Lecture 3

Thermodynamic Processes on Gases

- Types of thermodynamic processes – isochoric, isobaric, isothermal, isentropic, polytropic and throttling processes, equations representing the Processes, Derivation of work done, change in internal energy, change in entropy, rate of heat transfer for the above processes

Types of thermodynamic processes

- Whenever one or more of the properties of a system change, a change in the state of the system occurs.
- The path of the succession of states through which the system passes is called the **thermodynamic process**.

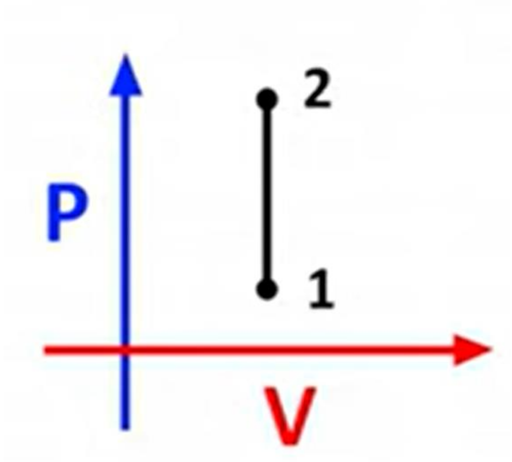
Types of Thermodynamic Processes

- **Isochoric process**
- **Isobaric process**
- **Isothermal process**
- **Isentropic process**
- **Polytropic process** - the plot of Log P vs. Log V is a straight line, PV^n = constant
- **Throttling process** - a process in which enthalpy is constant, $h_1 = h_2$, work= 0, and which is adiabatic, Q = 0

Isochoric Process

In isochoric process the change in volume of thermodynamic system is zero. A volume change is zero, so the work done is zero.

- Volume of the system = Constant
- Change in volume = 0
- If, change in volume = 0, then work done is zero.



- 1. work done

$$W = \int_1^2 pdV$$

dv =0 (Constant volume)

$$W = 0$$

- 2.Change in internal Energy

$$\Delta U = mC_v(T_2 - T_1)$$

- 3.Heat transfer $Q = \Delta U + W$ (according to First law)

$$Q = \Delta U$$

$$Q = mC_v(T_2 - T_1)$$

Isochoric Process

4. Change in enthalpy

$$\Delta H = mC_p(T_2 - T_1)$$

5.PVT relations

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\boxed{\frac{T_2}{T_1} = \frac{P_2}{P_1}}$$

Example 1

2 kg of a gas at 50°C is heated at constant volume until the pressure is doubled. Determine;
(1)Final temperature
(2) Change in internal energy
(3) Change in enthalpy
Take c_p=1.005 kJ/kgK & γ=1.69 for the gas.

Solution:

Data Given;

Mass = m= 2kg

T_1= 50°C = 323K

$V_1 = V_2$

$p_2 = 2p_1$

c_p=1.005 kJ/kgK

$\gamma=1.69$

To find;

(1) T_2= ?

(2) dU= ?

(3) dH= ?

From p,V & T relationship,

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right) = 323 \left(\frac{2}{1}\right) = 646K = 373°C$$

Data Given;

