}\right) t$$

$$\therefore \qquad t = \frac{H\,(P_1 + P_2)}{P_1 P_2} = \frac{H\left(\frac{H}{t_1} + \frac{H}{t_2}\right)}{\frac{H}{t_1} \times \frac{H}{t_2}} = \frac{\frac{1}{t_1} + \frac{1}{t_2}}{\frac{1}{t_1 t_2}}$$

$$= t_1 + t_2 = 10 + 15 = 25 \text{ minutes}$$

**Q.17. An electric kettle working at 220 V heats up 2 litres of water from 20°C to 100°C in 2 minutes. What is the current drawn by the kettle?**

**Ans.** Heat produced by kettle = Heat taken by water

or
$$\frac{V I t}{J} = mc\,(\theta_2 - \theta_1)$$

or
$$\frac{220 \times I \times (2 \times 60)}{4.2} = 2000 \times 1 \times (100 - 20) \quad \therefore \quad I = 25.45 \text{ A}$$

**Q.18. A house wiring supplied by 220 V line is protected by 9 A fuse. Find the maximum number of 60 W bulbs that can be turned on.**

**Ans.** In a house, electrical appliances are connected in parallel across the supply line.

Current drawn by one lamp, $I = P/V = 60/220 = 3/11$ A

$$\therefore \text{ Max. no. of lamps that can be turned on } = \frac{9}{I} = \frac{9}{3/11} = 33$$

**Q.19. The resistance of the filament of a bulb is 100 Ω. How much heat in calories will be produced by it in half hour if it is connected to 200 V supply?**

**Ans.** Heat produced
$$= \frac{V^2}{R}\, t = \frac{(200)^2}{100} \times (30 \times 60) \text{ J} = 72 \times 10^4 \text{ J}$$

$$= \frac{72 \times 10^4}{4.2} \text{ cal} = 1.7 \times 10^5 \text{ cal}$$

**Q.20. A 500 W heater is designed to operate on a 220 V line. If the line voltage drops to 200 V, what is the percentage drop in the heat output?**

**Ans.**
$$P = \frac{(220)^2}{R}\ ;\ P' = \frac{(200)^2}{R} \quad \therefore \quad \frac{P'}{P} = \left(\frac{200}{220}\right)^2$$

$$\text{\%age drop in heat output} = \left(\frac{P - P'}{P}\right) \times 100 = \left(1 - \frac{P'}{P}\right) \times 100 = \left[1 - \left(\frac{200}{220}\right)^2\right] \times 100 = 17.36\%$$

## LONG ANSWER QUESTIONS

1. What is heating effect of electric current? Explain the cause of heating effect of electric current. **[Refer to Arts. 7.1 and 7.2]**
2. Derive an expression for heat produced in a conductor carrying electric current. **[Refer to Art. 7.3]**
3. Discuss three applications of heating effect of electric current. **[Refer to Art. 7.6]**
4. Explain the terms (*i*) electric power and (*ii*) electrical energy. **[Refer to Arts. 7.4 and 7.5]**

# COMPETITION SUCCESS MATERIAL

## Useful Concepts/Information

1. Electric Power, $P = I^2 R = \dfrac{V^2}{R}$ watts

   Electrical energy consumed, $W = I^2 R\, t = \dfrac{V^2}{R} t$ joules

   The above formulas apply *only* to resistors and to devices (*e.g.* electric bulb, heater, electric kettle *etc.*) where all electrical energy consumed is converted into heat.

2. Electric power, $P = VI$ watts

   Electrical energy consumed, $W = VIt$ joules.

   These formulas apply to any type of load including the one mentioned in point 1 above.

3. 1 kWh = 1kW × 1 hr = 1000 W × 3600 s = $36 \times 10^5$ Ws or joules ;

   1kcal = 4180 J ; 1 kWh = 860 kcal.

4. Power of electric bulb, $P = V^2/R$ or $R \propto 1/P$ for constant $V$. Thus for the same voltage rating, the bulb with smaller power will have more resistance.

5. A 60W electric bulb means that it converts 60 J of electrical energy into heat in one second.

6. In a house, electrical appliances are connected in parallel.

7. When electrical appliances are connected in parallel, the working of each appliance is independent of the other.

8. When a number of electric bulbs of powers $P_1$, $P_2$, $P_3$......rated at the same voltage $V$ are connected in parallel across voltage $V$, then total power $P_T$ is

   $$P_T = P_1 + P_2 + P_3 + \ldots\ldots$$

9. When a number of electric bulbs of powers $P_1$, $P_2$, $P_3$.... rated at the same voltage $V$ are connected in series across voltage $V$, then total power is

   $$\frac{1}{P_T} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + \ldots\ldots$$

10. The brightness of an electric bulb is directly proportional to the rate of production of heat by the bulb.

11. When two bulbs of different wattage but of the same rated voltage $V$ are connected in series across voltage $2V$, then bulb of smaller rating (*i.e.* wattage) will fuse.

12. In series combination of electric bulbs, the brightness of 60W bulb is more than that of 100W bulb.

13. When the electric bulbs are connected in series, the rates of heats generated in them are directly proportional to their resistances ($\because$ current in them is the same).

14. The electrical energy supplied to the heating appliance forms the *input energy*. The heat obtained from the device is the *output energy*.

    $$\text{Efficiency, } \eta = \frac{\text{Output energy}}{\text{Input energy}}$$

15. Two coils of resistances $R_1$ and $R_2$ take time $t_1$ and $t_2$ respectively to produce a given amount of heat $H$ when connected to the same supply.

(*i*) If the coils are connected in series across the same supply, then time $t$ taken to produce the same amount of heat $H$ is $t = t_1 + t_2$.

(*ii*) If the coils are connected in parallel across the same supply, then time $t$ taken to produce the same amount of heat $H$ is

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} \quad \text{or} \quad t = \frac{t_1 t_2}{t_1 + t_2}$$

**16.** $n$ equal resistors are connected in series across a given supply and dissipate power $P$. If these resistors are connected in parallel across the same supply, they will dissipate power $= n^2P$.

**17.** If $H$ is the heat lost per second per unit surface area of a fuse wire, then,

$$H = \frac{I^2 \rho}{2\pi^2 r^3} \quad \text{or} \quad I^2 \propto r^3 \quad \therefore \quad I \propto r^{3/2}$$

Note that fusing current $I$ is independent of the length of the wire.

**18.** When two resistors $R_1$ and $R_2$ are connected in parallel across a supply and power dissipated in them are $P_1$ and $P_2$ respectively, then,

$$P_1R_1 = P_2R_2$$

**19.** Electrical appliances in a house are connected in parallel because different appliances have the same voltage rating but different current rating.

**20.** The hot resistance of an electric bulb is more than 10 times its cold resistance.

## MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

**1.** An electric heater has a resistance of 12Ω and is operated from 220 V power line. If no heat escapes from it, how much time is required to raise the temperature of 40 kg of water from 15°C to 80°C ? **[CBSE PMT 2001]**

(*a*) 2400 sec (*b*) 2708 sec
(*c*) 2500 sec (*d*) 2600 sec

**2.** An electric fan and a heater are marked as 100 W, 220 V and 1000 W, 220 V respectively. The resistance of heater is **[CPMT 1990]**

(*a*) equal to that of fan
(*b*) greater than that of fan
(*c*) lesser than that of fan
(*d*) zero

**3.** A 100W, 220V bulb is connected across 110V supply. The actual power consumed by the bulb will be **[AFMC 2000]**

(*a*) 200 W (*b*) 50 W
(*c*) 100 W (*d*) 25 W

**4.** Same length of two identical wires are first connected in series and then in parallel across the same supply. The rate of amount of heat produced in the two cases are in the ratio **[AIIMS 2000]**

(*a*) 4 : 1 (*b*) 1 : 4
(*c*) 1 : 2 (*d*) 2 : 1

**5.** A battery of e.m.f. 10 V and internal resistance 0.5Ω is connected across a variable resistance $R$. The value of $R$ for which the power delivered to it is maximum is **[CBSE PMT 1992]**

(*a*) 0.5Ω (*b*) 1Ω
(*c*) 2Ω (*d*) 0.25Ω

**6.** If a high power heater is connected to electric mains, then the bulbs in the house become dim because there is **[Pb PMT 2000]**

(*a*) current drop (*b*) potential drop
(*c*) no current drop (*d*) no potential drop

**7.** A wire of resistance 20Ω is covered with ice and a voltage of 210V is applied across the wire. Then rate of melting of ice is **[AFMC 1997]**

(*a*) 4.2 g/sec (*b*) 2.4 g/sec
(*c*) 1.2 g/sec (*d*) 6.6 g/sec

**8.** A heater at 220 V heats a volume of water in 5 minutes time. If the voltage is 110 V, the time taken to heat the same volume of water will be **[AFMC 1993]**

(a) 5 minutes (b) 8 minutes
(c) 20 minutes (d) 10 minutes

9. The resistance of two bulbs of the same voltage are in the ratio 1 : 2. On connecting them in parallel, the power consumption will be in the ratio **[MP PMT 1994]**

(a) 2 : 1 (b) 1 : 1
(c) 1 : 4 (d) 1 : 2

10. If current flowing in a conductor increases by 1%, then power consumption will increase by **[AFMC 1996]**

(a) 10% (b) 2%
(c) 1% (d) 100%

11. A 5°C rise in temperature is observed in a conductor by passing a current. When the current is doubled, the rise in temperature will be approximately **[CBSE PMT 1998]**

(a) 10°C (b) 12°C
(c) 16°C (d) 20°C

12. An electric iron which when hot has a resistance of 80Ω is used on a 200V source. Calculate the electric energy consumed in kilowatt-hour if it is used for 2 hours. **[CPMT 1997]**

(a) 2 kWh (b) 1 kWh
(c) 8 kWh (d) 32 kWh

13. If 2.2 kW power is transmitted through a 10 Ω line at 22000V, the power loss in the form of heat will be **[MP PMT 1998]**

(a) 1W (b) 10 W
(c) 0.1W (d) 100 W

14. Two resistance filaments of the same length are connected first in series and then in parallel. Find the ratio of power dissipated in both cases assuming that equal current flows in the main circuit. **[AIIMS 2000]**

(a) 4 : 1 (b) 1 : 4
(c) 1 : 2 (d) 2 : 1

15. A heating coil is labelled 100W, 220V. The coil is cut into two equal pieces which are joined in parallel to the same source. The energy now liberated per second is **[CBSE PMT 1995]**

(a) 25 $Js^{-1}$ (b) 50 $Js^{-1}$
(c) 200 $Js^{-1}$ (d) 400 $Js^{-1}$

## ANSWERS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

| | | | | |
|---|---|---|---|---|
| **1.** (b) | **2.** (c) | **3.** (d) | **4.** (b) | **5.** (a) |
| **6.** (b) | **7.** (d) | **8.** (c) | **9.** (a) | **10.** (b) |
| **11.** (d) | **12.** (b) | **13.** (c) | **14.** (a) | **15.** (d) |

## HINTS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. Heat required to raise the temperature of 40 kg of water from 15°C to 80°C is

$$H = mc\theta = (40 \times 10^3) \times 1 \times (80 - 15) = 26 \times 10^5 \text{ cal}$$

Now $$H = \frac{V^2 t}{R \times 4.2} \text{ cal or } 26 \times 10^5 = \frac{(220)^2 \times t}{12 \times 4.2}$$

∴ $$t = \frac{26 \times 10^5 \times 12 \times 4.2}{(220)^2} = 2708 \text{ sec}$$

2. $P = V^2/R$. For the same $V$, $R \propto 1/P$. Therefore, resistance of heater is lesser than that of fan.

3. Resistance of the bulb, $$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\,\Omega$$

Actual power consumed by bulb, $$P' = \frac{(V')^2}{R} = \frac{(110)^2}{484} = 25\text{W}$$

4. Let $R$ be the resistance of each wire. When the wires are connected in series, the equivalent resistance $R_s = R + R = 2R$. When the wires are connected in parallel, the

equivalent resistance $R_P = R \times R/\ (R + R) = R/2$. Heat produced per second = $V^2/R$ or $H \propto 1/R$ .

$$\therefore \quad \frac{H_S}{H_P} = \frac{R_P}{R_S} = \frac{R/2}{2R} = \frac{1}{4} \quad \therefore\ H_S : H_P = \mathbf{1 : 4}$$

5. According to maximum power transfer theorem, maximum power is transferred from the source (*i.e.* battery) to the load when the load resistance is equal to the internal resistance of the source. Therefore, the required value of $R$ is **0.5 Ω.**

6. When a high power heater is connected to the electric mains, the heater draws a large current from the mains and there is appreciable **potential drop** in the line. As a result, the voltage across the bulbs decreases and they become dim ($P = V^2/R$).

7. Heat produced by the wire in 1 sec, $H = \frac{V^2}{R} = \frac{(210)^2}{20} = 2205\ \text{J/sec}$. Let $m$ gram/sec of ice melts due to heat produced by wire. Then,

$$mL = 2205 \text{ or } m = \frac{2205}{L} = \frac{2205}{335} = \mathbf{6.6\ g/sec}$$

Here $L$ is the latent heat of fusion of ice and is equal to 335 J/g.

8. Heat produced, $H = \frac{V^2 t}{R}$. For same $R$, $H \propto V^2\, t$.

$$\therefore \quad V_1^2 t_1 = V_2^2 t_2 \text{ or } (220)^2 \times 5 = (110)^2 \times t_2 \quad \therefore\ t_2 = 5 \times \frac{(220)^2}{(110)^2} = \mathbf{20\ minutes}$$

9. $$P = \frac{V^2}{R}. \text{ For the same } V,\ P \propto 1/R .$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{R_2}{R_1} = \frac{2}{1} \text{ or } P_1 : P_2 = \mathbf{2 : 1}$$

10. $P = I^2R$ and $P_1 = (1.01I)^2\, R = 1.02I^2R = 1.02P$

$$\therefore\ \%\ \text{increase in power} = \frac{P_1 - P}{P} \times 100 = \frac{1.02P - P}{P} \times 100 = \mathbf{2\%}$$

11. $$\text{H} \propto I^2 \quad \therefore\ \frac{H_2}{H_1} = \left(\frac{2I}{I}\right)^2 = 4. \text{ Now } \theta \propto H$$

$$\therefore \quad \frac{\theta_2}{\theta_1} = \frac{H_2}{H_1} = 4 \text{ or } \theta_2 = 4\theta_1 = 4 \times 5 = \mathbf{20°\ C}$$

12. Electric energy consumed = $\frac{V^2 t}{R} = \frac{(200)^2 \times 2}{80} = 1000\ \text{Wh}$

$$= 1000 \times 10^{-3}\ \text{kWh} = \mathbf{1\ kWh}$$

13. Current in the line, $I = \frac{P}{V} = \frac{2.2 \times 10^3}{22000} = \frac{1}{10}\ \text{A}$

$\therefore$ Power loss in the line = $I^2R = (1/10)^2 \times 10 =$ **0.1 W**

**14.** When connected in series, the equivalent resistance $R_S = R + R = 2R$ and when connected in parallel, equivalent resistance $R_P = R \times R/(R + R) = R/2$.

$$\therefore \quad \frac{P_S}{P_P} = \frac{I^2 R_S}{I^2 R_P} = \frac{I^2 \times 2R}{I^2 \times R/2} = 4 \quad \therefore P_S : P_P = \mathbf{4 : 1}$$

**15.** Let $R$ be the resistance of the heating coil. Then $P = \frac{V^2}{R} = \frac{(220)^2}{R} = 100\,\text{W}$.

When the heating coil is cut into two equal parts, the resistance of each part $= R/2$. When these are connected in parallel, equivalent resistance $= R/4$.

$$\therefore \text{ New power, } P' = \frac{V^2}{R/4} = \frac{(220)^2}{R/4} = 4 \times \frac{(20)^2}{R} = 4P = 4\times100 = 400\text{W} = \mathbf{400\ Js^{-1}}$$

## MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

**1.** A fuse wire with a radius of 1mm blows at 15 A. If the fuse wire of the same material should blow at 30A, the radius of the wire must be **[Haryana CEET 1991]**

(*a*) $4^{1/3}$ mm (*b*) $\sqrt{2}$ mm
(*c*) 0.5 mm (*d*) 8.0 mm

**2.** A condenser having a capacity of 2μF is charged to 200V and then the plates of the capacitor are connected to a resistance wire. The heat produced is **[KCET 1992]**

(*a*) $2 \times 10^{-2}$ J (*b*) $4 \times 10^{-2}$ J
(*c*) $4 \times 10^{4}$ J (*d*) $4 \times 10^{10}$ J

**3.** Heat developed in a wire of resistance $R$ by a current of $I$ amperes for time $t$ seconds is **[MP PET 1993]**

(*a*) $\frac{I^2 t}{4.2R}$ calories (*b*) $\frac{Rt}{4.2I^2}$ calories
(*c*) $\frac{I^2 Rt}{4.2}$ calories (*d*) $\frac{I^2 R}{4.2t}$ calories

**4.** A 2 kW boiler used for 1 hour everyday consumes electrical energy in thirty days equal to **[Haryana CEET 1993]**

(*a*) 120 units (*b*) 60 units
(*c*) 15 units (*d*) none of the above

**5.** In an ordinary electric heater if the length of the coil is reduced to half, a given quantity of water will boil in **[Haryana CEET 1993]**

(*a*) more time (*b*) same time
(*c*) less time
(*d*) a time which depends on the resistivity of the wire

**6.** An electric bulb rated for 500 W, 100 V is used in a circuit having a 200 V supply. The resistance $R$ that must be connected in series with the bulb so that the bulb delivers 500 W is **[CEE Delhi 2002]**

(*a*) 80 Ω (*b*) 40 Ω
(*c*) 10 Ω (*d*) 20 Ω

**7.** A coil develops heat of 800 cal/sec when 20V is applied across its end. The resistance of the coil is (1cal = 4.2J) **[MP PET 1994]**

(*a*) 0.12Ω (*b*) 1.12Ω
(*c*) 0.14Ω (*d*) 1.4Ω

**8.** In the circuit shown in Fig. 7.12 the heat produced by 6Ω resistance due to current flowing in it is 60 cal/sec. The heat generated in 3Ω resistance per second will be **[MP PET 1996]**

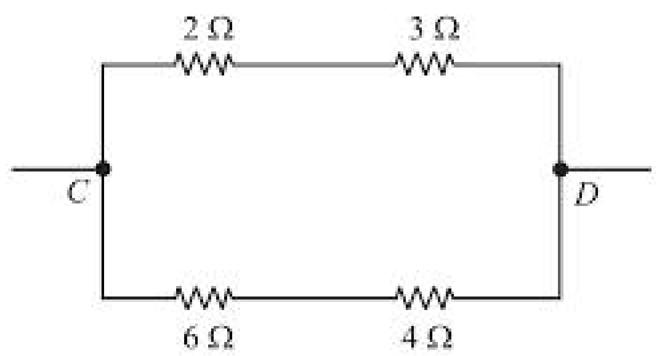



**Fig. 7.12**

(*a*) 30 calories (*b*) 60 calories
(*c*) 100 calories (*d*) 120 calories

**9.** An electric kettle takes 4A at 220V. How much time it will take to boil 1 kg of water from room temperature of 20°C? The temperature of boiling water is 100°C. **[CEE Delhi 1999]**

(*a*) 12.65 min (*b*) 6.36 min
(*c*) 9.45 min (*d*) 16.78 min

10. The resistor $R_1$ dissipates power $P$ when connected to a generator. If a resistor $R_2$ is inserted in series with $R_1$ the power dissipated by $R_1$ **[Haryana CEET 1998]**

(*a*) increases (*b*) decreases

(*c*) remains the same

(*d*) may increase or decrease depending upon the values of $R_1$ and $R_2$

11. Two bulbs when connected in parallel to a source take 60 W each. The total power consumed when they are connected in series with the same source is **[AMU 1998]**

(*a*) 15 W (*b*) 60 W

(*c*) 120 W (*d*) 30 W

12. A constant voltage is applied between two points of a metallic wire. Some heat is developed in it. The heat developed is doubled if **[AMU 1998]**

(*a*) both the length and the radius of the wire are doubled

(*b*) both the length and the radius of the wire are halved

(*c*) the radius of the wire is doubled

(*d*) the length of the wire is doubled

13. An electric kettle has two heating elements. One brings water to boiling in 10 minutes and the other in 15 minutes. If the two elements are connected in parallel, water will boil in **[KCET 2000]**

(*a*) 8 minutes (*b*) 5 minutes

(*c*) 6 minutes (*d*) 25 minutes

14. Time taken by 836W heater to heat 1 litre of water from 10°C to 40°C is **[BHU 2000]**

(*a*) 150 sec (*b*) 100 sec

(*c*) 50 sec (*d*) 200 sec

15. If two bulbs of 25 W and 30 W, each rated at 220 V, are connected in series with 440V supply, which bulb will fuse? **[MP PET 2000]**

(*a*) 30 W bulb (*b*) 25 W bulb

(*c*) neither of them (*d*) both of them

## ANSWERS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

| | | | | |
|---|---|---|---|---|
| **1.** (*a*) | **2.** (*b*) | **3.** (*c*) | **4.** (*b*) | **5.** (*c*) |
| **6.** (*d*) | **7.** (*a*) | **8.** (*d*) | **9.** (*b*) | **10.** (*b*) |
| **11.** (*d*) | **12.** (*a*) | **13.** (*c*) | **14.** (*a*) | **15.** (*b*) |

## HINTS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. Fusing current, $I \propto r^{3/2}$. $\therefore$ $\dfrac{I_1}{I_2} = \left(\dfrac{r_1}{r_2}\right)^{3/2}$ or $\dfrac{r_2}{r_1} = \left(\dfrac{I_2}{I_1}\right)^{2/3}$

$$\therefore \quad r_2 = r_1 \times \left(\frac{I_2}{I_1}\right)^{2/3} = 1 \times \left(\frac{30}{15}\right)^{2/3} = 1 \times (2)^{2/3} = \mathbf{4^{1/3}\ mm}$$

2. Energy stored in the capacitor $= \dfrac{1}{2}CV^2 = \dfrac{1}{2} \times 2 \times 10^{-6} \times (200)^2 = 4 \times 10^{-2}$ J

Energy stored in the capacitor is dissipated as heat in the resistance wire. Therefore, heat produced is **$4 \times 10^{-2}$ J.**

3. Heat produced = $I^2 Rt$ joules = $\dfrac{I^2 Rt}{4.2}$ **calories**

4. Electrical energy consumed per day = $P \times t = 2 \times 1 = 2$ kWh = 2 units

Electrical energy consumed in 30 days = $2 \times 30$ = **60 units**

5. Power, $P = V^2/R$. When the length of the coil is reduced, the resistance $R$ of the wire is decreased. Therefore, power will increase and the **water boils in less time.**

6. Rated current of the bulb, $I = 500/100 = 5A$. In order that the bulb delivers 500 W, current through it should be 5A and voltage across it should be 100 V as shown in Fig. 7.13. Therefore voltage across $R = 200 - 100 = 100V$.

$$\therefore \quad R = V/I = 100/5 = \mathbf{20\Omega}$$

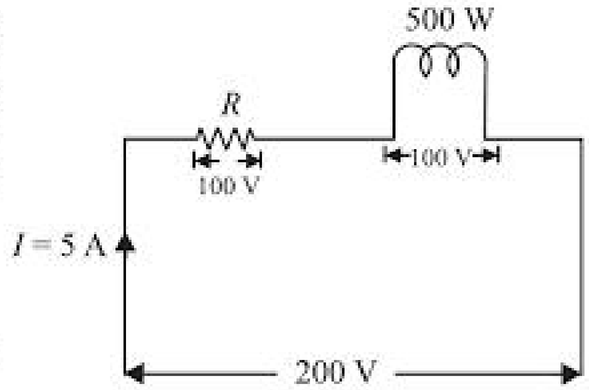


**Fig. 7.13**

7. $P = 800$ cal/sec $= 800 \times 4.2$ J/sec $= 800 \times 4.2$ W

$$\therefore \quad R = \frac{V^2}{P} = \frac{(20)^2}{800 \times 4.2} = \mathbf{0.12\ \Omega}$$

8. Let $I$ be the current through $6\Omega$ resistor. Then $P = I^2 R$ or $60 \times 4.2 = I^2 \times 6$. Therefore, $I = \sqrt{42}$ A. Now p.d. across $CD = \sqrt{42} \times 10$ V.

Current through $3\Omega$ resistor, $I' = \dfrac{\sqrt{42} \times 10}{5} = 2\sqrt{42}$ A

$\therefore$ Heat generated in $3\Omega$/sec $= \dfrac{(I')^2 \times 3}{4.2} = \dfrac{(2\sqrt{42})^2 \times 3}{4.2} = \mathbf{120\ cal}$

9. Let $t$ sec be the required time. Electrical energy supplied is

$$W = VIt \text{ joules} = 220 \times 4 \times t \text{ joules}$$

Heat required to raise the temperature of 1 kg (= 1000g) of water from 20°C to 100°C is

$$H = mc\theta = 1000 \times 1 \times (100 - 20) \text{ cal} = 1000 \times 80 \times 4.2 \text{ J}$$

Since $W = H$

$$\therefore \quad 220 \times 4 \times t = 1000 \times 80 \times 4.2 \text{ or } t = \frac{1000 \times 80 \times 4.2}{220 \times 4} = 381.8\text{s} = \mathbf{6.36\ min}$$

10. Power dissipated by $R_1, P = I^2 R_1$. When a resistor $R_2$ is connected in series with $R_1$ current decreases and hence the power dissipated by $R_1$ **decreases.**

11. $P_1 = P_2 = 60$W. When they are connected in series, the total power $P_T$ is

$$P_T = \frac{P_1 P_2}{P_1 + P_2} = \frac{60 \times 60}{60 + 60} = \mathbf{30\ W}$$

12. Heat produced per second, $H = \dfrac{V^2}{R} = \dfrac{V^2}{\rho \frac{l}{A}} = \dfrac{V^2 A}{\rho l} \quad \therefore H \propto \dfrac{A}{l}$

If **both the length and the radius of the wire are doubled,** the ratio $A/l$ becomes 2 times the initial value. Therefore, option (*a*) is correct.

13. Let $P_1$ and $P_2$ be the dissipating power of the first and second heating elements respectively. Then,

$$H = P_1 t_1 = P_2 t_2 = (P_1 + P_2)t$$

$$\therefore \quad t = \frac{H}{P_1 + P_2} = \frac{H}{H/t_1 + H/t_2} = \frac{t_1 t_2}{t_1 + t_2} = \frac{10 \times 15}{10 + 15} = \mathbf{6\ minutes}$$

14. Let $t$ seconds be the required time. Electrical energy supplied is

$$W = P \times t = 836t \text{ joules} = \frac{836t}{4.2} \text{ cal.}$$

Heat required to raise the temperature of 1 litre (=1000 g) of water from 10°C to 40°C is $H = mc\theta = 1000 \times 1 \times (40 - 10) = 1000 \times 30$ cal.

$$\therefore \quad \frac{836t}{4.2} = 1000 \times 30 \text{ or } t = \frac{1000 \times 30 \times 4.2}{836} = \textbf{150 sec}$$

**15.** Resistance of bulb $R = V^2/P$. Clearly, the resistance of 25W bulb will be more than that of 30W bulb. When the bulbs are connected in series across 440V supply,

Voltage across 25W bulb + Voltage across 30W bulb = 440V

Since current is the same and resistance of 25W bulb is more than that of 30W bulb, the voltage across 25W bulb will be greater than 220V and that across 30W bulb will be less than 220V. Therefore, **25W bulb will fuse.**

## NUMERICAL PROBLEMS FOR COMPETITIVE EXAMINATIONS

**1. A potential difference of 10 V is applied across a 2.5Ω resistor. Calculate the current, the power dissipated and the energy transformed into heat in 5 minutes.**

**Hint.**

$$I = \frac{V}{R} = \frac{10}{2.5} = \textbf{4 A}$$

$$P = VI = 10 \times 4 = \textbf{40 W}$$

$$H = \frac{P\,t}{J} = \frac{40 \times (5 \times 60)}{4.2} = \mathbf{2.86 \times 10^3\ cal}$$

**2. The maximum power rating of a 20Ω resistor is 2 kW (*i.e.*, this is the maximum power the resistor can dissipate as heat without melting or changing in some undesirable way). Would you connect this resistor directly across a 300 V d.c. source of negligible internal resistance ?**

**Hint.** Maximum power rating of resistor = 2 kW

When this resistor is connected to 300 V d.c. source, the power consumed or rate of production of heat

$$= \frac{V^2}{R} = \frac{(300)^2}{20} = 4500 \text{ W} = 4.5 \text{ kW}$$

Since the power consumed exceeds the maximum power rating of the resistor, the 20Ω resistor **should not be connected** directly across the 300 V d.c. source.

**3. An electric bulb rates for 500 W at 100 V is used in a circuit having 200 V supply. Calculate the resistance $R$ that one must put in series with the bulb so that the bulb delivers 500 W. [I.I.T.]**

**Hint.** Resistance of bulb, $R_b = \dfrac{V^2}{P} = \dfrac{(100)^2}{500} = 20\ \Omega$

Rated current of bulb, $I = \dfrac{V}{R_b} = \dfrac{100}{20} = 5$ A

In order that bulb delivers 500 W, the voltage across the bulb should be 100 V and current through it should be 5 A. When the bulb is used in a circuit having 200 V supply, a resistance $R$ must be put in series with the bulb so that circuit current is 5 A and voltage across bulb is 100 V.

$\therefore$ Voltage across $R = 200 - 100 = 100$ V

$$\text{Value of } R = \frac{100}{5} = 20\ \Omega$$

**4. The resistance of a 240 V and 200 W electric bulb when hot is 10 times the resistance when cold. Find its resistance at room temperature. If the working temperature of the filament is 2000°C, find the temperature co-efficient of the filament.**

**Hint.** Resistance of hot bulb, $R_2 = \dfrac{V^2}{P} = \dfrac{(240)^2}{200} = 288\ \Omega$

$\therefore$ Resistance of the bulb at room temperature is $R_1 = \dfrac{R_2}{10} = \dfrac{288}{10} = 28.8\Omega$

Temperature co-efficient of the filament is $\alpha = \dfrac{R_2 - R_1}{R_1\, t} = \dfrac{288 - 28.8}{28.8 \times 2000}$

$$= 4.5 \times 10^{-3} \text{ per °C}$$

**5. An electric motor operating at 100 V d.c. supply draws a current of 10 A. Calculate the resistance of the windings of the motor if its efficiency is 25%.**

**Hint.** Since efficiency of the motor is 25%, power loss due to heat is 75%.

$\therefore$ Power loss $= 0.75 \times EI = 0.75 \times 100 \times 10 = 750$ W

Now $I^2R = 750$

$$\therefore \quad R = \frac{750}{I^2} = \frac{750}{(10)^2} = 7.5\ \Omega$$

**6. A fuse of lead wire has an area of cross-section 0.2 mm². On short circuiting, the current in the fuse wire is 30 A. How long after the short circuiting will the fuse begin to melt ? For lead, specific heat = 0.032 cal g⁻¹ °C⁻¹, melting point = 327°C, density = 11.34 g cm⁻³ and resistivity = 22 × 10⁻⁶ Ω cm. The initial temperature of the wire is 20°C. Neglect heat losses. [I.I.T.]**

**Hint.** Suppose length of fuse wire is $l$ metre.

Resistance of fuse wire, $R = \rho \dfrac{l}{A}$

Here $\rho = 22 \times 10^{-6}\ \Omega$ cm $= 22 \times 10^{-8}\Omega$ m; $A = 0.2$ mm$^2 = 2 \times 10^{-7}$ m$^2$

$$\therefore \quad R = 22 \times 10^{-8} \times \frac{l}{2 \times 10^{-7}} = 1.1\, l$$

If $t$ is the time required for melting the fuse, then heat developed in time $t$ is

$$H = \frac{I^2 Rt}{4.2} = \frac{(30)^2 \times (1.1\, l) \times t}{4.2} \text{ cal}$$

$$\therefore \quad H = 235.7\ lt \text{ cal} \qquad \ldots (i)$$

Heat gained by fuse wire is $H = mc\theta$

Here $m$ = mass of wire $= Al \times$ density $= 2 \times 10^{-7} \times l \times 11.34 \times 10^3$ kg

$c = 0.032 \times 10^3$ cal kg$^{-1}$ °C$^{-1}$; $\theta = 327 - 20 = 307$°C

$$\therefore \quad H = (2 \times 10^{-7} \times l \times 11.34 \times 10^3) \times (0.032 \times 10^3) \times 307 = 22.28\ l$$

# N.C.E.R.T. TEXTBOOK EXERCISES

## NCERT CHAPTER 3 : CURRENT ELECTRICITY

**Q.1. The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω what is the maximum current that can be drawn from the battery?**

**Ans.** Here, $E = 12$ V; $r = 0.4\ \Omega$

The current drawn from the battery will be maximum when external resistance in the circuit is zero.

$$\therefore \qquad I_{max} = \frac{E}{r} = \frac{12}{0.4} = \mathbf{30\ A}$$

**Q.2. A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?**

**Ans.** Here, $E = 10$ V ; $r = 3\ \Omega$ ; $I = 0.5$ A

Let $R$ be the resistance of the resistor. Then,

$$I = \frac{E}{R+r} \quad \text{or} \quad 0.5 = \frac{10}{R+3} \qquad \therefore R = \mathbf{17\ \Omega}$$

Terminal voltage, $V = I\,R = 0.5 \times 17 = \mathbf{8.5\ V}$

*Alternatively,* $V = E - I\,r = 10 - 0.5 \times 3 = \mathbf{8.5\ V}$

**Q.3.** (*a*) **Three resistors 1 Ω, 2 Ω and 3 Ω are combined in series. What is the total resistance of the combination?**

(*b*) **If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.**

**Ans.** Here, $R_1 = 1\ \Omega$ ; $R_2 = 2\ \Omega$ ; $R_3 = 3\ \Omega$

(*a*) Now, $R_S = R_1 + R_2 + R_3 = 1 + 2 + 3 = \mathbf{6\ \Omega}$

(*b*) Circuit current, $I = \dfrac{E}{R_S} = \dfrac{12}{6} = 2$ A

P.D. across $R_1 = I\,R_1 = 2 \times 1 = \mathbf{2\ V}$

P.D. across $R_2 = I\,R_2 = 2 \times 2 = \mathbf{4\ V}$

P.D. across $R_3 = I\,R_3 = 2 \times 3 = \mathbf{6\ V}$

**Q.4.** (*a*) **Three resistors, 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total resistance of the combination?**

(*b*) **If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.**

**Ans.** $R_1 = 2\ \Omega$ ; $R_2 = 4\ \Omega$ ; $R_3 = 5\ \Omega$ ; $V = 20$ volt

(*a*)
$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{or} \quad \frac{1}{R_P} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$$

$$\therefore \qquad \frac{1}{R_P} = \frac{10+5+4}{20} = \frac{19}{20} \quad \text{or} \quad R_P = \mathbf{\frac{20}{19}\ \Omega}$$

(*b*) The voltage across each resistor is $V = 20$ volt.

Current through $R_1 = V/R_1 = 20/2 = \mathbf{10\ A}$

Current through $R_2 = V/R_2 = 20/4 = \mathbf{5\ A}$

Current through $R_3 = V/R_3 = 20/5 = \mathbf{4\ A}$

Total current drawn from the battery is

$$I = 10 + 5 + 4 = \mathbf{19\ A}$$

*Alternatively,* $$I = \frac{V}{R_P} = \frac{20}{20/19} = \mathbf{19\ A}$$

**Q.5. At room temperature (27.0 °C), the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature co-efficient of the material of the resistor is $1.70 \times 10^{-4}$ °C$^{-1}$.**

**Ans.** Here, $R_{27} = 100\ \Omega$ ; $R_t = 117\ \Omega$ ; $\alpha = 1.70 \times 10^{-4}\ °C^{-1}$ ; $t = ?$

Now $$R_t = R_{27}\,[1 + \alpha\,(t - 27)]$$

or $$117 = 100\,[1 + 1.70 \times 10^{-4}\,(t - 27)]$$

$$\therefore \quad t = \frac{117 - 100}{100 \times 1.7 \times 10^{-4}} + 27 = 1000 + 27 = \mathbf{1027\ °C}$$

**Q.6. A negligibly small current is passed through a wire of length 15 m and uniform cross-section $6.0 \times 10^{-7}$ m$^2$, and its resistance is measured to be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment?**

**Ans.** Here, $l = 15$ m ; $A = 6.0 \times 10^{-7}$ m$^2$ ; $R = 5.0\ \Omega$

Let $\rho$ be the resistivity of the material. Then,

$$\rho = \frac{RA}{l} = \frac{5.0 \times 6.0 \times 10^{-7}}{15} = \mathbf{2.0 \times 10^{-7}\ \Omega\ m}$$

**Q.7. A silver wire has resistance of 2.1 Ω at 27.5 °C, and a resistance of 2.7 Ω at 100 °C. Determine the temperature co-efficient of resistance of silver.**

**Ans.** $R_{27.5} = 2.1\ \Omega$ ; $R_{100} = 2.7\ \Omega$ ; $\alpha = ?$

Now $$R_{100} = R_{27.5}\,[1 + \alpha\,(100 - 27.5)]$$

$$\therefore \quad \alpha = \frac{R_{100} - R_{27.5}}{R_{27.5} \times (100 - 27.5)} = \frac{2.7 - 2.1}{2.1 \times (100 - 27.5)} = \mathbf{0.0039\ °C^{-1}}$$

**Q.8. A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0°C? Temperature co-efficient of resistance of nichrome averaged over the temperature range involved is $1.70 \times 10^{-4}$ °C$^{-1}$.**

**Ans.** The statement of the problem suggests that current is 3.2 A at 27 °C and 2.8 A at a higher temperature $t$ °C.

$\therefore$ $R_{27} = 230/3.2 = 71.87\ \Omega$ ; $R_t = 230/2.8 = 82.143\ \Omega$

Now $$\alpha = \frac{R_t - R_{27}}{R_{27}\,(t - 27)}$$

$$\therefore \quad t - 27 = \frac{R_t - R_{27}}{R_{27} \times \alpha} = \frac{82.143 - 71.875}{71.875 \times 1.7 \times 10^{-4}} = 840.35\ °C$$

$$\therefore \quad t = 840.35 + 27 = \mathbf{867.35\ °C}$$

**Q.9. Determine the current in each branch of the network shown in Fig. 3.01**

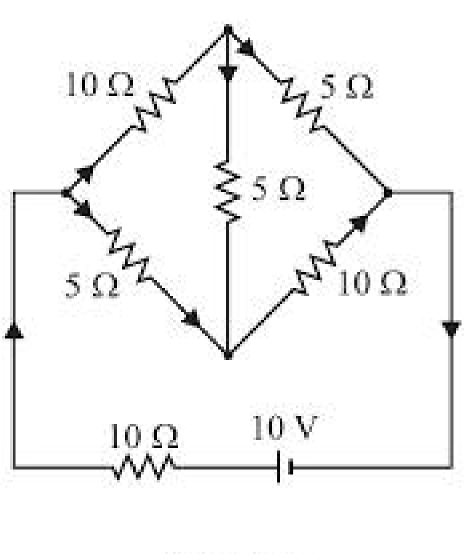



**Fig. 3.01**

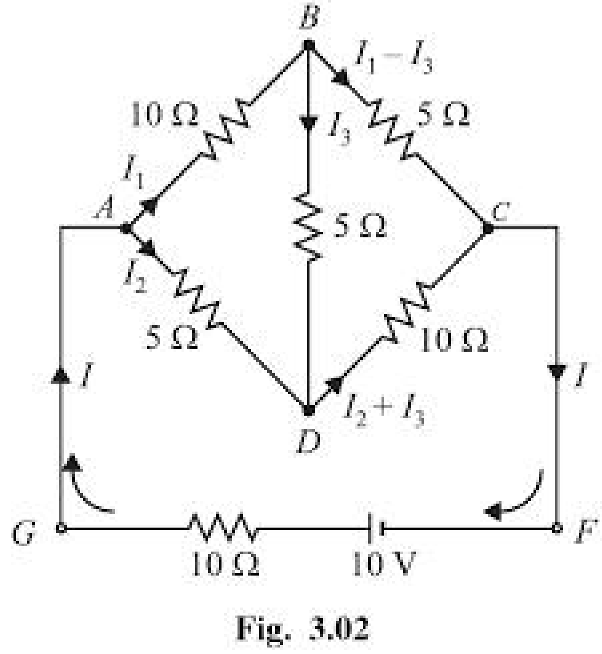



**Fig. 3.02**

**Ans.** Mark the currents in the various branches as shown in Fig. 3.02. Since there are three unknown currents ($I_1$, $I_2$ and $I_3$), three loops will be considered.

**Loop ABDA :** Applying Kirchhoff's voltage law to this loop, we get,

$$-10\, I_1 - 5\, I_3 + 5\, I_2 = 0$$

or
$$10\, I_1 + 5\, I_3 - 5\, I_2 = 0 \qquad \text{... } (i)$$

**Loop BCDB :** Applying Kirchhoff's voltage law to this loop, we get,

$$-5\, (I_1 - I_3) + 10\, (I_2 + I_3) + 5\, I_3 = 0$$

or
$$5\, I_1 - 10\, I_2 - 20\, I_3 = 0 \qquad \text{... } (ii)$$

**Loop ADCFGA :** Applying Kirchhoff's voltage law to this loop, we get,

$$-5\, I_2 - 10\, (I_2 + I_3) + 10 - 10\, (I_1 + I_2) = 0 \qquad (\because I_1 + I_2 = I)$$

or
$$10\, I_1 + 25\, I_2 + 10\, I_3 = 10 \qquad \text{... } (iii)$$

Solving eqs. (*i*), (*ii*) and (*iii*), we get,

$$I_1 = \frac{4}{17}\text{ A} \; ; I_2 = \frac{6}{17}\text{ A} \; ; I_3 = -\frac{2}{17}\text{ A}$$

$\therefore$ Currents in the various branches are:

$$I_{AB} = I_1 = \mathbf{\frac{4}{17}\ A} \; ; I_{BC} = I_1 - I_3 = \mathbf{\frac{6}{17}\ A} \; ; I_{DC} = I_2 + I_3 = \mathbf{\frac{4}{17}\ A}$$

$$I_{AD} = I_2 = \mathbf{\frac{6}{17}\ A} \; ; I_{BD} = I_3 = \mathbf{-\frac{2}{17}\ A}$$

$$\text{Total current, } I = I_1 + I_2 = \frac{4}{17} + \frac{6}{17} = \mathbf{\frac{10}{17}\ A}$$

**Q.10.** (*a*) **In a metre bridge [Fig. 3.03], the balance point is found to be at 39.5 cm from the end *A*, when the resistor *Y* is of 12.5 Ω. Determine the resistance of *X*. Why are the connections between resistors in a Wheatstone or metre bridge made of thick copper strips?**

(*b*) **Determine the balance point of the bridge above if *X* and *Y* are interchanged.**

(*c*) **What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?**

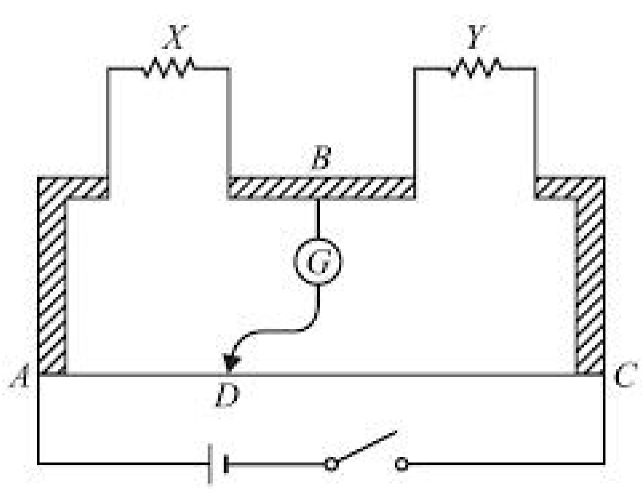



**Fig. 3.03**

**Ans.** (*a*) Here, $AD = 39.5$ cm ; $DC = 100 - 39.5 = 60.5$ cm ; $Y = 12.5\ \Omega$ ; $X = ?$

Since the bridge is balanced, product of opposite arms is equal *i.e.*,

$$X \times DC = Y \times AD \quad \text{or} \quad X = \frac{Y \times AD}{DC} = \frac{12.5 \times 39.5}{60.5} = \mathbf{8.16\ \Omega}$$

Connections are made by thick copper strips to minimise the resistance of connections which are not accounted for in the above formula.

$$\therefore \qquad \frac{m_2}{m_1} = \frac{m_{Cu}}{m_{Al}} = \frac{1}{0.46} = 2.16$$

Thus copper wire is 2.16 times heavier than aluminium wire. Therefore, **aluminium wire is lighter.**

It is clear that for the same length and same resistance, aluminium wire is lighter than copper wire. For this reason, aluminium wires are used for overhead power transmission lines. This leads to overall economy.

**Q.17. What conclusion can you draw from the following observations on a resistor made of alloy manganin?**

| Current (A) | Voltage (V) | Current (A) | Voltage (V) |
|---|---|---|---|
| 0.2 | 3.94 | 3.0 | 59.2 |
| 0.4 | 7.87 | 4.0 | 78.8 |
| 0.6 | 11.8 | 5.0 | 98.6 |
| 0.8 | 15.7 | 6.0 | 118.5 |
| 1.0 | 19.7 | 7.0 | 138.2 |
| 2.0 | 39.4 | 8.0 | 158.0 |

**Ans.** It is clear from the above table that ratio $V/I$ for different readings is the same, *i.e.*, resistance value remains the same. In other words, as the current increases from 0 to 8 A, the temperature increases but the resistance of manganin wire does not change. Hence, the resistivity of alloy manganin is nearly independent of temperature.

**Q.18. Answer the following questions :**

(*a*) **A steady current flows in a metallic conductor of non-uniform cross-section. Which of these quantities is constant along the conductor: current, current density, electric field, drift speed?**

(*b*) **Is Ohm's law universally applicable for all conducting elements? If not, give examples of elements which do not obey Ohm's law.**

(*c*) **A low voltage supply from which one needs high currents must have very low internal resistance. Why?**

(*d*) **A high tension (HT) supply of, say, 6 kV must have a very large internal resistance. Why?**

**Ans.** (*a*) Only current through the conductor of non-uniform area of cross-section is constant as all other quantities (*viz.*, current density, electric field and drift speed) vary inversely with area of cross-section.

(*b*) No, Ohm's law is not applicable to all conducting elements. For example, vacuum diode, semiconductor diode are non-ohmic elements.

(*c*) The maximum current that can be drawn from a voltage supply is given by ;

$$I_{max} = \frac{E}{r} \quad \text{..... where } r = \text{internal resistance of supply}$$

Therefore, in order to get large current from low voltage supply (*i.e.*, smaller $E$), the internal resistance $r$ of the voltage supply should be small.

(*d*) If the circuit containing HT supply gets short circuited accidentally, the current is limited only by the internal resistance of the supply ($\because I = E/r$). If the internal resistance ($r$) is not large, the circuit current may exceed the safe value and cause considerable damage to the circuit.

**Q.19. Choose the correct alternative ;**

(*a*) **Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.**

(*b*) **Alloys usually have much (lower/higher) temperature co-efficients of resistance than pure metals.**

(*c*) **The resistivity of the alloy manganin is nearly independent of/increases rapidly with increase of temperature.**

(*d*) **The resistivity of a typical insulator (*e.g.*, amber) is greater than that of a metal by a factor of the order of ($10^{22}/10^{3}$).**

**Ans.** (*a*) greater (*b*) lower (*c*) is nearly independent of (*d*) $10^{22}$.

**Q.20.** (*a*) **Given *n* resistors each of resistance *R*, how will you combine them to get the (*i*) maximum and (*ii*) minimum effective resistance? What is the ratio of the maximum to minimum resistance?**

(*b*) **Given the resistances of 1 Ω, 2 Ω, 3 Ω, how will you combine them to get an equivalent resistance of (*i*) (11/3) Ω, (*ii*) (11/5) Ω, (*iii*) 6 Ω, (*iv*) (6/11) Ω?**

(*c*) **Determine the equivalent resistance of networks shown in Fig. 3.04.**

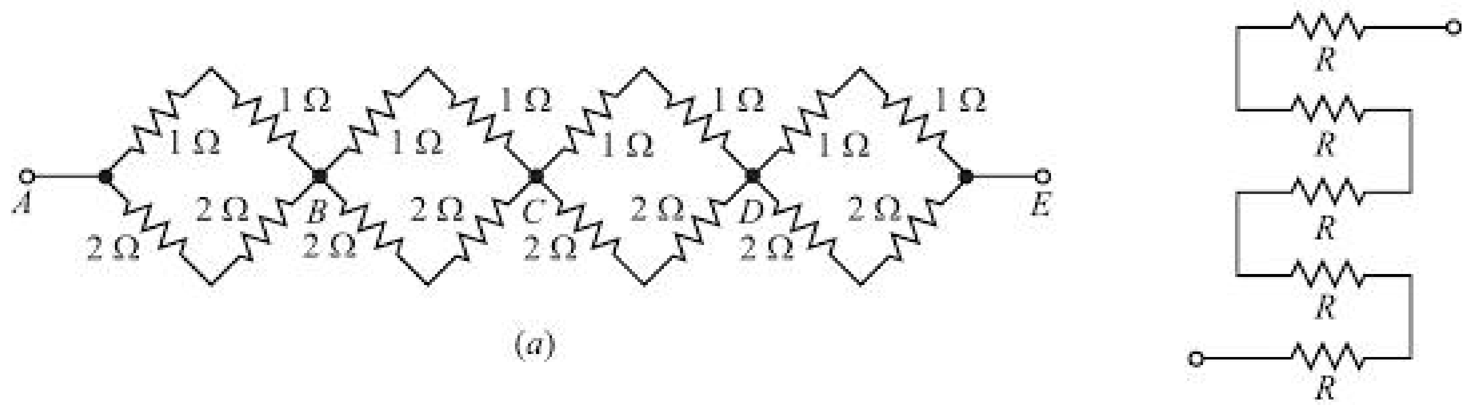



**Fig. 3.04**

**Ans.** (*a*) (*i*) For maximum effective resistance, the *n* resistors must be connected in series.

∴ Maximum effective resistance, $R_S = R + R + R + \ldots.\ n \text{ terms} = \boldsymbol{nR}$

(*ii*) For minimum effective resistance, the *n* resistors must be connected in parallel. It is given by:

$$\frac{1}{R_P} = \frac{1}{R}+\frac{1}{R}+\frac{1}{R} + \ldots..\ n \text{ terms} = \frac{n}{R}$$

∴ Minimum effective resistance, $R_P = \boldsymbol{\frac{R}{n}}$

Ratio of maximum to minimum resistance is

$$\frac{R_S}{R_P} = \frac{nR}{R/n} = \boldsymbol{n^2}$$

(*b*) Here $R_1 = 1\ \Omega$ ; $R_2 = 2\ \Omega$ ; $R_3 = 3\ \Omega$

(*i*) In order to obtain an equivalent resistance of 11/3 Ω, the parallel combination of 1 Ω and 2 Ω resistors should be connected in series with 3 Ω resistor as shown in Fig. 3.05 (*i*). The equivalent resistance is

$$R_{eq} = R_P + R_3 = \frac{1\times 2}{1+2} + 3 = \frac{11}{3}\ \Omega$$

(*ii*) In order to obtain an equivalent resistance of 11/5 Ω, the parallel combination of 2 Ω and 3 Ω resistors should be connected in series with 1 Ω resistor as shown in Fig. 3.05 (*ii*). The equivalent resistance is

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3} + R_1 = \frac{2\times 3}{2+3} + 1 = \frac{11}{5}\,\Omega$$

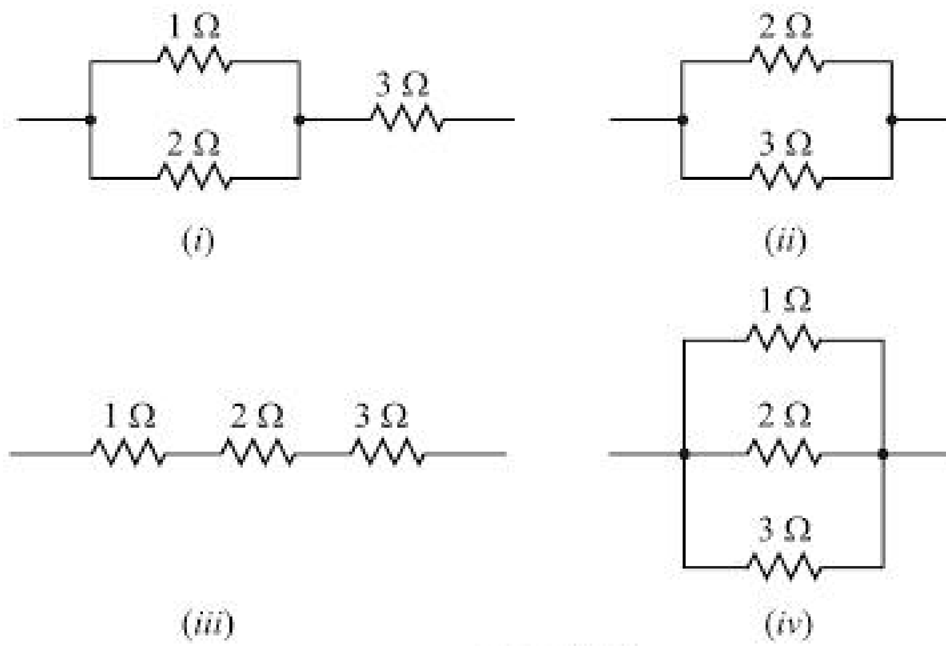



**Fig. 3.05**

(*iii*) In order to obtain an equivalent resistance of 6 Ω (maximum), the three resistors should be connected in series as shown in Fig. 3.05 (*iii*). The equivalent resistance is

$$R_{eq} = R_1 + R_2 + R_3 = 1 + 2 + 3 = 6\ \Omega$$

(*iv*) In order to obtain an equivalent resistance of 6/11 Ω (minimum), the three resistors should be connected in parallel as shown in Fig. 3.05 (*iv*). The equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6} \quad \therefore \quad R_{eq} = \frac{6}{11}\ \Omega$$

(*c*) (*i*) Refer to Fig. 3.04 (*a*). This network is a series combination of four identical units. One such unit is shown in Fig. 3.06. The equivatent resistance of this unit is

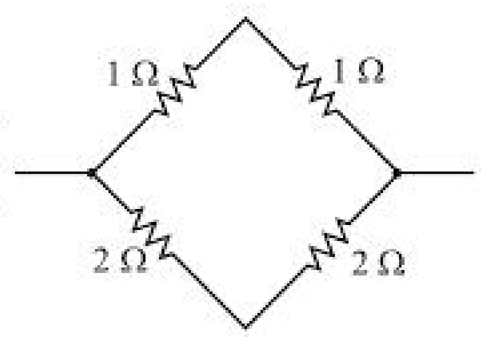



**Fig. 3.06**

$$R'_{eq} = (1\ \Omega + 1\ \Omega) \parallel (2\ \Omega + 2\ \Omega)$$

$$= (2\ \Omega) \parallel (4\ \Omega) = \frac{2 \times 4}{2 + 4} = \frac{4}{3}\ \Omega$$

∴ Equivalent resistance of the network shown in Fig. 3.04 (*a*) is

$$R_{eq} = 4\ R'_{eq} = 4 \times \frac{4}{3} = \mathbf{\frac{16}{3}\ \Omega}$$

(*ii*) The network shown in Fig. 3.04 (*b*) is a series combination of 5 resistors, each of resistance *R*. Therefore, equivalent resistance of this network is $R_{eq}$ = **5 *R*.**

**Q.21. Determine the current drawn from a 12 V supply with internal resistance 0.5 Ω by the infinite network shown in Fig. 3.07. Each resistor has 1 Ω resistance.**

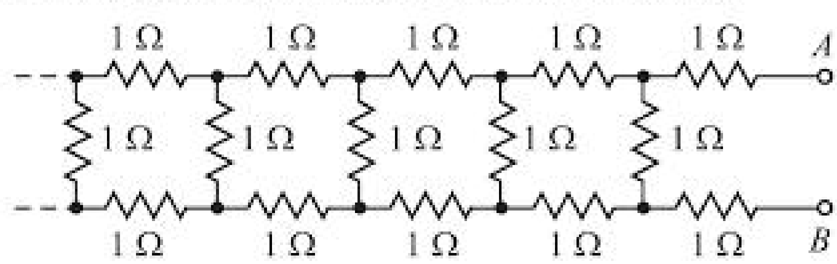



**Fig. 3.07**

**Ans.** Let the resistance of the infinite network shown in Fig. 3.07 be *X*. The network consists of infinite units of three resistors of 1 Ω, 1 Ω and 1 Ω. The addition of one more such unit across *AB* will not affect the total resistance, *i.e.*, it should still remain *X*. Therefore, the network obtained by adding one more unit of three resistances would appear as shown in Fig. 3.08.

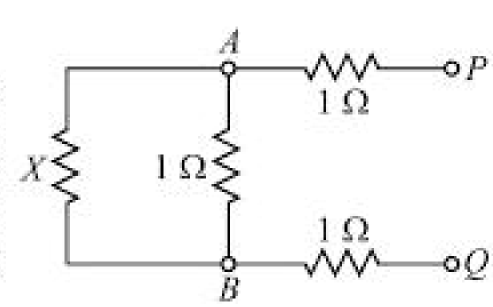



**Fig. 3.08**

Now $$R_{AB} = X \parallel 1\ \Omega = \frac{X \times 1}{X+1} = \frac{X}{X+1}$$

Also $$R_{PQ} = R_{AB} + 1\ \Omega + 1\ \Omega = \frac{X}{X+1} + 2$$

Therefore, $R_{PQ}$ must be equal to the resistance $X$ of the network.

$$\therefore \quad X = 2 + \frac{X}{1+X}$$

or $$X^2 - 2X - 2 = 0 \quad \text{or} \quad X = 1 \pm \sqrt{3}$$

As the value of resistance cannot be negative,

$$\therefore \quad X = 1 + \sqrt{3} = 2.732\ \Omega$$

$$\therefore \quad \text{Circuit current, } I = \frac{E}{X+r} = \frac{12}{2.732 + 0.5} = \mathbf{3.713\ A}$$

**Q.22. Figure 3.09 shows a potentiometer with a cell of 2.0 V and internal resistance 0.40 Ω maintaining a potential drop across the resistor wire *AB*. A standard cell which maintains a constant e.m.f. of 1.02 V (for very moderate currents upto a few A) gives a balance point at 67.3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown e.m.f. *E* and the balance point found similarly, turns out to be at 82.3 cm length of the wire.**

(*a*) **What is the value *E*?**

(*b*) **What purpose does the high resistance of 600 kΩ have?**

(*c*) **Is the balance point affected by this high resistance?**

(*d*) **Is the balance point affected by the internal resistance of the driver cell?**

(*e*) **Would the method work in the above situation if the driver cell of the potentiometer had an e.m.f. of 1.0 V instead of 2.0 V?**

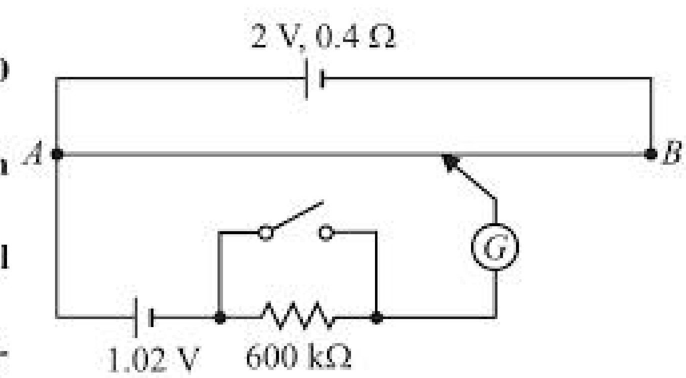



**Fig. 3.09**

(*f*) **Would the circuit work well for determining an extremely small e.m.f., say of the order of a few mV (such as the typical e.m.f. of a thermocouple)? If not, how will you modify the circuit?**

**Ans.** (*a*) $E_1 = 1.02$ V ; $l_1 = 67.3$ cm ; $E_2 = E = ?$ ; $l_2 = 82.3$ cm

Now, $$\frac{E_2}{E_1} = \frac{l_2}{l_1} \quad \therefore \quad E_2 = E_1 \times \frac{l_2}{l_1} = 1.02 \times \frac{82.3}{67.3} = \mathbf{1.247\ V}$$

(*b*) The purpose of using high resistance (600 kΩ in this case) is to allow only a very small current through the galvanometer when the balance point has not been obtained.

(*c*) No, balance point is not affected by the presence of high resistance (600 kΩ). It is because no current flows through the cell at the balance point.

(*d*) No, balance point is not affected by the internal resistance of the driving cell.

(*e*) No, the arrangement will not work. If the e.m.f. of the driver cell is less than that of the other cell, the balance point will not be obtained on the potentiometer wire *AB*.

(*f*) The circuit is not suitable for measuring extremely small e.m.f. It is because in such a case, the balance point will be very close to end *A* and the percentage error in measurement will be very high. The circuit is modified by putting a suitable high resistance in series with 2.0 V so that current in the potentiometer wire is decreased. This decreases the potential gradient (fall of voltage/cm) along the wire and the balance point will occur at a larger distance from end *A*.

**Q.23. Figure 3.010 shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor $R = 10.0\Omega$ is found to be 58.3 cm, while that with the unknown resistance $X$ is 68.5 cm. Determine the value of $X$. What might you do if you failed to find a balance point with the given cell of e.m.f. $E$ ?**

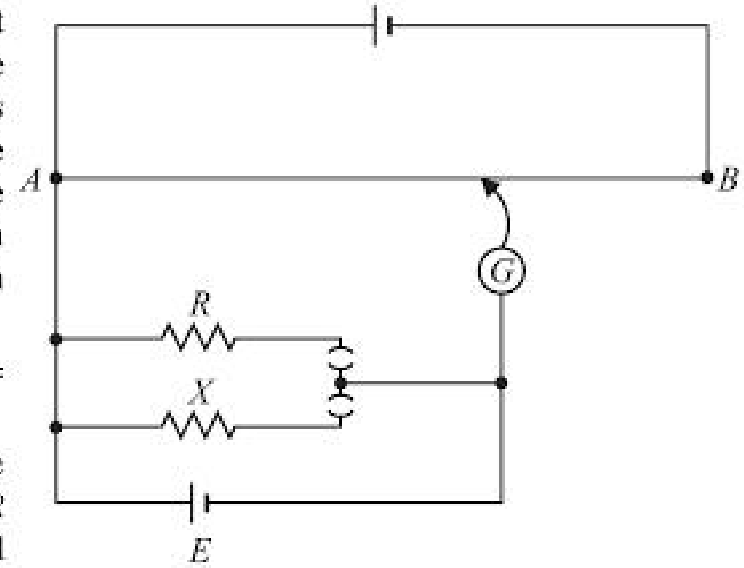



**Fig. 3.010**

**Ans.** Here, $R = 10.0\ \Omega$ ; $l_1 = 58.3$ cm ; $X = ?$ ; $l_2 = 68.5$ cm

Let $I$ be the current in the potentiometer wire and $E_1$ and $E_2$ be the potential drops across $R$ and $X$ respectively. The p.d. across standard resistor $R$ ($E_1 = I\,R$) is balanced against length $l_1$ (= 58.3 cm) of the potentiometer wire $AB$. Similarly, p.d. across unknown resistance $X$ ($E_2 = I\,X$) is balanced against length $l_2$ (= 68.5 cm) of the potentiometer wire.

$$\therefore \qquad \frac{l_1}{l_2} = \frac{E_1}{E_2} = \frac{I\,R}{I\,X} = \frac{R}{X}$$

$$\text{or} \qquad \frac{l_1}{l_2} = \frac{R}{X} \quad \therefore \quad X = R\left(\frac{l_2}{l_1}\right) = 10.0 \times \frac{68.5}{58.3} = \mathbf{11.75\ \Omega}$$

If there is no balance point, it means that potential drop across $R$ or $X$ is greater than the p.d. across the potentiometer wire $AB$. To obtain the balance point with the given cell $E$, one of the following two methods may be used :

(*i*) Reduce the current in $R$ and $X$. This can be done by putting a suitable resistance in series with $R$ and $X$.

