

MS 6222: APPLIED MATHEMATICS III

LECTURE 1

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INTRODUCTION

To be covered;

- Conversion of radians and degrees and vice-versa
- Arc length
- Area of a sector
- Trigonometrical ratios
- Trigonometric Identities

CONVERSION OF RADIAN AND DEGREES AND VICE-VERSA

From $180^\circ \approx \pi$ rad

$$\bullet \theta_{\text{radian}} = \frac{\pi}{180^\circ} \theta^\circ$$

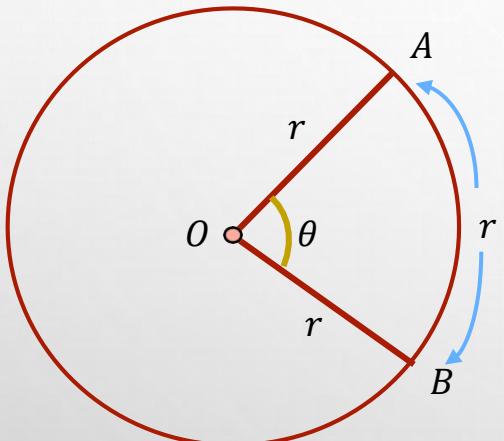
$$\bullet \theta_{\text{degree}} = \frac{180^\circ}{\pi} \theta_{\text{rad}}$$

CONVERT DEGREES TO RADIANS AND VICE-VERSA

$$180^\circ = \pi \text{ radians}$$

WHAT IS A RADIAN ?

- ❖ If the arc AB has a length r , then $\angle AOB$ is 1 radian (1° or 1 rad.)



$$2\pi \text{ radians} = 360^\circ$$

$$\text{so } \pi \text{ radians} = 180^\circ$$

It follows that $1 \text{ rad} = 57.295 \dots \dots \dots ^\circ$

$$\diamond \quad 1 \text{ radian} = \frac{180^\circ}{\pi}$$

- ❖ A radian is the angle subtended at the centre of a circle by an arc whose length is equal to that of the radius of the circle.

EXAMPLE 1

Convert $\frac{5\pi}{9}$ rad into degrees

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Solution $\frac{5\pi}{9}$ rad = $\frac{5}{9} \times 180^0 = 100^0$

EXAMPLE 2

Convert 135^0 into radians

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Convert 135^0 into radians

Solution $135^0 = 135 \times \frac{\pi}{180} \text{ rad} = \frac{9}{10} \pi \text{ rad}$

CONVERSION OF RADIANS AND DEGREES AND VICE-VERSA

ACTIVITY 1:

Convert 270° into radians.

CONVERSION OF RADIANS AND DEGREES AND VICE-VERSA

ACTIVITY 1:

Convert 270° into radians.

solution:

$$\theta_{\text{radian}} = \frac{\pi}{180^\circ} \theta^\circ$$

$$\theta_{\text{radian}} = \frac{\pi}{180^\circ} \times 270^\circ$$

$$\theta_{\text{radian}} = \frac{3\pi}{2} \text{ rad.}$$

CONVERSION OF RADIANS AND DEGREES AND VICE-VERSA

ACTIVITY 2:

Convert 0.785 radians into degrees.

CONVERSION OF RADIANS AND DEGREES AND VICE-VERSA

ACTIVITY 2:

Convert 0.785 radians into degrees.

$$\theta_{\text{degree}} = \frac{180^\circ}{\pi} \theta_{\text{rad}}$$

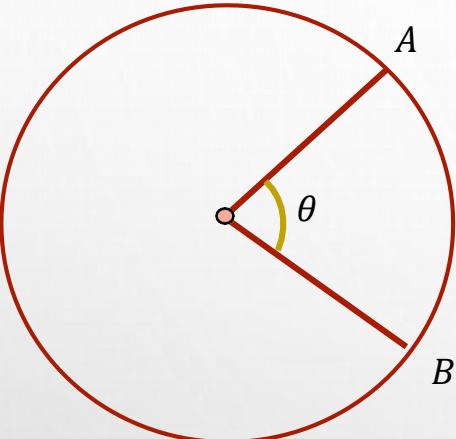
$$\theta_{\text{degree}} = \frac{180^\circ}{\pi} \times 0.785$$

$$\theta_{\text{degree}} = 44.98^\circ$$

ARC LENGTH

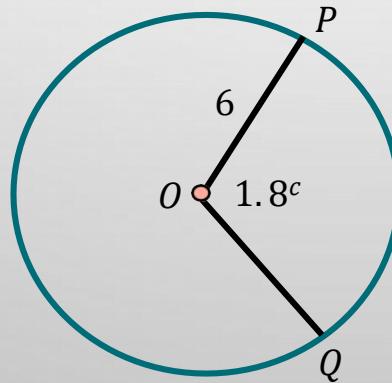
- The arc length, $S = \frac{\pi r}{180^\circ} \theta_{degree}$
- The arc length, $s = r\theta_{radian}$