Mass = m= 2kg

$T_1 = 50°C = 323K$

$V_1 = V_2$

$p_2 = 2p_1$

$c_p = 1.005$ kJ/kgK

$\gamma = 1.69$

Step 2

Change in internal energy,

$$dU = mc_v(T_2 - T_1)$$

$$= 2 \text{ x } 0.5947(373 - 50)$$

$$= \underline{384.176 \text{ kJ}}$$

Where…

$$c_v = c_p/\gamma$$

$$= 1.005/1.69$$

$$= 0.5947 \text{kJ/kgK}$$

Step 3

Change in enthalpy,

$$dH = mc_p(T_2 - T_1)$$

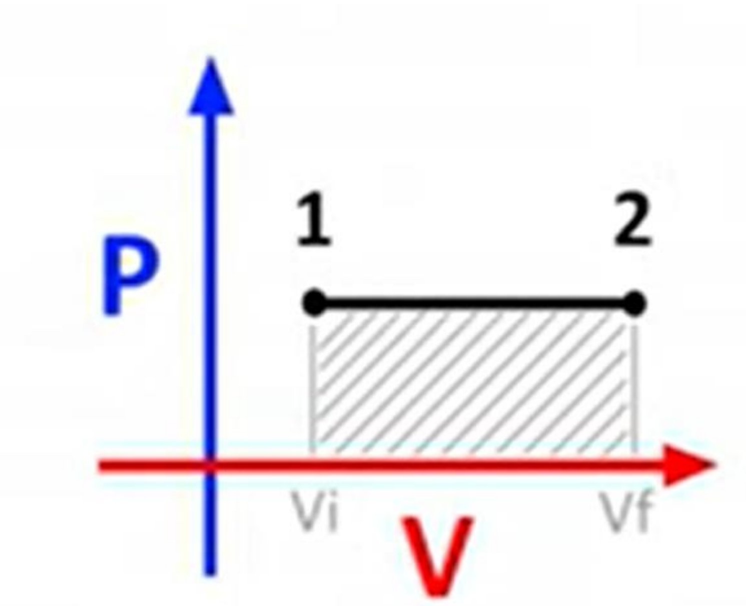
$$= 2 \text{ x } 1.005(373 - 50) = \underline{649.23 \text{ kJ}}$$

Isobaric process

- The pressure remains constant during this process. So,

  $$W=P(V_2-V_1)$$

- So, if volume increases, work done is positive, else negative.



- 1. work done

$$W=\int_1^2 pdV$$

$$W=p\int_1^2 dV$$

$$\boxed{W=p(V_2-V_1)}$$

$$W=p_2V_2-p_1V_1$$

$$W=mRT_2-mRT_1$$

$$W=mR(T_2-T_1)$$

- 2.Change in internal Energy

$$\Delta U= mC_v(T_2-T_1)$$

Isobaric process

3.Heat transfer (according to First law)

$$Q = \Delta U + W$$

$$Q = mC_v(T_2 - T_1) + mR(T2 - T1)$$

$$Q = m(Cv + R)(T2 - T1)$$

$$Q = mC_p(T2 - T1)$$

4. Change in enthalpy

$$\Delta H = mC_p(T_2 - T_1)$$

5.PVT relations

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{T_2}{T_1} = \frac{V_2}{V_1}$$

Example 2

- Find the value of cp & y for a gas whose constant is 0.324 kJkgK & c is 0.84 kJ/kgK. If2.25 kg of this gas with an initial volume of 1.15 m? undergoes a process during which its pressure remains constant at 7 bar whereas temp. becomes 280^{o}C at the end of the process. Calculate;(1)Change in int. energy(2)Change in enthalpy(3)Transferred heat and(4)Work done

Data Given:

R= 0.324 kJ/kgK

c_v= 0.84 kJ/kgK

m = 2.25kg

V_1= 1.15m^3

p_1= p_2= 7 bar

T_2= 280°C= 553K

To find;

dU = ?

dH = ?

δQ = ?

δW= ?

Step 1 :

For Initial temp. of the gas,

$p_1V_1 = mRT_1$

$$T_1 = \frac{p_1V_1}{mR}$$

$$= \frac{7 \text{ x } 10^5 \text{ x } 1.15}{2.25 \text{ x } 324}$$

$T_1 = 1104.25K = 831.25°C$

Step 2 :

$c_p = R + c_v \quad = 0.324 + 0.84 \quad = 1.164$ kJ/kgK

and,

$$\gamma = \frac{c_p}{c_v} \quad = \frac{1.164}{0.84} \quad = 1.3857 \text{ kJ/kgK}$$

Step 3 :

Change in internal energy,

$dU = mc_v(T_2 - T_1)$

$= 2.25 \times 0.84(280 - 831.25)$

$= \underline{-1041.867 \text{ kJ}}$

Thus, internal energy decreases.