(*ii*) Increase the voltage of the driving cell, *i.e.*, cell connected between points $A$ and $B$. This will increase the p.d. across potentiometer wire $AB$ and hence the potential gradient along the wire.

**Q.24. Figure 3.011 shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.**

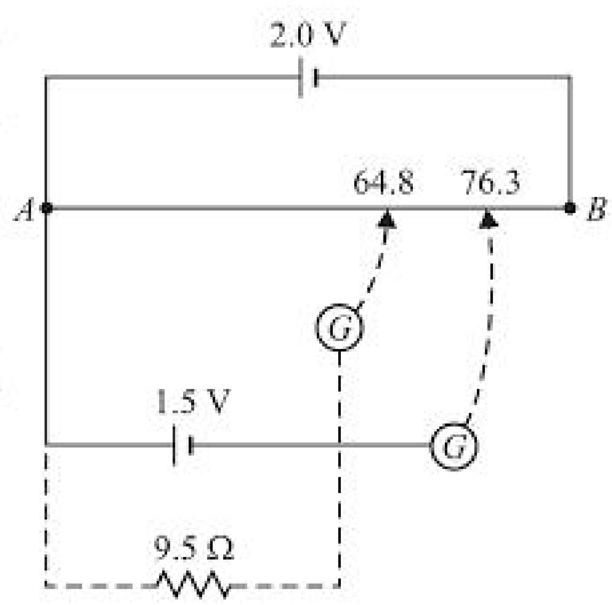



**Fig. 3.011**

**Ans.** Here, $l_1 = 76.3$ cm ; $l_2 = 64.8$ cm ; $R = 9.5\ \Omega$

The formula of the internal resistance $r$ of a cell by potentiometer method is

$$r = \frac{l_1 - l_2}{l_2} \times R = \frac{76.3 - 64.8}{64.8} \times 9.5 = \mathbf{1.7\ \Omega}$$

# Unit III

## MAGNETIC EFFECTS OF CURRENT AND MAGNETISM

- **Magnetic Field Due to Electric Current**
- **Motion of Charged Particles in Electric and Magnetic Fields**
- **Magnets and Earth's Magnetism**
- **Classification of Magnetic Materials**
- **N.C.E.R.T. Textbook Exercises**

# 8

# Magnetic Field Due to Electric Current

## INTRODUCTION

The first discovery of any connection between electric current and magnetism was made by Oersted in 1820. On one occasion at the end of his lecture, he inadvertently placed a wire carrying current parallel to a compass needle. To his surprise, the needle was deflected. Upon reversing the current in the wire, the needle deflected in the opposite direction. Oersted concluded that the compass deflection was due to the magnetic field established around the current carrying conductor. *The production of magnetism from electric current* (which we call **electromagnetism**) has opened a new era. The operation of all electrical machinery is due to the applications of magnetic effects of electric current in one form or the other. In this chapter, we shall discuss about the magnitude and direction of magnetic field due to various conductor arrangements and their practical applications.

## 8.1. OERSTED'S DISCOVERY

The magnetic effect of electric current was discovered by Oersted in 1820. He verified the magnetic effect of electric current by the following simple experiment.

(*i*) Fig. 8.1 (*i*) shows a conducting wire $AB$ above a magnetic needle parallel to it. So long as there is no current in the wire, the magnetic needle remains parallel to the wire *i.e.* there is no deflection in the magnetic needle.

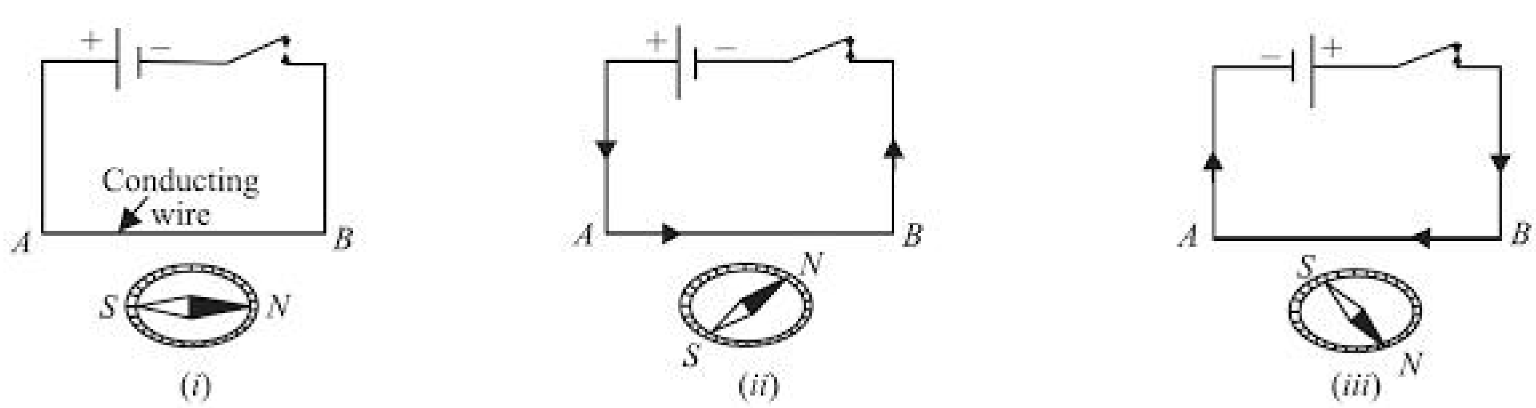



**Fig. 8.1**

(*ii*) As soon as the current flows through the wire $AB$ [See Fig. 8.1 (*ii*)], the needle is deflected. When the current in wire $AB$ is reversed [See Fig. 8.1 (*iii*)], the needle is deflected in the opposite direction. *This deflection is a convincing proof of the existence of a magnetic field around a current-carrying conductor.*

(*iii*) On increasing the current in the wire $AB$, the deflection of the needle is increased and vice-versa. *This shows that magnetic field strength increases with the increase in current and vice-versa.*

It is clear from Oersted's experiment that *a current carrying conductor produces a magnetic field around it. The larger the value of current in the conductor, the stronger is the magnetic field and vice-versa.*

**Ampere's swimming rule.** The direction of deflection of the magnetic needle due to current in the wire is given by *Ampere's swimming rule*. It is stated below :

*Imagine a man swimming along the wire in the direction of the flow of current with his face always turned towards the magnetic needle so that the current enters through his feet and leaves at his head. Then N–pole of the magnetic needle will be deflected towards his left hand.*

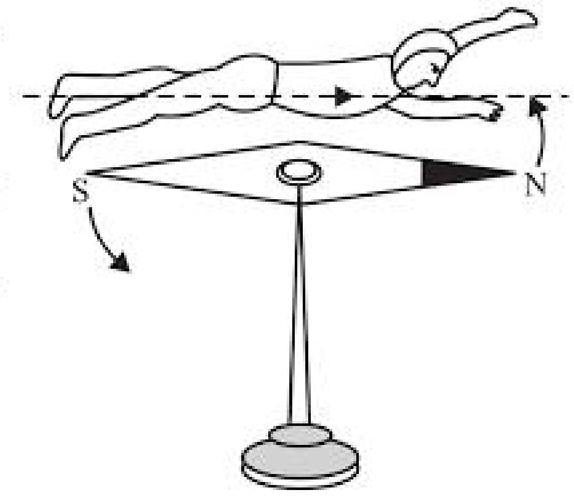


**Fig. 8.2**

Fig. 8.2 illustrates Ampere's swimming rule. This rule can be remembered with the help of the word **SNOW**. It means that if current flows from **S**outh to **N**orth and the wire is held **O**ver the needle, the north pole of the needle is deflected towards **W**est.

## 8.2. MAGNETIC FIELD

*The space around a current carrying conductor (or a magnet) where magnetic effects can be experienced is called a* **magnetic field**.

The magnetic field is represented by magnetic lines of force which form closed loops. The greater the current through the conductor, the stronger is the magnetic field and *vice-versa*. The magnetic field disappears as soon as the current is switched off or charges stop moving.

## 8.3. MAGNETIC FLUX DENSITY ($\vec{B}$)

The strength of the magnetic field is measured by a quantity called **magnetic flux density**. It is a **vector quantity** (*i.e.*, it has magnitude and direction) and is represented by the symbol $\vec{B}$. Note that magnetic flux density is simply called **magnetic field** $\vec{\boldsymbol{B}}$.

The following points may be noted carefully :

(*i*) The magnetic flux density is a measure of field concentration *i.e.*, number of lines in each square metre of the field. It is, therefore, the appropriate quantity to describe the magnetic field.

(*ii*) Just as electric field around a charge is described by the field vector $\vec{E}$, similarly the magnetic field around a current carrying conductor is described by the magnetic field vector $\vec{B}$.

(*iii*) The unit of magnetic flux density is **tesla (T)** or **Wb/m$^2$**.

## 8.4. DEFINITION OF MAGNETIC FIELD $\vec{B}$

Recall that we defined electric field $\vec{E}$ at a point in space as the force acting on a unit positive charge placed at that point. Similarly, we define $\vec{B}$ (strength of magnetic field) in terms force acting on a moving charge (or current) due to the magnetic field.

Consider a positive charge $+q$ moving in a uniform magnetic field $\vec{B}$ with a velocity $\vec{v}$. Let the angle between $\vec{v}$ and $\vec{B}$ be $\theta$ as shown in Fig. 8.3. It has been found experimentally that magnetic field exerts a force $\vec{F}_m$ on the charge. The magnitude $F_m$ of this force depends on the following factors :

(*i*) $F_m \propto q$

(*ii*) $F_m \propto B$

**(iii)** $F_m \propto v \sin\theta$

Combining the above factors, we get,

$$F_m \propto q\, v\, B \sin\theta$$

or $$F_m = k\, q\, v\, B \sin\theta$$

where $k$ is a constant of proportionality. The unit of $B$ is so defined that $k = 1$.

$$\therefore \quad F_m = q\, v\, B \sin\theta \qquad ...(i)$$

Eq. ($i$) can be written in the vector form as :

$$\vec{F}_m = q\left(\vec{v} \times \vec{B}\right)$$

Clearly, $\vec{F}_m$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$ as shown in Fig. 8.3.

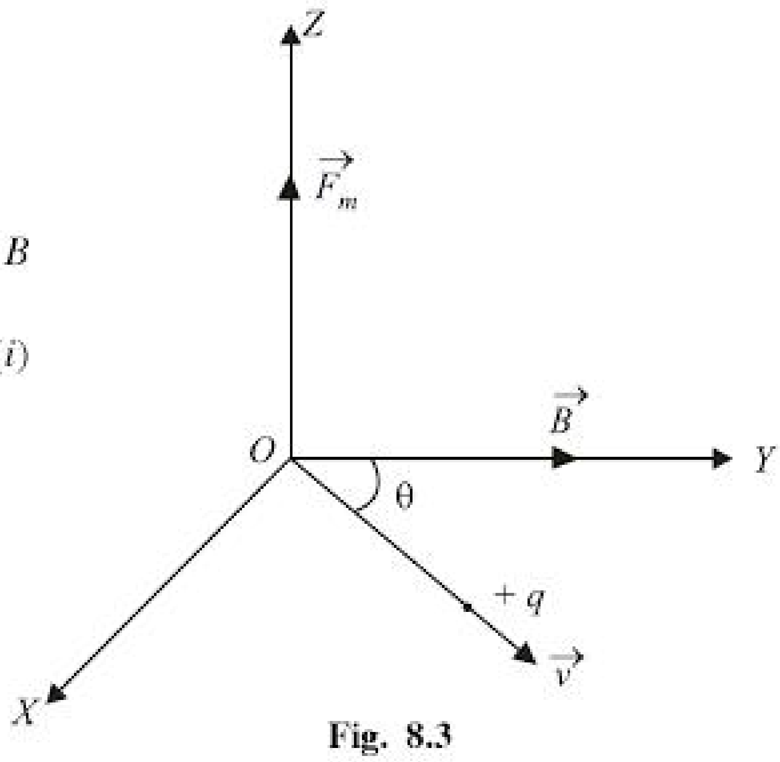



**Fig. 8.3**

**Definition of $\vec{B}$.** $F_m = q\, v\, B \sin\theta$

If $v = 1$, $q = 1$, $\theta = 90°$, then,

$$F_m = 1 \times 1 \times B \times \sin 90° \quad \therefore \quad F_m = B$$

*Hence* **magnetic field** $(\vec{B})$ *at a point in space is equal to the force experienced by a unit charge moving with a unit velocity perpendicular to the direction of magnetic field at that point.*

**Direction of magnetic force $\vec{F}_m$.** We have seen that magnetic force $\vec{F}_m$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. The force could be upward as in Fig. 8.3 or downward since both directions are perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. The direction (*i.e.*, sense) of force is given by the right-hand rule.

**Right-hand rule.** *Orient your right hand so that your outstretched fingers point along the direction of motion of *positively charged particle (conventional current); the orientation should be such that when you bend your fingers, they must point along the direction of the magnetic field* ($\vec{B}$). *Then your extended thumb will point in the direction of force on the charged particle.*

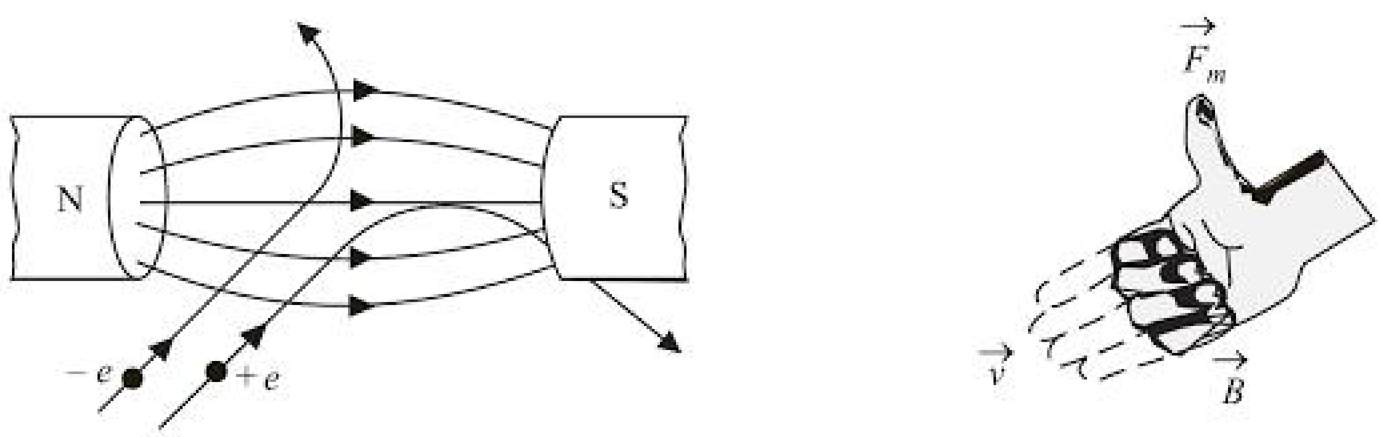



**Fig. 8.4**

Let us apply right-hand rule for an electron (negatively charged) entering the magnetic field as shown in Fig. 8.4. The direction of positive charge will be exactly opposite. Applying right-hand rule, it is clear that direction of force on the electron will be vertically upward. For a positively charged particle, it will be vertically downward. Note that direction of magnetic field means from $N$-pole to $S$-pole.

* For negatively charged particle, the force is exactly in the opposite direction.

Now $$\vec{F}_m = q(\vec{v} \times \vec{B})$$

This rule may be remembered as $\vec{v}$ *(direction of conventional current) swept into* $\vec{B}$.

## 8.5. SOME CASES OF MAGNETIC FORCE $\vec{F}_m$

Consider an electric charge $q$ moving with a velocity $\vec{v}$ through a magnetic field $\vec{B}$. Then the magnetic force $\vec{F}_m$ on the charge is given by ;

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

Magnitude of force, $F_m = q\,v\,B\sin\theta$

(*i*) **When $\theta = 0°$ or $180°$**, $\sin\theta = 0 \quad \therefore \quad F_m = q\,v\,B\,(0) = 0$

*Hence a charged particle moving parallel (or antiparallel) to the direction of magnetic field experiences no force.*

(*ii*) **When $\theta = 90°$**, $\sin\theta = 1 \quad \therefore \quad F_m = q\,v\,B$ ... maximum value

*Hence force experienced by a charged particle is maximum when it is moving perpendicular to the direction of magnetic field.*

(*iii*) **When $v = 0$** *i.e.* the charged particle is at rest,

$$F_m = q\,(0)\,B\sin\theta = 0$$

*If a charged particle is at rest in a magnetic field, it experiences no force.*

(*iv*) **When $q = 0$**; $F_m = 0$

*Hence electrically neutral particle (e.g., neutron) moving in a magnetic field experiences no force.*

**Note.** The magnetic force $\vec{F}_m$ acts perpendicular to velocity vector $\vec{v}$ (as well as $\vec{B}$). This means that a uniform magnetic field can neither speed up nor slow down a moving charged particle; it can change only the direction of $\vec{v}$ and not magnitude of $\vec{v}$. Since the magnitude of $\vec{v}$ does not change, *the magnetic force does not change the kinetic energy of the charged particle.*

## 8.6. UNITS AND DIMENSIONS OF $\vec{B}$

The SI unit of magnetic field $(\vec{B})$ is **1 tesla** (*i.e.*, 1T).

Now $$F_m = q\,v\,B\sin\theta$$

or $$B = \frac{F_m}{q\,v\sin\theta}$$

If $q = 1\text{C}$, $v = 1\text{ms}^{-1}$, $\theta = 90°$ and $F_m = 1\text{N}$, then,

$$B = \frac{1}{1 \times 1 \times 1} = 1\text{ T}$$

*Hence the strength of magnetic field at a point is **1 T** if a charge of 1C when moving with a velocity of 1 ms$^{-1}$ at right angles to the magnetic field, experiences a force of 1N at that point.*

Note that magnetic field of earth at its surface is about $10^{-4}$ T. On the other hand, strong electromagnets can produce magnetic fields of the order of 2T.

**Note.** $$B = \frac{F_m}{q\,v\sin\theta}$$

$$\therefore \quad 1\text{T} = \frac{1\text{ N}}{1\text{C} \times 1\text{ ms}^{-1}} = \frac{1\text{ N}}{1\text{As} \times 1\text{ ms}^{-1}} = 1\text{ NA}^{-1}\text{ m}^{-1} \quad (\because 1\text{ C} = 1\text{ A} \times 1\text{ s})$$

**Dimensions of $\vec{B}$.** $$B = \frac{F_m}{q\,v\,\sin\theta}$$

$\therefore$ $$\text{Dimensions of } B = \frac{MLT^{-2}}{AT[LT^{-1}]} = MA^{-1}T^{-2}$$

**Example 8.1. A proton is moving northwards with a velocity of $5 \times 10^6\ ms^{-1}$ in a magnetic field of 0.1T directed eastwards. Find the force on the proton. Charge on proton $= 1.6 \times 10^{-19}$C.**

**Solution.** $F_m = q\,v\,B\sin\theta$ ...magnitude

Here, $q = 1.6 \times 10^{-19}\text{C}$ ; $B = 0.1\text{T}$ ; $v = 5 \times 10^6\ ms^{-1}$ ; $\theta = 90^\circ$

$\therefore$ $F_m = (1.6 \times 10^{-19}) \times (5.0 \times 10^6) \times 0.1 \times \sin 90^\circ = 8 \times 10^{-14}$ **N**

By right-hand rule for cross product, the direction of magnetic force is vertically downward.

**Example 8.2. A copper wire has $1.0 \times 10^{29}$ free electrons per cubic metre, a cross-sectional area of 2.0 mm² and carries a current of 5.0A. The wire is placed at right angle to a uniform magnetic field of strength 0.15 T. Calculate the force acting on each electron.**

**Solution.** $I = n\,e\,A\,v_d$ $\therefore$ Drift velocity, $v_d = \dfrac{I}{n\,e\,A}$

Here $n = 1.0 \times 10^{29}\ m^{-3}$ ; $e = 1.6 \times 10^{-19}\text{C}$ ; $A = 2\ mm^2 = 2 \times 10^{-6}\ m^2$ ; $I = 5.0\text{A}$

$$\therefore \quad v_d = \frac{5.0}{(1.0\times10^{29})\times(1.6\times10^{-19})\times(2\times10^{-6})} = 1.56\times10^{-4}\ ms^{-1}$$

Force on each electron, $F_m = q\,v_d\,B\sin\theta$

Here $q = 1.6 \times 10^{-19}$ C ; $v_d = 1.56 \times 10^{-4}\ ms^{-1}$ ; $B = 0.15$ T ; $\theta = 90^\circ$

$\therefore$ $F_m = (1.6 \times 10^{-19}) \times (1.56 \times 10^{-4}) \times (0.15) \times \sin 90^\circ$

$= 3.75 \times 10^{-24}$ **N**

**Example 8.3. An $\alpha$-particle of mass $6.65 \times 10^{-27}$ kg is travelling at right angles to a magnetic field with a speed of $6 \times 10^5\ ms^{-1}$. The strength of the magnetic field is 0.2T. Calculate the force on the $\alpha$-particle and its acceleration.**

**Solution.** Force on $\alpha$-particle, $F_m = q\,v\,B\sin\theta$.

Here $q = 2 \times 1.6 \times 10^{-19}$ C ; $v = 6 \times 10^5\ ms^{-1}$; $B = 0.2$ T ; $\theta = 90^\circ$

$\therefore$ $F_m = (2 \times 1.6 \times 10^{-19}) \times (6 \times 10^5) \times (0.2) \times \sin 90^\circ = 3.84 \times 10^{-14}$ **N**

Acceleration of $\alpha$-particle, $$a = \frac{F_m}{m} = \frac{3.84\times10^{-14}}{6.65\times10^{-27}} = 5.77 \times 10^{12}\ ms^{-2}$$

## PROBLEMS FOR PRACTICE

1. A proton moving with a speed of $3.4 \times 10^7\ ms^{-1}$ enters a magnetic field in a direction perpendicular to the field. The strength of magnetic field is 2.0T. Calculate the force acting on the proton and the acceleration produced in it (mass of proton $= 1.67 \times 10^{-27}$ kg and charge on proton $= + 1.6 \times 10^{-19}$ C). **[$1.09 \times 10^{-11}$ N ; $6.5 \times 10^{15}\ ms^{-2}$]**
2. An electron in a TV camera tube is moving at $7.20 \times 10^6\ ms^{-1}$ in a magnetic field of strength 83.0mT. Without knowing the direction of the field, what will be the greatest and least magnitudes of force the electron could feel due to the field? **[$9.56 \times 10^{-14}$ N; Zero]**
3. An electron is moving vertically upward with a speed of $2.0 \times 10^8\ ms^{-1}$. What will be the magnitude of the force on the electron exerted by a horizontal magnetic field of 0.5T directed towards west? **[$1.6 \times 10^{-11}$ N towards north]**

4. A proton experiences the greatest force as it moves with a speed of $2.0 \times 10^7$ ms$^{-1}$ towards east in a magnetic field. The force is vertically upward and has a magnitude of $1.09 \times 10^{-12}$ N. What is the magnitude and direction of magnetic field?

**[0.34 T towards north]**

5. The electron density of copper is $8.0 \times 10^{28}$ electrons per m$^3$. A copper wire of length 1m and area of X-section 8.0mm$^2$ is carrying current. It is placed at right angles to a magnetic field of $5 \times 10^{-3}$ T and experiences a force of $8.0 \times 10^{-2}$ N. Calculate the drift velocity of free electrons in the wire. **[$1.56 \times 10^{-4}$ ms$^{-1}$]**

6. A charge of 3C is moving with velocity $\vec{v} = (4\hat{i} + 3\hat{j})$ ms$^{-1}$ in a magnetic field $\vec{B} = (4\hat{i} + 3\hat{j})$ Wb/m$^2$. Find the force acting on the test charge. **[Zero]**

7. A proton enters a magnetic field of 4 T intensity with a velocity of $2.5 \times 10^6$ ms$^{-1}$ at an angle of 30° with the field. Find the magnitude of the force on the proton. Charge on proton = $1.602 \times 10^{-19}$ C. **[$8 \times 10^{-13}$ N]**

## 8.7. BIOT-SAVART LAW

A conductor carrying current $I$ produces a magnetic field around it. We can consider the current carrying conductor to be consisting of infinitesimally small **current elements* $I\vec{dl}$; each current element contributing to magnetic field. *Biot-Savart law gives us expression for the magnetic field at a point due to a current element.*

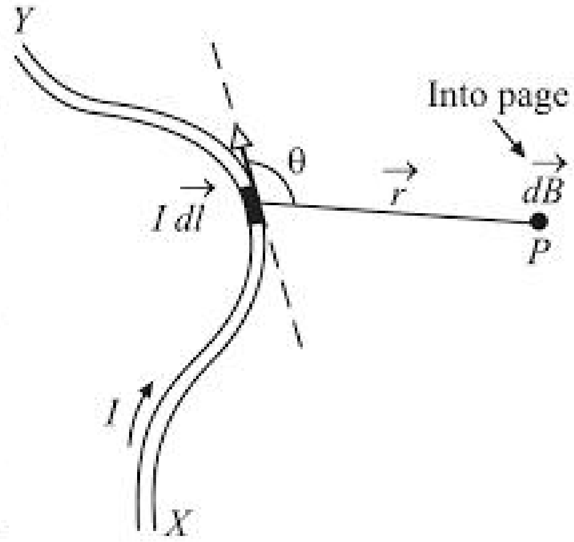



Fig. 8.5

Consider a current element $I\vec{dl}$ of a conductor $XY$ carrying current $I$. Let $P$ be the point where the magnetic field $\vec{dB}$ due to the current element is to be found. Suppose $\vec{r}$ is the position vector of point $P$ from the current element $I\vec{dl}$ and θ is the angle between $\vec{dl}$ and $\vec{r}$.

According to Biot-Savart law, the magnitude $dB$ of magnetic field at point $P$ due to the current element depends upon the following factors :

(*i*) $dB \propto I$ (*ii*) $dB \propto dl$ (*iii*) $dB \propto 1/r^2$ (*iv*) $dB \propto \sin\theta$

Combining all these four factors, we get,

$$dB \propto \frac{I\,dl\sin\theta}{r^2}$$

or

$$dB = K\frac{I\,dl\sin\theta}{r^2}$$

where $K$ is a constant of proportionality. Its value depends on the medium in which the conductor is situated and the system of units adopted.

For free space and SI units, $K = \frac{\mu_0}{4\pi} = 10^{-7}$ Tm A$^{-1}$

where $\mu_0$ = Absolute permeability of free space = $4\pi \times 10^{-7}$ Tm A$^{-1}$

* The current element $I\vec{dl}$ is a vector. Its direction is tangent to the element and acts in the direction of flow of current in the conductor.

$$\therefore \qquad dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl \sin\theta}{r^2} \qquad \text{... } (i)$$

Eq. (*i*) is known as *Biot-Savart law* and gives the magnitude of the magnetic field at a point due to small current element $I\vec{dl}$. **Note that Biot-Savart law holds strictly for steady currents.**

**In vector form.**

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\,(\vec{dl} \times \vec{r})}{r^3} \qquad \text{...}(ii)$$

The Biot-Savart law is analogous to Coulomb's law. Just as the charge $q$ is the source of electrostatic field, similarly, the source of magnetic field is the current element $I\vec{dl}$.

**Direction of $\vec{B}$.**

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\,(\vec{dl} \times \vec{r})}{r^3}$$

The direction of $\vec{dB}$ is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$. By right-hand rule for the cross product, the field is directed *inward.*

**Magnetic field due to whole conductor:** Eq. (*ii*) gives the magnetic field at point $P$ due to a small current element $I\vec{dl}$. The total magnetic field at point $P$ is found by summing (integrating) over all current elements :

$$\vec{B} = \int \vec{dB} = \int \frac{\mu_0}{4\pi} \frac{I\,(\vec{dl} \times \vec{r})}{r^3}$$

where the integration is taken over the entire conductor in which current $I$ flows.

**Special cases.**

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl \sin\theta}{r^2}$$

(*i*) **When $\theta = 0°$** *i.e.*, point $P$ lies on the axis of the conductor, then,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl \sin 0°}{r^2} = 0$$

Hence there is no magnetic field at any point on the thin linear current carrying conductor.

(*ii*) **When $\theta = 90°$** *i.e.*, point $P$ lies at a perpendicular position *w.r.t.* current element, then,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl \sin 90°}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl}{r^2} \qquad \text{.... } \textit{Maximum value}$$

Hence magnetic field due to a current element is maximum in a plane passing through the element and perpendicular to its axis.

(*iii*) **When $\theta = 0°$ or $180°$,** $dB = 0$ ....*Minimum value*

**Important points about Biot-Savart law.** This law has the following salient features :

(*i*) Biot-Savart law is valid for symmetrical current distributions.

(*ii*) Biot-Savart law cannot be proved experimentally because it is not possible to have a current carrying conductor of length $dl$.

(*iii*) Like Coulomb's law in electrostatics, Biot-Savart law also obeys inverse square law.

(*iv*) The direction of $\vec{dB}$ is perpendicular to the plane containing $I\vec{dl}$ and $\vec{r}$.

## 8.8. BIOT-SAVART LAW VERSUS COULOMB'S LAW IN ELECTRO-STATICS

According to Coulomb's law in electrostatics, the electric field due to a charge element $dq$ at a distance $r$ is given by ;

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2}$$

According to Biot-Savart law, the magnetic field due to a current element $I\vec{dl}$ at a distance $r$ is given by ;

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2}$$

From the above two equations, we note the following points of similarities and dissimilarities.

**Similarities**

(*i*) Both laws obey inverse square law.

(*ii*) Both the fields (magnetic field and electrostatic field) obey superposition principle.

(*iii*) Both the fields are long range fields.

**Dissimilarities**

(*i*) The electrostatic field is produced by a scalar source *viz* electric charge $dq$. However, the magnetic field is produced by a vector source *viz* current element $I\,\vec{dl}$.

(*ii*) The direction of the electrostatic field is along the displacement vector *i.e.*, the line joining the source and field point. However, the direction of magnetic field is perpendicular to the plane containing current element $I\,\vec{dl}$ and displacement vector $\vec{r}$.

(*iii*) In Biot-Savart law, the magnitude of magnetic field $dB \propto \sin\theta$. where $\theta$ is the angle between current element $I\,\vec{dl}$ and displacement vector $\vec{r}$. However, there is no angle dependence in Coulomb's law for electrostatics.

## 8.9. MAGNETIC FIELD DUE TO STRAIGHT CONDUCTOR CARRYING CURRENT

Consider a straight conductor $XY$ lying in the plane of the paper and carrying current $I$ in the $X$ to $Y$ direction as shown in Fig. 8.6. It is desired to find the magnetic field at point $P$ located at a perpendicular distance $a$ from the conductor *i.e.*, $PQ = a$.

Consider a small current element $I\,\vec{dl}$ of the conductor at $O$. Let $\vec{r}$ be the position vector of point $P$ from the current element and $\theta$ be the angle between $\vec{dl}$ and $\vec{r}$ (*i.e.*, $\angle POQ = \theta$). Let us further assume that $QO = l$.

According to Biot-Savart law, the magnitude $dB$ of magnetic field at point $P$ due to the current element $I\,\vec{dl}$ is given by ;

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2} \qquad ...(i)$$

To get the total magnetic field $B$, we must integrate eq. (*i*) over the whole conductor. As we move along the conductor, the quantities $dl$, $\theta$ and $r$ change. The integration becomes much easier if we express everything in terms of angle $\phi$ shown in Fig. 8.6.

In the right angled triangle $PQO$, $\theta = 90° - \phi$

$$\therefore \quad \sin\theta = \sin(90° - \phi) = \cos\phi$$

$$\text{Also,} \quad \cos\phi = \frac{a}{r} \quad \text{or} \quad r = \frac{a}{\cos\phi}$$

$$\text{Further,} \quad \tan\phi = \frac{l}{a} \quad \text{or} \quad l = a\tan\phi$$

$$\text{or} \quad dl = a\sec^2\phi\, d\phi$$

Putting the values of $\sin\theta$, $dl$ and $r$ in eq. (*i*), we have,

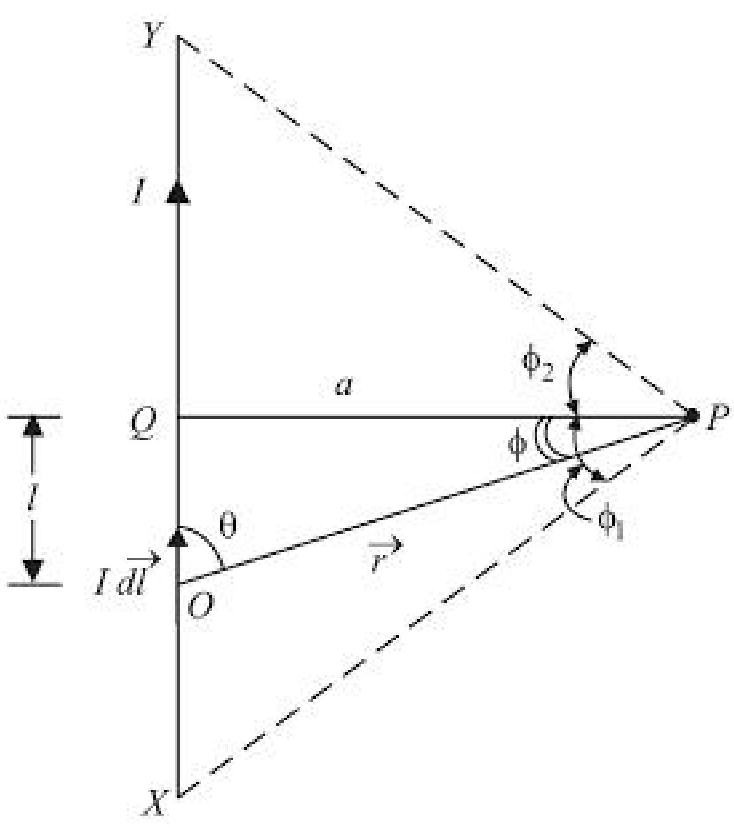



**Fig. 8.6**

(*i*) Right-hand grip rule (*ii*) Maxwell's cork screw rule

**(*i*) Right-hand grip rule.** *According to this rule, imagine that you grip the straight conductor with your right hand with thumb pointing in the direction of current. Then curled fingers point in the direction of the magnetic lines of force.* It is sometimes called *right-hand thumb rule.*

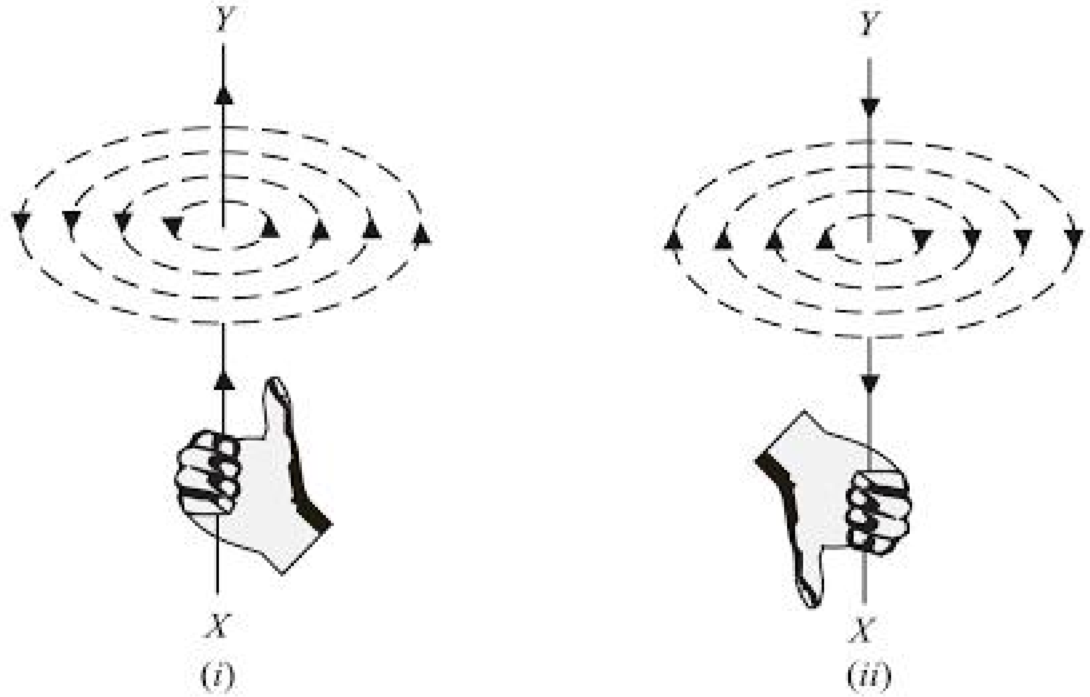



**Fig. 8.7**

Thus referring to Fig. 8.7 (*i*), the current is flowing in the conductor from $X$ to $Y$. Therefore, by right-hand grip rule, the direction of magnetic field lines is anticlockwise. In Fig. 8.7 (*ii*), current is flowing from $Y$ to $X$ and according to right-hand grip rule, the direction of field lines is clockwise.

**(*ii*) Maxwell's cork screw rule.** *According to this rule, if we imagine that a right handed screw is rotated along the conductor so that it advances in the direction of flow of current, then the direction in which the thumb rotates gives the direction of magnetic lines of force.*

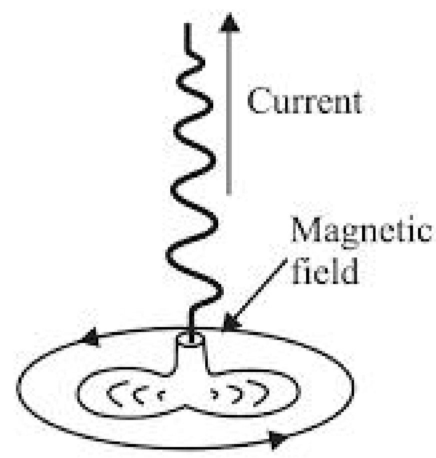



**Fig. 8.8**

Thus referring to Fig. 8.8, the current in the conductor is flowing in the upward direction. By Maxwell's cork screw rule, the direction of magnetic lines of force is anticlockwise.

## 8.10. MAGNETIC FIELD AT THE CENTRE OF CURRENT CARRYING CIRCULAR COIL

Consider a circular coil of radius $r$ and carrying current $I$ in the direction shown in Fig. 8.9. Suppose the loop lies in the plane of paper. It is desired to find the magnetic field at the centre $O$ of the coil. Suppose the entire circular coil is divided into a large number of current elements, each of length $dl$. According to Biot-Savart law, the magnetic field $\vec{dB}$ at the centre $O$ of the coil due to current element $I\vec{dl}$ is given by ;

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,(\vec{dl} \times \vec{r})}{r^3}$$

where $\vec{r}$ is the position vector of point $O$ from the current element. The magnitude of $\vec{dB}$ at the centre $O$ is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,r\sin\theta}{r^3} = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2} \qquad ...(i)$$

The direction of $\vec{dB}$ is perpendicular to the plane of the coil and is directed inwards. Since each current element contributes to the magnetic field in the same direction, the total magnetic field $B$ at the centre $O$ can be found by integrating eq. ($i$) around the loop *i.e.*

$$B = \int dB = \int \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}$$

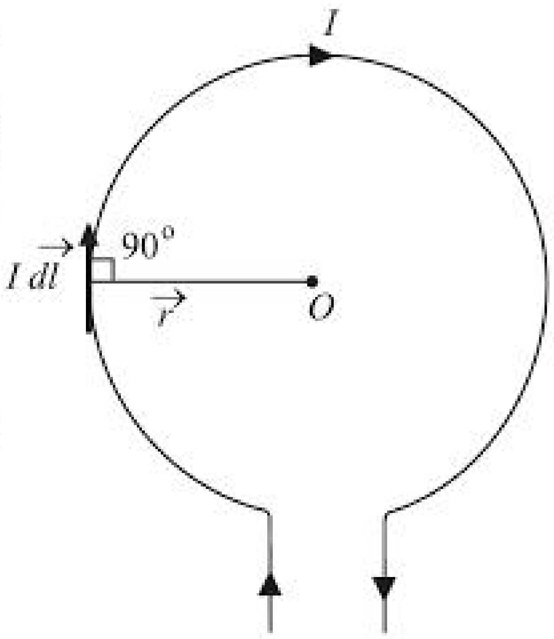


Fig. 8.9

For each current element, angle between $\vec{dl}$ and $\vec{r}$ is 90°. Also distance of each current element from the centre $O$ is $r$.

$$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{I \sin 90^\circ}{r^2} \int dl$$

Now, $\quad \int dl = $ Total length of the coil $= 2\pi r$

$$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{I}{r^2} (2\pi r)$$

or

$$B = \frac{\mu_0 I}{2r}$$

If the coil has $n$ turns, each carrying current in the same direction, then contributions of all the turns are added up.

$$\therefore \quad B = \frac{\mu_0 n I}{2r}$$

**Direction of magnetic field.** Fig. 8.10 shows the magnetic lines of force due to current carrying circular coil. The magnetic field lines are circular near the conductor. As we move radially towards the centre of the coil, the lines of force become less and less curved. At the centre of the coil, the magnetic lines of force are almost straight, parallel and perpendicular to the plane of the coil. This shows that magnetic field is uniform near the centre of the coil. For the direction of current in the circular coil in Fig. 8.10, the direction of magnetic field $\vec{B}$ at the centre of the coil is perpendicular to the plane of the coil directed inwards. This can be ascertained by *Right-hand palm rule.*

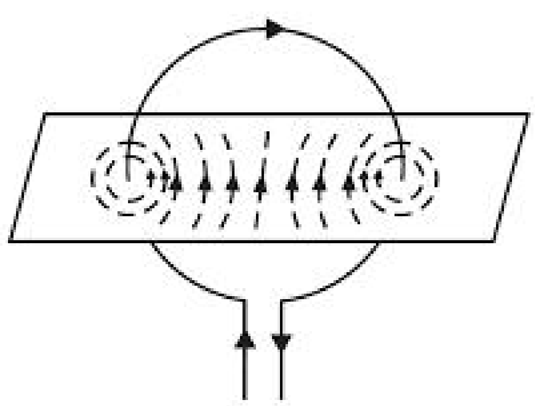


Fig. 8.10

**Right-hand palm rule.** *Orient the thumb of your hand perpendicular to the grip of the fingers such that curvature of the fingers points in the direction of current in the circular coil. Then thumb will point in the direction of the magnetic field near the centre of the circular coil.*

## 8.11. MAGNETIC FIELD ON THE AXIS OF CIRCULAR COIL CARRYING CURRENT

Consider a circular coil of radius $a$, centre $O$ and carrying a current $I$ in the direction shown in Fig. 8.11. Let the plane of the coil be perpendicular to the plane of the paper. It is desired to find the magnetic field at a point $P$ on the axis of the coil such that $OP = x$.

Consider two small current elements, each of length $dl$, located diametrically opposite to each other at $Q$ and $R$. Suppose the distance of $Q$ or $R$ from $P$ is $r$ *i.e.* $PQ = PR = r$.

$$\therefore \quad r = \sqrt{a^2 + x^2}$$

Let $\quad \angle QPO = \alpha = \angle RPO$

According to Biot-Savart law, the magnitude of magnetic field at $P$ due to current element at $Q$ is given by ;

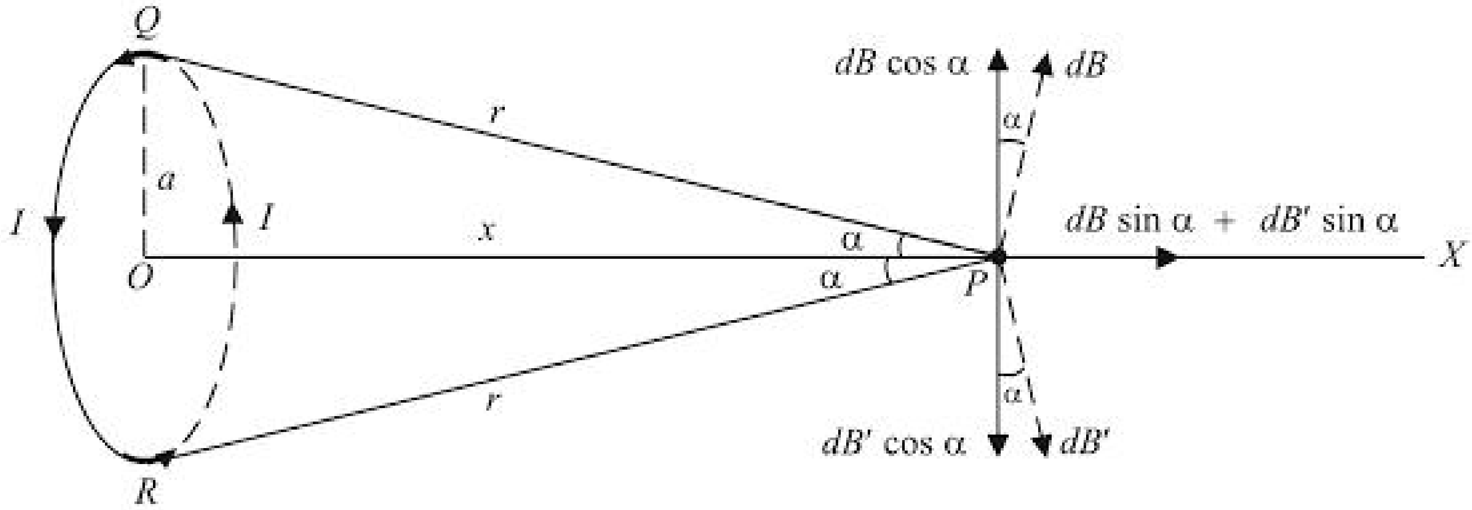



**Fig. 8.11**

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin 90^\circ}{r^2} \qquad (\because\ \theta^* = 90^\circ)$$

or

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{(a^2 + x^2)} \qquad ...(i)$$

The magnetic field at $P$ due to current element at $Q$ is in the plane of paper and at right angles to $\vec{r}$ and in the direction shown.

Similarly, magnitude of magnetic field at point $P$ due to current element at $R$ is given by ;

$$dB' = \frac{\mu_0}{4\pi}\frac{I\,dl}{(a^2 + x^2)} \qquad ...(ii)$$

It also acts in the plane of paper and at right angle to $\vec{r}$ but in opposite direction to $dB$.

From eqs. $(i)$ and $(ii)$, $dB = dB' = \dfrac{\mu_0\, Idl}{4\pi(a^2 + x^2)}$

Resolving $\vec{dB}$ and $\vec{dB'}$ into rectangular components [See Fig. 8.11], it is clear that vertical components ($dB\cos\alpha$ and $dB'\cos\alpha$) will be equal and opposite and thus cancel each other. However, components along the axis of the coil ($dB\sin\alpha$ and $dB'\sin\alpha$) are added and act in the direction $PX$. This is true for all the diametrically opposite elements of the circular coil. Therefore, when we sum up the contributions of all the current elements of the coil, the perpendicular components will cancel. Hence the resultant magnetic field at point $P$ is the vector sum of all the components $dB\sin\alpha$ over the entire coil.

$$\therefore \qquad B = \int dB\sin\alpha = \int \frac{\mu_0\, I\, dl\sin\alpha}{4\pi(a^2 + x^2)} = \frac{\mu_0 I\sin\alpha}{4\pi(a^2 + x^2)}\int dl$$

Now

$$\sin\alpha = \frac{a}{\sqrt{a^2 + x^2}} \quad \text{and} \quad \int dl = 2\pi a$$

$$\therefore \qquad B = \frac{\mu_0\, I\, a^2}{2(a^2 + x^2)^{3/2}} \text{ along } PX \qquad ...(iii)$$

If the circular coil has $n$ turns, then,

$$B = \frac{\mu_0\, n\, I\, a^2}{2(a^2 + x^2)^{3/2}} \text{ along } PX \qquad ...(iv)$$

* The radius vector $QP$ of each current element is perpendicular to it so that $\theta = 90^\circ$ in each case.

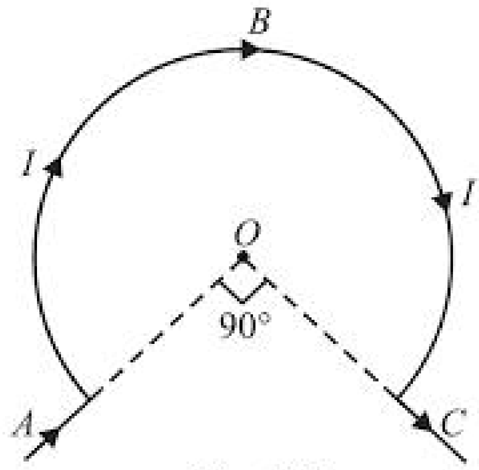


**Fig. 8.18**

**Solution.** Here $I = 10$ A ; $r = 3$ cm $= 3 \times 10^{-2}$ m

Magnetic field induction at the centre of a circular coil is

$$B = \frac{\mu_0 I}{2r} \quad \text{... } r \text{ is the radius of coil}$$

The circular coil subtends an angle of 360° at the centre $O$ of the coil.

∴ Angle subtended by the arc $ABC$ of wire at $O$ is

$$\theta = 360° - 90° = 270°$$

∴ Magnetic field induction at $O$ due to arc $ABC$ of wire is

$$B = \frac{\mu_0 I}{2r} \times \frac{270°}{360°} = \frac{4\pi \times 10^{-7} \times 10}{2 \times 3 \times 10^{-2}} \times \frac{270°}{360°} = \mathbf{1.57 \times 10^{-4}\ T}$$

Example 8.12. **A straight wire carrying a current of 12 A is bent into a semicircular arc of radius 2.0 cm as shown in Fig. 8.19 (*i*). What is the direction and magnitude of the magnetic field at the centre of the arc? What will be the magnitude and direction of the magnetic field at the centre if the wire were bent into a semicircular arc of the same radius but in opposite direction?**

**Solution.** Magnitude of the magnetic field at the centre $O$ of the semicircular arc is given by;

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2r}\right) = \frac{\mu_0 I}{4r}$$

Here $\mu_0 = 4\pi \times 10^{-7}$ T A$^{-1}$m, $I = 12$ A, $r = 2$ cm $= 0.02$ m

$$\therefore \quad B = \frac{(4\pi \times 10^{-7}) \times 12}{4 \times 0.02} = \mathbf{1.88 \times 10^{-4}\ T}$$

Since the values of $I$ and $r$ in both cases are the same, the magnitude of magnetic field at the centre $O$ will be the same in the two cases. However, the direction of magnetic field will be different. In Fig. 8.19 (*i*), the field is perpendicular to the plane of the paper and directed upwards. For Fig. 8.19 (*ii*), the field is perpendicular to the plane of the paper and is directed inward. In both cases, it is assumed that wire is in the plane of the paper.

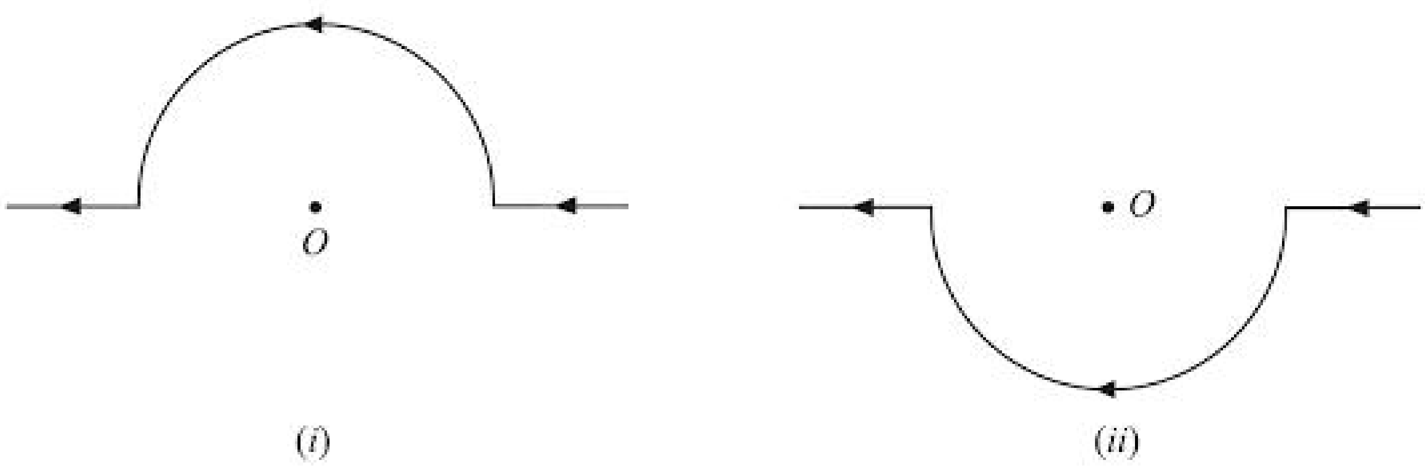


**Fig. 8.19**

Note. In both cases, the straight conductor portions do not contribute to any magnetic field at the centre $O$.

Example 8.13. **The electron of hydrogen atom moves along a circular path of radius 0.5 × $10^{-10}$ m with a uniform speed of 4.0 × $10^6$ ms$^{-1}$. Calculate the magnetic field produced by the electron at the centre ($e$ = 1.6 × $10^{-19}$ C).**

**Solution.** Number of revolutions made by the electron in 1 second is

$$f = \frac{v}{2\pi r} = \frac{4 \times 10^6}{2\pi \times 0.5 \times 10^{-10}} = 1.27 \times 10^{16}$$

Current, $I = \frac{f\,e}{t} = \frac{(1.27 \times 10^{16}) \times (1.6 \times 10^{-19})}{1} = 2.04 \times 10^{-3}$ A

Magnetic field produced by the electron at the centre is

$$B = \frac{\mu_0 I}{2r}$$

Here $\mu_0 = 4\pi \times 10^{-7}$ T A$^{-1}$ m ; $I = 2.04 \times 10^{-3}$ A ; $r = 0.5 \times 10^{-10}$ m

$$\therefore \quad B = \frac{(4\pi \times 10^{-7}) \times (2.04 \times 10^{-3})}{2 \times (0.5 \times 10^{-10})} = \mathbf{25.6\ T}$$

**Example 8.14. A cell is connected across points *A* and *B* of a uniform circular conductor. Prove that the magnetic field at its centre *O* will be zero.**

**Solution.** Figure 8.20 shows the conditions of the problem. The two paths *ADB* and *ACB* are in parallel. Let $I_1$ and $l_1$ be the current and length of path *ADB*; $I_2$ and $l_2$ being that for path *ACB*. If $k$ is the resistance per unit length, then,

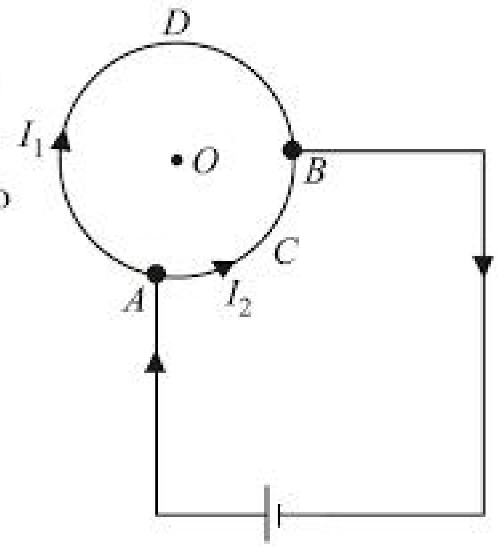



**Fig. 8.20**

$$(kl_1)I_1 = (kl_2)I_2 \quad \text{... p.d. is same across two paths}$$

or $$I_1 l_1 = I_2 l_2$$

According to Biot-Savart law, the magnetic field at $O$ due to current in path *ADB* is given by ;

$$B_1 = \frac{\mu_0}{4\pi}\int \frac{I_1\,dl \sin 90°}{r^2} = \frac{\mu_0 I_1}{4\pi r^2}\int dl = \frac{\mu_0 I_1 l_1}{4\pi r^2}$$

Similarly, $$B_2 = \frac{\mu_0 I_2 l_2}{4\pi r^2} \quad \text{... for path } ACB$$

Since $I_1 l_1 = I_2 l_2$ ; $B_1 = B_2$

By right-hand palm rule, the directions of $B_1$ and $B_2$ are opposite. Hence the resultant magnetic field at $O$ is zero.

**Example 8.15. A circular coil of 100 turns has a radius of 10cm and carries a current of 5A. Determine the magnetic field (*i*) at the centre of the coil (*ii*) at a point on the axis of the coil at a distance of 5cm from the centre of the coil.**

**Solution.** (*i*) Magnetic field at the centre of the coil is

$$B = \frac{\mu_0 n I}{2r}$$

Here $\mu_0 = 4\pi \times 10^{-7}$ T A$^{-1}$ m ; $n = 100$; $I = 5$A; $r = 10$ cm $= 0.1$m

$$\therefore \quad B = \frac{(4\pi \times 10^{-7}) \times 100 \times 5}{2 \times 0.1} = \mathbf{3.14 \times 10^{-3}\ T}$$

(*ii*) Magnetic field on the axis of the coil at a distance $x$ from the centre is

$$B = \frac{\mu_0 n I a^2}{2(a^2 + x^2)^{3/2}}$$

Here, $\mu_0 = 4\pi \times 10^{-7}$ T A$^{-1}$ m ; $n = 100$ ; $I = 5$ A ; $a = 0.1$ m ; $x = 0.05$ m

$$\therefore \quad B = \frac{(4\pi \times 10^{-7}) \times (100) \times 5 \times (0.1)^2}{2[(0.1)^2 + (0.05)^2]^{3/2}} = \mathbf{2.25 \times 10^{-3}\ T}$$

**Example 8.16. Fig. 8.21 shows a right angled isosceles triangle *PQR* having its base equal to *a*. A current of *I* amperes is passing downwards along a thin straight wire cutting the plane of the paper normally as shown at *Q*. Likewise, a similar wire carries an equal current passing normally upwards at *R*. Find the magnitude and direction of magnetic flux density *B* at *P*. Assume the wires to be infinitely long.**

**Solution.** Referring to Fig. 8.21,

$$\angle PQR = \angle PRQ = 45°$$

$$\therefore \quad PQ = PR = QR \sin 45° = \frac{a}{\sqrt{2}}$$

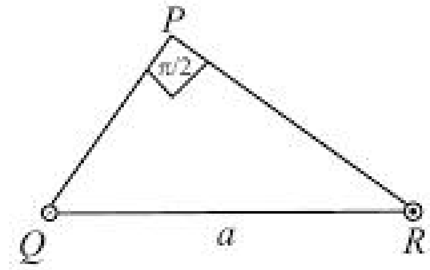



Fig. 8.21

Magnetic flux density $B_1$ at point $P$ due to current through the conductor at point $Q$ is

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{2I}{PQ} = \frac{\mu_0}{4\pi} \frac{2\sqrt{2}\, I}{a} \text{ along } PR$$

Magnetic flux density $B_2$ at point $P$ due to current through the conductor at point $R$ is

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{2I}{PR} = \frac{\mu_0}{4\pi} \frac{2\sqrt{2}\, I}{a} \text{ along } PQ$$

Since $B_1$ and $B_2$ are at right angles, the resultant magnetic flux density $B$ at point $P$ is

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{4\pi} \frac{I}{a} \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = \frac{\mu_0}{4\pi} \times \frac{4I}{a} = \frac{\mu_0 I}{\pi a}$$

The direction of $B$ is along the right bisector of $\angle QPR$ towards the base $QR$ of the triangle.

Example 8.17. **An electric current *I* is flowing in a circular wire of radius *a*. At what distance from the centre on the axis of circular wire will the magnetic field be 1/8th of its value at the centre?**

**Solution.** Magnetic field $B$ at the centre of the circular coil is

$$B = \frac{\mu_0 I}{2a}$$

Suppose at a distance $x$ from the centre on the axis of the circular coil, the magnetic field is $B/8$.

$$\therefore \quad \frac{B}{8} = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

$$\text{or} \quad \frac{\mu_0 I}{2a} \times \frac{1}{8} = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

$$\text{or} \quad \frac{1}{8a} = \frac{a^2}{(a^2 + x^2)^{3/2}} \quad \text{or} \quad (a^2 + x^2)^{3/2} = 8\, a^3$$

$$\therefore \quad a^2 + x^2 = 4\, a^2 \quad \text{or} \quad x = \sqrt{3}\, a$$

Example 8.18. **A long straight wire carries a current of 50 A. An electron moving at $10^7$ ms$^{-1}$ is 5 cm from the wire [See Fig. 8.22]. Find the force acting on the electron if velocity is directed (*i*) towards the wire (*ii*) parallel to the wire (*iii*) perpendicular to the directions defined by (*i*) and (*ii*).**

**Solution.** The magnetic field produced by current carrying long wire at a distance $a$ (= 5 cm = $5 \times 10^{-2}$ m) is

$$B = \frac{\mu_0 I}{2\pi a} = \frac{\mu_0}{2\pi} \times \frac{I}{a}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{50}{5 \times 10^{-2}} = 2 \times 10^{-4} \text{ tesla}$$

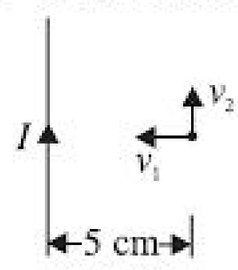



Fig. 8.22

The field is directed downward perpendicular to the plane of the paper.

(*i*) The velocity $\vec{v_1}$ is towards the wire. Therefore, the angle between $\vec{v_1}$ and $\vec{B}$ is 90°.