CALCULATE ARC LENGTHS USING RADIANS



Example 3

Find the arc length of the minor arc PQ of a circle where the radius is 6 cm and $\angle PQR = 1.8$ rad



$$\frac{AB}{\text{circumference}} = \frac{\theta}{2\pi}$$

$$\frac{AB}{2\pi} = \frac{\theta}{2}$$

$$AB = \frac{\theta}{2\pi} \times 2\pi r = r\theta$$

$$\boxed{\text{Arc length} = r\theta}$$

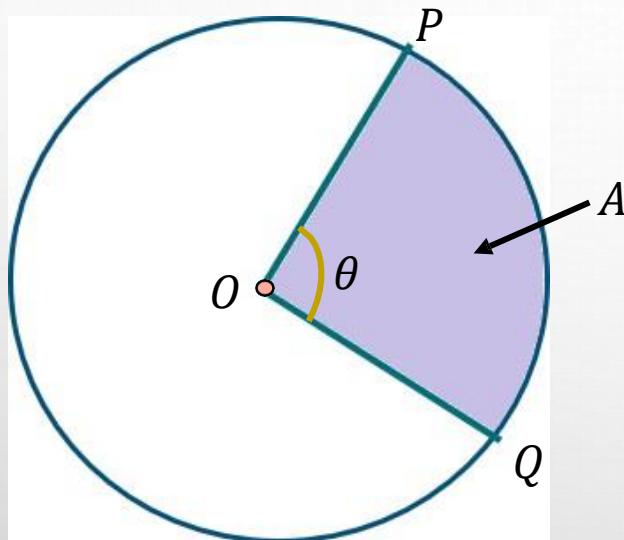
Solution

$$\begin{aligned}\text{Arc length} &= r\theta \\ &= 6 \times 1.8 \\ &= 10.8 \text{ cm}\end{aligned}$$

AREA OF A SECTOR

- Area of a sector, $A = \frac{\pi r^2}{360^\circ} \theta_{\text{degree}}$
- Area of a sector, $A = \frac{r^2}{2} \theta_{\text{radian}}$

CALCULATE SECTOR AREA USING RADIANS



$$\frac{\text{area of sector } POQ}{\text{area of circle}} = \frac{\theta}{2\pi}$$

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

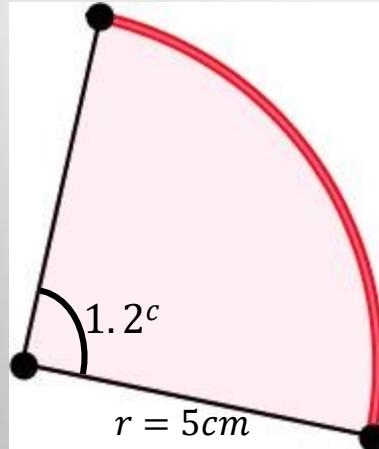
$$A = \frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2} r^2 \theta$$

$\text{Area of a sector} = \frac{1}{2} r^2 \theta$

EXAMPLE 4

Find:

- The area and
- The perimeter of the sector of the circle shown in the diagram below



Solution

$$\begin{aligned}\text{a) area} &= \frac{1}{2}r^2\theta \\ &= \frac{1}{2} \times 5^2 \times 1.2 \\ &= 15\text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{b) perimeter} &= r\theta + r + r \\ &= 5 \times 1.2 + 5 + 5 \\ &= 16\text{ cm}\end{aligned}$$

AREA OF A SECTOR

ACTIVITY 3:

An automatic garden spray produces a spray to a distance of 6.4 m and revolves through an angle α which may be varied. If the desired spray catchment area is to 25 m^2 , to what should angle α be set? Provide your answer to nearest degrees.