Step 4 :

Change in enthalpy,

$dH = mc_p(T_2 - T_1)$

$= 2.25 \times 1.164(280 - 831.25)$

$= \underline{-1443.73 \text{kJ}}$

Step 5 :

Heat transferred is $\delta Q = dH = \underline{-1443.73} \text{ kJ}$

Thus, Heat is rejected.

Step 6 :

Work done on the gas,

$\delta Q = \delta W + dU$

Thus,

$\delta W = \delta Q - dU$

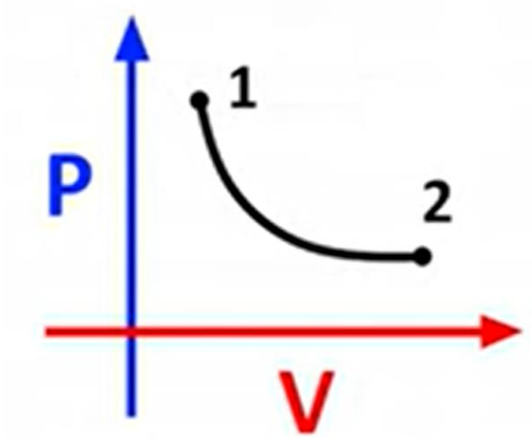
$= -1443.73 - (-1041.867)$

$\delta W = \underline{-401.87 \text{ kJ}}$

Negative signed means Work energy is supplied to the system

Isothermal process

- It is a thermodynamic process in which temperature remains constant.
- This process is accomplished by keeping the system in thermal equilibrium with a large heat bath during the process.



- 1. work done

$$W = \int_1^2 p dV$$

$$pV = C$$

$$p = \frac{C}{V}$$

$$W = \int_1^2 \frac{C}{V} dV$$

$$= C\int_1^2 \frac{1}{V} dV$$

$$= C(\ln V)_1^2$$

$$= C(\ln V_2 - V_1)$$

$$\boxed{W = pV(\ln \frac{V_2}{V_1})}$$

$$\boxed{= p_1 V_1 \ln(\frac{p_1}{P_2})}$$

Isothermal process

2.Change in internal Energy

$$\Delta U = mC_v(T_2 - T_1) \qquad \Delta U = 0 \qquad (T_1 = T_2)$$

3.Heat transfer (according to First law)

$$Q = \overset{0}{\cancel{\Delta U}} + W$$

$$O = W$$

4. Change in enthalpy

$$\Delta H = mC_p(T_2 - T_1) \qquad \Delta H = 0$$

5.PVT relations

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$P_1 V_1 = P_2 V_2$$

$$\frac{P_2}{P_1} = \frac{V_1}{V_2}$$

Example 3

- A quantity of gas is expanded isothermally from initial condition of 0.1m³ and 735 kPa to a final pressure of 118 kPa. Find:(1)Final volume(2)Work done(3)Heat transferred

Data Given:

V_1=0.1 m^3

T_1=T_2

p_1=735 kPa= 7.35 bar

p_2=118 kPa=1.18 bar

To find:

V_2 = ?

δW = ?

δQ = ?

