MATH 3190 Homework 4

Maximum Likelihood Estimators

Due 3/28/2022

Here you will practice what you learned in the maximum likelihood estimation. Please turn this in as an RMarkdown document. You can either add your solution in Latex or you can write it by hand and input a scanned version or picture into the R Markdown. ‘Turn it in’ by uploading to your GitHub repository.

\begin{enumerate} (20 points) Suppose $\mathbf{x} = (x_1, \dots, x_N)^T$ follow a Poisson distribution with a parameter $\lambda > 0$ and p.m.f. given by

$$P(x = k|\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}.$$

Answer the following questions: \begin{enumerate}
Using {}, plot the Poisson pmf for $k = 0, 1, \dots, 10$ when $\lambda = 5$.

```
library(tidyverse)
```

```
## Warning: package 'tidyverse' was built under R version 4.0.3
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.0 --
```

```
## v ggplot2 3.3.5     v purrr   0.3.4
## v tibble  3.0.3     v dplyr   1.0.4
## v tidyr   1.1.2     v stringr 1.4.0
## v readr   1.3.1     v forcats 0.5.0
```

```
## Warning: package 'ggplot2' was built under R version 4.0.5
```

```
## Warning: package 'tidyr' was built under R version 4.0.3
```

```
## Warning: package 'purrr' was built under R version 4.0.3
```

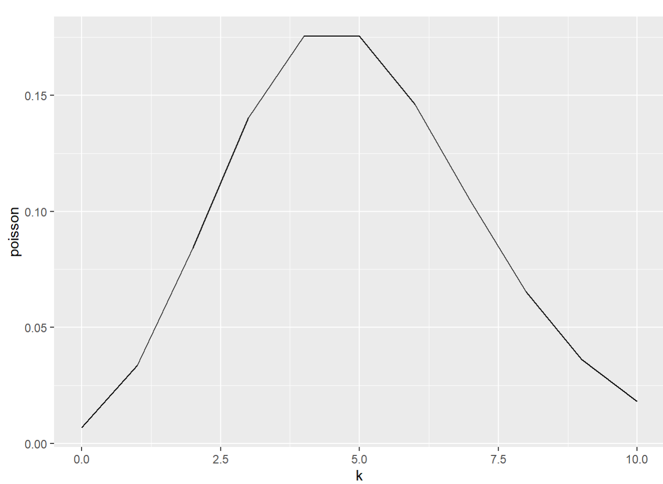
```
## Warning: package 'dplyr' was built under R version 4.0.3
```

```
## Warning: package 'forcats' was built under R version 4.0.3
```

