Lecture 6

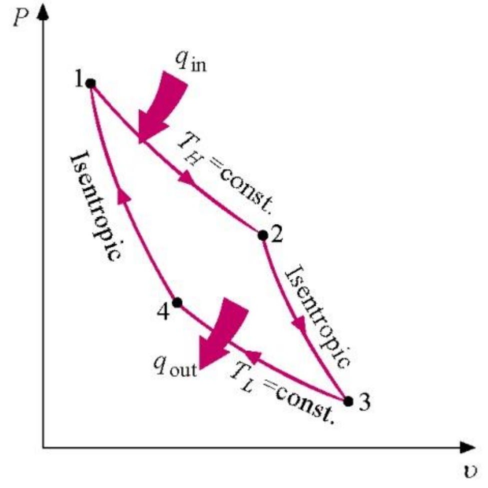
Air Standard Power Cycle

Air Standard Power Cycle

- Carnot cycle, air standard Otto cycle, diesel cycle, Joule cycle and Rankine cycle, comparison of air standard cycles with Carnot cycle.

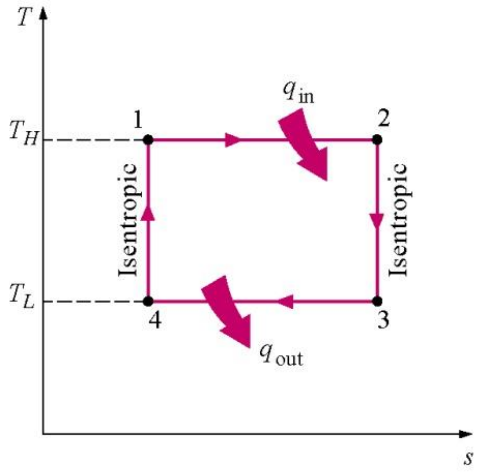
Carnot Cycle

The Carnot cycle was introduced in Chapter 5 as the most efficient heat engine that can operate between two fixed temperatures T_H and T_L. The Carnot cycle is described by the following four processes.



Carnot Cycle

Process	Description
1-2	Isothermal heat addition
2-3	Isentropic expansion
3-4	Isothermal heat rejection
4-1	Isentropic compression



Note the processes on both the *P-v* and *T-s* diagrams. The areas under the process curves on the *P-v* diagram represent the work done for closed systems. The net cycle work done is the area enclosed by the cycle on the *P-v* diagram. The areas under the process curves on the *T-s* diagram represent the heat transfer for the processes. The net heat added to the cycle is the area that is enclosed by the cycle on the *T-s* diagram. For a cycle we know $W_{\text{net}} = Q_{\text{net}}$; therefore, the areas enclosed on the *P-v* and *T-s* diagrams are equal.

$$\eta_{th,\,Carnot} = 1 - \frac{T_L}{T_H}$$

We often use the Carnot efficiency as a means to think about ways to improve the cycle efficiency of other cycles. One of the observations about the efficiency of both ideal and actual cycles comes from the Carnot efficiency: Thermal efficiency increases with an increase in the average temperature at which heat is supplied to the system or with a decrease in the average temperature at which heat is rejected from the system.

Air-Standard Assumptions

In our study of gas power cycles, we assume that the working fluid is air, and the air undergoes a thermodynamic cycle even though the working fluid in the actual power system does not undergo a cycle.

To simplify the analysis, we approximate the cycles with the following assumptions:

•The air continuously circulates in a closed loop and always behaves as an ideal gas.

•All the processes that make up the cycle are internally reversible.

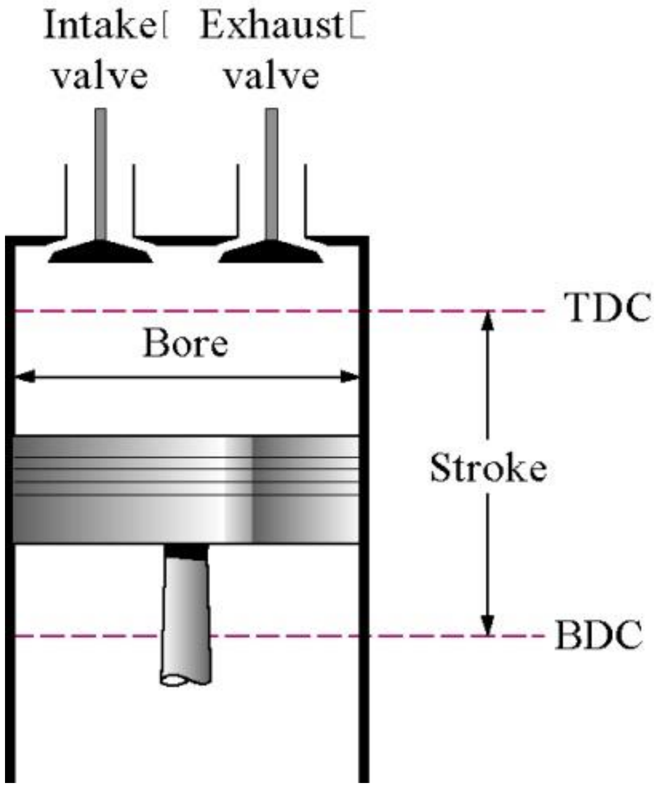
•The combustion process is replaced by a heat-addition process from an external source.

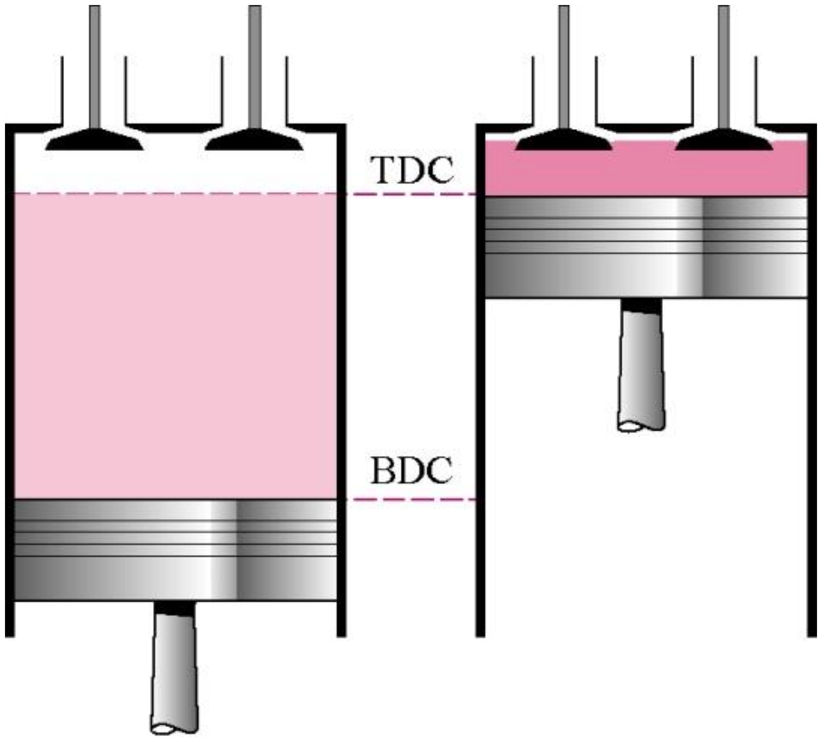
•A heat rejection process that restores the working fluid to its initial state replaces the exhaust process.