Step 1

To find V_2,

$$p_1V_1=p_2V_2$$

$$7.35 \text{ x } 0.1=1.18 \text{ x } V2$$

$$V_2= \underline{0.6229 \text{ m}^3}$$

Step 2

For Isothermal process,

$$W.D. = p_1V_1[\ \log_e V_2/ V_1]$$

$$W.D. = 7.35 \text{ x } 100 \text{ x } 0.1 [\ \log_e (0.6229/0.1)]$$

$$\therefore\ W.D.=\underline{134.445 \text{ kJ}}$$

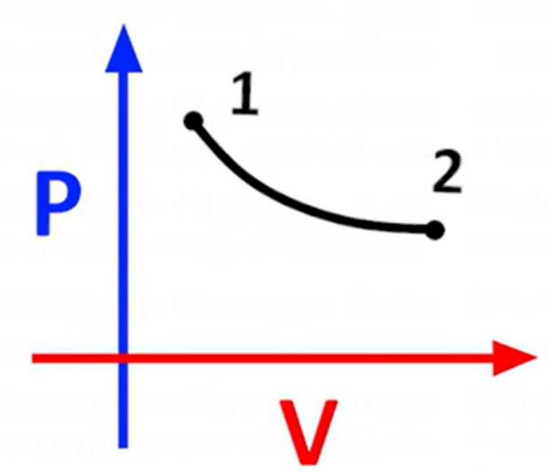
Step 3

For Isothermal process,

$$\delta Q = \delta W = \underline{134.445 \text{ kJ}}$$

Isentropic process

- In an **isentropic/adiabatic process**, the system is insulated from its environment so that although the state of the system changes, no heat is allowed to enter or leave the system.
- A process in which the entropy of the fluid remains constant.
- Will occur if the process the system goes through is reversible and adiabatic.
- Can also be called a constant entropy process



- 1. work done

$$W = \int_1^2 p dV$$

$$pV^\gamma = C$$

$$p = \frac{C}{V^\gamma}$$

$$W = \int_1^2 \frac{C}{V^\gamma} dV$$

$$= C\int_1^2 \frac{1}{V^\gamma} dV$$

$$= C\left(\frac{V^{-\gamma+1}}{-\gamma+1}\right)_1^2$$

$$= C\left(\frac{V_2^{-\gamma+1}}{-\gamma+1} - \frac{V_1^{-\gamma+1}}{-\gamma+1}\right) = C\left(\frac{V_2^{-\gamma+1} - V_1^{-\gamma+1}}{-\gamma+1}\right)$$

Isentropic process

$$W = pV^{\gamma}(\frac{V_2^{-\gamma+1} - V_1^{-\gamma+1}}{-\gamma+1}) = (\frac{p_2 V_2^{\gamma} V_2^{-\gamma+1} - p_1 V_1^{\gamma} V_1^{-\gamma+1}}{-\gamma+1})$$

$$W = (\frac{p_2 V_2 - p_1 V_1}{-\gamma+1})$$

$$W = (\frac{p_1 V_1 - p_2 V_2}{\gamma-1})$$

$$W = (\frac{mRT_1 - mRT_2}{\gamma-1})$$

$$W = \frac{mR(T_1 - T_2)}{\gamma-1}$$

$$W = mC_v(T_1 - T_2)$$

Isentropic process

2.Change in internal Energy

$$\Delta U = mC_v(T_2 - T_1)$$

3.Heat transfer (according to First law)

$$Q = \Delta U + W$$

$$\Delta U = -W$$

4. Change in enthalpy

$$\Delta H = mC_p(T_2 - T_1)$$

5.PVT relations

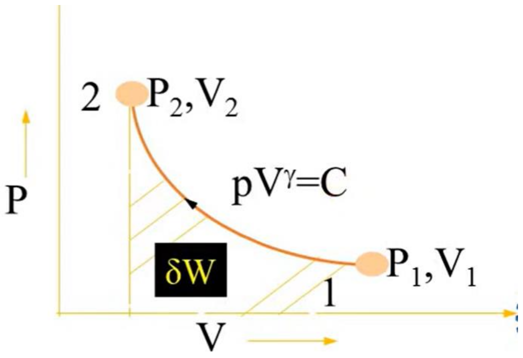
$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\boxed{\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}} \qquad \boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}} \qquad \boxed{\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}}$$

Example 4

A perfect gas is compressed adiabatically from a state of 1.93 pa, volume 0.1 m? and temperature -4^{o}C to a pressure of 5.84 MPa. Find;(1)Temp.& Volume at the end of Compression(2)Change in enthalpy(3)Work transfer with direction(4)Heat transfer(5)Change in internal energy

Take, γ=1.4&c_p=1.005 kJ/kg K.