$\therefore$ Force on the electron is

$$\vec{F} = q\,(\vec{v_1} \times \vec{B})$$

or $$F = ev_1\, B \sin 90^\circ = (1.6 \times 10^{-19}) \times (10^7) \times (2 \times 10^{-4}) \times 1 = \mathbf{3.2 \times 10^{-16}\ N}$$

By right hand rule for cross product, the *force is parallel to the current.*

*(ii)* When the electron is moving parallel to the wire, angle between $\vec{v_2}$ and $\vec{B}$ is again 90°. Therefore, force is again $3.2 \times 10^{-16}$ N. However, by right hand rule for cross product, the *force is directed radially away from the wire.*

*(iii)* When the electron is moving perpendicular to the directions defined by (*i*) and (*ii*), the angle between $\vec{v}$ and $\vec{B}$ is 0 or $\pi$. Therefore, $\sin\theta = 0$. Hence, force acting on the electron is **zero**

**Example 8.19. Use Biot-Savart law to obtain an expression for the magnetic field at the centre of a coil in the form of a square of side 2*a* carrying a current *I*.**

**Solution.** Fig. 8.23 shows the conditions of the problem. Let $O$ be the centre of the square $ABCD$ of side $2a$ carrying current $I$. The magnetic field at the centre $O$ due to finite length of wire $AB$ is

$$B_1 = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$$

$$= \frac{\mu_0 I}{4\pi a}(\sin 45^\circ + \sin 45^\circ) \quad (\because \theta_1 = \theta_2 = 45^\circ)$$

$$= \frac{\sqrt{2}\,\mu_0 I}{4\pi a}$$

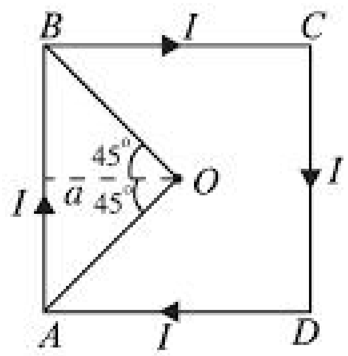



**Fig. 8.23**

By symmetry, the magnetic field at $O$ due to each side

$$= \frac{\sqrt{2}\,\mu_0 I}{4\pi a}$$

$\therefore$ Net magnetic field at $O$ due to current carrying square is

$$B = 4B_1 = 4 \times \frac{\sqrt{2}\,\mu_0 I}{4\pi a} = \boldsymbol{\frac{\sqrt{2}\,\mu_0 I}{\pi a}}$$

**Example 8.20. In Bohr's model of hydrogen atom the electron circulates around nucleus on a path of radius 0.51 Å at a frequency of 6.8 × $10^{15}$ rev/second. Calculate the magnetic field induction at the centre of the orbit. What is the effective dipole moment ?**

**Solution.** The circulating electron is equivalent to circular current loop carrying current $I$ given by ;

$$I = \frac{dq}{dt} = \frac{e}{1/f} = ef = (1.6 \times 10^{-19}) \times (6.8 \times 10^{15}) = 1.1 \times 10^{-3}\ \text{A}$$

Magnetic field at the centre due to this current is

$$B_{centre} = \frac{\mu_0 I}{2r} = \frac{(4\pi \times 10^{-7}) \times (1.1 \times 10^{-3})}{2 \times (0.51 \times 10^{-10})} = \mathbf{14\ T}$$

Effective dipole moment is $M = nI\,A$

Here $n = 1$ (for single electron); $I = 1.1 \times 10^{-3}$ A; $A = \pi r^2$

$$\therefore \quad M = 1 \times (1.1 \times 10^{-3}) \times [\pi \times (0.51 \times 10^{-10})^2] = \mathbf{9 \times 10^{-24}\ Am^2}$$

## PROBLEMS FOR PRACTICE

1. A horizontal overhead power line carries a current of 90A in east to west direction. What is the magnitude and direction of magnetic field due to the current 1.5m below the wire? **[1.2 × $10^{-5}$ T *towards south*]**

2. A long straight wire is turned into a loop of radius 10 cm as shown in Fig. 8.24. If a current of 8 A is passed, then find the value of magnetic field at the centre $O$ of the loop. **[3.4 × $10^{-5}$ T *perpendicular to plane of paper pointing upward*]**

   [**Hint.** The magnetic field at $O$ due to straight wire is perpendicular to the plane of paper and is directed downward. However, field due to circular loop is directed in opposite direction.]

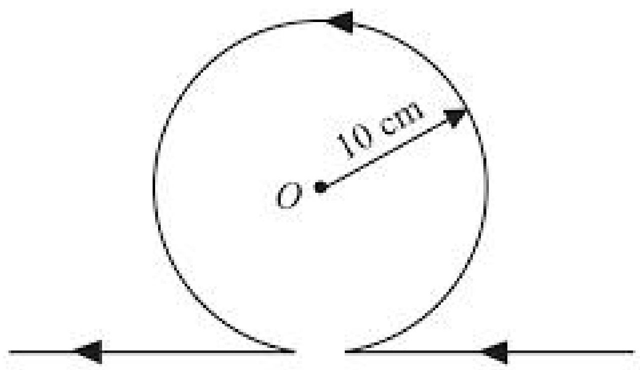



**Fig. 8.24**

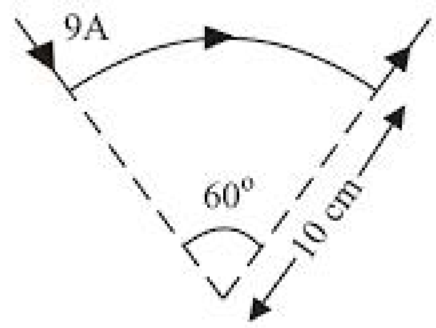



**Fig. 8.25**

3. A circular segment of radius 10 cm subtends an angle of 60° at its centre. A current of 9A is flowing through it. Find the magnitude and direction of magnetic field produced at the centre [See Fig. 8.25].

   **[9.42 × $10^{-6}$ T *perpendicular to the plane of paper pointing downward*]**

   [**Hint.** The magnetic field at the centre of a single turn circular coil is

$$B = \frac{\mu_0 I}{2r} \quad \text{... } r \text{ is the radius of coil.}$$

   For the given arc, $B = \dfrac{60^\circ}{360^\circ}\left(\dfrac{\mu_0 I}{2r}\right)$]

4. A long wire having a semicircular loop of radius $a$ carries a current $I$ amperes as shown in Fig. 8.26. Find the magnetic field at the centre of the semicircular arc. $\left[\dfrac{\boldsymbol{\mu_0 I}}{\boldsymbol{4a}}\right]$

   [**Hint.** The straight portions $AB$ and $DE$ do not contribute to any magnetic field at $O$. Therefore, magnetic field at $O$ is only due to semicircular loop.]

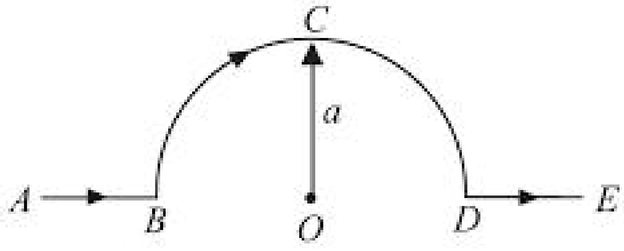



**Fig. 8.26**

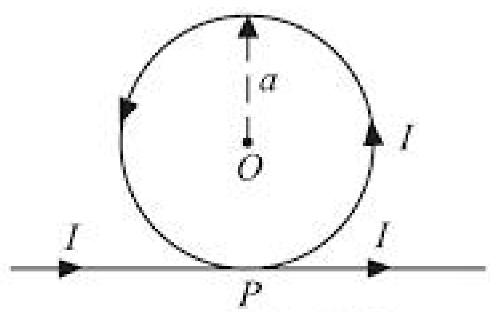



**Fig. 8.27**

5. The wire shown in Fig. 8.27 carries a current $I$. What will be the magnitude and direction of magnetic field at the centre $O$? Assume that various portions of wire do not touch each other at $P$.

$$\left[\frac{\mu_0 I}{2a}\left(1+\frac{1}{\pi}\right) \textit{ perpendicular to the plane of paper directed upward}\right]$$

[**Hint.** The magnetic field due to straight conductor and that due to circular part aid each other at $O$.]

**6.** Two concentric circular coils $X$ and $Y$ of radii 16 cm and 10 cm respectively lie in the same vertical plane containing the north-south direction. Coil $X$ has 20 turns and carries a current of 16A; coil $Y$ has 25 turns and carries a current of 18A. The sense of current in $X$ is anticlockwise and in $Y$ clockwise for an observer looking at coils facing west. Determine the magnitude and direction of net magnetic field due to the coils at their centre. **[$1.6 \times 10^{-3}$ T towards East]**

**7.** A long straight wire carries a current of 50A. An electron moving at $10^7$ ms$^{-1}$ is 5.0 cm from the wire. Find the force on the electron if it is moving parallel to the wire. **[$3.2 \times 10^{-16}$ N]**

**Hint.** Magnetic field at a distance '$a$' from a wire is given by;

$$B = \frac{\mu_0}{2\pi}\frac{I}{a}$$

Now, $F = B\,q\,v$ $\quad (\because \theta = 90^\circ)$

Note that magnetic field is perpendicular to the plane of conductor. When electron is moving parallel to the wire angle between $\vec{v}$ and $\vec{B}$ is 90°.]

**8.** A circular coil of wire of radius 5 cm having 500 turns carries a current of 1 A. Calculate the magnetic field (*i*) at the centre of coil and (*ii*) at a point at a distance of 12 cm from the centre along the axis of the coil.

**[(*i*) $6.28 \times 10^{-3}$ T; (*ii*) $3.58 \times 10^{-20}$ T]**

**9.** A long straight wire carries a current of 4 A. A proton travels with a velocity of $4 \times 10^4$ ms$^{-1}$ parallel to the wire 0.2 m from it and in a direction opposite to the current. What is the force which the magnetic field of current exert on the motion of proton? **[$2.56 \times 10^{-20}$ N]**

**10.** A long straight conductor carries a current of 100 A. At what distance from the conductor is the magnetic field caused by the current equal to $0.5 \times 10^{-4}$ T?

**[40 cm]**

**11.** A circular loop of one turn carries a current of 5 A. If the magnetic field at the centre is 0.2 mT, find the radius of the loop. **[1.57 cm]**

**12.** Show that the magnetic field along the axis of a current carrying circular coil of radius $a$ at a distance $x$ ($x \ll a$) from the centre of the coil is smaller by a factor $1.5\, x^2/a^2$ than the field at the centre of the coil.

## 8.12. AMPERE'S CIRCUITAL LAW

Just as Gauss's law is an alternative form of Coulomb's law in electrostatics, similarly we have Ampere's circuital law as an alternative form of Biot-Savart law in magnetostatics. Ampere's circuital law gives the general relation between a current in a wire of any shape and the magnetic field produced around it and may be stated as under :

*The line integral of magnetic field $\vec{B}$ around any closed path in vacuum/air is equal to $\mu_0$ times the total current enclosed by that path i.e.*

$$\oint \vec{B}.\,\vec{dl} = \mu_0 I$$

where $\oint \vec{B}.\,\vec{dl}$ = Line integral of $\vec{B}$ around the closed path

$I$ = Current enclosed by that path

This result holds good irrespective of the size and shape of the closed path enclosing the current. ***This law is true for steady currents only.*** The real importance of this law lies in the fact that it relates the magnetic field to current in a direct and mathematically elegant way.

**Proof.** Consider a long straight conductor carrying current $I$ in the direction shown in Fig. 8.28. Due to current in the conductor, the magnetic lines of force are concentric circles centred on the conductor. We know that magnitude of the magnetic field at a point $P$ at a perpendicular distance $r$ from the conductor is given by ;

$$B = \frac{\mu_0 I}{2\pi r}$$

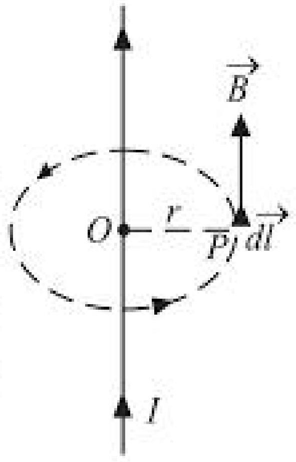



Fig. 8.28

Consider a magnetic line of force of radius $r$. On this closed path, the magnitude of $\vec{B}$ is same everywhere. Further, the direction of $\vec{B}$ at every point is along tangent to the circle *i.e.* angle between $\vec{dl}$ and $\vec{B}$ at every point on this path is zero *i.e.* $\cos\theta = 1$. Therefore, line integral of $\vec{B}$ over this closed path is given by ;

$$\oint \vec{B}.\vec{dl} = \oint B\, dl \cos 0° = \oint B\, dl$$

$$= \oint \frac{\mu_0 I}{2\pi r} dl \qquad \left(\because\ B = \frac{\mu_0 I}{2\pi r}\right)$$

$$= \frac{\mu_0 I}{2\pi r}\oint dl = \frac{\mu_0 I}{2\pi r} \times 2\pi r = \mu_0 I$$

$$\therefore \qquad \oint \vec{B}.\vec{dl} = \mu_0 I$$

This proves ampere's circuital law.

In order to use this law, it is necessary to choose a closed path for which it is possible to determine the line integral of $\vec{B}$. Therefore, this law can be used for calculating magnetic field *only* for current distributions with sufficient symmetry. Where it is not possible to use this *law, we employ Biot-Savart law to determine $\vec{B}$.

## 8.13. APPLICATIONS OF AMPERE'S CIRCUITAL LAW

For charge distribution with sufficient symmetry, we used Gauss's law to solve for the electric field in a simple way. Similarly, for current distribution with sufficient symmetry, we can use Ampere's law to solve for the magnetic field. In order to illustrate the use of Ampere's law, we shall consider the following cases :

(*i*) Magnetic field due to a long straight conductor carrying current.

(*ii*) Magnetic field due to a solenoid carrying current.

(*iii*) Magnetic field due to current in a toroid.

## 8.14. MAGNETIC FIELD DUE TO STRAIGHT CONDUCTOR CARRYING CURRENT

Consider a long straight conductor carrying current $I$ in the direction shown in Fig. 8.29. It is desired to find the magnetic field $\vec{B}$ at a point $P$ at a perpendicular distance $r$ from the conductor. The magnetic lines of force are concentric circles centred at the conductor. We choose a circle of radius $r$ as the closed path. The magnitude of $\vec{B}$ is the same everywhere on this closed

* It may be noted that Ampere's law holds even if there is no symmetry. However, in such cases, it is generally not possible to find the line integral of $\vec{B}$ over a closed path. Instead, we can use Biot-Savart law in such situations.

path. Further, the angle between $\vec{B}$ and $\vec{dl}$ is 0° everywhere on this path. Therefore, applying Ampere's circuital law to this closed path, we have,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

or $$\oint B\, dl \cos 0° = \mu_0 I$$

or $$B \oint dl = \mu_0 I$$

or $$B \times 2\pi r = \mu_0 I$$

$\therefore$ $$B = \frac{\mu_0 I}{2\pi r}$$

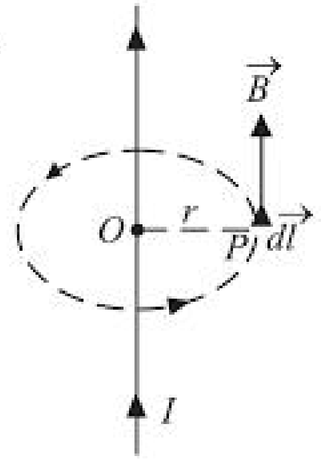


**Fig. 8.29**

This is the same expression as was obtained in section. 8.9. The reader may note the ease with which the result is obtained by using Ampere's law.

## 8.15. MAGNETIC FIELD DUE TO A LONG CURRENT CARRYING SOLENOID

*A long coil of wire consisting of closely packed loops is called a* **solenoid.**

The word solenoid comes from a Greek word meaning "tube-like". By a long solenoid we mean that the length of the solenoid is very large as compared to its diameter. Fig. 8.30 shows the magnetic field lines due to an air-cored solenoid carrying current. Note that the magnetic field resembles that of a bar magnet. Inside the solenoid, the magnetic field is uniform and parallel to the solenoid axis. Outside the solenoid, the magnetic field is very small as compared to the field inside and may be assumed zero. It is because the same number of field lines that are concentrated inside the solenoid spread out into very vast space outside.

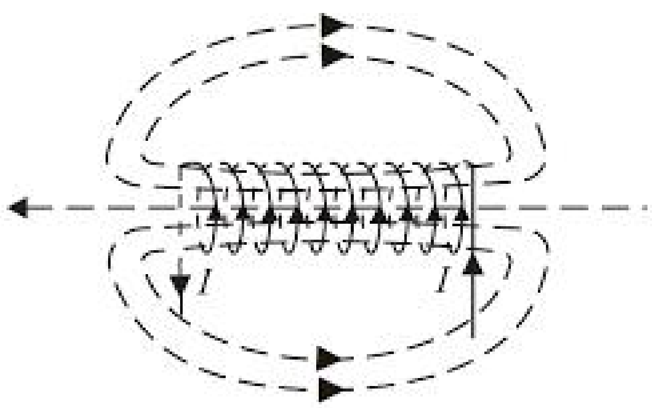


**Fig. 8.30**

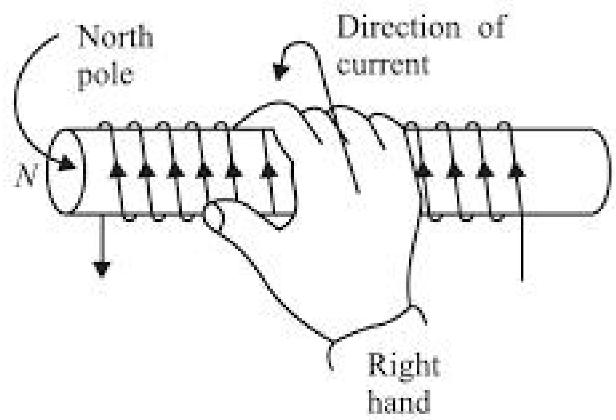


**Fig. 8.31**

The magnetic polarity of any end of the solenoid can be determined by *Ampere's right hand rule.*

**Ampere's right hand rule.** *Imagine to grasp the solenoid with right hand so that the fingers are curled in the direction of current. Then the thumb stretched parallel to the axis of the solenoid will point towards the N-pole end of the solenoid* [See Fig. 8.31]

**Magnetic field $\vec{B}$ inside a solenoid.** Consider a long air-cored solenoid having closely packed coils as shown in Fig. 8.32.

Let $I$ = Current through the solenoid

$n$ = number of turns per unit length

$B$ = magnitude of magnetic field inside the solenoid

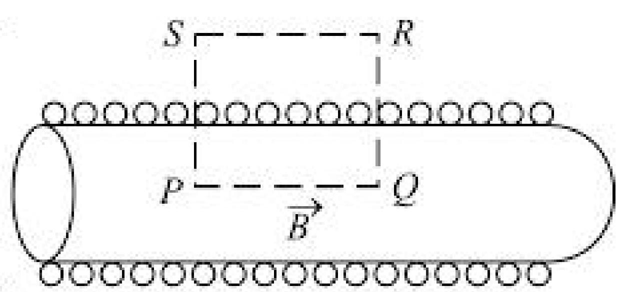


**Fig. 8.32**

In order to use Ampere's circuital law to determine the magnetic field inside a solenoid, we choose a rectangular closed [See Fig. 8.32] path *PQRSP* where $PQ = l$. The line integral of $\vec{B}$ over the closed path *PQRSP* is given by ;

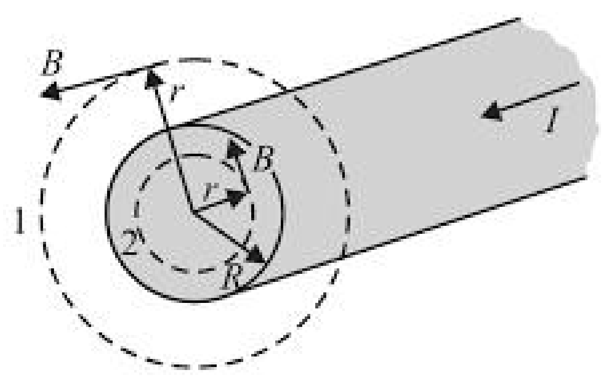



Fig. 8.35

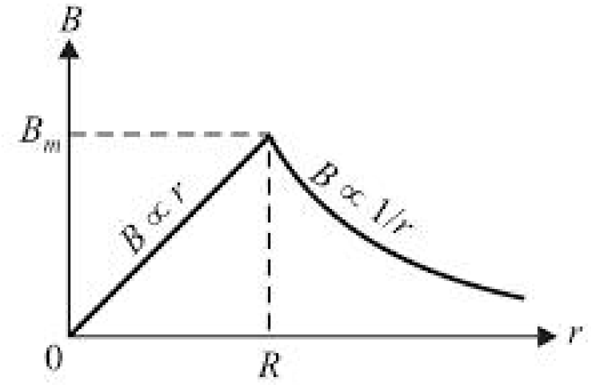



Fig. 8.36

**(*ii*) *B* at a point inside the conductor.** Let us now choose a circular path of radius $r$ ($r < R$) centred on the conductor *i.e.*, path 2 in Fig. 8.35. Again $\vec{B}$ is same everywhere on this path and angle between $\vec{B}$ and $\vec{dl}$ is zero everywhere on this path.

$$\therefore \quad \oint \vec{B}.\vec{dl} = \mu_0 \times \text{Current enclosed by closed path}$$

The current enclosed by this path is less than $I$ and is given by ;

$$I' = \frac{I}{\pi R^2} \times \pi r^2 = \frac{I\,r^2}{R^2}$$

$$\therefore \quad \oint \vec{B}.\vec{dl} = \mu_0 \times I'$$

$$\text{or} \quad B \times 2\pi\, r = \mu_0 \times \frac{I\,r^2}{R^2}$$

$$\therefore \quad B = \frac{\mu_0\, I\, r}{2\pi\, R^2} \qquad \text{... } (ii)$$

Fig. 8.36 shows the graph between magnetic field $B$ and distance $r$ from the axis of the cylinder. From eq. (*ii*) above, $B \propto r$ so that $B = 0$ for a point on the axis of the conductor and increases linearly with $r$ till $r = R$. At $r = R$ (*i.e.*, at the surface of the conductor) the value of $B$ is maximum ($B_m$). From eq. (*i*) above, $B \propto 1/r$ so that for $r > R$, the graph is a hyperbola.

**Example 8.22. A solenoid has a length of 1.23 m and inner diameter 4 cm. It has five layers of windings of 850 turns each and carries a current of 5.57A. What is the magnitude of the magnetic field at the centre of the solenoid?**

**Solution.** The magnitude of the magnetic field at the centre of a solenoid is given by ;

$$B = \mu_0\, n\, I$$

Here $\mu_0 = 4\pi \times 10^{-7}\ \text{T A}^{-1}\ \text{m}$; $n = \dfrac{5 \times 850}{1.23}$; $I = 5.57\text{A}$

$$\therefore \quad B = (4\pi \times 10^{-7}) \times \frac{(5 \times 850)}{1.23} \times 5.57 = 24.2 \times 10^{-3}\ \text{T}$$

**Note.** The magnitude of the magnetic field ($B = \mu_0 nI$) does not depend upon the diameter of the solenoid. Therefore, the above formula is applicable even if more layers of winding are added.

**Example 8.23. A toroid has a core (non-ferromagnetic) of inner radius 20 cm and outer radius 25 cm around which 1500 turns of a wire are wound. If current in the wire is 2A, calculate the magnetic field (*i*) inside the toroid, (*ii*) outside the toroid.**

**Solution.** Mean radius, $r = \dfrac{20 + 25}{2} = 22.5\ \text{cm} = 0.225\ \text{m}$

Mean length, $l = 2\pi\, r = 2\pi \times 0.225 = 1.413\ \text{m}$

(*i*) The magnitude of the magnetic field inside the toroid is given by ;

$$B = \mu_0 n I$$

Here, $\mu_0 = 4\pi \times 10^{-7}\ TA^{-1}m$ ; $n = \frac{1500}{1.413}$ ; $I = 2$ A

$\therefore$ $$B = (4\pi \times 10^{-7}) \times \left(\frac{1500}{1.413}\right) \times 2 = 0.003\ T$$

(*ii*) The magnetic field outside the toroid is zero. It is all inside the toroid.

Example 8.24. **A solenoid 1.5 m long and 4 cm in diameter possesses 10 turns/cm. A current of 5 A is flowing through it. Calculate the magnetic induction (*i*) inside and (*ii*) at one end on the axis of the solenoid.**

**Solution.** $$n = \frac{N}{l} = 10 \text{ turns/cm} = 10^3 \text{ turns/metre}$$

(*i*) Inside the solenoid, the magnetic induction is given by ;

$$B = \mu_0 n I = (4\pi \times 10^{-7}) \times (10^3) \times 5 = 2\pi \times 10^{-3}\ T$$

(*ii*) At the end of the solenoid, the magnetic induction is given by ;

$$B = \frac{\mu_0 n I}{2} = \frac{2\pi \times 10^{-3}}{2} = \pi \times 10^{-3}\ T$$

Example 8.25. **A long straight solid conductor of radius 5 cm carries a current of 2 A, which is uniformly distributed over its circular cross-section. Find the magnetic field induction at a distance of 3 cm from the axis of the conductor.**

**Solution.** As proved in example 8.21, the magnetic field induction inside a long straight solid conductor is given by ;

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

Here $I = 2$ A ; $R = 5$ cm $= 5 \times 10^{-2}$ m ; $r = 3$ cm $= 3 \times 10^{-2}$ m

$\therefore$ $$B = \frac{4\pi \times 10^{-7} \times 2 \times 3 \times 10^{-2}}{2\pi \times (5 \times 10^{-2})^2} = 4.8 \times 10^{-6}\ T$$

## PROBLEMS FOR PRACTICE

1. A 32 cm long solenoid, 1.2 cm in diameter, is to produce a 0.2 T magnetic field at its centre. If current in the solenoid is 3.7A, how many turns must the solenoid have? **[$1.4 \times 10^4$]**
2. A toroid has a core of inner radius 25 cm and outer 26 cm around which 3500 turns of a wire are wound. If current in the wire is 11A, calculate magnetic field (*i*) inside the toroid and (*ii*) outside the toroid. [(*i*) **$3 \times 10^{-2}$ T** (*ii*) **zero**]
3. A solenoid 60 cm long and of radius 4 cm has 3 layers of windings of 300 turns each. A 2 cm long wire of mass 2.5 g lies inside the solenoid near its centre normal to its axis; both the wire and the axis of solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of solenoid to an external battery which supplies a current of 6 A in the wire. What value of current in the windings of the solenoid can support the weight of wire? g = 9.8 $ms^{-2}$. **[108.3 A]**

   [Hint. The direction of current in the wire should be such that magnetic force acts vertically upwards.

$$B = \mu_0 \, n \, I = (4\pi \times 10^{-7}) \times \left(\frac{900}{0.6}\right) I = 6\pi \times 10^{-4} \; I \text{ Tesla}$$

$$F = I' B \, l = (6\pi \times 10^{-4}) \times I \times (6) \times 0.02 = 2.26 \times 10^{-4} \, I \text{ newton.}$$

Now $\quad F = mg \quad \text{or} \quad 2.26 \times 10^{-4} \, I = 2.5 \times 10^{-3} \times 9.8 \quad \therefore \; I = 108.3$ A]

4. A 10 cm long solenoid has a total of 400 turns of wire and carries a current of 2 A. Calculate the magnetic field inside near the centre. **[$10^{-2}$ T]**

5. A toroid coil of inner radius 15 cm and outer radius 17 cm is wound from 1200 turns of wire. What are (*i*) the maximum and (*ii*) minimum magnetic field strengths within the toroid when a current of 10A flows? [(*i*) **0.016 T** (*ii*) **0.014T**]

6. A long solenoid consists of 20 turns per cm. What current is necessary to produce a magnetic field of 20 mT inside the solenoid? **[8 A]**

7. A long straight solid conductor of radius 4 cm carries a current of 2 A, which is uniformly distributed over its circular cross-section. Find the magnetic field at a distance of 3 cm from the axis of the conductor. **[$7.5 \times 10^{-6}$ T]**

## CONCEPTUAL QUESTIONS

**Q.1. Distinguish between electric and magnetic fields.**

**Ans.** The following are the points of differences between electric and magnetic fields:

(*i*) The source of electric field is the electric charge while that of magnetic field is the current element $(I\vec{dl})$.

(*ii*) The electric field is associated with stationary as well as moving charges. However, magnetic field is associated only with a moving charge.

(*iii*) Electric lines of force originate from a positive charge and end at a negative charge *i.e.* they are discontinuous. However, magnetic lines of force form closed loops.

**Q.2. An electron is moving along *X*-axis in a magnetic field acting along *Y*-axis. What is the direction of magnetic force acting on it?**

**Ans.** The magnetic force $\vec{F_m}$ on a charge $q$ moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by ;

$$\vec{F_m} = q\,(\vec{v} \times \vec{B})$$

Therefore, $\vec{F_m}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. Hence $\vec{F_m}$ will act along the $Z$ axis.

**Q.3. An electron is not deflected while passing through a certain region of space. Can we conclude that there is no magnetic and electric field?**

**Ans.** $\vec{F_m} = e\,(\vec{v} \times \vec{B}) \quad \text{and} \quad \vec{F_e} = e\,\vec{E}$

It is given that $\vec{F_m} = 0$ and $\vec{F_e} = 0$

(*i*) For $F_m$ to be zero, either $v$ is parallel or antiparallel to $B$ or $B = 0$. In other words, there may or may not be magnetic field.

(*ii*) For $F_e$ to be zero, $E$ must be zero. Hence in this region, there is no electric field.

**Q.4. An electron can be deflected by magnetic as well as electric field. What is the difference in these deflections?**

$$\text{Current element} = I\,\vec{dl}$$

Current element produces magnetic field just as a stationary charge produces an electric field.

**Q.21. What is the significance of Biot-Savart law?**

**Ans.** It gives the magnetic flux density at a point due to a small current element.

**Q.22. Biot-Savart law cannot be tested directly. Why?**

**Ans.** Biot-Savart law cannot be tested directly because it is not possible to have a current carrying conductor of length $dl$.

**Q.23. Give expression for Biot-Savart law in vector form.**

**Ans.** According to Biot-Savart law, the magnetic flux density $(\vec{dB})$ at a point distant $r$ due to a current element of length $dl$ carrying current $I$ is given by ;

$$\vec{dB} = \frac{\mu_0}{4\pi} I \frac{(\vec{dl} \times \vec{r})}{r^3}$$

**Q.24. Does a current-carrying circular coil produce a uniform magnetic field?**

**Ans.** No. However, at the centre of the coil, the magnetic field is *nearly* uniform.

**Q.25. What is the effect of increasing the number of turns of a circular coil on the magnetic field at its centre?**

**Ans.** Magnetic field at the centre of circular coil, $B = \frac{\mu_0 n I}{2r}$. By increasing the number of turns $(n)$ of the coil, magnetic field $(B)$ is increased.

**Q.26. Name the rule used to find the direction of magnetic field at the centre of a current carrying circular coil.**

**Ans.** It is found by using *Right hand palm rule*.

**Q.27. Looking at a circular coil, the current is found to be flowing in anticlockwise direction. Predict the direction of magnetic field.**

**Ans.** The direction of magnetic field is perpendicular to the plane of the coil and directed towards the observer.

**Q.28. Fig. 8.39 shows a circular loop carrying current *I*. What is the direction of magnetic field?**

**Ans.** By right hand palm rule, the direction of magnetic field $(B)$ is perpendicular to the plane of the loop and is directed in outward direction.

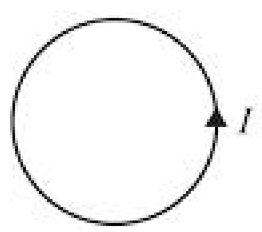



**Fig. 8.39**

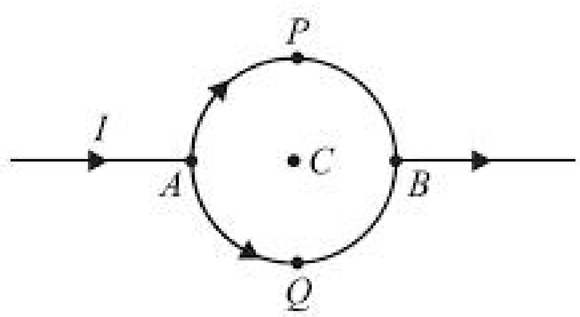



**Fig. 8.40**

**Q.29. What is the magnetic field at the centre *C* of the circular loop shown in Fig. 8.40? Here *APB* and *AQB* are semicircles.**

**Ans.** Zero. It is because magnetic field $(B)$ at the centre $C$ of the loop due to semicircle $APB$ is equal and opposite to that due to the semicircle $AQB$.

**Q.30. How does a current loop behave as a magnetic dipole?**

**Ans.** When a loop carries current, one face of the current loop behaves as a north pole and the other face as south pole. Therefore, current loop behaves as a magnetic dipole.

**Q.31. What is the magnetic moment associated with a coil of 1 turn, area of cross-section $10^{-4}$ m$^2$, carrying a current of 2 A?**

**Ans.** Magnetic moment, $M = n\,I\,A = 1 \times 2 \times 10^{-4} = 2 \times 10^{-4}$ A m$^2$

**Q.32. Equal currents *I*, *I* are flowing through infinitely long parallel wires. What will be the magnetic field at a point midway, when the currents are flowing in the same direction?**

**Ans.** Zero. It is because magnetic fields due to these wires are equal and opposite at the mid-point.

**Q.33. In the above question, what will be the magnetic field at a point midway, when the currents are flowing in the opposite directions?**

**Ans.** It will be double due to one current carrying wire. It is because now magnetic fields due to the two wires are in the same direction.

**Q.34. What will be the magnetic field (*i*) outside (*ii*) inside a long current carrying air-cored solenoid?**

**Ans.** (*i*) Magnetic field outside the solenoid is zero. (*ii*) Magnetic field well inside the solenoid is $B = \mu_0 nI$. Here $n$ is the number of turns per unit length of the solenoid.

**Q.35. What is the value of magnetic field inside an iron-cored solenoid?**

**Ans.** The value of magnetic field inside an iron-cored solenoid is $B = \mu_0\,\mu_r\,n\,I$. Here $\mu_r$ is the relative permeability of iron.

**Q.36. What is the magnetic field at each end of a very long air-cored current-carrying solenoid?**

**Ans.** It is given by ;$B = \mu_0\,n\,I/2$.

**Q.37. Why is the magnetic field due to a toroid the same as for a solenoid?**

**Ans.** It is because a toroid is a solenoid in the form of a closed ring.

**Q.38. Is Ampere's law true for varying current?**

**Ans.** No, it is true for steady currents only.

**Q.39. What is the importance of Ampere's law?**

**Ans.** It gives the relation between current and magnetic field mathematically.

## SHORT ANSWER QUESTIONS

**Q.1. How is a magnetic field produced?**

**Ans.** A magnetic field can be produced (*i*) by a magnet (*ii*) by a current-carrying conductor (*iii*) by changing electric field.

**Q.2. What type of field is produced by (*i*) a stationary charge (*ii*) moving charge?**

**Ans.** (*i*) A stationary charge produces electric field only (*ii*) A moving charge (electric current) produces electric as well as magnetic field.

**Q.3. Free electrons keep on moving in a wire even if it carries no current. Why does it not produce magnetic field?**

**Ans.** When a wire carries no current, the free electrons inside the wire are in random motion. Therefore, their average velocity is zero *i.e.*, free electrons do not have net velocity in a definite direction. In other words, there is no current in the wire. Therefore, no magnetic field is produced.

**Q.4. In what respect does a wire carrying current differ from a wire carrying no current?**

**Ans.** When a wire carries current, the electrons (*i.e.*, charges) move in a definite direction. Therefore, such a wire produces magnetic field. If a wire carries no current, the electrons in the wire are in random motion and their average velocity in a particular direction is zero. Therefore, such a wire does not produce any magnetic field.

**Q.5. State the rule to find the direction of magnetic field due to straight current carrying conductor.**

**Ans.** It is found by Right hand thumb rule which states as follows: Imagine that you grip the straight conductor with your right hand with thumb pointing in the direction of conventional current. Then curled fingers point in the direction of magnetic lines of force.

**Q.6. Is the source of magnetic field analogue to the source of electric field?**

**Ans.** No. The source of electric field is an electric charge. The source of magnetic field is not a magnetic charge because we cannot have a single magnetic pole. Instead, electric current (or a magnet) is the source of magnetic field.

**Q.7. In what ways is an electric field different from a magnetic field?**

**Ans.** (*i*) A charged particle always experiences force in an electric field whether the charged particle is at rest or is in motion. However, only a moving charged particle can experience a force in a magnetic field (*ii*) When electric field acts on a charged particle, its kinetic energy changes. However, there is no effect on the kinetic energy of the charged particle due to its motion inside the magnetic field.

**Q.8. Which one of the following will experience a greater force when projected with the same velocity $v$ perpendicular to the magnetic field; (*i*) $\alpha$-particle (*ii*) $\beta$-particle?**

**Ans.** Force, $F_m = q\ v\ B\ \sin\theta = q\ v\ B\ \sin 90^\circ = q\ v\ B$

(*i*) For $\alpha$-particle, $q = 2\ e\ \therefore\ F_\alpha = 2\ e\ v\ B$; (*ii*) For $\beta$-particle, $q = e\ \therefore\ F_\beta = e\ v\ B$. Therefore, $\alpha$-particle will experience greater force.

**Q.9. How will you identify whether the magnetic field at a point is due to earth or due to some current carrying conductor?**

**Ans.** If the freely suspended magnetic needle at that point always rests in north-south direction, then magnetic field is due to earth. If magnetic needle is in some other direction, then it means that some other magnetic field is acting on it.

**Q.10. An electron moving with a velocity of $10^7$ ms$^{-1}$ enters a uniform magnetic field of 1T along a direction parallel to the magnetic field. What would be its trajectory in this field?**

**Ans.** Magnetic force, $F_m = q\ v\ B\ \sin\theta$. Since the electron enters the magnetic field along a direction parallel to the magnetic field, $\theta = 0^\circ$. Therefore, $F_m = 0$. As a result, the trajectory of the electron remains unchanged *i.e.*, the trajectory will be a straight line.

**Q.11. A charged particle moves perpendicular to a magnetic field. How is its momentum affected?**

**Ans.** When a moving charged particle enters perpendicular to a magnetic field, it experiences a force $F_m$ ($= q\ v\ B\ \sin 90^\circ = q\ v\ B$). This force changes the direction of velocity $v$ of the charged particle but not the magnitude of $v$ ($\because F_m$ is always $\perp v$). Therefore, the momentum of the charged particle changes but its kinetic energy remains constant.

**Q.12. A charged particle enters a uniform magnetic field with its initial velocity directed (*i*) parallel to the field (*ii*) perpendicular to the field. Show that there is no change in the kinetic energy of the particle in both cases.**

**Ans.** (*i*) $F_m = q\ v\ B\ \sin\theta = q\ v\ B\ \sin 0^\circ = 0$. Since the force acting on the charged particle is zero, there is no change in the velocity or kinetic energy of the charged particle.

(*ii*) $F_m = q\ v\ B\ \sin 90^\circ = q\ v\ B$. Since $F_m$ is always perpendicular to $v$, this force changes only the direction of velocity ($v$) and not its magnitude. Therefore, there is no change in the kinetic energy of the particle.

**Q.13. What is the nature of magnetic field produced by a current carrying circular wire loop?**

**Ans.** The magnetic lines of force are pependicular to the plane of the circular wire loop. The lines of force are circular near the wire loop but practically straight near the centre of the loop. In the middle of the loop, the magnetic field is uniform for a short distance on either side.

**Q.14. State the rule to find the direction of magnetic field at the centre of current carrying circular wire loop?**

**Ans.** It is found by Right hand palm rule (Refer to Art. 8.10).

**Q.15. How will the magnetic field intensity at the centre of a circular loop carrying current change, if the current through the coil is doubled and the radius of the coil is halved?**

**Ans.** Magnetic field at the centre of circular coil, $B = \dfrac{\mu_0\, n\, I}{2\, r}\ \therefore\ B \propto \dfrac{I}{r}$

$$\frac{mv^2}{r} = q\,v\,B \quad \text{or} \quad r = \frac{mv}{qB} \qquad (\because \ \theta = 90°)$$

**10.** The direction of magnetic force $(\vec{F}_m)$ on a moving charge in a magnetic field can be found by right-hand rule for cross product given below:

*Orient your right hand so that your outstretched fingers point along the direction of motion of positively charged particle; the orientation should be such that when you bend your fingers, they must point along the direction of magnetic field* $(\vec{B})$. *Then your extended thumb will point in the direction of force on the charged particle.*

**11.** The Biot-Savart law can be stated as under :

The magnitude of magnetic flux density $dB$ at a point $P$ which is at a distance $r$ from a very short length $dl$ of a conductor carrying a current $I$ is given by ;

$$dB \propto \frac{I\,dl \sin\theta}{r^2}$$

or $$dB = K\,\frac{I\,dl \sin\theta}{r^2}$$

where $\theta$ is the angle between the short length $dl$ and the line joining it to point $P$ (See Fig. 8.47). The constant of proportionality $K$ depends on the medium around the conductor. In vacuum (or air) $K = \mu_0 / 4\pi$ .

$\therefore$ $$dB = \frac{\mu_0}{4\pi}\,\frac{I\,dl \sin\theta}{r^2}$$

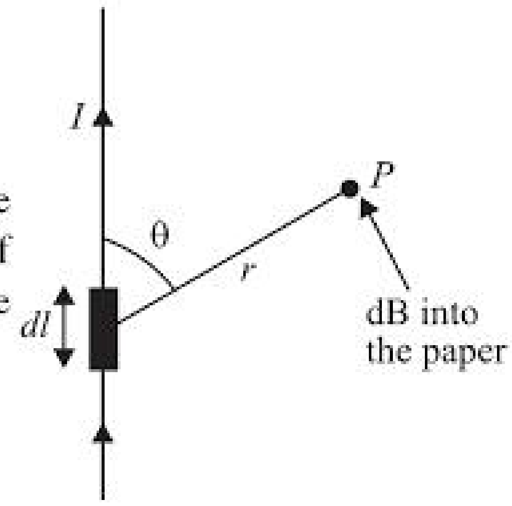



Fig. 8.47

In vector form, $$\vec{dB} = \frac{\mu_0}{4\pi}\,\frac{I\,(\vec{dl} \times \vec{r})}{r^3}$$

(*i*) The direction of $\vec{dB}$ is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$ . It can be found by right hand rule for cross product.

(*ii*) Biot-Savart law holds strictly for steady currents.

(*iii*) The quantity $I\,\vec{dl}$ is called *current element.*

(*iv*) This law cannot be tested directly because it is not possible to have a current-carrying conductor of length $dl$. However, it can be used to derive expressions for flux densities of real conductors and these give values which are in agreement with those determined by experiments.

**12.** The magnitude of magnetic field at the centre of a current-carrying circular coil of radius $r$ is

$$B = \frac{\mu_0 I}{2r} \qquad \text{.....}I \text{ is the current in the coil}$$

If the coil has $n$ turns each carrying current in the same direction, then,

$$B = \frac{\mu_0\,nI}{2r}$$

**13.** A current-carrying loop behaves as a *magnetic dipole*. The magnitude of magnetic dipole moment ($M$) of current-carrying loop is given by ;

$$M = nIA$$

$n$ = number of turns of the loop; $I$ = current ; $A$ = area of the loop

**14.** The magnitude of magnetic field at point $P$ located at a perpendicular distance $a$ from a *finite* straight conductor carrying current $I$ is [See Fig. 8.48]

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_2 + \sin\phi_1)$$

*For infinitely long conductor*, $\phi_1 = \phi_2 = \pi/2$ .

$$\therefore \qquad B = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\pi/2 + \sin\pi/2) = \frac{\mu_0}{4\pi}\frac{2I}{a}$$

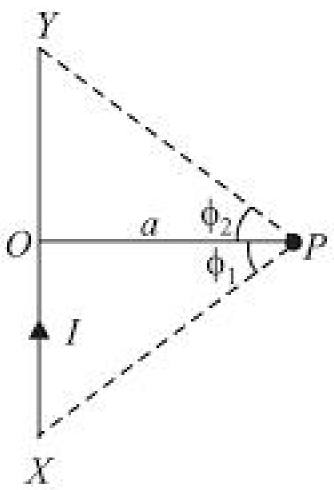



**Fig. 8.48**

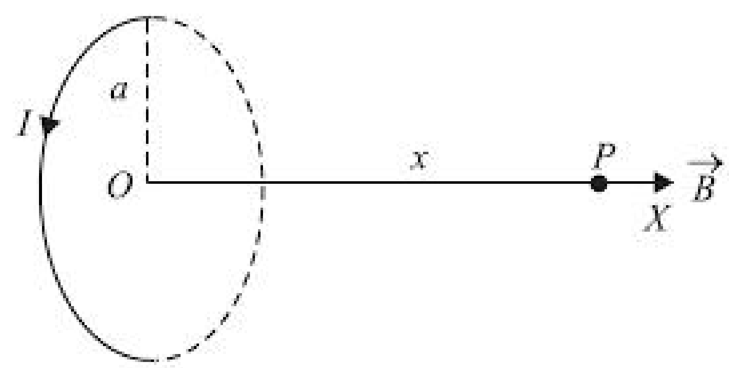



**Fig. 8.49**

**15.** Consider a circular coil of radius $a$, centre $O$ and carrying current $I$ as shown in Fig. 8.49. Let the plane of the coil be perpendicular to plane of the paper. The magnitude of the magnetic field at point $P$ on the axis of the coil ($OP = x$) is

$$B = \frac{\mu_0 n I a^2}{2(a^2 + x^2)^{3/2}} \quad \text{along } PX$$

$n$ = number of turns of the coil

(*i*) When point $P$ is at the centre of coil, $x = 0$ and $B$ becomes :

$$B = \frac{\mu_0 n I}{2a} \quad \text{.......... at the centre of the coil}$$

(*ii*) When point $P$ is far away from the centre of the coil, $x >> a$ so that:

$$B = \frac{\mu_0 n I a^2}{2x^3}$$

**16.** Ampere's circuital law states that the line integral of magnetic field $\vec{B}$ around any closed path in vacuum/air is equal to $\mu_0$ times the total current enclosed by that path [See Fig. 8.50]. *i.e.*

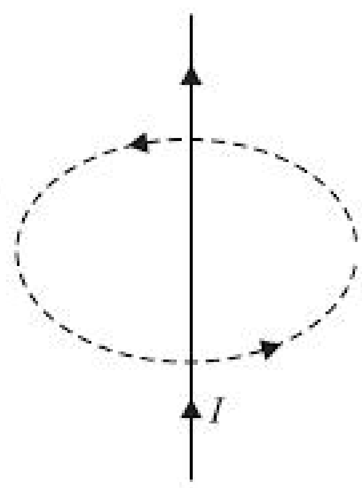



**Fig. 8.50**

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

or $$\oint B\, dl \cos\theta = \mu_0 I$$

Here $\theta$ = angle between $\vec{B}$ and $\vec{dl}$

(*i*) This law is *true for steady currents only.*

(*ii*) Ampere's law is an alternative to the Biot-Savart law but it can be used only in those situations where it is possible to find the line integral of the chosen path. For example, this law cannot be used in case of a plane circular coil, though it is true in this case.

17. The magnitude of magnetic field on the axis of a long air-cored solenoid is [See Fig. 8.51] given by ;

$$B = \mu_0 n I$$

$n$ = number of turns per metre

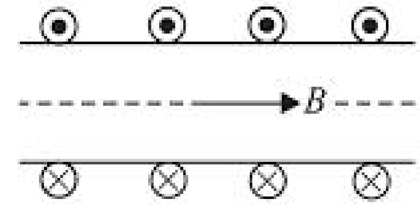



Fig. 8.51

(*i*) $\vec{B}$ is directed along the axis of the solenoid.

(*ii*) At either end of the long solenoid, $\vec{B}$ along the axis is $\mu_0 n I/2$ .

(*iii*) The magnetic field outside a solenoid is zero.

18. The magnitude of magnetic field due to an air-cored toroid is $B = \mu_0 n I$.

(*i*) This is the same expression as for solenoid because a toroid is a solenoid in the form of a ring.

(*ii*) Magnetic field is *only* confined to inside the toroid.

(*iii*) The magnetic field inside the toroid is constant and is always tangent to the field lines.

(*iv*) The magnetic field outside a toroid is zero.

19. The direction of the magnetic field due to various current-carrying conductor arrangements can be found by using the following rules:

(*i*) **Right-hand grip rule.** For a straight current-carrying wire, the magnetic field lines are a series of concentric circles centred on the wire [See Fig. 8.52]. The direction of the magnetic field lines can be found by using right-hand grip rule stated below:

*Grip the wire with your right hand with thumb pointing in the direction of the conventional current. Then curled fingers point in the direction of the magnetic field lines.* This rule is illustrated in Fig. 8.52.

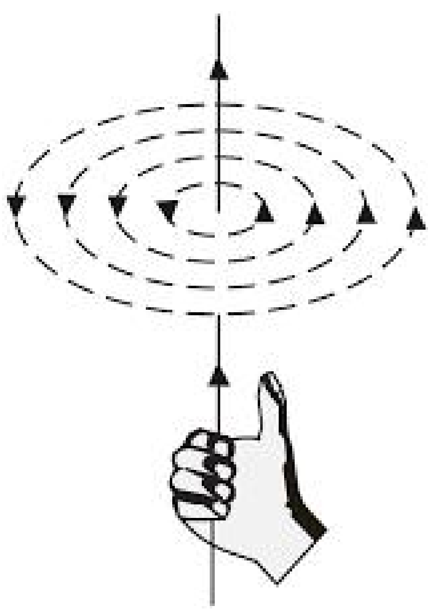


Fig. 8.52

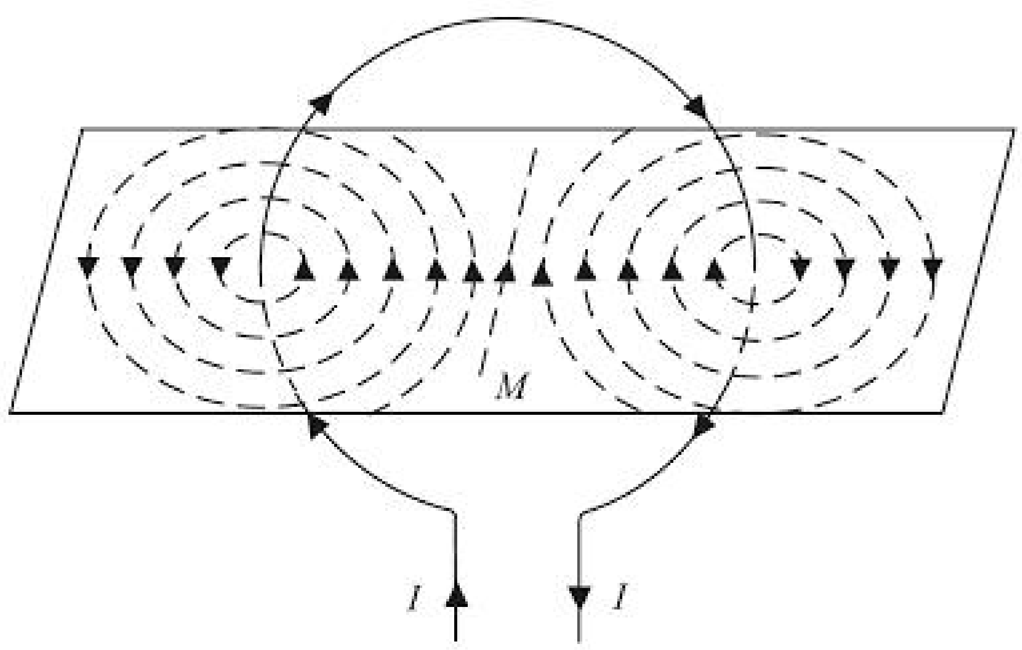



Fig. 8.53

(*ii*) **Right-hand palm rule.** The direction of magnetic field at the centre of a current-carrying circular coil can be found by right-hand palm rule stated below:

*Orient the thumb of your right hand perpendicular to the grip of the fingers such that curvature of the fingers points in the direction of current in the circular coil. Then thumb will point in the direction of magnetic field lines near the centre of the circular coil.* This rule is illustrated in Fig. 8.53.

(*iii*) **Right-hand fist rule.** The magnetic field at the centre of a circular coil is along the axis of the coil as shown in Fig. 8.54. The direction of magnetic field can be determined by right-hand fist rule stated below :

Therefore, the current in wire $B$ is 3 A, *directed upward perpendicular to the plane of the paper.*

**(ii)** The magnetic field produced by wire $A$ at $S$ is

$$B_1 = \frac{\mu_0 I_1}{2\pi \times AS} \text{ along } \overrightarrow{SB}$$

$$= \frac{(4\pi \times 10^{-7}) \times 9.6}{2\pi \times 1.6} = 12 \times 10^{-7} \text{ T along } \overrightarrow{SB}$$

The magnetic field produced by wire $B$ at $S$ is

$$B_2 = \frac{\mu_0 I_2}{2\pi \times BS} \text{ along } \overrightarrow{SA}$$

$$= \frac{(4\pi \times 10^{-7}) \times 3}{2\pi \times 1.2} = 5 \times 10^{-7} \text{ T along } \overrightarrow{SA}$$

Now $SA = 1.6$ m; $SB = 1.2$ m and $AB = 2$ m

Clearly, $(AB)^2 = (SA)^2 + (SB)^2$

It means that angle between $\vec{B_1}$ and $\vec{B_2}$ is 90°.

$\therefore$ Resultant magnetic field at $S$ is

$$B_S = \sqrt{B_1^2 + B_2^2} = \sqrt{(12 \times 10^{-7})^2 + (5 \times 10^{-7})^2} = 1.3 \times 10^{-6} \text{ T}$$

**5. A pair of stationary and infinitely long bent wires are placed in $XY$ plane as shown in Fig. 8.71. The wires carry currents of $i = 10$ A each as shown. The segments L and M are along the X-axis. The segments P and $Q$ are parallel to the Y-axis such that $OS = OR = 0.02$ m. Find the magnitude and direction of the magnetic induction at the origin $O$.** **[I.I.T.]**

**Hint.** Magnetic field at $O$ due to segment $L$

$$= \sum \frac{\mu_0}{4\pi} \frac{i\, dl \sin 180^\circ}{r^2} = 0$$

Magnetic field at $O$ due to segment $M$

$$= \sum \frac{\mu_0}{4\pi} \frac{i\, dl \sin 180^\circ}{r^2} = 0$$

Therefore, there is no contribution to magnetic field at $O$ due to segments $L$ and $M$.
Magnetic field at $O$ due to segment $P$ is

$$B_1 = \frac{\mu_0 i}{4\pi \times OR} (\sin \theta_1 + \sin \theta_2)$$

$$= \frac{\mu_0 i}{4\pi \times OR} \left( \sin 0^\circ + \sin \frac{\pi}{2} \right) = \frac{\mu_0 i}{4\pi \times OR} \text{ (upward)}$$

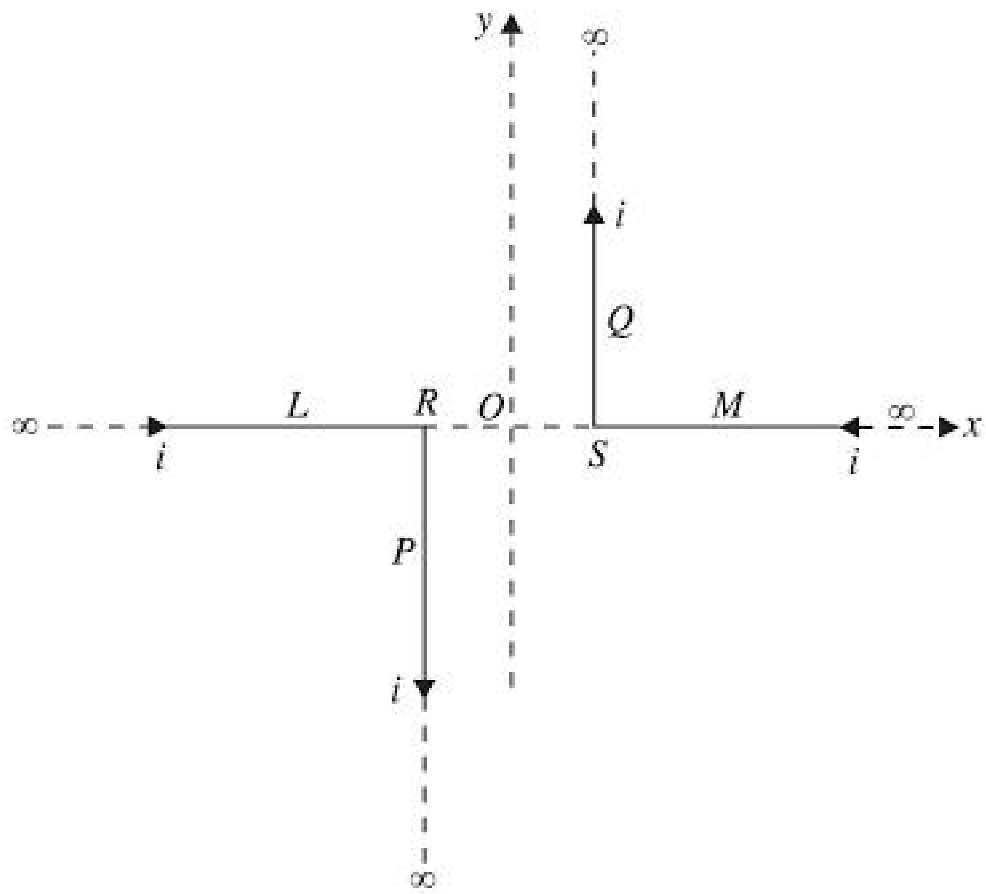



Fig. 8.71

Magnetic field at $O$ due to segment $Q$ is

$$B_2 = \frac{\mu_0 i}{4\pi \times OS}(\sin\theta_1 + \sin\theta_2)$$

$$= \frac{\mu_0 i}{4\pi \times OS}\left(\sin\frac{\pi}{2} + \sin 0^\circ\right) = \frac{\mu_0 i}{4\pi \times OS} \text{ (upward)}$$

$\therefore$ Net magnetic field at $O$ is

$$B = B_1 + B_2 = \frac{\mu_0 i}{4\pi \times OR} + \frac{\mu_0 i}{4\pi \times OS}$$

$$= \frac{\mu_0 i}{2\pi \times OR} \qquad (\because \; OR = OS = 0.02 \text{ m})$$

$$= \frac{(4\pi \times 10^{-7}) \times 10}{2\pi \times 0.02} = \mathbf{10^{-4}\ T}$$

**6. Two parallel long straight wires at a distance *d* apart are each carrying a current *I* in the same direction (See Fig. 8.72). Find the net magnetic field (*i*) midway between the wires (*ii*) at points P and Q.**

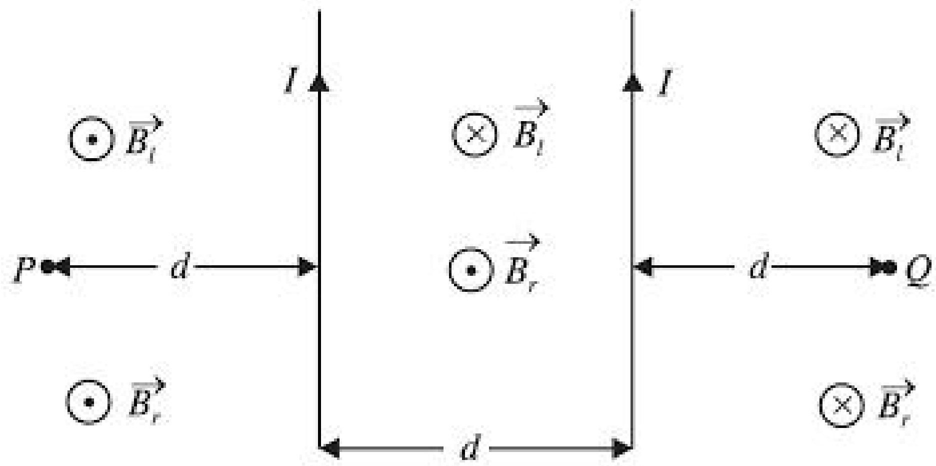



Fig. 8.72

**Hint.** (*i*) According to right hand grip rule, the field $\vec{B_l}$ due to the left-hand wire points into the page in the region between the wires. Similarly, the field $\vec{B_r}$ due to the right-hand wire points out of the page in this region. At the mid-point, the distance to either wire is the same so $\vec{B_r}$ and $\vec{B_l}$ are equal in magnitude. Since they are oppositely directed, their vector sum is zero. Hence, the net field mid-way between the wires is **zero**.

(*ii*) At point $P$, the fields due to both wires point out of the page. Therefore, net field at $P$ is

$$B_P = B_l + B_r = \frac{\mu_0 I}{2\pi d} + \frac{\mu_0 I}{2\pi\,(2d)}$$

$$= \frac{\mu_0 I}{2\pi d}\left[1 + \frac{1}{2}\right] = \frac{\mu_0 I}{2\pi d} \times \frac{3}{2}$$

$$= \frac{(4\pi \times 10^{-7})\, I}{2\pi d} \times \frac{3}{2} = \frac{3 \times 10^{-7} \times I}{d}$$

The net field at $Q$ is the same as at $P$. However, it is directed into the page.

**7. Calculate the magnetic field at the point $O$ for the closed circuit loop shown in Fig. 8.73. The loop consists of two straight portions and a circular arc of radius $R$ which subtends an angle $\theta$ at the centre of the arc.**

**Hint.** Note that the magnetic field at $O$ due to the straight segments $OA$ and $OC$ is identically zero since $\vec{dl}$ is parallel to $\hat{r}$ along these paths and, therefore, $\vec{dl} \times \hat{r} = 0$. This simplifies the problem because now we need to be concerned only with the magnetic field at $O$ due to the curved portion $AC$. Note that each element along the path $AC$ is at the same distance $R$ from $O$ and each gives a contribution $\vec{dB}$ which is directed into the paper at $O$. Furthermore, at every point on the path $AC$, we see that $\vec{dl}$ is perpendicular to $\hat{r}$ so that $|\vec{dl} \times \hat{r}| = dl$. Therefore, field at $O$ due to the segment $dl$ is

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl}{R^2}$$

Since $I$ and $R$ are constants, we can easily integrate this expression.

$$\therefore \quad B = \int dB = \frac{\mu_0 I}{4\pi R^2}\int dl = \frac{\mu_0 I}{4\pi R^2} \times l$$

Now $\theta = l/R$ or $l = \theta R$, where $\theta$ is in *radians*.

$$\therefore \quad B = \frac{\mu_0\, I}{4\pi R^2} \times \theta R$$

$$\text{or} \quad B = \frac{\mu_0\, I}{4\pi R}\,\theta$$

For example, if the loop subtends an angle $\theta = \pi/2$ rad, then $B$ at $O$ is

$$B = \frac{\mu_0 I}{8R}$$

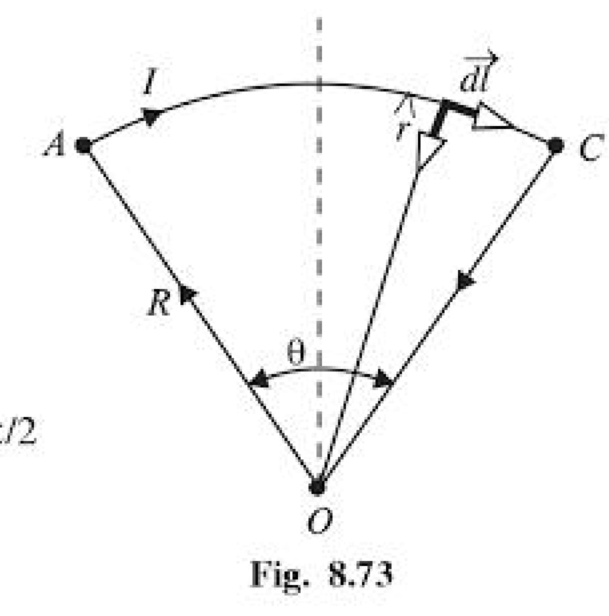


Fig. 8.73

If the loop were a full circle of radius $R$, then $\theta = 2\pi$ rad and $B$ at the centre of the loop is

$$B = \frac{\mu_0 I}{2R}$$

**8. A solenoid of length 0.4 m and having 500 turns of wire carries a current of 3 A. A thin coil of 10 turns of wire and of radius 0.01 m carries a current of 0.4 A. Calculate the torque required to hold the coil in the middle of the solenoid with its axis perpendicular to the axis of the solenoid.**

**Hint.** The magnetic field at the middle of the solenoid is

$$B = \mu_0 n I_1$$

Here $\mu_0 = 4\pi \times 10^{-7}$ H/m; $n = 500/0.4$; $I_1 = 3$ A

$$\therefore \quad B = (4\pi \times 10^{-7}) \times \left(\frac{500}{0.4}\right) \times 3 = 15\pi \times 10^{-4} \text{ T}$$

Torque on the coil placed in the middle of solenoid is

$$\tau = MB \sin \alpha$$

Here $M = n_2 I_2 A = 10 \times 0.4 \times \pi \times (0.01)^2$; $B = 15\pi \times 10^{-4}$ T; $\alpha = 90°$

$$\therefore \quad \tau = [10 \times 0.4 \times \pi \times (0.01)^2] \times (15\pi \times 10^{-4}) \times \sin 90° = \mathbf{5.91 \times 10^{-6} \text{ Nm}}$$

**9. A flat circular coil of 120 turns has a radius of 18 cm and carries a current of 3 A. What is the magnitude of magnetic field at a point on the axis of the coil at a distance from the centre equal to the radius of the coil ?**

**Hint.** Number of turns, $n = 120$, Radius of coil, $a = 18$ cm $= 0.18$ m

Axial distance, $x = 18$ cm $= 0.18$ m, Current in coil, $I = 3$ A

The magnitude of field at the desired point is

$$B = \frac{\mu_0 n I a^2}{2(a^2 + x^2)^{3/2}}$$

$$= \frac{(4\pi \times 10^{-7}) \times 120 \times 3 \times (0.18)^2}{2[(0.18)^2 + (0.18)^2]^{3/2}} = \mathbf{4.4 \times 10^{-4} \text{ T}}$$

**10. A toroidal coil has 3000 turns. The inner and outer diameters are 22 cm and 26 cm respectively. Calculate the magnetic field inside the coil when it carries a current of 5 A.**

**Hint.** Mean radius, $r = \frac{13 + 11}{2} = 12$ cm $= 0.12$ m

Number of turns per unit length is

$$n = \frac{N}{l} = \frac{3000}{2\pi r} = \frac{3000}{2\pi \times 0.12}$$

Magnetic field inside the toroidal coil is

$$B = \mu_0 n I = (4\pi \times 10^{-7}) \times \left(\frac{3000}{2\pi \times 0.12}\right) \times 5 = \mathbf{0.025 \text{ T}}$$

# 9

# Motion of Charged Particles in Electric and Magnetic Fields

## INTRODUCTION

We have seen that sub-atomic charged particles (*e.g.* protons, electrons, $\alpha$-particles etc.) experience forces when subjected to electric and/or magnetic fields. Since the mass of a sub-atomic particle is very small as compared to its charge, it is greatly influenced by electric and magnetic fields. In fact, the acceleration produced in the sub-atomic particles under the influence of electric and magnetic fields is so large that we neglect the effect of acceleration due to gravity. As we shall see, the motion of charged particles in electric and magnetic fields has provided a very powerful tool in the hands of scientists to make useful investigations. In this chapter, we shall study the behaviour of charged particles in electric and magnetic fields.