•The cold-air-standard assumptions apply when the working fluid is air and has constant specific heat evaluated at room temperature (25^oC or 77^oF).

Terminology for Reciprocating Devices

The following is some terminology we need to understand for reciprocating engines—typically piston-cylinder devices. Let's look at the following figures for the definitions of top dead center (TDC), bottom dead center (BDC), stroke, bore, intake valve, exhaust valve, clearance volume, displacement volume, compression ratio, and mean effective pressure.



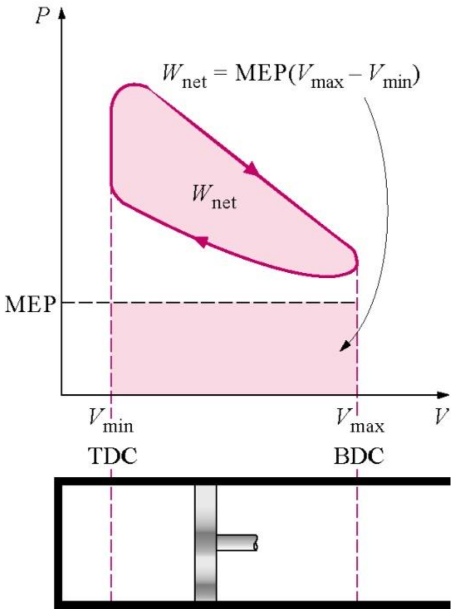


(*a*) Displacement volume   (*b*) Clearance volume

The compression ratio *r* of an engine is the ratio of the maximum volume to the minimum volume formed in the cylinder.

$$r = \frac{V\max}{V\min} = \frac{V_{BDC}}{V_{TDC}}$$

The mean effective pressure **(MEP*)*** is a fictitious pressure that, if it operated on the piston during the entire power stroke, would produce the same amount of net work as that produced during the actual cycle.



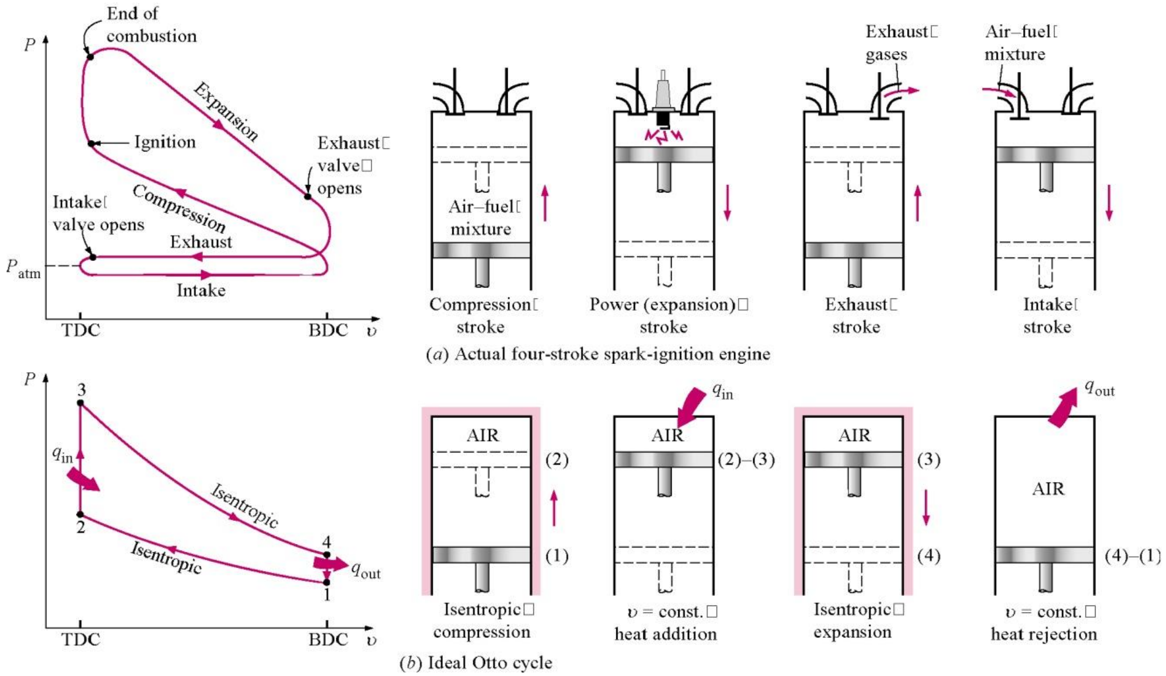
$$MEP = \frac{W_{net}}{V_{\max} - V_{\min}} = \frac{w_{net}}{v_{\max} - v_{\min}}$$

Otto Cycle: The Ideal Cycle for Spark-Ignition Engines

Consider the automotive spark-ignition power cycle.

Processes
Intake stroke
Compression stroke
Power (expansion) stroke
Exhaust stroke

Often the ignition and combustion process begins before the completion of the compression stroke. The number of crank angle degrees before the piston reaches TDC on the number one piston at which the spark occurs is called the engine timing. What are the compression ratio and timing of your engine in your car, truck, or motorcycle?

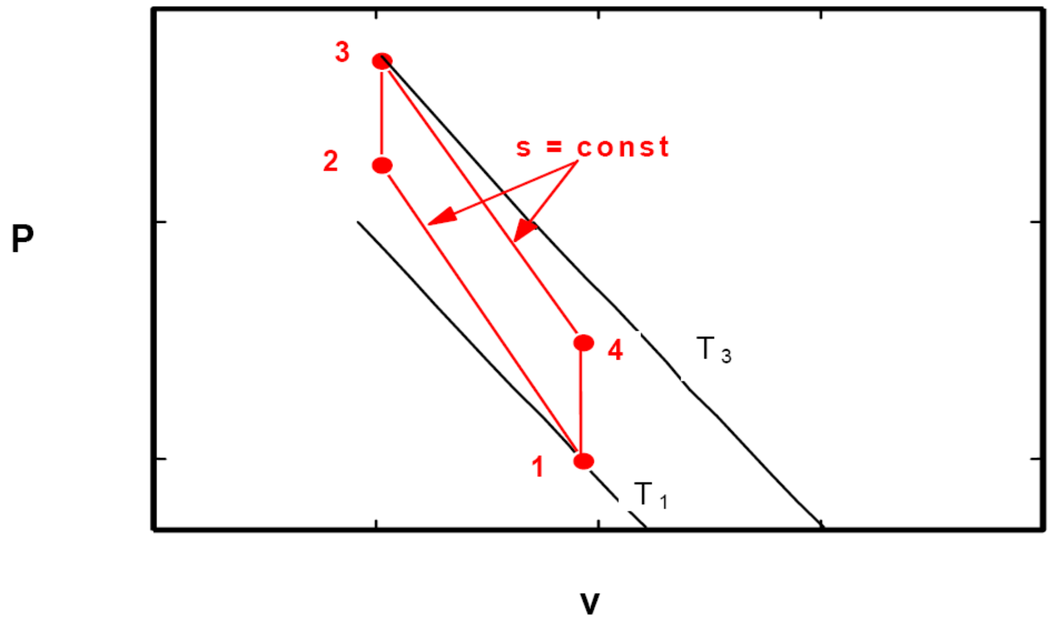


The air-standard Otto cycle is the ideal cycle that approximates the spark-ignition combustion engine.