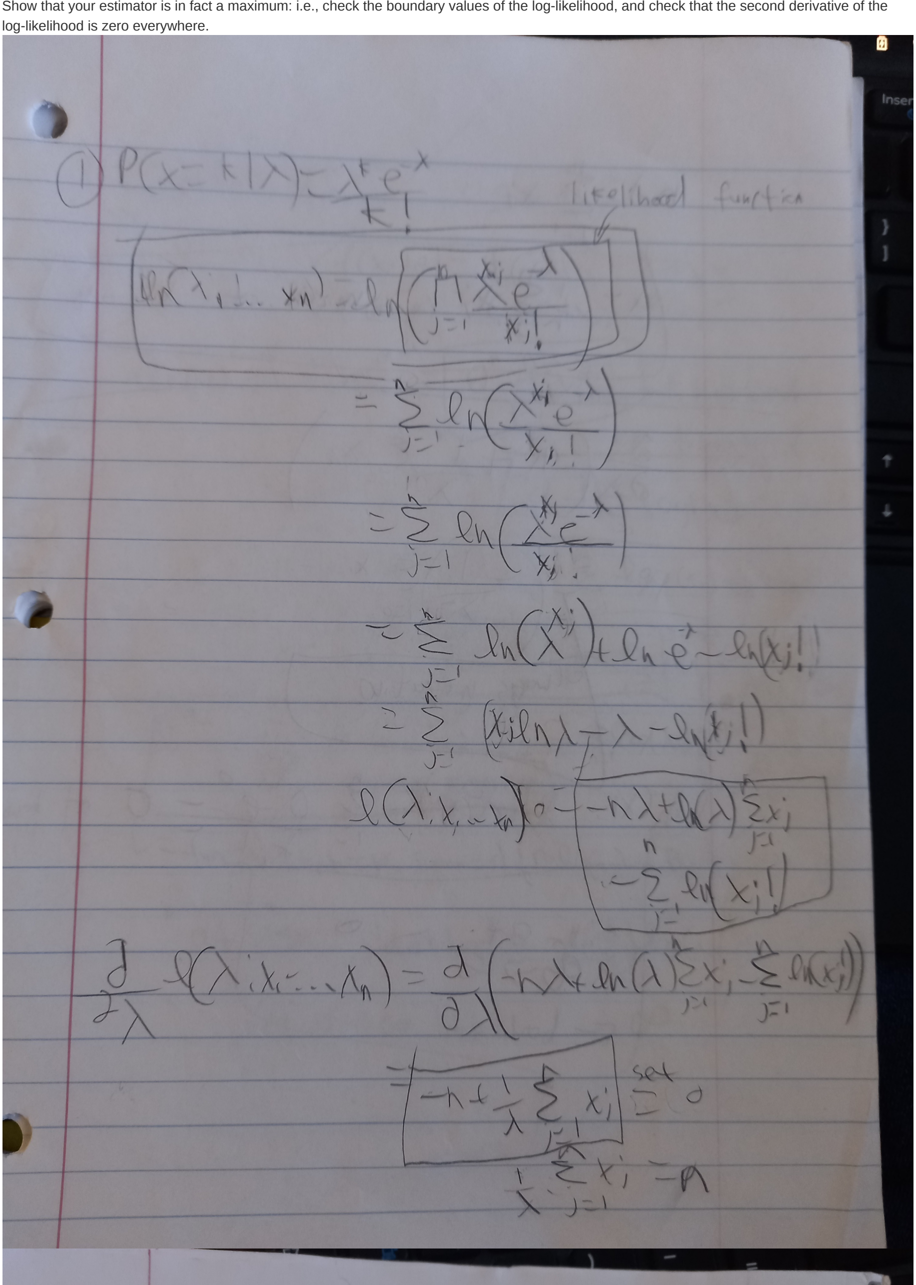
```
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

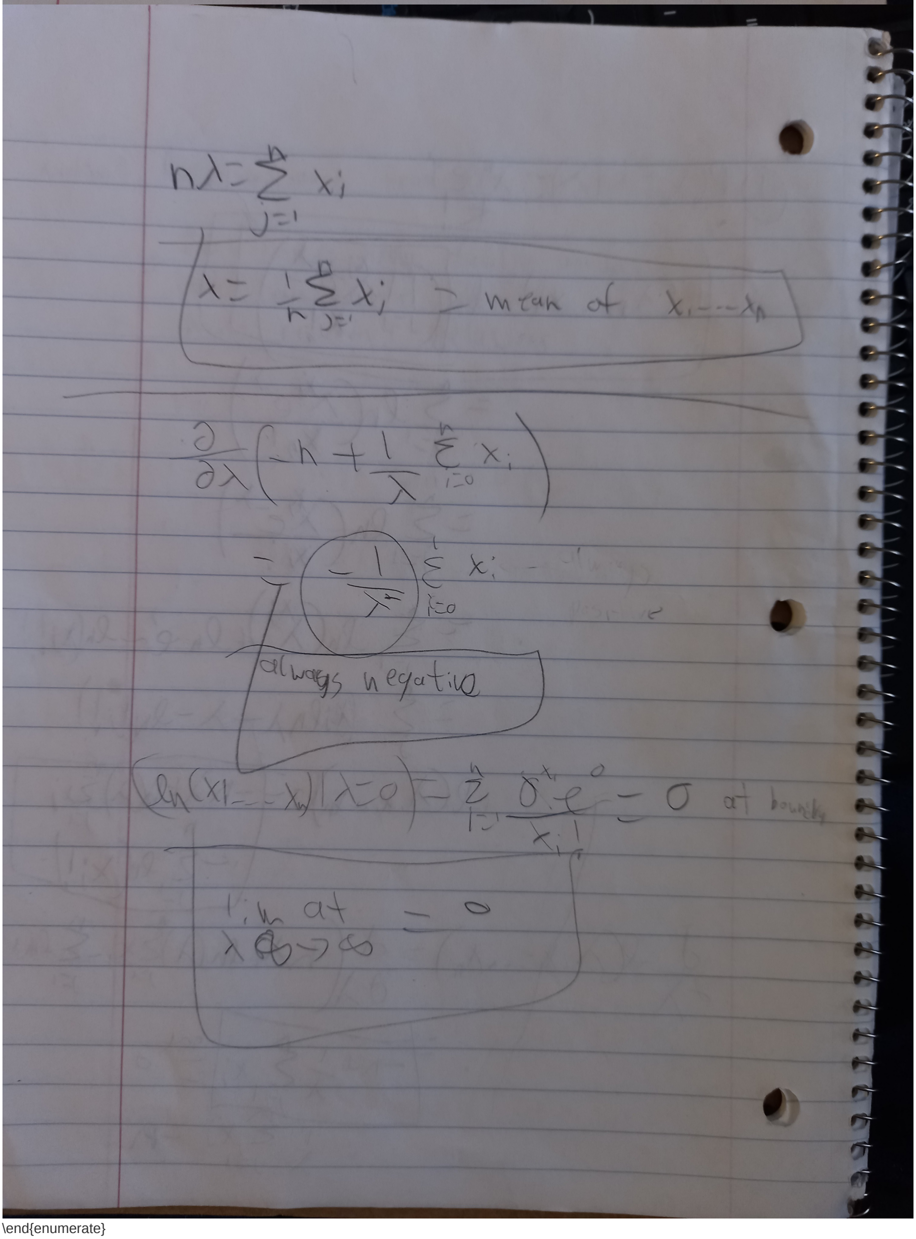
```
k <- 0:10
pos_dist <- tibble(k,"poisson" = dpois(k, lambda=5))