Data Given:

p_1= 1.93 MPa = 19.3 bar
V_1= 0.1m^3
T_1= – 4^{0}C= 269K
p_2= 5.84 MPa = 58.4 bar
γ=1.4
c_p=1.005 kJ/kg K

To find:

T_2 = ?
V_2 = ?
dh =?
δW = ?
δQ = ?
dU = ?

Step 1:

For final Volume & Temp.,

$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{1/\gamma}$$

$$V_2 = 0.1\left(\frac{19.3}{58.4}\right)^{1/1.4}$$

$$V_2 = \underline{0.04535\ m^3}$$

Also,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}$$

$$= 269\left(\frac{58.4}{19.3}\right)^{1.4-1/1.4}$$

$$T_2 = \underline{369.09\ K}$$

Step 2:

$$p_1V_1 = mRT_1$$

$$m = \frac{p_1V_1}{RT_1}$$

$$R = c_p(\gamma - 1)/\gamma$$

$$= 0.287\ kJ/kgK$$

$$m = \frac{19.3 \times 100 \times 0.1}{0.287 \times 269}$$

$$m = 2.5\ kg$$

Step 3:For change in enthalpy,

$$dH = mc_p(T_2 - T_1)$$

$$= 2.5 \times 1.005 \times (369.09 - 269)$$

$$dH = \underline{251.48\ kJ}$$

Step 4:

$$\delta W = \frac{p_1V_1 - p_2V_2}{\gamma - 1}$$

$$= \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{2.5 \times 0.287(269 - 369.09)}{1.4 - 1}$$

$$\delta W = \underline{-179.536\ kJ}$$

Step 5: For change in Internal energy,

$$dU = mc_v(T_2 - T_1)$$

$$= 2.5 \times (1.005/1.4) \times (369.09 - 269)$$

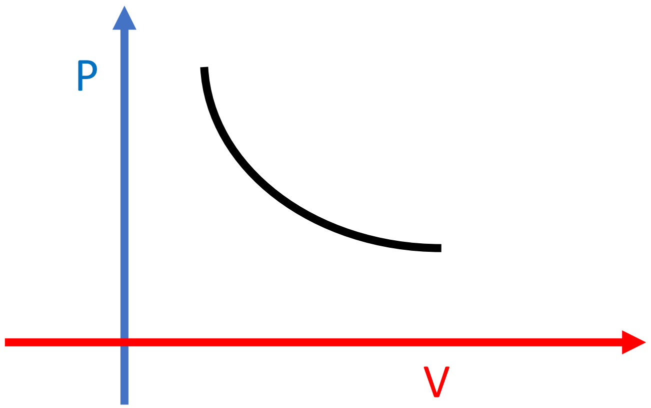
$$dU = \underline{179.536\ kJ}$$

<u>NOTE</u>: For Isentropic process, $\delta Q=0$ and thus

$$\delta W = -dU$$

Polytropic process

- Can occur when a gas undergoes a reversible process in which there is heat transfer
- A plot of Log P vs. Log V is a often a straight line
- PV^n = constant
- Example - the expansion of the combustion gasses in the cylinder of a water-cooled reciprocating engine



- 1. work done

$$W = \int_1^2 p\,dV$$

$$pV^n = C$$

$$p = \frac{C}{V^n}$$

$$W = \int_1^2 \frac{C}{V^n}\,dV$$

$$= C\int_1^2 \frac{1}{V^n}\,dV$$

$$= C\left(\frac{V^{-n+1}}{-n+1}\right)_1^2$$

$$= C\left(\frac{V_2^{-n+1}}{-n+1} - \frac{V_1^{-n+1}}{-n+1}\right) = C\left(\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1}\right)$$