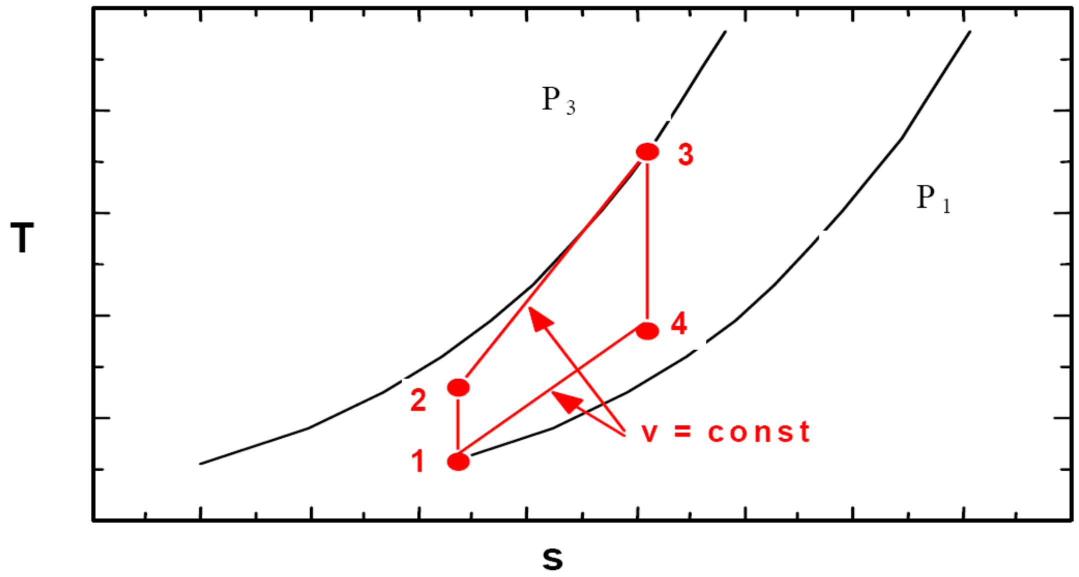
Process	Description
1-2	Isentropic compression
2-3	Constant volume heat addition
3-4	Isentropic expansion
4-1	Constant volume heat rejection

The *P-v* and *T-s* diagrams are

Air Otto Cycle P-v Diagram



Air Otto Cycle T-s Diagram



Thermal Efficiency of the Otto cycle:

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{Q_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Now to find Q_{in} and Q_{out}.

Apply first law closed system to process 2-3, V = constant.

$$Q_{net,\,23} - W_{net,\,23} = \Delta U_{23}$$

$$W_{net,\,23} = W_{other,\,23} + W_{b,\,23} = 0 + \int_2^3 P\,dV = 0$$

Thus, for constant specific heats,

$$Q_{net,\,23} = \Delta U_{23}$$

$$Q_{net,\,23} = Q_{in} = mC_v(T_3 - T_2)$$

Apply first law closed system to process 4-1, *V* = constant.

$$Q_{net,\,41} - W_{net,\,41} = \Delta U_{41}$$

$$W_{net,\,41} = W_{other,\,41} + W_{b,\,41} = 0 + \int_4^1 P\,dV = 0$$

Thus, for constant specific heats,

$$Q_{net,\,41} = \Delta U_{41}$$

$$Q_{net,\,41} = -Q_{out} = mC_v(T_1 - T_4)$$

$$Q_{out} = -mC_v(T_1 - T_4) = mC_v(T_4 - T_1)$$

The thermal efficiency becomes

$$\eta_{th,\,Otto} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$= 1 - \frac{mC_v(T_4 - T_1)}{mC_v(T_3 - T_2)}$$

$$\eta_{th,\,Otto} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4 / T_1 - 1)}{T_2(T_3 / T_2 - 1)}$$

Recall processes 1-2 and 3-4 are isentropic, so

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \quad and \quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1}$$

Since $V_3 = V_2$ and $V_4 = V_1$, we see that

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$or$$

$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

The Otto cycle efficiency becomes

$$\eta_{th,\,Otto} = 1 - \frac{T_1}{T_2}$$

Is this the same as the Carnot cycle efficiency?

Since process 1-2 is isentropic,

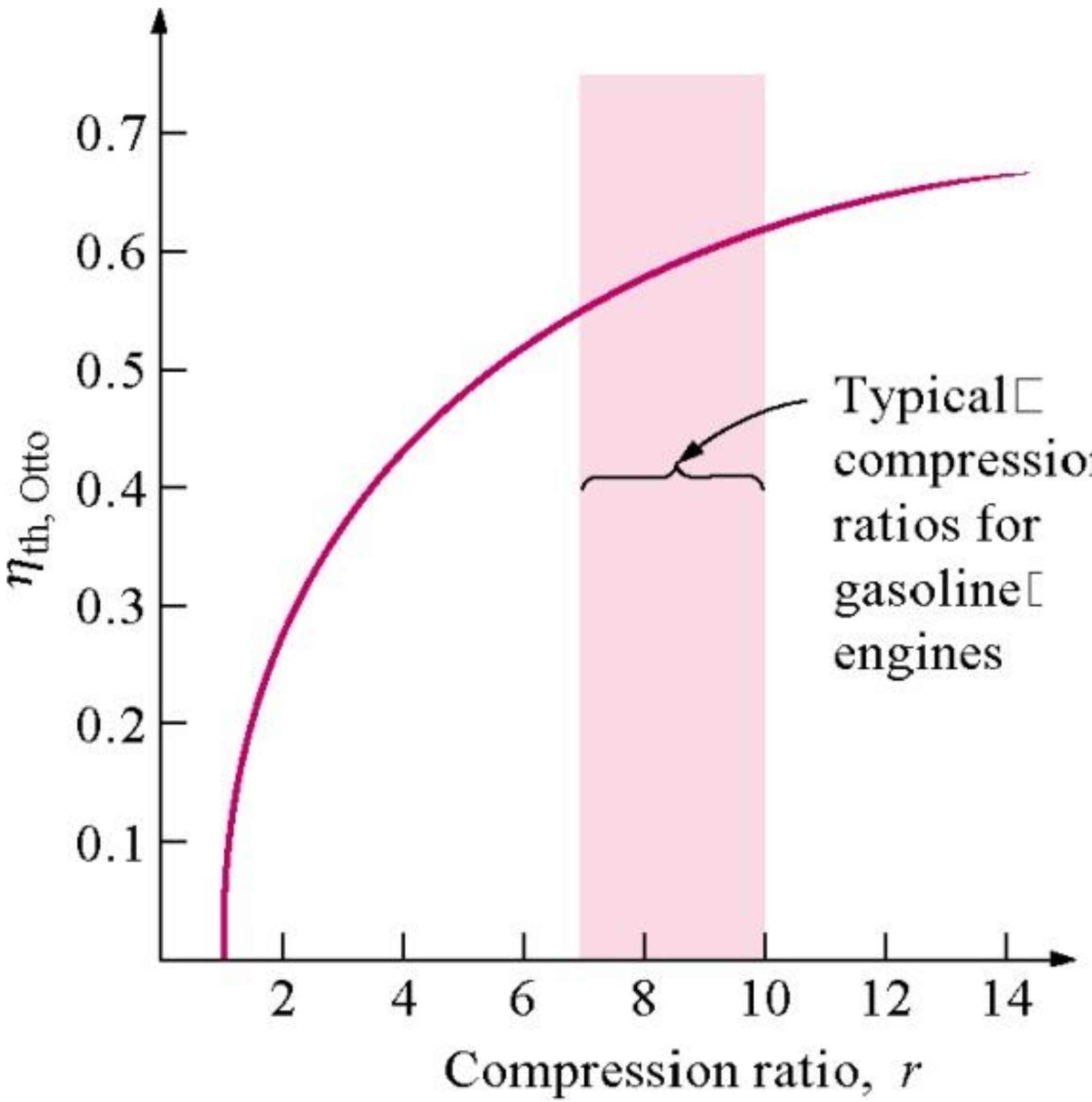
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k-1} = \left(\frac{1}{r}\right)^{k-1}$$