ggplot(data = pos_dist, aes(x = k, y = poisson)) +
  geom_line()
```



Asummining $\mathbf{x}$ is observed, give the likelihood $L(\lambda|\mathbf{x})$ and log-likelihood $l(\lambda|\mathbf{x})$ functions. Find the Maximum Likelihood Estimator (MLE) $\hat\lambda$ for λ.
Show that your estimator is in fact a maximum: i.e., check the boundary values of the log-likelihood, and check that the second derivative of the log-likelihood is zero everywhere.





\end{enumerate}

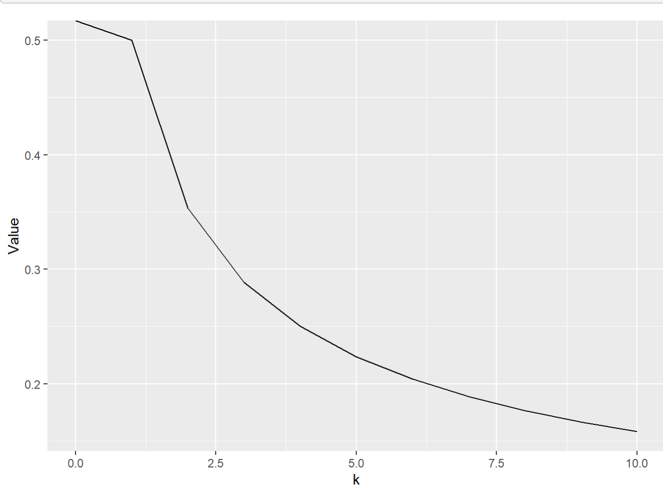
(20 points) Suppose $\mathbf{x} = (x_1, \dots, x_N)^T$ are iid random variables with p.d.f. given by

$$f(x|\theta) = \theta x^{\theta-1},\ 0 \le x \le 1,\ 0 < \theta < \infty.$$

\begin{enumerate}
Using {}, plot the pdf for an individual x_i given $\theta = 0.5$ and also for $\theta = 5$.

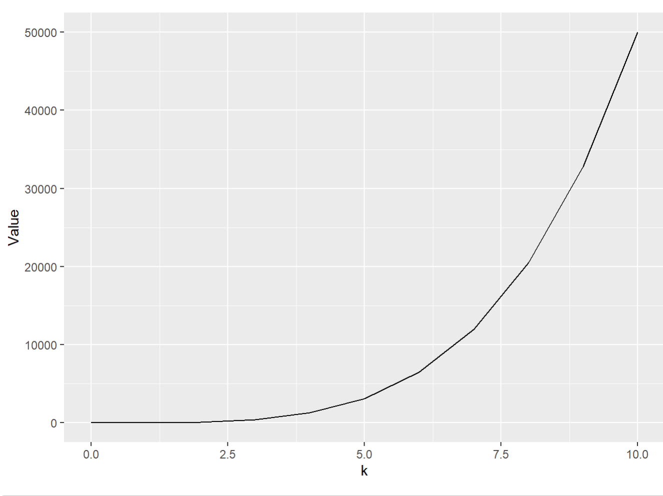
```
newDist <- function(th, x){th*x^(th-1)}
k = 0:10
new_dist <- tibble(k,"Value" = newDist(th = .5, k))