AREA OF A SECTOR

ACTIVITY 3:

Solution:

Given data

Distance, $r=6.4\text{m}$

Angle, α to be found

Area covered, $a = 25\text{m}^2$

AREA OF A SECTOR

ACTIVITY 3:

From; area of a sector = $\frac{\pi r^2 \theta}{360^\circ}$

$$25m^2 = \frac{\pi \times 6.4^2 \alpha}{360^\circ}$$

$$\alpha = \frac{25 \times 360}{6.4^2 \times \pi} = 69.9411371$$

$$\alpha = 70^\circ$$

Therefore, the angle α should be set at 70°

TRIGONOMETRIC RATIOS

Trigonometric ratios and their inverses (reciprocals) include;

- Sine,
- Cosine,
- Tangent,
- Cosecant,
- Secant and
- Cotagent.

TRIGONOMETRIC RATIOS

In relation to right-angle triangle these ratios are defined by SOHTOACAH

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

TRIGONOMETRIC RATIOS FOR ACUTE ANGLES

- Acute angle are all angles less than right angle
- Acute angles include special angles such as,
 $0^\circ, 30^\circ, 45^\circ, 60^\circ$, and 90° .
- Trigonometrical ratios for special angles can determined without using a calculator

TRIGONOMETRIC RATIOS

ACTIVITY 4:

To evaluate $\tan(\cos^{-1} 1/2)$ without using a calculator.

TRIGONOMETRIC RATIOS

ACTIVITY 4:

To evaluate $\tan(\cos^{-1} 1/2)$ without using a calculator.

Solution:

$$\text{Let } \theta = \cos^{-1} 1/2$$

$$\tan(\cos^{-1} 1/2) = \tan \theta$$

$$\text{From } \theta = \cos^{-1}(1/2), \quad \theta = 60^\circ$$

$$\text{From trigonometric ratios; } \tan 60^\circ = \sqrt{3}$$

$$\text{Therefore, } \tan(\cos^{-1} 1/2) = \sqrt{3}$$

TRIGONOMETRIC RATIOS

ACTIVITY 5:

➤ Solve for x, if $\tan(\cos^{-1} x) = \sin(\tan^{-1} 4)$

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Solution

$$\tan(\cos^{-1} x) = \sin(\tan^{-1} 4)$$

$$\tan(\cos^{-1} x) = \sin(75.96375653)$$

$$\tan(\cos^{-1} x) = \sin(75.96^\circ)$$

$$\tan(\cos^{-1} x) = 0.9701425$$

TRIGONOMETRIC RATIOS

ACTIVITY 5:

➤ Solve for x , if $\tan(\cos^{-1} x) = \sin(\tan^{-1} 4)$

Solution

$$\cos^{-1} x = \tan^{-1}(0.97)$$

$$\cos^{-1} x = 44.13174921$$

$$x = \cos(44.13^\circ)$$

$$x = 0.717740562 = 0.7177$$

Therefore, the value of $x = 0.7177$.

PYTHAGORAS THEOREM

Pythagoras theorem states that, “**in a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides** “.

➤ Mathematical formula

$$c^2 = a^2 + b^2$$

TRIGONOMETRIC IDENTITIES

Consider Pythagoras theorem in relation to trigonometric ratios;