where the compression ratio is $r = V_1/V_2$ and

$$\eta_{th,\,Otto} = 1 - \frac{1}{r^{k-1}}$$

We see that increasing the compression ratio increases the thermal efficiency. However, there is a limit on *r* depending upon the fuel. Fuels under high temperature resulting from high compression ratios will prematurely ignite, causing knock.



Example

An Otto cycle having a compression ratio of 9:1 uses air as the working fluid. Initially P_1 = 95 kPa, T_1 = 17°C, and V_1 = 3.8 liters. During the heat addition process, 7.5 kJ of heat are added. Determine all *T*'s, *P*'s, η_{th}, the back work ratio, and the mean effective pressure.

Process Diagrams: Review the *P-v* and *T-s* diagrams given above for the Otto cycle.

Assume constant specific heats with C_v = 0.718 kJ/kg ·K, *k* = 1.4. (Use the 300 K data from Table A-2)

Process 1-2 is isentropic; therefore, recalling that $r = V_1/V_2 = 9$,

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = T_1(r)^{k-1}$$

$$= (17+273)K(9)^{1.4-1}$$

$$= 698.4K$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^k = P_1(r)^k$$

$$= 95kPa(9)^{1.4}$$

$$= 2059\,kPa$$

The first law closed system for process 2-3 was shown to reduce to (your homework solutions must be complete; that is, develop your equations from the application of the first law for each process as we did in obtaining the Otto cycle efficiency equation)

$$Q_{in} = mC_v(T_3 - T_2)$$

Let $q_{in} = Q_{in} / m$ and $m = V_1/v_1$

$$v_1 = \frac{RT_1}{P_1}$$

$$= \frac{0.287\frac{kJ}{kg\cdot K}(290\,K)}{95\,kPa}\,\frac{m^3kPa}{kJ}$$

$$= 0.875\frac{m^3}{kg}$$

$$q_{in} = \frac{Q_{in}}{m} = Q_{in}\,\frac{v_1}{V_1}$$

$$= 7.5kJ\,\frac{0.875\,\frac{m^3}{kg}}{3.8\cdot 10^{-3}\,m^3}$$

$$= 1727\,\frac{kJ}{kg}$$

Then,

$$T_3 = T_2 + \frac{q_{in}}{C_v}$$

$$= 698.4K + \frac{1727\,\frac{kJ}{kg}}{0.718\,\frac{kJ}{kg\cdot K}}$$

$$= 3103.7\,K$$

Using the combined gas law ($V_3 = V_2$)

$$P_3 = P_2 \frac{T_3}{T_2} = 9.15\, MPa$$

Process 3-4 is isentropic; therefore,

$$T_4 = T_3 \left(\frac{V_3}{V_4} \right)^{k-1} = T_3 \left(\frac{1}{r} \right)^{k-1} = (3103.7)K \left(\frac{1}{9} \right)^{1.4-1}$$

$$= 1288.8\, K$$

$$P_4 = P_3 \left(\frac{V_3}{V_4} \right)^{k} = P_3 \left(\frac{1}{r} \right)^{k}$$

$$= 9.15\, MPa \left(\frac{1}{9} \right)^{1.4}$$

$$= 422\, kPa$$

Process 4-1 is constant volume. So the first law for the closed system gives, on a mass basis,

$$Q_{out} = mC_v(T_4 - T_1)$$

$$q_{out} = \frac{Q_{out}}{m} = C_v(T_4 - T_1)$$

$$= 0.718\frac{kJ}{kg \cdot K}(1288.8 - 290)K$$

$$= 717.1\frac{kJ}{kg}$$

The first law applied to the cycle gives (Recall Δu_{cycle} = 0)

$$w_{net} = q_{net} = q_{in} - q_{out}$$

$$= (1727 - 717.4)\frac{kJ}{kg}$$

$$= 1009.6\frac{kJ}{kg}$$

The thermal efficiency is

$$\eta_{th,\,Otto} = \frac{w_{net}}{q_{in}} = \frac{1009.6\,\frac{kJ}{kg}}{1727\,\frac{kJ}{kg}}$$

$$= 0.585 \;\; or \;\;\; 58.5\%$$

The mean effective pressure is

$$MEP = \frac{W_{net}}{V_{\max} - V_{\min}} = \frac{w_{net}}{v_{\max} - v_{\min}}$$

$$= \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1(1 - v_2/v_1)} = \frac{w_{net}}{v_1(1 - 1/r)}$$

$$= \frac{1009.6\,\frac{kJ}{kg}}{0.875\,\frac{m^3}{kg}(1 - \frac{1}{9})}\,\frac{m^3 kPa}{kJ} = 1298\ kPa$$

The back work ratio is (can you show that this is true?)

$$BWR = \frac{w_{comp}}{w_{\exp}} = \frac{\Delta u_{12}}{-\Delta u_{34}} = \frac{C_v(T_2 - T_1)}{C_v(T_3 - T_4)} = \frac{(T_2 - T_1)}{(T_3 - T_4)}$$