ggplot(data = new_dist, aes(x = k, y = Value)) +
  geom_line()
```

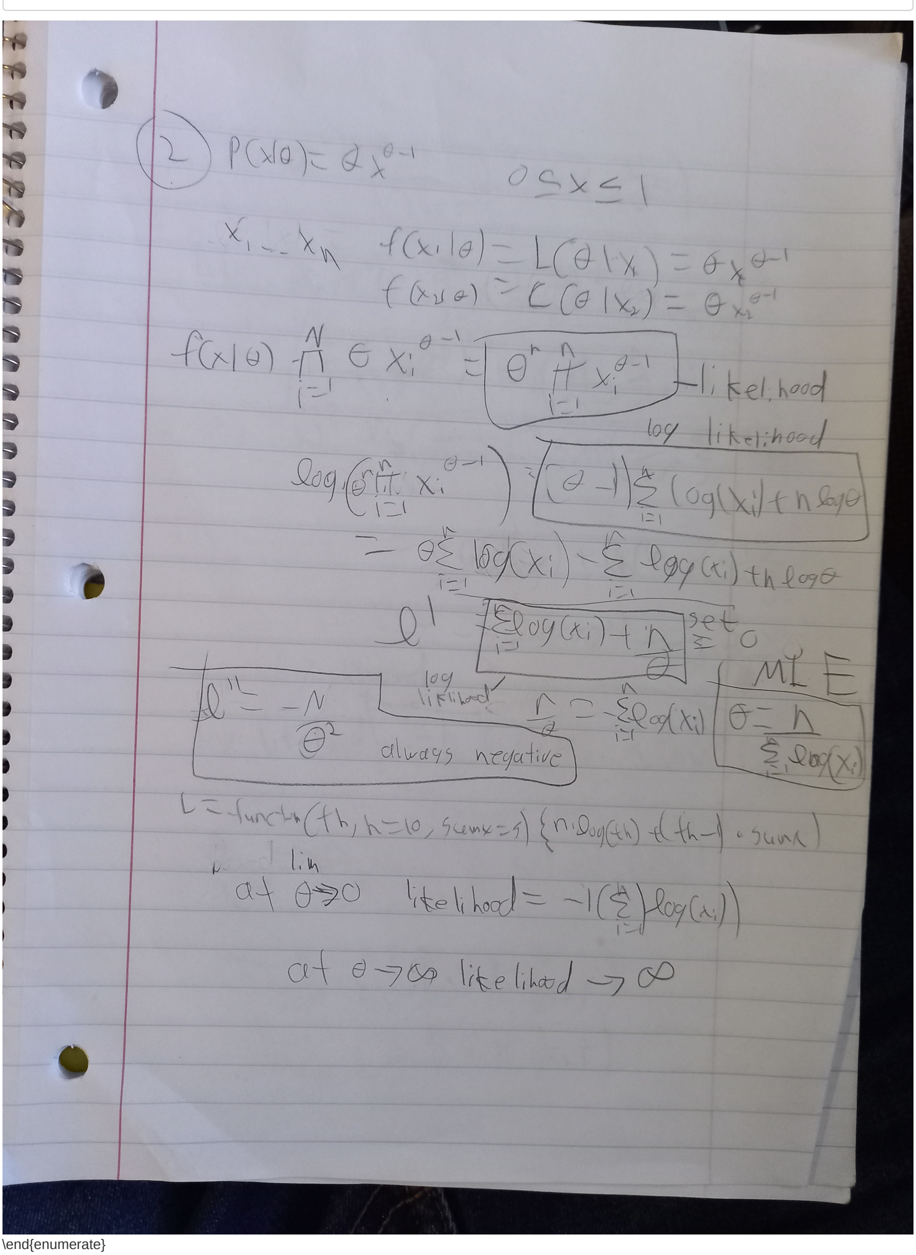