$$1. \sin^2 \theta + \cos^2 \theta = 1$$

$$2. 1 + \tan^2 \theta = \sec^2 \theta$$

$$3. 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

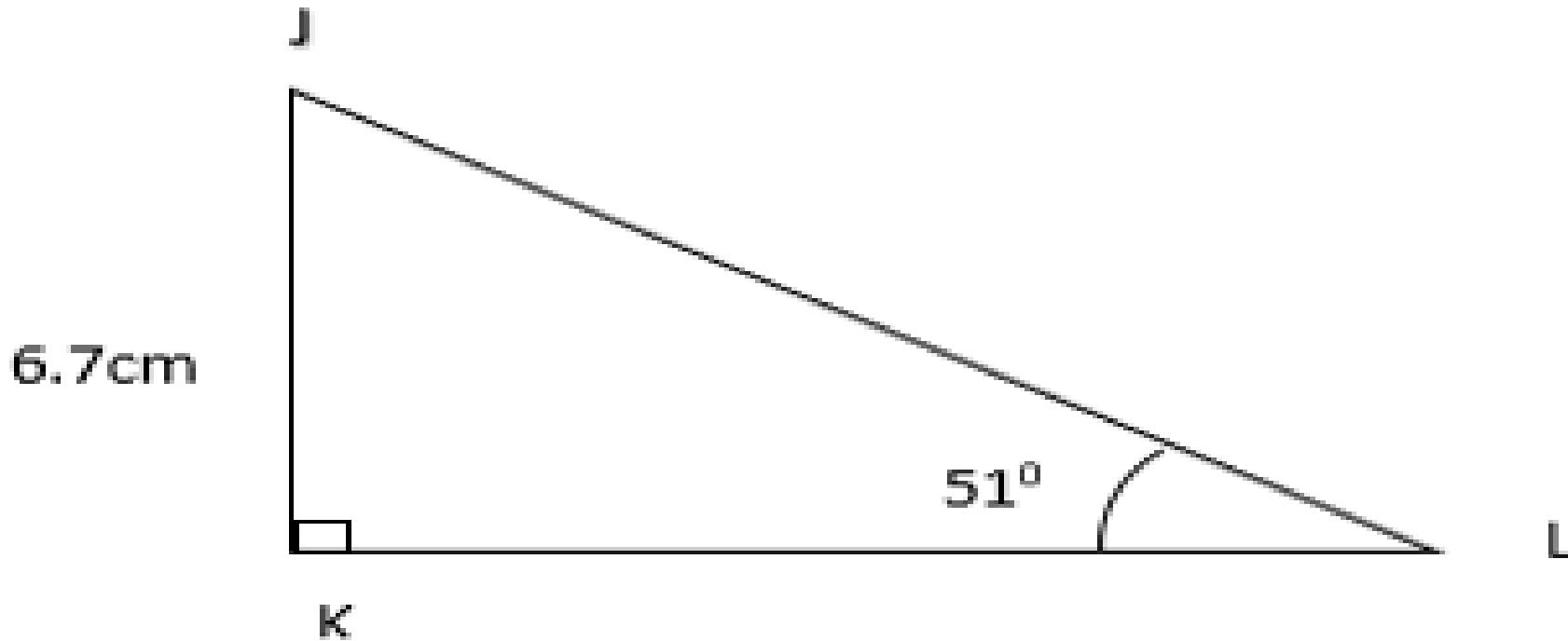
SOLUTIONS OF RIGHT-ANGLED TRIANGLE

In solving right-angled triangle, Pythagoras theorem and trigonometrical ratios are useful in determining and unknown sides and angles of right-angled triangles.

SOLUTIONS OF RIGHT-ANGLED TRIANGLE

Example:

Solve the triangle JKL in the figure below and find its area.



SOLUTION:

To solve the triangle JKL and to find its area

$$\text{Angle, } J = 180^\circ - 90^\circ - 51^\circ \text{ or } 90^\circ - 51^\circ = 39^\circ$$

$$\angle J = 39^\circ$$

For sides, JL and KL trigonometrical ratios and pythagoras theorem can be used.

$$\tan 51^\circ = \frac{6.7\text{cm}}{\text{kl}}, \text{KL} = \frac{6.7\text{cm}}{\tan 51^\circ} = 5.425553022$$

The length of kl is 5.4 cm.

SOLUTION:

Applying Pythagoras theorem to get magnitude of side JL

$$\overline{JL}^2 = 6.7^2 + 5.4^2$$

$$\overline{JL}^2 = 74.05$$

$$\overline{JL} = 8.605230967$$

$$\overline{JL} = 8.6 \text{ cm.}$$

SOLUTION:

$$\text{Area of triangle JKL} = \frac{1}{2}bh$$

$$\text{Area of triangle JKL} = \frac{1}{2} \times 5.4\text{cm} \times 6.7\text{cm}$$

$$\text{Area of triangle JKL} = 18.09\text{cm}^2$$

The area of triangle JKL is 18.09cm^2

SOLVING NON-RIGHT ANGLED TRIANGLE

SINE RULE, COSINE RULE AND TANGENT RULE

In solving non-right angled triangles, sine rule, cosine rule and tangent rule can be used to evaluate unknown sides and angles.

SOLVING NON-RIGHT ANGLED TRIANGLE

SINE RULE

$$\rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

SOLVING NON-RIGHT ANGLED TRIANGLE COSINE RULE

$$\square a^2 = b^2 + c^2 - 2bc\cos A$$

$$\square b^2 = a^2 + c^2 - 2ac\cos B$$

$$\square c^2 = a^2 + b^2 - 2ab\cos C$$

SOLVING NON-RIGHT ANGLED TRIANGLE

Tangent rule

$$\frac{a-b}{a+b} = \frac{\tan \frac{A-B}{2}}{\tan \frac{A+B}{2}}$$

$$\frac{b-c}{b+c} = \frac{\tan \frac{B-C}{2}}{\tan \frac{B+C}{2}}$$

$$\frac{c-a}{c+a} = \frac{\tan \frac{C-A}{2}}{\tan \frac{C+A}{2}}$$

SOLVING NON-RIGHT ANGLED TRIANGLE

EXAMPLE 1:

Solve the triangle XYZ, given that $\angle X = 65^\circ$, $\angle Y = 60^\circ$, $YZ = 7.6\text{cm}$ and then find its area

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Solve the triangle XYZ, given that $\angle X = 65^\circ$, $\angle Y = 60^\circ$, $YZ = 7.6\text{cm}$ and then find its area.