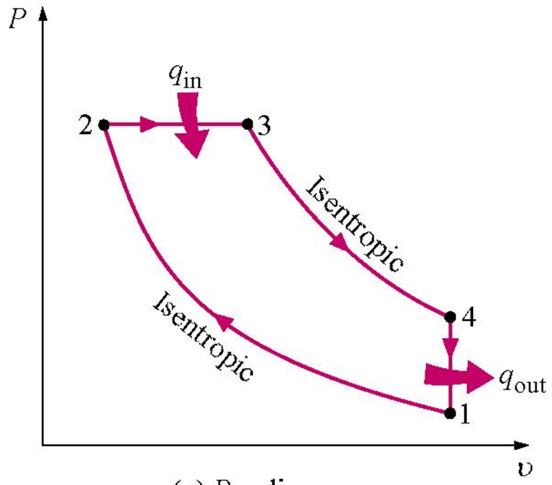
$$= 0.225 \ \ or \ \ 22.5\%$$

Air-Standard Diesel Cycle

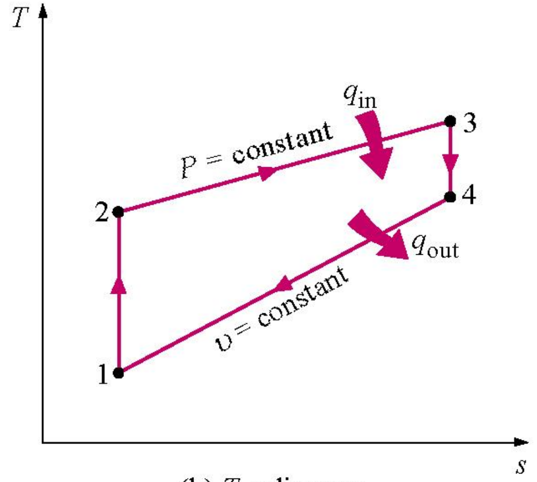
The air-standard Diesel cycle is the ideal cycle that approximates the Diesel combustion engine

Process	Description
1-2	Isentropic compression
2-3	Constant pressure heat addition
3-4	Isentropic expansion
4-1	Constant volume heat rejection

The *P-v* and *T-s* diagrams are



(a) P-υ diagram



(b) T-s diagram

Thermal efficiency of the Diesel cycle

$$\eta_{th,\,Diesel} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Now to find Q_{in} and Q_{out}.

Apply the first law closed system to process 2-3, P = constant.

$$E_{in} - E_{out} = \Delta E$$

$$Q_{net,\,23} - W_{net,\,23} = \Delta U_{23}$$

$$W_{net,\,23} = W_{other,\,23} + W_{b,\,23} = 0 + \int_2^3 P\,dV$$

$$= P_2(V_3 - V_2)$$

Thus, for constant specific heats

$$Q_{net,\,23} = \Delta U_{23} + P_2(V_3 - V_2)$$

$$Q_{net,\,23} = Q_{in} = mC_v(T_3 - T_2) + mR(T_3 - T_2)$$

$$Q_{in} = mC_p(T_3 - T_2)$$

Apply the first law closed system to process 4-1, V = constant (just as we did for the Otto cycle)

$$E_{in} - E_{out} = \Delta E$$

$$Q_{net,41} - W_{net,41} = \Delta U_{41}$$

$$W_{net,41} = W_{other,41} + W_{b,41} = 0 + \int_4^1 P dV = 0$$

Thus, for constant specific heats

$$Q_{net,41} = \Delta U_{41}$$

$$Q_{net,41} = -Q_{out} = mC_v (T_1 - T_4)$$

$$Q_{out} = -mC_v (T_1 - T_4) = mC_v (T_4 - T_1)$$

The thermal efficiency becomes

$$\eta_{th,\,Diesel} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$= 1 - \frac{mC_v (T_4 - T_1)}{mC_p (T_3 - T_2)}$$

$$\eta_{th,\,Diesel} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$= 1 - \frac{1}{k}\frac{T_1(T_4 / T_1 - 1)}{T_2(T_3 / T_2 - 1)}$$

What is T_3/T_2 ?

$$\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2} \quad \text{where } P_3 = P_2$$

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c$$

where r_c is called the cutoff ratio, defined as V_3 / V_2, and is a measure of the duration of the heat addition at constant pressure. Since the fuel is injected directly into the cylinder, the cutoff ratio can be related to the number of degrees that the crank rotated during the fuel injection into the cylinder.

What is T_4/T_1 ?

$$\frac{P_4 V_4}{T_4} = \frac{P_1 V_1}{T_1} \quad \text{where } V_4 = V_1$$

$$\frac{T_4}{T_1} = \frac{P_4}{P_1}$$

Recall processes 1-2 and 3-4 are isentropic, so

$$P_1 V_1^k = P_2 V_2^k \quad \text{and} \quad P_4 V_4^k = P_3 V_3^k$$

Since $V_4 = V_1$ and $P_3 = P_2$, we divide the second equation by the first equation and obtain

$$\frac{P_4}{P_1} = \left(\frac{V_3}{V_2}\right)^k = r_c^k$$

Therefore,

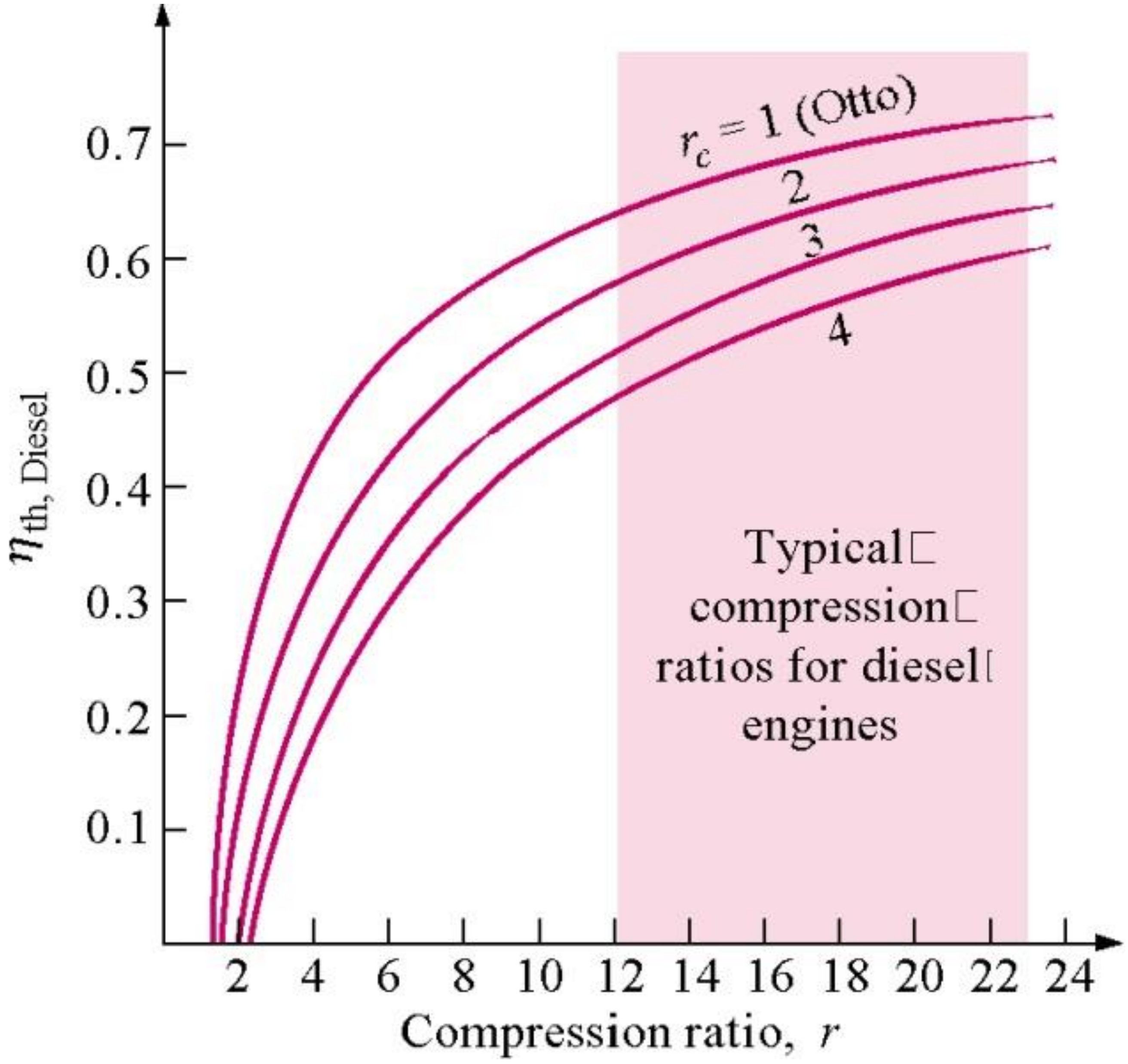
$$\eta_{th,\,Diesel} = 1 - \frac{1}{k}\frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$= 1 - \frac{1}{k}\frac{T_1}{T_2}\frac{r_c^k - 1}{(r_c - 1)}$$