```
 newDist <- function(th, x){th*x^(th-1)}
k = 0:10
new_dist <- tibble(k,"Value" = newDist(th = 5, k))


ggplot(data = new_dist, aes(x = k, y = Value)) +
  geom_line()
```




```
\item Give the likelihood $L(\theta|{\bf x})$ and log-likelihood $l(\theta|{\bf x})$ functions.
\item Find the Maximum Likelihood Estimator (MLE) $\hat\theta$ for $\theta$.
\item Show that your estimator is in fact a maximum: i.e., check the boundary values of the log-likelihood, and c
heck that the second derivative of the log-likelihood is zero everywhere.
```

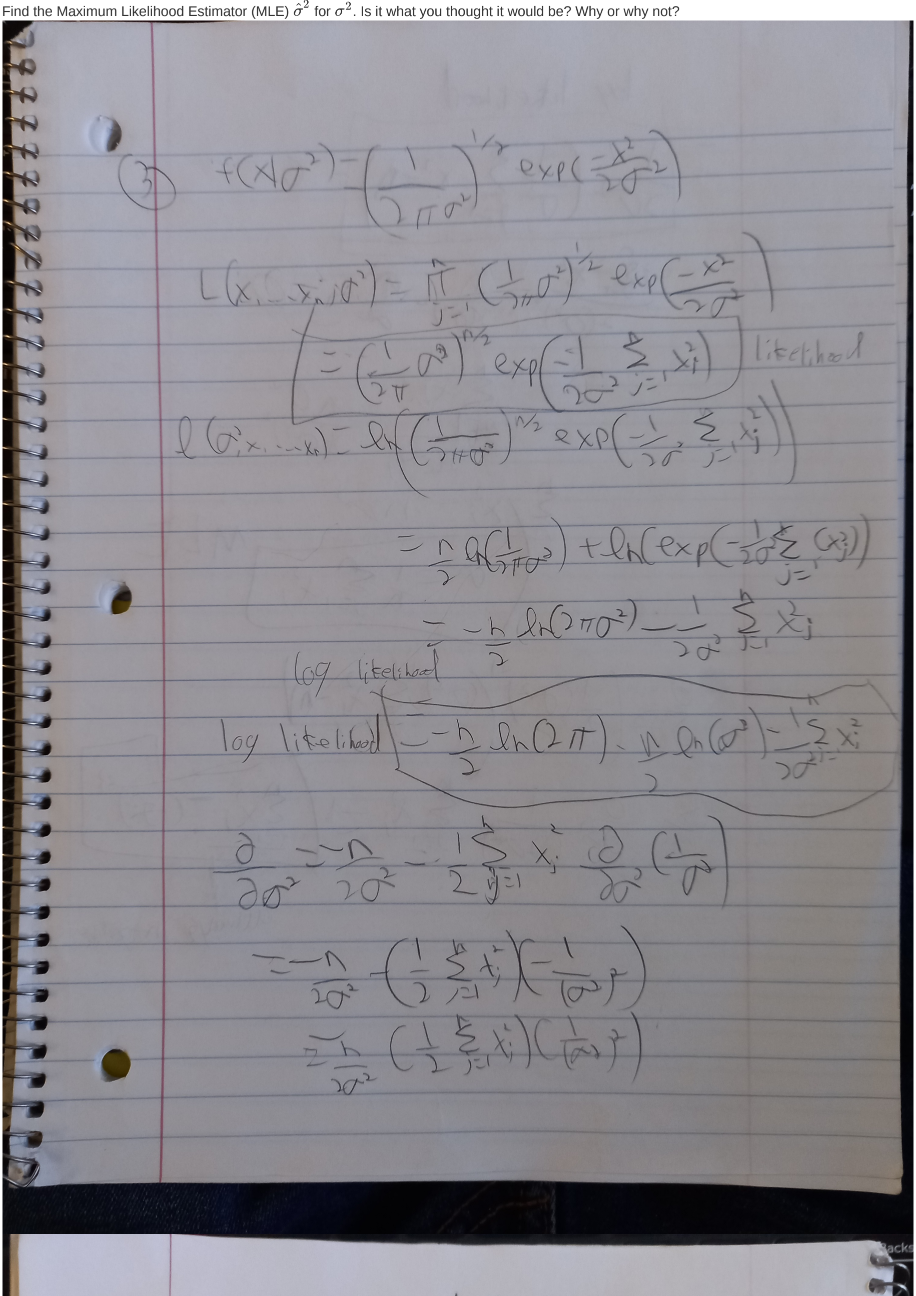


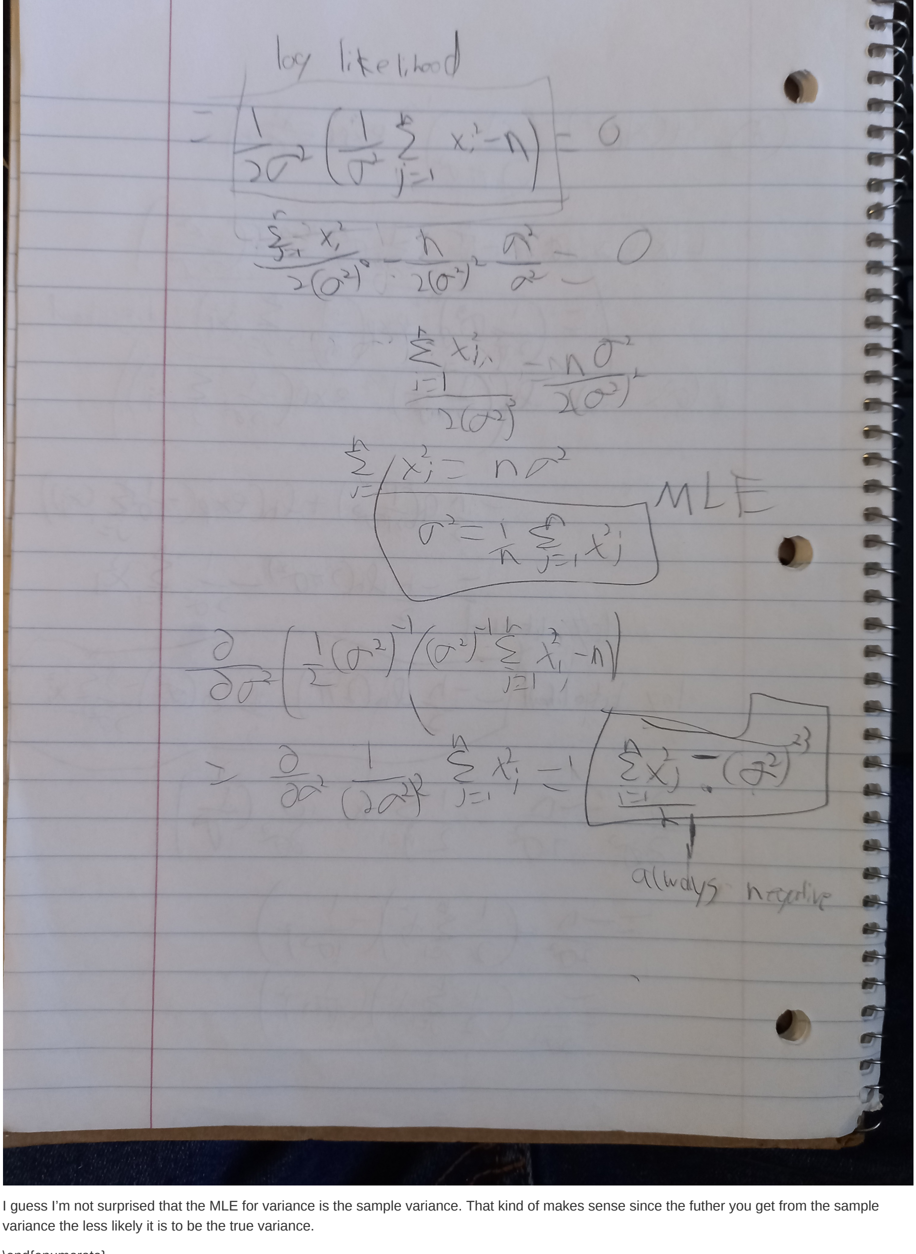
\end{enumerate}

(20 points) Suppose $\mathbf{x} = (x_1, \dots, x_N)^T$ are iid random variables from a $Normal(0, \sigma^2)$ distribution. The pdf is given by

$$f(x|\sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{1/2} e^{-\frac{x^2}{2\sigma^2}},\ -\infty < x < \infty,\ \sigma^2 > 0.$$

Find the Maximum Likelihood Estimator (MLE) $\hat\sigma^2$ for σ^2. Is it what you thought it would be? Why or why not?





I guess I’m not surprised that the MLE for variance is the sample variance. That kind of makes sense since the futher you get from the sample variance the less likely it is to be the true variance.

\end{enumerate}