SOLUTION

- $\angle Z = 180^\circ - 65^\circ - 60^\circ$
- $\angle Z = 55^\circ$

SOLVING NON-RIGHT ANGLED TRIANGLE

EXAMPLE 1:

Solve the triangle XYZ, given that $\angle X = 65^\circ$, $\angle Y = 60^\circ$, $YZ = 7.6\text{cm}$ and then find its area.

SOLUTION

FROM SINE RULE: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$\frac{\sin 65^\circ}{7.6} = \frac{\sin 60^\circ}{XZ} = \frac{\sin 55^\circ}{XY}$$

$$XZ = \frac{\sin 60^\circ}{\sin 65^\circ} \times 7.6 \text{ CM} = 7.262205130 = 7.3 \text{ CM}$$

SOLVING NON-RIGHT ANGLED TRIANGLE

EXAMPLE 1:

Solve the triangle XYZ, given that $\angle X = 65^\circ$, $\angle Y = 60^\circ$ $YZ = 7.6\text{cm}$ and then find its area.

SOLUTION

$$\overline{XZ} = \frac{\sin 60^\circ}{\sin 65^\circ} \times 7.6 \text{ CM} = 7.262205130 = 7.3 \text{ CM}$$

$$\overline{XY} = \frac{\sin 55^\circ}{\sin 60^\circ} \times 7.3\text{CM} = 6.904889738$$

$$\overline{XY} = 6.9 \text{ cm}$$

SOLVING NON-RIGHT ANGLED TRIANGLE

EXAMPLE 1:

Solve the triangle XYZ, given that $\angle X = 65^\circ$, $\angle Y = 60^\circ$ $YZ = 7.6\text{cm}$ and then find its area.

SOLUTION

$$\text{area of a triangle, } XYZ = \frac{1}{2} \times \overline{XZ} \times \overline{XY} \sin 65^\circ$$

$$\text{area of a triangle, } XYZ = \frac{1}{2} \times 7.3 \times 6.9 \sin 65^\circ$$

$$\text{area of a triangle, } XYZ = 22.82536162 \text{ cm}^2 = 23 \text{ cm}^2.$$

Therefore, the area of a triangle, $XYZ = 23 \text{ cm}^2$

SOLVING NON-RIGHT ANGLED TRIANGLE

Example 2: A computer room of 16 m width has a span roof that slopes at 36° on one side and 44° . Determine the lengths of roof.

SOLVING NON-RIGHT ANGLED TRIANGLE

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Solution:

Recalling sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$\frac{\sin 44^\circ}{L_1} = \frac{\sin 36^\circ}{L_2} = \frac{\sin 100^\circ}{16m}$$

$$L_1 = \frac{\sin 44^\circ}{\sin 100^\circ} \times 16m = 11.28599353 \text{ m} = 11.3 \text{ m}$$

$$L_2 = \frac{\sin 36^\circ}{\sin 100^\circ} \times 16m = 9.549644596 \text{ m} = 9.5 \text{ m}$$

Therefore, the lengths of the roof are 11.3m at 36° and 9.5m at 44° .

SOLVING NON-RIGHT ANGLED TRIANGLE

Example 3: find the value of an angle A in a triangle ABC which is such that, $a = 4$, $b = 7$ and $c = 4$.

SOLVING NON-RIGHT ANGLED TRIANGLE

Example 3: find the value of an angle A in a triangle ABC which is such that, a = 4, b = 7 and c = 4.

SOLUTION:

RECALL COSINE RULE: $a^2 = b^2 + c^2 - 2bc \cos A$

- $6^2 = 7^2 + 4^2 - 2 \times 7 \times 4 \cos A$

- $36 = 49 + 16 - 56 \cos A$

- $-29 = -56 \cos A$

- $\cos A = \frac{29}{56}$

- $A = \cos^{-1}\left(\frac{29}{56}\right)$

- $A = 58.81137767 = 58.8^\circ$

ASSIGNMENT

Derive the following;

1. Sine rules
2. Cosine rules
3. Tangent rules

Thank you for your participation

The end