$$= 1 - \frac{1}{r^{k-1}}\frac{r_c^k - 1}{k(r_c - 1)}$$

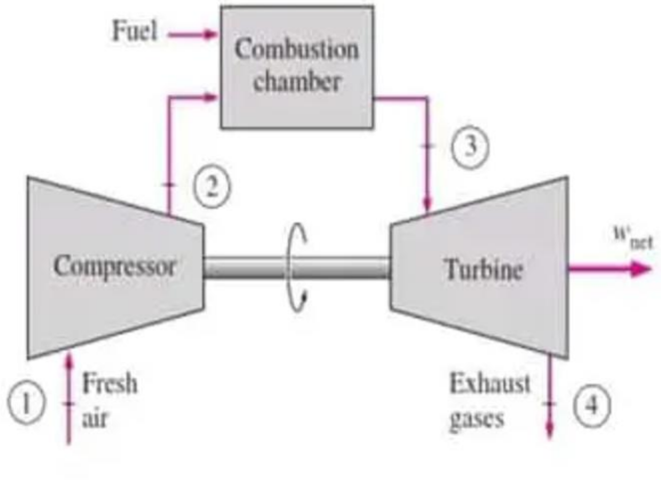
* What happens as r_c goes to 1? Sketch the *P-v* diagram for the Diesel cycle and show r_c approaching 1 in the limit.

When $r_c > 1$ for a fixed r, $\eta_{th,\,Diesel} < \eta_{th,\,Otto}$. But, since $r_{Diesel} > r_{Otto}$, $\eta_{th,\,Diesel} > \eta_{th,\,Otto}$.

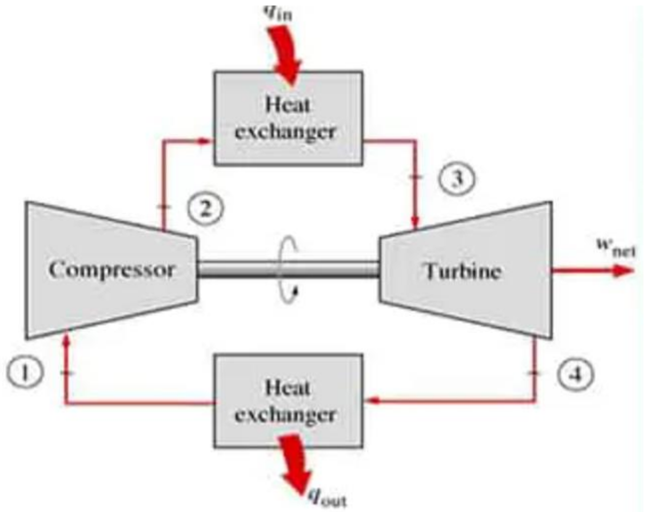


Brayton (Joule) Cycle

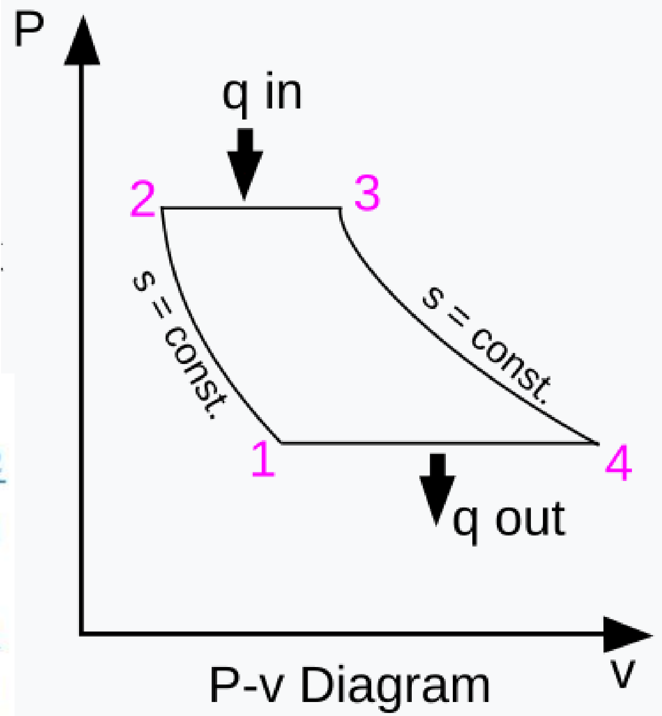
- Brayton cycle is the basic cycle for the simple gas turbine powerplant.
- The p-v, T-s flow diagram for this cycle is shown.
- The atmospheric air is first adiabatically compressed in a rotary compressor.