Polytropic process

$$W = pV^n \left(\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} \right) = \left(\frac{p_2 V_2^n V_2^{-n+1} - p_1 V_1^n V_1^{-n+1}}{-n+1} \right)$$

$$W = \left(\frac{p_2 V_2 - p_1 V_1}{-n+1} \right)$$

$$W = \left(\frac{p_1 V_1 - p_2 V_2}{n-1} \right)$$

$$W = \left(\frac{mRT_1 - mRT_2}{n-1} \right)$$

$$W = \frac{mR(T_1 - T_2)}{n-1}$$

Polytropic process

2.Change in internal Energy

$$\Delta U = mC_v(T_2 - T_1)$$

3.Heat transfer (according to First law)

$$Q = \Delta U + W$$

$$Q = mC_v(T_2 - T_1) + mR\,(T_2 - T_1)/(n-1) \longrightarrow Q = W\left(\frac{\gamma - n}{\gamma - 1}\right)$$

4. Change in enthalpy

$$\Delta H = mC_p(T_2 - T_1)$$

5.PVT relations

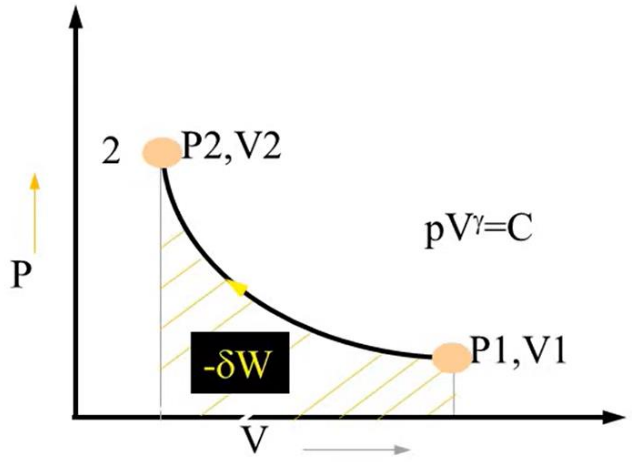
$$P_1V_1^{\,n} = P_2V_2^{\,n}$$

$$\boxed{\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n} \qquad \boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}} \qquad \boxed{\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}}$$

Example 5

The pressure & Temp, of air in the cylinder are 93.5 kPa and 45°C.The air is compressed according to the law $pV^{1.24}=C$ until the pressure becomes 640 kPa. The volume of air initially is 0.035 m³. Find;(1)Mass of air(2)Final temperature(3)Work required for compression(4)Heat rejected during compression

Take,γ=1.4 & R=0.287 kJ/kgK.



Data Given:

p_1=93.5 kPa = 0.935 bar

T_1=45°C = 318K

p_2=640 kPa = 6.4 bar

n = 1.24

V_1= 0.035m³

γ = 1.4

R=0.287 kJ/kgK

Find Out:

m = ?

T_2 = ?

W = ?

δQ = ?

Step 1: For mass of gas,

$$p_1V_1 = mRT_1$$

$$m = \frac{p_1V_1}{RT_1}$$

$$m = \frac{0.935 \times 100 \times 0.035}{0.287 \times 318}$$

$$m = \underline{0.03586\ kg}$$

Step 2: For final temp. of gas,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{n-1/n}$$