## 9.1. MOTION OF A CHARGED PARTICLE IN A UNIFORM ELECTRIC FIELD

Consider that a charged particle of charge $+q$ and mass $m$ enters at right angles to a uniform electric field of strength $E$ with velocity $v$ along $OX$-axis as shown in Fig. 9.1. The electric field is along $OY$-axis and acts over a horizontal distance $x$.

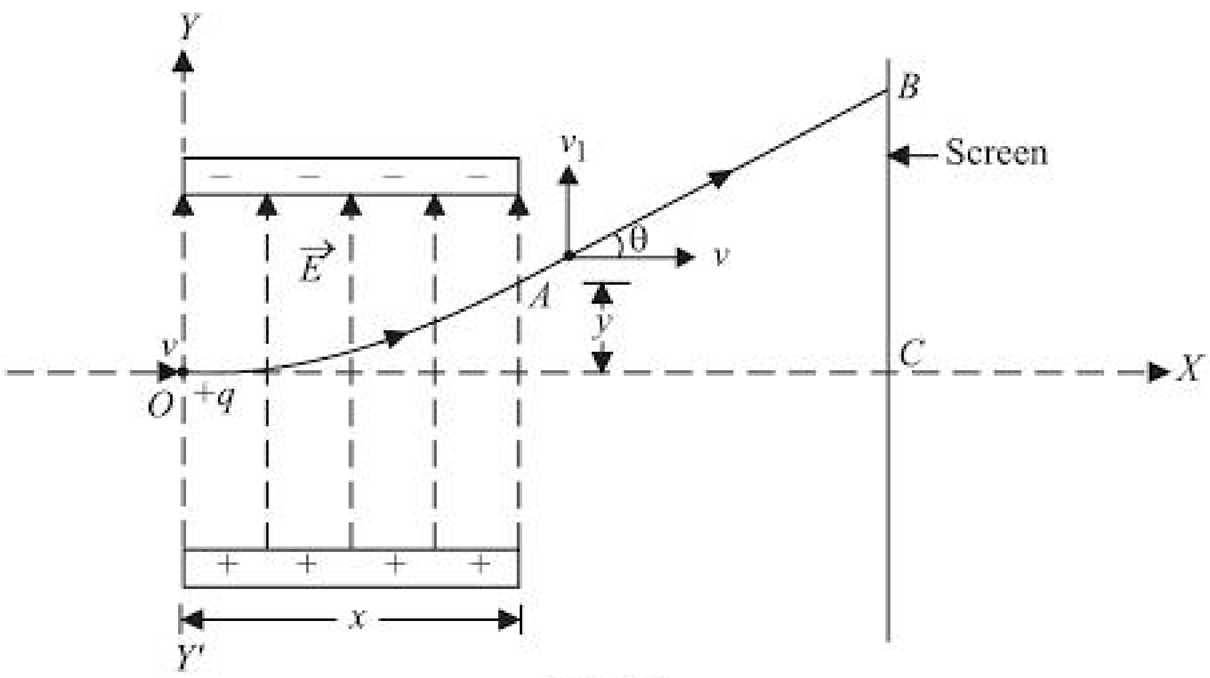



**Fig. 9.1**

Since the electric field is along $OY$-axis, no horizontal force acts on the charged particle entering the field. Therefore, the horizontal velocity $v$ of the charged particle remains the same throughout the journey. *The electric field accelerates the charged particle along OY-axis only.*

Force on the charged particle, $F = qE$ ... along $OY$

By right hand rule for cross product ( $\vec{v}$ swept into $\vec{B}$ ) the direction of force is horizontal from west to east.

Example 9.8. **A beam of protons with a velocity $4 \times 10^5$ ms$^{-1}$ enters a uniform magnetic field of 0.3 tesla at an angle of $60^\circ$ to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of rotation). Mass of proton = $1.67 \times 10^{-27}$ kg; charge on proton = $1.6 \times 10^{-19}$ C.**

**Solution.** Radius of helical path, $r = \dfrac{mv \sin\theta}{qB}$

Here $m = 1.67 \times 10^{-27}$ kg; $v = 4 \times 10^5$ ms$^{-1}$; $\theta = 60^\circ$; $q = 1.6 \times 10^{-19}$ C; $B = 0.3$ tesla

$$\therefore \quad r = \frac{(1.67\times10^{-27})\times(4\times10^5)\times\sin 60^\circ}{(1.6\times10^{-19})\times0.3} = 1.205\times10^{-2} \text{ m} = 1.205 \text{ cm}$$

$$\text{Pitch of helix} = (v\cos\theta)\,T = (v\cos\theta)\frac{2\pi r}{v\sin\theta}$$

$$= 2\pi r \cot\theta = 2\pi \times (1.205 \times 10^{-2}) \cot 60^\circ$$

$$= 4.37 \times 10^{-2} \text{ m} = 4.37 \text{ cm}$$

Example 9.9. **An $\alpha$-particle is describing a circle of radius 0.45 m in a field of magnetic induction 1.2 Wb/m$^2$. Find its speed, frequency of rotation and kinetic energy. What potential difference will be required which will accelerate the particle so as to give this much energy to it ? The mass of $\alpha$-particle is $6.8 \times 10^{-27}$ kg and its charge is $3.2 \times 10^{-19}$ C.**

**Solution.** The magnetic force (= $qvB$) provides the necessary centripetal force (= $mv^2/r$).

$$\therefore \quad qvB = \frac{mv^2}{r}$$

or
$$v = \frac{qBr}{m} = \frac{(3.2\times10^{-19})\times(1.2)\times(0.45)}{6.8\times10^{-27}} = 2.6 \times 10^7 \text{ ms}^{-1}$$

$$\text{Frequency of rotation, } f = \frac{v}{2\pi r} = \frac{2.6\times10^7}{2\pi\times0.45} = 9.2 \times 10^6 \text{ sec}^{-1}$$

$$\text{K.E. of } \alpha\text{-particle, } E_k = \frac{1}{2}mv^2 = \frac{1}{2}\times(6.8\times10^{-27})\times(2.6\times10^7)^2$$

$$= 2.3 \times 10^{-12} \text{ J} = \frac{2.3\times10^{-12}}{1.6\times10^{-19}} \text{ eV} = 14\times10^6 \text{ eV}$$

If $V$ is the accelerating potential of $\alpha$-particle, the kinetic energy = $q\,V$

or
$$14 \times 10^6 \text{ eV} = 2eV \qquad (\because \text{ Charge on } \alpha\text{-particle} = 2e)$$

$$\therefore \quad V = \frac{14\times10^6}{2} = 7 \times 10^6 \text{ volts}$$

Example 9.10. **A long straight wire $AB$ carries a current of 4A. A proton $P$ travels at $4 \times 10^6$ ms$^{-1}$ parallel to the wire, 0.2 m from it and in a direction opposite to the current as shown in Fig. 9.8. Calculate the force which the magnetic field of current exerts on the proton. Also specify the direction of force.**

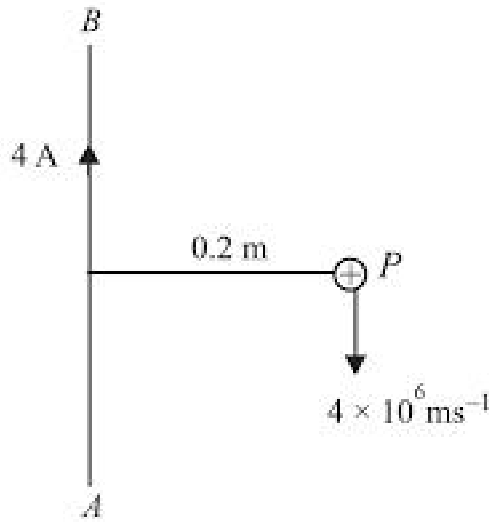



Fig. 9.8

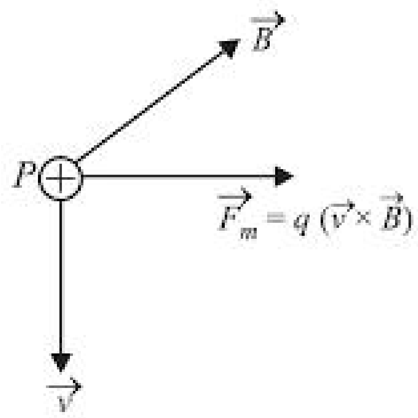



Fig. 9.9

**Solution.** Magnetic field at $P$ due to current-carrying wire $AB$ is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a} = \frac{10^{-7} \times 2 \times 4}{0.2} = 4 \times 10^{-6}\ \text{T}$$

By right-hand grip rule, the magnetic field at $P$ is directed perpendicular to the plane of the paper and in inward direction.

Force on proton, $F_m = q\ v\ B = (1.6 \times 10^{-19}) \times (\ 4 \times 10^6) \times 4 \times 10^{-6} =$ $2.56 \times 10^{-18}$ N

By right-hand rule for cross product, the force on the proton acts parallel to horizontal towards right.

Eample 9.11. **An $\alpha$-particle is moving in a magnetic field of $(3\hat{i} + 2\hat{j})$ tesla with a velocity of $5 \times 10^5\ \hat{i}$ ms$^{-1}$. What will be the magnetic force acting on the particle ?**

**Solution.** $\vec{B} = (3\hat{i} + 2\hat{j})$ tesla ; $\vec{v} = 5 \times 10^5\ \hat{i}$ ms$^{-1}$; $q = 2e = 2 \times 1.6 \times 10^{-19}$ C

Force on $\alpha$-particle, $\vec{F}_m = q\ (\vec{v} \times \vec{B}) = q\ [(5 \times 10^5\ \hat{i}) \times (3\hat{i} + 2\hat{j})]$

$$= q \times 10^6\ \hat{k} = 2 \times 1.6 \times 10^{-19} \times 10^6\ \hat{k} = 3.2 \times 10^{-13}\ \hat{k}\ \text{N}$$

Therefore, force on the $\alpha$-particle is $3.2 \times 10^{-13}$ N towards positive Z-direction.

Example 9.12. **A proton, a deuteron and an $\alpha$-particle, whose kinetic energies are same, enter perpendicularly to a uniform magnetic field. Compare the radii of their circular paths.**

**Solution.** Let 1, 2 and 3 be the suffix for proton, deuteron and $\alpha$-particle respectively. As per statement of question,

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_3 v_3^2$$

If $m_1 = m$, then, $m_2 = 2m$ and $m_3 = 4m$

$$\therefore \quad \frac{1}{2} m v_1^2 = \frac{1}{2} 2m v_2^2 = \frac{1}{2} 4m v_3^2$$

or $$v_1^2 = 2 v_2^2 = 4 v_3^2$$

or $$v_1 = \sqrt{2}\ v_2 = 2v_3$$

$$\therefore \quad v_2 = v_1 / \sqrt{2} \quad \text{and} \quad v_3 = v_1/2$$

$$\text{Radius of the path, } r = \frac{mv}{qB} \quad \left(\because qvB = \frac{mv^2}{r}\right)$$

If $q_1 = q$, then, $q_2 = q$ and $q_3 = 2q$

$$\therefore \qquad r_1 = \frac{m_1 v_1}{q_1 B} = \frac{m v_1}{qB}$$

$$r_2 = \frac{m_2 v_2}{q_2 B} = \frac{2m \times v_1 / \sqrt{2}}{qB} = \frac{\sqrt{2} m v_1}{qB}$$

$$r_3 = \frac{m_3 v_3}{q_3 B} = \frac{4m \times v_1 / 2}{2qB} = \frac{m v_1}{qB}$$

$$\therefore \qquad r_1 : r_2 : r_3 = \frac{m v_1}{qB} : \frac{\sqrt{2} m v_1}{qB} : \frac{m v_1}{qB}$$

or $\qquad r_1 : r_2 : r_3 = \mathbf{1 : \sqrt{2} : 1}$

Example 9.13. **An electron beam passes through a magnetic field of $2 \times 10^{-3}$ Wb/m$^2$ and an electric field of $3.4 \times 10^4$ Vm$^{-1}$, both acting simultaneously. If the path of the electron remains undeviated, calculate the speed of the electrons. If the electric field is removed, what will be the radius of the circular path? Mass of electron = $9.1 \times 10^{-31}$ kg.**

**Solution.** $B = 2 \times 10^{-3}$ Wb/m$^2$ ; $E = 3.4 \times 10^4$ Vm$^{-1}$

Since the electron beam remains undeviated,

$\therefore$ Magnetic force on electron = Electric force on electron

or $\qquad e\,v\,B = e\,E \qquad \therefore \qquad v = \frac{E}{B} = \frac{3.4 \times 10^4}{2 \times 10^{-3}} = \mathbf{1.7 \times 10^7\ ms^{-1}}$

When electric field is removed the magnetic force on the electron (= $evB$) provides the necessary centripetal force ($mv^2 / r$) to move the electron in a circular path of radius $r$.

$$\therefore \qquad e\,v\,B = \frac{m v^2}{r}$$

$\therefore$ Radius of circular path, $r = \frac{m\,v}{e\,B} = \frac{9.1 \times 10^{-31} \times 1.7 \times 10^7}{1.6 \times 10^{-19} \times 2 \times 10^{-3}} = \mathbf{4.8 \times 10^{-2}\ m}$

Example 9.14. **A potential difference of 600 V is applied across the plates of a parallel plate capacitor. The separation between the plates is 3mm. An electron projected vertically parallel to the plates with a velocity of $2 \times 10^6$ ms$^{-1}$ moves undeflected between the plates. Find the magnitude of magnetic field in the region between the plates.**

**Solution.** When the electron is undeflected, the force due to the electric field $F_e$ (= $e\,E$) is just balanced by the force due to the magnetic field $F_m$ (= $B\,e\,v$).

$$\therefore \qquad B\,e\,v = e\,E \qquad \text{or} \qquad Bv = \frac{V}{d} \qquad \left(\because E = \frac{V}{d}\right)$$

or $\qquad B = \frac{V}{dv}$

Here, $V = 600$ V ; $d = 3$ mm $= 3 \times 10^{-3}$ m ; $v = 2 \times 10^6$ ms$^{-1}$

$$\therefore \qquad B = \frac{600}{3 \times 10^{-3} \times 2 \times 10^6} = \mathbf{0.1\ T}$$

## PROBLEMS FOR PRACTICE

1. A particle of charge $q$ moves in a circular path of radius $r$ in a uniform magnetic field $\vec{B}$ . Show that its momentum is $q\ B\ r$.

   [Hint: $r = m\ v/q\ B$ $\therefore$ momentum $= m\ v = q\ B\ r$]

2. An electron is accelerated through a potential difference of 1500 V and is then allowed to enter a uniform magnetic field of $25 \times 10^{-3}$ T perpendicular to its direction of motion. Calculate the radius of the path described by the electron.

   **[1.65 cm]**

3. An $\alpha$-particle describes a circle of radius 0.45 m in a magnetic field of strength 1.2 T. Find (*i*) speed, (*ii*) frequency of rotation and (*iii*) kinetic energy. The mass of $\alpha$-particle is $6.8 \times 10^{-27}$ kg and its charge is $3.2 \times 10^{-19}$ C.

   **[(*i*) $2.6 \times 10^7$ ms$^{-1}$; (*ii*) $9.2 \times 10^6$ Hz; (*iii*) 14 MeV]**

4. A proton, a deuteron and $\alpha$-particle whose kinetic energies are the same enter at right angles to a uniform magnetic field. Compare the radii of their circular paths. **[$1 : \sqrt{2} : 1$]**

   [Hint: $m_d = 2m_p$ ; $m_\alpha = 4\ m_p$ ; $q_d = q_p$ ; $q_\alpha = 2q_p$]

5. An electron having 450 eV energy moves at right angles to a uniform magnetic field of $1.5 \times 10^{-3}$ T. Find the radius of the circular path described by the electron. Assume specific charge of the electron as $1.76 \times 10^{11}$ C kg$^{-1}$. **[4.8 cm]**

6. Find the flux density of the magnetic field to cause 62.5 eV electron to move in a circular path of radius 5cm. Given, $m = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C.

   **[$5.335 \times 10^{-4}$ T]**

7. A particle having a charge of 100 $\mu$C and mass of 10 mg is projected in a uniform magnetic field of 25 mT with a speed of 10 ms$^{-1}$ in a direction perpendicular to the field. What will be the period of revolution of the particle in the magnetic field?

   **[25s]**

8. What should be the minimum magnitude and direction of the magnetic field that must be produced at the equator of earth so that a proton may go round the earth with a speed of $1.0 \times 10^7$ ms$^{-1}$? Earth's radius $= 6.4 \times 10^6$ m.

   **[$1.63 \times 10^{-8}$ T perpendicular to the equator in a horizontal direction]**

9. A stream of electrons moving with a velocity of $6 \times 10^9$ cm s$^{-1}$ passes between parallel plates. An electric field of 30 Vcm$^{-1}$ has been applied across the plates. Calculate the strength of the magnetic field to keep the electrons undeflected.

   **[$5 \times 10^{-5}$ T]**

10. A proton is shot into the magnetic field $\vec{B} = 0.8\,\hat{j}\,$T with a velocity $(2 \times 10^6\ \hat{i} + 3 \times 10^6\ \hat{j})\,\text{ms}^{-1}$. Calculate the radius and pitch of the helix path followed by proton. **[$2.5 \times 10^{-2}$ m; 0.24 m]**

## 9.5. CYCLOTRON

In nuclear physics, we often *need high-energy subatomic particles (*e.g.* proton, deuteron, $\alpha$-particle etc.) to carry out several experiments. The question arises, how can this high acceleration be achieved? One way is to accelerate particles of charge $q$ through a large potential difference

* These high-energy subatomic particles are used to bombard nuclei, causing nuclear reactions, which are studied to obtain information about the nucleus.

the walls of the dees. One way to deal with the increase in mass with speed is to increase the magnitude of magnetic field ($B$) as the ion speeds up. Such a device is called *synchrotron*.

(*ii*) Cyclotron is suitable only for accelerating heavy particles such as protons, $\alpha$-particles, etc. It is not suitable for accelerating electrons. It is because the mass of electron is very small and hence gains speed quickly due to small increase in energy. As a result, the relativistic variation of mass makes electrons out of step with the oscillating electric field.

(*iii*) Cyclotron cannot accelerate uncharged particles (*e.g.*, neutrons).

(*iv*) For very high kinetic energy (*e.g.* 500GeV), it is impossible to design magnetic field system.

Example 9.15. **A cyclotron is to accelerate protons to a kinetic energy of 5.0 MeV. If the strength of magnetic field in the cyclotron is 2.0T, find (*i*) the frequency needed for the applied alternating voltage and (*ii*) radius of the cyclotron.**

**Solution.** (*i*) Cyclotron frequency, $f = \dfrac{q\,B}{2\pi\, m}$

Here $q = 1.6 \times 10^{-19}$ C; $B = 2.0$ T; $m = 1.66 \times 10^{-27}$ kg

$$\therefore \qquad f = \frac{(1.6\times10^{-19})\,(2)}{2\pi\times1.66\times10^{-27}} = \mathbf{3.0 \times 10^7\ Hz}$$

(*ii*) Required K.E. of protons, $K = 5$ MeV $= (5 \times 10^6)\,(1.6 \times 10^{-19})$ J

$$\text{Now} \qquad K = \frac{1}{2}\,m\,v^2$$

$$\therefore \qquad v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\times(5\times10^6)\,(1.6\times10^{-19})}{1.66\times10^{-27}}} = 3.1\times10^7\ \text{ms}^{-1}$$

$$\text{Required radius, } r = \frac{m\,v}{q\,B} = \frac{(1.66\times10^{-27})\,(3.1\times10^7)}{(1.6\times10^{-19})\times(2)} = \mathbf{0.16\ m}$$

Example 9.16. **A small cyclotron of maximum radius $r_0 = 0.5$ m accelerates protons in a 1.7 T magnetic field. Calculate the kinetic energy (maximum) of protons when they leave the cyclotron.**

**Solution.** The K.E. of protons as they leave the cyclotron is given by ;

$$\text{K.E.} = \frac{B^2\,q^2\,r_0^2}{2m}$$

Here, $B = 1.7$ T ; $q = 1.6 \times 10^{-19}$ C ; $r_0 = 0.5$ m; $m = 1.66 \times 10^{-27}$ kg

$$\therefore \qquad \text{K.E.} = \frac{(1.7)^2\,(1.6\times10^{-19})^2\,(0.5)^2}{2\times1.66\times10^{-27}} = 5.5\times10^{-12}\ \text{J} = \frac{5.5\times10^{-12}}{1.6\times10^{-19}}\ \text{eV}$$

$$= 34 \times 10^6\ \text{eV} = \mathbf{34\ MeV}$$

Note that magnitude of voltage applied to the dees does not affect the final energy; but the higher the voltage, the fewer revolutions are required to bring the protons to final energy.

Example 9.17. **The pole faces of a cyclotron magnet are 120 cm in diameter; the field between the pole faces is 0.8 T. The cyclotron is used to accelerate the $\alpha$-particles. Calculate the kinetic energy in eV and the speed of an $\alpha$-particle as it emerges from the cyclotron. Also find the frequency of alternating voltage that must be applied to the dees of the cyclotron.**

**Solution.** Kinetic energy of $\alpha$-particle as it emerges from the cyclotron is

$$\text{K.E.} = \frac{B^2 q^2 r_0^2}{2m}$$

Here, $B = 0.8$ T ; $q = 2\ (1.6 \times 10^{-19})$ C ; $r_0 = 1.2/2 = 0.6$ m; $m = 4\ (1.66 \times 10^{-27})$ kg

$$\therefore \quad \text{K.E.} = \frac{(0.8)^2\ (3.2 \times 10^{-19})^2\ (0.6)^2}{2 \times 4\ (1.66 \times 10^{-27})}\ \text{J}$$

$$= \frac{(0.8)^2\ (3.2 \times 10^{-19})^2\ (0.6)^2}{2 \times 4\ (1.66 \times 10^{-27})\ (1.60 \times 10^{-19})}\ \text{eV}$$

$$= 11.0 \times 10^6\ \text{eV} = \mathbf{11\ MeV}$$

$$\text{Speed of } \alpha\text{-particle, } v = \frac{q B r}{m}$$

$$= \frac{2(1.6 \times 10^{-19})\ (0.8)\ (0.6)}{4\ (1.66 \times 10^{-27})} = \mathbf{2.3 \times 10^7\ ms^{-1}}$$

Frequency of applied alternating voltage is

$$f = \frac{q B}{2\pi m} = \frac{2\ (1.6 \times 10^{-19})\ (0.8)}{2\pi \times 4\ (1.66 \times 10^{-27})} = \mathbf{0.61 \times 10^7\ Hz}$$

Example 9.18. **A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating the protons ? If the radius of the 'dees' is 60 cm, what is the kinetic energy in MeV of the proton beam produced by the accelerator ?**

**Solution.** $f = 10$ MHz $= 10^7$ Hz ; $q = 1.6 \times 10^{-19}$ C; $r_0 = 60$ cm $= 0.6$ m; $m = 1.67 \times 10^{-27}$ kg

$$\text{Cyclotron frequency, } f = \frac{q B}{2\pi m}$$

$$\therefore \text{ Operating magnetic field, } B = \frac{2\pi m f}{q} = \frac{2\pi \times 1.67 \times 10^{-27} \times 10^7}{1.6 \times 10^{-19}} = \mathbf{0.66T}$$

When the proton beam emerges, the kinetic energy of protons is maximum.

$$\therefore \quad K.E._{max} = \frac{B^2 q^2 r_0^2}{2\ m} = \frac{(0.66)^2 \times (1.6 \times 10^{-19})^2 \times (0.6)^2}{2 \times 1.67 \times 10^{-27}} = 1.2 \times 10^{-12}\ \text{J}$$

$$= \frac{1.2 \times 10^{-12}}{1.6 \times 10^{-13}}\ \text{MeV} = \mathbf{7.5\ MeV}$$

## PROBLEMS FOR PRACTICE

1. What strength of magnetic field is used in a cyclotron in which protons make $2.1 \times 10^7$ revolutions per second? **[1.4 T]**
2. A small cyclotron of maximum radius 0.5 m accelerates $\alpha$-particles in a 1.7T magnetic field. What maximum energy could they attain? What would their speed be? **[34 MeV; 4.1 × $10^7$ ms$^{-1}$]**
3. Deuterons are accelerated in a cyclotron that has an oscillatory frequency of $10^7$Hz and a dee radius of 50 cm. Calculate the strength of magnetic field needed to accelerate the deuterons. What is the energy of deuterons emerging from the cyclotron? Mass of a deuteron $= 3.34 \times 10^{-27}$ kg; charge on deuteron $= 1.6 \times 10^{-19}$ C. **[1.31 T; 10.29 MeV]**

## 9.7. FORCE ON CURRENT CARRYING CONDUCTOR PLACED IN UNIFORM MAGNETIC FIELD

We know that a moving charge in a magnetic field experiences a force. Now electric current in a conductor is due to the drifting of free electrons in a definite direction in the conductor. When such a current carrying conductor is placed in a uniform magnetic field, each free electron experiences a force. Since the free electrons are constrained in the conductor, the conductor itself experiences a force. Hence, a current carrying conductor placed in a magnetic field experiences a force.

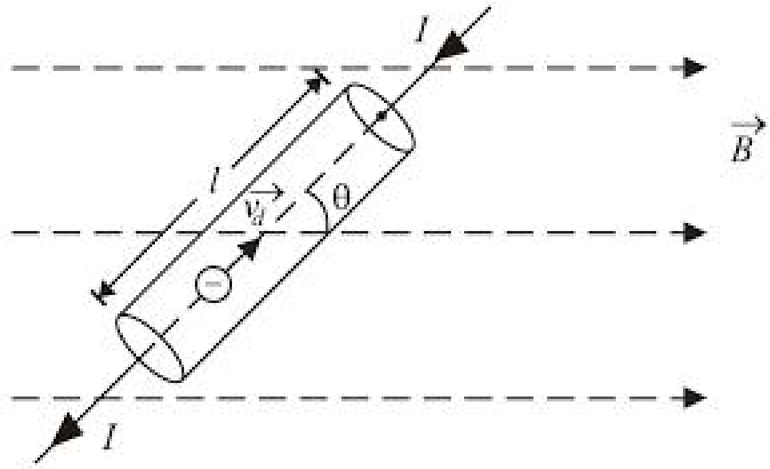



Fig. 9.12

**Expression for force.** Consider a conductor of length $l$ and area of cross-section $A$ placed at an angle $\theta$ to the direction of a uniform magnetic field $\vec{B}$ as shown in Fig. 9.12.

Let $I$ = current in the conductor

$\vec{v}_d$ = drift velocity of free electrons

$n$ = electron density in the conductor

$-e$ = charge on each electron

Force on each electron, $\vec{f} = -e\,(\vec{v}_d \times \vec{B})$

There are $n\,A\,l$ free electrons in the length $l$ of the conductor. Therefore, total force $\vec{F}$ acting on the length $l$ of the conductor is given by ;

$$\vec{F} = n\,A\,l\,\vec{f} = n\,A\,l\,[-e\,(\vec{v}_d \times \vec{B})]$$

$$= e\,n\,A\,[-l\,\vec{v}_d \times \vec{B}]$$

If $I\vec{l}$ represents a current element vector in the direction of current flow, then vectors $\vec{l}$ and $\vec{v}_d$ will have opposite directions so that :

$$-l\,\vec{v}_d = v_d\,\vec{l}$$

$$\therefore \quad \vec{F} = e\,n\,A\,v_d\,(\vec{l} \times \vec{B})$$

But $\quad e\,n\,A\,v_d = I$ = Current in the conductor

$$\therefore \quad \vec{F} = I\,(\vec{l} \times \vec{B}) \qquad ...(i)$$

or $$F = I\,l\,B\sin\theta \qquad ...(ii)$$

where $\theta$ is the smaller angle between $I\,\vec{l}$ and $\vec{B}$.

**Special Cases.** $F = IlB\sin\theta$

*(i)* **When $\theta = 0°$ or $180°$;** $\sin\theta = 0$

$\therefore \quad F = IlB\,(0) = 0$ ...minimum value

*Thus, if a current carrying conductor is placed parallel to the direction of the magnetic field, the conductor will experience no force.*

*(ii)* **When $\theta = 90°$;** $\sin\theta = 1$

$\therefore \quad F = IlB$ ...maximum value

$$= \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{20 \times 16}{0.1} \times 0.15 = 0.96 \times 10^{-4}\ \text{N}$$

$\therefore$ Net force on the loop, $F = F_1 - F_2 = 10^{-4}\ (2.4 - 0.96) = \mathbf{1.44 \times 10^{-4}\ N}$

Therefore, the net force on the loop is directed *towards* the current-carrying straight conductor *XY*. If the direction of current in the loop is reversed, the magnitude of net force on the loop remains the same (*i.e.*, $F = 1.44 \times 10^{-4}$ N) but its direction will be away from the current-carrying straight conductor *XY*.

Example 9.23. **On a smooth inclined plane inclined at 30° with the horizontal, a thin current-carrying metal rod is placed parallel to the horizontal ground. The plane is located in a uniform magnetic field of 0.15 T in the vertical direction. For what value of current can the rod remain stationary? The mass per unit length of the rod is 0.30 kg m$^{-1}$.**

**Solution.** Fig. 9.17 shows the conditions of the problem. The forces acting on the current-carrying metallic rod are:

(*i*) Weight of the rod ($m\,g$) acting vertically downward.

(*ii*) The magnetic force $F$ ($= B\,I\,l$) acting horizontally.

The force $m\,g \sin\theta$ tends to roll the rod down the plane while the force $B\,I\,l \cos\theta$ tends to move it up along the plane. For the rod to be stationary, these two forces must be equal *i.e.*

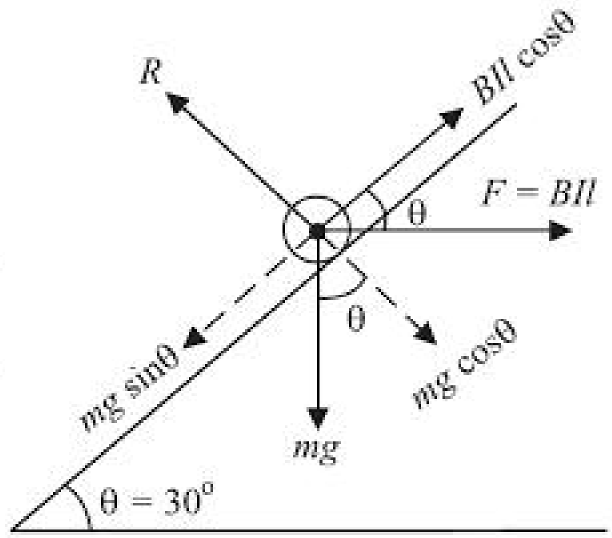



**Fig. 9.17**

$$BIl\cos\theta = mg\sin\theta$$

or $$I = \frac{mg\tan\theta}{B\,l}$$

Here, $m = 0.3 \times 1$ kg; $\theta = 30°$; $B = 0.15$ T; $g = 9.8$ ms$^{-2}$

$$\therefore \quad I = \frac{(0.3 \times l) \times 9.8 \times \tan 30°}{0.15 \times l} = \mathbf{11.32\ A}$$

Example. 9.24. **The horizontal component of the earth's magnetic field at a certain place is $3.0 \times 10^{-5}$ T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is (*a*) east to west; (*b*) south to north?**

**Solution.** Here, $B = 3.0 \times 10^{-5}$ T ; $I = 1$A

(*a*) When current is flowing from east to west, $\theta = 90°$.

$\therefore$ Force on the conductor per unit length is

$$F = \frac{I\,l\,B\sin\theta}{l} = I\,B\sin\theta = 1 \times 3.0 \times 10^{-5} \times \sin 90° = \mathbf{3 \times 10^{-5}\,Nm^{-1}}$$

By Fleming's left-hand rule, the direction of force will be downwards.

(*b*) When current is flowing from south to north, $\theta = 0°$.

$\therefore$ Force on the conductor per unit length is

$$F = IB\sin\theta = 1 \times 3.0 \times 10^{-5} \times \sin 0° = \mathbf{0}$$

Example 9.25. **A horizontal straight wire 5 cm long of mass 1.2 gm$^{-1}$ is placed perpendicular to a uniform magnetic field of 0.6 T. If resistance of the wire is 3.8 Ω m$^{-1}$, calculate the p.d. that has to be applied between the ends of the wire to make it just self-supporting.**

**Solution.** The current ($I$) in the wire is to be in such a direction that magnetic force acts on it vertically upward. To make the wire self-supporting, its weight should be equal to the upward magnetic force *i.e.*

$$BIl = mg \quad (\because \theta = 90^\circ)$$

or $$I = \frac{m\,g}{B\,l}$$

Here, $$m = 1.2 \times 10^{-3}\ l;\ B = 0.6\ \text{T}\ ;\ g = 9.8\ \text{ms}^{-2}$$

$\therefore$ $$I = \frac{(1.2 \times 10^{-3}\, l) \times 9.8}{0.6 \times l} = 19.6 \times 10^{-3}\ \text{A}$$

Resistance of the wire, $$R = 0.05 \times 3.8 = 0.19\ \Omega$$

$\therefore$ Required P.D., $$V = I\,R = (19.6 \times 10^{-3})\ 0.19 = \mathbf{3.7 \times 10^{-3}\ V}$$

**Example 9.26. A conductor of length 2m carrying current of 2A is held parallel to an infinitely long conductor carrying current of 10A at a distance of 100mm. Find the force on small conductor.**

**Solution.** Here, $I_1 = 2\text{A}$ ; $I_2 = 10\text{A}$ ; $r = 100\text{mm} = 0.1$ m ; $l = 2\text{m}$

Force on unit length of short conductor by the long conductor is given by ;

$$f = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$$

$\therefore$ Force on length $l$ ( = 2m ) of short conductor by the long conductor is

$$F = f \times l = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r} \times l$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{2 \times 10}{0.1} \times 2 = \mathbf{8 \times 10^{-5}\ N}$$

The force will be attractive if the direction of current is the same in the two conductors and it will be repulsive if the conductors carry currents in the opposite directions.

## PROBLEMS FOR PRACTICE

1. What is the force on a wire of length 4 cm placed inside a solenoid near its centre making an angle of 60° with the axis? The wire carries a current of 12 A and the magnetic field due to solenoid has a magnitude of 0.25 T. **[0.1 N]**

   [**Hint:** $F = I\,l\,B \sin\theta$. Note the field due to a solenoid near its centre is along its axis.]

2. A horizontal wire 0.1 m long carries a current of 5 A. Find the magnitude of field which can support the weight of the wire assuming that its mass is $3 \times 10^{-3}$ kg m$^{-1}$.

   **[$5.88 \times 10^{-3}$ T]**

3. A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5 A is set up in the rod through the wires.

   (*i*) What magnetic field should be set up normal to the conductor in order that tension in the wires is zero?

   (*ii*) What will be the total tension in the wires if the direction of current is reversed, keeping the magnetic field the same as before? Neglect the mass of the wires.

   **[(*i*) 0.26 T; (*ii*) 1.176 N]**

copper wire. The coil is wound on an aluminium former inside which is a soft iron cylinder $C$; the coil is so placed that it is free to rotate without touching the cylinder. The coil is suspended by means of a phosphor bronze wire $W$ in a strong magnetic field; the field is set up by attaching soft-iron curved pieces to the poles of a powerful magnet. The combination of curved pole pieces and the soft iron produces a *radial magnetic field in the air gap [See Fig. 9.23]. The current is led into and out of the coil $PQRS$ through the suspensions at $X$ and $Y$. The deflection of the coil is shown by a beam of light reflected by the mirror $M$ (attached to the suspension wire) to a scale in front of the instrument. The motion of the coil is controlled by the **spring $S$ *i.e.*, this spring provides the controlling torque.

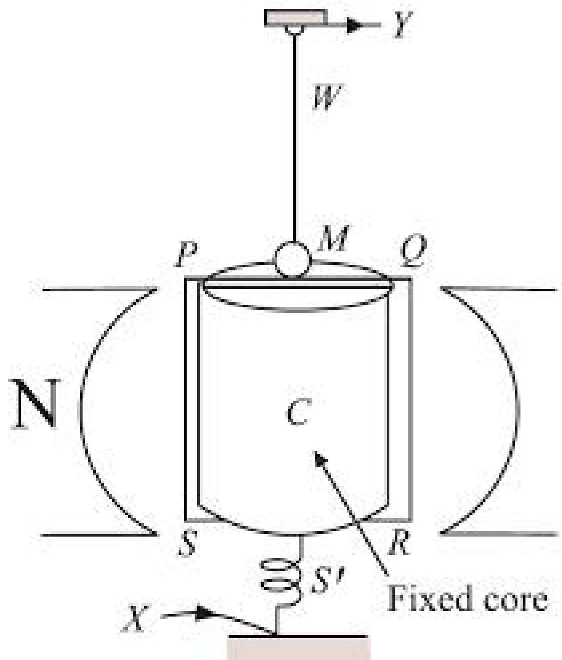



**Fig. 9.22**

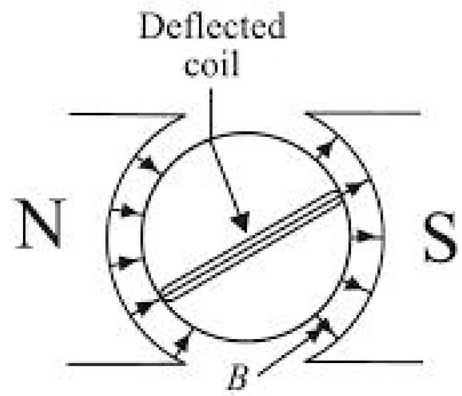



**Fig. 9.23**

**Working.** When current to be detected is passed through the coil, the coil experiences a torque. Under the influence of this deflecting torque ($T_d$), the coil begins to turn. As the coil turns, the phosphor bronze wire gets twisted. This results in an oppositely directed torque (called restoring torque $T_c$). The restoring torque is directly proportional to the angle $\theta$ through which the wire is twisted *i.e.*

$$T_c = k\,\theta$$

where $k$ is a constant of proportionality and is known as *torsion constant* of the suspension. Its unit is Nm per degree.

The coil turns until the restoring torque becomes equal to the deflecting torque. In other words, in the equilibrium position of the coil, $T_d = T_c$.

**Theory.** Let $n$ = number of turns in the coil

$A$ = area of the coil ($= l \times b$)

$I$ = current through the coil

$B$ = strength of the magnetic field

Since the field is radial, the plane of the coil is always parallel to the field (*i.e.* $\theta = 0°$). Therefore, the coil experiences a constant deflecting torque given by ;

$$\text{Deflecting torque, } T_d = n\,I\,A\,B \quad ...(i) \qquad (\because \cos\theta = 1)$$

If in the equilibrium position the coil deflects through an angle $\theta$, then,

$$\text{Restoring torque, } T_c = k\,\theta \quad ...(ii)$$

* Since the magnetic field is radial, the plane of the coil will always be parallel to the field *i.e.*, $\theta = 0°$); regardless of the orientation of the coil. Hence, the coil experiences a maximum and constant torque.

** The spring serves three purposes viz. (*i*) it provides the passage of current through rectangular coil, (*ii*) it keeps the coil in position and (*iii*) it produces the restoring torque on the twisted coil.

In the final deflected position of the coil,

$$T_d = T_c \quad \text{or} \quad n\,I\,A\,B = k\,\theta$$

or $$I = \frac{k\,\theta}{n\,A\,B}$$

or $$I = G\,\theta$$

where $$G = \frac{k}{n\,A\,B} = \text{a constant for a galvanometer}$$

$$\therefore \quad I \propto \theta$$

*Hence, the deflection of the coil is directly proportional to the current through it.* As a result, the moving coil galvanometer has a linear scale. This is an important advantage because the instrument can be accurately calibrated.

## 9.14. SENSITIVITY OF MOVING COIL GALVANOMETER

A galvanometer is said to be sensitive if a small current passed through it produces a large deflection. As we shall see, a galvanometer can be converted into an ammeter or voltmeter. Accordingly, it has two types of sensitivity *viz.*,

1. Current sensitivity 2. Voltage sensitivity

**1. Current sensitivity.** *It is defined as the deflection produced in the galvanometer when a unit current flows through it.*

If the deflection produced in the galvanometer is $\theta$ when current $I$ is passed through it, then,

$$\text{Current sensitivity, } S_I = \frac{\theta}{I} = \frac{n\,B\,A}{k} \qquad \left(\because\ I = \frac{k\theta}{n\,B\,A}\right)$$

The unit of current sensitivity is rad $A^{-1}$ or div. $A^{-1}$.

**2. Voltage sensitivity.** *It is defined as the deflection produced in the galvanometer when a unit potential difference is applied across the two terminals of the galvanometer.*

If the deflection produced in the galvanometer is $\theta$ when a potential difference $V$ is applied across its two terminals, then,

$$\text{Voltage sensitivity, } S_V = \frac{\theta}{V}$$

If $R$ is the resistance of the galvanometer, then, $V = IR$.

$$\therefore \quad S_V = \frac{\theta}{IR} = \frac{n\,B\,A}{k\,R} \qquad \left(\because\ I = \frac{k\theta}{n\,B\,A}\right)$$

The unit of voltage sensitivity is rad $V^{-1}$ or div. $V^{-1}$.

**Increasing sensitivity of galvanometer.** We have seen that :

$$\text{Current sensitivity, } S_I = \frac{n\,B\,A}{k}\ ; \quad \text{Voltage sensitivity, } S_V = \frac{n\,B\,A}{k\,R}$$

The current sensitivity of a galvanometer can be increased by ;

(*i*) Increasing the number of turns $n$ in the coil

(*ii*) Increasing the strength of magnetic field $B$

(*iii*) Increasing the area $A$ of the coil

(*iv*) Decreasing torsion constant $k$ of the spring

The high voltage sensitivity requires the same features as the high current sensitivity, together with low coil resistance ($R$).

**Limitations of increasing sensitivity.** The current or voltage sensitivity of a galvanometer cannot be increased to any extent. It is due to the following reasons :

(*i*) If $n$ is made very large, resistance of the galvanometer increases sufficiently as well as the coil becomes bulky. This tends to decrease the sensitivity. For these reasons, the optimum value of $n$ should be used.

(*ii*) These is an upper limit to the value of $A$ (area of coil) because it must not be so large that the instrument becomes bulky. Further, a coil of large area swings about its equilibrium position for a long time.

(*iii*) The small value of $k$ means weak suspension. Further, if the value of $k$ is too small, the coil would swing about its equilibrium position for a long time.

Therefore, the sensitivity (current or voltage) of a galvanometer can be increased by making magnetic field $B$ as large as possible. This is achieved by using a narrow air gap and a strong permanent magnet (typically $B = 0.5$T). The additional advantage of high value of $B$ is that the instrument is unaffected by external magnetic fields (*e.g.* Earth's field $= 5 \times 10^{-4}$ T).

## 9.15. ADVANTAGES OF MOVING COIL GALVANOMETER

A moving coil galvanometer essentially consists of two parts *viz. a permanent magnet* which provides the magnetic field and a *current carrying rectangular coil*. This arrangement has the following advantages :

(*i*) The sensitivity of the galvanometer can be increased by increasinig $n$, $B$ and $A$ while decreasing the value of $k$.

(*ii*) The instrument has a linear scale.

(*iii*) Since the instrument uses high value of $B$, the deflection is almost uninfluenced by the external magnetic fields.

(*iv*) As the coil is wound on a nonmagnetic metallic frame, damping is produced by eddy currents. As a result, the coil quickly assumes the final deflected position.

(*v*) The instrument can be used for a.c. measurements if a rectifier is used.

Example 9.31. **A moving coil galvanometer has a coil of 125 turns. The coil has an equivalent area of 400 mm$^2$. The coil lies in a radial field of 0.5 T.**

**(*i*) Calculate the torque exerted when a current of 20 mA flows through the coil.**

**(*ii*) Given that suspension offers a restoring torque of $40 \times 10^{-6}$ Nm/degree, calculate the angular deflection of the coil.**

**Solution.** (*i*) Torque exerted, $T = n\,I\,B\,A$

Here $n = 125$ ; $I = 20 \times 10^{-3}$ A; $B = 0.5$ T ; $A = 400 \times 10^{-6}$ m$^2$

$\therefore \quad T = (125) \times (20 \times 10^{-3}) \times (0.5) \times (400 \times 10^{-6}) = 5 \times 10^{-4}$ Nm

(*ii*) $\quad T = k\,\theta$

$$\therefore \quad \text{Deflection, } \theta = \frac{T}{k} = \frac{5\times10^{-4}}{40\times10^{-6}} = 12.5°$$

Example 9.32. **The coil of a moving coil galvanometer has an area of $5 \times 10^{-5}$ m$^2$. It consists of 60 turns of fine wire and is in a magnetic field of $9 \times 10^{-3}$ T. The restoring torque constant for the suspension wire is $3 \times 10^{-9}$ Nm per degree. Assuming the magnetic field to be radial, what is the maximum current that can be measured by this galvanometer if the scale can accommodate 36° deflection?**

**Solution:** Suppose $I_m$ is the coil current that corresponds to full-scale deflection (*i.e.*, $\theta = 36°$).

$$\therefore \quad n\,I_m\,B\,A = k\,\theta \quad \text{or} \quad I_m = \frac{k\,\theta}{n\,B\,A}$$

[**Hint:** Current sensitivity $S_I = \frac{nBA}{k}$. Since $B$, $A$ and $k$ are constant, $S_I \propto n$.

$\therefore \quad \frac{S'_I}{S_I} = \frac{n'}{n}$ or $1.5 = \frac{n'}{n}$ or $n' = 1.5\ n = 1.5 \times 20 = 30$]

**5.** Compare the current sensitivity and voltage sensitivity of the following moving coil galvanometers:

| | | | | |
|---|---|---|---|---|
| *Meter A* | $n = 30$; | $A = 1.5 \times 10^{-3}\ m^2$; | $B = 0.25$ T; | $R = 20\ \Omega$ |
| *Meter B* | $n = 35$; | $A = 2 \times 10^{-3}\ m^2$; | $B = 0.25$ T; | $R = 30\ \Omega$ |

**[9 : 14 ; 27 : 28]**

## 9.16. CONVERSION OF GALVANOMETER INTO AMMETER

The resistance of the galvanometer and the current needed to produce *full-scale deflection* (*f.s.d.*) are very small. Typical values of these parameters are $G = 20\ \Omega$ and $I_g = 1$mA as shown in Fig.9.24. This means that if 1mA current is passed through the coil of this galvanometer, the pointer will show full-scale deflection. Since the coil of galvanometer is very delicate, any attempt to pass current more than this limiting value (*i.e.* 1 mA in this case) would result in the burning out of the instrument coil due to excessive heat.

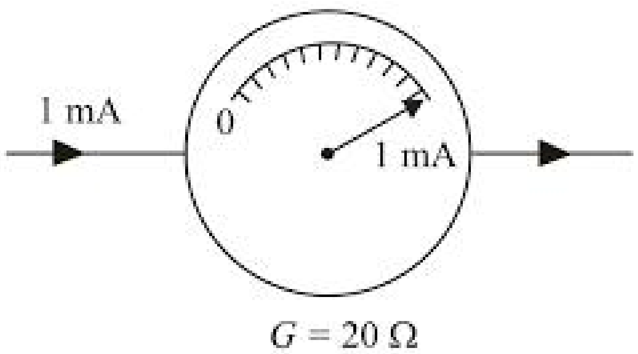



**Fig. 9.24**

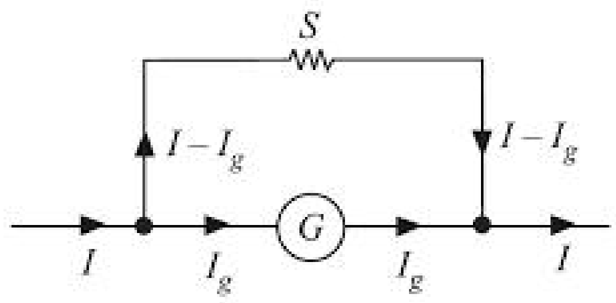



**Fig. 9.25**

The above galvanometer can measure currents upto 1mA. However, in practice, we have to measure large currents. In that case, a suitable low resistance $S$ (called **shunt**) is connected in parallel with the galvanometer. This arrangement converts galvanometer into ammeter.

*The shunted galvanometer is called an* **ammeter.**

The shunt diverts most of the current to be measured and hence the name shunt. Since the value of shunt is very small, the resistance of an ammeter is very low — a basic *requirement for an ammeter.

**Value of shunt $S$.** The value of shunt is chosen according to the maximum current we wish to measure. Suppose we want to read $I$ amperes at full-scale using a galvanometer having full-scale deflection current $I_g$ and resistance $G$. This means that when circuit current is $I$, we want current $I_g$ through the galvanometer as shown in Fig. 9.25. For this purpose, we connect a shunt $S$ of suitable value so that $I - I_g$ current flows through the shunt. Since the potential difference across the shunt is the same as across the galvanometer,

$$\therefore \quad (I - I_g)\ S = I_g\ G$$

$$\text{or} \quad S = \left(\frac{I_g}{I - I_g}\right) G \quad ...(i)$$

Thus by connecting shunt of resistance $S$ in parallel with the galvanometer given by eq. (*i*), we have an ammeter of range $0 - I$ amperes. For correct indications, we shall have to replace $0 - I_g$ mA scale by $0 - I$ amperes scale.

* An ammeter is connected in series with the circuit element whose current we wish to measure. Therefore, an ammeter should have very low resistance so that on connecting it in the circuit, there is negligible change in the circuit resistance and hence circuit current.

**Resistance of ammeter.** The resistance of ammeter (*i.e.* shunted galvanometer) is

$$\text{Resistance of ammeter, } R_m = \frac{G\,S}{G+S}$$

Clearly, the value of $R_m$ will be less than $S$. Since the value of $S$ is very small, the ammeter resistance will also be very low. Thus shunt has not only extended the current range but it has also lowered the resistance of the ammeter.

**Note.** Let $n$ = number of scale divisions in the galvanometer

$K$ = current for one scale deflection in the galvanometer

$\therefore$ Full-scale deflection current, $I_g = nK$

**Uses of shunt.** *A* **shunt** *is a low resistance connected in parallel with a galvanometer.* It has the following uses :

(*i*) A shunt is used to convert a galvanometer into an ammeter.

(*ii*) A shunt may be used to increase the range of an ammeter.

(*iii*) While determining the null point in metre bridge and potentiometer experiments, a shunt is used to protect the galvanometer from damage due to excessive current.

## 9.17. CONVERSION OF GALVANOMETER INTO VOLTMETER

Consider a galvanometer having full-scale deflection current $I_g$ = 1mA and resistance $G = 20\ \Omega$ [See Fig. 9.26]. When a potential difference of 20mV is applied across the galvanometer, current through its coil is 1mA and it will read full scale. This means that this galvanometer can read only upto 20 mV. However, in practice, the potential differences to be measured are usually greater than this value. In that case, a suitable high resistance $R$ (called **multiplier**) is connected in series with the galvanometer. This arrangement converts galvanometer into voltmeter.

*A galvanometer in series with a high resistance R (called multiplier) is called a* **voltmeter.**

The value of the series resistance $R$ is of the order of kilo-ohms. Since the value of series resistance is very large, the resistance of a voltmeter ( $= R + G$ ) is very high—a basic *requirement for a voltmeter.

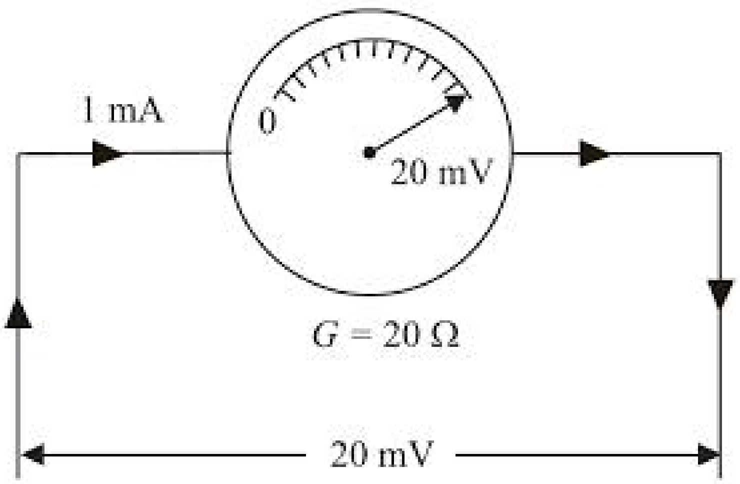



**Fig. 9.26**

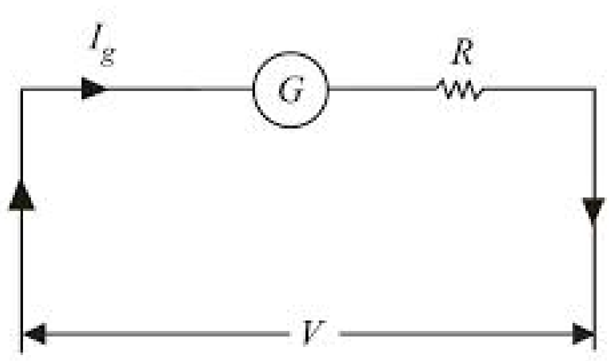



**Fig. 9.27**

**Value of series resistance *R*.** The value of series resistance is chosen according to the maximum voltage we wish to measure. Suppose we want to read $V$ volts at full scale using a galvanometer having full-scale deflection current $I_g$ and resistance $G$. This means that when potential difference across the voltmeter is $V$ volts, we want that current through the galvanometer should be $I_g$ as shown in Fig. 9.27. For this purpose, we connect a suitable high resistance $R$ in series with galvanometer so that current through the galvanometer is $I_g$. Referring to Fig. 9.27 and applying Ohm's law, we have,

* A voltmeter is connected in parallel with the circuit component across which potential difference is to be measured. It should have high resistance so that on connecting it in the circuit, there is negligible change in the circuit resistance and hence circuit current.

$$I_g = \frac{V}{G+R}$$

or
$$G + R = \frac{V}{I_g}$$

∴
$$R = \frac{V}{I_g} - G \qquad ...(i)$$

Thus by connecting a high resistance $R$ in series with the galvanometer given by eq. (*i*), we have a voltmeter of range 0 – $V$ volts. For correct indications, we shall have to replace 0 – $I_g G$ mV scale by 0 – $V$ volt scale.

**Resistance of voltmeter.** The resistance of voltmeter (*i.e.* a galvanometer in series with a high resistance) is given by ;

Resistance of voltmeter, $R_m = G + R$

Since the value of $R$ is large, the resistance of the voltmeter will be very high. Thus the series resistance has not only extended the voltage range but it has also increased the resistance of the voltmeter.

**Example 9.35. A galvanometer has a coil of resistance 5 ohms and requires 15 mA for full-scale deflection. How will you convert it into (*i*) an ammeter of range 0 – 1 A and (*ii*) voltmeter of range 0 – 15 V? Determine the resistance of the meter in each case.**

**Solution.** Galvanometer resistance, $G = 5\ \Omega$

Full-scale deflection current, $I_g$ = 15 mA = 0.015 A

(*i*) **As ammeter.** The galvanometer can be converted into ammeter of range 0 – 1A by connecting a low resistance $S$ in parallel with the galvanometer as shown in Fig. 9.28. The value of $S$ can be calculated as under. Referring to Fig. 9.28,

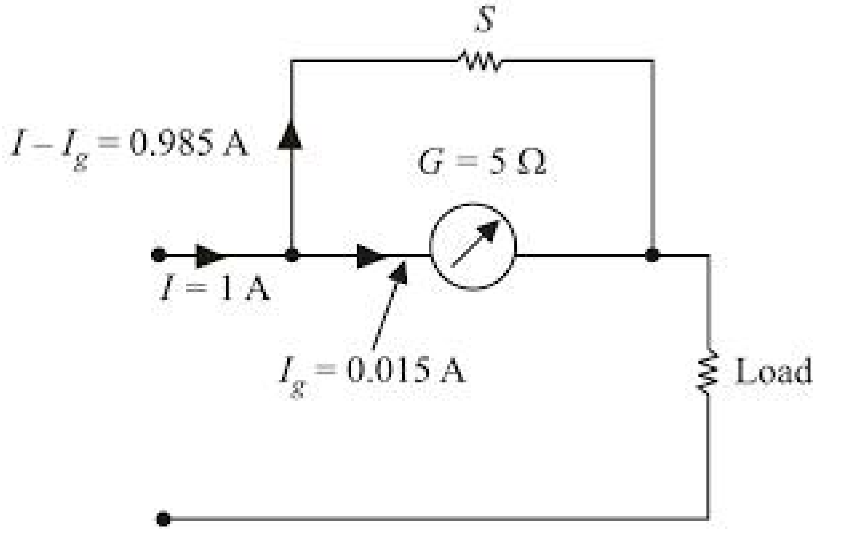



**Fig. 9.28**

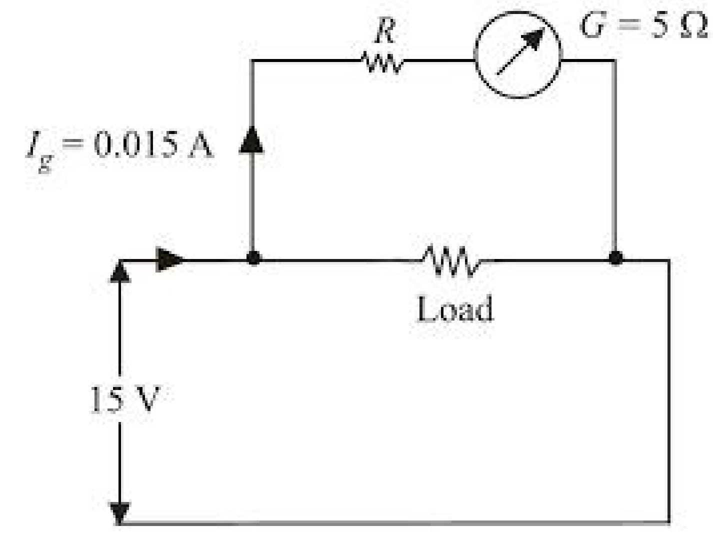



**Fig. 9.29**

$$I_g G = (I - I_g) S$$

∴
$$S = \frac{I_g G}{I - I_g} = \frac{0.015 \times 5}{1 - 0.015} = \mathbf{0.076\ \Omega}$$

$$\text{Resistance of the ammeter} = \frac{G S}{G + S} = \frac{5 \times 0.076}{5 + 0.076} = \mathbf{0.075\ \Omega}$$

Note that with shunt connection, the resistance of the instrument (*i.e.* ammeter) is decreased too much. This is profitable because an ammeter should have as low resistance as possible. *In fact, an ideal ammeter should have zero resistance.*

*(ii)* **As voltmeter.** The galvanometer can be converted into voltmeter of range 0 – 15 V by connecting a high resistance $R$ in series with it as shown in Fig. 9.29. The value of $R$ can be determined as under. Referring to Fig. 9.29.

$$V = I_g (R + G)$$

or
$$R = \frac{V}{I_g} - G = \frac{15}{0.015} - 5 = 995\ \Omega$$

Resistance of the voltmeter $= R + G = 995 + 5 = 1000\ \Omega$

Note that with the introduction of series resistance $R$, the resistance of the instrument (*i.e.* voltmeter) is increased considerably. *In fact, an ideal voltmeter should have infinite resistance.*

**Example 9.36. A galvanometer gives full-scale reading of 25 mA when a p.d. across its terminals is 75 mV. How it can be used (*i*) as an ammeter of range 0 – 100 A and (*ii*) as a voltmeter of range 0 – 750 V?**

**Solution.** Galvanometer resistance, $G = \frac{75\text{ mV}}{25\text{ mA}} = 3\ \Omega$

Full-scale meter current, $I_g = 25\text{ mA} = 0.025\text{ A}$

*(i)* **As ammeter.** The galvanometer can be converted into ammeter of range 0 – 100A by connecting a low resistance $S$ in parallel with it.

$$S = \frac{I_g G}{I - I_g} = \frac{0.025 \times 3}{100 - 0.025} = 0.00075\ \Omega$$

*(ii)* **As voltmeter.** The galvanometer can be converted into voltmeter by connecting a high resistance $R$ in series with it.