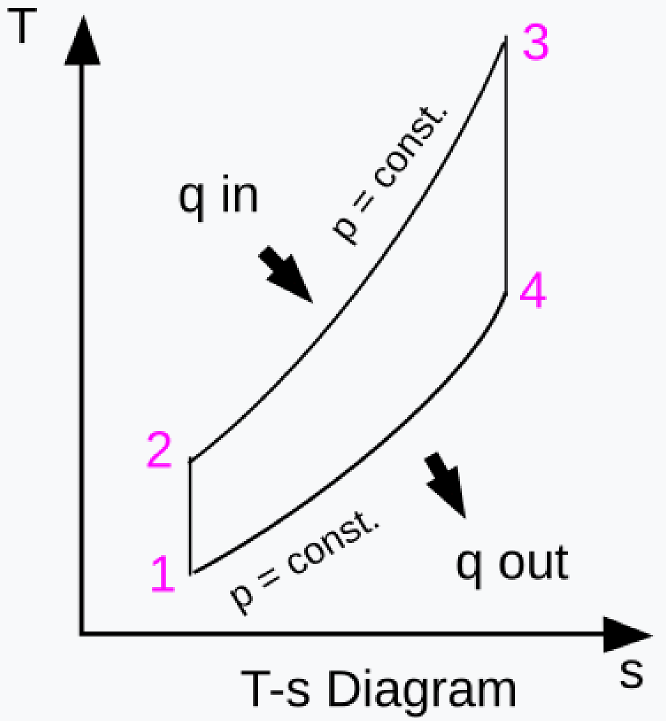


BRAYTON OPEN CYCLE



BRAYTON CLOSED CYCLE





Entropy Eq.: $s_2 = s_1$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \quad \text{and} \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{k}$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1} \quad \text{and} \quad \frac{P_3}{P_4} = \left(\frac{V_4}{V_3}\right)^{k}$$

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_{s=\text{const.}} = \frac{v_{r2}}{v_{r1}}$$

1 – 2 Isentropic compression in the compressor.

2 – 3 Constant pressure heat addition.

3 – 4 Isentropic expansion in the turbine.

4 – 1 Constant pressure heat rejection.

Assuming constant specific heats, the thermal efficiency of the cycle

$$\eta_{th} = 1 - \frac{Q_L}{Q_H}$$

$$\text{or } \eta_{th} = 1 - \frac{h_4 - h_1}{h_3 - h_2}$$

$$= 1 - \frac{C_P(T_4 - T_1)}{C_P(T_3 - T_2)}$$

The thermal efficiency can be written as

$$\eta_{th} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

$$= 1 - \frac{T_1\left[\frac{T_4}{T_1} - 1\right]}{T_2\left[\frac{T_3}{T_2} - 1\right]}$$

Now $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ and $\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$

Since, $p_2 = p_3$ and $p_1 = p_4$, it follows

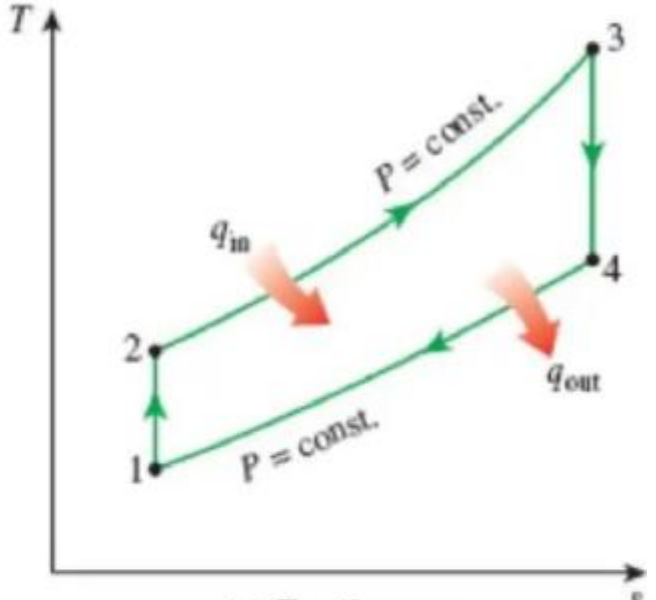
$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Hence, $\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}} = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}$

Where r_p is the pressure ratio. It can be seen that increasing the pressure ratio can increase the efficiency of the Brayton cycle.

Example

A gas-turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The gas temperature is 300 K at the compressor inlet and 1300 K at the turbine inlet. Utilizing the air-standard assumptions, determine (*a*) the gas temperature at the exits of the compressor and the turbine, (*b*) the back work ratio, and (*c*) the thermal efficiency.



(*a*) *T-s* diagram

Process 1–2 (isentropic compression of an ideal gas):

$$T_1 = 300 \text{ K} \rightarrow h_1 = 300.19 \text{ kJ/kg}$$
$$P_{r1} = 1.386$$

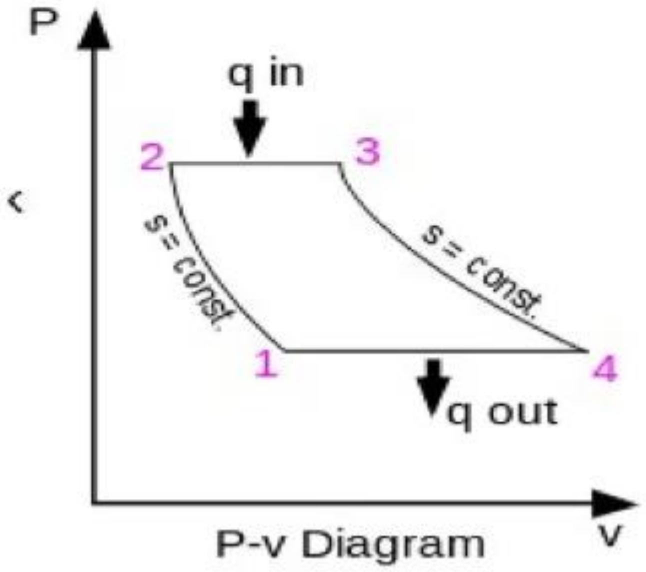
$$P_{r2} = \frac{P_2}{P_1} P_{r1} = (8)(1.386) = 11.09 \rightarrow T_2 = 540 \text{ K} \quad \text{(at compressor exit)}$$
$$h_2 = 544.35 \text{ kJ/kg}$$

$$T_3 = 1300 \text{ K} \rightarrow h_3 = 1395.97 \text{ kJ/kg}$$
$$P_{r3} = 330.9$$

$$P_{r4} = \frac{P_4}{P_3} P_{r3} = \left(\frac{1}{8}\right)(330.9) = 41.36 \rightarrow T_4 = 770 \text{ K} \quad \text{(at turbine exit)}$$
$$h_4 = 789.37 \text{ kJ/kg}$$

$$w_{comp,in} = h_2 - h_1 = 544.35 - 300.19 = 244.16 \text{ kJ/kg}$$

$$w_{turb,out} = h_3 - h_4 = 1395.97 - 789.37 = 606.60 \text{ kJ/kg}$$



(*b*) To find the back work ratio,

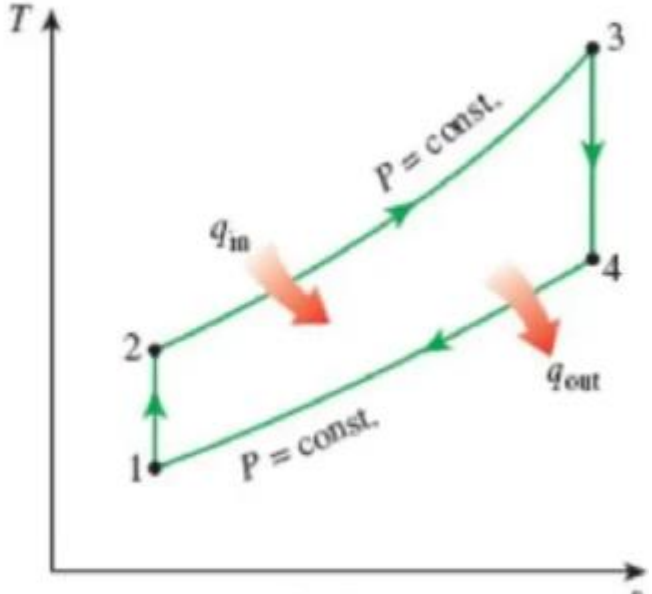
$$r_{bw} = \frac{w_{comp,in}}{w_{turb,out}} = \frac{244.16 \text{ kJ/kg}}{606.60 \text{ kJ/kg}} = \mathbf{0.403}$$