$$= 318 \left(\frac{6.40}{0.935}\right)^{1.24-1/1.24}$$

$$T_2 = \underline{461.44\ K}$$

$$= \underline{188.44°C}$$

Step 3: For work done,

$$\delta W = \frac{p_1V_1 - p_2V_2}{n-1}$$

$$= \frac{mR(T_1 - T_2)}{n-1}$$

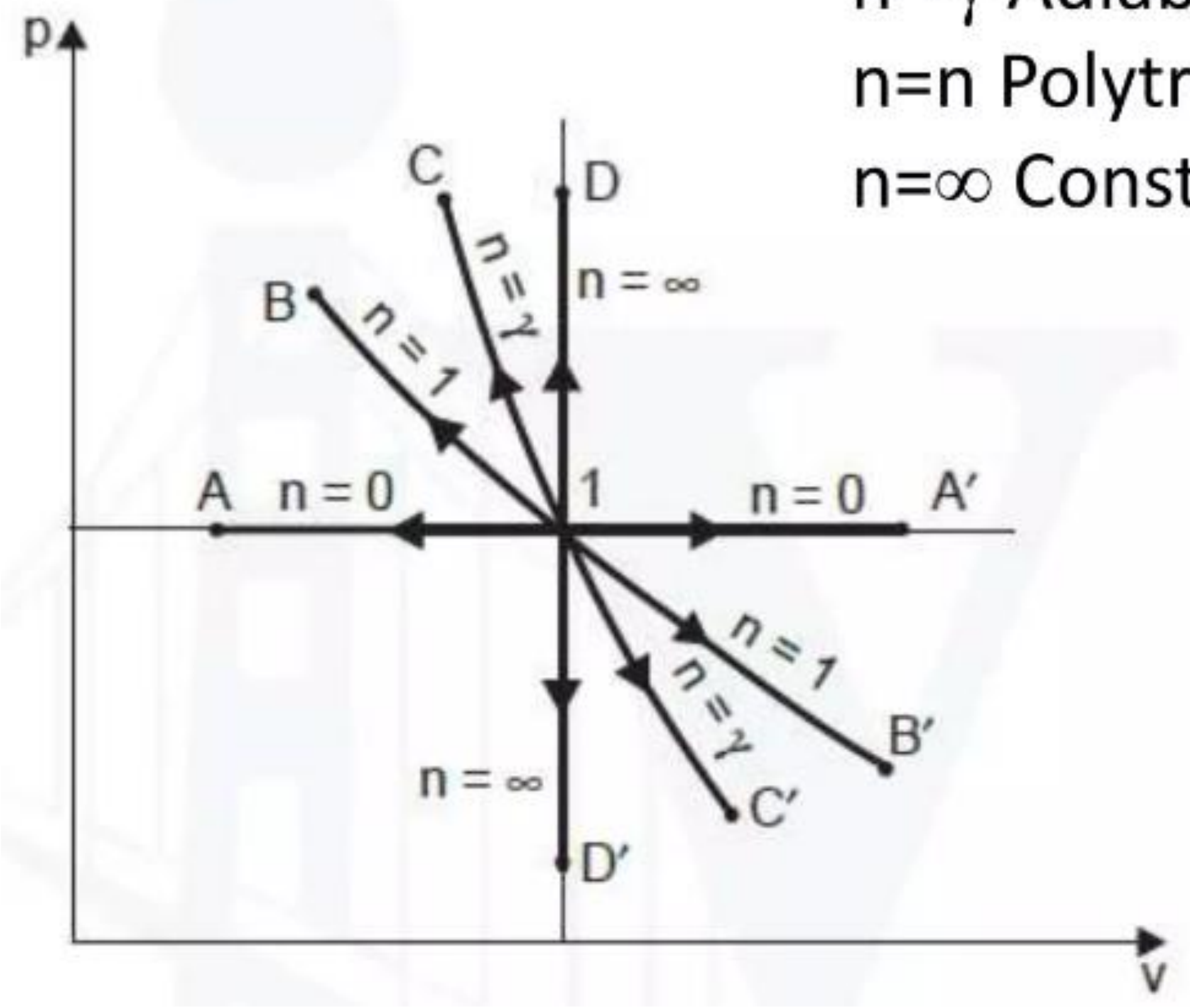
$$= \frac{0.03586 \times 0.287(318 - 461.44)}{1.24 - 1}$$

$$\delta W = -6.15\ kJ$$

Step 4: Heat transfer is given by,

$$\delta Q = \frac{\gamma - n}{\gamma - 1} \times WD \qquad = \frac{(1.4 - 1.24)}{(1.4 - 1)} \times -6.15$$