Required series resistance, $R = \frac{V}{I_g} - G = \frac{750}{0.025} - 3 = 29997\ \Omega$

**Example 9.37. In the circuit (Fig. 9.30) the current is to be measured. What is the value of the current if the ammeter shown (*a*) is a galvanometer with a resistance $R_G = 60.00\ \Omega$; (*b*) is a galvanometer described in (*a*) but converted to an ammeter by a shunt resistance $r_S = 0.02\ \Omega$ ; (*c*) is an ideal ammeter with zero resistance?**

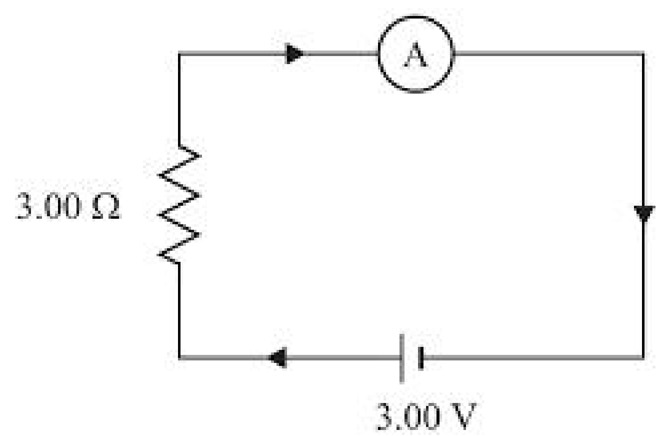



**Fig 9.30**

**Solution.** *(a)* Total circuit resistance, $R_T = R_G + 3.00 = 60 + 3 = 63\ \Omega$

$\therefore$ Circuit current, $I = 3/63 = 0.048\text{ A}$

*(b)* Resistance of the galvanometer when converted to ammeter is

$$R_m = \frac{R_G\, r_S}{R_G + r_S} = \frac{60 \times 0.02}{60 + 0.02} = 0.02\Omega$$

Total circuit resistance = 0.02 + 3 = 3.02 Ω

∴ Circuit current, $I$ = 3/3.02 = **0.99A**

(*c*) For an ideal ammeter, $R_m$ = 0.

∴ Circuit current, $I$ = 3/3 = **1A**

Example 9.38. **A resistance of 1980Ω is connected in series with a voltmeter after which the scale division becomes 100 times larger. Find the resistance of the voltmeter.**

**Solution.** Let $R_m$ be the resistance of the voltmeter. Suppose $I_g$ is the full-scale deflection current and $n$ is the number of divisions on the voltmeter scale.

If each division reads $v$ volts, then,

$$\frac{I_g R_m}{n} = v \qquad ...(i)$$

When a resistance of 1980 Ω is connected in series with the voltmeter, then each division reads 100$v$.

$$\therefore \quad \frac{I_g (R_m + 1980)}{n} = 100\ v \qquad ...(ii)$$

Dividing eq. (*ii*) by eq. (*i*), we get,

$$R_m + 1980 = 100\ R_m$$

$$\therefore \quad R_m = 1980/99 = \mathbf{20\Omega}$$

Example 9.39. **A 35V d.c. supply is connected across a resistance of 600 Ω in series with an unknown resistance *R*. A voltmeter having a resistance of 1200 Ω is connected across 600 Ω and shows a reading of 5 V. Calculate the value of unknown resistance *R*.**

**Solution.** Fig. 9.31 shows the conditions of the problem. The reading of the voltmeter is equal to p.d. across points $A$ and $B$. The total resistance of parallel resistors is

$$R_P = \frac{1200 \times 600}{1200 + 600} = 400\ \Omega$$

$$\therefore \quad \text{Circuit current, } I = \frac{V_{AB}}{R_P} = \frac{5}{400} = 0.0125\ \text{A}$$

P.D. across $R$ = 35 − 5 = 30 V

∴ Value of $R$ = 30/$I$ = 30/0.0125 = **2400 Ω**

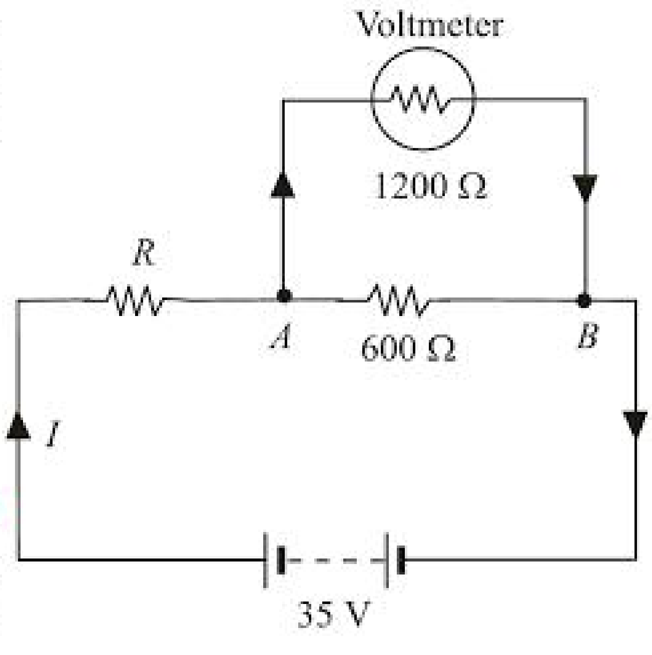



**Fig. 9.31**

Example 9.40. **Two resistors of 400 Ω and 800 Ω are connected in series with a battery of 6V of negligible resistance. It is desired to measure current in the circuit by means of an ammeter of resistance 10 Ω. What is the reading of the ammeter? If a voltmeter of resistance 10,000 Ω is connected across 400 Ω resistor, what will be the reading of the voltmeter?**

**Solution.** Reading of ammeter: Fig. 9.32 shows the conditions of the problem. The reading of the ammeter will be equal to the magnitude of circuit current.

Total circuit resistance, $R_T$ = 400 + 800 + 10 = 1210 Ω

∴ Circuit current, $I$ = $V/R_T$ = 6/1210 = **0.00496 A**

Therefore, the ammeter will read 0.00496 A.

Reading of voltmeter. Fig. 9.33 shows the conditions of the problem. The reading of the voltmeter will be equal to p.d. between points $A$ and $B$. Total resistance of parallel resistors is

Now $\qquad R_V = R + R_A$

$\therefore \qquad R = R_V - R_A = 1 - 0.015 = 0.985\ \Omega$

Thus, the given ammeter can be converted into 1 V voltmeter by connecting a 0.985 Ω resistor in series with the ammeter as shown in Fig. 9.38.

## PROBLEMS FOR PRACTICE

1. What is the resistance of a voltmeter on the 50 V scale if the meter sensitivity is 30,000 Ω/V? **[1.5 MΩ]**
2. A galvanometer has an internal resistance of 30 Ω and deflects full scale for a 60 μA current. Describe how to use this galvanometer to make (*i*) an ammeter to read currents up to 15 A and (*ii*) a voltmeter to give full-scale deflection of 3000V.
**[(*i*) $1.2 \times 10^{-4}$ Ω in parallel; (*ii*) $5.0 \times 10^{7}$ Ω in series]**
3. The current in 6 Ω resistor in the circuit shown in Fig. 9.39 is 3 A. What is the reading of voltmeter and ammeter? **[18 V; 5.25 A]**

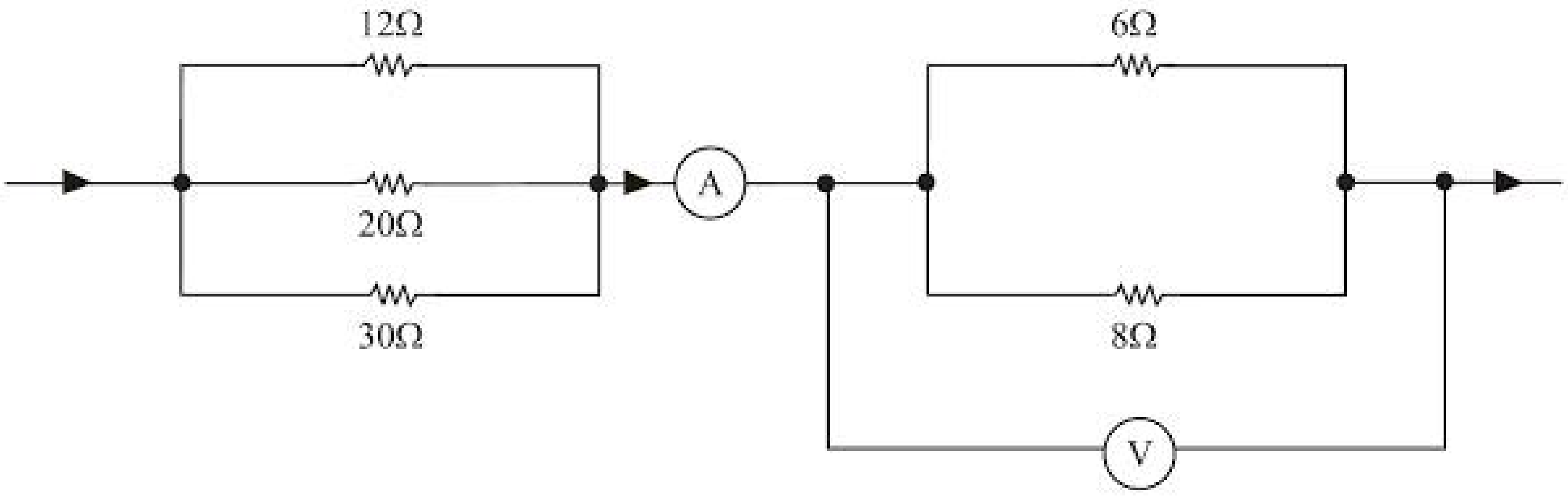



**Fig. 9.39**

4. It is desired to measure voltage across the 40 Ω resistor shown in Fig. 9.40. What reading would an ideal voltmeter give? A voltmeter with a resistance of 1000 Ω? **[4V ; 3.9V]**

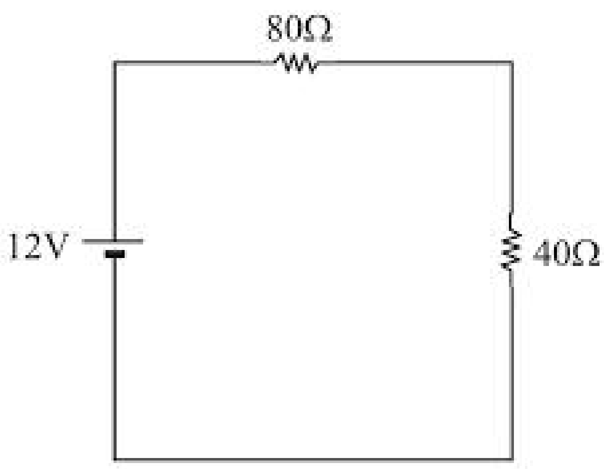



**Fig. 9.40**

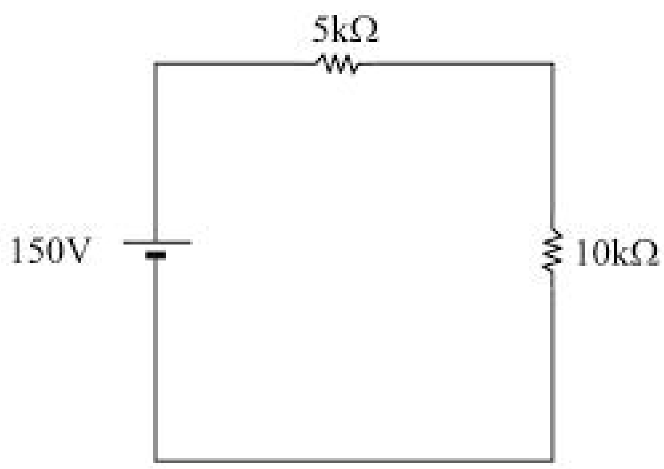



**Fig. 9.41**

5. A voltmeter with a resistance of 200,000 Ω is used to measure the voltage across the 10 k Ω resistor in the circuit shown in Fig. 9.41. By what percentage is the measurement error because of finite resistance of the voltmeter? **[1.7% low]**
6. A galvanometer has a resistance of 96Ω and it is desired to pass 4% of the total current through it. Calculate the value of shunt resistance. **[4Ω]**

7. Two resistance coils of 100Ω and 200Ω respectively are connected in series across 100V. A moving coil voltmeter of 200Ω is connected in turn across each coil. What will it read in each case? **[25V; 50V]**

8. A galvanometer of resistance 40Ω gives a deflection of 5 divisions per mA. There are 50 divisions on the scale. Calculate the maximum current that can pass through it when a shunt resistance of 2Ω is connected. **[210 mA]**

9. A voltmeter reads up to 3V. Its resistance is 200Ω. It is to be used to measure a potential difference, which may be as large as 60V. What measures you would take to protect the voltmeter? **[Connecting 3800Ω resistance in series]**

10. A resistance of 900Ω is connected in series with a galvanometer of resistance 100Ω. A potential difference of 1V produces a deflection of 100 divisions in the galvanometer. Find the figure of merit of galvanometer. **[$10^{-5}$ A div.$^{-1}$]**

## CONCEPTUAL QUESTIONS

**Q.1. Imagine that the room in which you are seated is filled with a uniform magnetic field pointing vertically downward. At the centre of the room, an electron is released with a certain speed in the horizontal direction. Describe the motion and path of the electron in the field.**

**Ans.** The magnetic field exerts a force on the electron in the horizontal plane. The direction of this force can be found by right-hand rule for cross product. Under the given conditions, the electron will keep on revolving clockwise in a circular path with constant speed.

**Q.2. A current carrying solenoid tends to contract. Why?**

**Ans.** The turns of the solenoid are parallel and carry current in the same direction. Since parallel conductors carrying current in the same direction attract each other, a current carrying solenoid tends to contract.

**Q.3. Suppose the magnetic field in a moving coil galvanometer is not radial. What will be its effect?**

**Ans.** If the magnetic field in a moving coil galvanometer is not radial, then,

$$n\,I\,A\,B\cos\theta = k\,\theta$$

or

$$I = \frac{k}{nAB}\frac{\theta}{\cos\theta} = G\frac{\theta}{\cos\theta}$$

where $G = \dfrac{k}{nAB}$ = a constant for a galvanometer

$$\therefore \quad I \propto \theta/\cos\theta$$

Therefore, the scale of the instrument is non-linear and it would be difficult to calibrate or read accurately.

**Q.4. The net charge in a current carrying conductor is zero, even then it experiences a force in a magnetic field. Why?**

**Ans.** In a current carrying conductor, free electrons move with drift velocity and hence magnetic force acts on them. Since the positive ions are stationary, they do not experience any force.

**Q.5. There are two identical galvanometers. One is converted into an ammeter and the other into a milliammeter. Which of the shunts will be of larger resistance ?**

**Ans.** Value of shunt, $S = \dfrac{I_g G}{I - I_g}$

Since $I_g$ and $G$ are constant, the value of shunt resistance will be larger for that meter which has lower current range $(0 - I)$. Hence milliammeter will have a shunt of greater resistance.

**Q.6. There are two identical galvanometers. One is converted into voltmeter and the other into millivoltmeter. Which meter will have smaller resistance?**

**Ans.** Value of series resistance, $R = \dfrac{V}{I_g} - G$

Since $G$ and $I_g$ are constant, the resistance of that meter will be small which has lower voltage range $(0 - V)$. Hence, millivoltmeter will have smaller resistance.

**Q.7. A voltmeter, an ammeter and a cell are connected in series. It is observed that ammeter practically shows no deflection. Why?**

**Ans.** The circuit resistance becomes very high due to the large resistance of the voltmeter. As a result, very small current will flow in the circuit. This small current on passing through the coil of the voltmeter will produce some deflection. However, in case of ammeter, most of this small current will flow through the shunt. Consequently, the deflection of the ammeter will be practically nil.

**Q.8. A 100-V voltmeter of resistance 20 kΩ is connected in series with a high resistance *R*. When this circuit is connected to a 110 V supply, the voltmeter reads 5V. What is the value of *R*?**

**Ans.** It is a series circuit consisting of resistances of 20 kΩ and $R$. Since voltmeter reads 5 V, it means p.d. across 20 kΩ is 5V.

$$\therefore \quad \text{Circuit current, } I = \frac{5\text{ V}}{20\text{ k}\Omega} = 0.25 \times 10^{-3}\text{ A}$$

$$\text{P.D. across } R = 110 - 5 = 105\text{ V}$$

$$\therefore \quad \text{Value of } R = \frac{105}{0.25 \times 10^{-3}} = 420 \times 10^{3}\ \Omega = 420\text{ k}\Omega$$

**Q.9. A circular loop of radius 0.1m carries a current of 1 A and is placed in a uniform magnetic field of 0.5 T. The magnetic field is perpendicular to the plane of the loop. What is the force experienced by the loop?**

**Ans.** A current carrying loop is equivalent to a magnetic dipole. A magnetic dipole does not experience any force in a uniform magnetic field. Therefore, this current carrying loop will not experience any net force.

**Q.10. A body is suspended from the lower end of a vertical spring. A direct current is passed through the spring. What will happen?**

**Ans.** The direction of current in each turn of the spring will be the same. Since parallel currents in the same direction attract each other, the turns come closer. As a result, the body will be lifted upward. The same thing will happen if the direction of current in the spring is reversed.

**Q.11. Can you increase or decrease the range of a voltmeter?**

**Ans.**
$$R = \frac{V}{I_g} - G \quad \text{or} \quad V = I_g\ (R + G)$$

Since the values of $I_g$ and $G$ are constant, the range of voltmeter $(0 - V)$ depends upon the value of $R$. The greater the value of series resistance $R$, the larger is the range of the voltmeter.

(*i*) The range of a voltmeter can be increased by connecting a suitable high resistance in series with it.

(*ii*) The range of a voltmeter can be decreased by reducing its resistance. This can be achieved by putting a suitable resistance in parallel with the voltmeter.

**Q.12. When is the equilibrium of a current carrying coil stable and unstable?**

**Ans.** The torque on a current carrying coil placed in a uniform magnetic field is

$$\vec{\tau} = \vec{M} \times \vec{B}$$

The coil will be in stable equilibrium when the angle between $\vec{M}$ and $\vec{B}$ is zero. For any other angle, it will be in unstable equilibrium.

**Q.13. How is radial magnetic field produced in a moving-coil galvanometer?**

**Ans.** The combination of the curved pole pieces and the soft iron cylinder produces a radial magnetic field in the air gap. Therefore, no matter what its orientation, the coil is always parallel to the field and experiences a constant torque $\tau$ (= $n\ I\ A\ B$ cos $\theta$) whose magnitude is

$$\tau = n\ I\ A\ B\ (\because\ \theta = 0°)$$

**Q.14. What will be the path of a charged particle moving along the direction of a uniform magnetic field ?**

**Ans.** When a charged particle moves along the direction of a uniform magnetic field, it experiences no force ($F_m = q\ v\ B \sin\theta$ and $\theta = 0°$). Therefore, the charged particle will move along its original straight path.

**Q.15. A charged particle moving in a perpendicular uniform magnetic field penetrates a layer of lead and thereby loses one half of its energy. How will the radius of curvature of the path change ?**

**Ans.** In a perpendicular uniform magnetic field, the moving charged particle will describe circular path of radius $r = m\ v\ /\ q\ B$.

$$\text{K.E. of the particle, } E_K = \frac{1}{2} m\ v^2 \quad \text{or} \quad v = \sqrt{\frac{2E_K}{m}}$$

$$\therefore \qquad r = \frac{m\ v}{q\ B} = \frac{m}{q\ B} \times \sqrt{\frac{2E_K}{m}} = \frac{\sqrt{2m\ E_K}}{q\ B}$$

It is clear that $r \propto \sqrt{E_K}$. When K.E. is halved, the radius is reduced to $1/\sqrt{2}$ of the initial value.

**Q.16. A beam of α-particles and of protons, of the same velocity *v*, enters a uniform magnetic field at right angles to the field lines. The particles describe circular paths. What is the ratio of radii of the two paths?**

**Ans.** Radius of α-particle path, $r_1 = \dfrac{m_1\ v}{q_1\ B}$

Radius of proton path, $r_2 = \dfrac{m_2\ v}{q_2\ B}$

$$\therefore \qquad \frac{r_1}{r_2} = \frac{m_1\ v}{q_1\ B} \times \frac{q_2\ B}{m_2\ v} = \frac{m_1}{m_2} \times \frac{q_2}{q_1} = 4 \times \frac{1}{2} = 2$$

Therefore, radius of α-particle path is twice that of proton's path.

**Q.17. When a charged particle is projected perpendicular to a uniform magnetic field, what happens?**

**Ans.** When a charged particle is projected perpendicular to a uniform magnetic field,

(*i*) its path is circular in a plane perpendicular to $\vec{B}$ and $\vec{v}$ .

(*ii*) its speed and kinetic energy remain the same.

**Q.33. What is the direction of force on the conductor of length $\vec{dl}$ carrying current $I$ in the magnetic field $\vec{B}$ ?**

**Ans.** $\vec{F} = I(\vec{dl} \times \vec{B})$ . Therefore, direction of $\vec{F}$ is perpendicular to the plane containing $I\,\vec{dl}$ and $\vec{B}$ .

**Q.34. What is the nature of force when two parallel conductors carry currents (*i*) in the same direction and (*ii*) in opposite directions?**

**Ans.** (*i*) Force is attractive (*ii*) Force is repulsive.

**Q.35. If the distance between two parallel current carrying conductors is doubled, how is the force between them affected?**

**Ans.** $F \propto 1/r$ where $r$ is the distance between the conductors. If $r$ is doubled, $F$ becomes one-half of the initial value.

**Q.36. The force between two parallel current carrying conductors is $F$. If current in each conductor is doubled, what is the value of force between them?**

**Ans.** Force $\propto I_1 I_2$. If current in each conductor is doubled, the force becomes 4 times *i.e.* $4F$.

**Q.37. Calculate the force per unit length on a long straight wire carrying a current of 4A below a parallel wire carrying 6A. Distance between the wires is 3 cm.**

**Ans.** Force per unit length, $F = \dfrac{\mu_0}{2\pi}\dfrac{I_1 I_2}{r} = \dfrac{4\pi \times 10^{-7}}{2\pi} \times \dfrac{4 \times 6}{3 \times 10^{-2}} = 1.6 \times 10^{-4}$ N/m

**Q.38. What is the expression for torque on a closed current loop placed in a magnetic field $\vec{B}$ ?**

**Ans.** When a closed current loop is suspended in a magnetic field, the torque on the loop is given by:

$$\tau = n\, I\, A\, B \cos\theta$$

Here $n$ is the number of turns of the loop, $A$ the area of the loop, $I$ the current in the loop and $\theta$ the angle which the plane of the loop makes with the direction of the magnetic field.

**Q.39. A current carrying loop free to turn is placed in a uniform magnetic field $\vec{B}$ . What will be its orientation in the equilibrium position?**

**Ans.** The current loop will orient itself in such a way that torque acting on it becomes zero. Thus in the equilibrium position, the plane of loop will be perpendicular to the direction of the magnetic field (*i.e.* $\theta = 90°$ so that $\tau = n\, I\, A\, B \cos\theta$ is zero).

**Q.40. Does the torque on a planar current loop in a magnetic field change when its shape is changed without changing its geometrical area?**

**Ans.** Torque, $\tau = n\, I\, A\, B \cos\theta$. Note that torque is independent of the shape if area $A$ is unchanged. Hence the torque on a planar current loop does not change when its shape is changed without changing its geometrical area.

**Q.41. What is the (*i*) net force (*ii*) torque on a current loop suspended in a uniform magnetic field?**

**Ans.** (*i*) Zero (*ii*) May or may not be zero.

**Q.42. What is the (*i*) net force (*ii*) torque on a current loop suspended in a non-uniform magnetic field?**

**Ans.** (*i*) Not zero (*ii*) May or may not be zero.

**Q.43. A planar current loop of a given perimeter is to be suspended in a given uniform magnetic field. What should be the shape of the loop for maximum torque?**

**Ans.** The loop should be circular in shape. It is because for a given perimeter, a circle has the maximum area ( $\because \tau = n\, I\, A\, B \cos\theta$ ).

**Q.44. What is a moving coil galvanometer?**

**Ans.** A moving coil galvanometer is an instrument used to detect and measure small electric currents.

**Q.45. What is the principle of moving coil galvanometer?**

**Ans.** Its working is based on the principle that when a current carrying coil is placed in a magnetic field, the coil experiences a torque. The value of torque ($\tau = n\, I\, A\, B \cos\theta$) depends upon the strength of current ($I$) for a given coil and magnetic field.

**Q.46. Why are pole pieces of galvanometer made concave?**

**Ans.** Concave pole pieces produce strong, uniform and radial magnetic field.

**Q.47. What is the function of radial magnetic field in a galvanometer?**

**Ans.** Due to the radial field, the plane of the coil is always parallel to the magnetic field (*i.e.* $\theta = 0°$), regardless of the orientation of the coil. As a result, the coil experiences a maximum and constant torque ($\tau = n\,I\,A\,B\,\cos\theta$). The radial field also provides linear scale ( $\because \tau \propto I$).

**Q.48. What is the function of soft iron cylinder between the poles of a moving coil galvanometer?**

**Ans.** Soft iron is very good conductor of magnetic flux. Therefore, soft iron cylinder makes the magnetic field strong and radial.

**Q.49. Why is the coil wrapped on a conducting frame in a moving coil galvanometer?**

**Ans.** The eddy currents induced in the conducting frame provide the retarding torque (or damping torque). This helps the coil to come to the final deflected position quickly.

**Q.50. What is a dead beat galvanometer?**

**Ans.** If on passing current through the coil of the galvanometer, the coil comes to rest at once to the final deflected position, it is called a dead beat galvanometer.

**Q.51. Why the earth's magnetic field does not affect the working of a moving coil galvanometer?**

**Ans.** A moving coil galvanometer uses a powerful magnet. The earth's magnetic field ($\simeq 5 \times 10^{-4}$T) is very weak compared to that of the galvanometer magnet ($\simeq$ 1T).

**Q.52. How will you convert a moving coil galvanometer into an ammeter?**

**Ans.** A galvanometer is converted into an ammeter by connecting a suitable small resistance (called *shunt*) in parallel with the galvanometer.

**Q.53. What is a shunt? State its SI unit.**

**Ans.** A small resistance connected in parallel with a galvanometer is called a shunt. Its SI unit is ohm.

**Q.54. Is the resistance of an ammeter greater than or less than that of the galvanometer of which it is formed?**

**Ans.** An ammeter is a galvanometer with a low resistance (*shunt*) in parallel with it. Since the resistance of shunt is very small compared to the resistance of the galvanometer, the resistance of ammeter is always less than that of the galvanometer of which it is formed.

**Q.55. How will you convert a moving coil galvanometer into a voltmeter?**

**Ans.** A galvanometer is converted into a voltmeter by connecting a very high resistance (called *multiplier*) in series with the galvanometer.

**Q.56. Why should an ammeter have low resistance?**

**Ans.** For measuring current in a circuit, an ammeter is connected in series with the circuit. In order that the insertion of ammeter in the circuit does not affect the circuit current, it should have as low resistance as possible.

**Q.57. Why should a voltmeter have a very high resistance?**

**Ans.** For measuring voltage across two points in a circuit, a voltmeter is connected across those points. In order that insertion of voltmeter does not affect the circuit current, it should have as high resistance as possible.

**Q.58. What is the resistance of (*i*) ideal ammeter (*ii*) ideal voltmeter?**

**Ans.** (*i*) Zero (*ii*) Infinite.

**Q.59. A galvanometer of resistance $G$ is converted into an ammeter by using a shunt of resistance $S$. What is the resistance of the ammeter?**

**Ans.** Since $R$ and $S$ are in parallel, ammeter resistance = $\dfrac{GS}{G + S}$.

**Q.60. Which has greater resistance (*i*) milliammeter or ammeter (*ii*) millivoltmeter or voltmeter?**

**Ans.** (*i*) Milliammeter (*ii*) voltmeter.

**Q.61. What happens when an ammeter is connected in parallel with a circuit?**

**Ans.** Since the resistance of an ammeter is very small, the effective resistance of the circuit is greatly reduced. As a result, a large current flows in the circuit.

**Q.62. What happens when a voltmeter is connected in series with a circuit?**

**Ans.** Since the resistance of a voltmeter is very high, the effective resistance of the circuit is greatly increased. As a result, a very small current flows in the circuit.

**Q.63. A galvanometer has a full-scale deflection current of $I_g$. Can it be converted into an ammeter of range $I < I_g$?**

**Ans.** No. Shunt resistance, $S = \dfrac{I_g \times G}{I - I_g}$

For $I < I_g$, the value of $S$ turns to be negative which is not possible.

**Q.64. State two properties of the material of the wire used for suspension of the coil in a moving coil galvanometer.**

**Ans.** (*i*) Its restoring torque per unit twist should be small. This makes the galvanometer very sensitive.

(*ii*) The tensile strength should be high so that it does not break under the weight of the coil.

**Q.65. Define current sensitivity of a moving coil galvanometer and state its SI unit.**

**Ans.** The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer when a unit current passes through it. The SI unit of current sensitivity is radian/ampere (rad. $A^{-1}$).

**Q.66. Define voltage sensitivity of a moving coil galvanometer and state its SI unit.**

**Ans.** The voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer when a unit voltage is applied across its two terminals. The SI unit of voltage sensitivity is radian/volt (rad. $V^{-1}$).

**Q.67. A galvanometer having a resistance of 8Ω is shunted by a wire of resistance 2Ω. If the total current is 1A, find current through the shunt.**

**Ans.** Current through shunt, $I_S = I \times \dfrac{8}{2+8} = 1 \times \dfrac{8}{2+8} = 0.8\text{ A}$

**Q.68. A 0–20 mA galvanometer has a resistance of 20Ω. To convert it into voltmeter of range 0–10V, what value of resistance is to be connected in series with it?**

**Ans.** Galvanometer resistance $G = 20\Omega$ ; $F.S.D.$ current, $I_g = 20\text{mA} = 20 \times 10^{-3}\text{A}$

Now $\quad I_g = \dfrac{V}{R+G} \quad$ or $\quad R = \dfrac{V}{I_g} - G = \dfrac{10}{20 \times 10^{-3}} - 20 = 480\Omega$

**Q.69. The resistance of a galvanometer is 25Ω and it requires 50 μA for full-scale deflection. What is the value of shunt resistance required to convert it into an ammeter of 5A?**

**Ans.** $G = 25\Omega$ ; $I_g = 50\ \mu\text{A} = 50 \times 10^{-6}\text{ A}$ ; $I = 5\text{A}$ ; $S = ?$

Value of shunt, $S = \dfrac{I_g\, G}{I - I_g} = \dfrac{(50 \times 10^{-6}) \times 25}{5 - 50 \times 10^{-6}} = 2.5 \times 10^{-4}\Omega$

**Q.70. A voltmeter of resistance 998Ω is connected across a cell of e.m.f. 2V and internal resistance 2Ω. What is the percentage error in the reading of the voltmeter?**

**Ans.** The total circuit resistance = 998 + 2 = 1000Ω. Therefore, circuit current, $I$ = 2/1000 = 0.002A and voltage drop in the cell = $Ir$ = 0.002 × 2 = 0.004 V.

∴ Error = 0.004V and % error = $\dfrac{0.004}{2} \times 100 = 0.2\%$

## SHORT ANSWER QUESTIONS

**Q.1. What is the path of a charged particle entering a uniform electric field at some angle with the field direction?**

**Ans.** Parabolic path. The reason is simple. The velocity of the charged particle can be resolved into two rectangular components—one along the field and the other perpendicular to the field. The velocity component along the field does not affect its path while the velocity component perpendicular to the field will make the charged particle to follow a parabolic path.

**Q.2. An electron and a proton are freely situated in an electric field. Will the electric force on them be equal? Will their acceleration be equal? Explain.**

**Ans.** Force on a charged particle in an electric field is $F = qE$. Since the magnitude of charge ($q$) on an electron is the same as that on a proton, the electric force on them will be equal.

Acceleration of electron, $a_e = \dfrac{eE}{m_e}$; Acceleration of proton, $a_p = \dfrac{eE}{m_p}$. Since mass of electron ($m_e$) is less than the mass of proton ($m_p$), the acceleration of electron will be greater than that of proton.

**Q.3. An electron beam moving with a uniform velocity is gradually diverging. When it is accelerated to a high velocity, it starts converging. Explain.**

**Ans.** Moving electrons experience electrostatic repulsion as well as magnetic attractions. Normally, the repulsive force is much greater than the magnetic attraction and hence the beam diverges. However, as the electrons acquire high velocity, the magnetic attraction predominates and the beam starts converging.

**Q.4. Why does not a moving charged particle entering a uniform electric field at right angles to the direction of field follow a circular path?**

**Ans.** The charged particle in an electric field experiences a force in the plane of the field. The force acts at right angles to the path of the charged particle only at its entry. Subsequently, this force no longer remains at right angles to its path. Therefore, the charged particle does not follow the circular path.

**Q.5. An electric field changes the kinetic energy of a charged particle while the magnetic field does not. Why?**

**Ans.** In an electric field, the force on the charged particle (whether at rest or in motion) is not always at right angles to the motion of the charge. Therefore, the speed and hence kinetic energy of the charged particle changes. However, a moving charge in a magnetic field experiences a force that is always at right angles to the motion of the charged particle. As a result, the speed and hence kinetic energy of the charged particle does not change.

**Q.6. Why does a moving charged particle entering a magnetic field at right angles to the direction of the field follow a circular path?**

**Ans.** When a moving charged particle enters at right angles to the direction of magnetic field, it experiences a force which always acts perpendicular to the direction of velocity of the charged particle. Therefore, the magnitude of velocity remains constant and only direction of the velocity changes. Thus, the force on the charged particle acts as a centripetal force and causes the charged particle to move in a circular path.

**Q.7. A moving charged particle enters a magnetic field making an angle $\theta$ ($\neq 90°$) with the direction of the field. What will be the path of the charged particle?**

**Ans.** Helical path. The reason is simple. The velocity of the charged particle can be resolved into two rectangular components—one along the field and the other perpendicular to the field. The velocity component perpendicular to the field causes the charged particle to move in a circular path while the velocity component along the field causes it to move it in the direction of field. The combination of these two motions causes the charged particle to move in a helical path.

**Q.8. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields, which are parallel to each other. What will be the nature of the path followed by the charged particle?**

**Ans.** Under the effect of electric field, the charged particle will move in a straight line along the direction of electric field. Since the motion of the charged particle is parallel to the magnetic field, the magnetic field will not affect its motion. Therefore, the charged particle will follow a straight line path.

**Q.9. An electron beam passes through a region of crossed electric and magnetic fields of intensities $E$ and $B$ respectively. For what value of electron speed will the beam remain undeflected?**

**Ans.** Let $v$ be the required speed of the electron. The electric force on the electron is $F_e = eE$ and the magnetic force on the electron is $F_m = evB$. The beam will remain undeflected when electric force on electron is equal and opposite to that due to the magnetic field *i.e.*

$$F_e = F_m \quad \text{or} \quad eE = evB \qquad \therefore v = \frac{E}{B}$$

**Q.10. The frequency of revolution of a charged particle in a cyclotron does not depend on the speed of the particle. Give physical explanation.**

**Ans.** Radius of circular path of charged particle is $r = mv/qB$ *i.e.* $r \propto v$. Therefore, decrease in time spent inside the dee due to increasing velocity of the charged particle is exactly compensated by the increase in the radius of the circular path. Consequently, the time period and frequency of the charged particle are independent of the speed of the particle.

**Q.11. Two parallel wires carrying currents in the same direction attract each other while the two beams of electrons travelling in the same direction repel each other. Why?**

**Ans.** In case of parallel wires, only attractive magnetic interaction takes place. However, in case of electron beams, electrostatic repulsion is greater than the attractive magnetic interaction.

**Q.12. A stream of protons is moving parallel to a stream of electrons. Will the two streams tend to come closer or move apart?**

**Ans.** The behaviour of the two streams will depend upon their speed. If the two beams are moving with small speeds, they will come closer because electrostatic attraction is stronger than the magnetic repulsive force. Reverse happens should the speeds of the beams become large.

**Q.13. Through two parallel wires $A$ and $B$, 10 amperes and 2 amperes are passed respectively in opposite directions. If wire $A$ is infinitely long and the length of wire $B$ is 2m, what is the force on wire $B$ which is situated at 10cm from wire $A$?**

**Ans.** Force on 1m length of wire $B$ is given by ;

$$F = \frac{\mu_0}{2\pi}\frac{I_A I_B}{r} = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{10 \times 2}{0.1} = 4 \times 10^{-5} \text{ N}$$

$\therefore$ Force on 2m length of wire $B = 2F = 2 \times 4 \times 10^{-5} = 8 \times 10^{-5}$ N

**Q.14. In Fig. 9.45, the straight wire $AB$ is fixed while the loop $PQRS$ is free to move under the influence of electric currents flowing in them. In which direction does the loop begin to move and why?**

**Ans.** There is no effect of wires $QR$ and $PS$ on wire $AB$. There is a force of attraction between $AB$ and $PQ$ and a force of repulsion between $AB$ and $RS$. Since $PQ$ is closer to $AB$ than $RS$, the force of attraction is greater than the force of repulsion. Therefore the loop begins to move towards $AB$.

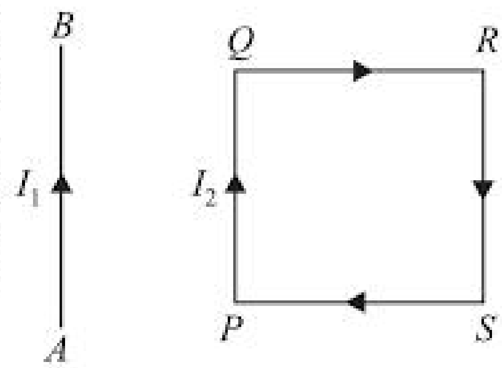


**Fig. 9.45**

**Q.15. What is a radial magnetic field? How is it produced in a moving coil galvanometer?**

**Ans.** When the magnetic lines of force point along the radii of a circle, it is called a radial magnetic field. The radial field is produced in a moving coil galvanometer by placing a soft iron cylinder between concave pole pieces.

**Q.16. What is the need of radial magnetic field in a moving coil galvanometer?**

**Ans.** Torque on a coil is $\tau = n\,I\,A\,B\cos\theta$ where $\theta$ is the angle between the plane of the coil and the direction of magnetic field. By making magnetic field radial in a moving coil galvanometer, the plane of the coil will always be parallel to the field (*i.e.* $\theta = 0°$), regardless of the position of the coil. Therefore, coil experiences a maximum and constant torque *i.e.* $\tau = nIAB$. Since $n$, $A$ and $B$ are constant, $\tau \propto I$. Due to this reason, the scale of a moving coil galvanometer is linear (*i.e.* deflection $\propto I$).

**Q.17. When an ammeter is put in a circuit, it reads less than the actual current in the circuit. Why?**

**Ans.** When an ammeter is put in the circuit, the circuit current decreases slightly due to the resistance of the ammeter. Therefore, it reads less than the actual current in the circuit.

**Q.18. When a voltmeter is put across a part of a circuit, it reads less than the actual voltage across that part. Why?**

**Ans.** When a voltmeter is put across a part of the circuit, the voltmeter draws a very small current ($\because$ its resistance is high). Therefore, current through the part under consideration is reduced slightly, resulting in lesser voltage value than the actual value.

**Q.19. How can we (*i*) increase (*ii*) decrease the range of a voltmeter?**

**Ans.** (*i*) The range of a voltmeter can be increased by connecting a suitable high resistance in series with it. (*ii*) The range of a voltmeter can be decreased by connecting a suitable resistance in parallel with it.

**Q.20. How can we (*i*) increase (*ii*) decrease the range of an ammeter?**

**Ans.** (*i*) The range of an ammeter can be increased by connecting a suitable low resistance in parallel with it. (*ii*) We cannot decrease the range of an ammeter.

**Q.21. Can a moving coil galvanometer be used to detect an alternating current in a circuit?**

**Ans.** A moving coil galvanometer measures the average value (d.c.) of current. Since the average value of an alternating current over a complete cycle is zero, a moving coil galvanometer cannot detect alternating current in a circuit.

**Q.22. Which has the lowest resistance: ammeter, voltmeter or galvanometer and why?**

**Ans.** Out of the three, ammeter has the lowest resistance. A galvanometer is converted into an ammeter by connecting a low resistance in parallel with it so that the effective resistance of ammeter is less than that of galvanometer ( $\because$ effective resistance is less than the least value of the two resistances). A galvanometer is converted into a voltmeter by connecting a high resistance in series with it so that a voltmeter has very high resistance than that of the galvanometer.

**Q.23. Which one of the two, an ammeter or milliammeter, has a higher resistance and why?**

**Ans.** A milliammeter will have a higher resistance than that of an ammeter.

$$\text{Shunt resistance, } S = \frac{I_g G}{I - I_g} \qquad \text{...}G \text{ is galvanometer resistance}$$

It is clear that meter of smaller current range (*i.e.* milliammeter) must have greater value of shunt resistance. Now effective resistance of meter = $GS/G + S$. Since value of $S$ (shunt resistance) is more for milliammeter, it will have a higher resistance than that of ammeter.

**Q.24. Why is an ammeter connected in series in a circuit?**

**Ans.** An ammeter is used to measure current in a circuit. Therefore, current to be measured (or a definite fraction of it) must pass through the ammeter. For this reason, an ammeter is connected in series in a circuit.

**Q.25. Why is a voltmeter always connected in parallel with a cricuit element across which voltage is to be measured?**

**Ans.** Like an ammeter, a voltmeter is also a current-operated device. Therefore, current proportional to the voltage to be measured must pass through the voltmeter. For this reason, a voltmeter is connected in parallel with the circuit element across which voltage is to be measured.

**Q.26. A straight wire, of length $L$, carrying a current $I$, stays horizontally in mid air in a region where there is a uniform magnetic field $\vec{B}$. The linear mass density of wire is $\lambda$. Obtain the magnitude of this magnetic field.**

**Ans.** Since the straight wire stays horizontal, the weight $mg$ of the wire acting vertically downwards is equal and opposite to the magnetic force $ILB$ sin90° acting vertically upward on the wire *i.e.*

$$m\,g = ILB \sin 90° \quad \text{or} \quad (\lambda\, L)g = ILB \quad \therefore \quad B = \lambda\, g/I$$

**Q.27. How does (*i*) an ammeter (*ii*) a voltmeter differ from a galvanometer?**

**Ans.** (*i*) An ammeter consists of a galvanometer with a low resistance (called *shunt*) connected in parallel with it. Therefore, an ammeter is a low resistance galvanometer.

(*ii*) A voltmeter is a galvanometer in series with a very high resistance (called *multiplier*). Therefore, a voltmeter is a high resistance galvanometer.

**Q.28. Two wires of equal lengths are bent in the form of two loops. One of the loops is square shaped whereas the other loop is circular. They are suspended in a uniform magnetic field and the same current is passed through them. Which loop will experience greater torque and why?**

**Ans.** Torque ($\tau$ ) on the loop is directly proportional to the area ($A$) of the loop *i.e.* $\tau \propto A$. For a given length, a circular loop has greater area than that of a square loop. Therefore, circular loop will experience greater torque.

**Q.29. A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape?**

**Ans.** It assumes circular shape with its plane perpendicular to the field to maximise magnetic flux. For a given perimeter, the area of a circle is maximum. For this reason, the loop of irregular shape changes to circular shape.

**Q.30. A current carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium? Show that in this orientation, the flux of total field (external field + field produced by the loop) is maximum.**

**Ans.** Torque on current loop is $\tau = I A B \cos \theta$ where $\theta$ is the angle between the plane of the loop and direction of the magnetic field. The current loop will be in stable equilibrium if torque on it is zero. This will be so when $\theta = 90^\circ$. Therefore, the current loop will orient itself so that its plane is perpendicular to the direction of the field. In this position, the magnetic field produced by the current loop will be in the same direction as the external magnetic field. Therefore, magnetic flux due to total field is maximum.

## LONG ANSWER QUESTIONS

1. Show that the path of a charged particle moving in uniform electric field with initial velocity perpendicular to the field is parabolic in the electric field. **[Refer to Art. 9.1]**
2. Show that the path of a charged particle moving in a uniform magnetic field with initial velocity perpendicular to the field is circular in the magnetic field. **[Refer to Art. 9.2]**
3. Show that the path of a charged particle moving in a uniform magnetic field with initial velocity making an angle $\theta$ to the direction of the field is helical in the magnetic field. **[Refer to Art. 9.2]**
4. Describe the principle, construction and working of a cyclotron. **[Refer to Art. 9.5]**
5. Describe the principle, construction and working of a moving coil galvanometer. **[Refer to Art. 9.13]**
6. Derive an expression for the torque acting on a rectangular current carrying loop placed in a uniform magnetic field. **[Refer to Art. 9.12]**
7. Explain how will you convert a galvanometer into (*i*) an ammeter (*ii*) a voltmeter ? **[Refer to Arts. 9.16 and 9.17]**
8. Write a short note on (*i*) uses of shunt (*ii*) advantages of a moving coil galvanometer. **[Refer to Arts. 9.16 and 9.15]**
9. Define current and voltage sensitivity of a galvanometer. Suggest methods to improve the sensitivity of a galvanometer. **[Refer to Art. 9.14]**
10. Find the expression for force acting on a current carrying conductor placed in a uniform magnetic field. **[Refer to Art. 9.7]**
11. Derive an expression for the force acting between two long straight parallel conductorrs carrying currents in the same direction. **[Refer to Art. 9.8]**
12. Derive an expression for maximum force experienced by a straight conductor carrying current when placed in a uniform magnetic field. **[Refer to Art. 9.7]**

# COMPETITION SUCCESS MATERIAL

## Useful Concepts/Information

1. The acceleration produced in the subatomic particles (*e.g.* protons, electrons, $\alpha$-particles etc.) under the influence of electric and magnetic fields is so large that we can neglect the effect of acceleration due to gravity.

2. When a particle of charge $q$ is placed in a uniform electric field of magnitude $E$, the electric force on the particle is $F = qE$. If $m$ is the mass of the particle, then acceleration of the particle is

$$\text{Acceleration, } a = \frac{F}{m} = \frac{qE}{m}$$

(*i*) Since $E$ is uniform (*i.e.* constant in magnitude and direction), acceleration $a$ is constant.

(*ii*) If the charge $q$ is positive, the acceleration will be in the direction of electric field. If the charge is negative, the acceleration will oppose the electric field.

(*iii*) While the charge is *in the uniform electric field, its *path is parabolic.* Once the charge has left the field, its path is linear. The situation is analogous to that of a projectile moving in a uniform gravitational field.

(*iv*) *An electric field changes the kinetic energy of the charged particle.*

3. The magnetic force acting on a charged particle moving in a uniform magnetic field is always perpendicular to the velocity of the particle. *Therefore, the work done by the magnetic force is zero since the displacement of the charge is always perpendicular to the magnetic force.*

4. When a particle of charge $q$ moving with velocity $\vec{v}$ enters at *right angles* to a uniform magnetic field $\vec{B}$, the magnetic force $\vec{F}_m$ on the charged particle is given by ;

$$\vec{F}_m = q(\vec{v} \times \vec{B}) \quad \text{or} \quad F_m = qvB \quad (\because \theta = 90°)$$

The direction of the magnetic force $\vec{F}_m$ is in the direction of $\vec{v} \times \vec{B}$ which by definition of cross product is perpendicular to both $\vec{v}$ and $\vec{B}$.

(*i*) The magnetic force $\vec{F}_m$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.

(*ii*) The particle undergoes an acceleration of magnitude $a$ given by ;

$$a = \frac{F}{m} = \frac{qvB}{m}$$

(*iii*) Because $\vec{F}_m$ is always perpendicular to $\vec{v}$ and $\vec{B}$, the acceleration $a$ is also perpendicular to $\vec{v}$ and $\vec{B}$. *Therefore, the particle moves in a circle whose plane is perpendicular to the magnetic field as shown in Fig* 9.46.

(*iv*) The force $\vec{F}_m$ is a centripetal force which changes only the direction of $\vec{v}$ while the speed remains constant. Since force $\vec{F}_m$ is in the radial direction and has magnitude of $qvB$, we can equate this to the mass $m$ of the particle multiplied by the centripetal acceleration $v^2/r$.

* Except when the charge moves parallel to the field lines.

$$\therefore \qquad F_m = qvB = \frac{mv^2}{r}$$

$\therefore$ Radius of path, $r = \dfrac{mv}{qB}$

Thus the radius of the path is directly proportional to the momentum $mv$ of the particle and inversely proportional to the magnetic field $B$ and charge $q$.

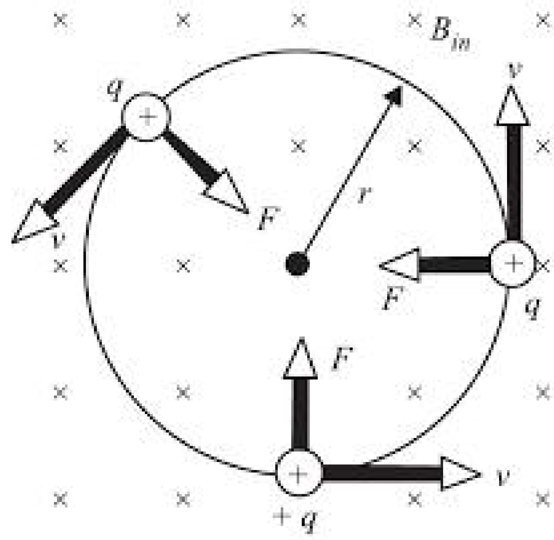



**Fig. 9.46**

(*v*) The angular frequency $\omega$ of the rotating particle is given by ;

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

(*vi*) The time period $T$ of its motion (*i.e.* time for one revolution) is equal to the circumference of the circle divided by the speed of the particle.

$$\therefore \qquad T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

*Note that $\omega$ and $T$ of the circular motion of the particle are independent of the speed $v$ of the particle and the radius $r$ of the orbit.*

(*vii*) Since the work done by the magnetic force $\vec{F}_m$ is zero, *the magnetic field does not change the kinetic energy of the charged particle.*

5. If a charged particle moves in a uniform magnetic field with a velocity $\vec{v}$ at an angle $\theta$ to $\vec{B}$, the particle follows a helical path as shown in Fig. 9.47.

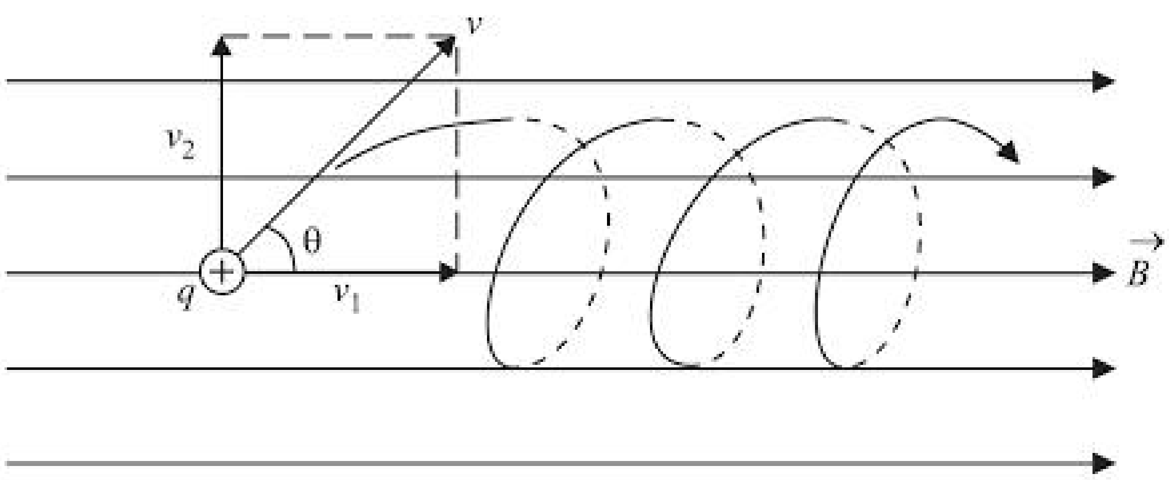



**Fig. 9.47**

(*i*) The perpendicular component $v_2$ (= $v$ sin$\theta$) moves the charged particle in a circular path while the horizontal component $v_1$ (= $v$ sin $\theta$) moves it in the direction of magnetic field. In other words, the charged particle covers circular path as well as linear path.

(*ii*) The perpendicular component of velocity (*i.e.* $v_2 = v \sin\theta$) determines the parameters of the circular motion while the horizontal component of velocity (*i.e.* $v_1 = v\cos\theta$) decides the *pitch of helix i.e.* linear distance covered by the charged particle when it completes one circular motion.

(*iii*) $F_m = ma$ or $qv_2 B = m\dfrac{v_2^2}{r}$ or $r = \dfrac{mv_2}{qB}$

Radius of circular path, $r = \dfrac{mv \sin\theta}{qB}$ $(\because v_2 = v \sin \theta)$

(*iv*) Since time period ($T$), frequency ($f$) and angular frequency ($\omega$) of a charged particle moving in a uniform magnetic field are independent of speed ($v$) of the particle and radius ($r$) of the path, these values remain the same as in point 4 above.

$$\therefore \qquad T = \frac{2\pi m}{qB};\ \omega = \frac{qB}{m};\ f = \frac{qB}{2\pi m}$$

(*v*) Pitch of helix, $d = v_1 T = v \cos\theta \left(\dfrac{2\pi m}{qB}\right)$

**6.** The total force experienced by a moving charged particle when both electric and magnetic fields are present is called *Lorentz force*. Suppose a charged particle of charge $q$ moves with velocity $\vec{v}$ in the presence of both an electric field $\vec{E}$ and a magnetic field $\vec{B}$. The charge will experience an electric force $q\vec{E}$ and a magnetic force $q(\vec{v} \times \vec{B})$. The total force $\vec{F}$ on the charge is

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

**7.** A cyclotron is a machine that can accelerate the charged particles to high velocities. A cyclotron employs a uniform magnetic field and an electric field. The magnetic field is perpendicular to the plane of the dees.

(*i*) The magnetic field maintains the charged particles in circular paths while electric field imparts them energy periodically.

(*ii*) The operation of a cyclotron is based on the fact that the period of motion of the charged particle ($T = 2\pi m/qB$) in a uniform magnetic field is independent of the speed ($v$) of the particle and radius ($r$) of its path.

(*iii*)
$$F_m = qvB = \frac{mv^2}{r}$$

$$\therefore \text{ Radius of path, } r = \frac{mv}{qB}$$

$$\text{Time period, } T = \frac{\text{Circumference}}{\text{Speed}} = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \qquad \left(\because \frac{r}{v} = \frac{m}{qB}\right)$$

$$\text{Frequency, } f = \frac{1}{T} = \frac{qB}{2\pi m}$$

If the voltage on the dees is reversed at a frequency equal to half of this value, it will be synchronised with the charged particle's arrival at the gap regardless of how fast the charged particle is moving or how large a radius its path has.

(*iv*) Maximum K.E. of positive ion $= \dfrac{B^2 q^2 r_0^2}{2m}$

Here $r_0$ is the maximum radius of the circular path of the ion.

(*v*)
$$2qVN = \frac{B^2 q^2 r_0^2}{2m}$$

$N$ = number of complete revolutions; $V$ = max. p.d. across dees

**8.** When a straight conductor of length $l$ carrying current $I$ is placed in a uniform magnetic field $\vec{B}$, the force on the conductor is given by:

$$\vec{F} = I(\vec{l} \times \vec{B}) \text{ or } F = IlB \sin\theta : \theta = \text{angle between } \vec{l} \text{ and } \vec{B}$$

(*i*) The direction of $\vec{F}$ is perpendicular to the plane containing $\vec{l}$ and $\vec{B}$ and can be found by using right-hand rule for cross product.

(*ii*) The above formula applies *only* to a straight conductor in a uniform magnetic field.

(*iii*) If current carrying conductor is parallel to the direction of magnetic field ($\theta = 0°$ or $180°$), the conductor will experience no force.

(*iv*) When $\theta = 90°$ ; $\sin\theta = 1$ $\quad \therefore F = IlB$ .... maximum value

**9.** If two parallel straight conductors carry currents $I_1$ and $I_2$, each conductor exerts a force on the other as shown in Fig. 9.48.

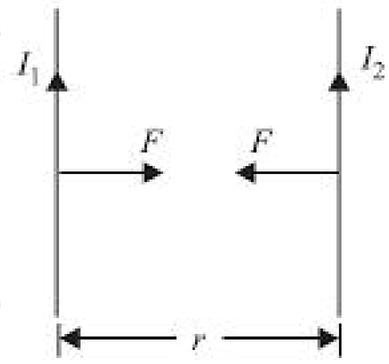


**Fig. 9.48**

Force per unit length, $F = \dfrac{\mu_0}{2\pi}\dfrac{I_1 I_2}{r}$

(*i*) The force per unit length is directly proportional to both currents and inversely to the distance $r$ between them.

(*ii*) If currents are in the same direction, the conductors attract each other; if currents are in opposite directions, the conductors repel each other.

**10.** *The net magnetic force on any closed current loop in a uniform magnetic field is zero.*

**11.** When a current loop is placed in a uniform magnetic field, a net torque acts on it which tends to rotate the current loop.

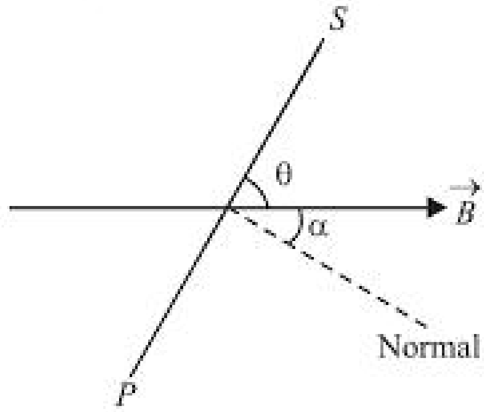


**Fig. 9.49**

(*i*) If $\theta$ is the angle which the plane of current loop makes with the direction of $\vec{B}$ [See Fig. 9.49], the torque on the current loop is given by ;

Torque, $\tau = nIAB \cos\theta$

$n$ = turns of loop ; $I$ = current through the loop;

$A$ = area of the loop; $B$ = magnitude of flux density

(*ii*) If normal to the plane of the coil makes an angle $\alpha$ with the direction of $\vec{B}$ [See Fig. 9.49], the torque on the loop is given by ;

$$\tau = nIAB \cos(90° - \alpha) = nIAB \sin\alpha \qquad (\because \theta + \alpha = 90°)$$

(*iii*) If $\theta = 0°$ or $\alpha = 90°$, the $\tau = nIAB$ ....... maximum value. If $\theta = 90°$ or $\alpha = 0°$, then $\tau = 0$ ...... minimum value.

(*iv*) We can express torque in vector notation as :

$$\vec{\tau} = \vec{M} \times \vec{B}$$

where $M = nIA$ = magnetic moment of the current loop

(*v*) When $\theta = 90°$ (*i.e.* plane of the coil is perpendicular to $\vec{B}$), $\tau = 0$. *Therefore, a current loop placed in a magnetic field rotates so that plane of current loop is perpendicular to* $\vec{B}$.

(*vi*) The torque formula $\tau = nIA\,B \cos\theta$ is valid for a planar loop of *any* shape.

12. A moving coil galvanometer is used to detect and measure small electric currents. Its operation is based on the principle that when a current-carrying coil is placed in a uniform magnetic field, the coil experiences a torque.

(*i*) The current-carrying coil is placed in a radial magnetic field. Therefore, no matter what its orientation, the plane of the coil is always parallel to the field (*i.e.* $\theta = 0°$ in each position of the coil). As a result, constant torque $\tau = nIA\,B$ acts on the coil ($\because \cos\theta = 1$).

(*ii*) If the coil deflects through an angle $\theta$, then, Deflecting torque, $T_d = nIAB$; Restoring torque, $T_c = k\theta$.

In the equilibrium position, $T_d = T_c$ or $nIAB = k\,\theta$

$$\therefore \qquad I = \frac{k\theta}{nAB} = G\theta \text{ where } G = \frac{k}{nAB} = \text{a constant for a galvanometer}$$

Since $I \propto \theta$, deflection of the coil is directly proportional to the current through it. Therefore, it has a linear scale.

(*iii*) Current sensitivity of galvanometer, $S_1 = \dfrac{\theta}{I} = \dfrac{nBA}{k} \quad \left(\because I = \dfrac{k\theta}{nBA}\right)$

Voltage sensitivity of galvanometer, $S_V = \dfrac{\theta}{V} = \dfrac{\theta}{IR} = \dfrac{nBA}{kR} \quad \left(\therefore \dfrac{\theta}{I} = \dfrac{nBA}{k}\right)$

Here $R$ is the resistance of the galvanometer.

(*iv*) The sensitivity of the galvanometer can be increased by increasing $n$, $B$ and $A$ while decreasing the valve of $k$.

(*v*) The resistance of galvanometer $G \simeq 20\ \Omega$ and full-scale deflection current $I_g \simeq 1\text{mA}$.

13. An ammeter is connected in series with the circuit element whose current we wish to measure. When a suitable low resistance $S$ (called **shunt**) is connected in parallel with a galvanometer, it becomes an ammeter. *Therefore, an ammeter is a shunted galvanometer as shown in* Fig 9.50.

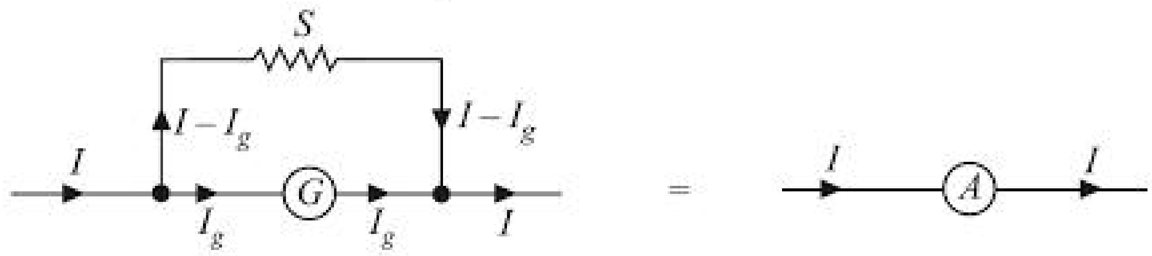



**Fig. 9.50**

(*i*) $(I - I_g)\,S = I_g\,G$ or $S = \left(\dfrac{I_g}{I - I_g}\right)G$

$I$ = Maximum current to be measured; $I_g$ = Full-scale deflection current

$S$ = Shunt resistance; $G$ = Resistance of galvanometer

(*ii*) Resistance of ammeter, $R_m = G \parallel S = \dfrac{GS}{G + S}$

An ammeter should have very low resistance so that on connecting it in the circuit, there is negligible change in the circuit resistance (and hence the circuit current).

(*iii*) An ideal ammeter has zero resistance.

14. A voltmeter is connected in parallel with the circuit component across which potential difference is to be measured. When a suitable high resistance $R$ (called **multiplier**) is connected in series with a galvanometer, it becomes a voltmeter. *Therefore, a voltmeter is a galvanometer in series with a high resistance as shown in Fig.*9.51.

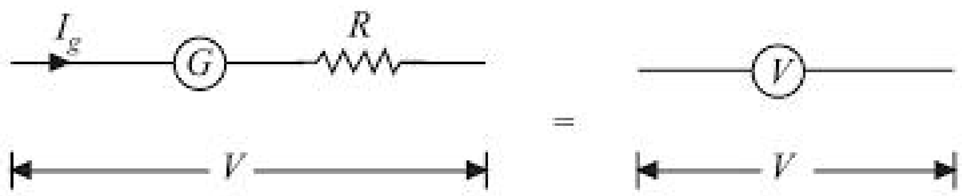



**Fig. 9.51**

(*i*) $I_g = \dfrac{V}{G + R}$ or $R = \dfrac{V}{I_g} - G$

$R$ = Series resistance; $G$ = Galvanometer resistance

$I_g$ = Full-scale deflection current; $V$ = Max. p.d. to be measured.

(*ii*) Resistance of voltmeter, $R_m = G + R$

A voltmeter should have a very high resistance so that on connecting it in the circuit, there is negligible change in the circuit resistance (and hence the circuit current).

(*iii*) An ideal voltmeter has infinite resistance.

15. When the resistance of an ammeter is not given, it is assumed that it has zero resistance. In that case, we can replace the ammeter by a wire.