(*c*) The thermal efficiency of the cycle is the ratio of the net power output to the total heat input:

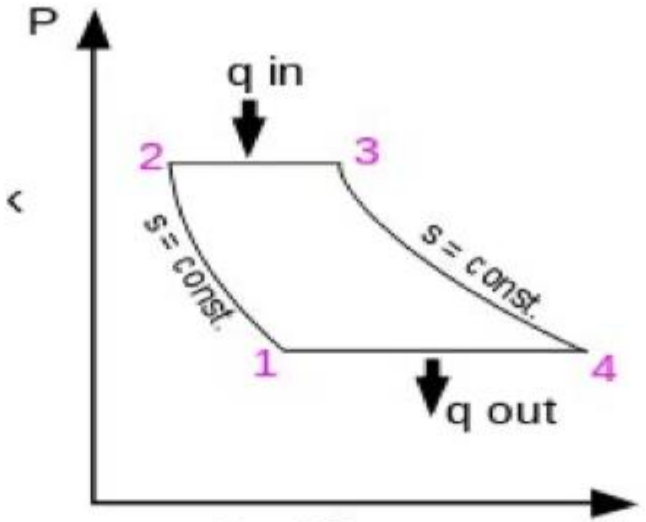
$$q_{in} = h_3 - h_2 = 1395.97 - 544.35 = 851.62 \text{ kJ/kg}$$

$$w_{net} = w_{out} - w_{in} = 606.60 - 244.16 = 362.4 \text{ kJ/kg}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{362.4 \text{ kJ/kg}}{851.62 \text{ kJ/kg}} = \mathbf{0.426} \text{ or } \mathbf{42.6\%}$$



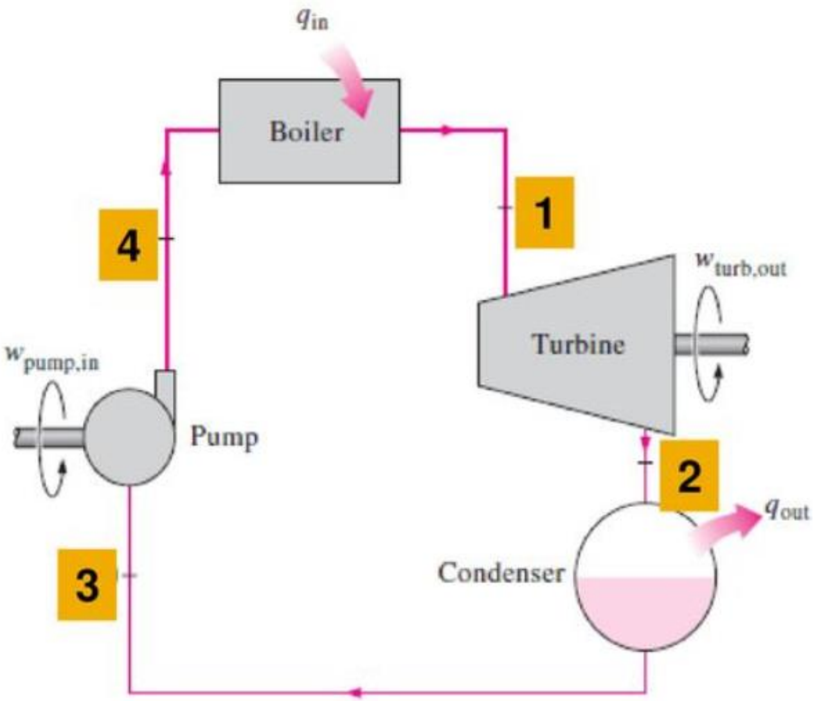
(*a*) *T-s* diagram



P-v Diagram

Rankine cycle

- The Rankine cycle is the fundamental operating cycle of all power plants where an operating fluid is continuously evaporated and condensed.



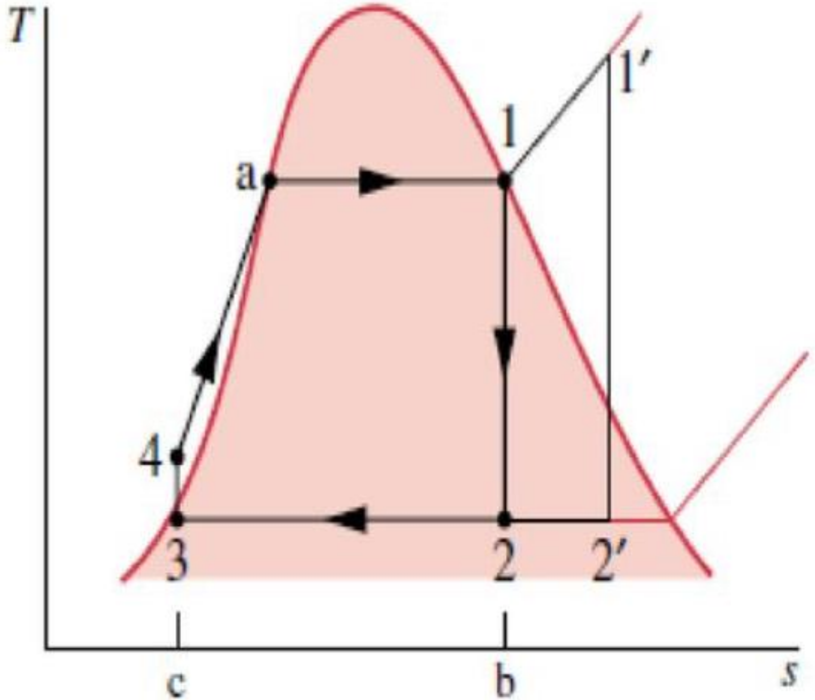
Processes in RANKINE cycle:

- ► Turbine: Isentropic
- ► Condenser: Isobaric
- ► Pump: Isentropic
- ► Boiler: Isobaric

$$\eta = \frac{W_T - W_P}{Q}$$

$$\eta = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}$$

$$\eta = \frac{(h_1 - h_2)}{h_1 - h_4}$$



- Efficiency of the Rankine cycle similarly depends on the average temperature at which the heat is transferred to and from the working fluid.
- Any change that increases the average temperature at which heat is transferred to the working fluid will increase the efficiency of the Rankine cycle.