$$\delta Q = -2.46\ kJ$$



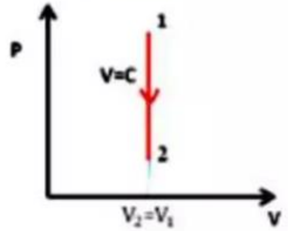
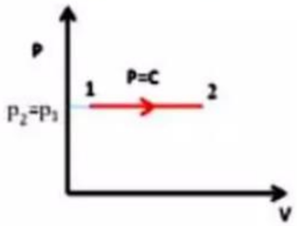
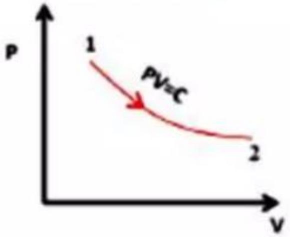
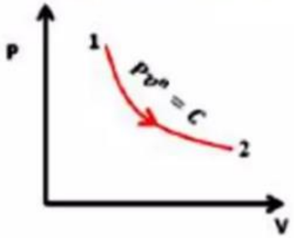
$n=0$ Constant Pressure process

$n=1$ Constant Temperatures

$n=\gamma$ Adiabatic process

$n=n$ Polytropic process

$n=\infty$ Constant Volume process

Process	Units	Constant Volume (V=C)	Constant Pressure (p=C)	Isothermal (pV=C)	Adiabatic Process ($pV^{\gamma} = C$)	Polytropic Process ($pV^{n} = C$)
p-V Diagram						
Work Done	kJ	W=0	$W = p_1(V_2 - V_1)$	$W = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$	$W = \frac{p_1V_1 - p_2V_2}{\gamma - 1}$	$W = \frac{p_1V_1 - p_2V_2}{n - 1}$
Change In Internal Energy	kJ	$\Delta U = mC_v(T_2 - T_1)$	$\Delta U = mC_v(T_2 - T_1)$	$\Delta U = 0$	$\Delta U = -W$	$\Delta U = Q - W$
Heat Transfer	kJ	$Q = mC_v(T_2 - T_1)$	$Q = mC_p(T_2 - T_1)$	$Q = W = p_1V_1 \ln\left(\frac{V_2}{V_1}\right)$	$Q = 0$	$Q = \left[\frac{\gamma - n}{(\gamma - 1)}\right] \times W$
p , V , T Relations		$\frac{T_2}{T_1} = \frac{p_2}{p_1}$	$\frac{T_2}{T_1} = \frac{V_2}{V_1}$	$\frac{V_2}{V_1} = \frac{p_1}{p_2}$	$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$	$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{n}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$

- Where C=Constant, 1 and 2 indicates as initial and final state,
- Equation for Ideal Gas : $pV = mRT$
- Pressure 'p' – kN/m^2 (1 bar =1 X 10^2 kN/m^2)
- Volume 'V' - m^3
- Mass 'm' - kg
- Gas Constant 'R' - 0.287 kJ/kg.K
- Temperature 'T' - K (0°C = 273K)
- Enthalpy 'h' = $u + Pv$ or $h = C_pT$

- $pv = RT$

Specific Volume $'v' = \frac{V}{m}$ m^3/kg

- $p\dot{V} = \dot{m}RT$

Volume flow rate $'\dot{V}' - m^3/s$
Mass flow rate '$\dot{m}$' – kg/s

- $p\bar{V} = \bar{n}RT$

Molar volume V- $m^3/kgmol$
Number of moles 'n'

For Ideal Gas (Air):
Specific Heat 'C_p' = 1.005 kJ/kg.K, 'C_v' = 0.718 kJ/kg.K
For Water :
Specific Heat 'C_p' = 4.186 kJ/kg.K,

Index of Expansion $'n' = \frac{\ln\left(\frac{p_2}{p_1}\right)}{\left(\ln\frac{V_1}{V_2}\right)}$

Gas constant = Universal Gas constant / Molecular weight

$$R = \frac{\bar{R}}{M}$$