16. When the resistance of a voltmeter is not given, it is assumed that it has infinite resistance. In that case, we can remove the voltmeter from the circuit.

17. Current multiplying power of shunt = $I/I_g$.

18. A voltmeter has a resistance $R$ ohms and a range of $V$ volts. Then $R = V/I_g$ or $nR = nV/I_g$. Therefore, in order to increase the range of voltmeter $n$ times, its total resistance should be increased $n$ times. The resistance to be connected in series is $R' = nR - R = (n - 1)\ R$.

19. An ammeter has a resistance of $R$ ohms and range $I$ amperes. The value of resistance used in parallel to convert it into an ammeter of range  is

$$S = \frac{GI_g}{I - I_g} = \frac{GI}{nI - I} = \frac{G}{n - I}$$

## MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. A circular loop of area 0.01 m$^2$ carrying a current of 10A is held perpendicular to a magnetic field of intensity 0.1T. The torque acting on the loop is **[Pb PMT 2000]**

(*a*) zero (*b*) 0.01 Nm
(*c*) 0.001 Nm (*d*) 0.8 Nm

2. A charge having $e/m = 10^8$ C/kg and with velocity $3 \times 10^5$ m/s enters into a uniform magnetic field $B = 0.3$ T at an angle of 30° with the direction of the field. The radius of curvature will be **[CBSE PMT 2000]**

(*a*) 0.01 cm (*b*) 0.5 cm
(*c*) 1 cm (*d*) 2 cm

3. A current-carrying coil is subjected to a uniform magnetic field. The coil will orient so that its plane becomes **[CBSE PMT 1992]**

(*a*) inclined at 45° to the magnetic field
(*b*) inclined at any arbitrary angle to the magnetic field
(*c*) parallel to the magnetic field
(*d*) perpendicular to the magnetic field

4. A coil carrying electric current is placed in a uniform magnetic field. Then, **[CBSE PMT 1993]**

(*a*) torque acts on the coil

(*b*) e.m.f. is induced in the coil

(*c*) both (*a*) and (*b*) are correct

(*d*) none of the above

5. A charge moving with velocity $v$ in $X$ direction is subjected to a field of magnetic induction $B$ in the negative $X$ direction. As a result, the charge will **[CBSE PMT 1993]**

(*a*) retard along $X$-axis

(*b*) start moving in a circular path in $YZ$ plane

(*c*) remain unaffected

(*d*) move in a helical path around $X$-axis

6. To convert a galvanometer of moving coil type into a voltmeter, we connect **[AFMC 1993]**

(*a*) high resistance in parallel

(*b*) high resistance in series

(*c*) low resistance in parallel

(*d*) low resistance in series

7. The path executed by a charged particle whose motion is perpendicular to a uniform magnetic field is **[CPMT 1996]**

(*a*) a straight line (*b*) an ellipse

(*c*) a helix (*d*) a circle

8. To make the magnetic field radial in a moving coil galvanometer **[MP PMT 1997]**

(*a*) poles are cylindrically cut

(*b*) the number of turns in the coil is increased

(*c*) coil is wound on aluminium former

(*d*) magnet is taken in the form of horse-shoe

9. Through two parallel wires $A$ and $B$, 10 amperes and 2 amperes of currents are passed respectively in opposite directions. If wire $A$ is infinitely long and the length of wire $B$ is 2m, the force on wire $B$ which is situated at 10 cm from $A$ will be **[MP PMT 1994]**

(*a*) $8 \times 10^{-5}$ N (*b*) $4 \times 10^{-7}$ N

(*c*) $4 \times 10^{-5}$ N (*d*) $4\pi \times 10^{-7}$ N

10. A charged particle moving in a magnetic field experiences a force **[MP PMT 1994]**

(*a*) in the direction of the magnetic field

(*b*) in the direction opposite to the magnetic field

(*c*) in the direction perpendicular to both magnetic field and velocity of the particle

(*d*) none of the above

11. If a particle of charge $10^{-12}$ C moving along the $X$-direction with a velocity of $10^5$ m/s experiences a force of $10^{-10}$ N in $Y$-direction due to a magnetic field, then the minimum value of magnetic field is **[MP PMT 1994]**

(*a*) $6.25 \times 10^3$ T in $Z$-direction

(*b*) $10^{-15}$ T in $Z$-direction

(*c*) $6.25 \times 10^{-3}$ T in $Z$-direction

(*d*) $10^{-3}$ T in Z-direction

12. A galvanometer having a resistance of 8 Ω is shunted by a wire of resistance of 2 Ω. If the total current is 1A, then current through the shunt is **[CBSE PMT 1997]**

(*a*) 1.2 A (*b*) 0.5 A

(*c*) 0.8 A (*d*) 0.3A

13. A long straight wire carries a current of 10A. An electron travels with a velocity of $5 \times 10^6$ m/s parallel to the wire 0.1 m from it and in a direction opposite to the current. What force does the magnetic field of current exert on the electron ? **[CBSE PMT 1995]**

(*a*) $1.5 \times 10^{-15}$ N (*b*) $1.6 \times 10^{-17}$ N

(*c*) $2.0 \times 10^{-12}$ N (*d*) zero

14. A 0-20 mA galvanometer has a resistance of 20 Ω. To convert it into voltmeter of range 0–10V, the resistance to be connected in series with it is **[AFMC 1997]**

(*a*) 280 Ω (*b*) 380 Ω

(*c*) 480 Ω (*d*) 580 Ω

15. Two parallel wires of length 9 m each are separated by a distance of 0.15 m. If they carry equal currents in the same direction and exert a force of $30 \times 10^{-7}$ N on each other, then value of current must be **[Pb PMT 1998]**

(*a*) 0.5 A (*b*) 1.5 A

(*c*) 2.25 A (*d*) 3.25 A

### ANSWERS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

| | | | | |
|---|---|---|---|---|
| **1.** (*a*) | **2.** (*b*) | **3.** (*d*) | **4.** (*a*) | **5.** (*c*) |
| **6.** (*b*) | **7.** (*d*) | **8.** (*a*) | **9.** (*a*) | **10.** (*c*) |
| **11.** (*d*) | **12.** (*c*) | **13.** (*b*) | **14.** (*c*) | **15.** (*a*) |

## HINTS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. Torque, $\tau = nIAB\ cos\theta$ . Here $\theta$ is the angle which the plane of the loop makes with the direction of the magnetic field.
Since $\theta = 90°$, $\tau =$ **0.**

2. Radius of curvature, $r = \dfrac{mv \sin\theta}{qB} = \dfrac{m}{q} \times \dfrac{v \sin\theta}{B}$

Here $\dfrac{e}{m} = \dfrac{q}{m} = 10^8$ C/kg; $v = 3 \times 10^5$ ms$^{-1}$; $B = 0.3$T; $\theta = 30°$

$\therefore \quad r = 10^{-8} \times \dfrac{3 \times 10^5 \times \sin 30^o}{0.3} = 0.005$ m = **0.5 cm**

3. Torque on the coil, $\tau = nIAB \cos\theta$ where $\theta$ is the angle between the plane of the coil and the direction of the magnetic field. The coil will orient so that torque acting on it is zero. This will be so when $\theta = 90°$ . Therefore, the coil will orient itself so that its plane becomes **perpendicular to the magnetic field.**

4. A current carrying loop behaves as a magnetic dipole of dipole moment $M$ (= $nIA$) where $n$ = number of turns of the loop, $A$ = area of the loop and $I$ = current through the loop. When the current loop is placed in a uniform magnetic field, then **torque acts on it** and is given by ;
$\vec{\tau} = \vec{M} \times \vec{B}$ or $\tau = M\ B\sin\theta$ where $\theta$ is the angle between $\vec{M}$ and $\vec{B}$

5. Force on charge, $F_m = qvB\sin\theta$. Here angle between $\vec{v}$ and $\vec{B}$ is $\theta = 180°$. Therefore, $F =$ **0.**

6. The resistance of a moving coil galvanometer is about 20 Ω and it can safely carry a current upto 1mA. Therefore, it can measure voltages up to 1 mA × 20 Ω = 20 mV. If the meter is to measure voltages larger than this, say 1V, a high resistance $R$ (called *multiplier*) is connected in series with the galvanometer so that current through the meter coil does not exceed 1mA. Therefore to convert a galvanometer into voltmeter, a **high resistance is connected in series** with the galvanometer.

7. Force on the charged particle, $\vec{F}_m = q(\vec{v} \times \vec{B})$ or $F_m = qvB \sin\theta = qvB \sin 90°$ $= qvB$. Note that $\vec{F}_m$ is always perpendicular to $\vec{v}$ and has the same value. Also acceleration $\vec{a}$ of the particle is always perpendicular to $\vec{v}$ . Therefore, the particle must move in **a circle** with a constant speed $v$.

8. To make the magnetic field radial in a moving coil galvanometer, the **poles are cylindrically cut.** Since the magnetic field is radial, the angle $\theta$ between the plane of the coil and the direction of magnetic field is always 0°. Therefore torque on the coil, $\tau = nIA\ B \cos\theta = nIAB \cos 0° = nIAB$ is constant.

9. Force on 1 m length of wire $B$ , $F = \dfrac{\mu_0}{2\pi}\dfrac{I_A I_B}{r} = \dfrac{4\pi \times 10^{-7}}{2\pi} \times \dfrac{10 \times 2}{0.1} = 4 \times 10^{-5}$ N
Force on 2 m length of wire $B = 2 \times F = 2 \times 4 \times 10^{-5} =$ **$8 \times 10^{-5}$ N**

10. Force on a charged particle moving in a magnetic field is $\vec{F}_m = q\ (\vec{v} \times \vec{B})$. By definition of cross product, $\vec{F}_m$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$ . Therefore, **$\vec{F}_m$ is perpendicular to both $\vec{v}$ and $\vec{B}$** .

**11.** $F = qvB \sin\theta$. Here $\theta$ is the angle between $\vec{v}$ and $\vec{B}$.

Now $B = \dfrac{F}{q\,v \sin\theta}$. The value of $B$ will be minimum when $\theta = 90°$ *i.e.* $B$ acts along Z-axis.

$\therefore$ Minimum $B = \dfrac{F}{qv \sin 90°} = \dfrac{F}{qv} = \dfrac{10^{-10}}{10^{-12} \times 10^{5}}$ = **$10^{-3}$ T in Z-direction**

**12.** Fig. 9.52 shows shunt and the galvanometer.

Current through shunt

$$= \text{total current} \times \frac{\text{other resistance}}{\text{sum}}$$

$$= 1 \times \frac{8}{8+2} = \mathbf{0.8\ A}$$

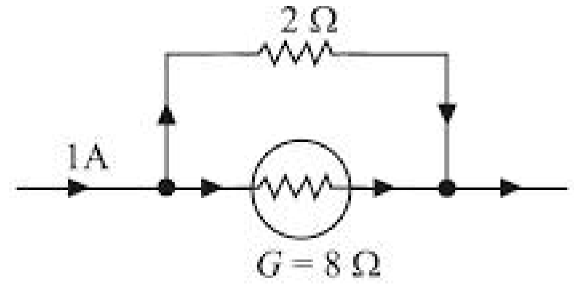



**Fig. 9.52**

**13.** Magnetic field at a distance $a$ (= 0.1 m) from a straight current carrying conductor is $B = \dfrac{\mu_0 I}{2\pi a} = \dfrac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.1} = 2 \times 10^{-5}$ T

The force exerted by the magnetic field due to current on the electron is

$F = qv\,B \sin\theta = (1.6 \times 10^{-19}) \times (5.0 \times 10^{-6}) \times (2 \times 10^{-5}) \times \sin 90° =$ **$1.6 \times 10^{-17}$ N**

**14.** Galvanometer resistance, $G = 20\ \Omega$ ; Full-scale deflection current, $I_g = 20$ mA $= 20 \times 10^{-3}$ A.

$$I_g = \frac{V}{R+G} \quad \text{or} \quad R = \frac{V}{I_g} - G = \frac{10}{20 \times 10^{-3}} - 20 = 500 - 20 = \mathbf{480\ \Omega}$$

**15.** $F = \dfrac{\mu_0}{2\pi} \dfrac{I^2 l}{r}$ or $30 \times 10^{-7} = \dfrac{4\pi \times 10^{-7}}{2\pi} \times \dfrac{I^2 \times 9}{0.15}$

$\therefore$ $I^2 = \dfrac{30 \times 10^{-7} \times 2\pi \times 0.15}{4\pi \times 10^{-7} \times 9} = 0.25$ or $I =$ **0.5 A**

## MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

**1.** A power line lies along the east-west direction and carries a current of 10 A. The force per metre due to earth's magnetic field of $10^{-4}$ is **[Roorkee 1992]**

(*a*) $10^{-5}$ N (*b*) $10^{-4}$ N
(*c*) $10^{-3}$ N (*d*) $10^{-2}$ N

**2.** A proton of energy 1 MeV describes a circular path in a plane at right angles to a uniform magnetic field of $6.28 \times 10^{-4}$ T. The mass of the proton is $1.7 \times 10^{-27}$ kg. The cyclotron frequency of the proton is very nearly equal to **[Haryana CEET 2001]**

(*a*) $10^7$ Hz (*b*) $10^6$ Hz
(*c*) $10^5$ Hz (*d*) $10^4$ Hz

**3.** An electron moves with a uniform velocity $v$ and enters a region of uniform magnetic field $B$. If $\vec{v}$ and $\vec{B}$ are parallel to each other, then electron **[EAMCET 1992]**

(*a*) will not move
(*b*) will move in a circular path
(*c*) will move in a direction perpendicular to $\vec{B}$
(*d*) will continue to move in the same direction

**4.** What is the reading of the voltmeter in Fig. 9.53? **[KCET 1997]**

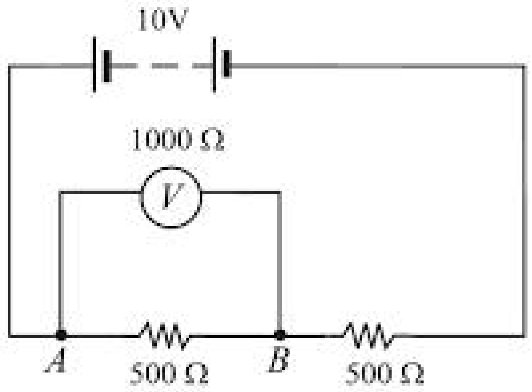



**Fig. 9.53**

(*a*) 3 V (*b*) 2 V
(*c*) 5 V (*d*) 4 V

5. A voltmeter of resistance 998 Ω is connected across a cell of e.m.f. 2V and internal resistance 2 Ω. The percentage error in the reading of the voltmeter is **[KCET 1999]**
(*a*) 1% (*b*) 2%
(*c*) 0.2 % (*d*) 0.1 %

6. In the circuit shown in Fig. 9.54, the point *B* is earthed. What is the potential at point *D*? **[KCET 1996]**

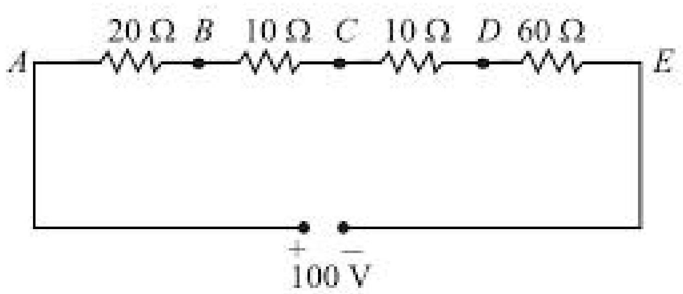



**Fig. 9.54**

(*a*) + 10 V (*b*) − 20 V
(*c*) 0 V (*d*) − 25 V

7. The deflection in a moving coil galvanometer is **[MP PET 1993]**
(*a*) directly proportional to the torsional constant
(*b*) directly proportional to the number of turns in the coil
(*c*) inversely proportional to the area of the coil
(*d*) inversely proportional to the current in the coil

8. The resistance of a galvanometer is 25 Ω and it requires 50 mA for full-scale deflection. The value of shunt resistance required to convert it into an ammeter of 5A is **[Haryana CEET 2000]**
(*a*) $2.5 \times 10^{-4}$ Ω (*b*) $1.25 \times 10^{-3}$ Ω
(*c*) 0.05 Ω (*d*) 2.5 Ω

9. A current of 10 A is flowing in a wire of length 1.5 m. A force of 15 N acts on it when it is placed in a uniform magnetic field of 2T. The angle between the direction of magnetic field and the direction of current is **[CEE Delhi 2000]**
(*a*) 45° (*b*) 60°
(*c*) 30° (*d*) 90°

10. An electron beam is moving horizontally towards east. If this beam passes through a uniform magnetic field directed upward, then in which direction will the beam be deflected? **[Haryana CEET 1999]**
(*a*) towards south (*b*) towards east
(*c*) towards west (*d*) towards north

11. If a proton be moving vertically upward and the magnetic force on it be acting towards north in a horizontal plane, then what will be the direction of the magnetic field? **[KCET 1998]**
(*a*) towards east in the horizontal plane
(*b*) towards west in the horizontal plane
(*c*) towards north in the horizontal plane
(*d*) towards south in the horizontal plane

12. In an ammeter, 5 per cent of main current passes through the galvanometer. If resistances of the galvanometer and shunt are *G* and *S* respectively, then, **[CEE Delhi 1998]**
(*a*) $S - G = \frac{1}{19}$ (*b*) $S = \frac{G}{19}$
(*c*) $G - S = \frac{1}{19}$ (*d*) $G = \frac{S}{19}$

13. In order to convert a milliammeter of range 1mA and resistance 1 Ω into a voltmeter of range 10 V, a resistance of how many ohms should be connected with it and in what manner? **[MP PET 2000]**
(*a*) 9999 Ω in series
(*b*) 999 Ω in parallel
(*c*) 999 Ω in series
(*d*) 9999 Ω in parallel

14. An α-particle and proton having same momentum enter into a region of uniform magnetic field and move in circular path. The ratio of the radii of curvature of their paths $r_\alpha/r_p$ in the field is **[AMU 1998]**
(*a*) 1 (*b*) 1/4
(*c*) 1/2 (*d*) 4

**15.** A rectangular coil of 50 turns carries a current of 2A and is placed in a magnetic field of 0.25 T as shown in Fig. 9.55. What is the magnitude of torque acting on the coil? **[CEE Delhi 2001]**

(*a*) 0.4 Nm (*b*) 0.2 Nm
(*c*) 0.1 Nm (*d*) 0.3 Nm

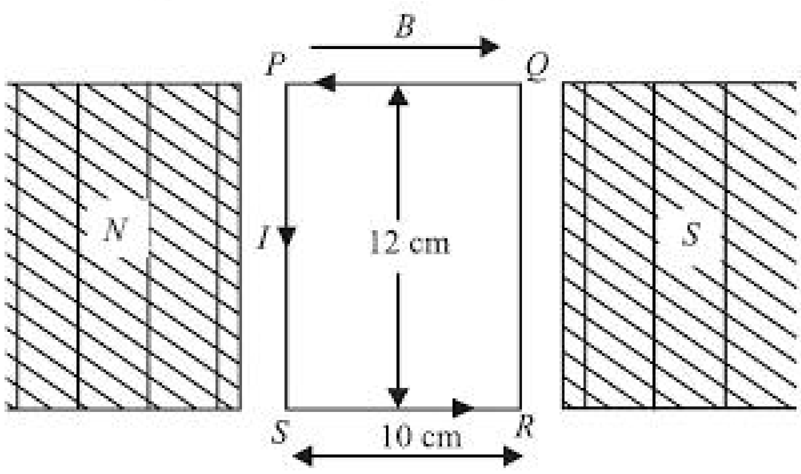



**Fig. 9.55**

**16.** Which of the following is likely to have the largest resistance? **[KCET 1998]**

(*a*) a moving coil galvanometer
(*b*) an ammeter of range 1A
(*c*) a copper wire of length 1m and diameter 3 mm
(*d*) a voltmeter of range 10 V

**17.** In Fig. 9.56, the ammeter reads 5A and voltmeter reads 50 V. The actual resistance *R* is **[MP PET 1997]**

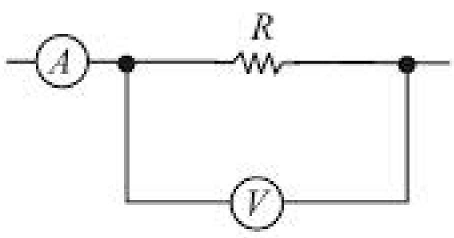



**Fig. 9.56**

(*a*) 10 Ω (*b*) greater than 10 Ω
(*c*) less than 10 Ω (*d*) none of above

**18.** In Fig. 9.57, the ammeter reads 5A and voltmeter reads 50 V. The actual resistance *R* is **[MP PET 1999]**

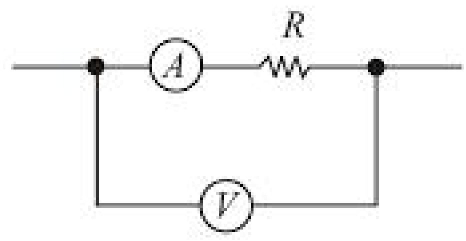



**Fig. 9.57**

(*a*) 10 Ω (*b*) greater than 10 Ω
(*c*) less than 10 Ω (*d*) none of above

**19.** A neutron, a proton, an electron and an $\alpha$-particle enter a region of uniform magnetic field with equal velocities. The magnetic field is perpendicular and directed into the paper. The track of the particles are labeled in Fig. 9.58. The electron follows the track **[KCET 1994]**

(*a*) *A* (*b*) *B*

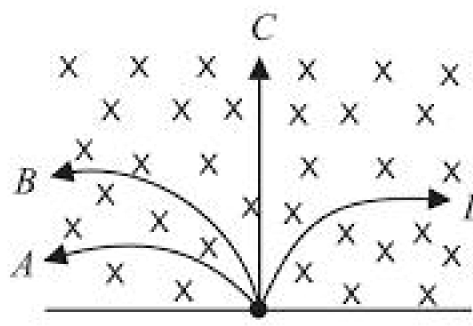



**Fig. 9.58**

(*c*) *C* (*d*) *D*

**20.** A cyclotron is acceleraing proton where the applied magnetic field is 2T and p.d. across the dees is 100kV. Then how many revolutions the proton has to complete to acquire a kinetic energy of 20 MeV? **[AMU 1999]**

(*a*) 200 (*b*) 300
(*c*) 150 (*d*) 100

## ANSWERS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

| | | | | |
|---|---|---|---|---|
| **1.** (*c*) | **2.** (*d*) | **3.** (*d*) | **4.** (*d*) | **5.** (*c*) |
| **6.** (*b*) | **7.** (*b*) | **8.** (*a*) | **9.** (*c*) | **10.** (*d*) |
| **11.** (*a*) | **12.** (*b*) | **13.** (*a*) | **14.** (*c*) | **15.** (*d*) |
| **16.** (*d*) | **17.** (*b*) | **18.** (*c*) | **19.** (*d*) | **20.** (*d*) |

## HINTS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. Force on wire, $F = IlB = 10 \times 1 \times 10^{-4} = \mathbf{10^{-3}\ N}$

2. Cyclotron frequency of proton, $f = \dfrac{qB}{2\pi m} = \dfrac{1.6 \times 10^{-19} \times 6.28 \times 10^{-4}}{2\pi \times 1.7 \times 10^{-27}} \simeq \mathbf{10^4\ Hz}$

3. Force on the electron moving in a magnetic field is $F = evB \sin\theta$. Here $\theta$ is the angle between $\vec{v}$ and $\vec{B}$. Since $\theta = 0°$ or $180°$, $F = 0$. Therefore, the motion of the electron will not be affected *i.e.* the electron **will continue to move in the same direction.**

4. The reading of the voltmeter is equal to the p.d. between points $A$ and $B$ ($i.e., V_{AB}$). It is easy to see that $R_{AB} = 1000/3$ ohms. The total circuit resistance is 2500/3 ohms.

$\therefore$ Circuit current, $I = 10 \div \dfrac{2500}{3} = \dfrac{3}{250}$ A

$\therefore \quad V_{AB} = IR_{AB} = \left(\dfrac{3}{250}\right) \times \dfrac{1000}{3} = \mathbf{4\ V}$

5. The total circuit resistance = 998 + 2 = 1000 Ω . Therefore, circuit current, $I = 2/1000$ = 0.002 A and voltage drop in the cell = $Ir$ =0.002 × 2 = 0.004 V.

$\therefore$ Error = 0.004 V and % error $= \dfrac{0.004}{2} \times 100 = \mathbf{0.2\ \%}$

6. The total circuit resistance = 100 Ω and circuit current, $I$ = 100/100 = 1A. Now $V_{BD} = V_B - V_D = IR_{BD} = 1 \times 20 = 20$ V. When point $B$ is earthed, $V_B = 0$. Therefore, $0 - V_D = 20$ or $V_D = \mathbf{-20\ V}$.

7. In a moving coil galvanometer, $I = \dfrac{k\theta}{nAB}$ or $\theta = \dfrac{nABI}{k} \quad \therefore \theta \propto n$

8. Fig. 9.59 shows the conditions of the problem.

$$S \times (I - I_g) = I_g G$$

$$\therefore \quad S = \frac{I_g G}{I - I_g} = \frac{(50 \times 10^{-6}) \times 25}{5 - (50 \times 10^{-6})}$$

$$= \mathbf{2.5 \times 10^{-4}\ \Omega}$$

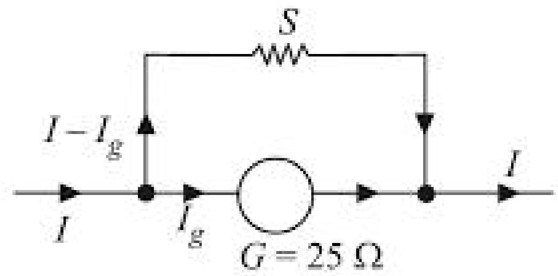



**Fig. 9.59**

9. $$F = IlB \sin\theta$$

or $$\sin\theta = \frac{F}{IlB} = \frac{15}{10 \times 1.5 \times 2} = 0.5 \quad \therefore \quad \theta = \mathbf{30°}$$

10. By right-hand rule for cross product, the beam will be deflected **towards north.**

11. By right-hand rule for cross product, the direction of magnetic field will be **towards east in the horizontal plane.**

12. Fig. 9.60 shows the conditions of the problem. Let the main current be $I$. Then as per given conditions, $0.95I$ flows through the shunt and the remaining $0.05I$ passes through the galvanometer. As voltage across shunt and galvanometer are equal,

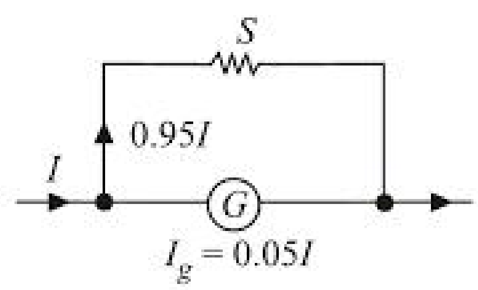



**Fig. 9.60**

$\therefore \quad S \times 0.95I = G \times (0.05\ I) \quad$ or $\quad \dfrac{S}{G} = \mathbf{\dfrac{1}{19}}$

**13.** Fig. 9.61 shows the conditions of the problem. The full-scale deflection current is $I_g = 1$ mA $= 1 \times 10^{-3}$ A and meter resistance is $R_m = 1\ \Omega$. In order to convert this milliammeter into a voltmeter of range 10V, we shall have to connect a resistance of $R$ ohms in series as shown in Fig. 9.61.

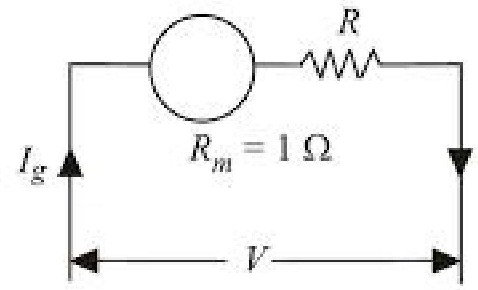



**Fig. 9.61**

$$I_g = \frac{V}{R + R_m} \text{ or } R = \frac{V}{I_g} - R_m = \frac{10}{1 \times 10^{-3}} - 1 = 10000 - 1 = 9999\ \Omega$$

**14.** The conditions of the problem suggest that the two particles enter at right angles to the magnetic field.

Radius of curvature, $r = \frac{mv}{qB}$. Since $mv$ and $B$ are constant, $r \propto \frac{1}{q}$.

$$\therefore \qquad \frac{r_\alpha}{r_p} = \frac{q_p}{q_\alpha} = \frac{e}{2e} = \frac{1}{2}$$

**15.** Torque on the coil, $\tau = nIAB \cos\theta$

Here $n = 50$; $I = 2$A ; $A = 12 \times 10 = 120$ cm$^2$ $= 120 \times 10^{-4}$ m$^2$; $B = 0.25$ T; $\theta = 0°$

$$\therefore \qquad \tau = 50 \times 2 \times 120 \times 10^{-4} \times 0.25 \times \cos 0° = 0.3 \text{ Nm}$$

**16.** A moving coil galvanometer has a small resistance (25 Ω to 50 Ω). An ammeter consists of a moving coil galvanometer in parallel with a suitable very low resistance (*i.e.*, shunt) so that resistance of an ammeter is far less than that of a galvanometer. A copper wire has a very low resistance due to its low value of resistivity. However, the resistance of a voltmeter ($= R + G$) is very high due to the large value of series resistance $R$. Hence, out of the given options, voltmeter has the largest resistance.

**17.** The measured resistance is $= 50/5 = 10\ \Omega$. The actual current flowing through $R = 5 - I_v$. Therefore, actual value of resistance $R = 50/(5 - I_v)$ as shown in Fig. 9.62. Hence, actual value of resistance is greater than 10 Ω.

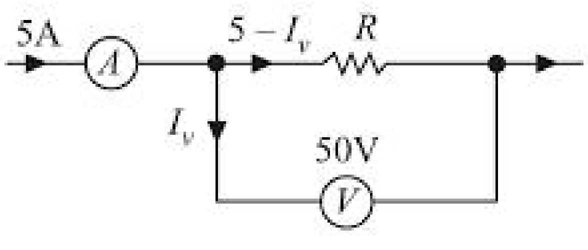



**Fig. 9.62**

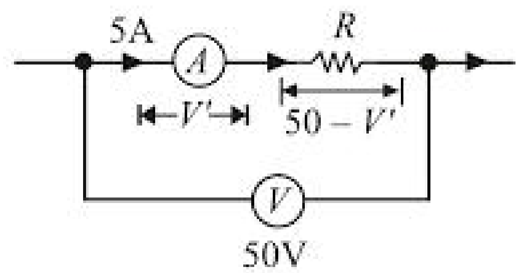



**Fig. 9.63**

**18.** The measured resistance is $50/5 = 10\ \Omega$. There is a voltage drop $V'$ across ammeter so that actual p.d. across $R = 50 - V'$ [See Fig. 9.63]. Therefore, actual value of resistance is

$$R = \frac{50 - V'}{5}$$

Hence, the actual value of $R$ is less than 10 Ω.

**19.** By right-hand rule for cross product, the electron follows the track D.

**20.** In half revolution, the proton gains a kinetic energy = 100 keV = 0.1 MeV. In one complete revolution, energy gained by proton = 2 × 0.1 = 0.2 MeV. Therefore required number of revolutions $N = 20/0.2 = 100$.

## NUMERICAL PROBLEMS FOR COMPETITIVE EXAMINATIONS

1. **An electron beam passes through a magnetic field of $2 \times 10^{-3}$ Wb/m$^2$ and an electric field of $1.0 \times 10^{4}$ V/m, both acting simultaneously. The path of the electrons remaining undeviated. Calculate the speed of the electrons. If the electric field is removed, what will be the radius of the electron path?**

**Hint.** Since the electron beam passes undeflected in simultaneous electric fields $\vec{E}$ and $\vec{B}$ respectively, $\vec{E}$, $\vec{B}$ and $\vec{v}$ must be mutually perpendicular.

Force on electron due to electric field, $F_e = eE$

Force on electron due to magnetic field, $F_m = evB$

As per the given condition, $\vec{F}_e$ and $\vec{F}_m$ are equal in magnitude but opposite in direction.

$$\therefore \quad evB = eE$$

$$\text{or} \quad v = \frac{E}{B} = \frac{1.0 \times 10^4}{2 \times 10^{-3}} = 5 \times 10^6 \text{ ms}^{-1}$$

If electric field is removed, the electrons will move in circular path due to the magnetic field. The radius $r$ of the path is given by ;

$$\frac{mv^2}{r} = evB$$

$$\text{or} \quad r = \frac{mv}{eB} = \frac{(9.1 \times 10^{-31}) \times (5 \times 10^6)}{(1.6 \times 10^{-19}) \times (2 \times 10^{-3})} = 1.43 \times 10^{-2} \text{ m} = 1.43\text{cm}$$

2. **A proton is to circulate the earth along the equator with a speed of $1.0 \times 10^7$ ms$^{-1}$. Find the minimum magnetic field which should be created at the equator for this purpose. The mass of proton = $1.7 \times 10^{-27}$ and radius of earth = $6.37 \times 10^6$ m.**

**Hint.** In order that the proton circulates the earth along the equator, the magnetic field induction should be horizontal and perpendicular to the equator. Let the minimum value of magnetic field be $B$. The magnetic force provides the necessary centripetal force.

$$\therefore \quad \frac{mv^2}{r} = qvB$$

$$\text{or} \quad B = \frac{mv}{qr} = \frac{(1.7 \times 10^{-27}) \times (1.0 \times 10^7)}{(1.6 \times 10^{-19}) \times (6.37 \times 10^6)} = 1.67 \times 10^{-8} \text{ Wb/m}^2$$

3. **A flat coil is made by wrapping wire around a bottle and then gently slipping it off. The resulting coil of 25 turns of wire has a radius of 5.5 cm.**
   **(*i*) What is the magnetic moment of the coil when it carries a current of 1.5 A?**
   **(*ii*) What is the maximum torque exerted on the coil by the earth's magnetic field of $5 \times 10^{-5}$ T when the coil conducts a current of 1.5 A?**

**Hint.** (*i*) The magnitude of magnetic moment is given by ;

$$M = nIA$$

Here $n = 25$ turns; $I = 1.5$ A; $A = \pi r^2 = \pi \times (5.5 \times 10^{-2})^2$

$$\therefore \quad M = (25) \times (1.5) \times \pi \times (5.5 \times 10^{-2})^2 = 0.36 \text{ Am}^2$$

(*ii*) In vector notation, torque is given by ;

$$\vec{\tau} = \vec{M} \times \vec{B}$$

7. **A proton moves with a speed of $8 \times 10^6$ ms$^{-1}$ along the $x$-axis. It enters a region where there is a magnetic field of magnitude 2.5 T directed at an angle of 60° to the $x$-axis and lying in the $xy$ plane (See Fig. 9.65). Calculate the initial force and acceleration of the proton.**

**Hint.** The force on the proton is given by ;

$$F = qvB \sin\theta$$
$$= (1.6 \times 10^{-19}) \times (8 \times 10^6) \times (2.5) \times \sin 60^\circ$$
$$= \mathbf{2.77 \times 10^{-12}\ N}$$

Note that since $\vec{v} \times \vec{B}$ is in the positive $z$ direction and since the charge is positive, the force $\vec{F}$ is in the positive $z$ direction.

Since the mass of proton is $1.67 \times 10^{-27}$ kg, its initial acceleration is

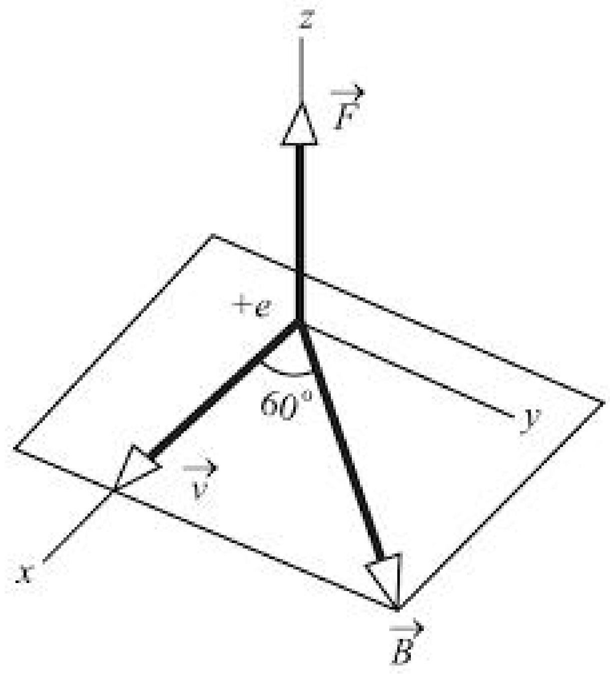



Fig. 9.65

$$a = \frac{F}{m} = \frac{2.77 \times 10^{-12}}{1.67 \times 10^{-27}} = \mathbf{1.66 \times 10^{15}\ ms^{-1}}$$

Note that acceleration is in the direction of force, *i.e.*, in the positive z direction.

8. **In Fig. 9.66, a 20V battery is shown hooked up to an electrical device by means of two parallel wires that are 80 cm long and 2 cm apart. The device has 4 Ω resistance and all other wires have negligible resistance compared to this. Calculate the magnetic force the wires exert on each other. Is the force repulsive or attractive?**

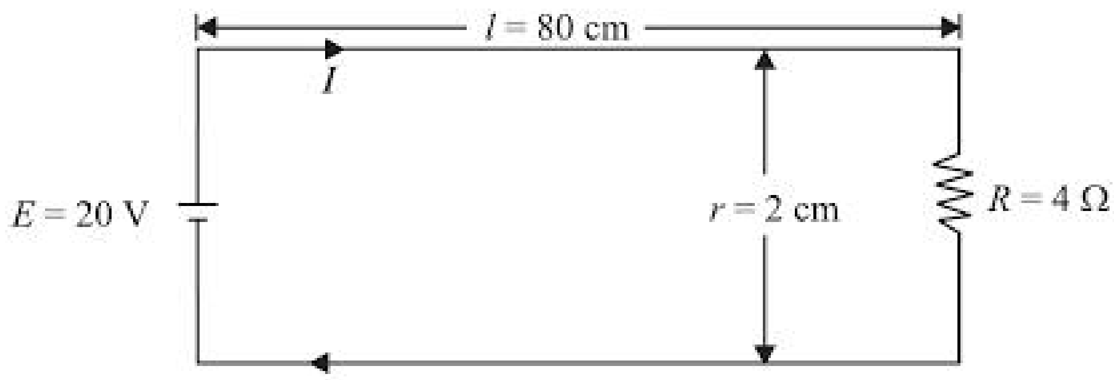



Fig. 9.66

**Hint.** Circuit current, $I = \frac{E}{R} = \frac{20}{4} = 5$ A

The current is flowing around a closed circuit. Therefore, the current in the two wires has the same magnitude and is oppositely directed.

∴ Force on each wire is

$$F = \frac{\mu_0}{2\pi} \times \frac{I_1 I_2}{r} \times l$$

Here $I_1 = I_2 = 5$ A; $r = 2$ cm $= 0.02$ m; $l = 80$ cm $= 0.8$ m

$$\therefore \quad F = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{5 \times 5}{0.02} \times 0.8 = \mathbf{2 \times 10^{-4}\ N}$$

The constant $\mu_0$ is a very small number and so the magnetic force between currents is very small unless the currents are extremely large or distances between them very small. Since the currents in the wires are oppositely directed, the force is **repulsive.**

### Properties of Magnetic Lines of Force

(*i*) *Each magnetic line of force forms a closed loop i.e.* outside the magnet, the direction of a magnetic line of force is from north pole to south pole and it continues through the body of the magnet to form a closed loop (See Fig. 10.5).

(*ii*) *The direction of magnetic flux density* $(\vec{B})$ *at a point is that of the tangent to the magnetic field line at that point.*

(*iii*) *No two magnetic lines of force can intersect each other.* If two magnetic lines of force intersect, there would be two directions of magnetic field at that point which is not possible.

(*iv*) *Where the magnetic lines of force are close together, the magnetic field is strong and where they are well spaced out, the field is weak.*

(*v*) *The larger the number of magnetic field lines crossing per unit normal area, the larger is the magnetic flux density* $(\vec{B})$.

(*vi*) *Magnetic lines of force contract longitudinally and widen laterally.*

(*vii*) *Magnetic lines of force are always ready to pass through iron or other magnetic material, in preference to passing through air, even though their closed paths are made longer thereby.*

## 10.6. UNIFORM AND NON-UNIFORM MAGNETIC FIELD

(*i*) **Uniform magnetic field.** *The magnetic field in a region is uniform if it has the same magnetic flux density* $(\vec{B})$ *and the same direction at all points in the region.*

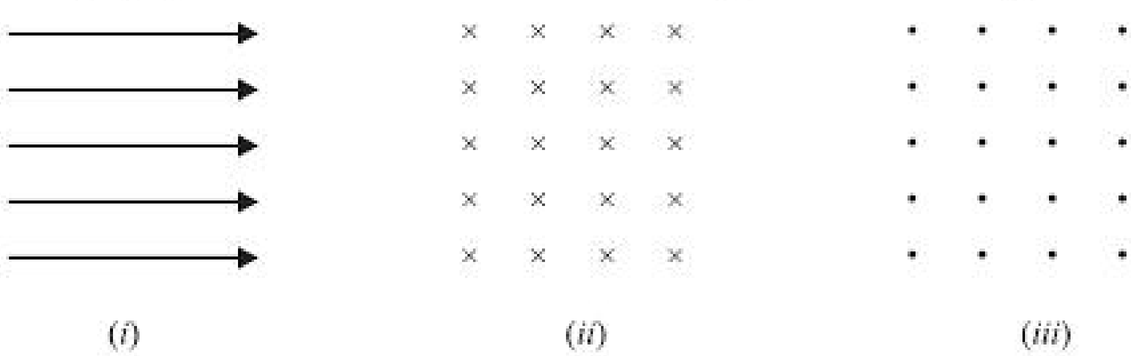



**Fig. 10.6**

For example, the earth's magnetic field is nearly uniform. This means that magnitude and direction of magnetic flux density at every point on earth's surface is nearly the same.

A uniform magnetic field acting in the plane of the paper is represented by parallel and equidistant magnetic lines of force as shown in Fig. 10.6 (*i*). A uniform magnetic field acting perpendicular to the plane of the paper and directed *downwards* is represented by equally spaced crosses as shown in Fig. 10.6 (*ii*). However, a uniform magnetic field acting perpendicular to the plane of the paper and directed *upwards* is represented by equally spaced dots as shown in Fig. 10.6 (*iii*). It is reminded that crowded magnetic lines of force means stronger magnetic field and vice-versa.

(*ii*) **Non-uniform magnetic field.** *The magnetic field in a region is non-uniform if the magnitude or/and direction of magnetic flux density varies from point to point in the region.*

For example, the magnetic field due to a bar magnet is non-uniform *i.e.* magnitude and direction of magnetic flux density varies from point to point. A non-uniform magnetic field is represented by converging or diverging magnetic lines of force. Fig 10.7 (*i*) shows lines of force due to non-uniform magnetic field where direction of magnetic field varies from point to point. In Fig. 10.7 (*ii*), both magnitude and direction of magnetic field are not constant.

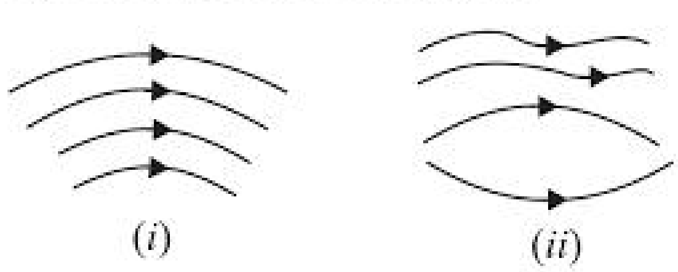



**Fig. 10.7**

## 10.7. MAGNETIC DIPOLE

*A* **magnetic dipole** *consists of two unlike magnetic poles of equal strength separated by a small distance.*

For example, a bar magnet, a solenoid, a compass needle, a current loop etc. behave as magnetic dipoles. The two poles of a magnetic dipole (or a magnet) called north and south poles are always of equal strength and of opposite nature. The distance between the two poles of a bar magnet is called the magnetic length of the magnet.

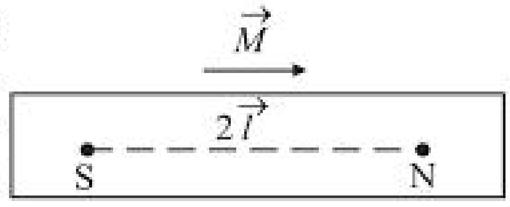



**Fig. 10.8**

Magnetic length is a vector directed from S-pole of the magnet to its N-pole and is represented by $2\vec{l}$ as shown in Fig. 10.8.

**Magnetic dipole moment.** *The* **magnetic dipole moment** *of a magnetic dipole is the product of the strength of its either pole (m) and magnetic length i.e.,*

Magnetic dipole moment = Strength of either pole × Magnetic length

or
$$\vec{M} = m(2\vec{l})$$

The magnetic dipole moment $(\vec{M})$ is a vector directed from south pole to north pole as shown in Fig. 10.8. As we shall see, the SI unit of magnetic dipole moment is **ampere metre$^2$ (Am$^2$)** or **joule per tesla (JT$^{-1}$)**. Clearly, the SI unit of pole strength is **Am.**

## 10.8. MAGNETIC FLUX DENSITY AT A POINT DUE TO BAR MAGNET

We know that *magnetic flux density* $(\vec{B})$ *at a point in a magnetic field is defined as the force experienced by a unit north pole placed at that point.* The direction of $\vec{B}$ is the direction along which the unit north pole would move if free to do so.

**(*i*) When the point lies on the axial line of the magnet (End-on Position)**

Consider a point $P$ located on the axial line of a bar magnet of magnetic length $2l$ and pole strength $m$. It is desired to find the magnetic flux density $(\vec{B})$ at point $P$ which is at a distance $d$ from the centre of the magnet [See Fig. 10.9]. Imagine a unit $n$-pole placed at $P$.

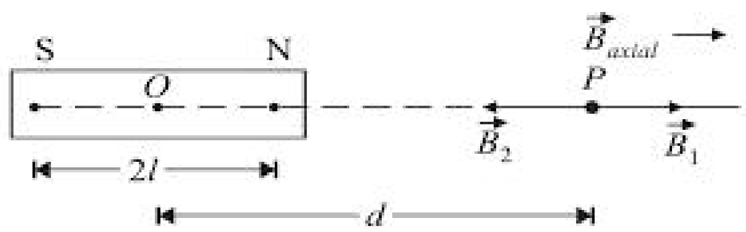



**Fig. 10.9**

∴ Magnetic flux density at $P$ due to $N$-pole of the magnet is

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(d-l)^2} \quad \text{...along } NP$$

Magnetic flux density at $P$ due to $S$-pole of the magnet is

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{(d+l)^2} \quad \text{...along } PS$$

∴ Net magnetic flux density at $P$ due to the magnet is

$$B_{axial} = B_1 - B_2 = \frac{\mu_0}{4\pi}\left[\frac{m}{(d-l)^2} - \frac{m}{(d+l)^2}\right]$$

$$= \frac{\mu_0}{4\pi}\left[\frac{4\,m\,d\,l}{(d^2 - l^2)^2}\right] = \frac{\mu_0}{4\pi}\left[\frac{(m \times 2l)2d}{(d^2 - l^2)^2}\right]$$

$$\therefore \qquad B_{axial} = \frac{\mu_0}{4\pi}\frac{2\,M\,d}{(d^2 - l^2)^2} \qquad (\because M = m \times 2l)$$

**If the bar magnet is very short,** $d >> l$ so that $l^2$ can be neglected as compared to $d^2$.

$$\therefore \qquad B_{axial} = \frac{\mu_0}{4\pi}\frac{2\,M\,d}{d^4}$$

$$\text{or} \qquad B_{axial} = \frac{\mu_0}{4\pi}\frac{2M}{d^3} \qquad \text{......along } NP$$

Clearly, *the magnetic field at any point on the axial line of a magnetic dipole is in the same direction as that of its magnetic dipole moment i.e., from S-pole to N-pole* [See Fig. 10.9].

**(*ii*) When the point lies on the equatorial line of the magnet (Broad side-on position)**

Let $P$ be a point on the equatorial line of a bar magnet where flux density $(\vec{B})$ due to the magnet is to be found out. Referring to Fig. 10.10, $OP = d$; $SN = 2l$; $SP = NP = \sqrt{d^2 + l^2}$.

Magnetic flux density at $P$ due to $N$-pole of the magnet is

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(d^2 + l^2)} \qquad \text{...along } NP \text{ produced}$$

Magnetic flux density at $P$ due to $S$-pole of the magnet is

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{(d^2 + l^2)} \qquad \text{...along } PS$$

It is clear that $B_1 = B_2$.

$\therefore$ Resultant magnetic flux density at $P$ due to the magnet is

$$B_{eqa} = 2B_1\cos\theta = 2\left[\frac{\mu_0}{4\pi}\frac{m}{(d^2 + l^2)}\cos\theta\right]$$

$$= 2\frac{\mu_0}{4\pi}\cdot\frac{m}{(d^2 + l^2)}\cdot\frac{l}{\sqrt{d^2 + l^2}}$$

$$\left(\because \cos\theta = \frac{l}{\sqrt{d^2 + l^2}}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{(m \times 2l)}{(d^2 + l^2)^{3/2}}$$

$$\text{or} \qquad B_{eqa} = \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}} \qquad \text{...along } PR$$

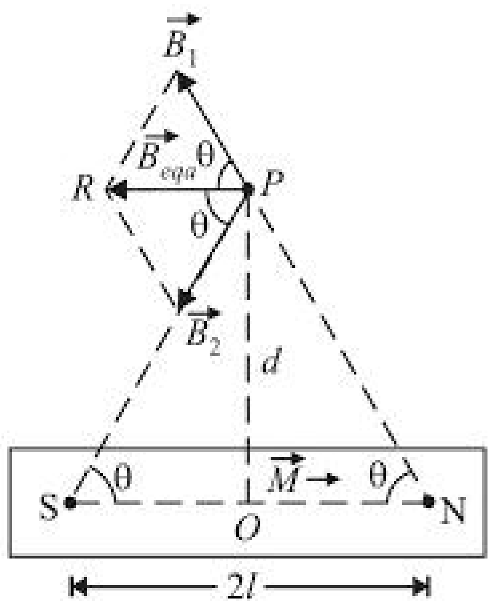



**Fig. 10.10**

**If the bar magnet is very short**, $d >> l$ so that $l^2$ can be neglected as compared to $d^2$.

$$\therefore \qquad B_{eqa} = \frac{\mu_0}{4\pi}\cdot\frac{M}{d^3} \qquad \text{.....along } PR$$

Clearly, *the magnetic field at any point on the equatorial line of a magnetic dipole is in a direction opposite to that of its magnetic dipole moment i.e., from N-pole to S-pole* [See Fig. 10.10].

Again, $$M = \frac{\tau}{B\sin\theta}$$

$\therefore$ SI unit of $M$ $= \frac{1\,\text{Nm}}{1\text{T}\times 1} = \mathbf{NmT^{-1}}$ or $\mathbf{JT^{-1}}$ or $^*\mathbf{Am^2}$

## 10.12. POTENTIAL ENERGY OF A MAGNETIC DIPOLE IN A UNIFORM MAGNETIC FIELD

When a magnetic dipole (*e.g.* bar magnet, current loop etc) is placed in an external magnetic field, torque acts on the dipole which tends to align the dipole in the direction of magnetic field. Therefore, work must be done to change the orientation (*i.e.*, position) of the dipole against this torque. This work done is stored in the magnetic dipole as potential energy of the dipole. Therefore, a magnetic dipole possesses potential energy in a magnetic field depending upon its position in the field.

Consider a magnetic dipole of moment $\vec{M}$ held at an angle $\theta$ to the direction of a uniform magnetic field $\vec{B}$. Then torque acting on the dipole is

$$\tau = M\,B\,\sin\theta$$

If the dipole is rotated through a very small angle $d\theta$ against this torque, then small amount of work done is

$$dW = \tau\,d\theta = MB\sin\theta\,d\theta$$

Total work done in rotating the dipole from $\theta_1$ to $\theta_2$ is

$$W = \int_{\theta_1}^{\theta_2} M\,B\sin\theta\,d\theta$$

Since $M$ and $B$ are constant, they can be taken out of integral.

$$\therefore \quad W = MB\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta = MB\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

or $$W = -MB(\cos\theta_2 - \cos\theta_1) \qquad \ldots (i)$$

Let us **suppose that the magnetic dipole is initially at right angles to the magnetic field $\vec{B}$ (*i.e.*, $\theta_1 = 90°$) and is then brought to a position making an angle $\theta$ to the direction of field (*i.e.*, $\theta_2 = \theta$).

Therefore, eq. (*i*) becomes : $$W = -MB(\cos\theta - \cos 90°)$$

$$\therefore \quad W = -MB\cos\theta$$

This work done is stored in the dipole as its potential energy ($U$).

$\therefore$ Potential energy of dipole, $U = -MB\cos\theta$

In vector form, $$U = -\vec{M}\cdot\vec{B}$$

Note that $\theta$ is the angle between dipole moment $\vec{M}$ and the magnetic field $\vec{B}$.

---

* $\text{JT}^{-1} = \frac{\text{J}}{\text{T}} = \frac{\text{Nm}}{\text{NA}^{-1}\,\text{m}^{-1}} = \text{Am}^2$

** The potential energy of the dipole is zero when $\theta_1 = 90°$ *i.e.*, when the dipole is at right angles to the magnetic field $\vec{B}$.

**Special cases.** The following three cases are discussed :

(*i*) **When $\theta = 0°$** ; $U = -MB \cos 0° = -MB$ ...minimum.

Thus the potential energy of the dipole is minimum when $\vec{M}$ is *parallel* to $\vec{B}$. This is the position of **stable equilibrium**.

(*ii*) **When $\theta = 90°$** ; $U = -MB \cos 90° = 0$

Thus the potential energy of the dipole is zero when $\vec{M}$ is *perpendicular* to $\vec{B}$.

(*iii*) **When $\theta = 180°$** ; $U = -MB \cos 180° = +MB$ ...maximum

Thus the potential energy of the dipole is maximum when $\vec{M}$ is antiparallel to $\vec{B}$. This is the position of **unstable equilibrium**.

## 10.13. WORK DONE IN ROTATING A MAGNETIC DIPOLE IN UNIFORM MAGNETIC FIELD

Suppose a magnetic dipole of dipole moment $M$ ($= m \times 2l$) is rotated in a uniform magnetic field $\vec{B}$ through an angle $\theta$ from its stable equilibrium position as shown in Fig. 10.14. The potential energy of the dipole in this position is $U = -M\,B \cos\theta$.

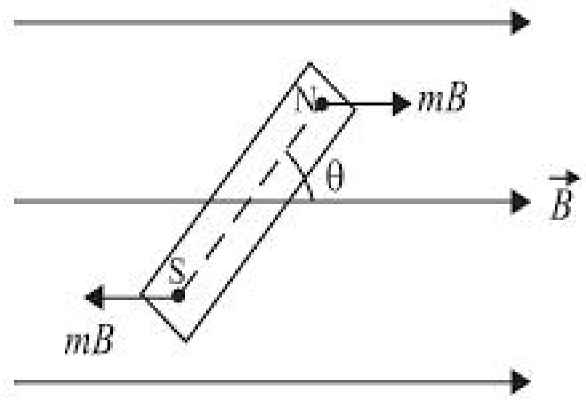



**Fig. 10.14**

(*i*) When the dipole is in stable equilibrium position (*i.e.*, $\theta = 0°$), then its potential energy is

$$U_i = -MB \cos 0° = -MB$$

(*ii*) When the dipole is rotated through an angle $\theta$ from its stable equilibrium position, then its potential energy is

$$U_f = -MB \cos\theta$$

The external work done in rotating the dipole through an angle $\theta$ is equal to the increase in potential energy of the dipole.

$$\therefore \quad \text{Work done} = U_f - U_i = (-MB\cos\theta) - (-MB)$$

$$= -MB\cos\theta + MB = MB(1-\cos\theta)$$

or

$$\text{Work done} = MB(1-\cos\theta)$$

Note that work done in rotating the dipole through 90° from the direction of external magnetic field is $MB$ and that for 180° rotation, it is $2MB$.

## 10.14. GAUSS'S LAW IN MAGNETISM

According to Gauss's law in electrostatics, the surface integral of electric field $\vec{E}$ over a closed surface $S$ is equal to $1/\varepsilon_0$ times the net charge $q$ enclosed by the surface *i.e.*,

$$\oint_S \vec{E}\cdot\vec{dS} = \frac{q}{\varepsilon_0}$$

An isolated magnetic pole does not exist. It is because magnetic poles exist in pairs of equal and opposite strengths *i.e.*, as a magnetic dipole. This means that the surface can enclose only a magnetic dipole *i.e.*, a pair of equal and opposite magnet poles so that net pole strength enclosed by the surface is zero. Hence Gauss's law for magnetism is

$$\oint_S \vec{B}\,.\,\vec{dS} = 0$$

*Therefore,* **Gauss's law in magnetism** *states that the surface integral of magnetic field* $\vec{B}$ *over a closed surface is always zero.*

Now the surface integral of magnetic field $\vec{B}$ over a closed surface gives the magnetic flux through that surface. Therefore, Gauss's law in magnetism can also be stated as under :

*The net magnetic flux through any closed surface is always zero.*

In terms of magnetic field lines, this law tells us that the number of magnetic field lines leaving any closed surface is always equal to the number of magnetic field lines entering it. It other words, magnetic field lines form closed loops.

Gauss's law in magnetism tells us the following facts :

(*i*) Isolated magnetic poles (called *monopoles*) do not exist.

(*ii*) The most elementary magnetic element is a magnetic dipole or current loop. All magnetic phenomena can be explained in terms of an arrangement of magnetic dipoles and/or current loops.

**Analogy between electric and magnetic dipoles.** The equation for magnetic field $\vec{B}$ due to a magnetic dipole (or bar magnet) can be obtained from the electric field $\vec{E}$ due to an electric dipole by making the following replacements :

$$\vec{E} \longrightarrow \vec{B}\ ;\ \vec{p} \longrightarrow \vec{M}\ ;\ \frac{1}{4\pi\varepsilon_0} \longrightarrow \frac{\mu_0}{4\pi}$$

The table below shows the analogy between electric and magnetic dipoles.

| *Physical quantity* | *Electrostatics* | *Magnetism* |
|---|---|---|
| Free space constant | $1/\varepsilon_0$ | $\mu_0$ |
| Dipole moment | $\vec{p}$ | $\vec{M}$ |
| Axial field | $\frac{1}{4\pi\varepsilon_0}\cdot\frac{2\vec{p}}{r^3}$ | $\frac{\mu_0}{4\pi}\cdot\frac{2\vec{M}}{r^3}$ |
| Equatorial field | $-\frac{1}{4\pi\varepsilon_0}\cdot\frac{\vec{p}}{r^3}$ | $-\frac{\mu_0}{4\pi}\cdot\frac{\vec{M}}{r^3}$ |
| Torque in external field | $\vec{p}\times\vec{E}$ | $\vec{M}\times\vec{B}$ |
| P.E. in external field | $-\vec{p}\cdot\vec{E}$ | $-\vec{M}\cdot\vec{B}$ |

Here $r$ is the distance of the point $P$ from the centre of the electric or magnetic dipole.

**Example 10.1. Two poles, one of which is 5 times as strong as the other, exert on each other a force equal to 0.8 × $10^{-3}$ kgf, when placed 10 cm apart in air. Find the strength of each pole.**

**Solution.** Let $m_1$ and $m_2$ be the pole strengths of the weaker and stronger poles respectively. If $m_1 = m$, , then $m_2 = 5\ m$.

Now $$F = \frac{\mu_0}{4\pi}\cdot\frac{m_1 m_2}{d^2}$$

Here $F = 0.8 \times 10^{-3}$ kgf $= 0.8 \times 10^{-3} \times 9.8$ N; $d = 10$ cm $= 0.1$ m

$$\therefore\quad 0.8 \times 10^{-3} \times 9.8 = \frac{10^{-7} \times m \times 5m}{(0.1)^2} \quad \text{or} \quad m = 12.52 \text{ Am}$$

$$\therefore\quad m_1 = \mathbf{12.52\ Am}\ ;\ m_2 = 5m = 5 \times 12.52 = \mathbf{62.6\ Am}$$

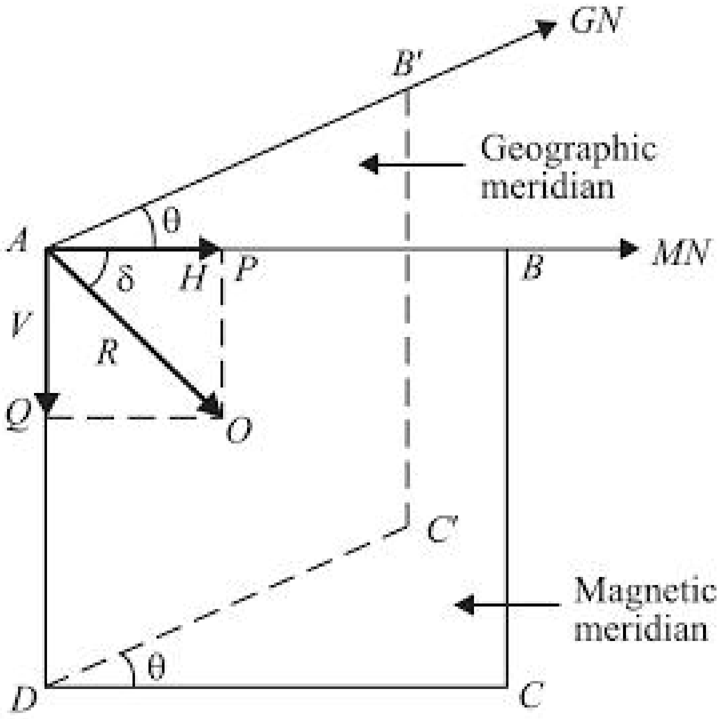



**Fig. 10.20**

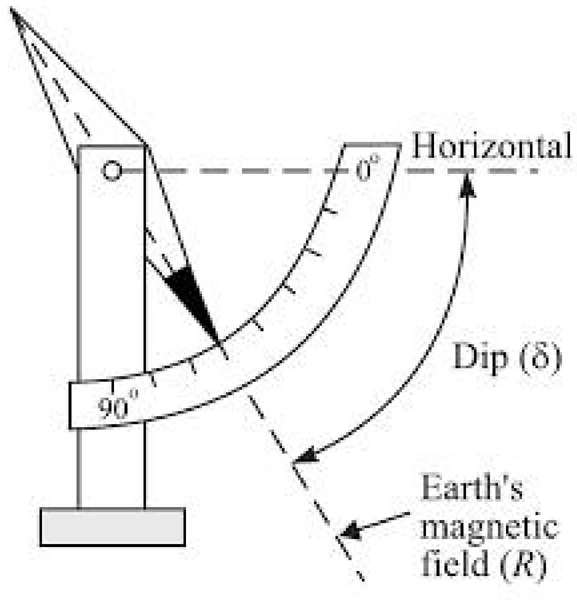



**Fig. 10.21**

2. **Magnetic dip(δ). Magnetic dip or Magnetic Inclination** *at a place is the angle which the direction of total earth's magnetic field (R) makes with the horizontal line in the magnetic meridian*. It is denoted by δ.

When a magnetic needle is so mounted that it is free to rotate in a vertical plane, it is called a *dip needle* (See Fig. 10.21). When the plane of rotation of the dip needle is in the magnetic meridian, the needle will orient itself in the direction of $R$, total intensity of earth's magnetic field. The angle δ between the needle and the horizontal (*i.e.*, angle between $AO$ and $AP$ in Fig. 10.20) is the angle of dip at that place. In the northern hemisphere, the north pole of the dip needle is depressed below the horizontal while in the southern hemisphere, south pole of the needle is pointing below the horizontal. The value of dip (δ) is different at different places on the surface of the earth.

At the earth's magnetic poles, the magnetic field of earth is perpendicular to earth's surface. Therefore, the value of dip is 90° at earth's magnetic poles; the dip needle becomes vertical at these locations. At the magnetic equator, $\delta = 0°$ so that dip needle becomes horizontal.

3. **Horizontal component of Earth's magnetic field.** *It is the component of earth's total magnetic field along horizontal direction in the magnetic meridian. It is denoted by H.*

Thus referring to Fig. 10.20, $R$ is the intensity of the earth's total magnetic field and δ is the angle of dip. Resolving $R$ into two rectangular components, we have,

$$\text{Horizontal component, } H = R\cos\delta \quad \text{... along } AP$$

$$\text{Vertical component, } V = R\sin\delta \quad \text{... along } AQ$$

$$\therefore \quad R^2\cos^2\delta + R^2\sin^2\delta = H^2 + V^2$$

$$\text{or} \quad R^2 = H^2 + V^2$$

$$\therefore \quad R = \sqrt{H^2 + V^2}$$

$$\text{Also} \quad \frac{R\sin\delta}{R\cos\delta} = \frac{V}{H}$$

$$\text{or} \quad \tan\delta = \frac{V}{H}$$

The value of horizontal component $H$ (= $R\cos\delta$) is different at different places on earth's surface. At the magnetic poles, $\delta = 90°$ so that $H = 0$. However, at the magnetic equator, $\delta = 0°$ so that $H = R$.

**Note.** Once we know the values of declination (θ), dip (δ) and horizontal component ($H$) of earth's magnetic field at a place, we can specify the strength and direction of earth's magnetic field at that place.

## 10.19. USEFUL INFORMATION ABOUT ANGLE OF DIP

$$H = R \cos \delta \quad \text{and} \quad V = R \sin \delta$$

(*i*) *At a place on the magnetic poles,* $\delta = 90°$.

$$\therefore \qquad V = R \quad \text{and} \quad H = 0$$

**Therefore, earth's magnetic field always has a horizontal component except at poles.** A freely suspended magnet at poles will stand vertical with its north pole pointing towards the magnetic north pole ($MN$) which is south pole of earth's magnet and vice-versa.

(*ii*) *At a place on the equator,* $\delta = 0°$

$$\therefore \qquad V = 0 \quad \text{and} \quad H = R$$

**Therefore, earth's magnetic field always has a vertical component except at the equator**. A freely suspended magnet at the equator will stand horizontal.

(*iii*) If a magnet is suspended in a vertical plane at θ° from the magnetic meridian, then relation between true angle of dip and apparent angle of dip can be found as under.

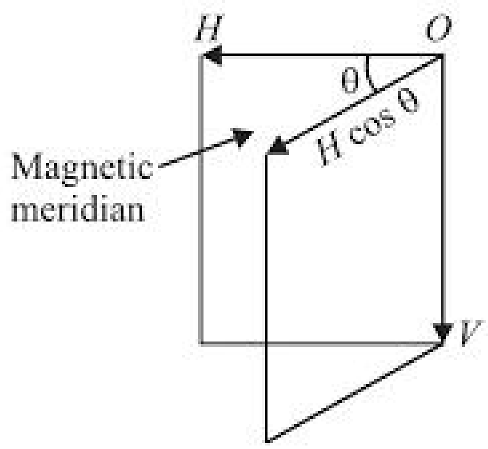



**Fig. 10.22**

Suppose $H$ and $V$ are the horizontal and vertical components of earth's magnetic field *in the magnetic meridian*. Then true angle of dip δ is given by ;

$$\tan \delta = \frac{V}{H} \qquad ...(i)$$

Now in a vertical plane at an angle θ from the magnetic meridian [See Fig. 10.22], the horizontal component is $H' = H \cos \theta$ while the vertical component is still $V$. Therefore, apparent angle of dip δ′ at this plane is given by ;

$$\tan \delta' = \frac{V}{H'} = \frac{V}{H \cos \theta} \qquad ...(ii)$$

Dividing eq. (*ii*) by eq. (*i*), we get,

$$\tan \delta' = \frac{\tan \delta}{\cos \theta} \qquad ...(iii)$$

Eq. (*iii*) gives the relation between apparent angle of dip (δ′) and true angle of dip (δ).

## 10.20. VARIATION OF EARTH'S MAGNETIC FIELD

Earth's magnetic field changes both in magnitude and direction due to (*i*) global variations and (*ii*) temporal variations.

**(*i*) Global variations (*i.e.*, from place to place).** Earth's magnetic field changes both in magnitude and direction from place to place.

(*a*) The magnitude of magnetic field of earth is small ($\simeq 4 \times 10^{-5}$T).

(*b*) At about 32,000 km above the surface of earth, the magnetic field of earth falls to about $10^{-6}$T.

(*c*) At distances greater than 32,000 km, the dipole field pattern of the earth's magnetic field gets severely distorted by solar winds. The solar wind consists of streams of charged particles that emerge continuously from the sun.

**Magnetic maps.** These are the charts which show the variation of the magnetic elements of earth's magnetic field (declination, angle of dip, horizontal component of earth's magnetic field) from place to place.

The lines joining the places of equal declination are called **isogonic lines** while the lines joining the places of equal dip are called **isoclinical lines**. However, the lines joining places having the same value of horizontal component of earth's magnetic field are called **isodynamic lines**.

***(ii)* Temporal variations (*i.e.*, from time to time).** Earth's magnetic field changes both in magnitude and direction as the time passes. These changes are of two types *viz* (*a*) *short-term changes* and (*b*) *long-term changes*.

In short-term changes, the position of the magnetic poles of the earth are found to change with time. For example, in a period of 240 years (1580 to 1820), the magnetic declination at London has changed by 35°. The long-term changes are on geological time ($\simeq 10^5$ to $10^6$ years). It has been found that earth's magnetic field reverses its direction every million years or so.

**Example 10.17. The declination at a place is 18° east of north. In what direction a ship should steer so that it reaches a place due east?**

**Solution.** The conditions of the problem are represented in Fig. 10.23. It is desired that the ship should steer along *GE*. Therefore, the ship should steer making an angle of 72° with the magnetic axis *i.e.* **72° east of north (magnetic)**.

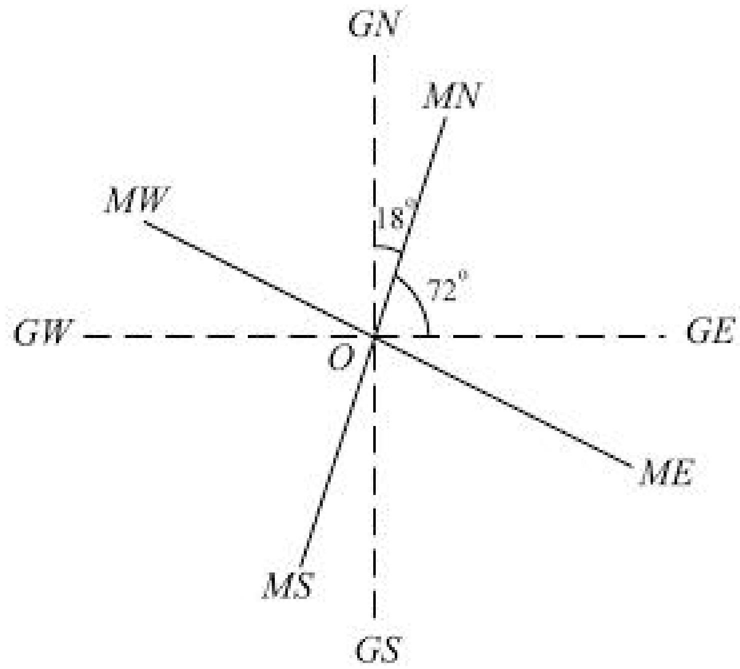



**Fig. 10.23**

**Example 10.18. A compass needle of magnetic moment 60 ampere-metre² is pointing geographical north at a certain place. It experiences a torque of $1.2 \times 10^{-3}$ Nm. The horizontal component of earth's magnetic field at that place is 40 micro weber/m². What is the angle of declination at that place?**

**Solution.** Suppose the angle of declination at the place is θ. Then, torque on needle is

$$\tau = M H \sin\theta$$

or $$\sin\theta = \frac{\tau}{MH} = \frac{1.2\times 10^{-3}}{60\times 40\times 10^{-6}} = 0.5$$

$$\therefore \quad \theta = 30^\circ$$

**Example 10.19. The vertical component of earth's magnetic field at a place is $\sqrt{3}$ times the horizontal component. What is the value of angle of dip at this place?**

**Solution.** Here, $V = \sqrt{3}\,H\,;\,\delta = ?$

Now, $$\tan\delta = \frac{V}{H} = \frac{\sqrt{3}\,H}{H} = \sqrt{3}$$

$$\therefore \quad \delta = 60^\circ$$

**Example 10.20. At a certain place, the horizontal component of earth's magnetic field is 0.28 gauss and its vertical component is 0.40 gauss. Find (*i*) angle of dip and (*ii*) earth's total magnetic field at that place.**

**Solution.** $H = 0.28$ G ; $V = 0.40$ G

(*i*) $$\tan\delta = \frac{V}{H} = \frac{0.40}{0.28} = 1.43$$

$$\therefore \quad \delta = 55^\circ$$

(*ii*) $$R = \sqrt{H^2 + V^2} = \sqrt{(0.28)^2 + (0.40)^2} = 0.49\ \text{G}$$

$$\therefore \quad I_1 = \frac{2rH}{\mu_0 n_1}\tan\theta_1 \text{ and } I_2 = \frac{2rH}{\mu_0 n_2}\tan\theta_2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{n_2}{n_1}\frac{\tan\theta_1}{\tan\theta_2} \text{ or } \frac{1}{2} = \frac{n_2}{n_1}\frac{\tan 30°}{\tan 45°}$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{2\tan 30°}{\tan 45°} = \frac{2\times 0.5774}{1} = 1.155$$

## PROBLEMS FOR PRACTICE

1. The coil in a tangent galvanometer is 11 cm in radius. How many turns of wire should be wound on it if a current of 70 mA is to produce a deflection of 45°? Given that horizontal component of earth's magnetic field is $0.32 \times 10^{-4}$ T. **[80]**
2. The reduction factor of a tangent galvanometer of 60 turns coil of radius 10 cm is 0.085 A. Calculate the horizontal component of earth's magnetic field. **[$0.32 \times 10^{-4}$ T]**
3. A tangent galvanometer of 50 turns of wire and of 8 cm mean radius is in a circuit with a silver voltameter and a battery. If the galvanometer shows a steady deflection of 45° and 0.185g of silver is deposited in 30 minutes, find the strength of the horizontal component of earth's magnetic field. E.C.E. of copper is 0.001118 g/C. **[$0.361 \times 10^{-4}$ T]**

   [Hint. For tangent galvanometer, $I = \frac{2rH}{\mu_0 n}\tan\theta$. For silver voltameter, $I = \frac{m}{zt}$]
4. Two tangent galvanometers have coil radii 7.5 and 10 cm, number of turns 15 and 10, resistances of 8 Ω and 12 Ω respectively. They are connected in parallel in a circuit. If the deflection of first galvanometer is 60°, find the deflection for the second. **[30°]**

   [Hint. $I_1 R_1 = I_2 R_2$]
5. A tangent galvanometer of resistance 30 Ω and reduction factor 0.1A is connected in series with an external resistance of 70 Ω and a battery of e.m.f. 3 V. What is the deflection of the galvanometer? **[$\tan^{-1}$ 0.3]**

## CONCEPTUAL QUESTIONS

**Q.1. Why is a bar magnet called a magnetic dipole?**

**Ans.** A current loop is called a magnetic dipole. The magnetic field produced by a bar magnet closely resembles that of a current loop. Therefore, bar magnet is sometimes called a magnetic dipole.

**Q.2. We cannot have isolated magnetic poles. Why?**

**Ans.** All magnetic fields are caused by electric currents. There is no way to divide up a current and obtain a single magnetic pole.

**Q.3. An iron nail is attracted by a magnet. What is the source of kinetic energy?**

**Ans.** Energy is required to make a magnet. A part of this energy is converted into kinetic energy.

**Q.4. If a compass needle be placed on the magnetic north pole of the earth, then how will it behave?**

**Ans.** At the magnetic north (or south) pole of the earth, the angle of dip is 90°. Therefore, only vertical component of earth's magnetic field acts; the horizontal component of earth's magnetic field being zero there. Consequently, the compass needle may stay in any direction.

**Q.4. Are the two poles of a magnet of equal strength?**

**Ans.** Yes, the two poles of a magnet are of equal strength.

**Q.5. Can we have a magnet with a single pole?**

**Ans.** No, a magnet has always two unlike poles (N,S) of equal strength.

**Q.6. What is the unit of pole strength?**

**Ans.** The unit of pole strength is Ampere-metre (Am).

**Q.7. What is the source of magnetism?**

**Ans.** Magnetism is of electrical origin. The electrons revolving in an atom behave as tiny current loops. These current loops give rise to magnetism.

**Q.8. What do you mean by a magnetic field?**

**Ans.** A magnetic field is the space around a magnet or the space around a current-carrying conductor in which a magnet experiences a force of attraction or repulsion.

**Q.9. How is magnetic field produced?**

**Ans.** A magnetic field can be produced (*i*) by a magnet, (*ii*) by a current-carrying conductor, (*iii*) by a moving charge and (*iv*) by changing electric field.

**Q.10. What is the sure test of magnetism?**

**Ans.** Repulsion is the sure test of magnetism. It is because a magnet can attract a magnet or a magnetic substance (*e.g.* iron) but only a magnet can repel another magnet.

**Q.11. Why do magnetic lines of force always form closed loops?**

**Ans.** Magnetic poles always exist in pairs. Therefore, magnetic lines of force run from N-pole to S-pole outside the magnet and from S-pole to N-pole inside the magnet. As a result, magnetic lines of force form closed loops.

**Q.12. What is the basic difference between electric and magnetic lines of force?**

**Ans.** An electric line of force starts from a positive charge and ends on a negative charge and does not form a closed loop. However, a magnetic line of force always forms a closed loop; it runs from N-pole to S-pole outside the magnet and from S-pole to N-pole inside the magnet.

**Q.13. State two ways to demagnetise a magnet.**

**Ans.** (*i*) By heating and (*ii*) By striking it on the ground again and again.

**Q.14. Define the term magnetic dipole moment.**

**Ans.** The magnetic dipole moment is the product of the strength of either pole ($m$) and magnetic length ($2\vec{l}$) of the magnet. It is represented by $\vec{M}$.

$$\vec{M} = m(2\vec{l})$$

**Q.15. What is the SI unit and direction of magnetic dipole moment?**

**Ans.** The SI unit of magnetic dipole moment is joule/tesla (J/T) or ampere metre$^2$ (Am$^2$). It is a vector quantity and its direction is from S-pole to N-pole of the magnetic dipole (or magnet).

**Q.16. A bar magnet is cut into two equal pieces, (*i*) transverse to its length and (*ii*) along its length. What happens to its pole strength and dipole moment?**

**Ans.** (*i*) Pole strength of each part remains the same as that of the original magnet. However, magnetic moment is halved because length is halved [$\because M = m\,(2l)$].

(*ii*) Pole strength of each part becomes half the pole strength of the original magnet; magnetic moment is also halved.

**Q.17. A magnetic dipole is situated in the direction of a magnetic field. What is its potential energy? If it is rotated through 180°, then what amount of work will be done?**

**Ans.** P.E. of dipole = $-MB\cos\theta$. When it is situated in the direction of magnetic field, $\theta = 0°$ so that its P.E. = $-MB$.

Work done = $MB\,(1 - \cos\theta) = MB\,(1 - \cos 180°) = MB\,(1 + 1) = 2MB$

**Q.18. When a magnet is brought near iron nails, the nails is sticking to it, translate as well as rotate. Why?**

**Ans.** The magnetic field due to the magnet is non-uniform. Therefore, the field exerts a net force and a net torque on the nails. As a result, the nails translate as well as rotate in sticking to the magnet.

**Q.19. When does a magnetic dipole possess (*i*) minimum potential energy and (*ii*) maximum potential energy inside a uniform magnetic field?**

**Ans.** P.E. of magnetic dipole, $U = -\vec{M}.\vec{B} = -MB\cos\theta$

(*i*) P.E. of magnetic dipole will be minimum when $\vec{M} \parallel \vec{B}$ *i.e.* $\theta = 0°$.

(*ii*) P.E. of magnetic dipole will be maximum when $\vec{M}$ and $\vec{B}$ are antiparallel *i.e.* $\theta = 180°$.

**Q.20. A short bar magnet placed with its axis making an angle θ with uniform magnetic field *B* experiences a torque. What is the magnetic moment of the magnet?**

**Ans.** Torque on the magnetic dipole, $\tau = MB \sin\theta \quad \therefore M = \tau/B \sin\theta$

**Q.21. A magnetic dipole is placed in the position of stable equilibrium in a uniform magnetic field. What is the potential energy of the magnetic dipole in this position?**

**Ans.** A magnetic dipole is in stable equilibrium when it has minimum P.E. Therefore, P.E. of dipole in stable equilibrium $= -M\,B\cos 0° = -M\,B$.

**Q.22. Why two magnetic lines of force cannot cross each other?**

**Ans.** It is because at the point of intersection, we can draw two tangents to the two magnetic lines of force. This would mean two directions of magnetic field at the same point which is not possible.

**Q.23. Why is a current loop considered a magnetic dipole?**

**Ans.** When current flows through a loop wire, one face of the loop becomes N-pole and the other S-pole. Therefore, a current loop behaves as a small magnet. For this reason, a current loop is considered a magnetic dipole.

**Q.24. Give some examples of magnetic dipole.**

**Ans.** A bar magnet, a compass needle, current loop *etc*.

**Q.25. What is terrestrial magnetism?**

**Ans.** It is the study of the properties of magnetic field of earth.

**Q.26. What is the strength of earth's magnetic field on the surface of earth?**

**Ans.** It is of the order of $10^{-4}$ tesla.

**Q.27. Name the parameters needed to completely specify the earth's magnetic field at a place on earth's surface.**

**Ans.** These are : (*i*) Magnetic declination (θ) at that place, (*ii*) Magnetic dip or inclination (δ) at that place, (*iii*) Horizontal component (*H*) of earth's magnetic field at that place.

**Q.28. What is the angle of dip at the (*i*) magnetic poles and (*ii*) magnetic equator?**

**Ans.** (*i*) $\delta = 90°$; (*ii*) $\delta = 0°$.

**Q.29. In which direction will a freely suspended magnet align itself when placed at the magnetic poles of the earth?**

**Ans.** The magnet will align itself perpendicular to earth's surface because there is only vertical component of earth's magnetic field at the poles.

**Q.30. If the horizontal and vertical components of earth's magnetic field are equal at a place, what is the angle of dip at that place?**

**Ans.** $\tan\delta = \dfrac{V}{H} = 1 \qquad \therefore \delta = 45°$

**Q.31. How does angle of dip vary as one moves from the magnetic equator to the magnetic pole?**

**Ans.** From 0° to 90°.

**Q.32. On earth's surface, where is the value of the vertical component of earth's magnetic field zero?**

**Ans.** Vertical component of earth's magnetic field, $V = R\sin\delta$. At the magnetic equator, $\delta = 0°$ so that $V = 0$. Therefore, vertical component of earth's magnetic field is zero at the magnetic equator.

**Q.33. Define neutral point in the magnetic field of a bar magnet.**

**Ans.** It is a point near the magnet where the magnetic field due to the magnet is equal and opposite to the horizontal component (*H*) of earth's magnetic field. Clearly, the resultant magnetic field at the neutral point is zero.

(*ii*) In case of magnet $Q$ [See Fig.10.32 (*ii*)], again net force on it is zero. Now torque on $Q$, $\tau = MB \sin\theta$. Here $\theta = 0^\circ$ so that $\tau = 0$.

As the net force and net torque on both the magnets are zero, therefore, both magnets are in equilibrium. Since P.E. of magnet $Q$ is minimum ($\because$ P.E. $= -MB\cos\theta = -MB\cos 0^\circ = -MB$), it is in stable equilibrium.

**Q.12. A magnet is held vertically on a horizontal board. How many neutral points are there on the horizontal board?**

**Ans.** One only. The reason is simple. The earth's magnetic field is from south to north. The magnetic field of the pole on the board is radial. Therefore, there can be only one point on the board where magnetic field due to the pole cancels horizontal component of earth's magnetic field.

**Q.13. What is the probable cause of earth's magnetic field?**

**Ans.** It is believed that the magnetism of earth is due to the molten charged metallic fluid in the core of earth. As the earth rotates about its axis, the charged fluid also rotates. This gives rise to circular electric currents inside the core of the earth. These electric currents are responsible for earth's magnetism.

**Q.14. Which direction would a compass needle point to if located right on the geomagnetic north or south pole?**

**Ans.** A compass needle is capable of rotating in a horizontal plane only. At the geomagnetic north or south pole, the horizontal component of earth's magnetic field is zero. As a result, torque on the compass needle is zero. Therefore, the compass will stay in any direction at the geomagnetic north or south pole.

**Q.15. A magnet is held vertically along its length at the equator and then released. Will it strike the ground head-on or fall flat on the ground?**

**Ans.** The magnet will fall flat on the ground. The reason is simple. At the equator, there is only horizontal component of earth's magnetic field. The torque on the magnet due to this horizontal component will make it to fall flat (horizontal) on the ground.

**Q.16. The horizontal component of earth's magnetic field at a place is $0.4 \times 10^{-4}$T. If angle of dip is 45°, what is the value of vertical component of earth's magnetic field at that place?**

**Ans.** $\tan\delta = \dfrac{V}{H}$ or $\tan 45^\circ = \dfrac{V}{H}$ $\therefore$ $V = H = 0.4 \times 10^{-4}$ T

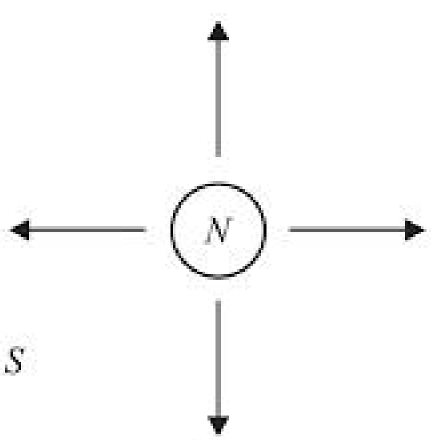
**Q.17. The vertical component of earth's magnetic field at a place is $\sqrt{3}$ times the horizontal component. What is the value of angle of dip at this place?**

**Ans.** $\tan\delta = \dfrac{V}{H} = \dfrac{\sqrt{3}H}{H} = \sqrt{3}$ $\therefore$ $\delta = \tan^{-1}\sqrt{3}$ or $\delta = 60^\circ$

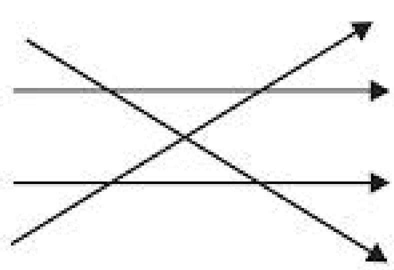
**Q.18. Why does a magnetic dipole possess potential energy when placed at some inclination with the direction of magnetic field?**

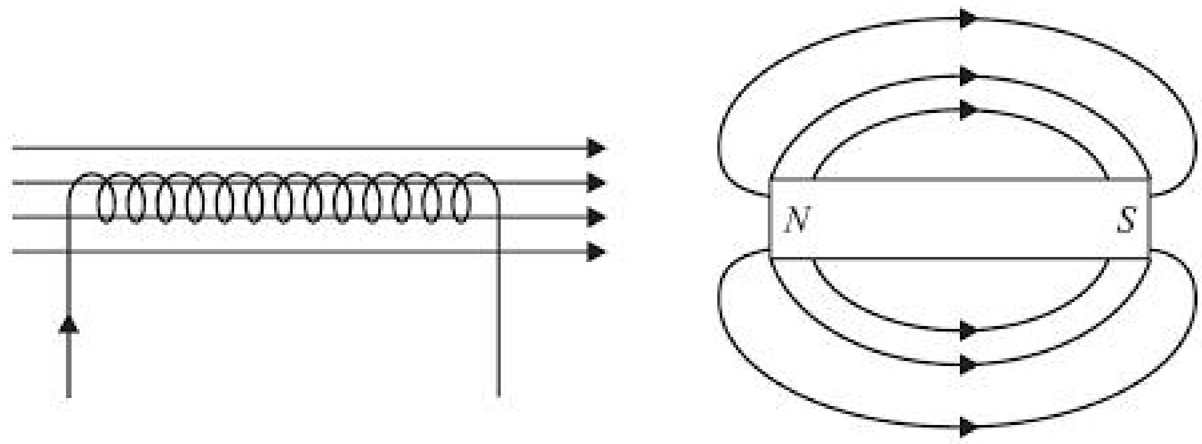
**Ans.** In the stable equilibrium position, a magnetic dipole always positions itself along the direction of magnetic field. When the magnetic dipole is displaced from the stable equilibrium position, a *restoring torque* acts on the dipole to bring it back to stable equilibrium position. The work done against this restoring torque is stored in the dipole in the form of its potential energy.

**Q.19. Identify the correct plotting of magnetic field lines in Fig. 10.33 with reasons.**



(*a*)


(*b*)




(*c*) (*d*)

**Fig. 10.33**

**Ans.** (*a*) **Wrong** because isolated magnetic poles do not exist.

(*b*) **Wrong** because magnetic field lines do not intersect or cross each other.

(*c*) **Wrong** because magnetic field lines cannot be straight outside the solenoid. The magnetic field lines must bend at the ends so as to form closed loops.

(*d*) **Correct** because magnetic field lines form closed loops and around both north and south poles, the net flux of the field is zero.

**Q.20. If magnetic monopoles existed, how would Gauss's law of magnetism be modified?**

**Ans.** According to Gauss's law of magnetism, the flux of $\vec{B}$ through any closed surface $S$ is always zero *i.e.*

$$\oint_S \vec{B} \cdot \vec{dS} = 0$$

If monopole existed, the magnetic flux would no longer be zero but equal to $\mu_0$ times the pole strength ($m$) enclosed by the closed surface $S$.

**Q.21. Give two differences between a magnetic dipole and an electric dipole.**

**Ans.** (*i*) A magnetic dipole consists of two equal and opposite poles which do not have any separate existence. However, an electric dipole consists of two equal and opposite charges that have a separate existence.

(*ii*) Poles of a magnetic dipole do not move while charges of an electric dipole can move.

## LONG ANSWER QUESTIONS

1. Discuss the important properties of a magnet. **[Refer to Art. 10.2]**
2. What is magnetic field? Give the important properties of magnetic lines of force. **[Refer to Art. 10.5]**
3. Derive an expression for the torque acting on a bar magnet held at an angle with the direction of magnetic field. **[Refer to Art. 10.11]**
4. Derive an expression for the magnetic field at a point on the axial line of a magnetic dipole. **[Refer to Art. 10.8]**
5. Derive an expression for the magnetic field at a point on the equatorial line of a magnetic dipole. **[Refer to Art. 10.8]**
6. What are uniform and non-uniform magnetic fields? How can they be represented? **[Refer to Art. 10.6]**
7. Find the potential energy of a magnetic dipole in a uniform magnetic field. **[Refer to Art. 10.12]**

8. Show that a current loop behaves as a magnetic dipole. What is the expression for its magnetic moment? **[Refer to Art. 10.9]**
9. What is Gauss's law in magnetism? Explain its significance. **[Refer to Art. 10.14]**
10. Give three evidences in support of earth's magnetism? What is the cause of earth's magnetism? **[Refer to Arts. 10.15 and 10.16]**
11. Explain the three magnetic elements of earth's magnetic field at a place. **[Refer to Art. 10.18]**
12. What is a neutral point? How will you find the magnetic moment of a magnet by locating its neutral point? **[Refer to Art. 10.21]**
13. State and prove tangent law in magnetism. **[Refer to Art. 10.22]**
14. Describe the construction and working of tangent galvanometer. **[Refer to Art. 10.23]**

$$= (4.19 \times 10^{23}) \times (1.8 \times 10^{-23}) = \mathbf{7.54\ Am^2}$$

$$\text{Torque, } \tau = MB \sin\theta = (7.54) \times (1.5) \times \sin 90^\circ = \mathbf{11.31\ Nm}$$

**2. A long vertical wire carries a steady current of 10 A flowing upward through it at a place where the horizontal component of earth's magnetic field is 0.3 gauss. What is the total magnetic induction at a point 5 cm from the wire due magnetic north of wire?**

**Hint.** Fig 10.42 shows the conditions of the problem. The magnetic field due to current carrying wire at a distance $a$ (= 5 cm = $5 \times 10^{-2}$ m) is

$$B_1 = \frac{\mu_0 I}{2\pi a}$$

$$= \frac{(4\pi \times 10^{-7}) \times 10}{2\pi \times 5 \times 10^{-2}}$$

$$= 0.4 \times 10^{-4}\ \text{T} = 0.4 \text{ gauss due west}$$

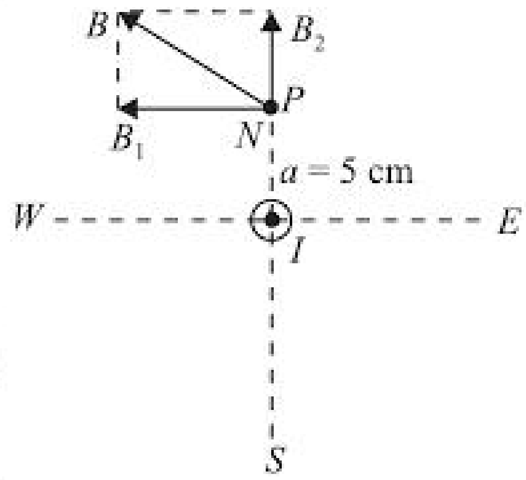


Fig. 10.42

The earth's horizontal component $B_2$ (= $H$ = 0.3 gauss) is towards north.

∴ Net magnetic field at the required point (*i.e.*, point $P$) is given by ;

$$B = \sqrt{(B_1)^2 + (B_2)^2} = \sqrt{(0.4)^2 + (0.3)^2} = \mathbf{0.5\ gauss}$$

**3. A current carrying planar loop suspended in a vertical plane normal to the magnetic meridian is in stable equilibrium. The horizontal component of earth's field is 0.32 G. Another horizontal magnetic field of magnitude 0.48 G parallel to the plane of the loop is set up along the magnetic west to east. Specify the new orientation of the loop when it comes to stable equilibrium. What happens if the current in the loop is reversed?**

**Hint.** Fig. 10.43 shows the conditions of the problem. It is clear that

$$\tan\beta = \frac{B_1}{B_2} = \frac{0.32}{0.48} = 0.6667$$

$$\therefore \quad \beta = 33^\circ\ 42'$$

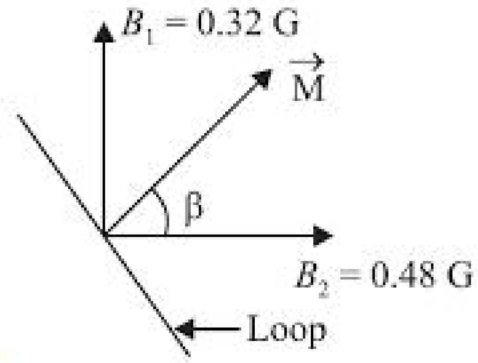


Fig. 10.43

Therefore, in stable equilibrium, the loop is **33° 42′, west of the magnetic meridian.** Angular orientation of the magnetic moment = 90° − 33° 42′ = 56° 18′ east of north.

If the current in the loop is reversed, this orientation is in unstable equilibrium. A slight disturbance will result in the rotation of the loop by 180° bringing it to stable equilibrium.

**4. What is the net magnetic moment of two identical magnets each of magnetic moment $M_0$ inclined at 60° to each other in Fig. 10.44?**

**Hint.** The net magnetic moment $M$ of the system is

$$M = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos\theta}$$

$$= \sqrt{M_0^2 + M_0^2 + 2M_0M_0\cos 60^0}$$

$$= \sqrt{3}\,M_0$$

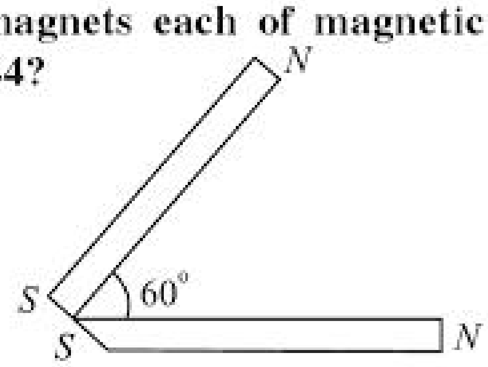


Fig. 10.44

5. **Two coils each of 100 turns are held such that one lies in the vertical plane and the other in the horizontal plane with their centres coinciding. The radius of the vertical coil is 0.2 m and that of the horizontal coil is 0.3 m. How would you neutralise the magnetic field of earth at their common centre ? What current is required to be passed through each coil ? Horizontal component of earth's field is 0.35 × $10^{-4}$ T and the angle of dip is 30°.** **[Roorkee]**

**Hint.** $n_1 = n_2 = 100$; $r_1 = 0.2$ m; $r_2 = 0.3$ m; $H = 0.35 \times 10^{-4}$ T; $\delta = 30°$

$\tan \delta = \frac{V}{H}$ $\therefore$ $V = H \tan \delta = 0.35 \times 10^{-4} \times \tan 30° = 0.2 \times 10^{-4}$ T

The field produced by vertical coil can neutralise the horizontal component of earth's magnetic field if

$$H = \frac{\mu_0 n_1 I_1}{2r_1} \quad \text{or} \quad I_1 = \frac{2Hr_1}{\mu_0 n_1} = \frac{2 \times (0.35 \times 10^{-4}) \times 0.2}{4\pi \times 10^{-7} \times 100} = \mathbf{0.11\ A}$$

The field produced by horizontal coil can neutralise the vertical component of earth's magnetic field if

$$V = \frac{\mu_0 n_2 I_2}{2r_2}$$

or

$$I_2 = \frac{2Vr_2}{\mu_0 n_2} = \frac{2 \times (0.2 \times 10^{-4}) \times 0.3}{4\pi \times 10^{-7} \times 100} = \mathbf{0.096\ A}$$

6. **A circular coil of radius 8.8 cm has 10 turns.**

   **(*i*) Find the magnetic induction at the centre produced by a current of 1 A passing through it.**

   **(*ii*) Find the resultant field at the centre when the plane of the coil is vertical and is (*a*) in the magnetic meridian, (*b*) perpendicular to the magnetic meridian. Horizontal component of earth's magnetic field = 0.4 × $10^{-4}$ T.**

**Hint.** (*i*) The magnetic field at the centre of a circular coil due to current flowing in it is

$$B = \frac{\mu_0 nI}{2r} = \frac{(4\pi \times 10^{-7}) \times 10 \times 1}{2\,(8.8 \times 10^{-2})} = \mathbf{0.7 \times 10^{-4}\ T}$$

The direction of $\vec{B}$ is along the axis of the coil.

(*ii*) (*a*) When the plane of the coil is vertical and in the magnetic meridian, the directions of $B$ and $H$ will be perpendicular as shown in Fig. 10.45. Therefore, the resultant field $R$ at the centre of the coil is

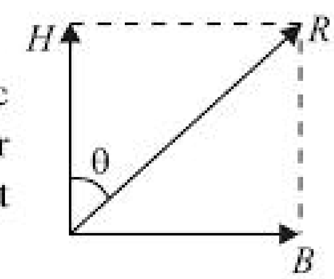


**Fig. 10.45**

$$R = \sqrt{B^2 + H^2} = \sqrt{(0.7 \times 10^{-4})^2 + (0.4 \times 10^{-4})^2}$$
$$= \mathbf{0.8 \times 10^{-4}\ T}$$

Its direction with magnetic meridian is $\theta = \tan^{-1} \frac{B}{H} = 60.3°$.

(*b*) When the plane of the coil is vertical and perpendicular to the magnetic meridian, the directions of $B$ and $H$ will be the same or opposite depending upon the direction of current in the coil. Therefore, the magnitude of the resultant field at the centre of the coil will be

$$R = B \pm H = 0.7 \times 10^{-4} \pm 0.4 \times 10^{-4}\ \text{T}$$
$$= \mathbf{1.1 \times 10^{-4}\ T} \quad \text{or} \quad \mathbf{0.3 \times 10^{-4}\ T}$$

**7. A thin magnet is cut into two equal parts by cutting it parallel to its length. What is the new magnetic moment of each part ? What is the time period of each part compared to that of original magnet if vibrated in the same magnetic field?**

**Hint.** If $M$ $(= ml)$ is the magnetic moment of the original magnet, then magnetic moment of each part

$$= \left(\frac{m}{2}\right)l = \frac{ml}{2} = \frac{M}{2}$$

Moment of inertia of original magnet is

$$I = \frac{m_0 l^2}{12} \qquad \ldots\ m_0 \text{ is the mass}$$

Moment of inertia of each part

$$= \frac{(m_0/2)l^2}{12} = \frac{I}{2}$$

$\therefore$ Time period of original magnet is

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

Time period of each part is given by ;

$$T' = 2\pi\sqrt{\frac{(I/2)}{(M/2)H}} = 2\pi\sqrt{\frac{I}{MH}} = T$$

Therefore, the time period remains *unchanged*.

**8. A closely wound solenoid of 1000 turns and area of cross-section $2 \times 10^{-4}$ m² carries a current of 2 A. It is placed with its horizontal axis at 30° with the direction of a uniform horizontal magnetic field of 0.16 T as shown in Fig. 10.46.**

**(*i*) What is the torque experienced by the solenoid due to the field?**

**(*ii*) If the solenoid is free to turn about the vertical direction, specify its orientation of stable and unstable equilibrium. What is the amount of work needed to displace the solenoid from its stable orientation to its unstable orientation?**

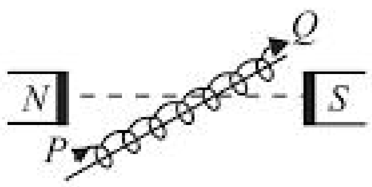



**Fig. 10.46**

**Hint.** The magnitude of magnetic moment of the solenoid is

$$M = nIA = 1000 \times 2 \times 2 \times 10^{-4} = 0.4 \text{ Am}^2$$

The direction of $\vec{M}$ will be along the axis of the solenoid. In the above figure, it is along $PQ$.

(*i*) The solenoid behaves as a bar magnet of magnetic moment 0.4 Am² with end $P$ as south pole and end $Q$ as north pole. The magnitude of torque experienced by it is

$$\tau = MB\sin\theta = (0.4) \times 0.16 \times \sin 30° = 0.032 \text{ Nm}$$

The direction of $\tau$ is vertically downward; its turning effect will try to bring the axis of the solenoid along $N - S$ direction of the external field.

(*ii*) When $\vec{M}$ and $\vec{B}$ are parallel (*i.e.*, *PQ* should be along *NS* with *P* facing *N* and *Q* facing *S*), the solenoid has stable orientation. However, when $\vec{M}$ is antiparallel to $\vec{B}$, the solenoid has unstable orientation (*QP* along *NS* with *Q* facing *N* and *P* facing *S*).

P.E. for stable equilibrium $= -MB\cos 0°$

$$= -0.4\times 0.16\times 1 = -0.064 \text{ J}$$

P.E. for unstable equilibrium $= -MB\cos 180°$

$$= -0.4\times 0.16\times(-1) = +0.064 \text{ J}$$

$\therefore$ Amount of work needed to displace the solenoid from its stable orientation to its unstable orientation is

Work needed, $W = +0.064 - (-0.064) = \mathbf{+0.128\ J}$

**9. A dip needle oscillating in a vertical plane makes 40 oscillations per minute in a magnetic meridian and 30 oscillations per minute in a vertical plane at right angles to the magnetic meridian. Find the angle of dip.**

**Hint.** When the dip needle lies in the magnetic meridian, it oscillates under the action of total intensity *R* of earth's field. When the dip needle is turned through right angles, the needle oscillates under the action of vertical component only ($\because$ horizontal component *H* in this plane is zero). If $T_1$ and $T_2$ are the periodic times in the two cases, then,

$$T_1 = 2\pi\sqrt{\frac{I}{MR}} \quad\text{and}\quad T_2 = 2\pi\sqrt{\frac{I}{MV}}$$

Here *I* is the moment of inertia of dip needle.

$$\therefore \qquad \frac{T_2}{T_1} = \sqrt{\frac{R}{V}}$$

$$\text{or} \qquad \frac{T_2^2}{T_1^2} = \frac{R}{V}$$

If $\delta$ is the angle of dip, then $V = R\sin\delta$.

$$\therefore \qquad \frac{T_2^2}{T_1^2} = \frac{R}{R\sin\delta} = \frac{1}{\sin\delta}$$

$$\text{or} \qquad \sin\delta = \frac{T_1^2}{T_2^2}$$

$$\text{Now} \qquad T_1 = \frac{60}{40} = 1.5\text{ s} \ ; \ T_2 = \frac{60}{30} = 2\text{ s}$$

$$\therefore \qquad \sin\delta = \frac{(1.5)^2}{(2)^2} = 0.565$$

$$\therefore \qquad \delta = \mathbf{34.4°}$$

**10. The values of the apparent angles of dip in two planes at right angles to each other are 30° and 45°. Calculate the true value of the angle of dip at that place.**

**Hint.** Given $\delta_1 = 30.0°$ ; $\delta_2 = 45°$

The true angle of dip $\delta$ is given by ;

$$\cot^2 \delta = \cot^2 \delta_1 + \cot^2 \delta_2 = \cot^2 30^\circ + \cot^2 45^\circ = 3 + 1 = 4$$

$$\therefore \qquad \cot \delta = 2 \quad \text{or} \quad \delta = \cot^{-1} 2 = 26.6^\circ$$

**11. A magnetic needle pivoted through its centre of mass and free to rotate in a plane containing a uniform magnetic field of 100 gauss is displaced slightly from its stable equilibrium position. The frequency of its angular oscillations of small amplitudes is measured to be 1.5 $s^{-1}$. If the moment of inertia of the needle about its axis of rotation is $0.75 \times 10^{-5}$ kg $m^2$, determine the magnetic moment of the needle.**

**Hint.** The frequency of small angular oscillations of a magnetic needle about its stable equilibrium is

$$f = \frac{1}{2\pi}\sqrt{\frac{MB}{I}} \quad \text{or} \quad f^2 = \frac{1}{4\pi^2} \times \frac{MB}{I}$$

or
$$M = \frac{4\pi^2 f^2 I}{B}$$

Here $f = 1.5\ s^{-1}$; $I = 0.75 \times 10^{-5}$ kg $m^2$; $B = 100$ gauss $= 100 \times 10^{-4}$ T

$$\therefore \qquad M = \frac{4\pi^2 \times (1.5)^2 \times 0.75 \times 10^{-5}}{100 \times 10^{-4}} = 0.067\ \text{Am}^2$$

**12. Two magnets of moment $M$ and $M\sqrt{3}$ are joined to form a cross (+). If this combination is suspended freely in a uniform magnetic field, what will be its orientation in the field?**

**Hint.** Suppose in the equilibrium position, the magnet of magnetic moment $M$ makes an angle $\theta$ with the direction of uniform magnetic field $B$. Then the magnet of moment $M\sqrt{3}$ will make an angle of $(90^\circ - \theta)$ with the direction of magnetic field.

Torque experienced by the first magnet

$$= MB \sin\theta$$

Torque experienced by the second magnet

$$= M\sqrt{3}\, B \sin(90^\circ - \theta) = M\sqrt{3}\, B\cos\theta$$

In the equilibrium position, we have,

$$MB\sin\theta = M\sqrt{3}\, B\cos\theta$$

or
$$\tan\theta = \sqrt{3} \quad \text{or} \quad \theta = 60^\circ$$

# 11
# Classification of Magnetic Materials

## INTRODUCTION

All substances are affected by magnetic fields; some attain weak magnetic properties and some acquire strong magnetic properties. The magnetic properties of the substances are explained on the basis of modern atomic theory. The atoms that make up any substance contain electrons that orbit around the central nucleus. Since the electrons are charged, they constitute an electric current and, therefore, produce a magnetic field. Thus an atom behaves as a magnetic dipole and possesses magnetic dipole moment. The magnetic properties of a substance depend upon the magnetic moments of its atoms. In this chapter, we shall focus our attention on the magnetic properties of various substances.

## 11.1. ATOM AS A MAGNETIC DIPOLE

In an atom, electrons revolve around the nucleus. The revolving electron is equivalent to a current loop. Since a current loop behaves as a magnetic dipole, it is clear that an atom is like a magnetic dipole and possesses definite magnetic dipole moment. Referring to Fig. 11.1, the electron is revolving anticlockwise so that current is clockwise. Therefore, the upper face of the current loop acts as south pole and lower face north pole.

**Expression for magnetic dipole moment.** Let us calculate the magnetic dipole moment of an atom due to the orbital motion of the electron. Fig. 11.1 shows an electron revolving in an orbit of radius $r$ with a uniform angular velocity $\omega$. The revolving electron is equivalent to a single-turn current loop. Therefore, magnetic moment $M$ of this current loop is given by ;

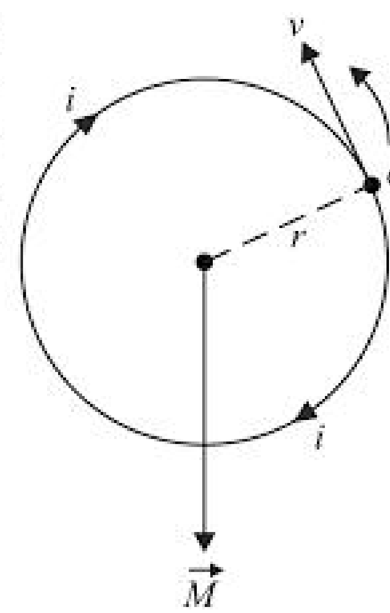


**Fig. 11.1**

$$M = i\,A$$

Now, $$i = \frac{e}{T} = \frac{e}{2\pi/\omega} = \frac{e\omega}{2\pi} \qquad \left(\because T = \frac{2\pi}{\omega}\right)$$

And Area of current loop, $$A = \pi r^2$$

$\therefore$ Magnetic moment of the atom is

$$M = \frac{e\omega}{2\pi} \times \pi r^2 = \frac{1}{2} e\omega r^2$$

or $$M = \frac{1}{2} evr \qquad ...(i) \quad (\because v = \omega r)$$

The current in the loop is clockwise. Therefore, by right hand palm rule, the magnetic dipole moment $\vec{M}$ is perpendicular to the plane of the loop and is directed in the downward direction as shown in Fig. 11.1.

According to Bohr's theory, the angular momentum of an electron ($mvr$) in a stationary orbit can have only those values which are integral multiples of $h/2\pi$, *i.e.*

$$mvr = \frac{nh}{2\pi} \quad \text{... (ii) where } n = 1, 2, 3, \ldots$$

From eqs. ($i$) and ($ii$), we have,

$$M = \frac{1}{2} e \frac{nh}{2\pi m}$$

or

$$M = n\left(\frac{eh}{4\pi m}\right) \quad \text{...(iii)}$$

Eq. ($iii$) gives the orbital magnetic moment of the electron revolving in the $n$th orbit.

The least value of the magnetic dipole moment of an electron due to orbital motion occurs when $n = 1$. This is called **Bohr magneton** and is represented by $\mu_B$.

$$\therefore \quad \mu_B = \frac{eh}{4\pi m}$$

$$\therefore \quad M = n\,\mu_B \quad \text{where} \quad \mu_B = \frac{eh}{4\pi m}$$

**Definition of $\mu_B$.** The magnetic moments of atoms are expressed in terms of $\mu_B$.

$$\mu_B = \frac{eh}{4\pi m} = \frac{(1.6\times10^{-19})\times(6.6\times10^{-34})}{4\pi\times(9\times10^{-31})}$$

$$= 9.27 \times 10^{-24}\ \text{Am}^2 \text{ or } \text{JT}^{-1}$$

*We may define* **Bohr magneton** *as the minimum magnetic dipole moment associated with an atom due to the orbital motion of an electron in the first orbit of the atom.*

The magnetic dipole moment of atoms is of the order of a few Bohr magnetons.

## 11.2. IMPORTANT TERMS USED IN MAGNETISM

The following terms are frequently used in describing the magnetic properties of the materials :

**($i$) Magnetic flux density ($\vec{B}$).** **Magnetic flux density** *in a material is a measure of the number of magnetic field lines passing per unit area of the material.*

The greater the number of magnetic field lines passing per unit area of the material, the greater is the magnetic flux density in the material and *vice versa.* The SI unit of magnetic flux density is **tesla** (T) or **Wb/m²**. The quantity $\vec{B}$ has several names *viz* **magnetic induction, magnetic field strength** and **magnetic flux density.**

**($ii$) Magnetic permeability.** The magnetic permeability of a material is a measure of its conductivity for magnetic field lines. The greater the permeability of a material, the greater is its conductivity for magnetic field lines and *vice-versa.* Since magnetic flux density ($B$) is the magnetic field lines passing per unit area of the material, it is a measure of magnetic permeability of the material.

Suppose magnetic flux density in vacuum/air is $B_0$. If vacuum/air is replaced by a material, suppose the magnetic flux density in the material becomes $B$. Then ratio $B/B_0$ is called the relative permeability ($\mu_r$) of the material.

# COMPETITION SUCCESS MATERIAL

## Useful Concepts/Information

1. The fundamental sources of all magnetic fields are the magnetic dipole moments associated with atoms.
2. An atomic dipole moment can arise both due to orbital motions of electrons and spinning of electrons.
3. All substances are affected by external magnetic fields and are classed as being **paramagnetic, ferromagnetic** and **diamagnetic.**
   (*i*) The atoms of ferromagnetic (*e.g.* iron, nickel, cobalt etc.) and paramagnetic (*e.g.* aluminium, antimony etc.) materials have permanent magnetic moments.
   (*ii*) Diamagnetic (*e.g.* copper, zinc, bismuth etc.) materials consist of atoms with no permanent magnetic moments.
4. When a paramagnetic or ferromagnetic material is placed in an external magnetic field, the dipole tends to align parallel to the field and this aligning increases the net field. In other words, these materials are magnetised in the direction of the applied external field.
   (*i*) The increase in the field is quite small in case of paramagnetic substances. It is because the dipoles are partially aligned in the presence of external magnetic field.
   (*ii*) The magnetic moments in a ferromagnetic substance interact strongly with neighbouring moments. This results in a high degree of alignment in the presence of external magnetic field. Therefore, a ferromagnetic material is strongly magnetised in the direction of external magnetic field.
5. Above a critical temperature, called **Curie temperature**, a ferromagnetic material becomes paramagnetic. It is because the thermal energy can be sufficiently large to overcome the strong interaction responsible for the parallel alignment of magnetic moments.
6. The Curie temperature of iron is 1043K.
7. Although the atoms of a diamagnetic material do not possess permanent magnetic dipole moment, yet they are feebly magnetised in a direction *opposite* to the applied external magnetic field. This is a consequence of Lenz's law. The presence of external magnetic field alters the orbital motion of the electrons and this results in an induced magnetism whose field opposes the external field.
8. Diamagnetism is present in all substances but is very weak in comparison with paramagnetism and ferromagnetism. Therefore, it can be observed only in those materials whose atoms do not have permanent magnetic dipole moments.
9. According to Curie's law, the intensity of magnetisation $I$ (magnetic moment per unit volume) of a paramagnetic material is directly proportional to applied magnetic field ($B$) and inversely proportional to absolute temperature ($T$) *i.e.*

$$I = C\frac{B}{T};\ C = \text{constant called Curie constant}$$

10. The magnitude of magnetic flux density $\vec{B_0}$ in an *air-cored* toroid of $n$ turns per unit length and carrying current $I$ is given by ;

$$B_0 = \mu_0 n I$$

11. When an air-core is replaced by a ferromagnetic material in a toroid as shown in Fig. 11.25, the magnetic flux density is greatly increased due to the magnetisation of the

ferromagnetic substance by the external magnetic field. The magnitude of total magnetic flux density $\vec{B}$ is given by ;

$$B = B_0 + B_M$$

Here $B_M$ is the magnitude of magnetic flux density due to magnetisation of ferromagnetic substance.

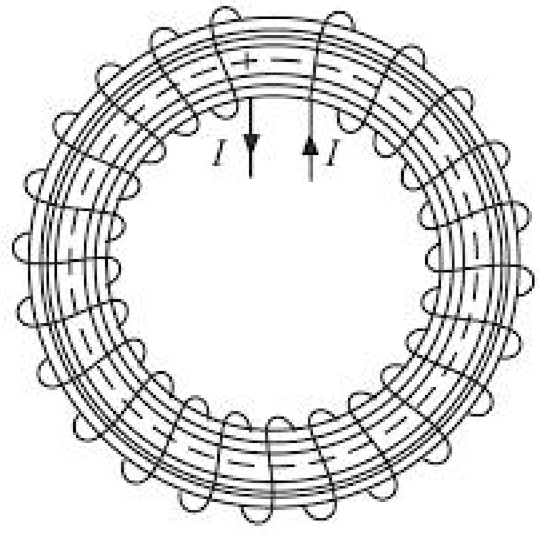



Fig. 11.25

**12.** The ratio $B/B_0$ is called relative permeability $\mu_r$ of the material *i.e.*

Relative permeability, $\mu_r = \dfrac{B}{B_0}$

Note $B_0$ is the flux density with air-core and $B$ the flux density with the material as core. Clearly, $\mu_r$ is a number and its value for air (or vacuum) is 1.

**13.** $$\mu_r = \frac{B}{B_0} \text{ or } B = \mu_r B_0 = \mu_r (\mu_0 n I) = \mu_0 \mu_r n I$$

$$\therefore \quad B = \mu_0 \mu_r \, nI = \mu \, n \, I$$

Here $\mu \; (= \mu_0 \mu_r)$ is the absolute permeability of the material.

**14.** The *magnetising force* or *magnetic intensity* $\vec{H}$ is the number of ampere-turns flowing per metre length of toroid (or magnetic circuit).

$$B = \mu \, n \, I$$

The quantity $nI$ represents ampere-turns per metre and is equal to $H$.

$$\therefore \quad B = \mu \; H$$

For air/vacuum, $B_0 = \mu_0 \; H$ ; For other materials, $B = \mu \; H$

**15.** The ratio $B/H$ $(= \mu = \mu_0 \; \mu_r)$ of a material is always constant — a reasonable approximation.

**16.** The materials can be classified on the basis of relative permeability $\mu_r$.

(*i*) For diamagnetic materials, $\mu_r$ is *slightly* less than 1.

(*ii*) For paramagnetic materials, $\mu_r$ is *slightly* greater than 1.

(*iii*) For ferromagnetic materials, $\mu_r$ is *very* large

Typical values are given below by way of illustration: diamagnetic : 0.99999 ; paramagnetic : 1.001 ; ferromagnetic : 10000.

**17.** Intensity of magnetisation $\vec{I}$ is a measure of the extent to which the material is magnetised by the magnetising force $\vec{H}$.

Intensity of magnetisation, $\vec{I} = \dfrac{\vec{M}}{V} = \dfrac{m \times 2l}{a \times 2l} = \dfrac{m}{a}$

where $\vec{M}$ = magnetic moment developed in the material

$V$ = volume of the material

$m$ = pole strength developed

$a$ = area of X-section of the material

$2l$ = length of the material

It is clear that unit of $\vec{I}$ is $Am^{-1}$.

18. Magnetic susceptibility of a material is the ratio of intensity of magnetisation ($I$) developed in the material to the applied magnetising force ($H$).

Magnetic susceptibility, $\chi_m = \dfrac{I}{H}$

The unit of $I$ is the same as that of $H$ so that $\chi_m$ is a number.

19. The relation between $\mu_r$ and $\chi_m$ is : $\mu_r = 1 + \chi_m$.

(*i*) For a diamagnetic material $\chi_m$ is negative having a very small value *e.g.* – 0.000015 for bismuth.

(*ii*) For a paramagnetic material, $\chi_m$ is positive having a very small value *e.g.* + 0.0001 for a typical paramagnetic substance.

(*iii*) For a ferromagnetic material, $\chi_m$ is very large *e.g.* about 8000 for soft iron.

(*iv*) For diamagnetic and paramagnetic materials, $\chi_m$ is nearly constant but it is not constant for ferromagnetic materials.

20. When a ferromagnetic material (*e.g.* iron) is subjected to a cycle of magnetisation (*i.e.* it is first magnetised in one direction and then in the other), the flux density ($B$) in the material lags behind the applied magnetising force ($H$). This phenomenon is called *hysteresis*.

21. When a ferromagnetic material is subjected to one cycle of magnetisation, the resulting $B$-$H$ curve is a closed loop called hysteresis loop.

(*i*) The hysteresis loop results because $B$ lags behind $H$.

(*ii*) The area enclosed by the hysteresis loop represents loss in energy. This loss in energy appears as heat in the material.

(*iii*) The energy loss due to hysteresis is supplied by the external field *i.e.* m.m.f. in the magnetic circuit.

(*iv*) The shape and size of the hysteresis loop largely depends upon the nature of the material.

22. Fig. 11.26 shows the hysteresis loops for soft iron and steel.

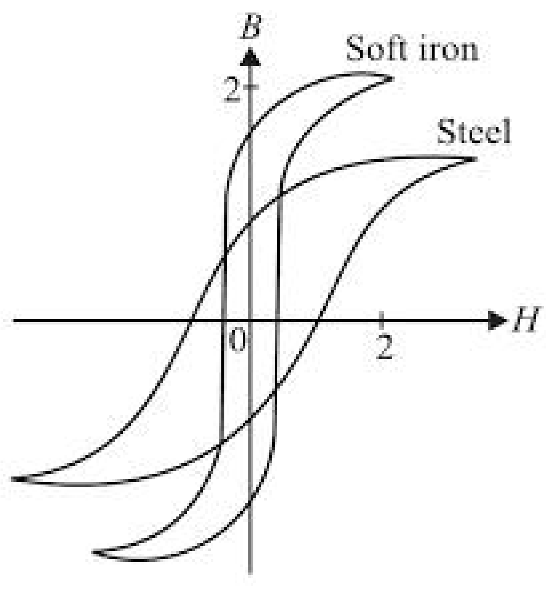



**Fig. 11.26**

From the shape and size of the loops, the following differences between soft iron and steel become apparent :

(*b*) Voltage sensitivity, $S_V = \frac{N\,B\,A}{k\,R} = \frac{S_I}{R}$

**For meter $M_1$**; $S_{VM_1} = \frac{S_{IM_1}}{R_1}$ ; **For meter $M_2$** ; $S_{VM_2} = \frac{S_{IM_2}}{R_2}$

$$\therefore \quad \frac{S_{VM_2}}{S_{VM_1}} = \frac{S_{IM_2}}{S_{IM_1}} \times \frac{R_1}{R_2} = 1.4 \times \frac{10}{14} = 1$$

**Q.11 In a chamber, a uniform magnetic field of 6.5 G (1 G = $10^{-4}$ T) is maintained. An electron is shot into the field with a speed of 4.8 × $10^6$ m $s^{-1}$ normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. ($e$ = 1.6 × $10^{-19}$ C, $m_e$ = 9.1 × $10^{-31}$ kg)**

**Ans.** Force on the moving electron due to magnetic field is

$$\vec{F}_m = e(\vec{v} \times \vec{B}) = e\,v\,B\sin\theta$$

Clearly, this force is perpendicular to $\vec{v}$ as well as $\vec{B}$. Therefore, this force will change the direction of velocity but not the magnitude of velocity (*i.e.* speed). This is the centripetal force acting on the electron and makes it to move in a circle.

Magnetic force on electron = Centripetal force on electron

$$\text{or} \quad e\,v\,B\sin 90^\circ = \frac{m_e v^2}{r}$$

$$\therefore \quad \text{Radius of path, } r = \frac{m_e v}{e\,B}$$

Here, $m_e = 9.1 \times 10^{-31}$ kg ; $v = 4.8 \times 10^6$ $ms^{-1}$ ; $e = 1.6 \times 10^{-19}$ C ; $B = 6.5 \times 10^{-4}$ T

$$\therefore \quad r = \frac{9.1 \times 10^{-31} \times 4.8 \times 10^6}{1.6 \times 10^{-19} \times 6.5 \times 10^{-4}} = 4.2 \times 10^{-2}\text{ m} = 4.2\text{ cm}$$

**Q.12 In Q.11, obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain.**

**Ans.** $e\,v\,B = \frac{m_e v^2}{r}$ or $e\,B = \frac{m_e v}{r} = \frac{m_e}{r}(r\,\omega) = m_e\omega = m_e \times 2\pi f$

$$\therefore \quad \text{Frequency, } f = \frac{e\,B}{2\pi\,m_e} = \frac{(1.6 \times 10^{-19}) \times (6.5 \times 10^{-4})}{2\pi \times 9.1 \times 10^{-31}} = 18.18 \times 10^6 \text{ Hz}$$

It is clear from the expression of frequency that frequency of revolution of electron is independent of the speed of electron.

**Q.13 (a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60° with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.**

**(b) Would your answer change, if the circular coil in (*a*) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)**

**Ans.** $I$ = 6.0 A; $n$ = 30 ; $B$ = 1.0 T, $\alpha$ = 60° ; $r$ = 8.0 cm = 8.0 × $10^{-2}$ m

$\therefore$ Area of the coil, $A = \pi r^2 = \pi \times (8.0 \times 10^{-2})^2 = 2.01 \times 10^{-2}$ $m^2$

(*a*) Deflecting torque, $\tau = n\,I\,A\,B\sin\alpha$

$= 30 \times 6.0 \times 2.01 \times 10^{-2} \times 1.0 \times \sin 60^\circ = 3.133$ Nm

$\therefore$ Counter torque = Deflecting torque = 3.133 Nm

(*b*) No. It is because torque on the planar loop is independent of its shape provided the area ($A$) of the loop remains the same.

## ADDITIONAL EXERCISES

**Q.14 Two concentric circular coils $X$ and $Y$ of radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil $X$ has 20 turns and carries a current of 16 A; coil $Y$ has 25 turns and carries a current of 18 A. The sense of the current in $X$ is anticlockwise, and clockwise in $Y$, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.**

**Ans.** Fig. 4.03 shows the conditions of the problem.

**For coil $X$.** $r$ = 16 cm = 0.16 m ; $n$ = 20 ; $I$ = 16 A

$\therefore$ Magnetic field at the centre of coil $X$ is

$$B_x = \frac{\mu_0 n I}{2r} = \frac{4\pi \times 10^{-7} \times 20 \times 16}{2 \times 0.16}$$

$$= 4\pi \times 10^{-4} \text{ T towards east}$$

As the current in coil $X$ is anticlockwise, the magnetic field $B_x$ is directed towards east.

**For coil $Y$.** $r$ = 10 cm = 0.1 m ; $n$ = 25 ; $I$ = 18 A

$\therefore$ Magnetic field at the centre of coil $Y$ is

$$B_y = \frac{\mu_0 nI}{2r} = \frac{4\pi \times 10^{-7} \times 25 \times 18}{2 \times 0.1} = 9\pi \times 10^{-4} \text{ T towards west}$$

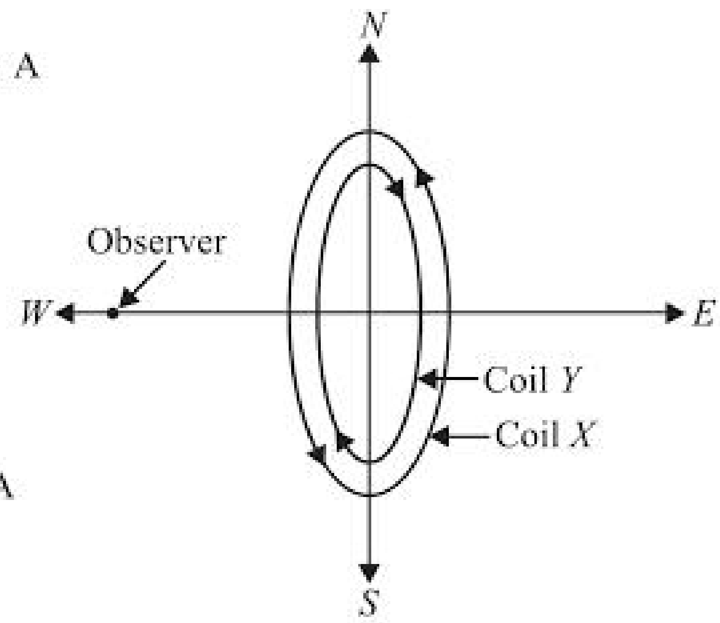



**Fig. 4.03**

As the current in coil $Y$ is clockwise, the magnetic field is directed towards west.

$\therefore$ Net magnetic field at the centre of two coils is

$$B = B_y - B_x = 9\pi \times 10^{-4} - 4\pi \times 10^{-4} = \mathbf{5\pi \times 10^{-4} \text{ T towards west}}$$

**Q.15 A magnetic field of 100 G (1 G = $10^{-4}$ T) is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about $10^{-3}$ m$^2$. The maximum current-carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m$^{-1}$. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic.**

**Ans.** Here, $B$ = 100 G = $100 \times 10^{-4}$ T = $10^{-2}$ T

Magnetic field inside a solenoid is

$$B = \mu_0 n I \quad \text{or} \quad n I = \frac{B}{\mu_0} = \frac{10^{-2}}{4\pi \times 10^{-7}} = 7957 \simeq 8000$$

The maximum current-carrying capacity of the given coil is 15 A. Let us take 10 A as the safe limit,

$$\therefore \qquad n = 8000/I = 8000/10 = 800 \text{ m}^{-1}$$

In order that magnetic field is uniform over a length of 10 cm, the solenoid may have a length of 50 cm (say 5 times of 10 cm) and area of cross-section $5 \times 10^{-3}$ m$^2$ (5 times the given value). With these design considerations, the suggested design particulars of the solenoid are :

$I$ = **10 A** ; Length = **50 cm** ; cross-sectional area = **$5 \times 10^{-3}$ m$^2$** ; $n$ = 800 × 0.5 = **400**.

**Q.16 For a circular coil of radius $R$ and $N$ turns carrying current $I$, the magnitude of the magnetic field at a point on its axis at a distance $x$ from its centre is given by ;**

$$B = \frac{\mu_0 I R^2 N}{2(x^2 + R^2)^{3/2}}$$

**(*a*) Show that this reduces to the familiar result for field at the centre of the coil.**

**(*b*) Consider two parallel co-axial circular coils of equal radius $R$, and number of turns $N$, carrying equal currents in the same direction, and separated by a distance $R$. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to $R$, and is given by ;**

Thus the torque is **$1.8 \times 10^{-2}$ Nm and acts along the negative *y*-axis.**

(*b*) Here, $I\vec{A} = 0.06\hat{i}$ Am$^2$ ; $\vec{B} = 0.3\hat{k}$ T

$\therefore$ Torque, $\vec{\tau} = I\vec{A} \times \vec{B} = 0.06\hat{i} \times 0.3\hat{k} = -1.8 \times 10^{-2}\hat{j}$ Nm

Thus the torque is **$1.8 \times 10^{-2}$ Nm and acts along the negative *y*-axis.**

(*c*) Here, $I\vec{A} = -0.06\hat{j}$ Am$^2$ ; $B = 0.3\hat{k}$ T

$\therefore$ Torque, $\vec{\tau} = I\vec{A} \times \vec{B} = -0.06\hat{j} \times 0.3\hat{k} = -1.8 \times 10^{-2}\hat{i}$ Nm

Thus the torque is **$1.8 \times 10^{-2}$ N and acts along the negative *x*-axis.**

(*d*) Here, $IA = 0.06$ Am$^2$ ; $B = 0.3$ T

$\therefore$ Torque, $|\vec{\tau}| = IAB = 0.06 \times 0.3 =$ **$1.8 \times 10^{-2}$ Nm**

The direction of torque will make an angle $30° + 90° = 120°$ with positive *x*-axis in anticlockwise direction or **240°** with positive *x*-axis in clockwise direction.

(*e*) Here, $I\vec{A} = 0.06\hat{k}$ Am$^2$ ; $\vec{B} = 0.3\hat{k}$ T

$\therefore$ Torque, $\tau = I\vec{A} \times \vec{B} = 0.06\hat{k} \times 0.3\hat{k} =$ **0**

(*f*) Here, $I\vec{A} = -0.06\hat{k}$ Am$^2$ ; $\vec{B} = 0.3\hat{k}$ T

$\therefore$ Torque, $\vec{\tau} = I\vec{A} \times \vec{B} = -0.06\hat{k} \times 0.3\hat{k} =$ **0**

The **resultant force is zero in all the six cases.**

The loop in **case (*e*) is in stable equilibrium.** It is because here $I\vec{A}$ and $\vec{B}$ are in the same direction *i.e.* angle between them is zero (*i.e.* $\theta = 0°$). On being disturbed slightly from this position, the restoring torque comes into play to bring the loop back to its original position.

**Note.** The loop in **case (*f*) is in unstable equilibrium.** It is because here angle between $I\vec{A}$ and $\vec{B}$ is 180° (*i.e.* $\theta = 180°$). If the loop is slightly disturbed from this position, it will not come to its original position.

**Q.25 A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the**

**(*a*) total torque on the coil,**

**(*b*) total force on the coil,**

**(*c*) average force on each electron in the coil due to the magnetic field?**

**(The coil is made of copper wire of cross-sectional area $10^{-5}$ m$^2$, and the free electron density in copper is given to be about $10^{29}$ m$^{-3}$.)**

**Ans.** Here, $n = 20$ ; $r = 10$ cm $= 0.10$ m ; $B = 0.10$ T ; $\alpha = 0°$ ; $I = 5.0$ A

Area of the circular coil, $A_C = \pi r^2 = \pi \times (0.10)^2$ m$^2$

(*a*) Torque on the coil, $\tau = nIA_C B \sin\alpha = nIA_C B \sin 0° =$ **0** $\qquad (\because \sin 0° = 0)$

(*b*) Total or net force on a planar current loop in a magnetic field is always **zero.**

(*c*) Force on each electron moving with drift velocity $v_d$ in the magnetic field is

$$F = ev_d B = eB\left(\frac{I}{neA}\right) \qquad (\because I = neAv_d)$$

Note that here $n$ is the electron density in copper and $A$ is the cross-sectional area of the copper wire.

$$\therefore \quad F = \frac{BI}{nA} = \frac{0.10 \times 5.0}{10^{29} \times 10^{-5}} = \mathbf{5 \times 10^{-25}\ N}$$

**Q.26 A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to the axis ; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire? $g = 9.8\ ms^{-2}$.**

**Ans.** Let $I$ be the required current in the solenoid.

**For solenoid.** $l = 60$ cm $= 0.60$ m ; $N = 3 \times 300 = 900$

Magnetic field along the axis of the solenoid is

$$B = \mu_0 n I = \mu_0 \left(\frac{N}{l}\right) I = 4\pi \times 10^{-7} \times \left(\frac{900}{0.60}\right) I = 6\pi \times 10^{-4}\ I\ \text{T}$$

**For wire.** $l' = 2.0$ cm $= 0.02$ m; $m = 2.5$ g $= 2.5 \times 10^{-3}$ kg ; $I' = 6.0$ A

The magnetic field $B$ due to solenoid acts perpendicular to the current carrying wire of length $l'$ ($= 0.02$ m). Therefore, magnetic force on the wire is

$$F = I'l'B = (6.0) \times (0.02) \times (6\pi \times 10^{-4}\ I)$$

The wire can be supported if the magnetic force $F$ is equal to the weight $mg$ of the wire *i.e.*,

$F = m\,g$ or $(6.0) \times (0.02) \times (6\pi \times 10^{-4}\ I) = 2.5 \times 10^{-3} \times 9.8$

$$\therefore \qquad I = \frac{2.5 \times 10^{-3} \times 9.8}{(6.0) \times (0.02) \times 6\pi \times 10^{-4}} = \mathbf{108.27\ A}$$

**Q.27 A galvanometer coil has a resistance of 12 Ω and the meter shows full scale deflection for a current of 3 mA, How will you convert the meter into a voltmeter of range 0 to 18 V?**

**Ans.** Here, $G = 12\ \Omega$ ; $I_g = 3$ mA $= 3 \times 10^{-3}$ A ; $V = 18$ volt ; $R = ?$

Now, $R = \dfrac{V}{I_g} - G = \dfrac{18}{3 \times 10^{-3}} - 12 = 6000 - 12 = \mathbf{5988\ \Omega}$

Thus above galvanometer can be converted into voltmeter of range 0–18 V by connecting 5988 Ω resistance in series with the galvanometer.

**Q.28 A galvanometer coil has a resistance of 15Ω and the meter shows full scale deflection for a current of 4 mA. How will you convert the meter into an ammeter of range 0 to 6 A?**

**Ans.** Here, $G = 15\ \Omega$ ; $I_g = 4$ mA $= 4 \times 10^{-3}$ A ; $I = 6$ A ; $S = ?$

$\therefore$ Value of shunt, $S = \dfrac{I_g\,G}{I - I_g} = \dfrac{4 \times 10^{-3} \times 15}{6 - 4 \times 10^{-3}} = \dfrac{6 \times 10^{-2}}{5.996} = \mathbf{0.01\ \Omega}$

Thus above galvanometer can be converted into an ammeter of range 0–6A by connecting a shunt of resistance 0.01 Ω in parallel with the galvanometer.

# N.C.E.R.T. TEXTBOOK EXERCISES

## NCERT CHAPTER 5 : MAGNETISM AND MATTER

**Q.1 Answer the following questions regarding earth's magnetism :**

**(*a*) A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.**

**(*b*) The angle of dip at a location in southern India is about 18°. Would you expect a greater or smaller dip angle in Britain?**

**(*c*) If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground ?**

**(*d*) In which direction would a compass point to, if located right on the geomagnetic north or south pole?**

**(*e*) The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment $8 \times 10^{22}$ $JT^{-1}$ located at its centre. Check the order of magnitude of this number in some way.**

**(*f*) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all?**

**Ans.** (*a*) The three independent quantities used to specify the earth's magnetic field are :

(*i*) Magnetic declination ($\theta$) (*ii*) Angle of dip ($\delta$) (*iii*) Horizontal component of earth's magnetic field ($H$)

(*b*) Britain is closer to the magnetic north pole. Therefore, angle of dip ($\delta$) is greater in Britain than in India. It is about 70° in Britain.

(*c*) Melbourne (Australia) is situated in southern hemisphere of earth where the earth's magnetic north pole lies. Therefore, magnetic lines of force of earth's magnetic field seem to come *out* of the ground at Melbourne.

(*d*) At geomagnetic poles, earth's total magnetic field is vertical ( $\because H = 0$). Since the compass needle is free to rotate in the horizontal plane only, the compass needle may point in any direction.

(*e*) Magnetic field at a point on the equatorial line of earth's magnetic dipole is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$$

Here $M = 8 \times 10^{22}$ $JT^{-1}$. For $d = R$ = Radius of earth = 6400 km = $6.4 \times 10^6$ m, we have,

$$B = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{8 \times 10^{22}}{(6.4 \times 10^6)^3} \text{T} = 0.3 \times 10^{-4} \text{ T} = 0.3 \text{ G}$$

This is approximately the observed value of earth's magnetic field.

(*f*) The earth's magnetic field is only approximately a dipole field. Therefore, local N-S poles may exist oriented in different directions due to the different deposits of magnetised materials.

**Q.2 Answer the following questions :**

**(*a*) The earth's magnetic field varies from point to point in space. Does it also change with time? If so, on what time scale does it change appreciably?**

**(*b*) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?**

**(*c*) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (*i.e.*, the source of energy) to sustain these currents?**

**(*d*) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?**

**Dimensions of magnetic flux.** We know that :

$$\phi = BA\cos\theta \text{ and } B = \frac{F}{qv}$$

$$\therefore \quad \phi = \frac{F}{qv} A\cos\theta$$

$$\therefore \quad [\phi] = \frac{[MLT^{-2}][L^2]}{[AT][LT^{-1}]} = \left[ML^2 T^{-2} A^{-1}\right]$$

## 12.3. ELECTROMAGNETIC INDUCTION

It has been found experimentally that when the magnetic flux linking a conductor (or coil) changes, an e.m.f. is induced in the conductor. This phenomenon is called *electromagnetic induction.*

*Hence the phenomenon of production of e.m.f. in a conductor (or coil) when the magnetic flux linking the conductor changes is called* *__electromagnetic induction.__*

If the conductor (or coil) forms a closed circuit, a current will flow in it. This is called *induced current*. Two things must be noted about electromagnetic induction. First, the basic requirement for electromagnetic induction is the *change* in the magnetic flux **linking the conductor (or coil). Secondly, the e.m.f. and hence current in the conductor (or coil) will persist so long as this change is taking place.

To demonstrate the phenomenon of electromagnetic induction, consider a coil $C$ of several turns connected to a centre zero galvanometer as shown in Fig. 12.5. If a permanent magnet is moved towards the coil, it will be observed that the galvanometer shows deflection in one direction. If the magnet is moved away from the coil, the galvanometer again shows deflection but in the opposite direction. In either case, *the deflection will persist so long as the magnet is in motion*. The production of e.m.f. and hence current in the coil $C$ is due to the fact that when the magnet is in motion (towards or away from the coil), the amount of magnetic flux linking the coil changes – the basic requirement for inducing current in the coil. If the movement of the magnet is stopped, though the flux is linking the coil, there is no change in flux and hence no e.m.f. is induced in the coil. Consequently, the deflection of the galvanometer reduces to zero.

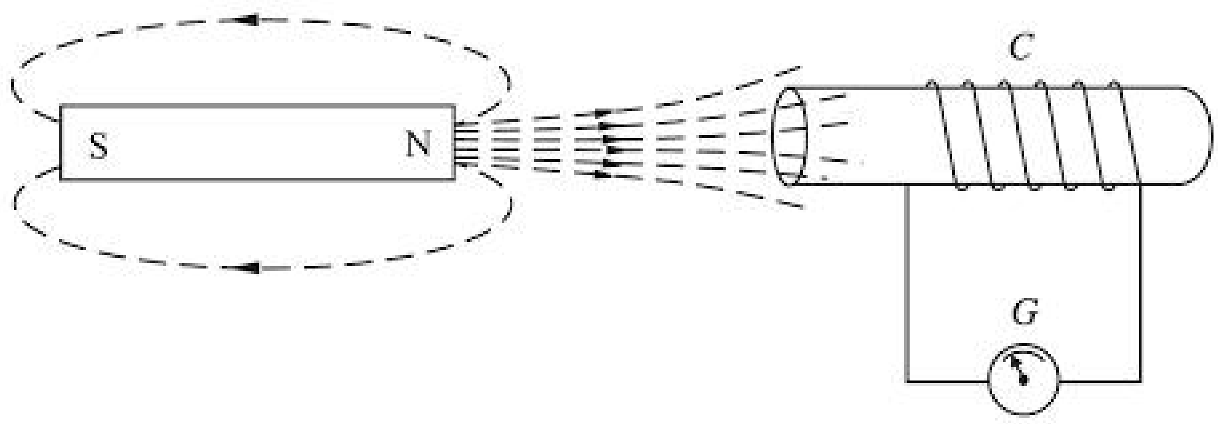



**Fig. 12.5**

It may be noted that basic requirement for inducing e.m.f. in a coil is not the magnetic flux linking the coil *but* the change in magnetic flux linking the coil. No change in magnetic flux, no e.m.f. is induced in the coil.

* So called because electricity is produced from magnetism (*i.e. electromagnetic*) and that there is no physical connection (*induction*) between the magnetic field and the conductor.

** Magnetic lines of force form closed loops. Flux linking the conductor means that flux embraces it *i.e.* it encircles the conductor.

*the direction of motion of the conductor, then the middle finger will point in the direction of induced current.*

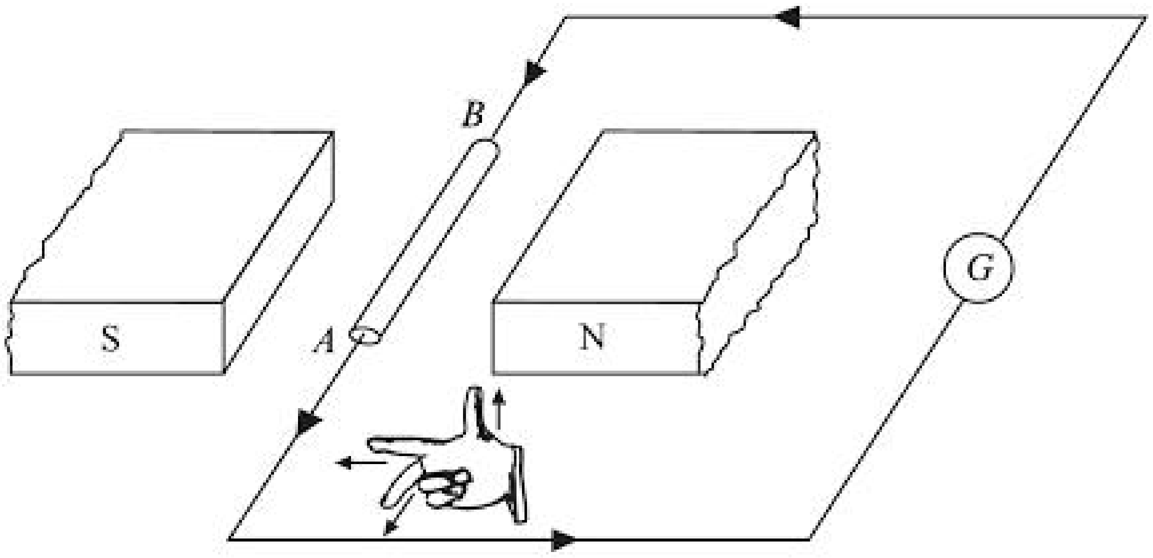



**Fig. 12.9**

Consider a conductor $AB$ moving upwards at right angles to a uniform magnetic field as shown in Fig. 12.9. Applying Fleming's right-hand rule, it is clear that the direction of induced current is from $B$ to $A$. If the motion of the conductor is downward, keeping the direction of magnetic field unchanged, then the direction of induced current will be from $A$ to $B$.

**Example 12.1. A coil of 100 turns is linked by a magnetic flux of 20 mWb. If this flux is reversed in a time of 2 ms, calculate the average e.m.f. induced in the coil.**

**Solution.** Initial magnetic flux linked with each turn, $\phi_1 = 20$ mWb

Final magnetic flux linked with each turn, $\phi_2 = -20$ mWb

Change in magnetic flux, $d\phi = \phi_2 - \phi_1 = -20 - (20)$

$= -40 \text{ mWb} = -40 \times 10^{-3}$ Wb

Time taken for the change, $dt = 2 \text{ ms} = 2 \times 10^{-3}$s

$$\therefore \quad \text{Induced e.m.f., } e = -N\frac{d\phi}{dt} = -100 \times \frac{-40 \times 10^{-3}}{2 \times 10^{-3}} = 2000 \text{ V}$$

**Example 12.2. A 10 cm diameter circular loop of wire is perpendicular to a uniform magnetic field of 0.35 T. It is removed from the field (moving perpendicular to the field) in 0.12 s. Calculate the average e.m.f. induced in the coil.**

**Solution.** Area of the loop, $A = \pi r^2 = \pi \times (5 \times 10^{-2})^2 = 25\pi \times 10^{-4}$ m$^2$

Initial flux linked with each turn, $\phi_1 = BA = (0.35) \times (25\pi \times 10^{-4})$ Wb

Final flux linked with each turn, $\phi_2 = 0$

$\therefore$ Change in magnetic flux, $d\phi = \phi_2 - \phi_1 = -(0.35) \times (25\pi \times 10^{-4})$ Wb

Time taken for the change, $dt = 0.12$ s

$$\therefore \quad \text{Induced e.m.f., } e = -N\frac{d\phi}{dt} = -1 \times \frac{-(0.35) \times (25\pi \times 10^{-4})}{0.12} = 0.02 \text{ V}$$

**Example 12.3. A square loop of side 10 cm and resistance 0.5 Ω is placed vertically in the east-west plane. A uniform magnetic field of 0.10 T is set up across the plane in the north-east direction. The magnetic field is decreased to zero in 0.70s at a steady rate. Determine the magnitudes of induced emf and current during this time-interval.**

**Solution.** Here, $A = 10 \times 10 = 100 \text{ cm}^2 = 10^{-2} \text{ m}^2$; $\theta = 45°$; $B_1 = 0.1$T;

$B_2 = 0$; $dt = 0.7$s; $R = 0.5\ \Omega$

Initial flux, $\phi_1 = B_1 A \cos\theta = 0.1 \times 10^{-2} \times \cos 45^\circ = \dfrac{10^{-3}}{\sqrt{2}}$ Wb

Final flux, $\phi_2 = B_2 A \cos\theta = 0 \times A \cos\theta = 0$

Change in flux $|d\phi| = |\phi_2 - \phi_1| = \dfrac{10^{-3}}{\sqrt{2}}$

$\therefore$ Magnitude of induced e.m.f. is

$$e = \frac{|d\phi|}{dt} = \frac{10^{-3}}{\sqrt{2} \times 0.7} = \mathbf{10^{-3}\ V}$$

$$\text{Current, } I = \frac{e}{R} = \frac{10^{-3}}{0.5} = \mathbf{2 \times 10^{-3}\ A}$$

**Example 12.4.** **At what rate would it be necessary for a single conductor to cut the flux in order that a current of 1.2 mA flows through it when 10 Ω resistor is connected across its ends?**

**Solution.** $e = N \dfrac{d\phi}{dt}$ ... in magnitude

Here $e = IR = 1.2 \times 10^{-3} \times 10 = 1.2 \times 10^{-2}$ V ; $N = 1$; $d\phi/dt = ?$

$$\therefore \quad \frac{d\phi}{dt} = \frac{e}{N} = \frac{1.2 \times 10^{-2}}{1} = \mathbf{1.2 \times 10^{-2}\ Wb/second}$$

**Example 12.5.** **A circular coil of radius 10 cm, 500 turns and resistance 2 Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. Estimate the magnitudes of the emf and current induced in the coil. Horizontal component of the earth's magnetic field at the place is $3.0 \times 10^{-5}$ T.**

**Solution.** Here, $r = 10$ cm $= 10^{-1}$ m; $B = 3.0 \times 10^{-5}$ T; $\theta_1 = 0^\circ$; $\theta_2 = 180^\circ$ ; $dt = 0.25$ s ; $N = 500$ ; $R = 2\ \Omega$

Initial flux through coil, $\phi_1 = BA\cos\theta_1 = 3.0 \times 10^{-5} \times \pi \times (10^{-1})^2 \times \cos 0^\circ = 3\pi \times 10^{-7}$ Wb

Final flux through coil, $\phi_2 = BA\cos\theta_2 = 3.0 \times 10^{-5} \times \pi \times (10^{-1})^2 \times \cos 180^\circ = -3\pi \times 10^{-7}$ Wb

Change in flux, $|d\phi| = |\phi_2 - \phi_1| = 6\pi \times 10^{-7}$ Wb

$\therefore$ Magnitude of e.m.f. induced in the coil is

$$e = N\frac{|d\phi|}{dt} = 500 \times \frac{6\pi \times 10^{-7}}{0.25} = \mathbf{3.8 \times 10^{-3}\ V}$$

$$\text{Current, } I = \frac{e}{R} = \frac{3.8 \times 10^{-3}}{2} = \mathbf{1.9 \times 10^{-3}\ A}$$

**Example 12.6.** **A fan blade of length 0.5 m rotates perpendicular to a magnetic field of $5 \times 10^{-5}$ T. If the e.m.f. induced between the centre and end of the blade is $10^{-2}$ V, find the rate of rotation of the blade.**

**Solution.** Let $n$ be the required number of rotations in one second. The magnitude of induced e.m.f. is given by ;

$$e = N\frac{d\phi}{dt} = N\frac{d}{dt}(BA) = B\frac{dA}{dt} \qquad [\because N = 1]$$

Here $dA$ is the area swept by the blade in one revolution and $dt$ is the time taken by the blade to complete one revolution.

Now, $e = 10^{-2}$ V; $B = 5 \times 10^{-5}$ T; $dA = \pi r^2 = \pi\,(0.5)^2 \text{m}^2$; $dt = \dfrac{1}{n}$

$$\therefore \quad 10^{-2} = 5\times 10^{-5} \times \frac{\pi \times (0.5)^2}{1/n}$$

$$\therefore \quad n = \frac{10^{-2}}{(5\times 10^{-5})\times \pi(0.5)^2} = \textbf{254.7 revolutions/second}$$

**Example 12.7. The magnetic flux passing perpendicular to the plane of the coil and directed into the paper (See Fig. 12.10) is varying according to the relation:**

$$\phi_B = 6t^2 + 7t + 1$$

**where $\phi_B$ is in mWb and t in seconds.**

**(*i*) What is the magnitude of induced e.m.f. in the loop when $t$ = 2 seconds?**

**(*ii*) What is the direction of current through the resistor *R*?**

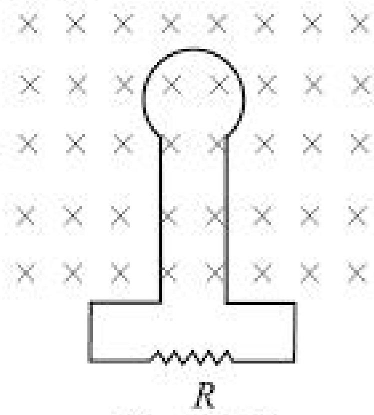



**Fig. 12.10**

**Solution.** $\phi_B = (6t^2 + 7t + 1)$ m Wb $= (6t^2 + 7t + 1) \times 10^{-3}$ Wb

(*i*) Magnitude of induced e.m.f. is

$$e = \frac{d\phi_B}{dt} = \frac{d}{dt}(6t^2 + 7t + 1)\times 10^{-3} = (12t + 7) \times 10^{-3} \text{ V}$$

At $t$ = 2 sec, $e = (12 \times 2 + 7) \times 10^{-3} = 31 \times 10^{-3}$ V = **31 mV**

(*ii*) According to Lenz's law, the direction of induced current will be such so as to oppose the change in flux. This means that direction of current in the loop will be such so as to produce magnetic field opposite to the given field. For this (*i.e.*, upward field), the current induced in the loop will be anticlockwise. Therefore, current in resistor *R* will be from left to right.

## PROBLEMS FOR PRACTICE

1. A square coil of side 5 cm contains 100 loops and is positioned perpendicular to a uniform magnetic field of 0.6 T. It is quickly removed from the field (moving perpendicular to the field) to a region where magnetic field is zero. It takes 0.1 s for the whole coil to reach field-free region. If resistance of the coil is 100 Ω, how much energy is dissipated in the coil ? **[$2.3 \times 10^{-3}$J]**

   [Hint: $e = N\dfrac{d\phi}{dt}$ ; $I = \dfrac{e}{R}$ ; Energy dissipated = $I^2R\,t$]

2. A flat search coil containing 50 turns each of area $2 \times 10^{-4}$ m$^2$ is connected to a galvanometer; the total resistance of the circuit is 100 Ω. The coil is placed so that its plane is normal to a magnetic field of flux density 0.25 T.

   (*i*) What is the change in magnetic flux linking the circuit when the coil is moved to a region of negligible magnetic field?

   (*ii*) What charge passes through the galvanometer? **[(*i*) $2.5 \times 10^{-3}$ Wb; (*ii*) 25 μC]**

3. The magnetic flux passing perpendicular to the plane of a coil and directed into the plane of the paper is varying according to the following equation :

   $$\phi = 5t^2 + 6t + 2$$

   where $\phi$ is in mWb and $t$ in seconds. Find the e.m.f. induced in the coil at $t$ = 1s. **[16 mV]**

4. A coil has an area of 0.04 m$^2$ and has 1000 turns. It is suspended in a magnetic field of $5 \times 10^{-5}$ Wb/m$^2$ perpendicular to the field. The coil is rotated through 90° in 0.2 s. Calculate the average e.m.f. induced in the coil due to rotation. **[0.01 V]**

   [Hint: When the coil is rotated through 90°, the magnetic flux linking it becomes zero.]

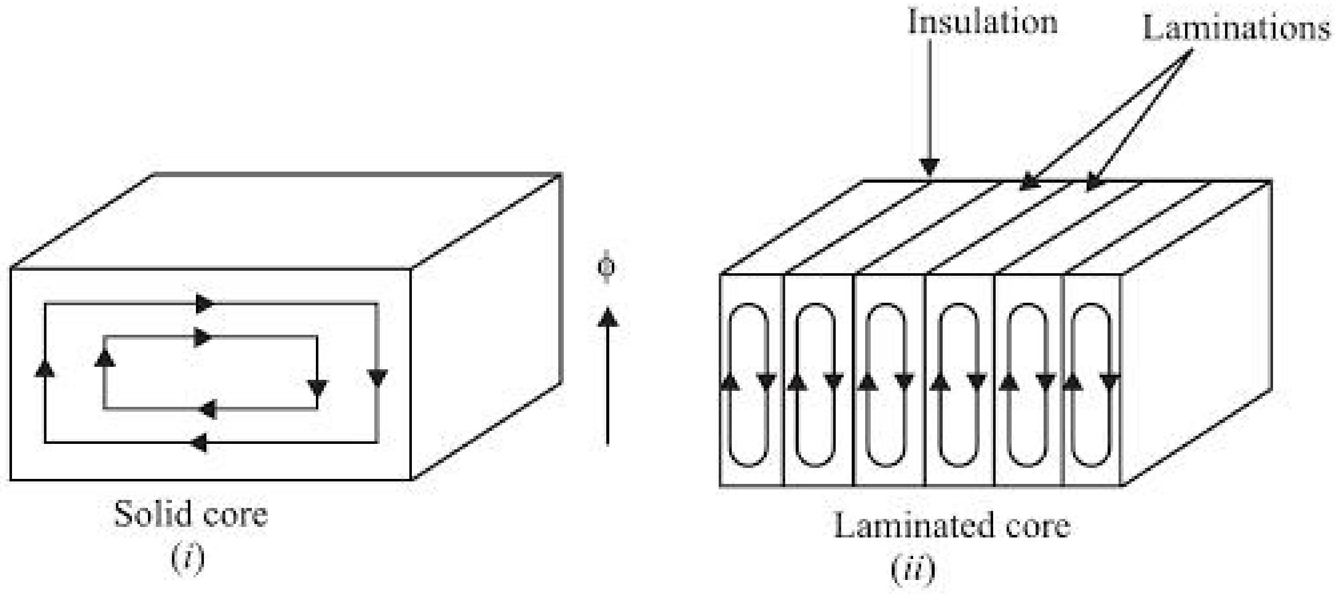



**Fig. 12.17**

To reduce the eddy currents, the cores are not taken as a single piece of iron [See Fig. 12.17(*i*)] but are split into thin sheets (called *laminations*) in planes parallel to magnetic field as shown in Fig. 12.17 (*ii*). Each lamination is insulated from the other by a layer of varnish. This arrangement reduces the area of each section and hence the induced e.m.f.($e$). It also increases the resistance ($R$) of eddy current paths since the area through which the currents can pass is smaller. Both these effects combine to reduce the magnitude of eddy currents. In electrical machines (motors, generators, transformers etc.), eddy currents are undesirable and must be kept to as low value as possible.

**Useful applications of eddy currents.** Eddy currents find many useful applications. Some of them are discussed below.

**(*i*) Eddy current damping.** In accordance with Lenz's law, eddy currents always flow in such a direction so as to oppose the motion which has produced them. Therefore, they can reduce the oscillations of a vibrating system.

A familiar example is that of a moving coil galvanometer. The coil of the galvanometer is generally wound on a metal frame. As the coil swings in the magnetic field of the instrument, eddy currents are induced in the frame. These eddy currents oppose the motion of the coil and hence the pointer attached to it. Consequently, the pointer quickly attains the final position without overshooting or oscillating violently. Thus eddy currents dampen (reduce) the oscillations of the pointer. This is known as *electromagnetic damping* or *eddy current damping*.

**(*ii*) Induction heating.** The heating effect of eddy currents can be used to heat/melt those substances which are conductors of electricity. The substance to be heated/melted is placed in a high-frequency magnetic field. The rapidly changing magnetic field induces large eddy currents. The heat thus produced melts the substance. This technique is known as induction heating and is being widely used to extract metals from their ores, preparation of certain alloys etc.

Another use of induction heating is in *diathermy*, where heat is applied to the human body for healing purposes. The high-frequency current is passed through a coil surrounding the affected part of the body. Since the body is a conductor (though a rather poor one), heat is produced deep within it, though the skin remains cool.

**(*iii*) Energy meters.** Eddy current braking is employed in energy meters. The aluminium disc of the energy meter rotates between the poles of two permanent horse shoe magnets. As the disc rotates and cuts across the magnetic fields of the magnets, eddy currents are produced in the disc. These eddy currents oppose the motion of the disc. As a result of this braking effect, the speed of the disc is directly proportional to the energy consumed.

**(*iv*) Electromagnetic Brakes.** Eddy current braking can be used to control the speed of electric trains. In order to reduce the speed of the train, an electromagnet is turned on that applies its field to the wheels. Large eddy currents are set up which produce the retarding effect.

## 12.10. SELF INDUCTION

When current in a coil increases or decreases, there is a change in the magnetic flux linking the coil. Hence an e.m.f. is induced in the coil (called *self induced e.m.f.* $e_s$) which opposes the change of current in the coil (Lenz's law). This phenomenon is called *self induction.*

*The phenomenon of production of opposing e.m.f.* ($e_s$) *in a coil when current through the coil changes is called* **self induction.** The self induced e.m.f. opposes the change of current in the coil.

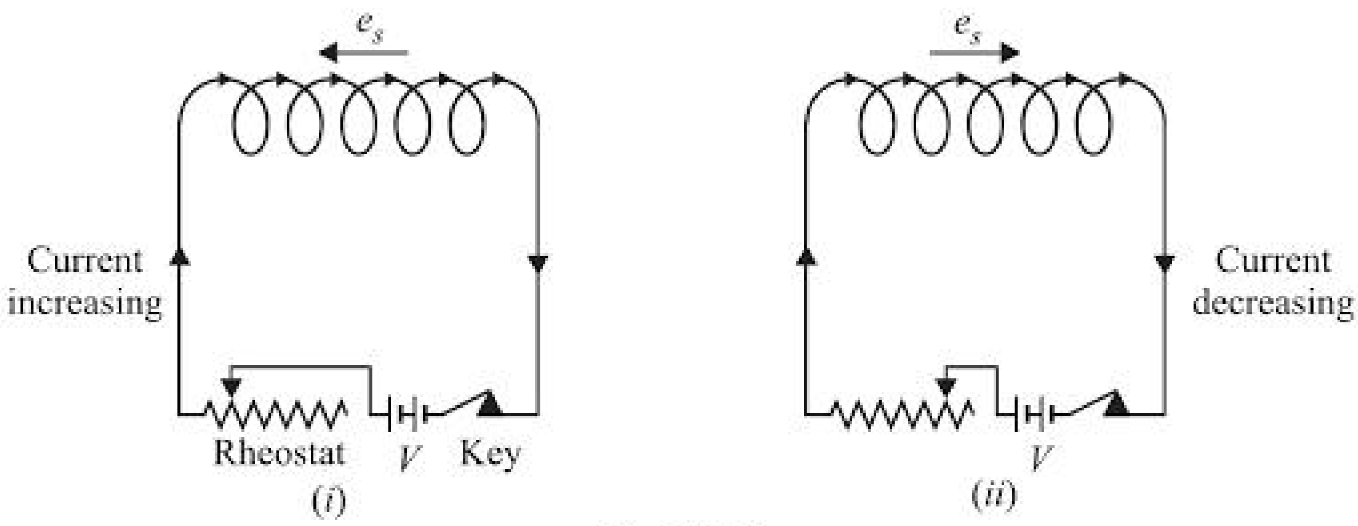



**Fig.12.21**

Thus when the current in the coil is increasing as shown in Fig. 12.21 (*i*), the self induced e.m.f. ($e_s$) is produced in the coil in such a direction so as to *oppose* the increase in current *i.e.*, the self induced e.m.f. acts in a direction opposite to that of the applied voltage $V$.

Similarly, when the current in the coil is decreasing as shown in Fig. 12.21.(*ii*), the self induced e.m.f. ($e_s$) is produced in the coil in such a direction so as to *oppose* the decrease in current *i.e.*, self induced e.m.f. acts in the direction of applied voltage $V$.

## 12.11. COEFFICIENT OF SELF INDUCTION (or SELF INDUCTANCE)

*The property of a coil (or circuit) by virtue of which it opposes any change in the amount of current flowing through it is called* **self inductance** *of the coil.* It is denoted by the symbol $L$.

Thus self inductance of a coil opposes the change of current (increase or decrease) through the coil. This opposition occurs because a changing current produces self induced e.m.f. ($e_s$) which opposes the change of current. That is why self inductance of a coil is called *electrical inertia* of the coil.

**Expressions for self inductance ($L$).** We now find two equivalent expressions for self inductance $L$ of a coil (or circuit).

(*i*) Consider a coil of $N$ turns carrying a current $I$. Suppose the magnetic flux linked with *each turn* of coil due to this current is $\phi$. Then the flux linkages with the coil will be $N\phi$.

It is found that : $N\phi \propto I$

or $$N\phi = LI \qquad ...(i)$$

where $L$ is a constant of proportionality and is called **coefficient of self induction** or **self inductance** of the coil.

Now, $$L = \frac{N\phi}{I} \qquad ...(ii)$$

*Hence* **coefficient of self induction** *of a coil is equal to the number of flux linkages with the coil when unit current is flowing through the coil.*

(*ii*) If changing current through a coil of $N$ turns produces self induced e.m.f. $e_s$, then,

$$e_s = -N\frac{d\phi}{dt} = -\frac{d}{dt}(N\phi)$$

From eq. (*i*), $$N\phi = LI$$

$\therefore$ $$e_s = -\frac{d}{dt}(LI) = -L\frac{dI}{dt}$$

or $$e_s = -L\frac{dI}{dt} \quad ...(iii)$$

where $dI/dt$ is the rate of change of current in the coil. The negative sign shows that self induced e.m.f. ($e_s$) is always in such a direction so as to oppose the change of current in the coil.

From eq. (*iii*), $$L = -\frac{e_s}{dI/dt}$$

If $dI/dt = 1$, then, $L = e_s$ (numerically).

*Hence* **coefficient of self induction** *of a coil is numerically equal to the self induced e.m.f. in the coil when the rate of change of current in the coil is unity.*

Either eq. (*ii*) or eq. (*iii*) can be used to find the inductance of a coil (or circuit) depending upon the given date. The value of self inductance ($L$) of a coil depends upon: (*a*) Dimensions of the coil, (*b*) number of turns of the coil and (*c*) nature of core material of the coil.

**Units of self inductance.** The SI unit of self inductance ($L$) is henry (H).

Now, $$e_s = L\frac{dI}{dt} \quad \text{...magnitude}$$

If $$\frac{dI}{dt} = 1\text{A/s} \quad \text{and} \quad e = 1\text{V, then, } L = 1\text{H.}$$

*Hence a coil (or circuit) has self inductance of* **1 henry** (H) *if current changing at the rate of 1 ampere per second through the coil induces an e.m.f. of 1V in it.*

Self inductance is a **scalar** quantity.

The smaller units of $L$ are millihenry (mH) and microhenry (μH).

$$1\text{ mH} = 10^{-3}\text{ H} \; ; \; 1\text{ μH} = 10^{-6}\text{ H}$$

**Dimensions of $L$.** From eq. (*iii*), magnitude of $e_s$ is

$$e_s = L\frac{dI}{dt}$$

$\therefore$ $$L = e_s \times \frac{dt}{dI}$$

Now $$e_s = \frac{\text{Work}}{\text{Charge}} = \frac{W}{q} = \frac{\text{ML}^2\text{ T}^{-2}}{\text{AT}}$$

$\therefore$ $$[L] = \left[\frac{\text{ML}^2\text{ T}^{-2}}{\text{AT}}\right]\left[\frac{\text{T}}{\text{A}}\right] = \left[\text{ML}^2\text{ T}^{-2}\text{ A}^{-2}\right]$$

## 12.12. INDUCTANCE OF A LONG SOLENOID

Consider a long air-cored solenoid of length $l$, area of cross-section $A$ and having total number of turns $N$. For a long solenoid, the magnetic field inside is constant. If the solenoid is carrying a current $I$, then magnetic field inside the solenoid is given by ;

$$B = \mu_0 n I = \frac{\mu_0 N I}{l} \quad \text{where} \quad n = \frac{N}{l}$$

Magnetic flux linked with each turn of solenoid is

$$\phi = B \times \text{Area of each turn} = \left(\frac{\mu_0 N I}{l}\right) \times A = \frac{\mu_0 N I A}{l}$$

$\therefore$ Inductance of solenoid, $L = \dfrac{N\phi}{I} = \dfrac{N}{I}\left(\dfrac{\mu_0 N I A}{l}\right) = \dfrac{\mu_0 N^2 A}{l}$

$$\therefore \qquad L = \frac{\mu_0 N^2 A}{l}$$

Note that this is the expression for air-cored solenoid. If the solenoid carries a core of relative permeability $\mu_r$, then,

$$L = \frac{\mu_0 \mu_r N^2 A}{l}$$

Thus inductance of a solenoid (or coil) is directly proportional to the square of the number of turns on the solenoid.

## 12.13. ENERGY STORED IN AN *INDUCTOR

Consider a source of e.m.f. connected to an inductor. As the current increases from zero, an e.m.f. is induced in the inductor. This e.m.f. opposes the growth of current (Lenz's law). Therefore, electrical energy must be supplied by the source of e.m.f. in setting up current in the inductor against this induced e.m.f. This supplied energy is stored in the magnetic field of the inductor.

Suppose at any instant the current in the inductor of inductance $L$ is $I$ and is increasing at the rate of $dI/dt$. Then magnitude of e.m.f. induced in the inductor is given by ;

$$e = L\frac{dI}{dt}$$

If the source of e.m.f. sends current $I$ through the inductor for a small time $dt$, then small amount of work done by the source is given by ;

$$dW = e\,I\,dt = LI\frac{dI}{dt}\cdot dt = LI\,dI$$

The total work done to increase the current from zero to the final value $I_0$ is given by ;

$$W = \int_0^{I_0} dW = \int_0^{I_0} LI\,dI = L\left|\frac{I^2}{2}\right|_0^{I_0}$$

or $$W = \frac{1}{2}LI_0^2$$

This work done is equal to the energy $U$ stored in the inductor.

$\therefore$ Energy stored in inductor, $U = \dfrac{1}{2}LI_0^2$ ...(*i*)

Eq. (*i*) gives the expression for the energy stored in an inductor when current through it increases from zero to the final value $I_0$.

The energy stored in the inductor will be in joules if inductance ($L$) and current ($I_0$) are in henry and amperes respectively. The following points may be noted :

(*i*) The energy stored in the inductor is supplied by the source of e.m.f.

(*ii*) The energy in an inductor can be considered to be stored in its magnetic field.

* A coil whose inductance is very large and resistance is negligible is called an **inductor** or **choke coil.**

*(iii)* When current in an inductor is constant (say $I_0$), the e.m.f. induced in $L$ is zero. However, the energy stored in the inductor is (1/2) $LI_0^2$.

Example 12.14. **When a direct current of 2.5 A is passed through a coil of 500 turns, the magnetic flux produced is 1.4 × 10⁻⁴ Wb. What is the inductance of the coil?**

**Solution.** Inductance of the coil, $L = \dfrac{N\phi}{I}$

Here, $N = 500$ ; $\phi = 1.4 \times 10^{-4}$ Wb ; $I = 2.5$ A

$$\therefore \quad L = \frac{(500)\times(1.4\times10^{-4})}{2.5} = 28 \times 10^{-3} \text{ H} = \textbf{28 mH}$$

Example 12.15. **If current in a 130 mH coil changes steadily from 20 mA to 28 mA in 140 ms, find the magnitude and direction of induced e.m.f.**

**Solution.** Inductance of the coil, $L = 130 \text{ mH} = 130 \times 10^{-3}$ H

Initial current in the coil = 20 mA

Final current in the coil = 28 mA

$\therefore$ Change in current, $dI = 28 - 20 = 8 \text{ mA} = 8 \times 10^{-3}$ A

Time taken for the change, $dt = 140 \text{ ms} = 140 \times 10^{-3}$ s

$$\text{Induced e.m.f.} = -L\frac{dI}{dt} = -130\times10^{-3}\times\frac{8\times10^{-3}}{140\times10^{-3}}$$

$$= \mathbf{-7.43 \times 10^{-3}\ V}$$

The direction of induced e.m.f. will be such so as to oppose the increase in current.

Example 12.16. **An air-cored solenoid having a diameter of 4 cm and a length of 60 cm is wound with 4000 turns. Find the inductance of the solenoid. What will be the inductance of the solenoid if it has an iron core of relative permeability 4000?**

**Solution.** Inductance of solenoid, $L = \dfrac{\mu_0 N^2 A}{l}$

Here, $\mu_0 = 4\pi \times 10^{-7}$ H/m ; $N = 4000$ ; $l = 0.6$ m ; A $= \left(\dfrac{\pi}{4}\right)\left(16\times10^{-4}\right) = 4\pi \times 10^{-4}$ m²

$$\therefore \quad L = \frac{(4\pi\times10^{-7})\times(4000)^2\times4\pi\times10^{-4}}{0.6} = \textbf{0.042 H}$$

$$L_{iron} = \mu_r L = 4000 \times 0.042 = \textbf{168 H}$$

Note that inductance of the solenoid has increased enormously (*i.e.* 4000 times) with the introduction of iron core.

Example 12.17. **Find the change in current in an inductor of 10H in which the e.m.f. induced is 300 V in 10⁻² second. Also find the change in magnetic flux.**

**Solution.** Here, $L = 10$ H; $e = 300$ V; $dt = 10^{-2}$s

Now, $$e = L\frac{dI}{dt}$$

$$\therefore \quad dI = \frac{e\times dt}{L} = \frac{300\times10^{-2}}{10} = \textbf{0.3 A}$$

Also $$e = \frac{d\phi}{dt}$$

$$\therefore \quad d\phi = e \times dt = 300 \times 10^{-2} = \textbf{3 Wb}$$

**Example 12.18. The inductance of a coil having 200 turns is 10 mH. Calculate the total flux linked with coil corresponding to a current of 4 mA. Also find the magnetic flux through the cross-section of the coil.**

**Solution.** The total flux linked with the coil is $N\phi$ where $\phi$ is the flux linked with each turn of the coil.

Now, $$N\phi = LI = (10 \times 10^{-3}) \times (4 \times 10^{-3}) = 4 \times 10^{-5} \text{ Wb}$$

Magnetic flux through the cross-section of the coil (*i.e.* through each turn of the coil) is

$$\phi = \frac{4\times10^{-5}}{N} = \frac{4\times10^{-5}}{200} = 2 \times 10^{-7} \text{ Wb}$$

**Example 12.19. A current of 20 mA is passing through a coil of inductance 500 mH. Find the magnetic energy stored in the coil. If the current is halved, find the new value of energy.**

**Solution.** For the first case, $U_1 = \frac{1}{2}L\,I_1^2 = \frac{1}{2}\,(500 \times 10^{-3}) \times (20 \times 10^{-3})^2 = 100 \times 10^{-6}$ J

For the second case, $U_2 = \frac{1}{2}L\,I_2^2 = \frac{1}{2}\,(500 \times 10^{-3}) \times (10 \times 10^{-3})^2 = 25 \times 10^{-6}$ J

**Example 12.20. A battery of e.m.f. 12 V is suddenly connected in series with a 30 Ω resistor and 220 mH inductor. When current in the circuit reaches half its maximum possible value, at what rate is (*i*) energy being delivered by the battery, (*ii*) energy being stored in the magnetic field of the inductor?**

**Solution.** Maximum current, $I_0 = E/R = 12/30 = 0.4$ A

Current at given instant, $I = I_0/2 = 0.4/2 = 0.2$ A

(*i*) Power being supplied by battery is

$$P = E\,I = 12 \times 0.2 = 2.4 \text{ W}$$

(*ii*) The energy stored in inductor ($L$) at any instant is given by ;

$$U = \frac{1}{2}LI^2$$

where $I$ is the current in the inductor at that instant.

∴ Rate at which energy is stored in magnetic field is

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2\right) = LI\frac{dI}{dt}$$

Rate of change of current at that instant is

$$\frac{dI}{dt} = \frac{E - IR}{L} = \frac{12 - 0.2\times30}{0.22} = \frac{6}{0.22}\ \text{As}^{-1}$$

$$\therefore \quad \frac{dU}{dt} = 0.22\times0.2\times\frac{6}{0.22} = 1.2 \text{ W}$$

**Example 12.21. The current in a coil of self inductance 2.0 H is increasing according to $i = 2\sin t^2$ amp. Find the amount of energy spent during the period when the current changes from 0 to 2A.**

**Solution.** Here, $L = 2.0$ H ; $i = 2 \sin t^2$

Increase in current, $i_0 = 2 - 0 = 2$ A

∴ Energy stored in the inductor is given by ;

$$U = \frac{1}{2}Li_0^2 = \frac{1}{2}\times2.0\times(2)^2 = 4\text{J}$$

Note that energy spent is 4J because energy stored in the inductor is equal to the energy spent by the source.

## PROBLEMS FOR PRACTICE

1. The self inductance of a coil is 3 mH and an electric current of 5 A is flowing through it. On switching off, the current reduces from 5 A to zero in 0.1 second. Calculate the average self induced e.m.f. in the coil. **[0.15 V]**
2. What is the inductance of a coil if it produces an e.m.f. of 6.5 V when current in it changes from –12 mA to +23 mA in 11 ms? **[2.04 H]**
3. A 35 V e.m.f. is induced in a 0.32 H coil by a current that rises uniformly from zero to $I_0$ in 2 ms. What is the value of $I_0$? **[0.22 A]**
4. A coil of inductance 0.5 H is connected to a 12 V battery. Calculate the rate of growth of current. **[24 As$^{-1}$]**

   [**Hint:** $e = L\dfrac{dI}{dt}$ or $\dfrac{dI}{dt} = \dfrac{e}{L} = \dfrac{12}{0.5} = 24\ \text{As}^{-1}$]
5. An air-cored solenoid having a diameter of 4 cm and a length of 50 cm is wound with 2000 turns. Calculate the inductance of the solenoid. **[12.62 mH]**
6. An average e.m.f. of 25 V is induced in an inductor when the current in it is changed from 2.5 A in one direction to the same value in the opposite direction in 0.1 sec. Find the self inductance of the inductor. **[0.5 H]**
7. The current in a solenoid of 240 turns, having a length of 12 cm and a radius of 2 cm, changes at the rate of 0.8 As$^{-1}$. Find the e.m.f. induced in it. **[$6 \times 10^{-4}$ V]**
8. A 5 H inductor carries a steady current of 2 A. How can a 50 V self induced e.m.f. be made to appear in the inductor? **[By decreasing current from 2 A to 0 A in 0.2s]**
9. A long solenoid of 10 turn cm$^{-1}$ has a small loop of area 1 cm$^2$ placed inside with the normal of the loop parallel to the axis. Calculate the voltage across the small loop, if the current in the solenoid is changed at a steady rate from 1 A to 2 A in 0.1s during the duration of the charge. **[1.257 μV]**
10. Calculate the inductance and energy stored in the magnetic field of an air-cored solenoid 50 cm long, 5 cm in diameter and wound with 1000 turns, if carrying a current of 5 A. **[0.005 H; 0.0625 J]**
11. The field winding of a machine consists of 8 coils in series, each containing 1200 turns. When the current is 3 A, flux linked with each coil is 20 mWb. Calculate (*i*) the inductance of the circuit (*ii*) the energy stored in the circuit (*iii*) the average value of induced e.m.f. if the circuit is broken in 0.1 s. **[(*i*) 64 H; (*ii*) 288 J; (*iii*) 1920 V]**

## 12.14. MUTUAL INDUCTION

Consider two coils $A$ and $B$ placed near each other as shown in Fig. 12.22(*i*). If current $I_1$, in coil $A$ changes, then changing magnetic flux links with coil $B$ and e.m.f. is induced in it. The e.m.f. induced in coil $B$ is called *mutually induced e.m.f.* ($e_M$). Similarly, when current in coil $B$ changes [See Fig. 12.22.(*ii*)], mutually induced e.m.f. ($e_M$) is produced in coil $A$. According to Lenz's law, the mutually induced e.m.f. will act in such a direction so as to oppose the cause producing it. Now the cause producing mutually induced e.m.f. in one coil is the change of current in the other coil. Therefore, $e_M$ will oppose the current in the coil that produces it. Thus in Fig. 12.22(*i*), $e_M$ in coil $B$ will send induced current in such a direction so as to oppose the

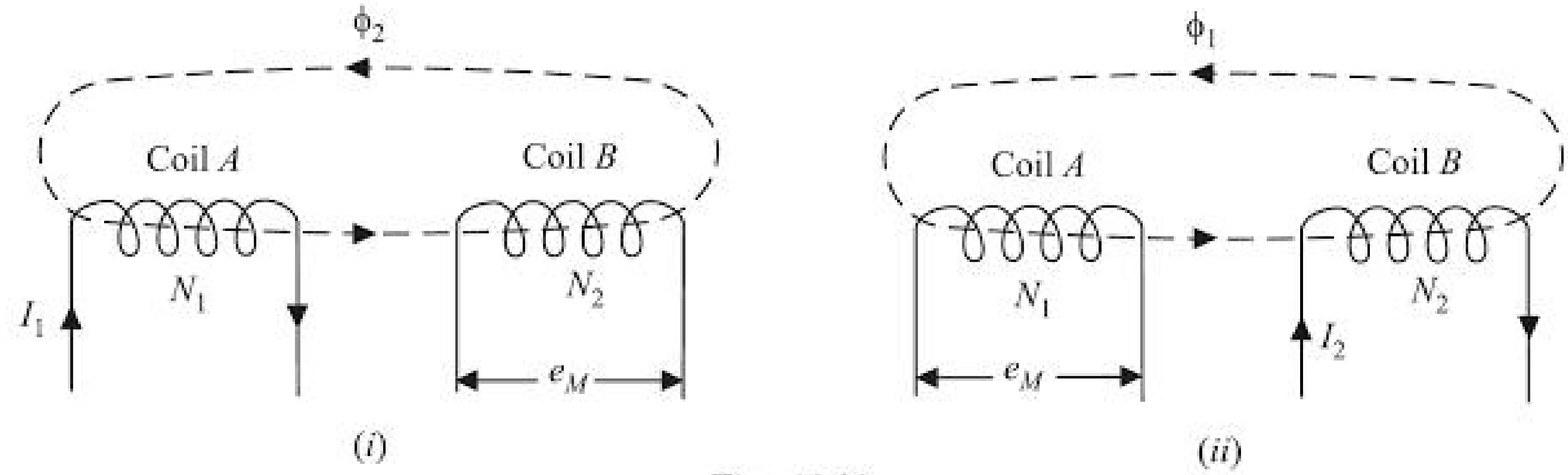



**Fig. 12.22**

change of current $I_1$ in coil $A$. Similarly, in Fig. 12.22(*ii*), $e_M$ in coil $A$ will oppose the change of current $I_2$ in coil $B$. This behaviour of two neighbouring coils is called *mutual induction*.

*Thus the property of two neighbouring coils by virtue of which each opposes any change of current flowing in the other by developing mutually induced e.m.f. is called* **mutual induction.**

(*i*) The mutually induced e.m.f. persists in a coil so long as the current in the other coil is changing. If the current in the coil becomes steady, the mutual flux also becomes constant and mutually induced e.m.f. drops to zero.

(*ii*) *The mutual induction between two coils depends upon (a) the size and shape of the two coils (b) their relative orientation (c) separation between the coils and (d) material of the core on which they are wound.*

## 12.15. COEFFICIENT OF MUTUAL INDUCTION (or MUTUAL INDUCTANCE)

We now find two equivalent expressions for mutual inductance of two coils.

(*i*) Consider two coils $A$ and $B$ of turns $N_1$ and $N_2$ respectively placed near each other. Suppose current flowing through coil $A$ is $I_1$. Let due to this current, the magnetic flux linked with *each turn* of coil $B$ be $\phi_2$. Then the flux linkages with the coil $B$ will be $N_2\,\phi_2$. It is found that :

$$N_2\,\phi_2 \propto I_1$$

or
$$N_2\phi_2 = MI_1 \qquad ...(i)$$

where $M$ is a constant of proportionality and is called **coefficient of mutual induction** or **mutual inductance** of the two coils.

Now
$$M = \frac{N_2\,\phi_2}{I_1} \qquad ...(ii)$$

If $I_1 = 1$, then, $M = N_2\,\phi_2$.

*Hence* **coefficient of mutual induction** *of two coils is equal to the number of flux linkages with one coil when a unit current flows in the other coil.*

(*ii*) If a changing current in coil $A$ produces mutually induced e.m.f. $e_M$ in coil $B$, then,

$$e_M = -N_2\frac{d\phi_2}{dt} = -\frac{d}{dt}(N_2\,\phi_2)$$

From eq. (*i*), $N_2\,\phi_2 = MI_1$

$$\therefore \quad e_M = -\frac{d}{dt}(MI_1) = -M\frac{dI_1}{dt}$$

or
$$e_M = -M\frac{dI_1}{dt} \qquad ...(iii)$$

where $dI_1/dt$ is the rate of change of current in coil $A$. The negative sign shows that direction of $e_M$ in coil $B$ is always such that it opposes any change of current in coil $A$.

From *eq*. (*iii*), $M = -\dfrac{e_M}{dI_1/dt}$

If $\dfrac{dI_1}{dt} = 1$ then, $M = e_M =$ (numerically).

*Hence* **coefficient of mutual induction** *of two coils is numerically equal to the mutually induced e.m.f. in one coil when the rate of change of current in the other coil is unity.*

**Units of mutual inductance.** The SI unit of mutual inductance is henry (H).

Now, $$e_M = M\frac{dI_1}{dt} \quad \text{...magnitude}$$

If $\dfrac{dI_1}{dt} = 1$ A/s and $e_M = 1$V, then, $M = 1$H.

*Hence mutual inductance between two coils is* **1 henry** *if current changing at the rate of 1 ampere per second in one coil induces an e.m.f. of 1V in the other coil.*

The dimensions of mutual inductance are the same as those of inductance.

## 12.16. COEFFICIENT OF COUPLING ($k$)

*The coefficient of coupling (k) between two coils is defined as the fraction of magnetic flux produced by the current in one coil that links the other coil.*

The coefficient of coupling has a maximum value of 1 (or 100%) when the entire magnetic flux of one coil links the other. If only one half the magnetic flux set up in one coil links the other, then coefficient of coupling ($k$) is 0.5 or 50%. It is clear that mutual inductance between two coils depends upon the coefficient of coupling. It can be proved that mutual inductance $M$ between two coils of inductances $L_1$ and $L_2$ [See Fig. 12.23] is given by ;

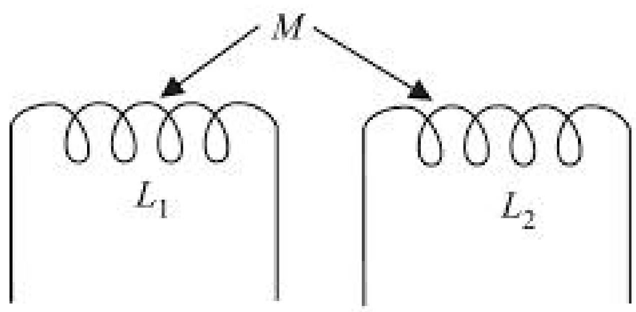



**Fig. 12.23**

$$M = k\sqrt{L_1 L_2}$$

Obviously, mutual inductance between the coils will be maximum when $k = 1$.

(*i*) If two coils $P$ and $S$ are wound on a soft-iron core as shown in Fig. 12.24 (*i*), the coefficient of coupling is maximum. Therefore, the mutual inductance between the coils is maximum.

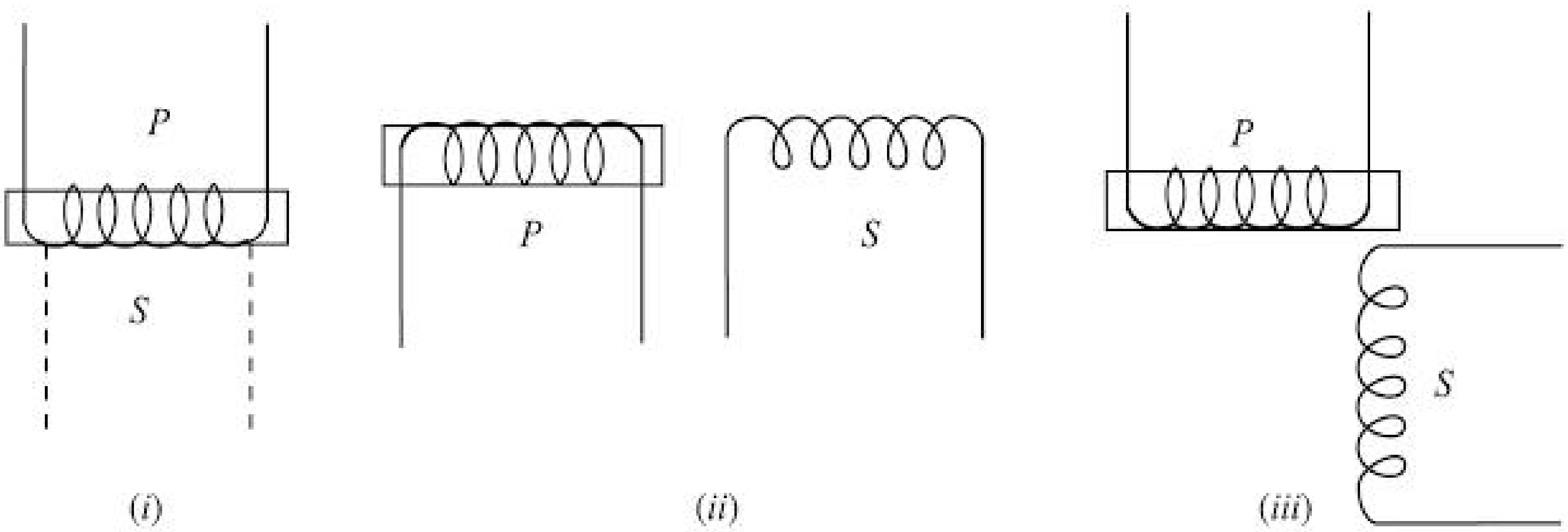



**Fig. 12.24**

(*ii*) If the two coils are placed so as to have a common axis as shown in Fig. 12.24 (*ii*), the coefficient of coupling is less than that of the above case. Therefore, the mutual inductance between the coils is large but not maximum.